Extrapolative Expectations and Capital Flows during Convergence

Cozzi, Guido and Davenport, Margaret

School of Economics and Political Science, University of St. Gallen

January 2017

Extrapolative Expectations and Capital Flows during Convergence

Guido Cozzi * Margaret Davenport †

March 30, 2017

Abstract How long shall a country take to learn the world technological frontier? What would happen if that country found the same difficulties in learning the true model of its economy? After all, countries catching up often experience life-changing transformations during the catch-up to a balanced growth path. We show that an open economy, learning rational expectations alongside foreign technology, may be characterized by excessive saving and current account surpluses, as often observed in the data and at odds with the standard open economy theoretical predictions, and not fully explained by standard adaptations such as habit formation. Moreover, such a learning process in a large developing country can upset the savings behavior of a fully rational expectations advanced country. In a US-China calibration, we show that this effect can be so strong as to explain important current account imbalances, the savings glut hypothesis, as well as the distribution of factor income.
JEL Classification: F21, F41, E03
Keywords: capital flows, extrapolative expectations, global imbalances, technological convergence

1 Introduction

The workhorse neoclassical model makes very clear predictions about the relationship between the current account and long term or persistent productivity growth (e.g. Obstfeld and Rogoff, 1996). In the face of persistent growth,

*guido.cozzi@unisg.ch, School of Economics and Political Science, University of St. Gallen, St. Gallen, Switzerland
†margaret.davenport@unisg.ch, Institute of Economics (FGN), University of St. Gallen, St. Gallen, Switzerland

consumption increases in excess of current output and is supplemented by capital inflows; similar patterns are present for investment decisions, which respond positively to expected future total factor productivity increases. Hence, lower savings and higher investments should be associated with faster technological progress. Faster growing economies should be on the receiving end of international capital flows and these flows should be increasing in productivity growth.

In spite of these predictions, not only large global imbalances, but also a negative relationship between persistent economic growth and capital inflows has emerged over a longer period starting in 1980s for non-developed economies: the allocation puzzle (Gourinchas and Jeanne, 2013; Alfaro, Kalemli-Ozcan, and Volosovych, 2014). This premise relies on three key features that deserve greater attention. First, the neoclassical model in relation to these puzzles subsumes perfectly functioning financial markets. Second, analysis often focuses on a representative agent framework, abstracting from life-cycle savings motives and demographic structure. Third, it relies on the ability of agents to rationally forecast the persistent growth component as well as future marginal utility of consumption.

While many explanations for global imbalances and the allocation puzzle focus on the first and second points, no model has instead exploited the potential open economy effects of a slow convergence to rational expectations, quite natural for an economy undergoing a transition towards a more balanced environment, and essential to consolidate agents' knowledge of the true model of their economy. In the present paper, we incorporate extrapolative expectations (where agents rely on their more recent experience) into an otherwise standard small open economy, dynamic general equilibrium (DGE) model that can be brought to the data. We use a reduced form of learning, where agents forming expectations are learning to rationally forecast their future marginal utility and the future marginal product of capital, that is endogenous to the path of development. When this occurs, we show that a negative pattern between realized economic growth and capital inflows may emerge that is strengthened for higher productivity growth, and thus it can explain the allocation puzzle.

This novel channel of persistence can also induce the emergence of global imbalances between two large regions that we calibrate to the China and United States.

The basic intuition behind our results is as follows. At the beginning of a transition to a balanced growth path associated with a higher productivity levels, agents must learn to forecast rationally along with the new model of their economy. As this occurs, they rely on more recent experience for forming their expectations. Agents at the beginning of transition tend to underestimate the extent of their marginal utility of consumption and the unconstrained consumer smooths consumption to a new level that reflects the increase in *perceived* marginal utility of income. Consumption is therefore too low and excess saving flows out of the economy. This mechanism is augmented with higher productivity growth and higher convergence. Our expectational mechanism is consistent with the well documented positive relationship between savings and economic growth, but goes a step further in that it offers an additional channel that supports causation running from economic growth to savings.

By focusing on the potential of our new channel in a transparent way, our paper purposefully neglects other important ingredients highlighted in the literature, which of course matter in the international capital market. Most notably, in a model of equilibrium global imbalances, Caballero, Farhi, and Gourinchas (2008a) show that uphill capital flows may emerge as a result of depressed supply of safe assets in developing economies. Mendoza, Quadrini, and Rios-Rull (2007) argue that it is a lack of supply for contingent claims in the presence of idiosyncratic risks and, as a consequence, precautionary savings motives that can explain current account surpluses of developing countries. Similar arguments have been made for developing economies on the investment side.[1]

However, in a panel of developed and developing economies, Gruber and

[1]Prasad, Rajan, and Subramanian (2007) argue that a lack of financial intermediation prevents domestic savings from being channeled into productive investments; and Song, Storesletten, and Zilibotti (2011) model financial frictions which prevent loans from being allocated to highly productive investment in the private sector as the source of capital outflows in the Chinese case.

Kamin (2009) do not find systematic evidence that measures of financial development such as credit to GDP ratio or capitalization of stock markets can explain the direction of observed capital flows.[2]

Central to the emergence of both imbalances and the puzzling allocation of capital, is an underlying positive relationship between savings and economic growth. The work of Carroll and Weil (1993) highlights the causal channel running from economic growth to household saving, which can be accounted for in a model of habit formation where agents adjust consumption slowly (Carroll and Weil, 1993; Carroll, Overland, and Weil, 2000).

At the root of our mechanism is indeed a positive correlation between saving rate and rate of growth, however, as we document in Section 8, habit formation is not enough to generate proximity of the model's predictions to the data. In other words, the extent of underconsumption observed from many fast-growing converging economies cannot be sufficiently explained with habit formation in an open economy model.

Models with an overlapping generation structure may also predict positive relationship between savings and growth, differing from the standard neoclassical model (Obstfeld and Rogoff, 2009). Recent research has focused on the demographic structure in explaining a feedback loop that allows causation in both directions. Mehlum, Torvik, and Valente (2013) in a two sector, overlapping generations (OLG) economy, derive the "savings multiplier" of economic growth that emerges via more redistribution to a young generation of savers and by increasing the cost of old age care. The model is able to explain why faster-growing economies exhibit higher saving rates absent any additional market imperfections.

Further, research in this area highlights implications of the interaction of economic growth and financial frictions in a life-cycle framework for private saving. Coeurdacier, Guibaud, and Jin (2012) consider demographic structure with economic growth differentials and heterogeneous borrowing constraints

[2] Furthermore, Gourinchas and Jeanne (2013), point out that the allocation puzzle is strongest for developing economies that are the most financially integrated (and perhaps equipped with more developed financial markets).

in a two country model calibrated for China and the US. Their model is able to replicate divergence of private savings rates, global imbalances and falling interest rates, while making predictions for the age-specific savings behavior in the presence of economic growth.

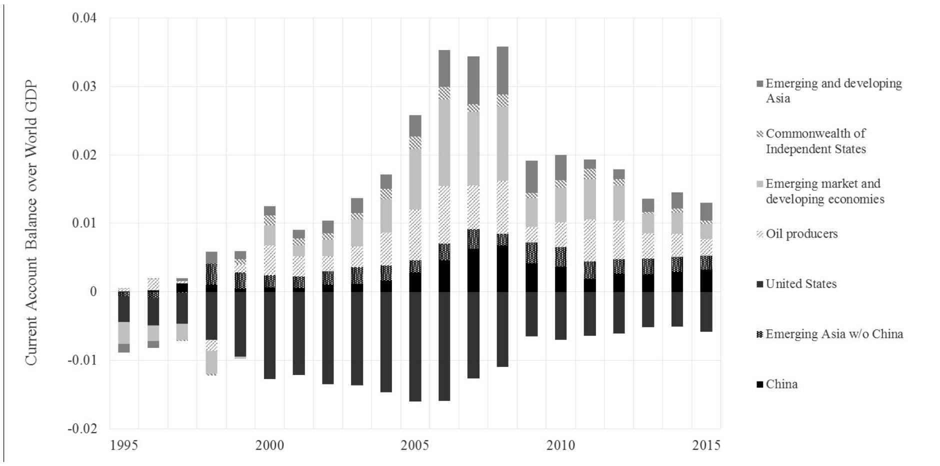
Our paper also relates to a growing literature in macroeconomics, which has found important support empirically for alternative expectation formations. Fuster, Laibson, and Mendel (2010) incorporate extrapolative expectations into a standard macro forecasting model and show that, in a wide variety of settings, observed macroeconomic variables such as consumption, inflation, output and investment are better approximated by the incorporation of expectations based on some form of extrapolation bias. Similar approaches have been used for statistical learning by Evans and Honkapohja (2001) and behavioral model by De Grauwe (2012).

Although we highlight an expectational mechanism, we view our approach to be complementary to the friction channels that may affect savings and investment decisions in developing economies. When agents use more recent experience to form expectations as transition begins, this represents a market friction between what is optimal given the true model and what agents judge to be optimal. This channel operates via a distortion in agents' optimal savings behavior. These implications are discussed in greater detail in Section 8.

2 Stylized Facts

Among the empirical facts well documented by the international economic literature has been the emergence of the global imbalances documented in Figure 1, which shows current account as a ratio to world GDP for a select group of economies. Current account imbalances emerged from the mid 1990s and increase towards the late 1990s after the east Asian financial crisis. For the United States deficits persist throughout the 2000s, with a significant improvement from 2006 to 2007 and a further improvement after the financial crisis. Current account deficits do not return to previous levels after the financial crisis.

Figure 1: Current Account Imbalances



Current account balances are shown as percentage of world GDP for various world regions from 1995 to 2015. Source: authors' calculations, WEO Database, IMF

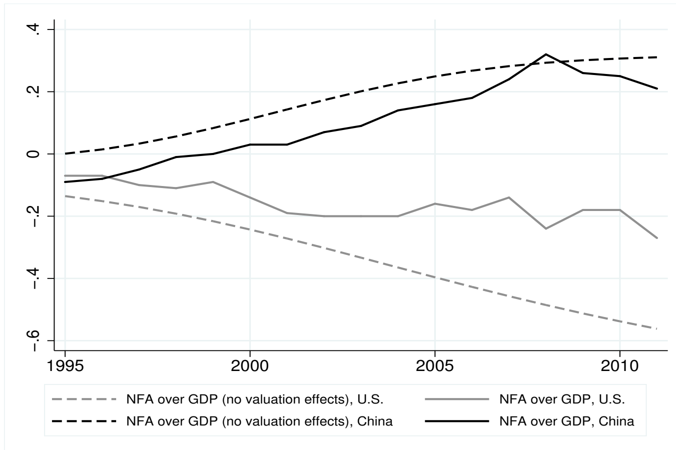
The US deficits are matched by current account surpluses from fast growing, emerging economies on the whole, most particularly developing Asia, oil producing economies, and former Soviet countries. A sizeable current account surplus arises in most of these economies starting in the late 1990s.

As a consequence of current account imbalances, large (in absolute value) net foreign asset positions, particularly for the United States and China, have emerged, shown in Figure 2. This figure shows both the reported net foreign asset position over GDP from Lane and Milesi-Ferretti (2007) and also the cumulative sum of capital account deficits for the two countries using the same dataset. Both series are filtered of business cycle frequencies using an Hodrick-Prescott filter.

The US and China held positions that were relatively small as a percentage of GDP until the mid-1990s, when current account imbalances between the two countries emerged. The growth in magnitude of these international investment positions accelerated into the 2000s and were reduced after the financial crisis. For the US case, however, the reduction is less pronounced relative to US GDP.

There is robust evidence of a negative relationship between economic growth and the capital account shown in Figure 3. This figure shows the relationship

Figure 2: Net International Investment Position 1995-2011, United States and China



The solid black and grey lines depicts the net foreign asset position over GDP for China and the United States, respectively. The dashed lines depict the net foreign asset position over GDP excluding valuation effects. This series is calculated by taking an initial net foreign asset position in 1981 (the first series available in Lane and Milesi-Ferretti (2007)) and adding subsequent current account surpluses in the years following. The series is then smoothed using the Hodrick-Prescott filter with smoothing parameter, $\lambda = 100$. Source: authors' calculations, EWN Mark II data from Lane and Milesi-Ferretti (2007)

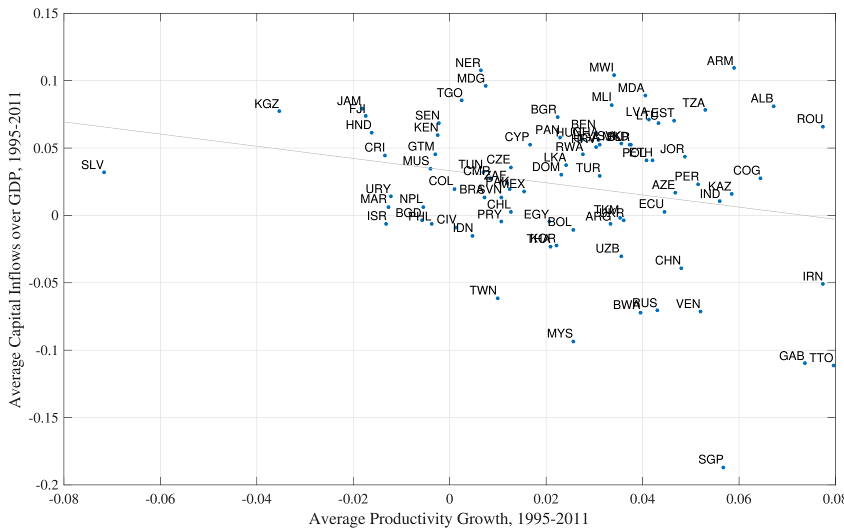
between the cross sectional average capital account over GDP and average trend total factor productivity growth over the period 1995 to 2011 for non-OECD countries.[3]

This evidence tells us that not only does capital flow out of fast growing, developing economies, this relationship is extenuated in faster growing, higher investing developing economies. This is an extension of the findings of Gourinchas and Jeanne (2013) for a sample including transition economies and for the time period 1995 to 2011.

