Positional Preferences: Efficiency and Distortions under Welfarist- and Paternalistic Governments

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Positional Preferences: Efficiency and Distortions under Welfarist- and Paternalistic Governments

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Abstract. This paper analyzes the distortionary effects of positional preferences when labor supply is exogenous under both a welfarist and a paternalistic government. Extending the prior literature, reference levels may be partially exogenous to the government (e.g., determined by consumption choices in a foreign country), and individuals may be positional with respect to wealth in addition to consumption. Neither consumption- nor consumption-cum-wealth positionality needs to cause inter-temporal distortions under either welfare criterion. We derive necessary and sufficient conditions for non-distortion of positional preferences. If those conditions are not satisfied, \textit{the same reference levels} of consumption and wealth can give rise to under-saving or to over-saving – depending on the extent to which the reference levels are exogenous to the government. Moreover, we provide conditions for which positional preferences for wealth and consumption imply over-consumption with respect to the welfarist criterion but, at the same time, over-saving with respect to the paternalistic criterion.

Keywords and Phrases: Status, keeping up with the Joneses, positional preferences, distortion, welfarist government, paternalistic government, \textit{Ak} model.

JEL Classification Numbers: D91, O40

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1 Introduction

This paper analyzes the distortionary effects of positional preferences with respect to both consumption and wealth.¹ By positional preferences we mean a situation in which households not only derive utility from own consumption and wealth, but also from own consumption and wealth relative to some consumption- and wealth reference levels. These reference levels are exogenous from the viewpoint of a household, and partially endogenous from the view of the government.² In particular, we have the following case in mind. For a given economy, a part of the reference level can be explained by choices within that economy. For example, the average consumption level in an economy might represent an important determinant for one’s consumption reference level.³ While an economy’s average consumption level is exogenous to any individual, it is fully endogenous to a social planner, that is, a government that designs an optimal allocation (or an optimal policy) according to some welfare criterion. However, there might be further determinants for reference levels that are not explained by the model itself. Most notably, these can be foreign consumption- or wealth levels, as transmitted by social media and television on a daily basis. We consider these determinants to be exogenous to a social planner. As argued in this paper, the “endogenous-exogenous composition” of the reference level turns out to be critical for the nature of a possible distortion (over- or under-saving) caused by positional preferences.

We develop necessary and sufficient conditions for positional preferences to be non-distortionary. In our simple Ak framework labor is exogenous. That is, the nature of the distortions we analyze is inter-temporal (in contrast to intra-temporal distortions when labor supply is elastic). In evaluating distortionary effects, we

¹ Different authors employ various terms, with slightly varying meanings, to describe positional preferences. These terms include (negative) consumption externality, relative wealth or consumption, jealousy, envy, keeping or catching up with the Joneses, external habits, positional concerns, conspicuous wealth or consumption.

² The case in which the reference levels are fully endogenous to a social planner (e.g., the mean consumption level of an economy) is a special case of our more general framework.

³ In models with homogeneous households, virtually all of the literature on positional preferences assumes the mean consumption level to represent an individual’s consumption reference level.
consider a paternalistic government in addition to a welfarist government. This is warranted, as several authors, including Sen (1979) or Harsanyi (1982), argue that it is questionable to include anti-social preferences, such as envy, in a social welfare function. Private preferences are suitable for a government objective function only if they are laundered (Goodin, 1986). Their arguments call for a paternalistic welfare function that does not consider positional preferences. However, other authors are more positive to include positional preferences in a welfare function (Piketty and Saez, 2013, p.453). In this paper we neither adopt a welfarist nor a paternalistic view. However, we are interested in studying the distortions possibly caused by positional preferences, as viewed through both lenses – that of a welfarist government as well as that of a paternalistic government. In this paper, we argue that – together with partial exogeneity of the reference levels as discussed above – the same reference levels can imply over-saving according to one welfare criterion, and, at the same time, under-saving according to the other welfare criterion.

But are positional preferences significant at all? We argue that they are. Social distinction or status is an important motivation of human behavior. This was already shown by Darwin (1871), who emphasized sexual selection besides natural selection. “To spread across the population, genes of sexual species not only need to survive in their natural and social environment, but also need to be or appear a more attractive mating partner than their same sex competitors.” (Truyts 2010, p.137) Clearly, Darwin was not the first to think about positional preferences. Philosophers have started to comment on positional preferences more than 2400 years ago. In his The Republic (Book II), Plato argues: Since ... appearance tyrannizes over truth and is lord of happiness, to appearance I must devote myself. This passage astoundingly resembles Darwin’s argument on sexual selection. In more recent times, Easterlin (1995) demonstrated that while national incomes have increased over the decades, happiness levels have not grown. One explanation for this Easterlin Paradox is that people have positional preferences, as emphasized by

An excellent, brief discussion of this question is given by Aronsson and Johansson-Stenman (2017).

We present several contributions with respect to the prior literature. First, we identify necessary and sufficient conditions for positional preferences not to impose a distortion (according to either a welfarist or a paternalistic government). In a framework without wealth-dependent preferences, the prior literature argues that, positional preferences have no impact on the steady state equilibrium – therefore also not a distortionary impact – once labor supply is exogenous.\(^5\) We show that this claim holds true only under the condition of constancy of the degree of positionality (as discussed in the proceeding section below). Once this condition is violated, consumption positionality is distortionary in spite of exogenous labor supply.\(^6\) Moreover, in contrast to Nakamoto (2009), we show that consumption positionality does not imply a distortion once preferences are wealth-dependent. Specifically, once the consumption positionality matches the wealth positionality, positional preferences do not cause an inter-temporal distortion.\(^7\) Second, we consider a paternalistic welfare criterion in addition to a welfarist one. In the context of optimal taxation, Aronsson and Johansson-Stenman (2017), Kanbur and Tuomala (2013), and Micheletto (2011) investigate optimal non-linear redistributive taxation.


\(^6\)We also show that existence of a balanced growth path does not imply the constancy-of-marginal-degree-of-positionality condition.

\(^7\)Few other papers address the distortionary effects of positional preferences. Alonso-Carrera et al. (2006) consider an Ak model in which habit-forming households exhibit positional preferences for consumption. Though, Alonso-Carrera et al. (2006) focus on the interaction between relative consumption and habits, this paper works out conditions when households are concerned with both relative consumption and relative wealth. Arrow and Dasgupta (2009) work out the conditions for the case of endogenous labor supply and positional concerns for both consumption and leisure. This paper focuses on positional preferences with respect to consumption and wealth, with exogenous labor supply. Ghosh and Wendner (2017) consider a functionally specified framework with wealth dependent preferences. They do not, however consider a general framework. None of the aforementioned papers considers partial exogenous reference levels or a paternalistic government.
in the presence of positional preferences, discussing a paternalistic welfare criterion. However, all of these papers are very much in the spirit of the optimal non-linear income tax tradition with heterogeneous households. None of these papers, though, consider either a dynamic setting, or a preference for wealth or partially exogenous reference levels. Third, we address the impact of the “endogenous-exogenous compositions” of the reference levels on the nature of distortions implied by positional preferences. To this end, we demonstrate that positional preferences may give rise to one distortion (like over-saving) according to a welfarist government, while giving rise to the opposite distortion (under-saving) according to a paternalistic government. To the best of our knowledge, no other authors have addressed and systematically investigated this case before.

To summarize, this paper extends the prior literature with respect to three dimensions. First, households may be positional with respect to wealth in addition to consumption. Second, a household’s reference levels is only partially explained by endogenous consumption- and savings choices within the economy. A part of the reference level is exogenous, e.g., determined by choices in a foreign country. As such, the unexplained part of the reference level is also exogenous to the government (social utility function). Third, the distortionary effect of positional preferences is evaluated according to both welfarist- and paternalistic welfare criteria. Exploiting this extended framework, we show three main results. The presence of consumption- and wealth positionality does in no way necessarily imply a distortion. Even the presence of wealth in the utility function does not imply that a consumption externality is distortionary, once the degree of positionality with respect to wealth matches the one with respect to consumption. Next, for given consumption- and wealth reference levels, our model gives rise to both under-saving and to over-saving – depending on the extend to which the reference levels are exogenous. Additionally, we provide conditions for which positional preferences for wealth and consumption imply under-saving with respect to the welfarist criterion but, at the same time, over-saving with respect to the paternalistic criterion.
This paper is structured as follows. Section 2 presents the endogenous growth model (with inelastic labor supply) for both the market economy and the social optima. Section 3 derives conditions for non-distortionarity of positional preferences. Moreover, for the cases in which these conditions are not satisfied, the section investigates the nature of distortions under welfarist- and paternalistic welfare criteria. Section 4 concludes the paper.

