A Game-Theoretic Analysis of the Waterloo Campaign and Some Comments on the Analytic Narrative Project

Mongin, Philippe

CNRS HEC Paris, LEMMA (Université Panthéon-Assas Labex MMEDII)

2017

A GAME-THEORETIC ANALYSIS OF THE WATERLOO CAMPAIGN AND SOME COMMENTS ON THE ANALYTIC NARRATIVE PROJECT[*]

February 2017

Philippe MONGIN[**]

Abstract: The paper has a twofold aim. On the one hand, it provides what appears to be the first game-theoretic modelling of Napoléon's last campaign, which ended dramatically on 18 June 1815 at Waterloo. It is specifically concerned with the decision Napoléon made on 17 June 1815 to detach part of his army and send it against the Prussians, whom he had defeated, though not destroyed, on 16 June at Ligny. Military strategists and historians agree that this decision was crucial but disagree about whether it was rational. Hypothesizing a zero-sum game between Napoléon and Blücher, and computing its solution, we show that dividing his army could have been a cautious strategy on Napoléon's part, a conclusion which runs counter to the charges of misjudgment commonly heard since Clausewitz. On the other hand, the paper addresses some methodological issues relative to "analytic narratives". Some political scientists and economists who are both formally and historically minded have proposed to explain historical events in terms of properly mathematical game-theoretic models. We liken the present study to this "analytic narrative" methodology, which we defend against some of objections that it has aroused. Generalizing beyond the Waterloo case, we argue that military campaigns provide an especially good opportunity for testing this new methodology.

Keywords: Napoléon, Waterloo, military history, rational choice theories, game theory, zero-sum two-person games, analytical narrative
JEL Classification Numbers: N43, C72, B49

1. Motivation and overview

First deployed in economics, the mathematical theories of rational choice – individual decision theory, game theory, social choice theory – have spread abundantly beyond their original realm. Certain schools of sociology and political science have aligned themselves with the use of such theoretical devices, sparking controversies that have become academic *topoi* in their respective disciplines. By contrast, history has offered strong resistance. Historians have not simply been unwilling to make use of these mathematical theories, they have been disinclined even to consider their potential, even to dismiss it. And this despite efforts that have been made to shake them out of their indifference. Both economics and political science have historically oriented subsections, which attempt to model historical events using the mathematical theories in question, with game theory generally occupying center stage. A major example is the game-theoretic work long pursued among political scientists as a means to analyze pivotal events in international relations, such as the outbreak of World War I or the 1962 Cuban missile crisis.[1] More recently, a group of scholars, again from political science and economics (Bates, Greif, Levi, Rosenthal and Weingast, 1998), have proposed an original reconciliation of the narrative form of history with the modeling form permitted by mathematical theories of rational choice. For this reconciliation, they propose the surprising and apparently oxymoronic tag "analytic narratives", which also provides the title for their book. Their five cases are drawn from institutional history: international regulation of the coffee trade in the 20th Century (Bates), municipal conflicts in medieval Genoa, conscription laws in 19th-Century Europe (Greif), tax systems in prerevolutionary Europe (Rosenthal), and the entry of new states to the American federation (Weingast). These cases represent challenges to economics and political science as well as to history, but one can detach what specifically concerns the latter.[2] In brief, the authors think of some historical events

---

[1] The 1914 crisis has been explored in game-theoretic detail by Zagare (2011). Regarding the Cuban crisis, the classic study by Allison (1971) was quickly followed by more advanced game-theoretic studies (one of the first being by Brams, 1975, ch. 1). Of lesser historical relevance are the many game-theoretic pieces written on deterrence in general, as in Schelling (1960) and followers. See O'Neill (1994) for early references along both lines.

[2] Bates et al. (1998) push analytic narratives in two directions at once, i.e., historical explanation and the neo-institutionalist analysis of institutional change (along the same lines as North, 1990). Here we interpret them only in the former sense, which the authors' own division of the issues permits (see their methodological introduction). Zagare (2011, 2015) and Brams (2011) understand the expression "analytic narratives" in the same way.

as raising interpretative and explanatory problems that cannot be resolved by narratives in classical form, and thus call for the importing of models – in their case, from the theory of games under extensive form. "Analytic narrative" thus refers to the change in the narrative genre that results from this importation. The proposed method cannot be said to be entirely new, given the aforementioned work in international relations, but the authors provide a clearer account of it than their predecessors.[3]

It is indeed an understatement to say that historians have expressed little interest in the proposal. In our experience, few are even aware of this brave attempt by rational choice modelers at bridging the gap with them.[4] This is all the more distressing since the illustrative studies carefully avoid irrelevant technicalities and make good reading. Although initiated independently of the analytic narrative project, the present work offers a partial answer to this communication problem. We also explore a specific historical event, not generalities; we adopt the formal mode of rational choice theory (more specifically game theory: although not games in extensive form); and finally we propose that narratives be reconciled with models. Given all these features, our work belongs to the analytic narrative genre. At the same time, it departs from the precedent set by Bates et al. by its choice of historical case. Ours is much more limited in scope, and avowedly less innovative than theirs; but we will try to turn this apparent lack of ambition into an asset, seeking to show that the gap with historians can be bridged more easily with studies of this kind.

Our historical case is Napoléon's last campaign in June 1815, which he eventually lost to Wellington and Blücher on the battlefield of Waterloo. One reason we had for turning to military studies is that they have often served as a touchstone for rational choice explanations. For instance, Pareto (1917-1919, §152) classes them alongside economic and technological studies as those which embody his concept of "logical action". Within military studies overall, the account of military campaigns appears to be the most amenable to simple rational choice explanations: for example Weber (1922a, p. 10; Eng.

---

[3] Besides the 1998 introduction, see Bates et al. (2000) and the further elucidations in Levi (2002, 2004).
[4] A rare counterexample is the review published in *History and Theory* by Downing (2000).

ed. p. 21) had this branch in mind when he illustrated his "instrumental rationality" ideal-type by Moltke's and Benedek's cogitations before the Sadowa battle. Even more clearly, since Jomini and Clausewitz founded the genre in the 19th Century, the authors of campaign narratives themselves have given substance to the view that military campaigns have a special susceptibility to rational choice explanation.

Capitalizing on such a well-understood connection, the analytic narrative of a military campaign could hope to relate to the preexisting body of historical work more easily than analytic narratives with more complex historical subjects. As a supporting argument for our choice of topic, some of the technical concepts of game theory – beginning with that of the strategy – obviously relate to the informal concepts used by the actors and narrators themselves. Admittedly, von Neumann and Morgenstern (1944) themselves did not pay attention to military affairs,[5] but their immediate post-war followers at the RAND Corporation and in US military organizations certainly did.[6] These followers initiated the long-lasting alliance between game theory and deterrence studies. Yet concerning war itself, rather than the ways of avoiding it, applications were surprisingly scarce. The most relevant one is a little-known application by Haywood (1950, 1954), a US general who analyzed two strategic decisions made in World War II in terms of von Neumann and Morgenstern's zero-sum two-person games with mixed strategies.[7] Our model will also resort to this basic tool.

Applications like these pay for their didactic facility with a clear disadvantage: they do not have the same demonstrative consequences as if they had taken on less tractable topics like medieval Genoa or the finances of prerevolutionary France.[8] For the present work, so be it. We will more easily renew the debate on analytic narratives, and in particular draw in historians, if our chosen starting point is relatively consensual. At the end of the day, we would be satisfied if we also achieved consensus on the bare

---

[5] Von Neumann and Morgenstern took their basic examples from parlor games; see Leonard (2010).
[6] See, e.g., Erikson et al. (2013).
[7] The two decisions analyzed by Haywood belong to the US Pacific campaign and the US Normandy campaign, respectively. Despite Brams's efforts (1975, ch. 1), which breathed new life into Haywood's work, this interesting precursor has remained virtually unnoticed.
[8] This objection was put to us by Margaret Levi; we try to answer it here and later in the text.

4

existential point that there is at least one class of indisputable applications of game theory to historical events, however unsurprising this class may seem.

Within the class of military campaigns, our specific case is itself scarcely original. An old chestnut from the strategy courses of military staff colleges in the 19th and early 20th Centuries, Napoléon's 1815 campaign has remained a subject of inexhaustible fascination for war historians up to the present. The rich bibliography in three major languages is certainly one attraction of the case, but an even stronger one is that, despite the availability of so much evidence, historians have been unable to come to agreement on why Napoléon experienced such a stupendous disaster. More precisely, what they disagree on is the *rationality* of this prominent actor. The role of game-theoretic modeling will thus be to illuminate and perhaps to arbitrate this disagreement. The issue goes back to an account the overthrown emperor dictated to his companions in exile on Sainte-Hélène, which was a plea *pro domo*. Among the texts which record this account, we have selected *Mémorial de Sainte-Hélène* by Las Cases, which is the most widely distributed and the most succinct.[9] Napoléon lays the blame for his defeat with marshals Grouchy and Ney, who he claims misjudged their strategic possibilities and did not properly follow his instructions. Clausewitz, however, who had access to the *Mémorial* as well as further French and German sources, reached the opposing conclusion that Napoléon should not be exculpated. The first genuine scholar of the Waterloo campaign, Clausewitz is also a passionate critic of Napoléon's handling of it. With various nuances, his position has carried the day; but the imperial argument, long upheld by French military writers, has not disappeared altogether. One finds it even today, and not only within France, endorsed by authors with sufficient credentials that it seems no less worth considering than the dominant view. So historians are in a deadlock, and after so much time there seems to be little hope that progress will be made – unless one radically changes perspective. This is how we defend the shift to an analytic narrative.

---

[9] Las Cases includes "Relation de la campagne de Waterloo, dictée par Napoléon" in *Mémorial de Sainte-Hélène* under the date of 26 August 1816. The other reports are Gourgaud's *La campagne de 1815* and Bertrand's *Cahiers de Sainte-Hélène*. The last work was published long after its author's death and played no role in the Waterloo controversy, unlike the first two which came out in 1823 and 1818 respectively.

Clausewitz's interpretation is to be found in the monograph *The Campaign of 1815 in France*, which his famous treatise *On War* has regrettably overshadowed.[10] In it, one can find an anticipated use of Weber's principle of instrumental rationality (henceforth, we say simply "the rationality principle"). By contrast, the concepts of ends and means that direct the classic definitions of war in the treatise are part of a more abstract teleology, which unlike the Weberian concept is detached from acting individuals. A great merit of the monograph is that, while following the principle of rationality throughout, it now and then surpasses the level of informality at which historians normally stop, suggesting *models* in the sense relevant here. We will in fact rebut Clausewitz's substantive interpretation of Waterloo, but praise his method, registering him as a definite precursor of the analytic narrative genre. It is a secondary contribution of the present paper to highlight an aspect of his work that does not seem to have been noticed.

The rest of the paper develops as follows. Section 2 reviews the main facts and interpretations of the Waterloo campaign, emphasizing those which matter for the model to follow. It will touch only lightly on the tactical aspects of the battles, and focus rather on the overall strategy pursued by Napoléon during the three days from 16 to 18 June 1815. This section intentionally reproduces the standard narrative mode of military historians.