Our parsimonious model, introduced in the following section, and numerical exercises, are able to replicate the salient features of the emerged stylized facts introduced above. We provide particular explanation for the imbalances

[3] The sample of 84 non-OECD countries is the same as that used in Gourinchas and Jeanne (2013) including transitional economies with available data from 1995. The slope of the line shown in the graph is -0.45 and is significant at the 5 percent level.

Figure 3: Negative Relationship between Average Capital Inflows and Productivity Growth



Average capital inflows over GDP are calculated as the negative average current account surplus over GDP across countries, smoothing using the Hodrick-Prescott filter with smoothing parameter of 100. Average productivity growth is calculated as an average growth rate of level of TFP measured using the approach outlined in Section 4. The sample is for 84 developing economies. The slope of the above line is -0.45 and is significant at the 5 percent level. Source: authors calculations using data from PWT 8.1 (Feenstra et al., 2013).

emerging between the United States and China since the 1990s and the evidence for a negative relationship between productivity growth and capital inflows over the same period.

3 Model - Small Open Economy

To make our results more transparent, but without loss of generality, we use a simple small open economy neoclassical growth model, with inelastic labor supply and no capital adjustment costs. Since we are interested in long-run relationship, we focus on deterministic trend growth and abstract from the stochastic growth component characteristic of the business cycle literature.

The economy is populated by an infinite number of identical households who maximize their expected lifetime utility with respect to consumption,

$$\hat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma} - 1}{1-\gamma}. \tag{1}$$

Population is held constant and normalized to 1, so that consumption corresponds to per capita consumption. We use in Eq. (1) a standard CRRA utility function in consumption only, where γ is the inverse elasticity of intertemporal substitution.

Households maximize utility with respect to consumption subject to the flow budget constraint given by

$$d_t = (1 + r_{t-1})d_{t-1} + c_t + i_t - y_t, \tag{2}$$

where d_t is the household's end of period t debt position, r_{t-1} is the cost of international borrowing, c_t is per capita consumption, y_t is per capita income and i_t is investment per capita.

The law of motion for capital stock is

$$k_{t+1} = i_t + (1 - \delta)k_t, \tag{3}$$

where δ is the depreciation rate of physical capital.

Output of a single good in the economy is produced using the following technology

$$y_t = k_t^{\alpha}(\xi_t h_t)^{1-\alpha}, \tag{4}$$

where h_t is inelastic labor supply, and ξ_t is a labor-augmenting technology parameter. We normalize inelastic labor supply to $h_t = 1$.[4] We discuss the time path for productivity parameter as a combination of trend growth and productivity convergence in the following section.

We assume the domestic interest rate, r_t is

$$r_t = \bar{r} + \phi(e^{\frac{\tilde{d}_t}{y_t} - \frac{\bar{d}}{y}} - 1). \tag{5}$$

[4]For now, we analyze the case where countries are allowed to differ only in their technological advancement.

The cost of borrowing is comprised of the world interest rate, reflecting the marginal product of capital minus the depreciation rate of economies that are on the balanced growth path, and a spread. The spread is increasing in the level of average household debt over income, $\frac{\bar{d}_t}{y_t}$, relative to some long term average, $\frac{\bar{d}}{y}$, where ϕ is a measure of the sensitivity of the interest rate with respect to debt to income ratio. Notice that while in equilibrium $\bar{d}_t = d_t$, households will take $\bar{d}_t$ as given in their utility maximization problem.

As in Schmitt-Grohe and Uribe (2003) and now standard in the literature, we incorporate this assumption in order to find a balanced growth path that does not rely on initial conditions of debt in each country.[5] This is a technical reason. However, as it becomes apparent in the quantitative results, the stationarity inducing assumption operates over the very long run, and impacts only marginally the dynamics we wish to analyze.[6] Furthermore, while generally used for business cycle analysis, the fact that the country spread should be increasing in the debt to GDP ratio finds support in literature on the long run determinants of country spreads (Kinoshita, 2006; Kumar and Baldacci, 2010).[7]

3.1 Productivity Growth

In the present paper, we analyze how the incorporation of learning to become rational along with the true model of the economy into a standard macro model impacts the predicted relationship between capital flows and long run convergence to a balanced growth path.

Growth in the economy is derived from exogenous growth of the productivity parameter, ξ_t. We assume the technological frontier grows at gross

[5] Without this closing assumption, the deterministic steady state depends on initial debt levels. This assumption is shown to be plausible over business cycle frequencies (Schmitt-Grohe and Uribe, 2003)

[6] For example, one may be concerned that the relative strength of income and substitution effects with falling interest rates may impact the direction of our results. Our results are robust to different values of the IES.

[7] It is important to note that the predictions of our model are not sensitive to the introduction of various closing methods used in the small open economy literature, including internal debt elastic interest rate and portfolio adjustment costs.

exogenous rate, g^*, which represents the long run, balanced growth factor of the most advanced economies. We highlight the case of convergence, or the economy catching up to the technological frontier. We formulate a catch-up parameter as in Gourinchas and Jeanne (2013) as follows,

$$\pi_t = \frac{\xi_t}{\xi_0 e^{g^* t}}. \tag{6}$$

This catch-up parameter measures the growth in the economy over t periods relative to that of the frontier.[8] With this formulation, we are able to describe technological convergence in a single parameter, where $\pi_t > 1$ for an economy that catches up to the technological frontier, $\pi_t < 1$ for an economy that is lagging behind, and $\pi_t = 1$ for countries growing at the rate of the frontier. It is important to note, that our model does not rely on full convergence to the productivity frontier; in fact, the economy may converge to any balanced growth path.

Rewriting Eq. (6) in terms of ξ_t and ξ_{t-1} and rearranging,

$$\xi_t = e^{g^*} \frac{\pi_t}{\pi_{t-1}} \xi_{t-1}. \tag{7}$$

The gross growth rate in the economy is comprised of a balanced growth component and a productivity frontier convergence or catch-up component.

In order to examine the transition of the economy during convergence, we assume the time path for catch-up,

$$\pi_t = (1-\psi)\pi_{t-1} + \psi\bar{\pi}, \tag{8}$$

with $0 < \psi < 1$. With this formulation, the economy catches up during a transition to a value of $\bar{\pi}$, to which the productivity parameter converges. In this sense, one may consider π_t to be the cumulative growth in excess of the frontier.

Considering this formulation of the productivity catch up, π_t is a concave

[8] One may rewrite Eq. (6) as the ratio of growth factors between the domestic economy and the frontier.

function of time: countries that begin to converge to the balanced growth path do so very quickly towards the beginning, and then experience slower long term convergence over time.

3.2 Equilibrium

Households choose optimal paths for c_t, d_t and k_{t+1} given initial conditions, $d_0 = \bar{d}$, $k_0 > 0$, taking the path of $\bar{d}_t$ as given, and subject to E q. (2) and the no Ponzi-game condition,

$$\lim_{j\to\infty} \frac{d_{t+j}}{\prod_{i=t-1}^{t+j-1}(1+r_{t+i})} \leq 0. \tag{9}$$

The necessary conditions for the above optimization problem are,

$$c_t^{-\gamma} = \lambda_t \tag{10}$$

$$\lambda_t = \beta(1+r_t)\hat{E}_t(\lambda_{t+1}) \tag{11}$$

$$\lambda_t = \beta\hat{E}_t\left[\lambda_{t+1}(\alpha\tilde{k}_{t+1}^{\alpha-1} + 1 - \delta)\right] \tag{12}$$

where $\tilde{k}_{t+1} = k_{t+1}/\xi_{t+1}$ is the capital stock per effective unit of labor, λ_t is the shadow price of consumption and where the $\hat{E}_t$ may correspond to the true conditional expectation for the rational case, or the extrapolative conditional forecast with learning. Eqs. (11) and (12) are the consumption Euler and capital demand equations, respectively. Eq. (10) is the definition of marginal utility from the CRRA utility function.

The above first order conditions together with Equations (2), (3), (4), (5) and given $k_0 > 0$ and $d_0 = \bar{d}$ describe the dynamical system completely.

3.3 Expectation Formation

The transitional dynamics during productivity convergence are largely determined by agents' expectations of future marginal utility of consumption relative to expected future return on savings and investment. Given the key role for the forward looking behavior of agents in the model, the assumptions on the formation of expectations on future variables are central to the dynamic predictions of the model.

The standard rational expectations assumption imposes very strict requirements on the computational ability of agents in the model. While we follow this approach in the stable environment of advanced countries, we instead assume that in countries experiencing a transitional path, agents are unable to perform the rational expectations predictions perfectly and instead rely on their more recent experience. More generally, we find the experiential interpretation particularly plausible in the absence of the balanced growth path.

In fact, in our small open economy model the economy begins transition with a technological growth away from balanced growth. When labor-augmenting technological progress begins its convergence process, agents in the economy must learn to use the rational expectations forecast along the transitional path to balanced growth. In such a situation, we find it natural to assume that agents may not trust their predictions about their future will to consume and supplement forecasts based on the true model of the economy with purely extrapolative beliefs, forming a forecast of future variables that is a convex combination of forecasts based on the true model and one that extrapolates on past values.

Within this framework, we postulate that the representative household's forecast at time t for λ_{t+1} is

$$\hat{E}_t \lambda_{t+1} = \mu_t \lambda_{t-1} + (1-\mu_t) E_t \lambda_{t+1}, \tag{13}$$

where, μ_t is the share of the household's forecast that is purely extrapolative.

Similarly, we postulate that firms form expectations of the right hand side of the capital demand equation, Eq. (12) according to

$$\hat{E}_t \left(\lambda_{t+1}(\alpha \tilde{k}_{t+1}^{\alpha-1} + 1 - \delta) \right) = \mu_t \lambda_{t-1}(\alpha \tilde{k}_{t-1}^{\alpha-1} + 1 - \delta) + (1-\mu_t) E_t \left(\lambda_{t+1}(\alpha \tilde{k}_{t+1}^{\alpha-1} + 1 - \delta) \right). \tag{14}$$

As a first step, we assume that μ_t evolves according to the reduced form autoregressive process,

$$\mu_t = \rho_\mu \mu_{t-1} + (1 - \rho_\mu)\bar{\mu}. \tag{15}$$

where the share of extrapolation that agents use in their forecasts converges to $\bar{\mu}$.

Assumption 1 *In the steady state, forecasts approach the fully rational forecast.* $\bar{\mu} = 0$.

We assume agents are becoming fully rational at a rate given by $(1-\rho_\mu)$.[9] As a consequence, the weight of extrapolation in the forecast converges from above to 0. Thus, this reduced form assumption for the evolution of μ_t represents a gradual learning of agents to forecast true model of the economy, as they converge to the balanced growth path.[10]

With this formulation, in an estimation exercise, we allow the data to determine the speed with which agents in the model converge to rationality. We could, alternatively, assume a fixed share of extrapolation in the forecast. However, we prefer this formulation so as not to force persistent extrapolation bias. Quite interestingly, the estimation returns a very persistent learning process.

With the incorporation of the forecast for λ_{t+1}, the dynamical system is completely described by,

$$\lambda_t = \beta(1 + r_t)[\mu_t \lambda_{t-1} + (1 - \mu_t)E_t \lambda_{t+1}] \tag{16}$$

$$\lambda_t = \beta \left[\mu_t \lambda_{t-1}(\alpha \tilde{k}_{t-1}^{\alpha-1} + 1 - \delta) + (1 - \mu_t)E_t \left(\lambda_{t+1}(\alpha \tilde{k}_{t+1}^{\alpha-1} + 1 - \delta)\right)\right] \tag{17}$$

together with Eqs. (2), (3), (4), (5), (15).

[9] One could allow agents to form "nowcasts", or assume agents may observe perfectly the current state of the economy. Agents using current period forecasts do not significantly change the results presented here, but change the shape of the transition slightly.

[10] The reduced form process for μ_t is easily micro-founded with adaptive learning.

4 Quantitative Exercise

In a deterministic setting, we simulate technological convergence of the economy to the frontier. We simulate this case both for the standard rational expectations model and for the case with learning. While, for exposition purposes, we simulate the full convergence of the Chinese economy, as we will see later, our results do not rely on unconditional convergence, but hold for convergence to any balanced growth path.

Along the balanced growth path, at the frontier, the technology parameter, ξ_t, grows at an gross exogenous rate, g^*. Therefore, we explain productivity convergence to the frontier using our catch-up parameter, π_t, that reaches a steady state level once convergence is attained.

We calculate total factor productivity using an approach common in growth and development account literature. Namely, from our production function

$$\xi_t = \left(\frac{y_t}{k_t^\alpha}\right)^{1/(1-\alpha)}, \tag{18}$$

where y_t and k_t are real GDP and capital stock per employed labor from PWT 8.1. Our measure for TFP is then the trend component of ξ_t, using a Hodrick-Prescott filter with smoothing parameter of 100.

The real GDP measure corrects for changing reference prices and PPP exchange rates over time and, as such, is comparable across countries and over time.[11]

The growth rate of the frontier is proxied by the long run, growth rate of trend total factor productivity in the United States of 1.016 from 1995 to 2011,

$$g^* = ln\left(\frac{\xi_{2011}}{\xi_{1995}}\right)/(2011-1995). \tag{19}$$

For sake of example, the simulation mimics the convergence of the People's

[11]We use the real GDP measure based on output side, the variable *rgdpo*, which takes into account the relative prices in exports and imports in addition to final goods in the calculation of PPPs over time. This will give a better measure of productivity changes over time, as it accounts for the affects of changing terms of trade on real GDP measure. We find this of particular importance in a sample, which includes emerging economies.