2 The Model

We consider a dynamic general equilibrium model of a closed economy that allows for fully endogenous growth. Endogenous growth stems from constant returns to capital ($Ak$ model). Time is considered to be continuous. There is a large number of households and firms, the respective number of which we normalize to unity. Households are homogeneous and exhibit positional preferences. They derive utility not only from own consumption but also from own consumption relative to some consumption reference level, and from own wealth relative to some wealth reference level.

2.1 Preferences

The representative household has preferences for consumption $c$, relative consumption $\Delta_c \equiv c - \bar{c}$, wealth $k$, and relative wealth $\Delta_k \equiv k - \bar{k}$. Relative consumption is given by individual consumption relative to some consumption reference level $\bar{c}$, and relative wealth is given by individual wealth relative to some wealth reference level $\bar{k}$. Both reference levels $(\bar{c}, \bar{k})$ are exogenous from the point of view of an individual household.

The consumption- and wealth reference levels are determined by two factors. The first factor is mean consumption, $\bar{c}^h$, and mean wealth, $\bar{k}^h$, in the economy (where superscript $h$ suggests home economy). As households are homogeneous, mean consumption and mean wealth represent natural determinants for the reference levels. Importantly, these determinants are endogenous from the point of view
of the government (social planner). The second factor is not explained by our model itself – it is exogenous: \( c^f, \bar{k}^f \). These “foreign” reference levels are determined by interaction via social media or by television broadcasting, none of which is explained endogenously in our model. Importantly, these determinants are exogenous even to a (welfarist or paternalistic) government. In what follows, we specify relative consumption \( \Delta_c \) and relative wealth \( \Delta_k \) as

\[
\Delta_c \equiv c - \bar{c}, \quad \bar{c} = \alpha^h \bar{c}^h + \alpha^f \bar{c}^f, \quad 1 \geq \alpha^h \geq 0, \quad \alpha^f \geq 0, \quad (1)
\]

\[
\Delta_k \equiv k - \bar{k}, \quad \bar{k} = \beta^h \bar{k}^h + \beta^f \bar{k}^f, \quad 1 \geq \beta^h \geq 0, \quad \beta^f \geq 0. \quad (2)
\]

Parameters \( \alpha^i \) and \( \beta^i \), \( i \in \{h, f\} \) determine the explained (endogenous) versus not explained (exogenous) parts of the positional reference levels. The standard case of fully endogenous mean value comparisons is implied by \( \alpha^h = 1, \alpha^f = 0 \) and \( \beta^h = 1, \beta^f = 0 \).

In this paper, both relative consumption and relative wealth enter the utility function. The instantaneous utility function is given by:

\[
u(c, \Delta_c, k, \Delta_k). \quad (3)
\]

In the standard model, \( u_i(c, \Delta_c, k, \Delta_k) > 0 \), and \( u_i(c, \Delta_c, k, \Delta_k) = 0 \) for some \( i \in \{\Delta_c, k, \Delta_k\} \), where a subindex refers to the partial derivative: \( u_x(.) \equiv \partial u(.) / (\partial x) \). If \( u_{\Delta_c}(c, \Delta_c, k, \Delta_k) > 0 \), preferences exhibit positional concerns for consumption.

For a given other’s consumption level (reference level), a rise in own consumption raises utility via the increase in relative consumption. If \( u_k(c, \Delta_c, k, \Delta_k) > 0 \), households derive utility not only from consumption, but also from wealth. Finally, If \( u_{\Delta_k}(c, \Delta_c, k, \Delta_k) > 0 \), preferences exhibit positional concerns for wealth. For a given wealth reference level, a rise in own wealth raises relative wealth, thereby it raises own utility. The time index \( t \) is suppressed, unless necessary to avoid ambiguities.

Throughout, we assume that the utility function (3) is strictly quasiconcave, twice continuously differentiable, strictly increasing in \( c \) and weakly increasing in all other arguments.
The intertemporal utility function, $U$, as viewed from date $t = 0$, is given by:

$$U = \int_{t=0}^{\infty} u(c, \Delta c, k, \Delta k) e^{-\rho t} dt, \quad \rho > 0,$$

where $\rho$ is the household’s constant pure rate of time preference.

### 2.2 Technology

A homogeneous output, $y$, is produced by capital according to the linear technology (Rebelo 1991):

$$y = A k, \quad A > 0$$

where $y$ is gross production per capita, and $k$ is capital per capita. The depreciation rate of capital is $\delta \in [0, 1]$. We assume $(A - \delta) \geq \rho$ to ensure nonnegative endogenous growth. Moreover, there is no population growth.

### 2.3 Market equilibrium

Let the superscript $m$ indicate a market (decentralized) equilibrium. Households choose a consumption stream so as to maximize intertemporal utility (4) subject to:

$$\dot{k}^m = y^m - c^m - \delta k^m = (A - \delta)k^m - c^m, \quad (6)$$

$$k_0^m \text{ given}, \quad (7)$$

$$\bar{c}^m, \bar{k}^m \text{ exogenous}, \quad (8)$$

$$\lim_{t \to \infty} \mu_t^m k_t^m e^{-\rho t} = 0. \quad (9)$$

Differential equation (6) reflects the flow budget constraint of the representative household. Restriction (7) is obvious; every household is required to base her plans on the initial value of her wealth. Notice that (6) and (7) hold for both the market framework and a social optimum (as discussed below). Restriction (8) reflects the fact that individual households consider the positionality reference levels as exogenous. Finally, (9) is the transversality condition.
For the market economy, the current value Hamiltonian is given by:

\[ H^m(c^m, \Delta c^m, k^m, \Delta k^m, \mu^m) = u(c^m, \Delta c^m, k^m, \Delta k^m) + \mu^m [(A - \delta) k^m - c^m], \]  

(10)

where the costate variable \( \mu^m \) represents the shadow price of capital. An interior solution implies the following first-order conditions:

\[ \mu^m = u(c^m, \Delta c^m, k^m, \Delta k^m), \]  

(11)

\[ \frac{\dot{\mu}^m}{\mu^m} = -[(A - \delta) - \rho] - \frac{u_k(c^m, \Delta c^m, k^m, \Delta k^m) + u_{\Delta k}(c^m, \Delta c^m, k^m, \Delta k^m)}{u(c^m, \Delta c^m, k^m, \Delta k^m) + u_{\Delta c}(c^m, \Delta c^m, k^m, \Delta k^m) + u_{\Delta k}(c^m, \Delta c^m, k^m, \Delta k^m)}. \]  

(12)

where we made use of the fact that \( \partial \Delta c / \partial c = \partial (c - \bar{c}) / \partial c = 1 \) and \( \partial \Delta k / \partial k = 1 \) from the point of view of an individual household. For the decentralized economy, an equilibrium path is characterized by (6), (7), (9), (11), and (12).

Before discussing the welfarist- and paternalistic governments’ problems, it turns out to be most useful to introduce the concept of the marginal degree of positionality (Johansson-Stenman et al., 2002), as a measure of how status concerned or positional an individual is. Specifically, the marginal degree of positionality with respect to consumption is defined by

\[ DOP_c \equiv \frac{u_{\Delta c}(c^m, \Delta c^m, k^m, \Delta k^m)}{u(c^m, \Delta c^m, k^m, \Delta k^m) + u_{\Delta c}(c^m, \Delta c^m, k^m, \Delta k^m)}. \]  

(13)

The degree of positionality defines the fraction of utility gain from an additional unit of consumption stemming from a rise in relative consumption \( \Delta c \). A value of zero indicates no positionality at all, while a value of unity indicates that only relative (not absolute) consumption matters.

Likewise, we define the marginal degree of positionality with respect to wealth by

\[ DOP_k \equiv \frac{u_{\Delta k}(c^m, \Delta c^m, k^m, \Delta k^m)}{u_k(c^m, \Delta c^m, k^m, \Delta k^m) + u_{\Delta k}(c^m, \Delta c^m, k^m, \Delta k^m)}. \]  

(14)

2.4 Welfarist Government

Let the superscript \( w \) indicate variables associated with a welfarist government’s choice problem. A welfarist government respects individual preferences. Specifically, it respects consumption and wealth positionality of households. In contrast
to individual households, the government takes into account that in equilibrium 
\( \bar{c}_h = c \) in (1), and \( \bar{k}_h = k \) in (2). However, both \( \bar{c}_f \) and \( \bar{k}_f \) are considered exogenous. That is, from the point of view of the government, \( \Delta_c = c(1 - \alpha^h) - \alpha_f \bar{c}_f \), and \( \Delta_k = k(1 - \beta^h) - \beta_f \bar{k}_f \).