Section 3 changes tone, proposing a model for Napoléon's crucial decision on 17 June 1815, the day after his victory over Blücher on 16 June at Ligny. That day he chose to send more than a third of his forces, under the command of Grouchy, against the retreating Prussians. All the historians agree that this division of the French army was the key to Wellington's victory on 18 June at Waterloo. Grouchy spent the fateful day at Wavre, baited by Blücher's rear guard, while the advance guard marched unimpeded to join Wellington in the midst of a still uncertain battle. The greatest question of the campaign, which involves Napoléon's rationality, is whether he could have made better

---

[10] *Der Feldzug von 1815 in Frankreich.* Posthumous like the others, this work appeared in 1835 in the *Hinterlassene Werke* edited by Marie von Clausewitz; it was written in 1827. Clausewitz's commentators do not spend much time on his campaign narratives. Aron (1976), for example, hardly mentions them, while Paret (1992, ch.9) is somewhat derogatory about them.

use of Grouchy's detachment. The model we propose to answer this question takes the form of a zero-sum game between Napoléon and Blücher. Despite the absence of Grouchy as an autonomous player, it adds precision to the competing hypotheses. In the end, we will side with the proNapoleonic minority against the antiNapoleonic majority led by Clausewitz. In favoring this conclusion, we exemplify the connection between rational choice theories and the *charity principle*, which requires considering an agent's reasons for an action in their strongest and most plausible form before evaluating them.[11]

Section 4 returns to analytic narratives, first summarizing the objections raised against them, and then examining how our model could possibly withstand them. At the same time, we take up the argument sketched in this introduction that the analytic narrative methodology can fruitfully be applied to military events, and particularly campaigns. The section will argue that this is a plausible application because some standard assumptions of formal rational choice theories appear to be satisfied in this context, and there is a continuum of rationalizations between the spontaneous ones evinced by the actors and those analytically developed by the modelers. Section 5 concludes, adding a brief comparison between analytic narratives and cliometrics.

2. The Waterloo Campaign: main facts and interpretations

In the spring of 1815, a coalition of the European powers was solidifying against France. Napoléon needed to annihilate the two armies already mounted – the English and the Prussian – as quickly as possible. Against Wellington's 93,000 Anglo-Dutch soldiers, who were preparing to meet Blücher's 118,000 Prussians in Belgium before invading France, Napoléon had only the 124,000 troops of the Armée du nord; his other forces covered the Rhine or garrisoned fortresses. The only way out was to reproduce his masterstroke from the Italian campaign: first defeat one army, then the other. All historians recognize this plan, and most of them, including Clausewitz, hold that it was

---

[11] Davidson (1980) is famous for emphasizing this connection.

the only one conceivable.[12] At first, the execution seemed promising. With his customary swiftness, Napoléon entered Charleroi on 15 June, forcing the Prussian advance guard to pull back northeast of the city. The allies had not yet joined forces, and each of them was in a rather unsatisfactory state. The Anglo-Dutch were widely spread out around Brussels and westward, as Wellington wanted at all costs to maintain communications with Ostend in that direction.[13] And Blücher was headquartered in Sombreffe, some 12 kilometers northeast of Charleroi, with only three corps; a fourth, commanded by Bülow, kept the rear guard and was useless for battle. In taking this forward position Blücher ran the risk of confronting Napoléon with insufficient forces. However, his decision becomes clearer in the light of the agreement he had reached with Wellington on May 3, to the effect that the allies would meet on the Quatre-Bras-Sombreffe line in the case of an offensive by Napoléon. This strategy ran afoul of the classical precept of maximal grouping before the engagement, but Blücher apparently believed that Wellington would join him in the heat of the battle.

The Prussians had occupied the hamlet of Ligny, which gave its name to the battle that they ended up fighting there alone over the afternoon and early evening of 16 June. Less famous than that of 18 June, this battle actually determined the succeeding chain of events, and it is with regards to its interpretation that the main hypotheses square off. The *Campaign*, to quote but one account, gives more space and emphasis to Ligny than it does to Waterloo.

Napoléon immediately recognized that Blücher's risky strategy offered him an opportunity to carry out his campaign plan. He won on 16 June following the two standard criteria of victory: lose fewer men than the adversary, and conquer the terrain of the battlefield. While very real, this victory was not yet decisive. Blücher managed to save most of his forces, some 90,000 men, in sufficient order to bring them back to his rear guard. So Blücher's initial error – leaving Bülow in reserve – would eventually turn

---

[12] *La campagne de France en 1815*, tr. Niessel, 1973, p. 37-43. From now on, all page references to Clausewitz are to his monograph and this French version, from which we translated the quotations.

[13] Hofschröer (1998-1999) stresses that Wellington had weakened himself in order to prepare for an attack from the west, which there was little reason to expect. The Duke had already faced the charge in his reaction to Clausewitz; see Bassford (1994, p. 42-45, and 2001 for a transcript of Wellington's comments).

to his and Wellington's advantage. In the *Mémorial*, Napoléon implies that the three Prussian corps engaged at Ligny escaped destruction through Ney's fault.[14] In fact, he had sent the marshal away in the north-west direction with the principal objective of holding the road from Charleroi to Brussels, which Wellington would have to use if he came in support of Blücher. Ney's group – about 25,000 men under his direct command – had the option of either attacking the Anglo-Dutch, or simply holding them back while taking on the Prussians from behind. The *Mémorial* mentions both tasks at once, which was probably too much to ask of poor Ney. At the field of Quatre-Bras, where he met the English forward guard, he carried out the former slowly and awkwardly, not even considering the latter. The corps of Drouet d'Erlon – 20,000 more men – was to come to Ligny or Quatre-Bras in case of need, but wandered pitifully from field to field without engaging; many have seen this as a turning point in the campaign.

Clausewitz defends Ney by arguing that the successive orders that Soult, the campaign's chief of staff, sent him in the name of the Emperor were incompatible. This analysis, which we will not develop here, brings out the rationality principle most clearly: "Ney absolutely completed his goals – to block the aid of Wellington. Bonaparte did not come to the idea of having him cooperate in the battle of Ligny until later, after having recognized Blücher's position…. Only today can we see [what Ney could have done], by bringing into our calculations all the fortuitous circumstances that could not be foreseen at the time" (Clausewitz, p. 105). Weber would do no better than this in distinguishing between *objective* rationality, which can be defined only by the retrospective observer, and *subjective* rationality, which is the only one accessible to the actors, and hence the only one pertinent for explaining their actions.[15]

Starting a movement that would turn out to be decisive, the Prussians did not back up along their natural line of communication, which was the Meuse river valley, but went farther northward, in the general direction of Louvain. They regrouped over the course of June at Wavre, a town situated on the river Dyle, mid-way between Ligny and Louvain.

---

[14] *Mémorial de Sainte-Hélène*, Garnier reprint, p. 237 (all page references to this edition).
[15] Cf. Weber (1922b, p. 435-439). The distinction between objective and subjective rationality has since become established; see, e.g., Popper's (1967) classic restatement.

This location allowed them to keep as many options open as possible. From there, Blücher could either organize a definitive retreat by reaching Liège by way of Louvain, or rejoin Wellington, who was a single day's march away. On the same day, Napoléon chose to separate his right wing, of some 30,000 men, which Grouchy had commanded at Ligny. With this detachment, the marshal could either mount a savage pursuit against the Prussians' rear, without worrying about what became of the rest of their army, or keep the Prussian army as a whole from meeting the Anglo-Dutch; or carry out both these two objectives of pursuit and blocking to the extent that they were compatible.

What actually occurred is that Grouchy set off after the Prussians, who intentionally slowed one of their corps, led by Thielemann. On 18 June, the marshal joined battle at Wavre against just this rear guard. Meanwhile, the advance guard, with Bülow and Pirch, marched unobstructed to Waterloo, and ploughed into the French right on the afternoon, early enough to help Wellington who was not in an easy position.[16] Having missed their chance the first time because of Wellington's inertia, the allies succeeded in concentrating their forces the second time thanks to Blücher's recovery. It is unlikely that Grouchy would have brought the French victory in a Waterloo battle that included Bülow and Pirch, but if he had been there instead of them, he would have given Napoléon the numerical advantage needed to defeat Wellington. As it was, each commander fielded 70,000 men, and equality favored Wellington, who had chosen to fight from a strong defensive position, as he had done to his advantage so often before.

As already indicated, a major problem of the campaign is to decide *what Napoléon intended to achieve with Grouchy's detachment*. And this is closely connected with another problem, which is to decide *how Napoléon interpreted the battle of Ligny*. To what extent did he overestimate the extent of his victory, and misjudge the direction of their retreat? Clausewitz (p. 107-109 and 146-148) claims that he made mistakes on both counts, and this has dominated the literature ever since. Let us review the evidence which bears on the two questions.

---

[16] Although Thielemann finally had to surrender Wavre, he had fulfilled his role by holding back the enemy for half a day. Clausewitz, then a colonel, served as his chief of staff.

Napoléon's initial orders to Grouchy on 17 June were oral, and neither the *Mémorial* nor the marshal's *Mémoires* are reliable enough to enable a reconstruction of them.[17] Clausewitz, in his chapter XXXVII, claims that Napoléon entrusted Grouchy with *a simple mission of pursuit*. This, together with an error that Napoléon made concerning the direction of the Prussian retreat, would cleanse the marshal of all responsibility for the next day's rout – busy in the east, Grouchy could not at the same time lend a hand to Napoléon. The conclusion seems inescapable if one accepts the premises; but Clausewitz has no more than hints to establish those premises, and to this day his supporters have not substantially improved the argument.[18] As he had done with Ney, Napoléon charges Grouchy with responsibility for the defeat, claiming in the *Mémorial* (p. 245) that he should have been on the Waterloo battleground. French military writers have often taken this position, while softening it with additional reproaches against Napoléon, and still more against his chief of staff Soult.[19] A good deal of this is transparent apologetics; however, there are also historians without any nationalist stake, such as the 20th-Century British general Fuller (1951-1956, ch. 18), who conclude that Grouchy ended up in a place he should not have been.[20] None in the present group of commentators accept Clausewitz's narrow interpretation of the orders of 17 June. By their reading, unsure of whether the Prussians had been truly beaten, Napoléon would have asked Grouchy to protect him from any possible intrusion by them into his next battle. He would thus have entrusted Grouchy with a *role of blocking or interposing at the same time as pursuing*. This wider interpretation obviates the problem of what precisely Napoléon thought of the direction of the Prussian retreat. There were two possibilities: either the entire Prussian force had moved east, in which case the chase would also serve as interposition; or else the enemy was dispersed, with some forces taking the dangerous way to the west, in which case *Grouchy should prioritize the objective of blocking over that of pursuit*. This

---

[17] Compiled by his descendants, Grouchy's *Mémoires* discuss these instructions at length, but the effort at exculpation is so blatant that it is impossible to take them seriously.

[18] Even the careful study by Hofschröer (1998-1999) falls far from making Clausewitz's case compelling.

[19] Mauduit (1847) eloquently illustrates the beginning of this line of interpretation, the first of many to blame the weakness of Soult and the staff in general.

[20] Here Fuller joins forces with Houssaye (1905-1906), a classic of the French rehabilitation literature.

line has no more solid proof than the other. What is known of the 17 June does not permit a clear winner in this interpretative contest.