Republic of China to the United States. While China is clearly not a small open economy, it has, with little doubt, been at the forefront of intrigue regarding the savings glut hypothesis and studies of excess saving during periods of high economic growth. Our choice of small open economy is for ease of exposition of our results. We perform an identical exercise (with essentially identical results) for a two region case in Section 7.

To calibrate the terminal catch-up parameter, $\bar{\pi}$, TFP is assumed to grow at the frontier by a constant rate, g^*,

$$\xi_t^* = e^{g^* t}\xi_0. \tag{20}$$

China is catching up over the simulation and TFP grows according to,

$$\xi_t = e^{g^* t}\frac{\pi_t}{\pi_0}\xi_0. \tag{21}$$

By definition, after full convergence the Chinese TFP growth should equal that of the US or frontier and therefore,

$$\frac{\bar{\pi}}{\pi_0} = \frac{\xi_0^*}{\xi_0}. \tag{22}$$

Normalizing π_0 to 1 and taking the ratio of our measure of US TFP to Chinese TFP, we calibrate $\bar{\pi} \approx 11.18$. This parameter measures the initial distance to the productivity frontier. That is, in 1995 the US level of TFP was roughly eleven times that of China.

The model is calibrated to annual data. The number of years that it takes the economy to converge to the frontier is governed by Eq. (8) and specifically, parameter ψ. From 1995 to 2011, Chinese trend TFP grows from $3,960.80$ to $8,546.70$, a factor of 2.16. From Eq. (8), we have after 17 years, the value of the catchup parameter $\pi_{17} = 1.68$. Plugging this into Eq. (8) for $t = 17$ we can solve for $\psi = 0.0043$.

Despite the high annual growth in trend TFP in China from 1995 to 2011 in excess of 3.6 percent, absent further differences in demographic patterns, capital to labor ratios, capital shares, the halfway point to convergence is attained after around 150 years. The length of catch-up would be naturally reduced should we examine the case of conditional convergence.

The world interest rate, $\bar{r}$ is taken as the interest rate along a balanced growth path. Combining Eqs. (10) and (11) and assuming that all variables grow at exogenous rate, g^*, along the balanced growth path, we have that $\bar{r} = (e^{g^*})^\gamma/\beta - 1$. Considering the frontier growth of 0.016, and assuming a subjective discount factor in the utility function of $\beta = 0.96$, this gives us a world interest rate of 5.81 percent, which is slightly lower than the value taken in Gourinchas and Jeanne (2013) and the long-run average return on capital markets as measured for developing countries of 6.9 percent (Caselli and Feyrer, 2007).

The remaining parameters are taken from standard values in the literature. We assume a positive rate of depreciation of 6 percent, a capital share is assumed to be $\alpha = 0.3$ and an inverse elasticity of substitution parameter γ of 1.[12] The parameter governing the elasticity of the interest rate with respect to average debt to GDP ratio ϕ is set to 0.035. This is in line with estimates of the sensitivity of country interest rates to debt to GDP ratios found in the literature (Kinoshita, 2006; Kumar and Baldacci, 2010).[13]

The initial share of extrapolation in the forecast and the speed of learning are latent. We estimate μ_0 and ρ_μ using a simulated method of moments approach, that minimizes the sum of squared distance between our model's predictions for current account balance over GDP and the empirical counterpart in China taken as trend current account over GDP from Lane and Milesi-Ferretti (2007) from 1995 to 2011. This estimation exercise results in an estimated $\mu_0 = 0.426$ and speed of learning of $\rho_\mu = 0.999$. The transitional

[12]Our assumption of log utility is not essential for our results. With $\gamma > 1$, the substitution effect is dominated by a negative income effect, resulting from falling interest rates, which exacerbates our results. With log utility, we are able to isolate the effects from our expectational channel and a positive wealth effect.

[13]A value of $\phi = 0.035$ means for a 1 percentage point increase in the debt to GDP ratio for China, interest rates increase around 3.5 basis points. This is consistent with estimates in the literature of the elasticity of country interest rates with respect to debt to GDP ratios, that long run country interest rates increase 2 to 7 basis points with an increase of one percentage point in debt to GDP ratio (Kinoshita, 2006; Kumar and Baldacci, 2010).

paths of μ_t is displayed in Figure 4.

All parameter values used in the simulation exercise are shown in Table 1.

Table 1: Parameters used for simulation

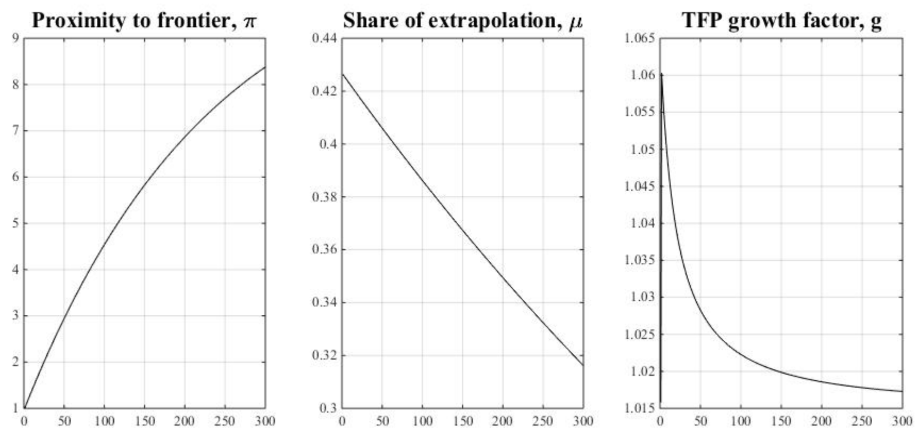
Calibrated Parameters			
Frontier TFP growth rate	g^*	1.016	Eq. (18) & Eq (19) & PWT 8.1 data
Initial convergence parameter	π_0	1.00	normalization
Final convergence parameter	$\bar{\pi}$	11.18	Eq. (21), Eq. (22) & PWT 8.1
Speed of tech. convergence	ψ	0.0043	from Eq. (8) & PWT 8.1
Parameters from literature			
Depreciation rate	δ	0.06	Caselli and Feyrer (2007)
Capital share of income	α	0.30	Gourinchas and Jeanne (2013)
Discount factor	β	0.96	Gourinchas and Jeanne (2013)
CRRA parameter	γ	1.00	log utility
Steady state debt/GDP ratio	$\bar{d}$	0	full repayment of debt in long run
Interest rate sensitivity	ϕ	0.035	Kinoshita (2006); Kumar and Baldacci (2010)
Estimated parameters			
Initial share of extrapolation	μ_0	0.426	estimated by simulated method of moments
Speed of learning	ρ_μ	0.999	estimated by simulated method of moments
Final extrapolation share	$\bar{\mu}$	0	rational agents along balanced growth path

The simulation begins at time $t = 0$, for the productivity catch-up by assuming an initial balanced growth path at a low initial productivity level relative to the frontier. At $t = 1$, convergence begins and total growth in the economy rises from 1.016 to in excess of 1.06. Figure 4 shows the transition of productivity growth and productivity catch-up.

The predictions of the neoclassical open economy–and rational expectations model–are well known. From an intertemporal perspective, forward looking agents in a country experiencing rapid technological convergence should borrow from abroad to increase current consumption in excess of current output. Therefore, absent any capital market frictions, we would expect there to be an immediate deterioration in the trade balance to GDP ratio, which recovers over time as output catches up to consumption.

The model's predictions for NFA position over GDP, current account and trade balance over GDP, capital stock over GDP and the interest rate are shown in Figure 5. The figure shows Chinese data for the empirical counter-

Figure 4: Share of Extrapolation, Simulation of Productivity Convergence



The solid lines above depict the transitional dynamics of the main driving parameters in the simulation. The left panel shows the movement of the proximity to frontier, given in Eq. (8). The halfway point to convergence is achieved, given the estimation of $\psi = 0.0043$ after around 150 years. The center panel shows the evolution of the share of extrapolation in the forecast for future marginal utility of consumption and marginal product of capital. The first order difference equation given in Eq. (15). This process is very slow moving, given an estimated parameter of $\rho_\mu = 0.999$. The third panel depicts the TFP growth factor in Eq. (7).

parts to the model in the grey dashed line.[14] The rational expectations (RE) model's transition is shown in the solid black line and the adaptive learning version in solid gray line. In the RE case, we observe a transition in the trade balance as expected: there is an initial deterioration in the trade balance, which is corrected over time, as the output growth catches up to the optimal consumption path.

Our model with learning during convergence, with the presence of extrapolation in the expectation formation, conveys a very different prediction.

[14]The trade balance to GDP ratio is taken from the World Bank's external balance of goods and services as a percentage of GDP and the NFA over GDP and current account over GDP series calculated from Lane and Milesi-Ferretti (2007) EWN Mark II database. Capital stock is taken from the real capital stock from PWT 8.1 and the interest rate is taken as the Chinese deposit rate from IMF's International Financial Statistics. All series are detrended using the Hodrick-Prescott filter with smoothing parameter 100. The NFA over GDP is absent valuation effects and taken as the cumulative sum of current account surpluses plus initial NFA position in 1980.

Quickly after the start of convergence, large trade surpluses emerge, rising to a maximum of around 5.5 percent of GDP after about 7 years, then falling thereafter, but remaining positive until 15 years after the start of convergence. Thereafter, trade deficits emerge that are persistent and more reflective of the RE predictions.

The RE prediction for external borrowing is also clear in the case of long run excess productivity growth. A growing country should increase international debt levels, borrowing from abroad in order to finance consumption in excess of output. In the learning case, we observe large net foreign asset positions relative to GDP emerging in this economy that reach 40 percent of GDP about 15 years after transition begins.

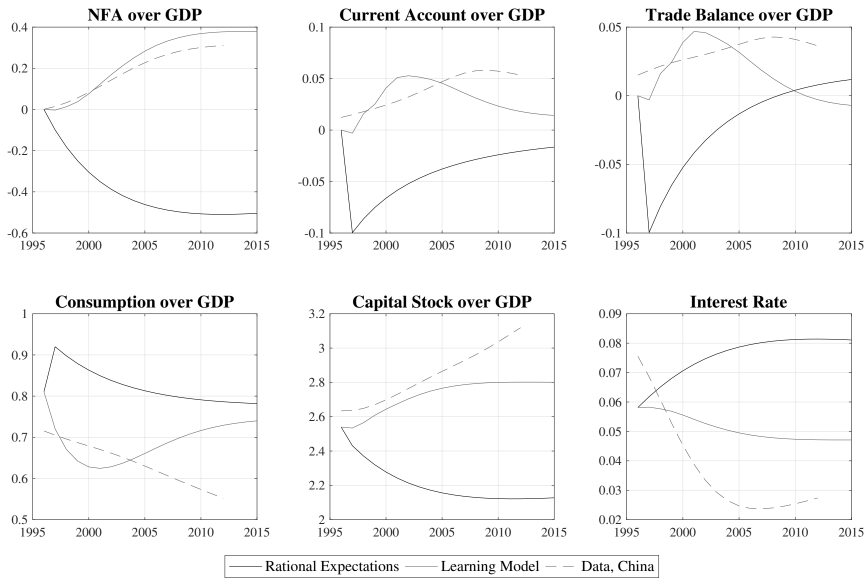
This excess saving is confirmed in the fact that consumption to GDP ratio falls after the start of convergence in the learning model. This effect is quite strong. In fact, in the presence of extrapolative expectations, in a converging economy, the expectation for future marginal product of capital would be lower, due to a lower capital stock per capita at the beginning of convergence. Indeed, we observe that the capital stock increases in the learning case, reflecting a higher investment. However, this is not enough to overturn the implication of excess saving on the external account.

The predictions of our model are very consistent with the data for China. The simple small open economy model, with parameters estimated directly from observables, is able to replicate the magnitude of observed trade balance over GDP of 4 percent almost exactly. Furthermore, the shape and magnitude NFA over GDP in our model is highly reflective of the emergence of large NFA position that has grown to 30 percent of GDP in China since the mid 1990s.

It is worth noting that the interest rate falls after convergence begins. While this is mechanically due to the formulation of the interest rate being debt elastic, this is consistent with evidence of falling rates of return to capital in China during the 1990s that increase thereafter (Bai, Hsieh, and Qian, 2006).

More generally, the simple learning during convergence model appears largely consistent with the data, particularly compared with the predictions in

Figure 5: Small Open Economy Results, Chinese Data



The solid, black line depicts the small open economy transition with rational expectations. The solid, gray line depicts the transition with extrapolative expectations. All simulated variables are in levels. The gray, dashed line depicts data for China from 1995 to 2011. All data is in levels and filtered using Hodrick Prescott filter with smoothing parameter 100.

the case of rational expectations. The simulation results for the main variables of our small open economy model are shown in Figure 16 in Appendix A.