The welfarist government chooses a consumption stream so as to maximize intertemporal utility

\[
U = \int_{t=0}^{\infty} u(c^w, \Delta_c^w, k^w, \Delta_k^w) e^{-\rho t} dt ,
\]

subject to

\[
\dot{k}_w = (A - \delta)k_w - c_w , \quad (16)
\]

\[
k_w^0 \text{ given}, \quad (17)
\]

\[
\bar{c}_h = c^w , \quad \bar{k}_h = k^w , \quad (18)
\]

\[
\lim_{t \to \infty} \mu^w t^\mu_k^w e^{-\rho t} = 0 , \quad (19)
\]

Restrictions (16) – (19) have the same interpretations as those given for the market economy. The main difference with respect to the decentralized framework is the fact that the social planner takes the reference levels (18) into account.

For the welfarist government, the current value Hamiltonian is given by:

\[
H^w(c^w, \Delta_c^w, k^w, \Delta_k^w, \mu^w) = u(c^w, \Delta_c^w, k^w, \Delta_k^w) + \mu^w [(A - \delta)k_w - c^w] . \quad (20)
\]

An interior solution implies the following first-order conditions:

\[
\mu^w = u_c(c^w, \Delta_c^w, k^w, \Delta_k^w) + (1 - \alpha^h) u_{\Delta_c}(c^w, \Delta_c^w, k^w, \Delta_k^w) , \quad (21)
\]

\[
\frac{\dot{\mu}^w}{\mu^w} = -[(A - \delta) - \rho] - \frac{u_k(c^w, \Delta_c^w, k^w, \Delta_k^w) + (1 - \beta^h) u_{\Delta_k}(c^w, \Delta_c^w, k^w, \Delta_k^w)}{u_c(c^w, \Delta_c^w, k^w, \Delta_k^w) + (1 - \alpha^h) u_{\Delta_c}(c^w, \Delta_c^w, k^w, \Delta_k^w)} , \quad (22)
\]

and an equilibrium path is characterized by (16), (17), (19), (21), and (22).

## 2.5 Paternalistic Government

Should the government accept positional concerns in its welfare criterion? Concerns for status and relative position are a form of jealousy and envy and one can question that such behavior has to be respected by the policy maker. In this subsection,
we set up the paternalistic government’s problem. The government knows that households care about status but it does not include positional preferences in the social welfare criterion. That is, the government’s and households’ preferences differ (cf. Kanbur et al. (2006) for an excellent discussion, in a survey article on non-welfarist optimal taxation).

In our framework, a paternalistic government does not fully respect individual preferences. In particular, it neglects positional preferences, that is, it considers relative consumption and wealth as exogenous: \( \Delta_c = \Delta_c \) and \( \Delta_k = \Delta_k \), where \( (\Delta_c, \Delta_k) \) is exogenous. In other words, the paternalistic government evaluates an equilibrium allocation as if households had no positional preferences at all. In the following, variables related to the paternalistic government’s choice problem are indicated by the superscript \( p \).

The welfarist government chooses a consumption stream so as to maximize intertemporal utility

\[
U = \int_{t=0}^{\infty} u(c^p, \Delta_c^p, k^p, \Delta_k^p) e^{-\rho t} dt, \tag{23}
\]

subject to

\[
\dot{k}^p = (A - \delta)k^p - c^p, \tag{24}
\]

\[
k^p_0 \text{ given}, \tag{25}
\]

\[
\Delta_c^p, \Delta_k^p \text{ exogenous}, \tag{26}
\]

\[
\lim_{t \to \infty} \mu^p_t k^p_t e^{-\rho t} = 0. \tag{27}
\]

Based on the current value Hamiltonian

\[
H^p(c^p, k^p, \mu^p) = u(c^p, \Delta_c^p, k^p, \Delta_k^p) + \mu^p [(A - \delta)k^p - c^p], \tag{28}
\]

an interior solution implies the following first-order conditions:

\[
\mu^p = u_c(c^p, \Delta_c^p, k^p, \Delta_k^p), \tag{29}
\]

\[
\frac{\dot{\mu}^p}{\mu^p} = -[(A - \delta) - \rho] - \frac{u_k(c^p, \Delta_c^p, k^p, \Delta_k^p)}{u_c(c^p, \Delta_c^p, k^p, \Delta_k^p)}. \tag{30}
\]
3 Positional preferences: efficiency and distortions

In this section, we address two cases. The first case is the special case in which households have positional preferences only with respect to consumption. We develop a necessary and sufficient condition for positional preferences to be non-distortionary and consider the type of distortion occurring when this condition is not satisfied. As a side, we develop an existence condition for a balanced growth path and show that existence does not imply efficiency (even for our framework with exogenous labor supply). The results developed are essential for analyzing the general second case, in which households have positional preferences with respect to both consumption and wealth. In the latter framework, we show that both the type of government (welfarist versus paternalistic) and the exogenous-endogenous composition of the reference levels play decisive roles for whether or not positional preferences are distortionary and if so, whether the distortion causes under- or over-saving.

3.1 Positional concerns with respect to consumption

In this subsection, we focus on the case: \( u_{\Delta k}(.) = 0 \). This is the case in which households are not concerned about others’ wealth levels, i.e., households may be positional with respect to consumption but not with respect to wealth. However, households may be concerned about own absolute wealth, in which case \( u_k(.) > 0 \).

In order to sharpen our results, we distinguish \( u_{\Delta k}(.) = 0 \) from the case \( u_k(.) > 0 \) in the following.

3.1.1 No preference for wealth: \( u_k(.) = 0 \)

We compare the equilibrium path of a market economy with those of the welfarist- and paternalistic governments. From (12), (22) and (30), we see that

\[
\frac{\dot{\mu}^m}{\mu^m} = \frac{\dot{\mu}^w}{\mu^w} = \frac{\dot{\mu}^p}{\mu^p} = -[(A - \delta) - \rho],
\]  
(31)
that is, the growth rates of the shadow prices are identical, and constant, in all three frameworks (decentralized, welfarist-, and paternalistic government). Efficiency then implies that $\mu^m = \phi^w \mu^w = \phi^p \mu^p$, with both $\phi^w$ and $\phi^p$ being constants.

From (11), (21) and (29), we can identify

$$\phi^w = \frac{u_c(c, \Delta_c, \ldots) + u_{\Delta_c}(c, \Delta_c, \ldots)}{u_c(c, \Delta_c, \ldots)} + (1 - \alpha^h) \frac{u_{\Delta_c}(c, \Delta_c, \ldots)}{u_c(c, \Delta_c, \ldots)}, \quad \phi^p = \frac{u_c(c, \Delta_c, \ldots) + u_{\Delta_c}(c, \Delta_c, \ldots)}{u_c(c, \Delta_c, \ldots)},$$

who are constant if and only if the utility function is homogeneous of some degree $R$ in $(c, \Delta_c)$, and the term $\Delta_c/c$ is constant.

**Assumption (A1).** The positionality term $\Delta_c/c$ is constant.

Considering the definition of $\Delta_c$ in (1), Assumption (A1) is equivalent to requiring the reference level $\bar{c}^f$ to be proportional to $c$. In fact, we assume

$$\bar{c}^f = \lambda^f c, \quad 0 < \lambda^c < \frac{1 - \alpha^h}{\alpha^f},$$

where the parameter restriction ensures that *in equilibrium* ("ex post") relative consumption $\Delta_c$ in fact increases in $c$. We consider Assumption (A1) to be a weak assumption, as it merely refers to a co-movement between the home- (endogenous) and foreign (exogenous) components of the consumption reference level.

**Assumption (A2).** Utility function $u(c, \Delta_c, \ldots)$ is homogeneous of degree $R < 1$ in $(c, \Delta_c)$.

Assumptions (A1) and (A2) together have a direct interpretation in terms of the degree of positionality. They are satisfied if and only if the degree of positionality with respect to consumption, $DOP_c$, is constant. In fact, considering (13), the degree of positionality is constant if and only if $u_{\Delta_c} = \kappa u_c$ with $\kappa$ being constant, in which case $\phi^w = (1 + \kappa)/[1 + \kappa(1 - \alpha^h)]$, and $\phi^p = (1 + \kappa)$. In the following, we assume (A1) and (A2), that is,

$$u_{\Delta_c} = \kappa u_c, \quad \kappa = \frac{u_{\Delta_c}(1, 1 - \alpha^h - \lambda^c \alpha^f, \ldots)}{u_c(1, 1 - \alpha^h - \lambda^c \alpha^f, \ldots)} > 0,$$
where we make use of the fact that both $u_c$ and $u_\Delta c$ are homogeneous of degree $(R - 1)$ by Euler’s theorem. The parameter restriction $R < 1$ ensures positivity of endogenous growth, as shown below.