The first written communication that follows the oral commands of 17 June is a letter dictated to Bertrand, received by Grouchy shortly after he set off. Both interpretations can find something in it, the first because it sends the marshal towards Gembloux, i.e., to the east, and, even worse, towards Namur, which distanced him from the Prussians; and the second because it directs him to report on Blücher's maneuvers and his possible intention to join Wellington.[21] From Gembloux, where he did not arrive before late evening, Grouchy replied to Napoléon with a revealing dispatch. This shows that he had at last understood that Wavre was one of the Prussian destinations, but not yet that it was the only one. Grouchy also brings up the possibility of an enemy movement towards Wellington, and adds that he would try to prevent it from occurring, which lends some support to the view that the conversation of 17 June had suggested interposition as a goal.[22] Although the marshal's letter arrived at 2:00 in the morning, the staff's reply was not sent before 10:00, in which we can see definite evidence of ill-functioning. On behalf of Napoléon, Soult commanded Grouchy to make all haste to Wavre, pushing back any Prussians he found as he approached the principal army. "His Majesty desires that you direct your movements to Wavre, in order to come closer to us, and to cooperate with our operations".[23] The minority line uses this sentence to argue that Napoléon wanted to have Grouchy participate in the battle of Waterloo (see, e.g., Fuller, 1951-1956, ch. 18). But Clausewitz countered the charge in advance, underlining that it was too late to send any orders to Grouchy; in fact, the marshal did not receive them until the afternoon of 18 June, by which time he had been trapped at Wavre by Thielemann, and Pirch had nearly reached Mont-Saint-Jean.

Regardless of what can be made of the last dispatch, somewhat confused and certainly too late, *the strategy was clear in itself*. Upon his arrival at Gembloux, Grouchy had to

---

[21] Cited by Mauduit (1847-2006, p. 142) and subsequent authors, Bertrand's letter is missing from Clausewitz, which weakens his chapter XXXVII.

[22] We use Mauduit's (1847-2006, p. 160-161) version of this letter. Fuller (p. 285-286) summarizes it accordingly, while Grouchy's *Mémoires* (LV, p. 58-59) distance themselves significantly from the text.

[23] The letter from Soult appears in Clausewitz (p. 141), as do all the subsequent dispatches.

arrange to block the Prussians' move towards Wellington instead of continuing to chase them. Fuller proposes an itinerary consisting of a march to Wavre from the west; thus, the marshal might intercept the first corps heading to Waterloo. Clausewitz (p. 143) also thinks that the westward march was the best strategy, agreeing for once with the *Mémorial* (p. 238-240), which first said that very thing.[24] It therefore appears that *on the level of objective rationality, all the interpreters are in agreement*. What divides them is how to apportion subjective rationality between the actors on the basis of their beliefs, and this conclusion is impossible to reach simply from the documents we have surveyed.

We will discuss the final battle only briefly. On 17 June, after the battle of Quatre-Bras, Wellington withdrew his troops to within about ten kilometers of Brussels, on the Mont-Saint-Jean plateau, whose value for defensive combat he had already spotted.[25] Partly hidden along the crest, the Anglo-Dutch could fire upon their opponents almost at leisure, while the attackers were hemmed in by solid buildings – farms and convents – in the center and on both flanks. On 18 June, rain delayed the French attack until 11:30, and hindered the artillery preparation that Napoléon was accustomed to implement before attacking. For this reason and others, the first offensive, directed against the center of the Anglo-Dutch line, was a complete rout. Several historians, including Fuller, conclude that with such a bad start Napoléon should have given up fighting the moment he heard of the arrival of the Prussians, that is around 3:30 p.m.[26] By moving to the defensive, he might have saved his army and fled with it back to France. But he did not. He tried to settle the outcome with a sequence of thrusts to the enemy's center, while simultaneously trying to close the gaps that the Prussians made in his right wing. While acknowledging the consistency of this battle plan, the specialists have judged it simplistic and moreover dangerous, given the frail right flank, and above all the stunningly feeble tactical execution. Leaving aside the full succession of attacks, we will single out the last and most famous, which is the engagement around 7:30 of the Old Guard, the last available

---

[24] Houssaye (1905-1961, p. 294-295) explains the desirable path. Grouchy would leave Gembloux to the west, marching to Mousty and Ottignies, where he would cross the Dyle and follow the river's left bank.

[25] It would be more accurate to call the battle after Mont-Saint-Jean, where it took place, than after the neighboring village of Waterloo, but Wellington wanted that name to be chosen. The Germans – Clausewitz among them – have long preferred to call the battle after the farm of Belle-Alliance, where Blücher and Wellington met on the evening of 18 June.

[26] Roberts (2005) puts the best moment for withdrawal even earlier.

reserve. Followed by many others, Clausewitz believes that this was an absolutely hopeless move: he goes as far as to claim that Napoléon no longer truly knew what he was doing (p. 158).

The moment has arrived for analytically reconsidering the campaign's main junctures. At three key moments – 17 June, around midday on 18 June, and in the final hours of the same day – Napoléon could have departed from the line of events that his previous decisions had set in motion, yet he did not. Was this evidence of inertia or lack of reflection – in which case he would no longer conform even to subjective rationality? Or is it a failure to correctly appreciate the situation at hand, in which case this form of rationality at least could be salvaged? Or is it the case that Napoléon did appropriately assess the situation by the criterion of objective rationality, and simply accepted the immense risks that this assessment made clear? Essentially, Clausewitz interprets the engagement of the Old Guard as irrationality pure and simple, and the dismembering of the army after Ligny as a subjectively rational action based on a false belief. He is more cautious in handling Napoléon's decision to continue the battle despite the threatening Prussian advance. At this point, he realizes that a taste for risk exacerbated by the circumstances may be consistent not only with subjective but also with objective rationality.[27]

The diagnosis is complicated by Napoléon's objectives, which were not of the usual military kind. He needed not just to win the campaign, but to win it *absolutely*; a weak victory would not have saved France from being invaded and his regime from collapsing. The two goals that Clausewitz usually assigns to war – destruction of the enemy forces and the political advantage that can be taken from the actions, whether victorious or not – were firmly bound together.[28] The Borodino battle of the Russian campaign, as reinterpreted in *On War* (IV, 12), makes this clear by way of contrast. There, Napoléon refused to engage his reserves against Kutuzov, consciously giving up a more complete

---

[27] See Clausewitz, p. 157. This is a brilliant insight for a time when the concept of risk-attitude was not yet separated from those of risk or uncertainty; see Mongin (2009).
[28] The tension between these two goals of war can be seen throughout *On War*, and Aron's (1976, ch. III) commentary brings it even more clearly to light.

victory that was otherwise within his reach. He was justified in holding back his limited forces, says Clausewitz, because he meant to enter Moscow in such obvious superiority that the tsar would beg him for peace. Borodino illustrates how a long-term political objective can diverge from the short-term military objective; by contrast, at Waterloo there was nothing to be gained from restraint, and the political and military objectives were strictly identical.[29]

Even the brutal sacrifice of the Guard is more ambiguous than it first appears. Recent military analysis permits us to reconsider the battle's last phase. The partial fall of the Anglo-Dutch center around 6:30 afforded Napoléon his best chance of the day. Had he launched the Guard precisely at this moment, rather than an hour later, fate, perhaps, would have turned in his favor.[30] This purely tactical reasoning should be contrasted with an interpretation that has sometimes been put forward: that taking defeat to be certain, Napoléon found it appropriate to finish his legend with some desperate, grandiose gesture. This is a wild suggestion, yet it is not incompatible with the former, purely tactical, interpretation; what both have in common is that they deepen the account of the actor's goals in order to dispel the impression that he acted irrationally.

The decision to divide the army created difficulties of a spatial and material nature that cannot be overcome simply by reconsidering Napoléon's ultimate goals. Since the unexpected northward movement of the Prussians made it impossible for Grouchy to carry out both the blocking and pursuit missions, one can attempt to salvage Napoléon's rationality by emphasizing either his misperception of the retreat (Clausewitz's solution), or his prioritizing interposition over pursuit in the orders to Grouchy (Fuller's). As we have seen, the conflicting hypotheses are loosely formulated and we have no firm evidence to rely on to decide between them. A proper model of the Emperor's choices should help on both scores. Not only will it make each alternative logically more definite, but, if it works well, it will discriminate between them, thus acting as a substitute for the missing data.

---

[29] Herbert-Rothe (2005) also compares the two battles of Waterloo and Borodino.
[30] This idea comes from Roberts (2005, p. 95).

3. A game-theoretic model of the decision of 17 June 1815

In the following, we model only the actions of Napoléon, Grouchy and Blücher, ignoring Wellington – a choice which can be defended on the ground that he remained fixed at Mont-Saint-Jean after bringing his men there on 17 June. In a more debatable simplification, we give Blücher only two possible actions:

B_1, march north, then go westward to join Wellington,

B_2, march north, then go eastward to return to Germany.

We omit a third possibility, B_3, which would consist of marching straight east to Germany. This brings the analysis closer to the actual choice of the Prussians, who did not take B_3 into consideration. The omission is more debatable from Napoléon's point of view, since he initially expected B_3 to occur. However, it would be awkward to formalize the revision of beliefs that took place on 17 and 18 June, and we will assume that, even on Clausewitz's interpretation, Napoléon is at all times uncertain between B_1 and B_2, instead of reaching this state of uncertainty only after initially believing in B_3.

No less schematically, two states of the world are possible:

E_1, Blücher is badly weakened,

E_2, Blücher is not badly weakened.

Before knowing which state is realized, Blücher therefore has four strategies at his disposal:

(B_i, B_j) = if E_1, then B_i; if E_2, then B_j, i, j, = 1, 2.

(By definition, a *strategy* is a function that associates actions to states of the world recognizable by the player.)

On the French side, we make another gross simplification by integrating Grouchy into Napoléon, treating them as though the latter were in fact the sole decider. It is somewhat paradoxical that this is less contestable from the marshal's own view point – his *Mémoires* describe him as a simple executor of orders – than from the point of view of Napoléon and his staff officers, who overloaded him with complex instructions. Not only

does it facilitate the game-theoretic analysis to bring the number of players to two, but we thereby eschew the difficulty of handling conditional instructions such as the following: chase the Prussian rear-guard if it does not appear that the advance-guard is moving to join the Anglo-Dutch force, drive westward in the opposite case. Yet the minority position à la Fuller would be best formalized by analyzing Grouchy just in terms of such strategies.

Thus fused with Grouchy, the player Napoléon has three possible actions:

S_1, keep the army together,

S_2, detach Grouchy's forces and send them to block Blücher's path to Wellington,

S_3, detach Grouchy's forces and send them in Blücher's pursuit.

Now technically reinterpreted, *blocking* (or *interposition*) means that the clash between Grouchy and Blücher will occur if Blücher goes west (case S_2B_1) and not if Blücher goes east (S_2B_2), while *pursuit* means that the clash will occur if Blücher goes the latter way (case S_3B_2) and not if he goes the former (S_3B_1). With these definitions, Grouchy's behavior becomes, as intended, mechanical. He rushes where Napoléon commands, and engages in battle or not depending on whether or not he meets Blücher there. Whereas Blücher learns in the interim stage which state of the world is realized, Napoléon does not, and his strategies are therefore constant functions across the states; that is, they are identical to his actions S_1, S_2, S_3.

The following probability parameters represent Napoléon's beliefs:

- k, the probability that Blücher is badly weakened by his defeat at Ligny;
- l, the probability of victory for Napoléon's consolidated army against a united Wellington and Blücher, *supposing Blücher was not badly weakened* (we will take l to be 0 in a simplified variation);
- l', the probability of victory for Napoléon's consolidated army against a united Wellington and Blücher *supposing Blücher was badly weakened*;
- l'', the probability of victory for Napoléon without Grouchy against a united Wellington and Blücher, *supposing Blücher was badly weakened*;

- *m*, the probability of victory for Napoléon without Grouchy, and against only Wellington, regardless of the state of Blücher's forces.