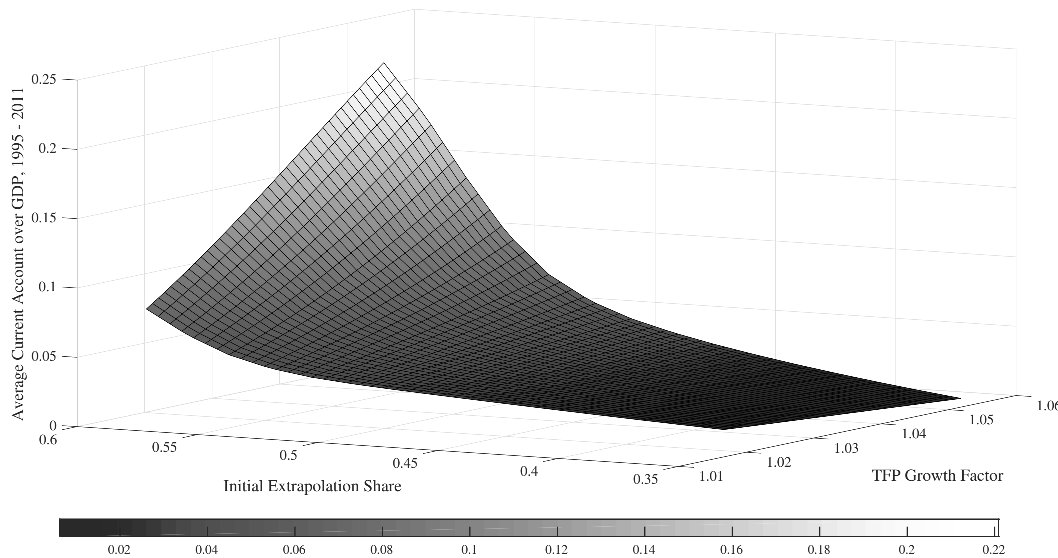
4.1 Evidence Across Countries

China has taken center stage in the discourse of global imbalances and savings glut until now. We are interested in what our model would predict for capital flows and allocation for different countries, with varying degrees of convergence. Furthermore, in order for the expectation channel to explain the allocation puzzle, our model must predict an increase in saving in excess of investment for higher growth in total factor productivity during convergence.

Our results are not isolated to the miraculous growth experience in China used in our simulation, but accommodate the diverse convergence experiences of developing economies. Figure 6 shows the model's predictions for average current account over GDP for the first 20 years of convergence for varying TFP growth values and initial extrapolation shares, keeping remaining model parameters fixed at the calibrated Chinese values.

For large enough values of the initial extrapolation share, as the extent of convergence increases, countries with extrapolation in their expectation formation experience greater capital outflows.

Figure 6: Average Current Account over GDP by Varying TFP Growth and Extrapolation Share



The three dimensional surface plot above depicts our model's predictions for average current account over GDP for varying levels of initial share of extrapolation in the forecast and TFP growth factor. All other parameters are kept at values estimated for China shown in Table 1. At values of initial extrapolation share below 0.43, the slope of the curve between TFP growth factor and average current account surplus over GDP is small in magnitude and negative. At values above 0.43, the slope turns positive and begins to increase in magnitude. This is due to the dominance of the investment channel over the consumption channel during initial years after convergence for countries further away from the frontier.

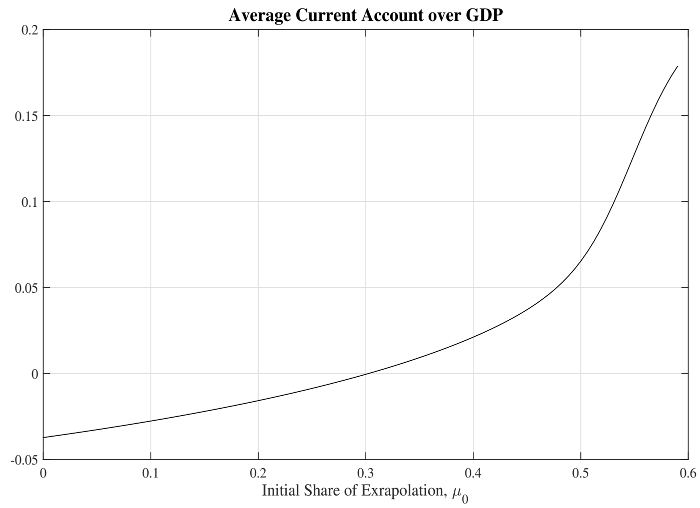
The non-monotonicity of the surface in Figure 6 results from two different channels of capital flows during technological convergence. First, a country very far away from the technological frontier, requires a higher amount of investment in order to converge to a higher steady state level of capital per effective worker. In the learning model, firm's expectations of future marginal product of capital are higher due to the extrapolation bias and investment is higher than in the rational expectation case. Second, households in the converging economy have higher expectations of future marginal utility of consumption and therefore, consume less than in the rational expectations case. If the consumption channel dominates the investment channel, we see increasing current accounts over GDP as the technological convergence increases.

Small open developing economies will also have diverse initial extrapolation forecasts. In our quantitative exercise of productivity convergence, initial forecasts are comprised of about one half extrapolative and one half based on the rational expectations forecast, reflected by an initial μ share of 0.426. The emergence of current account surpluses do not rely on a high value of this parameter specific to the Chinese case. Therefore, a lower initial μ_0 share can also accommodate countries starting out closer to the balanced growth path and, therefore, that are better able to forecast rationally.

Figure 7 shows the model's predictions for the average current account to GDP ratio over the first 20 years of convergence. The expectation mechanism in our model does not rely on agents being purely extrapolative, but predicts that current account surpluses may emerge with an initial weight of extrapolation in excess of 0.30, given a high convergence parameter of 11.18. Interestingly, the threshold μ_0 for which surpluses emerge is increasing in the convergence parameter. Given the fast TFP growth in China, this fact allows other developing economies with lower TFP growth to exhibit current account surpluses for much lower shares of extrapolation. Any extent of extrapolation increases the current account surplus predicted by our model relative to the rational expectations benchmark.

How do these model-simulated results reconcile with real development experiences of other countries over the same time period? We conduct our estimation exercise for μ_0 and ρ_μ across countries. That is, for a sample of 50 converging, non-OECD economies with data available from 1995 to 2011, we calculate directly, from our measure of TFP discussed above, $\bar{\pi}$ and convergence parameter ψ. Using these country-specific convergence parameters, we estimate μ_0 and ρ_μ using the same simulated method of moments estimation

Figure 7: Average Current Account over GDP with Varying Initial Share of Extrapolation



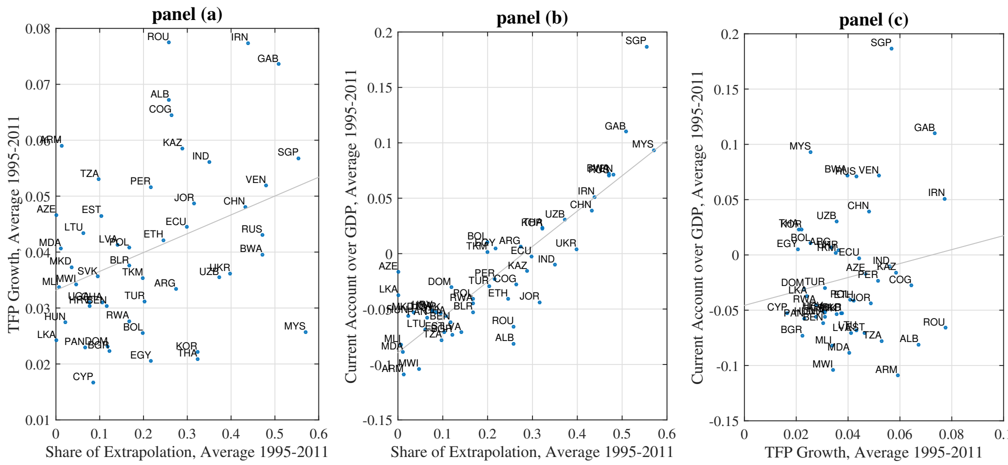
This figure depicts the average current account surplus over GDP for varying levels of initial extrapolation share. All other parameters are kept at values estimated for the Chinese economy listed in Table 1. For an initial distance to frontier given for China, there are increasing capital inflows in the initial extrapolation share. Thus, our distortion is positively related to capital inflows for a given extent of convergence. This graph results if we consider a vertical place from the x-axis in Figure 6 at the growth factor for China at 1.048. The positive relationship holds for all values of TFP growth factor.

as the baseline simulation introduced in Section 4.[15] A detailed discussion of this exercise is presented in Appendix B. The cross-country parameters used in the simulation as well as the estimated initial share of extrapolation for countries in our sample are shown in Table 3 in Appendix B.

Figure 8, shows the results of the cross-country simulation exercise. First, panel (a) shows a strong, positive relationship between the average extrapolation share in forecast and average productivity growth from 1995 to 2011. Countries that are growing faster during convergence, and therefore initially further away from balanced growth, tend to initially rely more on past experience when forming their expectations. The average share of extrapolation is

[15]We choose the sample period 1995-2011 for consistency with the Chinese experience, however, the results of this exercise also hold for the sample period 1990-2011.

Figure 8: Cross Country Evidence, 50 Converging Economies



Results of an estimated model with extrapolative expectations are shown for a sample of 50 developing economies that are converging to the world technological frontier. Panel (a) depicts a positive relationship between the average current account surpluses over GDP and average share of extrapolation in the forecast over the simulation period. Panel (b) shows a strong positive correlation between the average share of extrapolation and average TFP growth. Panel (c) depicts the empirical positive relationship between average current account surpluses over GDP and average TFP growth. The average share of extrapolation is taken as the longitudinal average of the share of extrapolation over the simulation period from Eq. (15).

calculated from simulated μ_t from Eq. (15). It is a function of both the initial share of extrapolation and the speed of convergence to rational expectations, $(1-\rho_\mu)$.

Panel (b) shows the empirical counterpart to Figure 6 using data across countries. Not only are higher extrapolation shares associated with higher TFP growth factors, but also greater capital outflows during convergence. Thus our mechanism appears to be able to explain, at least partially, the cross-sectional allocation of capital across converging economies.[16]

[16]Interestingly, the positive correlation between the extrapolation share and current accounts over GDP are not solely driven by current account surpluses, but also for those economy who exhibit smaller deficits than the neoclassical growth model with rational expectations would predict. This is consistent with Rothert (2016), who finds that a large portion of the allocation puzzle can be explained by the smaller magnitude of capital flows instead of the direction.

Finally, panel (c) reiterates our extended evidence of the negative relationship between capital inflows and productivity growth from our stylized facts for our sample of converging economies: there is a positive relationship between average current account surpluses over GDP and average TFP growth from 1995 to 2011.

5 Welfare Effects

How important is the excess saving that results during convergence for welfare? We compare the consumption equivalent changes in welfare during productivity convergence with rational expectations versus our model with learning.

The agent's productivity adjusted utility function is,

$$\tilde{U}_t = \sum_{j=0}^{\infty} \beta^j \left[ln(\tilde{c}_{t+j}) + \ln(\xi_{t+j}) \right], \tag{23}$$

where $\tilde{c}$ is productivity adjusted consumption.

We define productivity adjusted welfare, $\tilde{W}_t$, as

$$\tilde{W}_t \equiv \tilde{U}_t - \sum_{j=0}^{\infty} \beta^j \ln(\xi_{t+j}) = \sum_{j=0}^{\infty} \beta^j ln(\tilde{c}_{t+j}). \tag{24}$$

We may write the flow productivity welfare as,

$$\tilde{W}_t = \ln \tilde{c}_t + \beta \tilde{W}_{t+1}. \tag{25}$$

With our assumption of exogenous productivity growth, there is a steady state in levels for productivity adjusted consumption, $\tilde{c}$, which, solving Eq. (25) for the steady state $\tilde{c}$, is

$$\tilde{c} = \exp((1-\beta)\tilde{W}). \tag{26}$$

After the start of convergence, the value of $\tilde{W}_t$ represents the present discounted utility of productivity adjusted consumption during the whole transition. Therefore, by comparing the productivity adjusted consumption levels for the value of welfare in the rational expectations case and the learning case, we are able to evaluate the impact of varying expectation formation on the welfare of convergence.

With log preferences and same parameters taken in the quantitative exercise in the previous section, the presence of extrapolation in the forecast for the marginal utility of consumption reduces welfare to the equivalent of reducing consumption by about 3.5 percent–reducing consumption per effective unit of labor from 1.18 to 1.14.

6 Two Country World - Global Imbalances

In order to explore global imbalances in the context of our model, we recast the basic economy in terms of a two large economy model.

In the large open economy model, we assume two countries, Country A, a large industrialized country, and Country B, a large developing economy and analyze the interactions of these two economies during convergence to a balanced growth path. Country A is assumed to be at the frontier and Country B converges to the balanced growth path of the frontier. As such, the industrial economy grows at the long run rate of growth of the frontier, g^*. The developing economy catches up to the frontier and grows at a rate, $g_t > g^*$, during convergence.

Productivity evolves in Country A,

$$\xi_t^A = e^{g^*} \xi_{t-1}^A \tag{27}$$

and in Country B,

$$\xi_t^B = e^{g^*} \frac{\pi_t^B}{\pi_{t-1}^B} \xi_{t-1}^B. \tag{28}$$

The structure of the model is the same in each country apart from the following additional assumption.

Assumption 2 *Agents in an economy growing along the balanced growth path are fully rational.* $\mu_0^A = 0$.

As before, in the learning case, we assume that agents form their expectations based on a convex combination of extrapolative expectation and rational expectation of the marginal utility of consumption. Learning takes place during catch-up as agents learn to trust the true model of the economy. Given this assumption, the expectation formation for Country A at the frontier is purely derived from the true model of the economy and the learning case is present only in Country B.