**Proposition 1 ($u_\Delta k(\cdot) = u_k(\cdot) = 0$).**

If and only if Assumptions (A1) and (A2) are satisfied, the balanced growth path of the decentralized economy is efficient, that is, it coincides with the one implied by either the welfarist- or the paternalistic government.

The endogenous growth rate of consumption and capital, $g$, is given by

$$g^m = g^w = g^p = \frac{(A - \delta) - \rho}{(1 - R)[1 + \kappa(1 - \alpha h - \lambda c f)]}.$$ 

**Proof.** See appendix.

Proposition 1 reveals several important results. Primarily, the presence of consumption positionality does not imply a distortion. Whenever (A1) and (A2) are satisfied, positional preferences with respect to consumption do not introduce any distortion, as in Liu and Turnovsky (2005). This result is robust with respect to both welfare criteria (paternalistic and welfarist) and with respect to the exogenous-endogenous composition of the reference level. The result is not, however, robust with respect to having absolute wealth in the utility function, in which case consumption positionality always introduces a distortion (see below). Though, the result is “semi-robust” with respect to having wealth positionality. In this latter case, wealth positionality may outweigh consumption positionality. That is, consumption positionality can be non-distortionary in spite of having wealth in the utility function, as discussed below.\(^8\)

The parameter restrictions, particularly $R < 1$, ensure positivity of the endogenous growth rate. As seen in Proposition 1, the growth rate is sensitive with respect

\(^8\)A homogeneity condition was first introduced by Alonso-Carrera et al. (2006). However, while their paper concentrates on habits, this paper focuses on positional preferences regarding (relative) wealth. Proposition 1, though, is essential for the discussion of preferences for wealth. Interestingly, Arrow and Dasgupta (2009) discuss an equivalent condition to that of Alonso-Carrera et al. (2006) – however without being aware of their paper.
to the positionality parameters. In particular, the growth rate decreases in $\kappa$, and it increases in $\alpha^h$ and $(\lambda^c\alpha^f)$. The degree of positionality is given by $\kappa/(1 + \kappa)$ and rising in $\kappa$. A higher $\kappa$ (a higher $DOP_c$) raises consumption relative to savings in order to display “status.” Consequently, the saving rate falls, and so does the endogenous growth rate (as savings represent the endogenous growth engine). Next, a higher $\alpha^h$ or $(\lambda^c\alpha^f)$ directly impacts on $\Delta_c$. In particular, for a given increase in consumption, $\Delta_c$ grows the less the higher are $\alpha^h$ or $(\lambda^c\alpha^f)$. This, in turn, lowers the rate of decline of the marginal utility of consumption (in equilibrium), as shown in the appendix (Proof of Proposition 1, Step 1). The lower elasticity of marginal utility (in absolute terms) raises the optimal growth rate, by standard arguments of growth theory.

While consumption positionality has an impact on the equilibrium $(c/k, g)$, the impact is not distortionary, according to Proposition 1. We conclude this subsection by noting that existence of a balanced growth path does not imply efficiency.

**Proposition 2.** *Existence of a balanced growth path does not imply efficiency.*

**Proof.** See appendix.

Proposition 2 reveals two important findings. First, there exist balanced growth paths for which (A1) and (A2) are not satisfied. In fact, as shown in the appendix, existence of a balanced growth path requires the utility function to be of the following form:

$$v(c, \Delta_c) = \Delta_c^{-\gamma} \left[ \frac{c^{1-\theta}K^1 + K^2}{1-\theta} \right] + \Psi(\Delta_c), \quad K^1, K^2 \text{ constants.} \quad (33)$$

If and only if utility is of form (33), there exists a balanced growth path. This growth path, however, is efficient only under restrictions. For example, if $K^2 \neq 0$, utility function (33) is not homogeneous – thereby the necessary and sufficient efficiency conditions (A1) and (A2) are violated. One example for which consumption positionality does not introduce a distortion is: $K^1 = 1$, $K^2 = 0$, and $\Psi(\Delta_c)$ is homogeneous of degree $(1 - \gamma - \theta)$. 

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Second, even if labor supply is exogenous, consumption positionality can introduce an inter-temporal distortion. This latter result is in stark contrast to the literature analyzing consumption positionality in a neoclassical framework. The prior literature has shown that, in a neoclassical model, a consumption externality does not introduce any distortion as long as labor supply is exogenous.\footnote{Brekke and Howarth (2002, p.142) argue that “we have established that augmenting a standard neoclassical growth model to incorporate a concern for relative consumption has no impacts on long-run economic behavior.” Fisher and Hof (2000, p.249) show that the result that “relative consumption does not affect the long-run steady state...is robust with respect to the specification of the instantaneous utility function.” Liu and Turnovsky (2005, p.1106) state that “[w]ith exogenous labor supply, consumption externalities, which impact through the labor-consumption tradeoff, have no channel to affect steady state output” in a framework with neoclassical production. Rauscher (1997, p.38) argues that “conspicuous consumption does not affect the long-run steady state.”}

In what follows, we show that the result of Proposition 1 is not robust with respect to preferences for \textit{absolute} wealth (when, in addition, households are not wealth-positional).

### 3.1.2 Preference for wealth: $u_k(.) > 0$, $u_{\Delta k}(.) = 0$

In contrast to the previous subsection, we allow households to have a preference for wealth. Here, households care about own wealth, $u_k(.) > 0$, but they have no positional preference for wealth, $u_{\Delta k}(.) = 0$. We relax this assumption in the subsequent subsection. To sharpen results, though, we distinguish preferences for absolute wealth from \textit{positional} preferences for wealth (when $u_{\Delta k}(.) > 0$).

The steps followed here and in the next subsection resemble those of the above discussion — with the right adjustments, though. There is no change with respect to the marginal utilities of consumption, as given by (11), (21) and (29). With $u_k(.) > 0$, (12), (22) and (30) become:

\[
\frac{\dot{\mu}_m}{\mu_m} = -[(A - \delta) - \rho] - \frac{u_k(c^m, \Delta_c^m, k^m, \cdot)}{u_c(c^m, \Delta_c^m, k^m, \cdot) + u_{\Delta_c}(c^m, \Delta_c^m, k^m, \cdot)} \tag{34}
\]
\[
\frac{\dot{\mu}_w}{\mu_w} = -[(A - \delta) - \rho] - \frac{u_k(c^w, \Delta_c^w, k^w, \cdot)}{u_c(c^w, \Delta_c^w, k^w, \cdot) + (1 - \alpha_h) u_{\Delta_c}(c^w, \Delta_c^w, k^w, \cdot)} \tag{35}
\]
\[
\frac{\dot{\mu}_p}{\mu_p} = -[(A - \delta) - \rho] - \frac{u_k(c^p, \Delta_c^p, k^p, \cdot)}{u_c(c^p, \Delta_c^p, k^p, \cdot)} \tag{36}
\]
Following the above arguments, for these Euler equations to be satisfied, it is necessary that the right hand sides coincide. Inspection of the right hand sides of (34), (35) and (36), however, immediately shows the following. If \( u_{\Delta c}(c, \Delta c, k, .) \neq 0 \), then, if \( c_t^i = c_t^m \) and \( k_t^i = k_t^m \), \( i \in \{w, p\} \) for some \( t \), the right hand sides of (34) to (36) cannot be equal. We therefore conclude:

**Proposition 3** \((u_k(.) > 0, u_{\Delta k}(.) = 0)\).

Suppose households have a preference for absolute wealth. Then, if households are not wealth positional, consumption positionality always introduces a distortion.

**Proof.** See the discussion above. The restriction to preferences without wealth positionality is analyzed and discussed in the proceeding section. ■

Proposition 3 shows that positional concerns with respect to consumption are always distortionary, once households have a preference for absolute but not for relative wealth. This is true, even when Assumptions (A1) and (A2) are satisfied. The intuition for this result stems from the fact that the marginal rate of substitution of wealth for consumption always differs between the market equilibrium and the welfarist and paternalistic government equilibria. This is so, because households and the governments have different views with respect to the consumption reference levels. Individual households take the consumption reference level as exogenous, that is, \( \partial \Delta c / \partial c = 1 \). The welfarist government considers the marginal disutility from the endogenous part of reference consumption, that is, \( \partial \Delta c / \partial c = (1 - \alpha^k) \). The paternalistic government disregards relative consumption altogether, that is, \( \partial \Delta c / \partial c = 0 \). For this reason, the marginal rate of substitution of capital for consumption is the smallest for the market framework and differs from those of the government-frameworks. As a consequence, the Keynes-Ramsey rules differ between the market equilibrium and the governments’ optima and so do the respective consumption-to-capital ratios as well as the endogenous growth rates.