It is automatic to suppose that $l' > l$ and that $l', m > l''$. Other less obvious inequalities will have to be added to reach a solution.

The model assigns trivial values to all other relevant probability parameters. Thus, it gives a value of 1 to:

- the probability of victory for Napoléon's entire army against Wellington alone,
- the probability of Grouchy's victory against Blücher, *supposing that Blücher was badly weakened.*

And it gives a value of 0 to:

- the probability of victory for Napoléon, without Grouchy, against a united Wellington and Blücher, *supposing that Blücher was not badly weakened*;
- the probability of victory for Grouchy against Blücher, *supposing that Blücher was not badly weakened*.

It seems inelegant to have so many 0s and 1s; however, in experimenting with more general assumptions, we have found that these do not appreciably change the conclusions. And the *Mémorial* – although obviously a suspect source – does suggest we take extreme values here. For example, it claims that Grouchy's detachment was strong enough to "topple the Prussian rear-guard in whatever position it took" (p. 239). By this token, it is comparatively moderate to assign probability 1 to Grouchy's victory over Blücher conditional on Blücher being *weakened*. Still from the *Mémorial*, "if Grouchy had been on field and time had permitted the French army to deploy itself for battle", one after the other the Emperor would have undone the Anglo-Dutch and Prussian armies (p. 245). Again cautiously, we reserve this probability 1 of victory for the case of Napoléon's entire army fighting Wellington *alone*.

The model also includes the following utility values, which reflect Napoléon's evaluations, just as the probability values reflected his beliefs:

- a_1, the utility of victory against Wellington,

- a_2, the utility of victory against Blücher,
- b_1, the utility of defeat against Wellington,
- b_2, the utility of defeat against Blücher,
- c, the utility of no confrontation.

Nothing substantial is added if we assume that victories give positive utility, while defeats are negative:

$a_1 > 0 > b_1$, $a_2 > 0 > b_2$.

However, in a more debatable assumption, we will freely *sum* the numbers thus defined. In particular,

$a_1 + a_2$ = the utility of victory against Wellington and Blücher together

$b_1 + b_2$ = the utility of defeat against Wellington and Blücher together

In other words, Napoléon's victory against his two opponents at Mont-Saint-Jean would have the same value as his beating Wellington alone on this field, accompanied by Grouchy's beating Blücher's forces elsewhere; and similarly, *mutatis mutandis*, for the defeats of the French at the hands of both enemies.

We must still evaluate the situation in which Grouchy and Blücher do not meet. This receives the value $c = 0$ in the case (S_3B_1) where Blücher marches west and Grouchy pursues him in vain, and the value θa_2 – with θ a parameter between 0 and 1 – in the case (S_2B_2) where Blücher marches east and Grouchy engages in a futile block. The second case differs from the first in that a Prussian retreat without combat represents an additional victory for the French, albeit a much lesser one than would have occurred had Blücher been beaten on the field again.

All that remains in order to represent the situation as a *normal or strategic form game* is to define Napoléon's and Blücher's payoffs for the various outcomes. Using the probabilities k, l, l', l'', m and the utilities a_1, a_2, b_1, b_2, c, θa_2, we calculate Napoléon's payoffs by the customary rule of expected utility. Blücher's payoffs will be supposed to be algebraically opposite to Napoléon's. In technical terms, this is a *zero-sum game*,

which reflects the nature of this – although not every – military campaign.[31] One might argue, however, that the game is not classically zero-sum. Being also a game of incomplete information, it entails opposite values for expected utilities payoffs, which means that an assumption is made on the probabilities as well as the final payoffs.

We now sketch the resolution, leaving the details for the appendix. The argument will emphasize three expected utility payoffs, denoted V_1, V_2, V_3 in the game matrix below (the full matrix is given in the appendix):

	B_1B_1	B_1B_2	B_2B_1	B_2B_2
S_1	V_1			
S_2			V_2	
S_3	V_3			

The first step is to associate with each of Napoléon's strategies the minimum payoff it can bring, taking account of Blücher's response; this is *the strategy's security payoff*. For S_1, the minimizing strategy is B_1B_1 and the security payoff is V_1; for S_2, they are B_2B_1 and V_2; and for S_3, B_1B_1 and V_3. The last two conclusions follow from our definitional assumptions, but the first requires further optional assumptions regarding the utility values as well as l, l', θ. These boil down to an algebraically precise statement that l and l' are bounded from above.[32]

The second step compares the strategies S_1, S_2, S_3, supposing that each brings in its security payoff. The largest of the three numbers – his *maxmin* – is the greatest amount that Napoléon can guarantee himself, regardless of what Blücher does against him. We will assume that he plays the strategy associated with this value. The comparison between

---

[31] Following the previous analysis, at Borodino Napoléon did not aim at Kutuzov's total annihilation. Unless the payoffs are redefined, a zero-sum game would therefore not correctly represent the strategic interaction of the two adversaries. Haywood (1954) also underlines that not every battle is appropriately modeled as a zero-sum game.

[32] B_2B_1 minimizes the payoff of S_1 if and only if $(a_1 + \theta a_2 - b_1 - b_2)/(a_1 + a_2 - b_1 - b_2) > l$, l'. As θ grows, the central expression increases towards 1, thus binding l, l' less and less. This is not a very constraining assumption.

S_1 and S_2 depends on the inequality $V_1 < V_2$, which is equivalent to a joint restriction on k, l, m and the utility values. This restriction is pleasantly simplified when $l = 0$.[33] The comparison between S_3 and S_2 depends on the inequality $V_3 < V_2$, which follows from a joint restriction on k, m, θ and some utility values.[34] On the basis of these conditions, we conclude that V_2 is Napoléon's maxmin and that he plays S_2.

The third step is to investigate Blücher's strategies, B_1B_1, B_1B_2, B_2B_1, B_2B_2, calculating security payments for each, and finding the highest of these four numbers, i.e., Blücher's maxmin, as well as the corresponding strategy. Without further parameter restrictions, these are V_2 and B_2B_1. Making the same behavioral assumption as for Napoléon, we conclude that Blücher plays B_2B_1.

The resulting outcome (S_2, B_2B_1) satisfies *von Neumann and Morgenstern's solution concept for zero-sum two-person games*. This is not a genuinely interactive concept; rather, as the previous two paragraphs have illustrated, it applies an individual rationality argument twice over, rationality being identified with prudence (each player protects himself against the opponent's most damaging strategy). However, it is a well-known result – holding somewhat more generally than for zero-sum two-person games – that a solution so defined is also a *Nash equilibrium*, i.e., a pair of mutually optimal responses, and conversely.[35] That is to say, S_2 is Napoléon's best response to Blücher's choice of B_2B_1, and B_2B_1 is Blücher's best response to Napoléon's choice of S_2. We could have found the solution (S_2, B_2B_1) just by computing best responses, but this easier method would have been harder to justify in terms of individual rationality strictly conceived.

---

[33] We derive $V_1 < V_2$ from $m > k(1 - ld) + ld$, putting $d = (a_1 - b_1 + a_2 - b_2) / (a_1 - b_1)$. This is a substantial and constraining assumption, which is simplified as $m > k$ when $l = 0$. To ensure that the right-hand side is between 0 and 1, we also impose that $ld < 1$ – another bound on l – and that $m > 0$, $k < 1$.

[34] A sufficient condition for $V_3 < V_2$ is that $m > k(1 - \theta)a_2 / (1 - k)(a_1 - b_1)$. The right-hand side is less than 1 if $a_1 - b_1 > a_2$ and either $k < \frac{1}{2}$ or $\theta > k$. The first assumption is fully justified in the context of 17 June. Both of the latter two enter the historical explanation merely as conjectures. There are other sufficient conditions available.

[35] See Nash (1950) and Luce and Raiffa (1957, appendix 2).

The chief tool of zero-sum, two-player games is the minimax theorem, but this is not directly usable here.[36] We need to show that Napoléon's maxmin payoff is equal to the algebraic opposite of Blücher's maxmin payoff for the initial – so-called *pure* – strategies of the two players. The theorem would have secured the equality only for the more numerous – so-called *mixed* – strategies, which select randomly from between the pure strategies by means of some probability distribution. Hence the need for a computational proof (provided in the appendix).

In the limiting case $l = 0$, some of the parametric conditions vanish, while $V_1 > V_2$ becomes equivalent to the inequality $m > k$, which is easier to interpret in the form $1 - k > 1 - m$, since this emphasizes the risks taken by Napoléon. In words, *the risk that Blücher is not badly weakened must be greater than the risk of losing a duel against Wellington.* This comparison is the core of our account of Napoléon's deliberation on 17 June. Napoléon would first discard S_3 because this strategy led to the worst possible loss, greater than the loss incurred under S_1.[37] Then Napoléon comes to the truly difficult choice, that between S_1 and S_2, and resolves it in favor of S_2 after comparing the two risks just described. Had he really gone through these reflections, he would have acted *prudently*, not as the gambler of the legend. He had to face the unpleasant possibility that Blücher, having weathered Ligny better than expected, would defeat Grouchy; but he could at least exclude the worse possibility that Blücher would join forces with Wellington against him alone.

A passage of the *Mémorial* (p. 239) suggests the relatively low value for $1 - m$ we need for the reasoning: the Emperor's remaining forces were enough to "topple the Anglo-Dutch army" despite a slight numerical disadvantage. Unfortunately, it says nothing to suggest that $1 - k$ was large, except perhaps in the following, roundabout way. Had Napoléon believed the Prussians truly diminished, he would have turned different reproaches on Grouchy. In the already cited passage of p. 245, he describes himself

---

[36] Due to von Neumann (1928), this theorem owes its fame to von Neumann and Morgenstern (1944-1947).
[37] Observe that dominance reasoning does not suffice to eliminate S_3. Recall that one strategy is *dominated* by another if it returns a smaller payoff for all the opponents' responses, in all states of the world. Our game does not give dominated strategies to Napoléon, but it does to Blücher; see the appendix.

beating Wellington and Blücher one after the other, and he keeps the final victory over the Prussians for himself, leaving it to Grouchy to pin them down while he was finishing the Anglo-Dutch. Such a chain of events only makes sense if Blücher was not already annihilated by his defeat at Ligny.[38]

How does the model impact Clausewitz's position? Clausewitz maintains that Napoléon dispatched Grouchy for a chase (S_3) even though, from the model, his only sensible choice was between dispatching Grouchy for interposition (S_2) and keeping the army together (S_1). As can be checked, the conditions for getting the security payoffs associated with S_1, S_2, S_3 are mild or definitional, and maxmin reasoning excludes S_3 merely on the basis of a definitional inequality ($V_1 > V_3$ follows from $l' > l''$). Thus, Clausewitz should claim that Napoléon acted irrationally, contradicting his own suggestion that subjective rationality applies here (see section 2 on how Clausewitz allotted rationality and irrationality across the campaign). A more consistent but still dubious line would be to argue that Napoléon excluded S_3, but should have adopted S_1 instead of S_2. In the simplified version of the model, this is equivalent to reversing the fundamental inequality, i.e., putting $1 - m > 1 - k$. However, the *Campaign* does not properly support this restriction. It only says that Napoléon believed the Prussians to be badly damaged *and* was confident he could defeat Wellington without Grouchy. This amounts to taking small values for both $1 - k$ and $1 - m$, leaving the comparison indeterminate. Altogether, it is seems impossible to articulate Clausewitz's position in a satisfactory way.