The risk premium applies only to the case of Country B so that the interest rate at which Country A may borrow is r_t^A, whereas Country B's cost of borrowing is determined by a risk premium related to the level of debt over GDP relative to some average, $\bar{d}$,

$$r_t^B = r_t^A + \phi(e^{\frac{\bar{d}_t^B}{y_t^B} - \frac{\bar{d}}{y}} - 1). \tag{29}$$

Our choice of which country pays (receives) the positive (negative) premium does not affect the quantitative results in any calibrations.[17]

In addition, each country, $i = A, B$ has the production function,

$$y_t^i = k_t^{i\alpha}(\xi_t^i h_t^i)^{1-\alpha}. \tag{30}$$

Capital evolves in each economy according to,

$$k_{t+1}^i = i_t^i + (1-\delta)k_t^i \tag{31}$$

We assume that households maximize the same expected lifetime utility function as Eq. (1) subject to a budget constraint,

$$d_t^i = (1 + r_{t-1}^i)d_{t-1}^i + c_t^i + i_t^i - y_t. \tag{32}$$

[17] We have also tested whether rebating the premium back to Chinese households impacts our results. The impact is negligible.

Households in each country i choose c_t^i, d_t^i, k_{t+1}^i subject to Eq. (32), taking as given the path for $\bar{d}_t^B$ and given initial conditions, $k_0^A > 0$, $k_0^B > 0$ and d_0^A, d_0^B. The first order conditions for each country in the adaptive learning case are

$$c_t^{A^{-\gamma}} = \lambda_t^A \tag{33}$$
$$c_t^{B^{-\gamma}} = \lambda_t^B \tag{34}$$
$$\lambda_t^A = \beta(1 + r_t^A)E_t(\lambda_{t+1}) \tag{35}$$
$$\lambda_t^A = \beta\hat{E}_t[(\lambda_{t+1}^A)\alpha\tilde{k}_{t+1}^{A^{\alpha-1}} + (1-\delta)] \tag{36}$$
$$\lambda_t^B = \beta(1 + r_t^B)[\mu_t\lambda_{t-1}^B + (1-\mu_t)E_t\lambda_{t+1}^B] \tag{37}$$
$$\lambda_t^B = \beta\left[\mu_t\lambda_{t-1}^B(\alpha\tilde{k}_{t-1}^{B^{\alpha-1}} + 1 - \delta) + (1-\mu_t)E_t\left(\lambda_{t+1}^B(\alpha\tilde{k}_{t+1}^{B^{\alpha-1}} + 1 - \delta\right)\right] \tag{38}$$

To close the model, we impose a market clearing condition for international debt,

$$d_t^A = -d_t^B \tag{39}$$

since Country B can only borrow from abroad, the net foreign assets of country B must be equal to the net foreign liabilities of Country A.

As a consequence of the previous equation, the goods world market clearing condition holds:

$$c_t^A + c_t^B + i_t^A + i_t^B = y_t^A + y_t^B. \tag{40}$$

Eqs. (33)-(38) together with Eqs. (30), (31), (32), (29), (40), (39), (27) and (28) together with $k_0^A, k_0^B > 0$ and $d_0^A = d_0^B = \bar{d}$ describes our two-country world.

7 Global Imbalances

In our two country world, we simulate again a transition for the Chinese economy converging towards the productivity frontier. We perform an identical exercise to that in Section 4. The parameters used in the simulation exercise are listed in Table 2.

Table 2: Parameter Values for two-country simulation

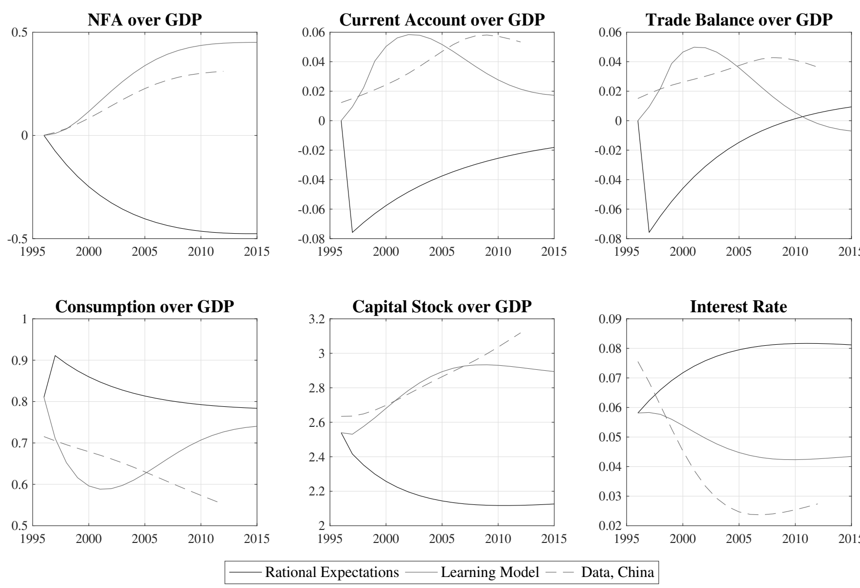
Parameters both Countries		
Depreciation Rate	δ	0.06
Capital share of income	α	0.30
Frontier growth rate	g^*	1.016
Discount factor	β	0.96
CRRA parameter	γ	1.00
Steady state debt/GDP ratio	$\bar{d}$	0
Parameters for China		
Final convergence parameter	$\bar{\pi}$	11.18
Speed of tech. convergence	ψ	0.0043
Initial share of extrapolation	μ_0	0.464

The interest rate along the balanced growth path is, $r^{US} = (e^{g^*})^{\gamma}/\beta - 1$. With frontier growth of 1.016, and assuming a subjective discount factor in the utility function, $\beta = 0.96$. This gives us an interest rate along the balanced growth path of 5.81 percent, which is consistent with slightly higher interest rates in the US around the start of our productivity convergence in 1995, which decrease (along with world interest rates) over time (Caballero et al., 2008a).

The main results for the US and China for this exercise are shown in Figures 17-18 in Appendix A. Figure 9 shows the main results for the simulation using Chinese data. The rational expectation model (the solid, black line) predicts an initial trade deficit over GDP as initially as China transitions to higher technology, consumption will increase above current output produced with the given capital stock. As the result of this debt-financed consumption in excess of output, China would be accumulating debt, generating a negative net foreign asset position.

If agents are tempted to rely more on their more recent experience in forming expectations of future marginal utility of consumption, a very different prediction materializes. A trade surplus emerges in the initial stages of convergence. Consumers tend not to trust the ability for future economic growth to enable them a sustained higher level of consumption and thus under-consume relative to output. This trade surplus persists for about 15 years before returning to a deficit, more consistent with rational expectation model.

Figure 9: Quantitative Results, Chinese Data

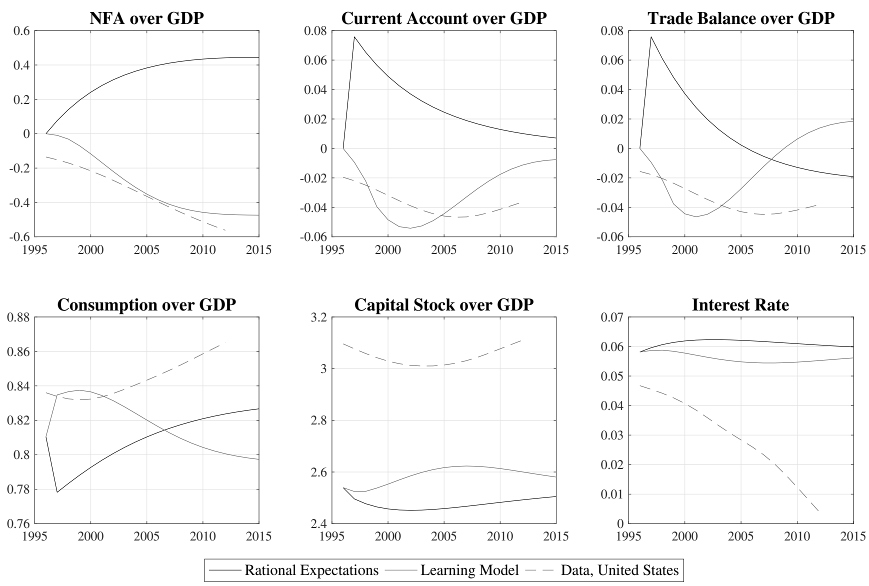


The solid, black line depicts the two large economy model results with rational expectations. The solid, gray line depicts the transition for the extrapolative expectations model. The simulated data are in levels. The dashed, gray line depicts data for China from 1995 to 2011. All data is filtered using Hodrick Prescott filter with smoothing parameter 100.

During initial convergence, excess savings is channeled abroad to US debt and Chinese net foreign assets over GDP expands quite significantly in excess of 45 percent of GDP after around 15 years of catch-up. Interestingly, the positive NFA position takes quite some time to unwind and Chinese net foreign assets remain positive well into transition.

The two-country model replicates well the size and evolution of NFA positions over GDP observed in the data–for both the Chinese and US cases shown in Figures 9 and 10, respectively. Our model offers an important prediction for global imbalances should China continue its transition: as consumers in China learn to trust the true model of technological convergence, the imbalances that have been at center stage of policy and academic debate since their emergence, will tend to disappear. China's consumers will learn to trust economic growth in the economy and imbalances will tend to diminish as they catch-up to rational expectations and the balanced growth path of frontier economies.

Figure 10: Quantitative Results, US data



The solid, black line depicts the two large economy model results with rational expectations. The solid, gray line depicts the transition for the extrapolative expectations model. The simulated data are in levels. The dashed, gray line depicts data for United States from 1995 to 2011. All data is filtered using Hodrick Prescott filter with smoothing parameter 100.

With the exception of capital stock over GDP in the US, our model replicates not only the current account to GDP ratio in China for which μ_0 and ρ_μ are estimated, but also other variables. Particularly noteworthy is the ability of the expectational channel to capture both the magnitude and time path of variables describing the Chinese economy during transition.

8 Discussion

In addition to the stylized facts that we can explain with our simple model, there are several larger implications from our findings. These are discussed in turn.

8.1 Savings and Investment Wedge Analysis

We perform a mapping of our novel distortion in agent's expectation formation process to a wedge in the agents savings and investment decisions. We calculate the wedge that results between optimal savings decision and our savings that varies over time.

Gourinchas and Jeanne (2013) perform a similar exercise, in which they introduce a savings wedge to the Euler equation in an otherwise standard neoclassical model in the form,

$$c_t{}^{-\gamma} = \beta(1+r_t)(1-\tau_{s,t})E_t(c_{t+1}{}^{-\gamma}). \tag{41}$$

This positive (negative) wedge essentially taxes (subsidizes) saving by altering the expected marginal utility of consumption.

Revisiting our agent's Euler equation, our distortion on expectation formation introduces a time-varying wedge that is a function of μ_t, and a backward looking component on the marginal product of capital,

$$c_t^{-\gamma} = \beta(1+r_t)(\mu_t c_{t-1}^{-\gamma} + (1-\mu_t)E_t(c_{t+1}^{-\gamma}). \tag{42}$$

In order to understand how our distortion might translate into a time varying wedge of the form in Eq. (41), we run our basic model convergence exercise to generate a series of consumptions $\{c_t\}_{t=0}^{\infty}$ and $\{\mu_t\}_{t=0}^{\infty}$, and use them to equalize Eqs. (41) and (42). This allows is to solve for the time-varying $\tau_{s,t}$ as,

$$\tau_{s,t} = \mu_t \left(1 - \frac{c_{t-1}^{-\gamma}}{E_t(c_{t+1}^{-\gamma})}\right). \tag{43}$$

Additionally, we introduce an investment distortion that drives a wedge between the social and private return to capital. We keep the form of this wedge similar to Gourinchas and Jeanne (2013) in the form,

$$c_t^{-\gamma} = \beta(1-\tau_{s,t})(1-\tau_{k,t})E_t(c_{t+1}^{-\gamma}(\alpha\tilde{k}_{t+1}^{\alpha-1}+1-\delta)). \tag{44}$$

Taking the savings wedge as given in each period, along with the basic model's sequences $\{c_t\}_{t=0}^{\infty}$, $\{\mu_t\}_{t=0}^{\infty}$, and $\{\tilde{k}_t\}_{t=0}^{\infty}$, we can calculate the investment wedges by setting Eq. (44) equal to the capital demand equation given in Eq. (17) from our model. The time-varying investment wedge that results for each t is

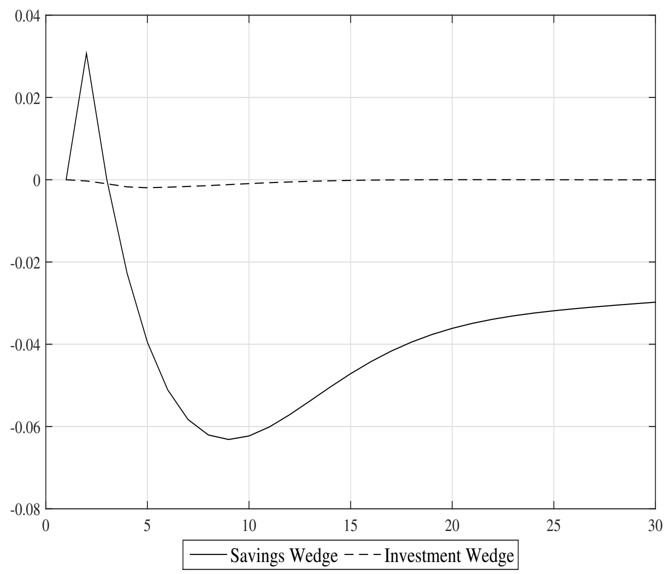
$$\tau_{k,t} = 1 - \frac{1}{(1-\tau_{s,t})}\left(1-\mu_t+\mu_t(\frac{c_{t-1}^{-\gamma}(\alpha\tilde{k}_{t-1}^{\alpha-1}+1-\delta)}{E_t(c_{t+1}^{-\gamma}(\alpha\tilde{k}_{t+1}^{\alpha-1}+1-\delta)}\right). \tag{45}$$

If we assume that the individual agents of this economy take sequences $\{\tau_{s,t}\}_{t=0}^{\infty}$ and $\{\tau_{k,t}\}_{t=0}^{\infty}$ as given when choosing their optimal plans, the equilibrium outcomes of the two models would be identical. The savings and investment wedges that result from this exercise are depicted in Figure 11.