Under the assumptions of Proposition 3, we finally address the question of whether positional preferences with respect to consumption give rise to over- or
over-saving. The most efficient way is to work out the endogenous growth rate for all three frameworks. Noting that \( c/k = (A - \delta) - g \), we argue that \( g^m < g^i \) implies over-consumption, and \( g^m > g^i, i \in \{w, p\} \) implies over-saving.

Without loss of generality, we assume that \( u(c, \Delta_c, k, .) \) is homogeneous of degree \( \hat{R} \) in \( k \).10 Moreover, we employ the following Lemma, which simplifies the the proceeding discussion enormously.

**Lemma 1.** Suppose, the endogenous growth rate is of the form

\[
g = \Omega^{-1} \{[(A - \delta) - \rho] + \Gamma c/k\}, \quad \Omega > 0, \quad \Gamma > 0.
\]

Then, \( \partial g/\partial \Gamma > 0 \).

**Proof.** In equilibrium, \( c/k = (A - \delta) - g \). Thus,

\[
\frac{dg}{d\Gamma} = \Omega^{-1} [(A - \delta) - g] \left( \frac{1}{1 + \Omega^{-1} \Gamma} \right) > 0,
\]

where the sign follows from positivity of \( c/k \). ■

The value of Lemma 1 is given by the fact that differences in \( \Gamma \) directly reveal differences in the endogenous growth rate \( g \), ceteris paribus. A higher \( \Gamma \) implies a higher growth rate. In light of the above discussion we conclude: \( \Gamma^m < \Gamma^i \) implies over-consumption, and \( \Gamma^m > \Gamma^i, i \in \{w, p\} \) implies over-saving. We are now ready to state:

**Proposition 4.** Suppose households have a preference for absolute wealth, but they are not wealth positional. Furthermore, assume (A1), (A2) and \( u(c, \Delta_c, k, .) \) is homogeneous of degree \( \hat{R} \) in \( k \). Then, consumption positionality introduces over-consumption. The distortion is stronger according to a paternalistic welfare criterion than for a welfarist welfare criterion. That is,

\[
\left( \frac{c_k}{k} \right)^m > \left( \frac{c_k}{k} \right)^w > \left( \frac{c_k}{k} \right)^p \Leftrightarrow g^m < g^w < g^p.
\]

**Proof.** See the appendix.

\[10\]Similarly to the proof of Proposition 2, existence requires constancy of \( u_{ckk}/u_{c} \). Given assumption (A2), the homogeneity requirement with respect to \( k \) is required for the existence of a balanced growth path.
Proposition 4 shows an intuitive result. Consumption positionality leads to over-consumption. However, this intuition is misleading, as the over-consumption result neither necessarily holds in a framework without a preference for absolute wealth, nor it necessarily holds in a framework in which individuals also have a positional preference for wealth.

With a preference for absolute wealth and no positional preference for wealth, though, the market equilibrium implies over-consumption for both welfare criteria. By not taking the externality into account, households overestimate the marginal utility of consumption. That is, they have a lower marginal rate of substitution of wealth for consumption than either the welfarist- or the paternalistic government. Consequently, in light of positional preferences with respect to consumption, households over-consume. In other words, households under-save, and the endogenous growth rate in the market economy is smaller than that for a welfarist- or paternalistic framework.

The difference between a welfarist- and a paternalistic government is given by the fact that the welfarist one considers the marginal disutility from the endogenous part of reference consumption, while the paternalistic one fully disregards relative consumption. So, ceteris paribus, marginal utility of consumption is higher for the welfarist government than for the paternalistic one. Consequently, the welfarist marginal rate of substitution of wealth for consumption is higher than the paternalistic one. This implies “over-consumption” of the welfarist government relative to the paternalistic one: \( (c/k)^w > (c/k)^p \). Equivalently, the saving rate is higher for the paternalistic equilibrium than for the welfarist one, implying \( g^p > g^w \).

Proposition 4 emphasizes two aspects of consumption positionality in the presence of preferences for absolute wealth. First, consumption positionality implies over-consumption, regardless of the respective welfare criterion. Second, consumption positionality implies a stronger distortion with respect to the paternalistic welfare criterion than with respect to the welfarist one. In what follows, we show that positional concerns with respect to wealth introduce a further distortion that
is capable of offsetting the distortionary effect of relative consumption under wealth
dependent preferences. Moreover, we show that according to a welfarist government
positional preferences may lead to one distortion (say over-consumption) while, at
the same time, they lead to the opposite distortion (over-saving) according to a
paternalistic welfare criterion.

3.2 Positional concerns with respect to consumption and
wealth

In contrast to the subsection above, here we consider the general case in which
households have positional preferences not only with respect to consumption but
also with respect to wealth. That is we consider \( u_{\Delta_c} > 0 \) and \( u_{\Delta_k} > 0 \). We follow
the methodology developed so far closely and introduce two more assumptions.

**Assumption (A3).** The positionality term \( \Delta_k/k \) is constant.

Considering the definition of \( \Delta_k \) in (2), Assumption (A3) is equivalent to requir-
ing the reference level \( \bar{k}^f \) to be proportional to \( k \). In fact, we assume

\[
\bar{k}^f = \lambda^k k, \quad 0 < \lambda^k < \frac{1 - \beta^h}{\beta^f},
\]

where the parameter restriction ensures that in equilibrium relative wealth \( \Delta_k \) in
fact increases in \( k \). Again, we consider Assumption (A3) to be a weak assump-
tion on the co-movement between the home- (endogenous) and foreign (exogenous)
components of the wealth reference level.

**Assumption (A4).** Utility function \( u(c, \Delta_c, k, \Delta_k) \) is homogeneous of degree \( \hat{R} \) in
\((k, \Delta_k)\).

Assumptions (A3) and (A4) together have a direct interpretation in terms of the
degree of positionality. More specifically, assume (A1) to (A4). Then, \( u_{\Delta_k} = \eta u_k \)
with \( \eta \) being a constant. Thus, the degree of positionality with respect to wealth,
as given in (14), is constant and given by \( DOP_k = \eta/(1 + \eta) \). Analytically,

\[
u_{\Delta_k} = \eta u_k, \quad \eta = \frac{u_{\Delta_k}(1, 1 - \alpha^h - \lambda^c \alpha^f, 1, 1 - \beta^h - \lambda^k \beta^f)}{u_c(1, 1 - \alpha^h - \lambda^c \alpha^f, 1, 1 - \beta^h - \lambda^k \beta^f)} > 0,\]

where we use (A1) – (A4) together with Euler’s theorem.

In the following, we adopt Assumptions (A1) to (A4). Based on the first-order conditions (11), (21), (29) as well as on the Euler equations (12), (22), (30), we state:

**Proposition 5** (Non-distortion of positional preferences when $u_\Delta(,) > 0$). Assume (A1) to (A4).

Paternalistic government: If and only if $\eta = \kappa$, the market equilibrium path is efficient (according to the paternalistic welfare criterium) and positional preferences do not introduce a distortion.

Welfarist government: If and only if

$$\frac{1 + \eta}{1 + \eta(1 - \beta^h)} = \frac{1 + \kappa}{1 + \kappa(1 - \alpha^h)}$$

the market equilibrium path is efficient (according to the welfarist welfare criterium) and positional preferences do not introduce a distortion.

**Proof.** See the appendix.

Proposition 5 provides necessary and sufficient conditions for positional preferences not to introduce a distortion. This result is in sharp contrast to the prior literature (Nakamoto 2009), which argues that positional preferences for consumption always cause a distortion when households have a preference for wealth. The proposition offers conditions under which positional preferences do not cause a distortion in spite of the fact that households have a preference for wealth.

The conditions in Proposition 5 ensure that the costate variables in the market framework and the governments’ frameworks grow at the same rates. This is the case when the distortion introduced by the consumption externality (positionality) is exactly counterbalanced by the distortion introduced by the wealth externality (positionality). In terms of the degrees of positionality, the proposition requires the marginal rate of substitution of wealth for consumption to be equal across the three frameworks (market, paternalistic, welfarist).
The reason for the conditions for the paternalistic government to differ from those for the welfarist government is that the latter does consider the endogenous parts of the reference levels in the marginal rate of substitution, while the former does not (we come back to this point below).

**Corollary 1** \((u_{\Delta c}(.)) = 0, u_{\Delta k}(.) > 0\).

If preferences are positional with respect to wealth but not with respect to consumption, the market equilibrium path is always inefficient, that is, it never coincides with the socially optimal one.

The corollary follows directly from the fact that \(\eta \neq \kappa = 0\). As preferences depend on relative wealth but not on relative consumption – as there is no countering positionality with respect to consumption – the positional preferences are always distortionary. Corollary 1 provides an interesting insight. While positional preferences for consumption alone need not be distortionary (cf. Proposition 1), positional preferences for wealth alone are always distortionary.