What now for the proNapoleonic position? The model clarifies the restrictions that it needs for its conclusion, i.e., that Napoléon rationally adopted S_2, and this time the available accounts roughly support the restrictions. For example, from what Fuller writes, the following becomes plausible: a low value for l, fairly large ones for l' and m, a weak one for k, and a moderate abatement θ. This delivers the solution S_2 not only in the simplified form of the model, but also in its general form. In sum, from the perspective of

---

[38] Inconclusive as they also are, two already discussed staff documents suggest a low k. On 17 June, Bertrand warns Grouchy about Blücher's remaining possible maneuvers, and Soult's dispatch of 18 June confirms that Napoléon was concerned about an offensive return of the Prussians.

this model, the proNapoleonic position offers a logical coherence that Clausewitz's lacks. This is not to deny that it involves a difficulty that the other does not. For it leaves unexplained the behavior of Grouchy, who, in our simple dichotomy of chase or block, undertook the former instead of the latter, for which he should have received more or less explicit commands. Thus, as reconstructed here, the position salvages Napoléon's rationality at the cost of wrecking Grouchy's. Note however that the other position, as reconstructed here, makes the opposite trade-off, which seems worse, given the comparative records of Napoléon and Grouchy.

We stressed earlier that the two interpretations can only hypothesize what Napoléon's commands to Grouchy truly were. Because of this empirical limitation, we used the model in *no less than three functions*, all of which involve the same set of parametric restrictions. First, the model permitted evaluating the strategies S_1, S_2, S_3; second, it established that Napoléon, acting rationally, ordered S_2 rather than S_3; and third, by the same rationality assumption, it explained this alleged fact as well as the observed fact that he did not order S_1. In standard methodological accounts, the explanatory use of rationality assumptions follows on after an allegedly sufficient observational stage; for example, as these accounts go, the role of preference maximization is only to explain the consumer's demand for a commodity, not to contribute to determining it empirically, since market data or questionnaires are sufficient for this purpose. But in our study, as in many other historical works, *what needs explaining is not fully observed*. Equivocal reports (here, the testimonies and dispatches) stand for the missing pieces of information (here, the oral commands). This is why we also used the model with the function – numbered two above – of clarifying the *explanandum*. This makes the explanatory process circular, in contradistinction with the consumers' demand case, but not necessarily viciously or inadequately so. For there is nothing sinister in circular reasoning if it makes overall sense of a sufficient amount of sufficiently diverse data (and it is important in this respect that Napoléon's rejection of S_1 can be observed). Still, reasoning in this style is probably better fitted to assessing the comparative value of existing

accounts and arguments than to providing a full-fledged explanation of the facts themselves.[39]

4. Campaign studies and the objections to analytic narratives

In this section, we return to the genre of analytic narratives and argue that it is well suited for the study of military topics of the kind exemplified here. As a preliminary step in this argument, we review the objections that have commonly been raised against Bates et al. (1998), and see how our model fares with respect to them. These objections come from historically minded political scientists and sociologists, not historians, yet may reveal why the latter have proved so impervious to the new genre.

There have been essentially three lines of criticism: (i) the historical events are poorly selected for the purposes of modeling; (ii) the chosen models fail to meet the appropriate game-theoretic standards; (iii) game theory and more generally rational choice theories, whether formal or not, are marred by so many conceptual difficulties and empirical failures that it is dubious to use them in the first place. Elster's (2000) fierce review of the 1998 book develops all three objections, and Bates, Greif, Levi, Rosenthal and Weingast (2000) reply to him on each count.[40] We agree with them that the grand objection (iii) is not really to the point. They write against Elster: "His real opponent is rational choice theory" (2000, p. 702). Indeed, why should one discuss analytic narratives at all if one has good reason to reject the theory on which it depends? To keep the discussion properly focused, we must it seems take for granted that rational choice theory, and more specifically its formal branches, has at least *some* degree of relevance. Only thus can one proceed to the more informative objections (i) and (ii). These would have to be evaluated by a detailed discussion of each case study in the book, but it transpires from the broad outline of the controversy that analytic narratives are indeed generally vulnerable to them.

---

[39] We pursue this hermeneutical line at greater length in Mongin (2009).

[40] We focus on this early debate because it puts the issues sharply, perhaps at the risk of oversimplification. For complementary viewpoints, see the collection of articles in *Social Science History* (2000), with another response by Bates et al., and the symposium in *Sociologica* (2007); see also the introductory comments in Zagare (2011).

Concerning (i), the choice of an event is poor if it involves imprecise boundaries, too many actors, or interactions that are too complex. It is also poor when the historical documentation is defective, even though the case may be simple in principle. There are more than one possible objections like (ii), but as far as game theory goes one can usefully distinguish between those which concern the chosen equilibrium concept, and those which concern the multiplicity of equilibria once that concept is fixed. Contrary to what Bates et al. (1998) and Levi (2002) suggest, it is not so clear that analytic narratives require extensive form games and the corresponding special equilibrium concepts. Their further suggestion that the multiplicity issue should be handled *ex post* by some kind of data-driven selection is also questionable; whenever possible, it seems better to reduce the number of equilibria *ex ante* by stronger modelling choices.

Our study will no doubt elicit objections similar to those just listed. To those who still take objection (iii) to heart, it is worth mentioning that our game-theoretic analysis has an advantage over others. We use the Nash equilibrium concept in a class of games – two-player, zero-sum – for which its appropriateness is least dubious, because equilibrium behavior is supported there by an *individual* rationality argument (specifically based on prudence). Moreover, our particular game has no other pure strategy equilibrium than the one calculated. This also answers one of the worries contained in objection (ii).

Concerning (ii), it must be added that the model includes a relatively rich description of Napoléon's uncertainties. For one thing, we have probabilized the issues in the payoff matrix, so as to reflect Napoléon's uncertainty regarding the outcome of his battles, once every strategic move has been made; and for another, we have endowed Blücher with strategies, rather than merely actions, in order to formally capture Napoléon's uncertainty regarding Blücher's strength after Ligny. The first form of uncertainty is exogenous, but the second is strategic and it structures the game, making it a *game of incomplete information*. This is so despite the fact that we do not probabilize Napoléon's beliefs on Blücher's types, as a full-fledged game of this class would require.[41] In the early

---

[41] This answers a technical question raised by Steve Brams and Françoise Forges.

literature of analytic narratives, only the first (exogenous) form of uncertainty occurs. To Elster's (2000, p. 293) complaint that they neglect uncertainty, Bates et al. (2000, p. 699-700) correctly reply that he should have distinguished between the two forms and recognized that some of their studies pay attention at least to the first. They express reservations on the second form, without fully explaining their grounds. Clearly, there is a tension between the adherence to the extensive form and the need for representing in-depth strategic uncertainty. Our choice of a strategic form is in part prompted by the greater ease with which this format can be extended in the uncertainty direction.[42]

It will no doubt be said that some of our technical assumptions are *ad hoc*, a criticism that borders on objections (ii) and (i). We have already considered one possibility of conflict between the game-theoretic model and the historical narrative: Grouchy should count among the strategic actors alongside Blücher and Napoléon. If this were done, the distinction between strategies and actions would also appear on the French side. Napoléon would have the choice of either remotely controlling Grouchy, or of delegating to him the power to act according to what he discovered on the terrain. In a game thus refined, the *ex post* inadequate choice of pursuit might become one of the equilibria instead of being a deviation from the single equilibrium. We do not dispute that these changes would be for the better, and we defend the current model in only two ways: it is a decent starting point to illustrate the method, and, within its limits, it seems sufficient to assess the conflicting interpretations.

Another possibility of conflict between the model and the narrative has to do with the actors' objectives, but that point does not have much force here. We have already argued for the appropriateness of the zero-sum assumption. The further assumption of additive utility seems defensible on the very same ground, i.e., that nothing short of a crushing victory in the campaign could fulfill Napoléon's objectives. The number and order of battles mattered little to him as long as he achieved this final result. However, we have

---

[42] However, some of Zagare's (2011) work uses perfect Bayesian equilibrium, and thus takes the step of integrating strategic uncertainty with the extensive form.

not taken the idea to its extreme, since we added only the final utilities – not the expected ones, which would have altered the conclusions significantly.

It remains to consider objection (i) more directly by discussing our case selection. The problem here is that a case may involve too much relational complexity – or, even if this is not the case, may be too poorly documented – for the formal theory to have any real bite. This is more likely to be realized when the event is taken broadly across time and space, and some of the cases in Bates et al. (1998) are indeed questionable on this score: Elster has a point here. Accordingly, we have selected a very small-scale event, whose context, the Waterloo campaign, is extremely well documented. We will now generalize from this example to the claim that military activities, and specifically military campaigns, offer a suitable topic for analytic narratives, in the sense of barring objection (i). This will involve us in surveying six relevant aspects of this topic.

First of all, the *hierarchical* nature of military organization makes it acceptable to concentrate the study on the decisions made by a few key individuals – typically, the general-in-command, his staff and principal lieutenants. In actual fact, the human material of an army never has the suppleness that makes top-down instructions fully effective. Our model integrates these "frictions" – Clausewitz's famous term (*On War*, I, VII) – by way of probabilizing the consequences of the actions of the decision-makers it selects. The limitation of this method is that it rules out some possibly relevant interactions. For example, to treat discipline among the rank-and-file as a stochastic phenomenon is to forget that it depends on a range of activities on the leaders' part – demonstrations of courage, promises, threats and exhortations; ideally, these activities should enter the definition of action sets in the model. However, the neglected interactions are not always significant to the same degree, and the empirical data may serve here as a touchstone. When an army threatens to fall apart modelers must address the relation between the leaders and the troops; when it obeys orders, as it did on 18 June before the tragic denouement, they may pass over it.

In the second place, the classic distinction between *strategy* and *tactics* offers a useful heuristic to select which decisions by the key actors should enter the model. According to standard military treatises, strategy concerns the organization of the movements of the army in a campaign, aimed at a battle or a linked collection of battles, in pursuit of the overall victory; tactics concerns the movements of the army within a given battle so as to win it.[43] Such a contrast reflects a means-ends hierarchy, a spatio-temporal difference – the movements of a campaign being more wide-ranging than those of a battle – and lastly an organizational difference – the general-in-command being solely responsible for strategy, whereas he shares or delegates responsibilities on tactics. The June 1815 sequence of events illustrates this threefold analysis neatly. On the basis of his overarching plan of campaign, Napoléon entrusted two subordinates, Ney and Grouchy, with the supervision of battles that were to be fought in distant locations. Since Napoléon was acting in accordance with the classic military distinction, a modeler can adopt it in turn. That is, the fact that *the actors themselves* act on a given concept guarantees that it is not merely convenient for the modeler to employ it, but also properly explanatory. The related point is that campaigns, rather than battles or other military events, are the natural objects of military analytic narratives.[44]

Third, the military goal is classically determined from without and once and for all – to win the battle or to achieve victory in the campaign, as the case may be. According to older military definitions, the former occurs with the final occupation of the field, and the latter with the conquest of a province or stronghold. The modern conception of a victorious campaign or battle is more abstract, holding it to be the destruction of the opposing forces, or, failing that, their significant weakening – along with their own admission of the fact if possible. This much is suggested by *On War*, although Clausewitz's precise meaning has been debated.[45] The plural understanding of victory gives rise to an ambiguity that can be worrying for the ordinary narrator, and hence also

---

[43] This is essentially the distinction made by *On War*, I, II.