Our model's consumption growth is mainly driven by a savings wedge. The time-varying saving wedge begins the simulation at zero, prior to the start of convergence. In the first period, it is positive at around 0.03 and quickly turns negative shortly after the start of convergence in the small open economy. The wedge stays negative, slowly tending towards zero with the extrapolative share of the forecast.

A positive wedge at the start of technological generates a less than one-to-one movement between the discounted future and current marginal utility of consumption. This is due to the fact that after the start of convergence, consumption actually falls relative to the initial consumption. The extrapolation bias introduced in our model generates a lower marginal utility of consumption during the first period of convergence and results in a positive $\tau_{s,t}$. Thereafter a negative $\tau_{s,t}$ indicates a subsidy on savings that operates by increasing the expected future marginal utility of consumption.

Figure 11: Saving and Investment Wedges, Small Open Economy Simulation



The solid, black line depicts the savings wedge in Eq. (43) and the dashed, black line depicts the investment wedge in Eq. (45).

In comparison to the savings wedge, the investment wedge is small in magnitude, and approaches zero after 15 periods from the start of convergence. Also slightly negative, the expectation formation tends to increase investment by creating a higher return on capital investments relative to the marginal product of labor. This stems from the fact that the expectation formation increases the expected future marginal product of labor with a lower initial capital stock (higher marginal product of capital).

8.2 Habit Formation

One might wonder if our results may be replicated using a more standard model feature of habit formation. Until our exercise, this point remained an open research question. In what follows, we include habit formation in the model and show the simulation results. As will become apparent, the inclusion of habit formation is not powerful enough to explain our stylized facts.

We keep the standard model introduced in Section 3, but include habit formation in a non-separable form into the preferences of households.

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t - \mu_2 c_{t-1})^{1-\gamma} - 1}{1-\gamma}, \tag{46}$$

where μ_2 governs the strength of habits and will ultimately slow the growth rate of consumption. We can see this from the consumption Euler equation,

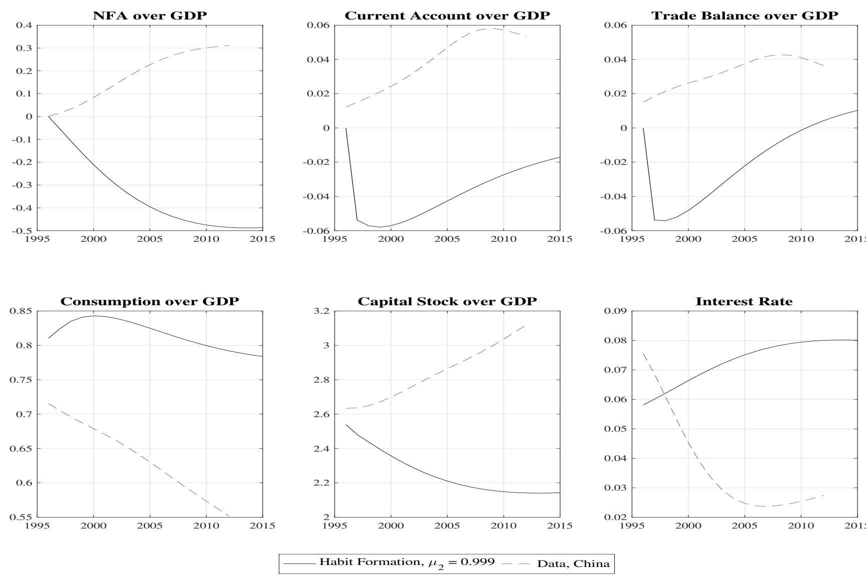
$$(c_t - \mu_2 c_{t-1})^{-\gamma} = \beta(1 + r_t)(c_{t+1} - \mu_2 c_t)^{-\gamma}. \tag{47}$$

An unconstrained household cares about the marginal utility derived from consumption levels and consumption growth. In a converging economy consumption growth will remain sticky due to the lower marginal utility that households derive from large shifts in consumption.

We test the suitability of the habit formation model to our data using the same estimation exercise as before, only with rational expectations assumption and habit formation. Our estimation exercise results in an intensity of habit parameter of $\mu_2 = 0.999$. The results of the quantitative exercise for carrying over the parameters from the initial exercise are shown for the small open economy and two large economy cases in Figures 12 and 19 in Appendix A, respectively.

Figure 12 shows the data for China in the dashed gray line and the model's prediction with habit formation in solid black line. What becomes immediately clear is that, despite the high intensity of habits, the model is unable to replicate our stylized facts. While habit formation is important and very successful in inducing persistence at business cycle frequencies, it does not allow model predictions to match systematic imbalances at lower frequencies, in which households rationally anticipate the effects of their consumption habits during the growth process. This suggests that learning plays a crucial role in understanding persistent imbalances in the growth and development process. Similar results hold for the two country model with habit formation. These

Figure 12: Quantitative Results Small Open Economy, Habit formation



The solid, black line depicts the small open economy model results with habit formation and rational expectations, with an estimated habit parameter, $\mu_2 = 0.999$. Simulated data are in levels. The gray, dashed line depicts data for China from 1995 to 2011. All data are in levels and filtered using Hodrick Prescott filter with smoothing parameter 100.

results are shown in Figure 19.

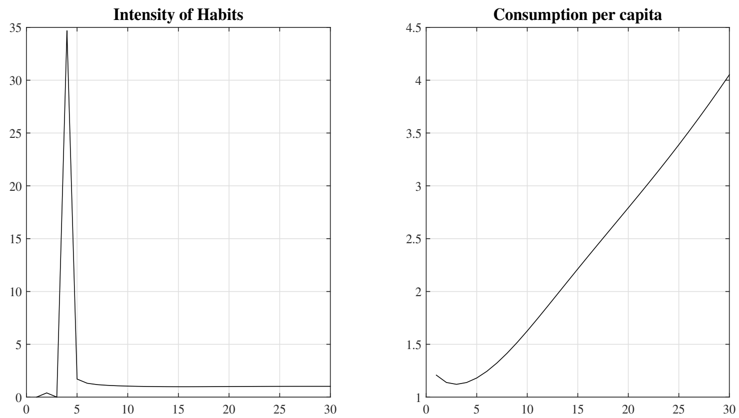
We next explore how our expectation distortion compares with the intensity of habits. Solving Eq. (47) for $\mu_{2,t}$, we use the consumption path from our simulation generated by Eq. (16) to calculate the following habit parameter that would replicate this simulated data,

$$\mu_{2,t} = \frac{(\beta(1+r_t))^{-1/\gamma} c_{t+1} - c_t}{(\beta(1+r_t))^{-1/\gamma} c_t - c_{t-1}}. \tag{48}$$

This time-varying intensity of habits parameter is that which justifies the simulated consumption growth in our model with extrapolation bias introduced in the expectation formation.

The path for the time varying habit parameter results from the non-

Figure 13: Time-varying Intensity of Habits



In the left panel the solid, black line depicts the intensity of habits that would replicate the consumption path generated by our model with extrapolative expectations for 30 periods after the start of technological convergence. The habit parameter is calculated from Eq. (48). The right panel depicts the path for consumption taken from our simulation of Eq. (16).

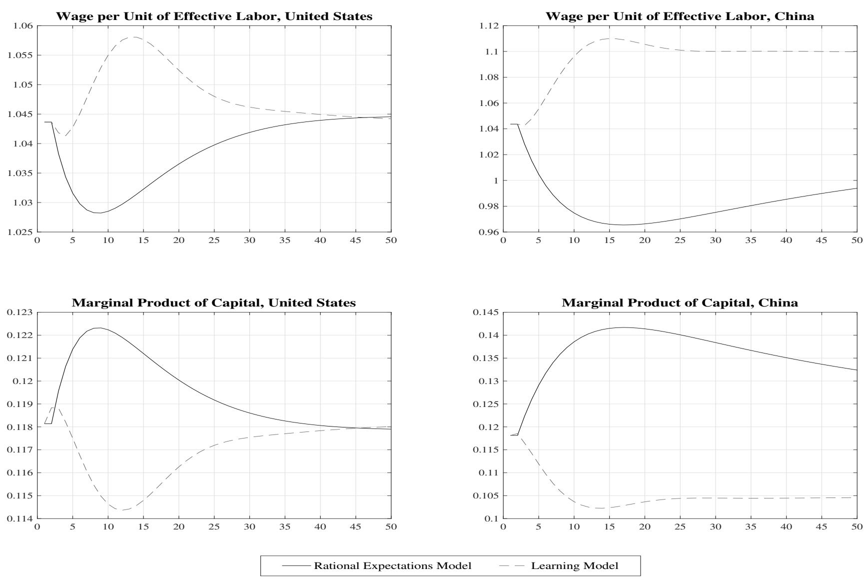
monotonic path for consumption shown in the second panel of Figure 13. The initial habit parameter is positive around 0.40 immediately after the start of technological convergence. This is due to a higher initial consumption prior to the start of convergence. The habit parameter becomes very large, around 35, in the fourth period due to consumption reaching a minimum after the start of convergence. Thereafter, the habit parameter stays very close but in excess of unity and slowly converges towards zero as the balanced growth path is attained.

8.3 Factor Income Distribution

Upon receiving news of convergence with rational expectations, even in an economy closed to world capital markets, households should rely on future economic growth to allow them to smooth consumption and therefore save less. Firms will increase investment, anticipating higher future marginal productivity. Therefore, the capital to labor ratio will determine the impact of convergence on the real wage.

In our baseline model of learning, both savings and investment will be in excess of their rational expectations counterparts at the start of convergence. If the savings channel dominates, the capital to labor ratio is higher in an economy converging to the frontier due to excess savings of households. The

Figure 14: Factor Income during Convergence

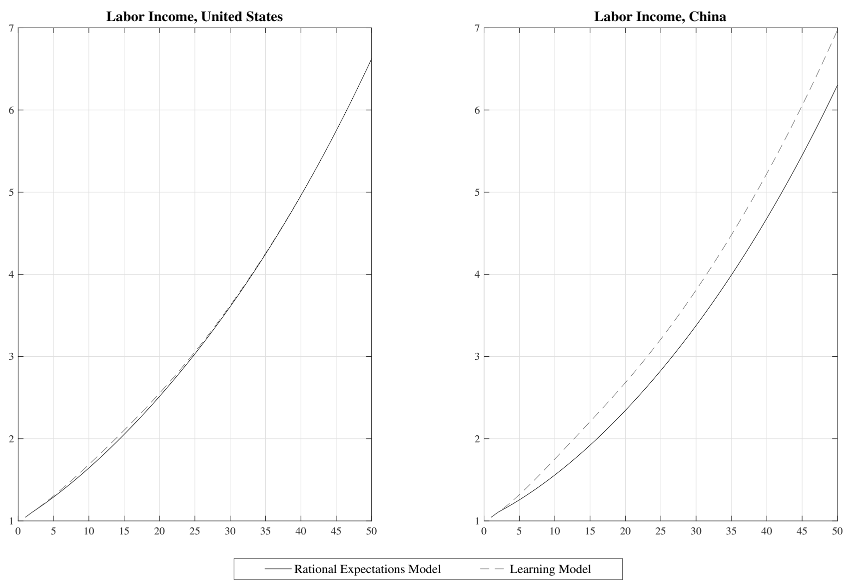


The solid, black line depicts the small open economy model predictions with rational expectations. The gray, dashed line shows the results for the small open economy model with extrapolative expectations. Wage per Unit of Effective Labor is calculated from Eq. (30) as $w_t = (1-\alpha)\tilde{y}_t$, where $\tilde{y}_t$ is output per effective unit of labor.

implications for factor incomes should be clear at this point. Real wages under rational expectations would have risen less in China since the 1990's than our model predicts. Therefore, the way expectations and learning takes place in converging countries has major implications for the distribution of factor income worldwide.

Figure 14 makes this point plain. It shows labor income per efficiency unit and the marginal product of capital for the US and China during the first 50 years after Chinese convergence. Given a common productivity scenario within each country for both model types, in the learning during convergence case, real labor income is larger with learning than with rational expectations. This has implications for the distribution of factor incomes in China during its transition experience.

Figure 15: Labor Income during Convergence



The solid, black line depicts the small open economy model predictions with rational expectations for labor income in the United States (left panel) and China (right panel). The gray, dashed line shows the results for the small open economy model with extrapolative expectations. Labor income is calculated from Eq. (4) as $w_t = (1 - \alpha) * y_t$, where $\tilde{y}_t$ is output per unit of labor. Given the trend growth for TFP, along the balanced growth path, once the economy converges, labor income will grow at a rate g^*.

What is also interesting is that in the converging economy, the differential effect of learning on real labor income seems to be quite large. Figure 15 depicts the real labor income in China and the US for first 50 years of convergence of the Chinese economy. In the Chinese case, real labor income is up to 10 percent higher with learning than under rational expectations. The differences are instead negligible in the frontier economy.

9 Conclusion

Technological catch-up entails a huge transformation in the converging countries' standards of living, often long before the country will have converged to a balanced growth path. During this sometimes dramatic transition, it is likely that agents in the economy fail to perfectly predict their and their offspring's future desire for consumption independently from their recent past experience. We therefore claim that a past of starvation or hardship will make a developing country's average household wish to save more for the future, simply because they may be incapable of imagining how close to satisfaction they and their children will be. This relative lack of self awareness–certainly conflicting with a textbook view of developing countries borrowing against an optimistic future–may also be justified by the higher complexity of rational expectations on an economy's transition towards a more stationary environment. As they get closer to a more balanced growth path, they will learn to trust the rational expectations view of the economy, and they will realize that so much saving is not necessary to guarantee an acceptable future.

