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2011

Online at <https://mpra.ub.uni-muenchen.de/78089/>  
MPRA Paper No. 78089, posted 03 Apr 2017 10:12 UTC

# A framework of coepetitive games: applications to the Greek crisis

David Carfi and Daniele Schilirò

## Abstract

In the present work we propose an original analytical model of coepetitive game. We shall apply this analytical model of coepetition (based on normal form game theory) to the Greek crisis, while conceiving this game theory model at a macro level. We construct two realizations of such model, trying to represent possible realistic macro-economic scenarios of the Germany-Greek strategic interaction. We shall suggest - after a deep and complete study of the two samples - feasible transferable utility solutions in a properly coepetitive perspective for the divergent interests which drive the economic policies in the euro area.

**Keywords.** Games and economics; competition; cooperation; coepetition.

## 1 Introduction

Greece, a country that represents just over 2% of gross domestic product of the Euro area, has led to a deep and widespread crisis in Europe. The Greek economy is locked into a serious and difficult crisis, due to its lack of competitiveness and is still at risk of insolvency, because of its public finance mismanagement. Although the EU Governments and the IMF have provided substantial financial assistance to cover the refinancing needs of Greece in exchange of a serious and tough austerity program, and even, the banks have agreed a heavy haircut on the Greek debts, the Greek economy is still on the brink of collapse. Germany, on the other hand, is the most competitive economy of the Euro Area and has a large trade surplus with Greece and other Euro partners; hence significant trade imbalances occur within the Euro Area.

The main purpose of our paper is to explore win-win solutions for Greece and Germany, involving a German increasing demand of a pre-determined Greek exports. We do not analyze the causes of the financial crisis in Greece and its relevant political and institutional effects on the European Monetary Union. Rather we concentrate on stability and growth, which should drive the economic policy of Greece and the other Euro countries.

**Organization of the paper.** The work is organized as follows:

- section 2 examines the Greek crisis, suggesting a possible way out to reduce the intra-eurozone imbalances through coepetitive solutions within a growth path;
- section 3 presents the original model of coepetitive game introduced in the literature by D. Carfi;
- section 4 proposes possible solutions concepts for the original model of coepetitive game;
- section 5, very briefly, provides the general ideas we follow, in this paper, for the applications of coepetitive games to the Eurozone context;
- section 6 provides a first sample of coepetitive game applied to the Eurozone context, showing possible coepetitive solutions; we propose the first sample in an intentionally simplified fashion (without direct strategic interactions among players) to emphasize the new role and procedures of coepetition;
- section 7 provides a second sample of coepetitive game applied to the Eurozone context, showing possible coepetitive solutions; we propose a linear model, with a direct strategic interactions among players;
- conclusions end up the paper.

Section 2 of this paper has been written by D. Schilirò while sections from 3 to 7 are due to D. Carfi, however, in strict joint cooperation. Abstract, introduction and conclusions were written by both authors, together.

## **2 The Greek Crisis and the coepetitive solution**

The deep financial crisis of Greece, which was almost causing the default of its sovereign debt, has revealed the weaknesses of Greek economy, particularly its lack of competitiveness, but also the mismanagement of the public finance and the difficulties of the banking sector.

### **2.1 The crisis and the Greek economy**

With the outbreak of the global crisis of 2008-2009, Greece relied on state spending to drive growth, so the country has accumulated a huge public debt. This has created deep concerns about its fiscal sustainability, and its financial exposition has prevented the Greek government to find capitals in the financial markets. In addition, Greece has lost competitiveness since joining the European Monetary Union and, because of that,

Greek's unit labor cost rose over 30 percent from 2000 to 2010. The austerity measures implemented by the Greek government are hitting hard the Greek economy, since its growth is continuing to be expected negative also this year (2012), making the financial recovery very problematic [Mussa (2010)]. Furthermore, Greece exports are much less than imports, so the trade balance shows a deficit above 10% of its GDP. Therefore, the focus of economic policy of Greece should become its productive system and growth must be the major goal for the Greek economy in a medium term perspective. This surely would help its re-equilibrium process.

## **2.2 The soundest European economy: Germany**

Germany, on the other hand, is considered the soundest European economy. It is the world's second-biggest exporter, but its wide commercial surplus is originated mainly by the exports in the Euro area, that accounts for about two thirds. Furthermore, since 2000 its export share has gradually increased vis-à-vis industrial countries. Thus Germany's growth path has been driven by exports. We do not discuss in this work the factors explaining Germany's increase in export share, but we observe that its international competitiveness has been improving, with the unit labor cost which has been kept fairly constant, since wages have essentially kept pace with productivity. Therefore the prices of the German products have been relatively low, favoring the export of German goods towards the euro countries and towards the markets around the world, especially those of the emerging economies (China, India, Brazil, Russia). Finally, since 2010 Germany has recovered very well from the 2008-2009 global crisis and it is growing at a higher rate than the others Euro partners.

