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Combining Semi-Endogenous and Fully Endogenous Growth: a Generalization*

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Abstract

This paper shows that combining the semi-endogenous and the fully endogenous growth mechanisms with a general CES aggregator, either growth process can prevail in the balanced growth path depending on their degree of complementarity/substitutability. Policy-induced long-run economic switches to the fully endogenous steady state as the R&D employment ratio surpasses a positive threshold are possible if the two growth engines are gross substitutes.

JEL classification: O30, O40.

Keywords: Strong scale effect; Semi-endogenous growth; Fully endogenous growth.

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1 Introduction

In a recent paper, Cozzi (2017) combined the semi-endogenous growth model by Jones (1995), Kortum (1997), and Segerstrom (1998) with the fully endogenous growth models without scale effect by Smulders and Van de Klundert (1995), Dinopoulos and Thompson (1998), Peretto (1998), Young (1998), and Howitt (1999). Each of these schools of thought captures an important element of the growth process. The semi-endogenous approach, while inheriting from Romer (1990) the idea that the aggregate flow of innovations increases in the size of the R&D employment, stresses the increase in the difficulty of generating a constant TFP growth¹. The scale free fully endogenous school of thought stresses that the number of innovations per sector increases with the fraction of the labour force which each sector employs in R&D, without assuming an increasing difficulty of R&D². They have different long-term predictions, though similar transitional predictions.

If both growth mechanisms have some empirical confirmation, Cozzi (2017) claims that they may both be present in the growth process, which would result from a weighted average of the semi-endogenous and the fully endogenous growth engines. Unlike one could expect, though, regardless of the weight of each growth engine in the aggregate growth process, Cozzi (2017) proves that only one would dictate the balanced growth path prediction: with high enough population growth rates the semi-endogenous solution will prevail in the long run, while with slowly increasing, constant, or shrinking population the fully endogenous solution will eventually dominate in the long run.

These results are striking, but they may depend on the simple averaging adopted by Cozzi (2017), while most likely the overall growth process could combine the two growth mechanisms in much more general ways. A natural way to generalize the process is by assuming a general constant elasticity of substitution (CES) aggregator of the two growth mechanisms. This generalization is meaningful because, as I will show in this paper, depending on whether the two growth mechanisms are gross complements or gross substitutes in the overall growth process opposite conclusions are obtained: if they are gross complement the semi-endogenous steady state growth rate prevails at all values of the population growth rate; if they are gross substitutes, the long run implications of the fully endogenous growth mechanism will dominate the long run provided population growth does not exceed an endogenous threshold level.³

This paper also differs from Cozzi (2017) also in minor aspects: it allows for heterogeneous innovation technology parameters; and it sets the model in discrete time, potentially useful for embedding the model in a business cycle framework.⁴

The paper is organized as follows: Section 2 extends Cozzi's (2017) main result and characterize long-run growth in terms of growth engines complementarity and substitutability. The final section concludes.

¹See Bloom, Jones, Van Reenen, and Webb (2017) for recent empirical support based on sector and firm microdata.

²See Ha and Howitt (2007) and Madsen (2008) for empirical support of this approach.

³Hence, Cozzi's (2017) simple average result emerges as a special case - perfectly substitutable growth engines - of the more general class of substitutability.

⁴Comin and Gertler (2006), Anzoategui, Comin, Gertler, and Martinez (2015), and Varga, Roeger, and In't Veld (2008 and 2016).

2 Growth Mechanics

As in Cozzi (2017), let us assume the following aggregate production function:

$$Y(t) = A_t L_{Yt}, \quad (1)$$

where Y_t is output at time t , A_t is total factor productivity, and L_{Yt} is labor employed in manufacturing. In a balanced growth path L_{Yt} is a constant fraction $0 < s_Y < 1$ of the total labour force L_t , which in turn grows at the - possibly negative - constant net rate n .

In a balanced growth path, the remaining fraction $s_A = 1 - s_Y$ of the labour force, $L_{At} = s_A L_t$, works in the R&D sector, which allows manufacturing total factor productivity (TFP) to grow according to the following general CES equation:

$$\frac{A_t - A_{t-1}}{A_{t-1}} \equiv g_{AY, \max} = \gamma \left\{ \alpha_{\text{sem}} \left[(A_{t-1})^{\varphi-1} (s_A L_{t-1})^{\lambda_1^A} \right]^\theta + (1 - \alpha_{\text{sem}}) \left[s_A^{\lambda_2^A} \right]^\theta \right\}^{\frac{1}{\theta}}. \quad (2)$$

where $\theta \leq 1$, $\frac{1}{1-\theta}$ is the elasticity of substitution, $\alpha_{\text{sem}} \in]0, 1[$ is the weight of semi-endogenous growth in the total factor productivity growth process, and I have allowed for potentially different "stepping on toes" (Romer, 1990, and Jones and Williams, 1998) parameters.