We conclude this section by analyzing the type of distortion (over-consumption or over-saving) caused by positional preferences. Interestingly, we will see that positional preferences may cause over-consumption according to one welfare criterion and, at the same time, over-saving according to the other welfare criterion.

As shown in the appendix, the endogenous growth rates are, as above, given by

\[
g^i = \Omega^{-1}\left\{[(A - \delta) - \rho] + \Gamma^i(c/k)^i\right\}, \quad i \in \{m, w, p\},
\]

\[
\Omega \equiv \left\{[(1 - R)(1 + \kappa (1 - \alpha^h - \lambda^c \alpha^f)] - \hat{R}[1 + \eta (1 - \beta^h - \lambda^k \beta^f)]\right\},
\]

\[
\Gamma^m = \frac{\hat{R}}{R} \frac{1 + \eta}{1 + \kappa}, \quad \Gamma^w = \frac{\hat{R}}{R} \frac{1 + \eta (1 - \beta^h)}{1 + \kappa (1 - \alpha^h)}, \quad \Gamma^m = \frac{\hat{R}}{R},
\]

where \(\Omega^{-1}\) represents the intertemporal elasticity of substitution (which is the same for the market-, welfarist- and paternalistic governments). The specific terms \(\Gamma^i(c/k)^i\) represent the respective marginal rates of substitution of wealth for consumption. As in the previous section, we can apply Lemma 1, that is \(\partial g^i/\partial \Gamma^i > 0\), in spite of endogeneity of the consumption-to-capital ratio \((c/k)^i\). In contrast to the
previous section, application of Lemma 1 does *not* yield a unique ranking among the $\Gamma^i$, thereby among $g^i$ and $(c/k)^i$, $i \in \{m, w, p\}$. The following proposition identifies all possible rankings or distortions.

**Proposition 6** (Welfare criteria, and the endogenous parts of $\bar{c}$ and $\bar{k}$). Assume (A1) to (A4). The positionality parameters give rise to four cases.

I. $\eta = \kappa$ and $\alpha^h = \beta^h$. There is no distortion according to either welfare criterion.

II. $\eta = \kappa$. There is no distortion according to the paternalistic welfare criterion. There is a distortion according to the welfarist welfare criterion when $\alpha^h \neq \beta^h$. Specifically, $\alpha^h > \beta^h$ implies over-consumption; $\alpha^h < \beta^h$ implies over-saving.

III. $\eta \neq \kappa$ and $\alpha^h = \beta^h$. There is a distortion according to both welfare criteria. Specifically, $\kappa > \eta$ implies over-consumption; $\kappa < \eta$ implies over-saving.

IV. $\eta \neq \kappa$ and $\alpha^h \neq \beta^h$. Specifically, let $\kappa = n\eta$ and $\beta^h = a\alpha^h$, with $n, a > 0$.

IV.1 Let $n, a > 1$ and $\eta > (n - a)/[n(a - 1)]$. Then positional preferences imply over-consumption according to the paternalistic welfare criterion and over-saving according to the welfarist welfare criterion.

IV.2 Let $n, a < 1$ and $\eta > (n - a)/[n(a - 1)]$. Then positional preferences imply over-saving according to the paternalistic welfare criterion and over-consumption according to the welfarist welfare criterion.

**Proof.** Case I follows directly from Proposition 5. Cases II to IV are based on the values of $\Gamma^i$ as shown above (and derived in the appendix): $\Gamma^m = \frac{R}{\kappa} \frac{1+\eta(1-\beta^h)}{1+\kappa(1-\alpha^h)}$, $\Gamma^w = \frac{R}{\kappa} \frac{1+\eta(1-\beta^h)}{1+\kappa(1-\alpha^h)}$, $\Gamma^p = \frac{R}{\kappa} \frac{1+\eta(1-\beta^h)}{1+\kappa(1-\alpha^h)}$. Application of Lemma 1 yields: $\Gamma^i \succeq \Gamma^j \iff g^i \succeq g^j \iff (c/k)^i \leq (c/k)^j$. We associate $(c/k)^m > (c/k)^i$, $i \in \{w, p\}$ with over-consumption and $(c/k)^m < (c/k)^i$, $i \in \{w, p\}$ with over-saving. 

If and only if the conditions given in Proposition 5 are not satisfied (Cases II to IV), positional preferences cause distortions. As long as $\eta = \kappa$ (Case II), there is
no distortion according to the paternalistic government. The reason is that individual households and the paternalistic government have the same marginal rate of substitution of wealth for consumption, as wealth positionality exactly offsets (equals) consumption positionality. The same does not hold for the welfarist government. By internalizing different amounts of the wealth- and consumption externalities \((\alpha^h \neq \beta^h)\), the welfarist government’s marginal rate of substitution of wealth for consumption differs from that of individual households. If, for example, \(\alpha^h > \beta^h\), the welfarist government has a higher marginal rate of substitution of wealth for consumption (by internalizing relative consumption by more than relative wealth) than individual households. Therefore, individual households choose a higher consumption-to-capital ratio than a welfarist government. In other words, households over-consume relative to the welfarist government. Over-consumption then implies a lower than optimal endogenous growth rate.

Case II gives rise to a most interesting observation. Suppose \(d\alpha^h = -d(\lambda^c\alpha^f)\) and \(d\beta^h = -d(\lambda^k\beta^f)\). Then changes in \((\alpha^h, \beta^h)\) do not change the reference levels \((\bar{c}, \bar{k})\). Starting from \(\alpha^h = \beta^h\), and perturbing either of these parameters yields either over-consumption (if \(\alpha^h\) is increased) or over-saving (if \(\beta^h\) is increased), for the same reference levels \((\bar{c}, \bar{k})\).

According to Case III, \(\eta \neq \kappa\) always introduces a distortion, though the distortion is stronger according to the paternalistic criterion as compared to the welfarist one (as the paternalistic government disregards positional preferences at all, while the welfarist government respects individual households’ preferences). Clearly, \(\kappa > \eta\) implies over-consumption, as individual households are more positional with respect to consumption than with respect to wealth compared with both governments. A parallel argument applies to the case in which \(\kappa < \eta\).

Case III implies a ranking according to which either \((c/k)^m < (c/k)^w < (c/k)^p\) or \((c/k)^m > (c/k)^w > (c/k)^p\). That is, positional preferences either imply over-consumption, or they imply over-saving.

Case IV shows that such rankings need not hold if both \(\kappa \neq \eta\) and \(\alpha^h \neq \beta^h\).
Specifically, positional preferences may imply over-consumption according to one welfare criterion and over-saving according to the other welfare criterion. Specifically, suppose $\kappa > \eta$ and $\beta^h > \alpha^h$ - with the latter inequality being proportionally larger. Then, following the above arguments, $\beta^h > \alpha^h$ implies over-saving according to the welfarist criterion, and $\kappa > \eta$ implies over-consumption according to both criteria. The condition given in the proposition ensures that the former effect dominates the latter effect, that is, there is over-consumption according to the paternalistic government (which does not care about $\alpha^h$ and $\beta^h$), and there is over-saving according to the welfarist government. A parallel argument holds for Case IV.2.

Cases IV.1 and IV.2 raise serious questions regarding the (optimal) policy responses to positional preferences. Should a government follow a welfarist- or a paternalistic welfare criterion upon which to base its policy analysis? There is no easy answer to this question, as discussed in the introduction. In fact, the question is a philosophical one, it is not a purely economic one. The difficulty comes with the fact that, depending on the answer to this question, a government should apply one set of optimal policies rather than another one. In particular, in the presence of positional preferences, under the conditions of Case IV.1, a paternalistic government should apply a consumption tax while a welfarist government should apply a tax on capital income in order to correct for the externalities. The reverse holds for Case IV.2.

4 Conclusions

In an endogenous growth context with exogenous labor supply, this paper addresses the research question of whether or not positional preferences are distortionary, and if so, whether they cause over-consumption or over-saving. The paper shows that the answer depends on three main factors: the characteristics of the utility function (homogeneity characteristics); the type of the welfare criterion (welfarist or paternalistic); the endogenous-exogenous composition of the reference levels for
consumption and wealth.

In our framework, labor supply is exogenous. In contrast to a neoclassical growth framework, as analyzed by the prior literature, we show that in an endogenous growth framework positional preferences may introduce inter-temporal distortions in spite of exogenous labor supply. Moreover, we prove that existence of a balanced growth path does not imply efficiency. Efficiency, however, is ensured by a homogeneity restriction.