This paper offers two simple theoretical and numerical examples of how the excess savings of a country catching up–both to the balanced growth path and to rational expectations modeling–could help to solve the allocation puzzle which characterizes many developing countries, and to contribute to explain important global imbalances, such as China's massive net foreign asset accumulation.

Quite remarkably in our two country world analysis, despite the US being assumed fully rational, the Chinese gradual catching up to fully rational expectations may even explain the US debt over accumulation. In this sense, our model gives an additional theoretical underpinning to the "savings glut hypothesis" formulated by Bernanke (2005).

It is important to remark that in order to make our point clear, we have operated under highly simplistic assumptions, most notably neglecting the important investment and saving frictions that plague developing countries (Gourinchas and Jeanne, 2013), as well as the process of financial development and exchange rate liberalization gradually taking place in a country such as China, highlighted by Caballero et al. (2008a,b) and Song et al. (2011, 2014). Hence we view our contribution as complementary to the analyses of these important issues.

References

Alfaro, L., S. Kalemli-Ozcan, and V. Volosovych (2014). Sovereigns, Upstream Capital Flows, And Global Imbalances. *Journal of the European Economic Association 12*(5), 1240–1284.

Bai, C.-E., C.-T. Hsieh, and Y. Qian (2006). The Return to Capital in China. Working Paper 12755, National Bureau of Economic Research.

Bernanke, B. S. (2005). The global saving glut and the U.S. current account deficit. Technical report.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2008a). An equilibrium model of global imbalances and low interest rates. *American Economic Review 98*(1), 358–93.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2008b). Financial crash, commodity prices, and global imbalances. Technical report.

Carroll, C., J. Overland, and D. Weil (2000). Saving and Growth with Habit Formation. *American Economic Review 90*(3), 341–355.

Carroll, C. D. and D. N. Weil (1993). Saving and Growth: A Reinterpretation. NBER Working Papers 4470, National Bureau of Economic Research, Inc.

Caselli, F. and J. Feyrer (2007). The Marginal Product of Capital. *The Quarterly Journal of Economics 122*(2), 535–568.

Coeurdacier, N., S. Guibaud, and K. Jin (2012). Credit Constraints and Growth in a Global Economy. CEPR Discussion Papers 9109, C.E.P.R. Discussion Papers.

De Grauwe, P. (2012). *Lectures on Behavioral Macroeconomics*. Princeton: Princeton University Press.

Evans, G. W. and S. Honkapohja (2001). *Learning and Expectations in Macroeconomics*. Princeton: Princeton University Press.

Feenstra, R. C., R. Inklaar, and M. Timmer (2013). The Next Generation of the Penn World Table. NBER Working Papers 19255, National Bureau of Economic Research, Inc.

Fuster, A., D. Laibson, and B. Mendel (2010). Natural expectations and macroeconomic fluctuations. *Journal of Economic Perspectives 24*(4), 67–84.

Gourinchas, P.-O. and O. Jeanne (2013). Capital Flows to Developing Countries: The Allocation Puzzle. *Review of Economic Studies 80*(4), 1484–1515.

Gruber, J. and S. Kamin (2009). Do differences in financial development explain the global pattern of current account imbalances? *Review of International Economics 17*(4), 667–688.

Kinoshita, N. (2006). Government debt and long-term interest rates.

Kumar, M. S. and E. Baldacci (2010, August). Fiscal Deficits, Public Debt, and Sovereign Bond Yields. IMF Working Papers 10/184, International Monetary Fund.

Lane, P. R. and G. M. Milesi-Ferretti (2007). The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970 to 2004. *Journal of International Economics 73*(2), 223 – 250.

Mehlum, H., R. Torvik, and S. Valente (2013, June). Chinaś Savings Multiplier. Working Papers 0013, Centre for Applied Macro- and Petroleum economics (CAMP), BI Norwegian Business School.

Mendoza, E. G., V. Quadrini, and J.-V. Rios-Rull (2007). Financial Integration, Financial Deepness and Global Imbalances. CEPR Discussion Papers 6149, C.E.P.R. Discussion Papers.

Obstfeld, M. and K. Rogoff (2009, December). Global Imbalances and the Financial Crisis: Products of Common Causes. CEPR Discussion Papers 7606, C.E.P.R. Discussion Papers.

Obstfeld, M. and K. S. Rogoff (1996). *Foundations of International Macroeconomics*, Volume 1 of *MIT Press Books*. The MIT Press.

Prasad, E., R. Rajan, and A. Subramanian (2007). Foreign capital and economic growth. NBER Working Papers 13619, National Bureau of Economic Research, Inc.

Rothert, J. (2016). On the savings wedge in international capital flows. *Economics Letters 145*, 126 – 129.

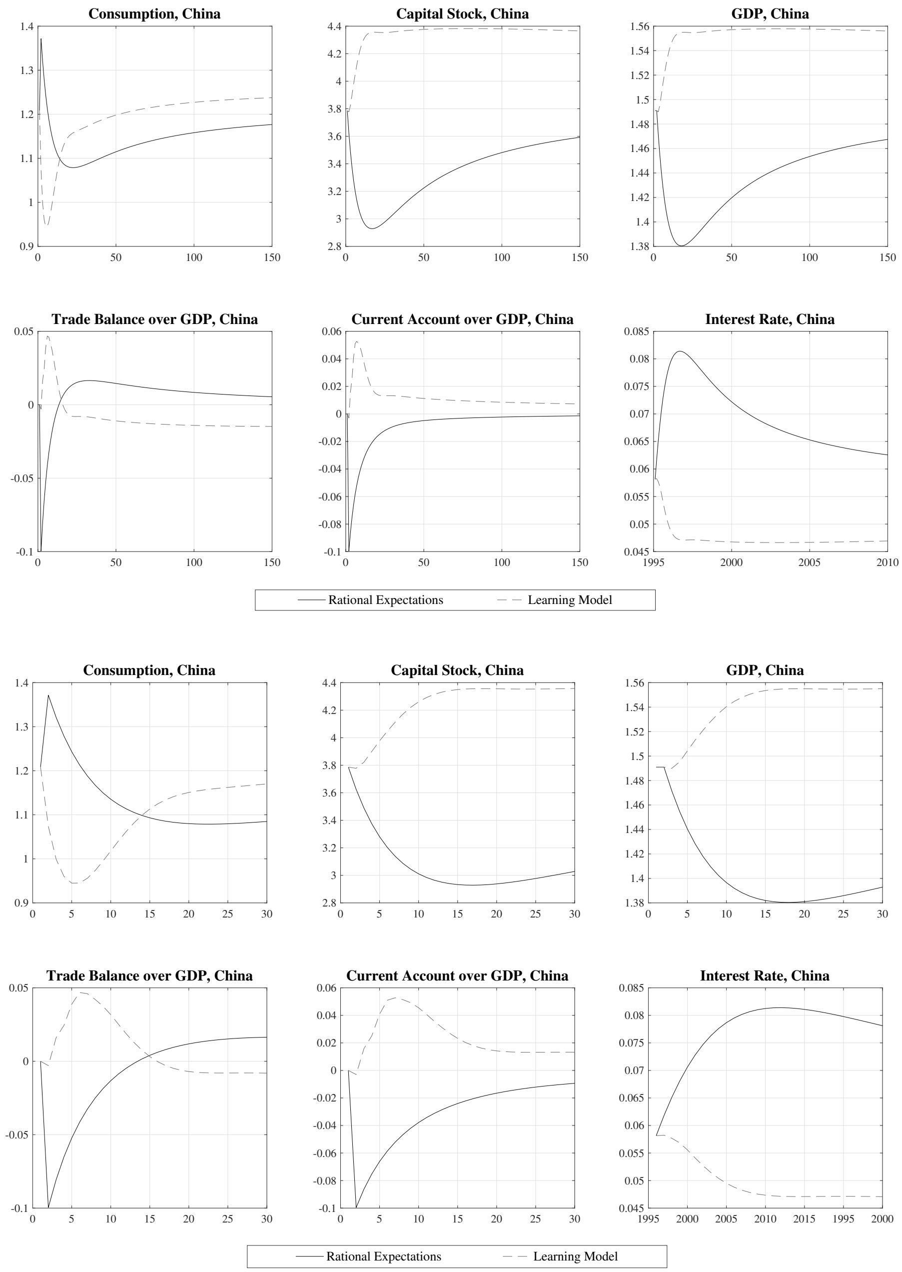
Schmitt-Grohe, S. and M. Uribe (2003). Closing small open economy models. *Journal of International Economics 61*(1), 163–185.

Song, Z., K. Storesletten, and F. Zilibotti (2011). Growing like china. *The American Economic Review 101*(1), 196–233.

Song, Z., K. Storesletten, and F. Zilibotti (2014). Growing (with Capital Controls) like China. *IMF Economic Review 62*(3), 327–370.