[44] When a single decision is at issue, as in Haywood's (1954) Pacific example, the distinctions between strategy and tactic, or campaign and battle, of course vanish.

[45] Arguing from *On War* (VIII), Aron (1976) concludes that Clausewitz promoted a novel conception of victory. But Paret (1992, p. 106) makes it clear that this is not entirely the case

for the analytic narrator.[46] But even if we grant that there are some exceptions, it remains broadly correct that the military context permits us to fix subjective goals relatively easily.

The fourth point is closely related to the third. Schematizing the *teleological* side of the reasons for action helps us to refine the *cognitive* side and to locate the desired explanation there. Formal rational choice theories attain their peak inferential capacity when they are precised in this way.[47] For example, by assuming that firms maximize their profits, economists are able to connect the oligopolistic structure of a market with the conjectures that each firm makes about its competitors' strategic moves. Or, again, by assuming that stock market traders maximize their expected utility, and that they are identically risk-averse, they can relate asset prices to the differences in the beliefs held by these traders. Both applications illustrate the logical power of models that postulate simple objectives and keep them fixed throughout. Historians of military campaigns follow this one-sided method in their own way when, proceeding from the assumption that victory is desirable to the leaders, they focus explanation on these leaders' beliefs and risk-attitudes. If their explanations seem incomplete or mutually discrepant, the role of the military analytic narrative is simply to sharpen them and facilitate adjudication between them.

Fifth, even a campaign decision of the highest degree of complexity is in principle to be assessed in terms of its final consequences on the field. An idealized general-in-command would apply backward reasoning, passing from the evaluation of the final consequences to that of the immediate consequences, and eventually of the initial decision. Seen from this *consequentialist* perspective, the choice made on 17 June was relevant only in relation to the chances of victory or defeat it established for 18 June. A decision of that kind necessarily also exhibits procedural features, such as complying with the art of war, or involving more or less panache; but from a consequentialist perspective these do not

---

[46] Even some Napoleonic events are not easy to classify. With Borodino, Eylau is the classic example of an ambiguous victory, and Tolstoy in *War and Peace* goes as far as to claim these two battles for the Russian camp.

[47] Davidson (2004, p. 26) claims that the explanatory asymmetry between desires and beliefs is structural, but others see it only as a contingent property of the given explanations.

matter unless they influence the final outcome. Many military campaigns can be described plausibly by ignoring such procedural features, and this again makes them a promising terrain for applying formal rational choice theories, which are consequentialist throughout.

The five features listed thus far should be contextualized. Both military activity itself and its strategic theorizing have changed dramatically since Napoléon's time.[48] The concept of the masterminded campaign that leads to decisive battles is now outdated. The indefinite battles of the First World War following the Marne, and the new concept of total war thus heralded, as well as the colonial wars, guerrilla combats and other 20th- and 21st-Century "dirty" wars – all shake the five features to a varying extent. In the other time direction, scholars have pointed out that Ancien Regime wars embodied specific conceptions of military activity that are equally divergent from the Napoleonic scheme. All this suggests that our case, being *too* well chosen, cannot support a general argument for treating military campaigns analytically.

This objection, however, misjudges the role of time. Military campaigns confront formal rational choice theories with a continuum of obstacles that are best stated abstractly. These theories apply more rigorously as the distinction between war and peace becomes clearer, as the goals in each camp come to be more closely aligned, as military decision-making adheres to a stricter hierarchy, and so on. By exploring this factor of dependency, one would likely discover that it is not the time period *per se* that decides whether or not a military analytic narrative is feasible. A more casual review of military history leads to the same conclusion, by suggesting promising examples in the mid-20th Century, the Ancien Regime or even Ancient Rome, as well as unpromising ones in the mid-19th Century.[49] Military historians adopting the methodology proposed here would not be in a very different position from economists, whose success or failure depends on the areas of social interaction to which they apply their maximizing and equilibrium assumptions.

---

[48] On these changes, see, among many others, Earle's (1943) collection or Aron's (1976) comments on Clausewitz's heritage.

[49] Compare the campaigns investigated by Fuller (1954-56) on a very broad time range. Some are evidently more amenable to rational choice modelling, and there is no such obvious time dependency as may seem at first glance.

The sixth and last feature is perhaps the single most important of the list. It is that military actions are *already* viewed as rational or irrational, before these qualifiers appear in scholarly work. The actors themselves are the first to adopt them, either *ex ante* or *ex post*; then, polarized in the same way, come the judgments of witnesses, memoir-writers, military instructors, academic historians. As a memoir-writer, Napoléon produced the first systematic study of the Waterloo campaign, which prompted those of Clausewitz and many further military instructors and academic historians; and on it went, right up to the current modest academic essay, which capitalizes abundantly on its predecessors. Each step has brought out new information as well as new questions, sometimes unexpected.[50] That interpretations develop in a wide reflective spiral is common in history, but that *individual rationality* should permeate each reflective stage is not so common, and this again singles out military campaigns as relevant objects of analytic narratives.

5. Conclusions

Analytic narratives face three main groups of objections. We have discarded those directed at rational choice theory as such, on the grounds that they are uninterestingly general; we have discussed some of the more technical ones relative to game theory; and finally we confronted head-on what seemed to us the crucial problem, i.e., the choice of historical topics suited for the analytic treatment. Taking the view that some cases are intrinsically more amenable to this treatment than others, we have offered campaign narratives as a favorable example. We emphasize, however, that we mean this only to be an example. The list of the previous section is only a step away from a yet more abstract argument, which would explore the six features in and of themselves, regardless of their military instantiations, and thus permit us to recognize other favorable examples. By and large, an analytic narrative works better when the following conditions are met: decision units are more definite and come closer to being concrete individuals; the distinction

---

[50] Largeaud (2008) provides a thorough account of how interpretations of Waterloo have succeeded – and to an extent generated – each other, on the French scene. One would welcome similar reviews for the British and German scenes.

between incidental and major decisions is clearer, and the latter are made more methodically; the decision-makers' ends are easier to grasp and separate from their beliefs; the reductive consequentialist approach applies more plausibly to the evaluations; and, last but not least, the antecedent informal narratives of the same case are more permeated with rationality considerations. Some cases in Bates et al. (1998) fit these general conditions better than others. However, we must defer this cross-examination and a full analysis of the conditions, leaving the present paper at its intermediate level of generality.[51] Hopefully, what has been said of military applications is sufficient to bring the point home that not every analytic narrative is marred by "excessive ambition", to quote Elster's (2000) derogatory slogan, and that historians should not persist with their diffidence about – or, worse, ignorance of – the new genre.

For the more specialized audience of this journal, a word may be added on how analytic narratives relate to cliometrics. Prominent among the shared characteristics are the orientation of the explanatory work towards specific historical events rather than any other *explananda*,[52] the problematizing form of the historical inquiry, as against the descriptive or classical narrative forms of traditional history,[53] and the application to history of formal models that are borrowed or adapted from economic theories broadly understood. The most salient difference has to do with the question of measurement. Cliometricians are typically busy constructing and investigating past economic quantities, with a view to giving a quantitative precision to their causal hypotheses. Analytic narrators have only dealt with qualitative data, such as changes in institutional patterns and weighty decisions made by individuals or organizations. Against earlier economic history, cliometrics asserted itself not so much by promoting the use of quantitative data, which was already developed in part of that history, as by recommending a powerful

---

[51] Some comments along the present lines can already be found in Mongin (2010, 2016). A troubling suggestion we make there is that analytic narratives tend to work well when traditional narratives of the same events already work well.

[52] Even though some cliometricians and analytic narrators endow their respective fields with an inductive potential. Thus, Diebolt and Haupert (2016) and Haupert (2016) connect cliometrics with the second German historical school, which promoted inductive economics, and Bates et al. (1998) express the hope that their models would be applicable across similar historical cases, thus revealing an inductive potential.

[53] See again Diebolt and Haupert (2016) and Haupert (2016) on this.

theoretical organization of the data that thus far was entirely missing.[54] We here have a very different contrast, since analytic narratives fully endorse the need for theory, but do not use the data in question. As a straightforward consequence, analytic narratives and cliometrics do not borrow from the same theories. The central tool of the former, game theory, is exemplary of the mathematics of the qualitative, and although it can combine with econometrics or other statistical tools, as some applied economic research testifies, this has not happened in analytic narratives. Cliometrics, for its part, makes very little use of game theory and other formal versions of rational choice theory when compared with the classic triad of microeconomics, macroeconomics and econometrics.[55]

However, the present state of affairs is perhaps only temporary. There are few analytic narratives in existence, and those which are yet to come may enlarge their concerns and techniques in the direction of cliometrics. As soon as the field takes a quantitative turn, it will become an integrated part of cliometrics.[56] To be faithful to their initial intent, these future works should remain genuine narratives, although not narratives of the form that cliometricians tend to disparage. The single most important message that the cliometricians could learn from their fellow analytic narrators is that there are many mansions in the Father's house, and not every narrative is adverse to theoretically inspired history.

REFERENCES

Allison, G.T. (1971), *Essence of Decision: Explaining the Cuban Missile Crisis*, 1ed. Little Brown (2ed. coauthored with P. Zelikow, Longman, 1999).
Aron, R. (1976), *Penser la guerre*, Paris, Editions Gallimard, 2 t. Eng. transl. by C. Booker and N. Stone, *Clausewitz : Philosophy of War*, Englewood Cliffs, NJ, Prentice-Hall, 1985.
Bassford, C. (1994), *Clausewitz in English: The Reception of Clausewitz in Britain and America*, 1815-1945, New York, Oxford University Press.

---

[54] This is a classic assessment; see, e.g., Heffer (1977) and McCloskey (1978).
[55] Some game-theoretic applications that border on cliometrics are mentioned in Greif (2002).
[56] Diebolt (2016) is already willing to include analytic narratives in cliometric works.

Bates, R.H., A. Greif, M. Levi, J.L. Rosenthal, B. Weingast (1998), *Analytic Narratives*, Princeton, Princeton University Press.

Bates, R.H., A. Greif, M. Levi, J.L. Rosenthal, B. Weingast (2000), "The Analytic Narrative Project", *American Political Science Review*, 94, p. 696-702.

Bertrand, H.G. (undated), *Cahiers de Sainte-Hélène*, ed. by P. Fleuriot de Langle, Paris, Albin Michel, 1949-1959, 3 vol.

Brams, S. (1975), *Game Theory and Politics*, New York, Free Press.

Brams, S. (2011), *Game Theory and the Humanities: Bridging Two Worlds*, Cambridge, MA, MIT Press.

Browning, B.M. (2000), "Economic Analysis in Historical Perspective", *History and Theory*, 39, p. 88-97.

Clausewitz, C. von (1832-1837), *Hinterlassene Werke*, 10 vol., Berlin, F. Dümmler (posthumous edition by Marie von Clausewitz).