When households exhibit a preference for absolute wealth (and not a positional preference for wealth), then consumption positionality always introduces over-consumption (under both a welfarist- and a paternalistic government). This result is not robust, though, with respect to a framework in which households also have preferences for relative wealth, i.e., they are wealth positional. We provide necessary and sufficient conditions for non-distortion of positional preferences in such a framework.

When households are positional, they consider reference points. These reference points are in part determined by the equilibrium in the home economy (e.g., the mean consumption level of the economy). However, they are also in part exogenous (e.g., mean consumption in a foreign country). It turns out that this “endogenous-exogenous composition” of reference levels plays a key role for whether positional preferences imply over-consumption or over-saving according to a welfarist government. The reasoning is as follows. Individual households consider reference levels as exogenous, while the welfarist government internalizes the endogenous parts of the reference levels. Depending on their endogenous-exogenous composition, the welfarist government can have a higher- or a lower marginal rate of substitution of wealth for consumption, than an individual household.

In addition to a welfarist government, we also consider a paternalistic one that does not include “anti-social preferences”, such as consumption- or wealth positionality in its social welfare function. We discuss two cases in particular. First, when the endogenous-exogenous composition is the same for both the consumption- and
the wealth reference level, the distortion of positional preferences, if any, is stronger under a paternalistic government than under a welfarist one. This is because the former completely disregards any positional preferences, while the latter considers (the endogenous) part of the reference levels. Second, we identify the cases for which distortions raised by positional preferences imply over-consumption according to the welfarist criterion, while implying over-saving according to the paternalistic criterion. These cases involve restrictions on both, the degrees of positionality with respect to consumption or wealth as well as the endogenous-exogenous compositions of the reference levels.

A number of further research questions suggest themselves. First of all, which optimal policy should be chosen in those cases for which distortions raised by positional preferences imply over-consumption according to one welfare criterion, while implying over-saving according to the other criterion. This question clearly is of interest in a much broader context than the one discussed in this paper. Second, if households are heterogeneous in terms of wealth, skills or preferences, what can systematically be said about distortionary effects of positional preferences and about optimal redistributive taxation? Third, how do the results presented in this study change in a model with an endogenous (e.g., wealth-driven) time preference?

Notwithstanding these limitations, we hope this study clarifies important aspects of distortionary effects of positional preferences, and can contribute to future discussions about the effects of positional preferences in economics.

Appendix

Proof of Proposition 1

We consider $k_0^m = k_0^w = k_0^p$. It is easy to show that – as in the standard $Ak$ framework – the dynamic system is one-dimensional, and the steady state is unstable. That is, there is no transitional dynamics, and consumption and capital grow at their balanced growth rates “from the beginning.” This argument follows standard textbook reasoning.
Step 1. Consumption and capital grow at the same rate. The endogenous growth rate of the market economy equals that of the (welfarist- and paternalistic) government: \( g^m = g^w = g^p \).

Given that consumption and capital grow at their balanced growth rates “from the beginning,” \( \dot{k}/k \) is constant, and (6) requires \( c \) to grow at the same rate as \( k \). Let \( g \) denote this growth rate. In the following we show that \( g = \dot{c}/c \) is the same for the decentralized economy as for the welfarist-/paternalistic government. As a matter of fact, although \( \mu^m \neq \mu^w \neq \mu^p \), from (11), (21), (29) it follows that the growth rates of the costate variables are the same: \( \dot{\mu}^m/\mu^m = \dot{\mu}^w/\mu^w = \dot{\mu}^p/\mu^p = \dot{u}_c/u_c \).12

That is,

\[ \dot{\mu}^i/\mu^i = \frac{u_{\Delta c}^i c}{u_c} = \frac{\Delta_c}{c}, \quad i \in \{m, w, p\}. \]

Next, we observe that (i) \( \Delta_c/c = (1 - \alpha^h - \lambda^c \alpha^f)\dot{c}/c \); (ii) \( u_{\Delta c} = u_{\Delta e} = \partial u_{\Delta e}/\partial c = \partial \kappa u_c/\partial c = \kappa u_{cc} \); (iii) by homogeneity of degree \( R \), \( u_{cc}/u_c = -(1 - R) \).

Considering (i) to (iii) together with (31) yields

\[ g_c = \left( \frac{\dot{c}}{c} \right)^i = \frac{(A - \delta) - \rho}{(1 - R)(1 + \kappa(1 - \alpha^h - \lambda^c \alpha^f))}, \quad i \in \{m, w, p\}. \]

Step 2. As \( g^m = g^w = g^p \) we have \( c_t^m = c_t^w = c_t^p \) for all \( t \geq 0 \). For \( t = 0 \), observe that \( k_0^m = k_0^w = k_0^p \). From (6), (16) and (24), \( c_0 = [(A - \delta) - g]k_0 \). As \( k_0^m = k_0^w = k_0^p \), it follows that \( c_0^m = c_0^w = c_0^p \). Finally, as the growth rates are identical, we also have \( c_t^m = c_t^w = c_t^p \) for all \( t > 0 \).

Step 3. The transversality conditions (TVC) are satisfied. Let \( \hat{u} \equiv u(1, 1 - \alpha^h - \lambda^c \alpha^f, \ldots) \). We have \( \mu^m = R(1 + \kappa)\hat{u}c^{R-1} \neq \mu^w = R(1 + \kappa(1 - \alpha^h))\hat{u}c^{R-1} \neq \mu^p = R\hat{u}c^{R-1} \). Next we consider \( \mu^i = \mu_0^i e^{(A-\delta)-\rho}t, \quad i \in \{m, w, p\}, c_t = c_0 e^{\rho t}, \) and \( k_t = k_0 e^{\rho t} \). Plugging these expressions into the respective TVC yields the following necessary and sufficient condition for the TVC (in all three frameworks) to be satisfied: \( (A - \delta) > g \). This condition, however, is satisfied in all three frameworks (market, welfarist, paternalistic), as \( c/k = (A - \delta) - g > 0 \).

12Observe that the growth rate \( \dot{u}_c/u_c \) is independent of whether or not \( \bar{c} \) or \( \bar{\Delta}_c \) is exogenous to the decision maker.
From steps 1 to 3 we conclude that all equilibrium paths are identical, therefore the decentralized equilibrium path is efficient according to either the welfarist or the paternalistic welfare criterion.

**Proof of Proposition 2**

Let $v(c, \Delta_c) \equiv u(c, \Delta_c, \ldots)$. Throughout we assume that (A1) is satisfied. Efficiency requires

$$\frac{v_{\Delta c}(c, \Delta_c)}{v_c(c, \Delta_c)} = \text{constant}, \quad (\text{EF})$$

and existence of a balanced growth path requires a constant growth rate of the costate variable:

$$\frac{\dot{\mu}}{\mu} = \frac{v_{cc}(c, \Delta_c)c}{v_c(c, \Delta_c)} \frac{\dot{c}}{c} + \frac{v_{\Delta c}(c, \Delta_c) \Delta_c}{v_c(c, \Delta_c) \Delta_c} \frac{\dot{\Delta}_c}{\Delta_c} = \text{constant}. \quad (\text{EX})$$

Both $\dot{c}/c$ and $\dot{\Delta}_c/\Delta_c$ are constant on a balanced growth path. The existence condition is satisfied if both elasticities of marginal utility are constant as well.

As demonstrated in the proof of Proposition 1, efficiency conditions (A1) and (A2) imply constancy of both elasticities of marginal utility. That is, $(\text{EF}) \Rightarrow (\text{EX})$. The reverse does not hold, though.

Assume (EX), and let $\theta$ and $\gamma$ denote the elasticities:

$$\theta \equiv -\frac{v_{cc}(c, \Delta_c)c}{v_c(c, \Delta_c)}, \quad \gamma \equiv -\frac{v_{\Delta c}(c, \Delta_c) \Delta_c}{v_c(c, \Delta_c)}. \quad (40)$$

From this information, we first derive the marginal utility of consumption. In the proceeding step, we use the result to infer the utility function.
Step 1. Marginal utility

\[
\frac{v_c(\Delta_c, \Delta_c)}{v_c(c, \Delta_c)} = \frac{d}{d \Delta_c} \ln v_c(c, \Delta_c) = -\frac{\gamma}{\Delta_c}
\]

\[
\Rightarrow \int \frac{d}{d \Delta_c} \ln v_c(c, \Delta_c) d\Delta_c = -\int \frac{\gamma}{\Delta_c} d\Delta_c = -\gamma \ln \Delta_c + \xi
\]

\[
\Rightarrow \ln v_c(c, \Delta_c) + \hat{f}(c) = -\gamma \ln \Delta_c + \xi
\]

\[
\Rightarrow \ln v_c(c, \Delta_c) = \ln(\Delta_c^{-\gamma}) + f(c), \quad f(c) \equiv \xi - \hat{f}(c)
\]

\[
\Rightarrow e^{\ln v_c(c, \Delta_c)} = e^{\ln(\Delta_c^{-\gamma})} e^{f(c)}
\]

\[
\Rightarrow v_c(c, \Delta_c) = \Delta_c^{-\gamma} e^{f(c)},
\]

where \(\xi\) and \(\hat{f}(c)\) are constants of integration.