When $\theta = 1$ the two sources of innovation are perfect substitutes as in Cozzi (2017).

For $\theta \leq 0$ the elasticity of substitution is no larger than 1, the two growth mechanisms in eq. (2) are gross complements, and each of them is essential. Therefore the steady state growth rate will be positive if and only if none of the two growth components is zero, which happens if and only if the first summand in the curly brackets is constant, which requires that $(A_{t-1})^{\varphi-1} s_A L_{t-1}^{\lambda_1^A}$ be constant. Therefore if population grows the steady state growth rate of A_t is

$$\bar{g}_{\text{sem}} \equiv \frac{\lambda_1^A n}{1 - \varphi}, \quad (3)$$

as in Jones (1995). If instead population is constant or shrinks the term $(A_{t-1})^{\varphi-1} s_A L_{t-1}^{\lambda_1^A}$, which will drag the whole growth rate expression (2) to zero in the long run. As a consequence the steady state TFP growth rate will be zero.

Therefore we have the following result:

Proposition 1. *If the two growth mechanisms are gross complements, the steady state TFP growth rate is zero if population is either constant or shrinking, while it is equal to \bar{g}_{sem} if population grows.*

If instead $\theta > 0$ the elasticity of substitution is larger than 1, and the two growth engines are gross substitutes. Then, similarly to Cozzi (2017), the steady state growth rate will be the fully endogenous growth expression

$$\bar{g}_{\text{endo}} \equiv \gamma (1 - \alpha_{\text{sem}})^{\frac{1}{\theta}} s_A^{\lambda_2^A}$$

if and only if

$$\frac{\lambda_1^A n}{1 - \varphi} < \gamma \left\{ 0 + (1 - \alpha_{\text{sem}}) \left(s_A^{\lambda_2^A} \right)^\theta \right\}^{\frac{1}{\theta}},$$

that is if and only if:

$$n < \frac{\gamma(1-\varphi)(1-\alpha_{\text{sem}})^{\frac{1}{\theta}}}{\lambda_1^A} s_A^{\lambda_2^A} \equiv \bar{n}_{\text{endo}}.$$

If instead $n \geq \bar{g}_{POP}$ the steady state growth rate will be equal to \bar{g}_{sem} .

Notice that the threshold population growth rate \bar{n}_{endo} is increasing in the R&D employment ratios s_A .

Summing up we have:

Proposition 2. *If the two growth mechanisms are gross substitutes, the steady state TFP growth rate is the fully endogenous value \bar{g}_{endo} if population growth rate is lower than \bar{n}_{endo} , while it is the semi-endogenous value \bar{g}_{sem} if population grows at a higher rate than \bar{n}_{endo} .*

If we fix the population growth rate, n , we can express Proposition 2 condition in terms of R&D employment ratio, and show that:

Corollary 2. *Under gross substitutability, the fully endogenous growth rate prevails in the balanced growth path if and only if:*

$$s_A > \left[\frac{\lambda_1^A n}{\gamma(1-\varphi)(1-\alpha_{\text{sem}})^{\frac{1}{\theta}}} \right]^{\frac{1}{\lambda_2^A}} \equiv s_{A\text{endo}}.$$

Notice that the higher the population growth rate the higher the R&D employment ratio required for the fully endogenous growth steady state growth rate to dominate. Theoretically, it is possible that for very high n , α_{sem} , λ_1^A , and φ , the implied $s_{A\text{endo}}$ be larger than 1, which is physically impossible for s_A to reach.

3 Conclusion

This paper has shown that if the aggregate TFP growth process results from a combination of the semi-endogenous and fully endogenous growth engines in a general CES function, only one growth paradigm will eventually dominate no matter the weight associated to each of them. More specifically, if the two growth mechanisms are gross complements in the aggregate TFP growth production function, the semi-endogenous growth engine prevails in the long run, while if they are gross substitutes the fully endogenous growth engine prevails for low enough population growth rates.

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