Clausewitz, C. von (1832-1834), *Vom Kriege*, in *Hinterlassene Werke*, vol. 1-3. New ed. by W. Hahlweg, Bonn, Dümmler, 1980. Eng. ed. and trans. by M. Howard and P. Paret, *On War*, New York, Everyman's Library, 1993.

Clausewitz, C. von (1835), *Der Feldzug von 1815 in Frankreich*, in *Hinterlassene Werke*, vol. 8, or in *Schriften, Aufsätze, Studien, Briefe*, ed. by W. Hahlweg, Göttingen, Vandenhoeck & Ruprecht, 1990, 2 vol., p. 936-1118. Eng. trans. in C. Bassford (ed.), *On Waterloo : The Campaign of 1815 in France by Carl von Clausewitz and a Memorandum on the Battle of Waterloo by the Duke of Wellington*, Boulder, Co., Westview Press, 2001. French tr. by M. Niessel, *La campagne de 1815 en France*, Paris, R. Chapelot, 1900. Reprint Paris, Champ Libre, 1973.

Davidson, D. (1980), *Essays on Actions and Events*, Oxford, Clarendon Press.

Davidson, D. (2004), *Problems of Rationality*, Oxford, Clarendon Press.

Diebolt, C. (2016), "Cliometrica After 10 years: Definition and Principles of Cliometric Research", *Cliometrica*, 10, p. 1-4

Diebolt, C. and M. Haupert (2016), "An Introduction to the *Handbook of Cliometrics*", in *Handbook of Cliometrics*, ed. by C. Diebolt, C. and M. Haupert, Berlin, Springer, p. i-xxii.

Downing, B.M. (2000), "Economic Analysis in Historical Perspective", *History and Theory*, 39, p. 88-97.

Earle, E.M. (1943) (ed.), *Makers of Modern Strategy*, Princeton, Princeton University Press.

Elster, J. (2000), "Rational Choice History: A Case of Excessive Ambition", *American Political Science Review*, 94, p. 685-695.

Erikson, P., J.L. Klein, L. Daston, R. Lemov, T. Sturm, and M.D. Gordin (2013), *How Reason Almost Lost Its Mind. The Strange Career of Cold War Rationality*, Chicago, University of Chicago Press,

Fuller, J.C. (1951-1956), *The Decisive Battles of the Western World and their Influence upon History*, London, Eyre & Spottiswoode.

Gourgaud, G. (1818), *Campagne de 1815*, Paris, P. Mongie Aîné.

Greif, A. (2002), "Game Theory and History", *Handbook of Game Theory*, ed. by R.J. Aumann and S. Hart, 3, ch. 52, Amsterdam, Elsevier, p. 1989-2024.

Grenier, J.Y., C. Grignon, P.M. Menger (2001) (eds) *Le modèle et le récit*, Paris, Editions de la Maison des sciences de l'homme.

Grouchy, E. de (1873-1874), *Mémoires du Maréchal de Grouchy. Publié par le Marquis de Grouchy*, Paris, E. Dentu, 1873-1874.

Haupert, M. (2016), "History of Cliometrics", in *Handbook of Cliometrics*, ed. by C. Diebolt, C. and M. Haupert, Berlin, Springer, p. 3-32.

Haywood Jr., O.G. (1950), "Military Decision and the Mathematical Theory of Games", *Air University Quarterly Review*, 4, p. 17-30.

Haywood Jr., O.G. (1954), "Military Decision and Game Theory", *Journal of the Operations Research Society of America*, 2, p. 365-385.

Heffer, J. (1977), "Une histoire scientifique : la nouvelle histoire économique", *Annales. Economies, Sociétés, Civilisations*, 32, p. 824-842.

Herbert-Rothe, A. (2005), "Clausewitz und Napoleon: Jena, Moskau, Waterloo", in *Clausewitz-Gespräch vom 1. Dezember 2005*, ed. by L. Souchon, Hamburg, Internationales Clausewitz-Zentrum, Führungsakademie der Bundeswehr.

Hofschröer, P. (1998-1999), *The Waterloo Campaign*, Londres, Greenhill Books, 2 vol.

Houssaye, H. (1905-1906), 1815, 3 vol., Paris, Perrin et Compagnie. Reprint of vol. 2, *Waterloo*, Paris, Librairie académique Perrin, 1961.

Largeaud, J.M. (2006), *Napoléon et Waterloo: la défaite glorieuse de 1815 à nos jours*, Paris, La Boutique de l'Histoire.

Las Cases, E. de (1823), *Le Mémorial de Sainte-Hélène*. Reprint Paris, Garnier, 2 vol., 1961. Eng. trans. *Memoirs of the Life, Exile and Conversations of the Emperor Napoleon by the Count of Las Cases*, New York, W.J. Widdleton, 4 vol., 1879,

Leonard, R. (2010), *Von Neumann, Morgenstern and the Creation of Game Theory: from Chess to Social Science*, New York, Cambridge University Press.

Levi, M. (2002), "Modeling Complex Historical Processes with Analytic Narratives", in R. Mayntz, *Akteure-Mechanismen-Modelle*, Campus, Schriften des Max-Planck-Institute für Gesellschatsforschung Köln, 42, p. 108-127.

Levi, M. (2004), "An Analytic Narrative Approach to Puzzles and Problems", in I. Shapiro, R. Smith and T. Masoud (eds), *Problems and Methods in the Study of Politics*, p. 201-226. New York, Cambridge University Press.

Luce, R.D and H. Raiffa (1957), *Games and Decisions*, New York, Wiley.

Mauduit, H. de (1847), *Histoire des derniers jours de la Grande Armée*. Reprint Paris, L.C.V. Services, Collection Mémoires d'Empire, 2006.

McCloskey, D. (1978), "The Achievements of the Cliometric School", *Journal of Economic History*, 38, p. 13-28.

Meade, E.M. (1943), *Makers of Modern Strategy*, Princeton University Press.

Mongin, P. (2008), "Retour à Waterloo. Histoire et théorie des jeux", *Annales. Histoire, sciences sociale*s, 63, 2008, p. 39-69

Mongin, P. (2009), "Waterloo et les regards croisés de l'interprétation", in A. Berthoz (ed.), *La pluralité interprétative*, Paris, Odile Jacob.

Mongin, P. (2010), "Analytic Narrative", in J.E. Alt, S. Chambers, G. Garrett, M. Levi, P. McClain (eds), *The Encyclopedia of Political Science*, 1, p. 48-51.

Mongin, P. (2016), "What Are Analytic Narratives?", HEC Paris Research Paper No. ECO/SCD-2016-1155. Available at SSRN: https://ssrn.com/abstract=2796567

Nash, J.F. (1950), "Equilibrium Points in n-Person Games", *Proceedings of the National Academy of Sciences*, 36, p. 286-162.

von Neumann, J. (1928), "Zur Theorie der Gesellschaftsspiele", *Mathematische Annalen*,

100, p. 295-320.

von Neumann, J. et O. Morgenstern (1944), *Theory of Games and Economic Behavior*, New York, Wiley (2nd rev. ed., 1947).

North, D.C. (1990), *Institutions, Institutional Change, and Economic Performance*, Cambridge, Cambridge University Press.

O'Neill, B. (1994), "Game Theory Models of Peace and War", ch. 29 in R.J. Aumann and S. Hart (eds), *Handbook of Game Theory with Economic Applications*, vol. 2, Amsterdam, Elsevier.

Paret, P. (1992), *Understanding War. Essays on Clausewitz and the History of Military Power, Princeton*, Princeton University Press.

Pareto, V. (1917-1919), *Traité de sociologie générale*. Reprinted as vol. XII in *Œuvres complètes*, Genève, Droz, 1968. Eng. tr. by A. Bongiorno and A. Livingston, *The Mind and Society: a Treatise on General Sociology*, Dover, New York, 19363.

Popper, K. (1967), "La rationalité et le statut du principe de rationalité", in E.M. Claassen (ed.), *Les fondements philosophiques des systèmes économiques*, Paris, Payot, p. 142-150. Eng. version "The Rationality Principle", in D. Miller (ed.), *Popper Selections*, Princeton, Princeton University Press, 1982, p. 357-365.

Roberts, A. (2005), *Waterloo*, London, HarperCollins.

Schelling, T. (1960), *The Strategy of Conflict*, Cambridge, Mass., Harvard University Press.

Weber, M. (1922a), *Wirtschaft und Gesellschaft*, Tübingen, Mohr et Siebeck. Eng. tr. by G. Roth, C. Wittich, E. Fischoff, *Economy and Society: An Outline of Interpretive Sociology*, University of California Press, 1978

Weber, M. (1922b), *Gesammelte Aufsätze zur Wissenschaftslehre*, Tübingen, Mohr et Siebeck. Partial Eng. tr. by E. A. Shils and H. A. Finch, *The Methodology of the Social Sciences*, New York, Free Press, 1949.

Zagare, F.C. (2011), *The Games of July: Explaining the Great War*, Ann Arbor, University of Michigan Press.

Zagare, F.C. (2015), "The Moroccan Crisis of 1905-1906: An Analytic Narrative", *Peace Economics, Peace Science and Public Policy*, 21, p. 327-350.

APPENDIX

As stated in the text, the zero-sum game between Napoleon and Blücher has a unique equilibrium (S_2, B_2B_1) if some optional conditions hold, beyond those which are automatically ensured by the definitions of final payoffs, probabilities and the abatement coefficient θ. The present appendix gives some details on the computation of this equilibrium.

Let us label Napoleon's payoffs in the following way:

$$B_1B_1 \quad B_1B_2 \quad B_2B_1 \quad B_2B_2$$

$$
\begin{array}{lcccc}
S_1 & V_{11} & V_{12} & V_{13} & V_{14} \\
S_2 & V_{21} & V_{22} & V_{23} & V_{24} \\
S_3 & V_{31} & V_{32} & V_{33} & V_{34}
\end{array}
$$

Each V_{ij} is obtained by an expected utility calculation:

$$V_{11} = k\big[l'(a_1 + a_2) + (1 - l')(b_1 + b_2)\big] + (1 - k)\big[l(a_1 + a_2) + (1 - l)(b_1 + b_2)\big]$$
$$V_{12} = k\big[l'(a_1 + a_2) + (1 - l')(b_1 + b_2)\big] + (1 - k)\big[a_1 + \theta a_2\big]$$
$$V_{13} = k(a_1 + \theta a_2) + (1 - k)\big[l(a_1 + a_2) + (1 - l)(b_1 + b_2)\big]$$
$$V_{14} = a_1 + \theta a_2$$
$$V_{21} = k\big[ma_1 + (1 - m)b_1 + a_2\big] + (1 - k)\big[ma_1 + (1 - m)b_1 + b_2\big]$$
$$V_{22} = k\big[ma_1 + (1 - m)b_1 + a_2\big] + (1 - k)\big[ma_1 + (1 - m)b_1 + \theta a_2\big]$$
$$V_{23} = k\big[ma_1 + (1 - m)b_1 + \theta a_2\big] + (1 - k)\big[ma_1 + (1 - m)b_1 + b_2\big]$$
$$V_{24} = ma_1 + (1 - m)b_1 + \theta a_2$$
$$V_{31} = k\big[l''(a_1 + a_2) + (1 - l'')(b_1 + b_2)\big] + (1 - k)(b_1 + b_2)$$
$$V_{32} = k\big[l''(a_1 + a_2) + (1 - l'')(b_1 + b_2)\big] + (1 - k)\big[ma_1 + (1 - m)b_1 + b_2\big]$$
$$V_{33} = k\big[ma_1 + (1 - m)b_1 + a_2)\big] + (1 - k)(b_1 + b_2)$$
$$V_{34} = k\big[ma_1 + (1 - m)b_1 + a_2)\big] + (1 - k)\big[ma_1 + (1 - m)b_1 + b_2)\big]$$

The definitions of k, m, l, l', l'', θ and the sign restrictions on a_1, b_1, a_2, b_2 imply a number of inequalities between the V_{ij}:

$V_{11} > V_{31}, \; V_{21} > V_{31}, \; V_{12} > V_{32}, \; V_{22} > V_{32}, \; V_{14} > V_{24},$
$V_{22} > V_{21}, \; V_{21} > V_{23}, \; V_{22} > V_{23}, \; V_{24} > V_{23},$
$V_{32} > V_{31}, \; V_{33} > V_{31}, \; V_{34} > V_{33}.$

The strategic analysis on Napoleon's side proceeds as follows. It is the case that:

$$V_{14} > V_{13} \text{ and } V_{12} > V_{11} \text{ iff } (*) \; \frac{a_1 + \theta a_2 - b_1 - b_2}{a_1 + a_2 - b_1 - b_2} > l,$$

and:

$V_{13} > V_{11}$ iff (*') $\dfrac{a_1 + \theta a_2 - b_1 - b_2}{a_1 + a_2 - b_1 - b_2} > l'$.