Step 2. Utility. Integrating the above with respect to \(c\) yields:

\[
\int \frac{d}{d c} v(c, \Delta_c) dc = \Delta_c^{-\gamma} \int e^{f(c)} dc
\]

\[
\Rightarrow v(c, \Delta_c) - \Psi(\Delta_c) = \Delta_c^{-\gamma} \varphi(c)
\]

\[
\Rightarrow v(c, \Delta_c) = \Delta_c^{-\gamma} \varphi(c) + \Psi(\Delta_c),
\]

where \(\Psi(\Delta_c)\) is a constant of integration. Next, we use constancy of the elasticity of marginal utility again, to obtain an expression for \(\varphi(c)\). Solving the differential equation yields:

\[
\varphi(c) = \frac{c^{1-\theta} K^1}{1 - \theta} + \hat{K}^2
\]

where \(K^1\) and \(\hat{K}^2\) are arbitrary constants of integration. Setting \(\hat{K}^2 = K^2/(1 - \theta)\) yields (33). □

Proof of Proposition 4

Our starting point is (11), (21), (29). Based on these first-order conditions, the growth rate of the respective costate variable is the same for all three, the market-, the welfarist- and the paternalistic framework:

\[
\frac{\dot{\mu}}{\mu} = \frac{\dot{u}_c}{u_c} = \frac{u_{cc} c \dot{c}}{u_c c} + \frac{u_{c\Delta_c} c \Delta_c}{u_c c} + \frac{u_{ck} k \dot{k}}{u_c k}.
\]
We assume (A1) and (A2). As shown above, the two elasticities of marginal utility of consumption with respect to \( c \) and \( \Delta c \) are constant. By homogeneity of degree \( \hat{R} \) in \( k \), elasticity \( u_{ck}k/u_c \) becomes \( u_{ck}k/u_c = \hat{R} \) and is also constant. Next, considering that along a balanced growth path \( \dot{k}/k = \dot{c}/c \), and \( \Delta c/\Delta c = (1 - \alpha^h - \lambda^c\alpha^f)\dot{c}/c \), as well as \( u_{\Delta c} = \kappa u_c \), we evaluate

\[ \frac{\dot{\mu}}{\mu} = -\Omega \dot{c}/c, \]  

where \( \Omega \equiv \{ (1-R)[1+\kappa(1-\alpha^h-\lambda^c\alpha^f)] - \hat{R} \} \), and \( \Omega^{-1} \) represents the intertemporal elasticity of substitution. It is important to notice that, in equilibrium, \( \Omega \) is the same for the market- as well as the welfarist- and paternalistic governments.

Next, we consider, the right hand sides of (34), (35), and (36). Observe that the homogeneity properties imply \( u_k/u_c = (\hat{R}/R)(c/k) \). Suppressing arguments, it is easy to derive

\[ \frac{u_k^m}{u_c^m + u_{\Delta c}^m} = \frac{u_k^m}{u_c^m(1 + \kappa)} = \frac{\hat{R}}{R(1 + \kappa)} \left( \frac{c}{k} \right)^m, \]  

\[ \frac{u_k^w}{u_c^w + (1-\alpha^h)u_{\Delta c}^w} = \frac{u_k^w}{u_c^w[1 + \kappa(1-\alpha^h)]} = \frac{\hat{R}}{R[1 + \kappa(1-\alpha^h)]} \left( \frac{c}{k} \right)^w, \]  

\[ \frac{u_k^p}{u_c^p} = \frac{\hat{R}}{R} \left( \frac{c}{k} \right)^p, \]

so that \( \Gamma^p > \Gamma^w > \Gamma^m \). Combining with (41) yields

\[ g^i = \Omega^{-1}\{[(A - \delta) - \rho] + \Gamma^i (c/k)^i \}, \quad i \in \{m, w, p\}. \]

We can directly apply Lemma 1 to infer

\[ g^p > g^w > g^m \iff \left( \frac{c}{k} \right)^p < \left( \frac{c}{k} \right)^w < \left( \frac{c}{k} \right)^m \]

where the latter inequalities follow from the fact that \( (c/k) = (A - \delta) - g \).

**Proof of Proposition 5**

Step 1. The homogeneity conditions (A1) – (A4) are equivalent to constancy of the marginal rates of substitution \( u_{\Delta c}/u_c, u_{\Delta k}/u_k, u_k/u_c \).
Step 2 (Paternalistic government). Following the reasoning above, the following must hold
\[
\frac{u_k + u_{\Delta k}}{u_c + u_{\Delta c}} = \frac{u_k}{u_c} \Leftrightarrow \frac{u_k}{u_c} \frac{1 + \eta}{1 + \kappa} \Leftrightarrow \eta = \kappa,
\]
where the first term on the left hand side represents the marginal rate of substitution of wealth for consumption in a market framework, and the term to the right of the equality sign represents the marginal rate of substitution of wealth for consumption for a paternalistic government. Together with (A1) – (A4), the condition is necessary and sufficient for positional preferences to be non-distortionary.

Step 3 (Welfarist government). Similarly, following the reasoning above, the following must hold:
\[
\frac{u_k + u_{\Delta k}}{u_c + u_{\Delta c}} = \frac{u_k + (1 - \beta^h)u_{\Delta k}}{u_c + (1 - \alpha^h)u_{\Delta c}} \Leftrightarrow \frac{u_k(1 + \eta)}{u_c(1 + \kappa)} = \frac{u_k[1 + \eta(1 - \beta^h)]}{u_c[1 + \kappa(1 - \alpha^h)]} \\
\Leftrightarrow \frac{1 + \eta}{1 + \kappa} = \frac{1 + \eta(1 - \beta^h)}{1 + \kappa(1 - \alpha^h)},
\]
which completes the proof. ■

Endogenous growth rate in the general case

We apply the same steps as in the proof of Proposition 4. However, here, we additionally take into account the positional preference for wealth.

Step 1. The growth rate of the costate variable becomes
\[
\frac{\dot{\mu}}{\mu} = \dot{u}_c = \frac{u_{\Delta c} c}{u_c c} + \frac{u_{\Delta k} k}{u_c k} \frac{\dot{\Delta}_c}{c} + \frac{u_{\Delta k} k}{u_c k} \frac{\dot{\Delta}_k}{k}.
\]
Considering (A1) – (A4) together with requirement that \(\dot{c}/c = \dot{k}/k\) for a balanced growth path, yields
\[
\frac{\dot{\mu}}{\mu} = -\Omega \frac{\dot{c}}{c} \quad \Rightarrow \quad g = -\Omega^{-1} \frac{\dot{\mu}}{\mu},
\]
where \(\Omega \equiv \{(1 - R)[1 + \kappa(1 - \alpha^h - \lambda^c \alpha^f)] - \hat{R}[1 + \eta(1 - \beta^h - \lambda^k \beta^f)]\}\), and \(\Omega^{-1}\) represents the intertemporal elasticity of substitution, as above.

Step 2. For calculating \(\dot{\mu}/\mu\), consider the right hand sides of (12), (22), and (30)
in order to evaluate the marginal rates of substitution of wealth for consumption. Specifically,

\[
\frac{u_k^m + u_{\Delta_k}^m}{u_k^c + u_{\Delta_c}^m} = \frac{u_k^m(1 + \eta)}{u_k^m(1 + \kappa)} = \frac{\hat{R}(1 + \eta)}{\hat{R}(1 + \kappa)} \left( \frac{c}{k} \right)^m, \\
\text{(market)}
\]

\[
\frac{u_k^w + (1 - \beta^h)u_{\Delta_k}^w}{u_k^w + (1 - \alpha^h)u_{\Delta_c}^w} = \frac{u_k^w[1 + \eta(1 - \beta^h)]}{u_k^w[1 + \kappa(1 - \alpha^h)]} = \frac{\hat{R}[1 + \eta(1 - \beta^h)]}{\hat{R}[1 + \kappa(1 - \alpha^h)]} \left( \frac{c}{k} \right)^w, \\
\text{(welfarist government)}
\]

\[
\frac{u_k^p}{u_k^c} = \frac{\hat{R}}{\hat{R}} \left( \frac{c}{k} \right)^p. \\
\text{(paternalistic government)}
\]

It is easy to see that the ranking of the expressions $\Gamma^i$, as derived for the proof of Proposition 4, does not hold here.

References