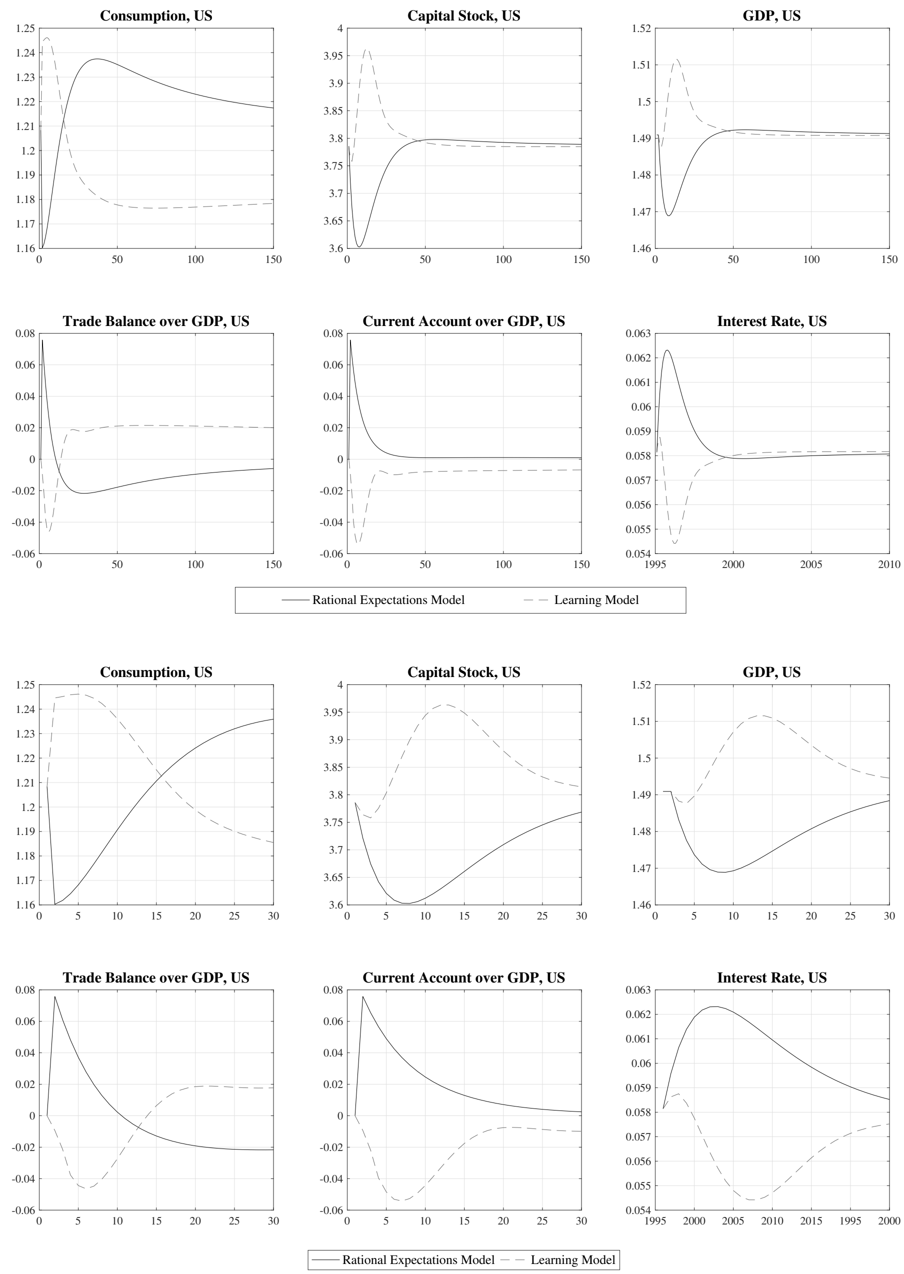
10 Appendix A

Figure 16: Small Open Economy Simulation of Productivity Convergence



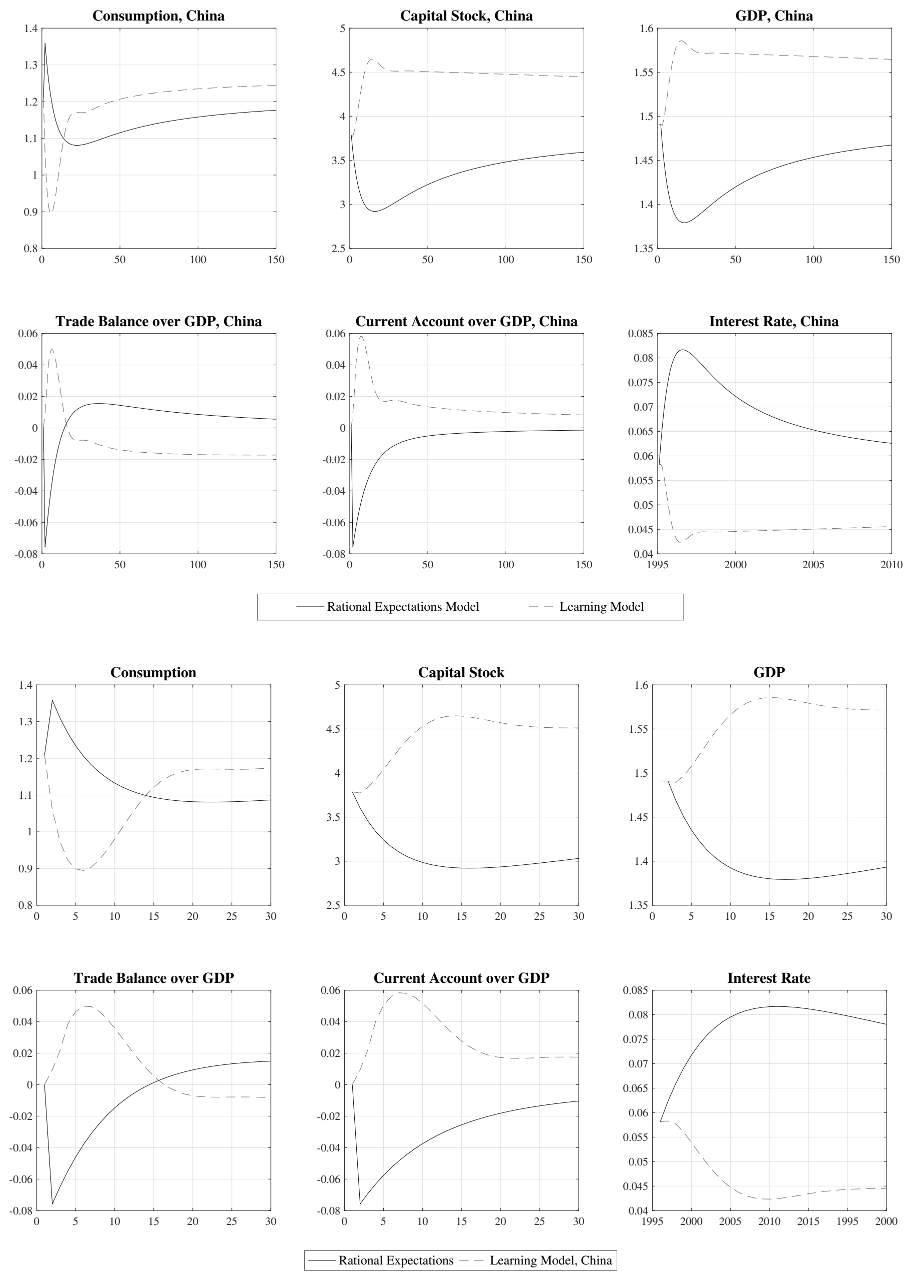
The top panel shows simulation results for 150 periods after the start of technological convergence. Consumption, Capital Stock and GDP are measured in efficiency units of labor. Trade Balance over GDP, Current Account over GDP and the Interest Rate are measured in levels. The lower panel depicts the various model predictions for the first 30 after convergence. The solid, black line depicts the small open economy model predictions with rational expectations. The gray, dashed line shows the results for the small open economy model with extrapolative expectations.

Figure 17: United States: Two Country Simulation of Productivity Convergence



The top panel shows simulation results for 150 periods after the start of technological convergence. Consumption, Capital Stock and GDP are measured in efficiency units of labor. Trade Balance over GDP, Current Account over GDP and the Interest Rate are measured in levels. The lower panel depicts the various model predictions for the first 30 after convergence. The solid, black line depicts the small open economy model predictions with rational expectations. The gray, dashed line shows the results for the small open economy model with extrapolative expectations.

Figure 18: China: Two Country Simulation of Productivity Convergence



The top panel shows simulation results for 150 periods after the start of technological convergence. Consumption, Capital Stock and GDP are measured in efficiency units of labor. Trade Balance over GDP, Current Account over GDP and the Interest Rate are measured in levels. The lower panel depicts the various model predictions for the first 30 after convergence. The solid, black line depicts the small open economy model predictions with rational expectations. The gray, dashed line shows the results for the small open economy model with extrapolative expectations.

Appendix B

In Section 8 we test the generalizability of our results to the convergence experience of other economies. Therefore, we conduct a model simulation exercises on a sample of converging economies.

From a total sample of 108 countries from PWT 8.1 with data available from 1995 to 2011, we calculate per employment output and capital stock, using (*emp*), real GDP (*rgdpo*) and capital stock (*rkna*). All variables are HP filtered with smoothing parameter $\lambda = 100$.

With these variables, TFP is calculated using Eq. (18). For each country, we calculate Average productivity growth from 1995 to 2011 using Eq. (19) and the convergence parameter, $\bar{\pi}^i$ from Eq. (22). Finally, the speed of technological convergence parameter, ψ^i is found using the solution to Eq. (8).

We start with the sample of developing economies from Gourinchas and Jeanne (2013). We include transitional economies of eastern Europe, with data available during our sample period.

We are interested in explaining convergence to a balance growth path from below. That is, we are interested in explaining capital flows for countries beginning from a level below that of the productivity frontier. As such, we exclude countries that are not converging during the period 1995 to 2011: that is with $\psi < 0$ or $\bar{\pi} < 1$.

Finally, we observe several countries that are very distant from the productivity frontier. As our model assumes perfectly functioning capital markets and no official role for government aid, we exclude economies exhibiting convergence parameters that exceed the 99th percentile. These countries are likely not to have access to international capital markets and receive a large portion of GDP in the form of official aid flows. This leaves us with a sample of 50 converging economies for which we carry out our simulation exercise.

We calculate the average capital inflows over the period 1995 to 2011 using data from Lane and Milesi-Ferretti (2007). Capital inflows are taken to be the negative of the current account measure in current US Dollars and GDP in current US Dollars. Both series are smoothed using an HP filter with smoothing parameter, $\lambda = 100$.

The resulting data as well as the estimation results for average extrapolation shares are listed below in Table 3.

Country	$\bar{\pi}$	ψ	g (TFP)	Avg. CA/Y	$\bar{\mu}$
Albania	8.64	0.011	0.067	-0.081	0.259
Argentina	3.42	0.009	0.033	0.007	0.273
Armenia	10.64	0.007	0.060	-0.109	0.013
Azerbaijan	5.60	0.009	0.047	-0.017	0
Belarus	3.79	0.010	0.038	-0.053	0.167
Benin	24.767	0.001	0.030	-0.062	0.117
Bolivia	10.22	0.001	0.026	0.010	0.199
Botswana	4.21	0.010	0.040	0.073	0.471
Bulgaria	3.624367	0.003	0.022	-0.073	0.122
China, People's Republic	11.18	0.004	0.048	0.039	0.430
Congo	18.1337	0.004	0.065	-0.028	0.264
Croatia	2.52	0.012	0.030	-0.051	0.077
Cyprus	1.80	0.001	0.017	-0.053	0.085
Dominican Republic	4.22	0.002	0.023	-0.030	0.119
Ecuador	8.19	0.005	0.044	-0.003	0.298
Egypt	4.09	0.002	0.021	0.005	0.218
Estonia	3.42	0.019	0.047	-0.070	0.103
Ethiopia	55.59	0.001	0.042	-0.041	0.245
Gabon	4.04	0.042	0.074	0.109	0.508
Ghana	15.08	0.001	0.031	-0.056	0.106
Hungary	2.67	0.008	0.027	-0.056	0.022
India	15.43	0.004	0.056	-0.010	0.350
Iran	5.20	0.032	0.077	0.051	0.439
Jordan	7.49	0.007	0.048	-0.044	0.316
Kazakhstan	5.71	0.015	0.059	-0.016	0.288
Latvia	3.65	0.013	0.041	-0.071	0.141
Lithuania	3.13	0.019	0.043	-0.069	0.062
Malawi	50.07	0.001	0.034	-0.104	0.046

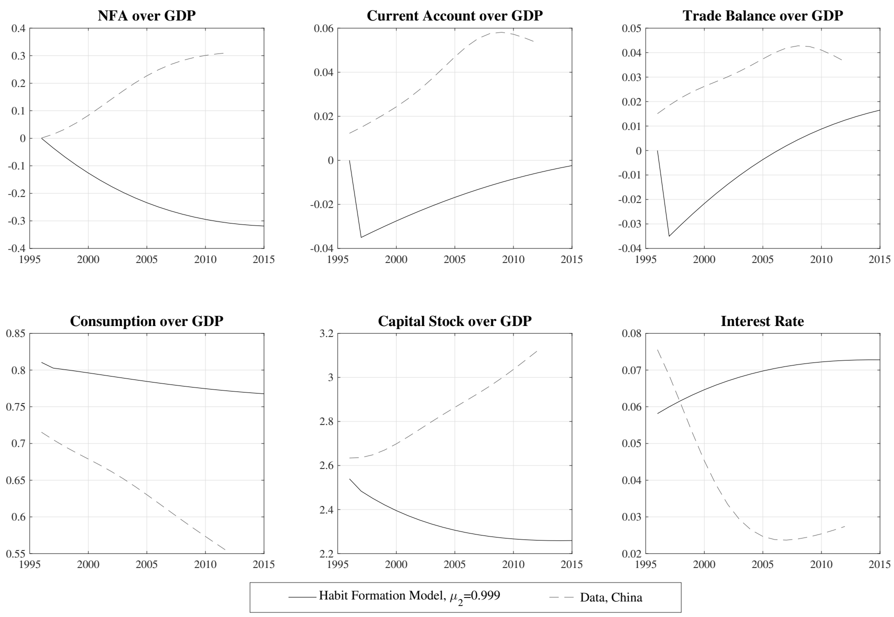
Malaysia	2.99	0.006	0.026	0.093	0.570
Mali	20.32	0.001	0.034	-0.082	0.006
Panama	2.51	0.005	0.023	-0.058	0.066
Peru	7.38	0.008	0.052	-0.023	0.216
Poland	3.00	0.018	0.041	-0.041	0.167
Republic of Korea	1.96	0.008	0.022	0.023	0.323
Republic of Moldova	12.83	0.003	0.041	-0.089	0.012
Romania	7.32	0.019	0.078	-0.066	0.259
Russian Federation	3.46	0.016	0.043	0.071	0.471
Rwanda	29.99	0.001	0.028	-0.045	0.167
Singapore	2.08	0.115	0.057	0.187	0.554
Slovakia	2.46	0.018	0.036	-0.053	0.095
Sri Lanka	5.27	0.002	0.024	-.0372	0.001
Thailand	6.49	0.001	0.021	0.023	0.323
Macedonia	4.04	0.009	0.037	-0.053	0.036
Turkey	1.86	0.024	0.031	-0.030	0.203
Turkmenistan	6.15	0.005	0.035	0.002	0.199
Uganda	25.33	0.001	0.031	-0.052	0.077
Ukraine	8.05	0.004	0.036	0.004	0.398
Tanzania	50.31	0.001	0.053	-0.078	0.098
Uzbekistan	6.08	0.005	0.035	0.030	0.372
Venezuela	4.37	0.016	0.052	0.072	0.480

Table 3: Variables for Cross Country Simulation

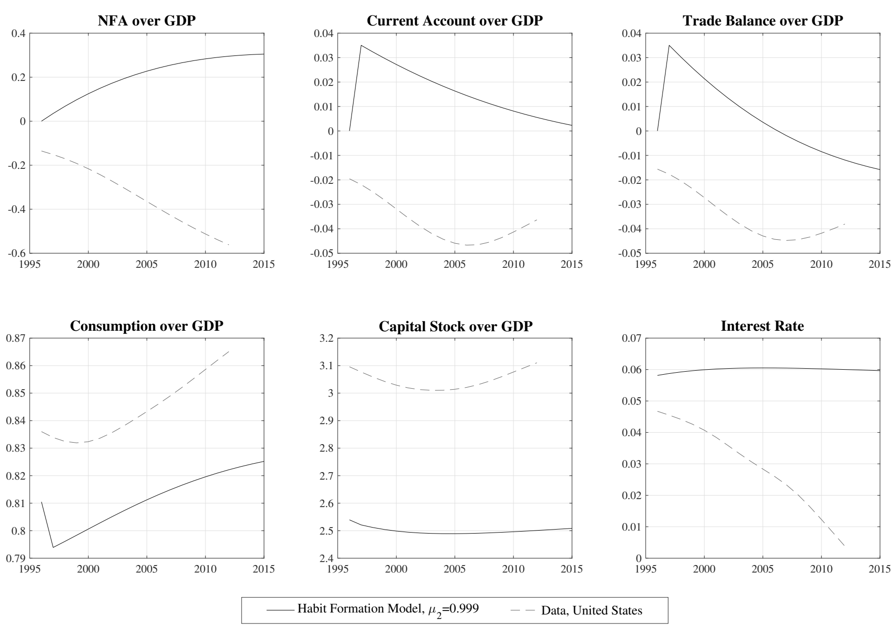
The table displays the key variables for our sample of 50 converging economies. The key convergence parameters calculated directly from the data are initial distance from frontier, $\bar{\pi}$, convergence speed parameter, ψ, growth rate of TFP, $\bar{g}$ and average current account over GDP from 1995 to 2011. The mean extrapolation share in the forecast, $\bar{\mu}$ is calculated directly from the simulated series for time varying extrapolation share, μ_t, using estimated parameters, μ_0 and ρ_μ, as mean share of extrapolation bias for the first 20 years after start of convergence.

Figure 19: Quantitative Results China and the US, model with habit formation

China



United States



Simulated variables are in levels. The solid, black line depicts the two economy model results with habit formation with a parameter of $\mu_2 = 0.999$ and rational expectations. The gray, dashed line shows data for China and the US. Data are in levels and smoothed using the Hodrick Prescott filter with parameter 100.