We assume both (*) and (*') to hold, thus ensuring that $V_{11} = V_1$ is the security payoff of S_1 (cf. fn. 32). By inspecting the definitional inequalities, we observe that $V_{23} = V_2$ is the security payoff of S_2 and that $V_{31} = V_3$ is the security payoff of S_3.

Now to compare the three values V_1, V_2, V_3. In view of (*'), the inequality $V_2 > V_1$ can be obtained from $V_2 > V_{13}$, which is equivalent to:
$$(m - k)(a_1 - b_1) > (l - kl)(a_1 - b_1 + a_2 - b_2),$$
or:

(**) $m > k(1 - ld) + ld$, with $d = \dfrac{a_1 - b_1 + a_2 - b_2}{a_1 - b_1}$.

This inequality makes sense only if the right-hand side is between 0 and 1, i.e., only if $m > 0$, $k < 1$, and

(**') $l < 1/d$,

which implies that $l < 1$. We impose these conditions (cf. fn. 33). Notice that (**) implies that $m > k$, a condition to which we return below.

Since $V_{33} > V_3$ holds, the inequality $V_2 > V_3$ can be obtained from $V_2 > V_{33}$, or equivalently:

(***) $m > (\dfrac{k}{1 - k})(1 - \theta)(\dfrac{a_2}{a_1 - b_1})$,

which requires that $k < 1$. The right-hand side is less than 1 under one of the two conditions:

(***') $k < \dfrac{1}{2}$, $a_2 < a_1 - b_1$,

or:

(***") $\theta > k$, $a_2 < a_1 - b_1$.

We assume (***) to hold, as well as either (***') or (***") (cf. fn. 33). Hence, V_2 is Napoleon's maxmin.

Here are the computations on Blücher's side. From what has just been shown in the last paragraphs, $-V_2$ is the security payoff of the conditional strategy B_2B_1. We will show that it is also the maxmin by checking that no other strategy can deliver a higher security payoff.

Concerning B_1B_1: from definitional inequalities, the security payoff is either $-V_{11}$ or $-V_{21}$. It cannot be $-V_{11}$ because $V_{11} > V_{21}$ would imply a cycle, given that $V_{21} > V_2 > V_{13} > V_{11}$; so it is $-V_{21}$, which cannot be the maxmin, given the first inequality in this sequence.
Concerning B_1B_2: definitional inequalities entail $-V_{12}$ or $-V_{22}$ being the security payoff, but neither can be the maxmin because $V_{22} > V_2$ holds (if $-V_{22}$ is the security payoff, it falls below $-V_2$, and the same if it is $-V_{12}$, since this implies $V_{12} > V_{22}$).
Concerning B_2B_2: again from definitional inequalities, either $-V_{14}$ or $-V_{24}$ is the security payoff, and by a similar argument, $V_{24} > V_2$ precludes either value from being the maxmin.

Thus, we conclude that the equilibrium of the game, in the von Neumann-Morgenstern sense, is (S_2, B_2B_1).

A Cournot-Nash equilibrium calculation would have reached the same conclusion somewhat differently and more quickly. It would have used the fact that Blücher's strategies B_2B_2 and B_1B_2 are dominated, respectively, by B_2B_1 and B_1B_1, once condition (*) is granted. So they are discarded from consideration for Napoleon too, and his strategy S_3 becomes dominated by S_2 from (***), (***') or (***"), and definitional inequalities. The game is now 2x2, and the remaining conditions, i.e., (*'), (**), (**'), ensure that (S_2, B_2B_1) is an equilibrium in the Cournot-Nash sense and that it is unique.

As mentioned in the text, the assumption that $l = 0$ simplifies the analysis. Then, (*), (*'), (**') are trivially satisfied. The binding conditions are (***), (***') or (***"), and (**), which reduces to the straightforward inequality $m > k$. Thus, in this limiting case, the necessary condition becomes sufficient.

Not all the probabilities can take extreme values. The conditions make it necessary that $m > 0$, $l < 1$ and $k < 1$, with k being further bounded from above and θ unrestrained between 0 and 1, or alternatively k and θ being mutually related. Notice that l'' is the least constrained parameter, being only subjected to the definitional inequalities l', $m > l''$.

It is trivial to find non-extreme values that satisfy all the conditions. For example, take as utility parameters:
$a_1 = 1, b_1 = -1, a_2 = 1/2, b_2 = -1/2, \theta = 1/2$,
and as probability parameters:
$l = 0.1, l' = 1/2, l'' = 1/3, k = 1/3, m = 2/3$.

BRUSSELS AND VICINITY, 1815
WATERLOO CAMPAIGN
Situation Midnight 17-18 June and
Movements Since 16 June

65

Only those secondary roads in the vicinity of Waterloo-Charleroi appear on this map. Also, only the forest south of Brussels is depicted because of its tactical significance. All other forests are omitted for purposes of clarity.
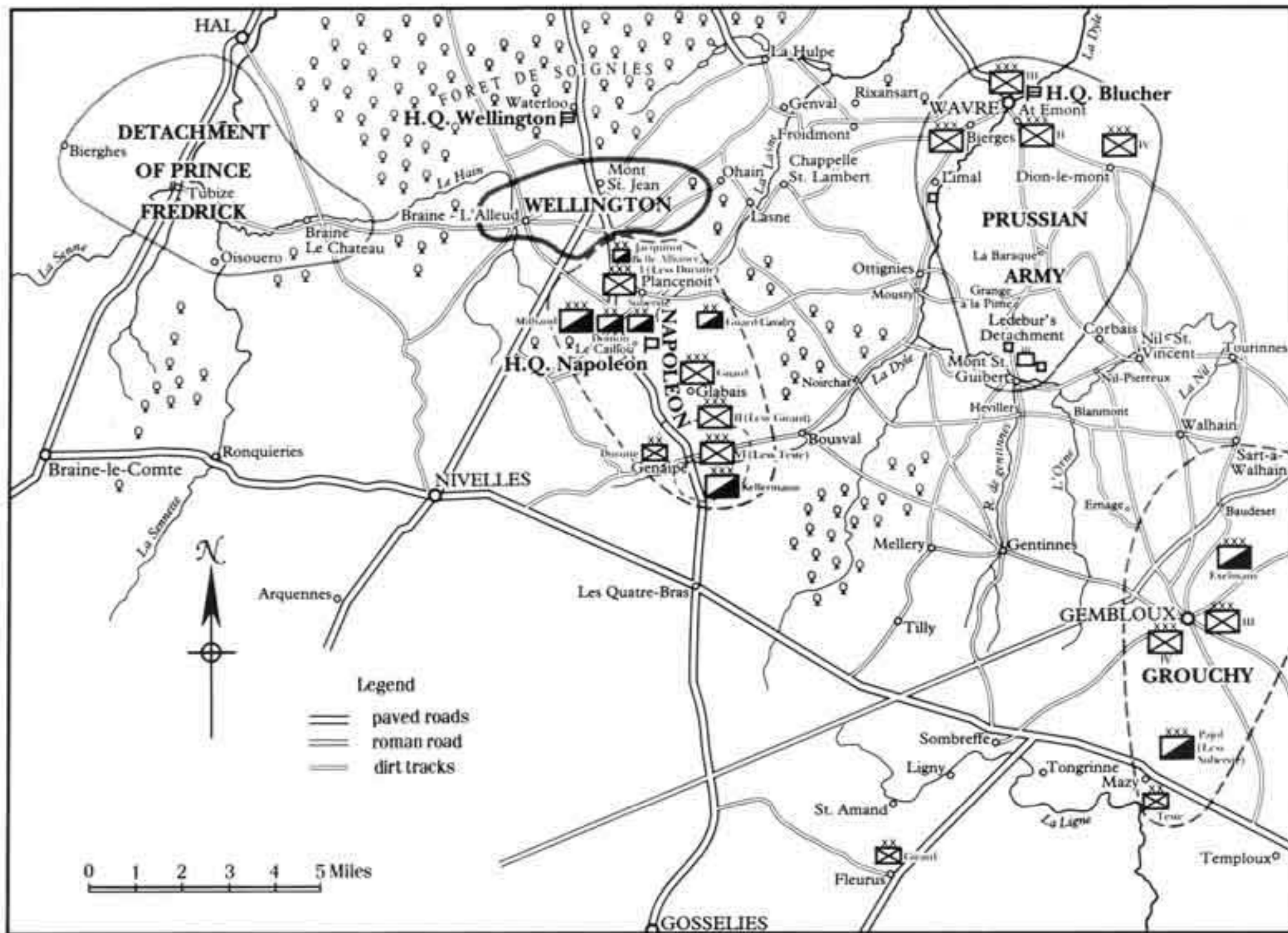
Artillery, Engineers and trains numbered 9,500 men.

The advance of Schwenzenberg and Kleist had not yet begun. These formations were still awaiting the arrival of the Russian army on the Rhine.

Mobilization of French forces continued during the campaign.

NORTH SEA

Ostend
Bruges
Antwerp
Ghent
Louvain
Maastricht
Brussels
FOREST DE SOIGNES
Lille
Tournay
Enghien
Hal
WELLINGTON (68,000)
ANGLO-DUTCH
Waterloo
Mont St. Jean
PRUSSIA
BLÜCHER (89,000)
Wavre
ZIETEN
BULOW
THIELMANN
PIRCH
Plancenoit
II HILL (25,534)
(17,000)
Ath
Braine le Comte
Nivelles
NORTH
NAPOLEON(-) (72,000)
Genappe
Quatre-Bras
Sombreffe
GROUCHY (+) (33,000)
Ligny
Mons
Fleurus
Namur
Charleroi
Malpaque
Thuin
Maubeuge
Beaumont
Dinant
Cambrai
Avesnes
Givet
Rocroi

Lys River
Escaut River
Scheldt River
Dyle River
Lasne River
Meuse River
Sambre River
Oise River
Meuse River

ELEVATION IN METERS
0   100   200   300   OVER

SCALE OF MILES
0   5   10   15   20

N

Positions of the Armies, 17 to 18 June