Therefore we share the view that Germany (and the other surplus countries of the Euro area) should contribute to overcome the crisis of Greek economy stimulating its demand of goods from Greece and relying less on exports towards the Euro area in general. Germany, as some economists as Posen [2010] and Abadi [2010] underlined, has benefited from being the anchor economy for the Eurozone over the last 11 years. For instance, in 2009, during a time of global contraction, Germany has been a beneficiary, being able to run a sustained trade surplus with its European neighbors. Germany is exporting to Greece far more than it imports.

## **2.3 A win-win solution for Greece and Germany**

Thus we believe that an economic policy that aims at adjusting government budget and trade imbalances and looks at improving the growth path of the real economy in the medium and long term in Greece is the only possible one to assure a stable re-balancing of the Greek economy and also to contribute to the stability of the whole euro area [Schilirò, 2011]. As we have already argued, German modest wage increases and weak domestic demand favored the export of German goods towards the euro countries.

We suggest, in accordance with Posen [2010], a win-win solution (a win-win solution is the outcome of a game which is designed in a way that all participants can profit from it in one way or the other), which entails that Germany, which still represents the leading economy, should re-balance its trade surplus and thus ease the pressure on the southern countries of the euro area, particularly Greece. Obviously, we are aware that this is a mere hypothesis and that our framework of coepetition is a normative model. However, we believe that a cooperative attitude must be taken within the members of the European monetary union. Thus we pursue our hypothesis and suggest a model of coepetitive game as an innovative instrument to analyze possible solutions to obtain a win-win outcome for Greece and Germany, which would also help the whole economy of the euro area.

## **2.4 Our coepetitive model**

The two strategic variables of our model are investments and exports for Greece, since this country must concentrate on them to improve the structure of production and its competitiveness, but also shift its aggregate demand towards a higher growth path in the medium term. Thus Greece should focus on innovative investments, specially investments in knowledge [Schilirò, 2010], to change and improve its production structure and to increase its production capacity and its productivity. As a result of that its competitiveness will improve. An economic policy that focuses on investments and exports, instead of consumptions, will address Greece towards a sustainable growth and, consequently, its financial reputation and economic stability will also increase. On the other hand, the strategic variable of our model for Germany are private consumption and imports.

The idea which is driving our model to solve the Greek crisis is based on a notion of coepetition where the cooperative aspect will prevail. Thus we are not talking about a situation in which Germany and Greece are competing in the same European market for the same products, rather we are assuming a situation in which Germany stimulates its domestic demand and, in doing so, will create a larger market for products from abroad. We are also envisaging the case where Germany purchases a greater quantity of Greek products, in this case Greece increases its exports, selling more products to Germany. The final results will be that Greece will find itself in a better position, but also Germany will get an economic advantage determined by the higher growth in the two countries. In addition, there is the important advantage of a greater stability within the European Monetary system. Finally our model will provide a new set of tools based on the notion of coepetition, that could be fruitful for the setting of the euro area economic policy issues.

## 2.5 The coopetition in our model

The concept of coopetition was essentially devised at micro-economic level for strategic management solutions by Brandenburger and Nalebuff [1995], who suggest, given the competitive paradigm [Porter, 1985], to consider also a cooperative behavior to achieve a win-win outcome for both players. Therefore, in our model, coopetition represents the synthesis between the competitive paradigm [Porter, 1985] and the cooperative paradigm [Gulati, Nohria, Zaheer, 2000; Stiles, 2001]. Coopetition is, in our approach, a complex theoretical construct and it is the result of the interplay between competition and cooperation. Thus, we suggest a model of coooperative games, applied at a macroeconomic level, which intends to offer possible solutions to the partially divergent interests of Germany and Greece in a perspective of a cooperative attitude that should drive their policies.

## 3 Coooperative games

### 3.1 Introduction

In this paper we develop and apply the mathematical model of a *coooperative game* introduced by David Carfi in [11] and [6]. The idea of coooperative game is already used, in a mostly intuitive and non-formalized way, in Strategic Management Studies (see for example Brandenburgher and Nalebuff).

#### 3.1.1 The idea.

A coooperative game is a game in which two or more players (participants) can interact *cooperatively and non-cooperatively at the same time*. Even Brandenburger and Nalebuff, creators of coopetition, did not define, precisely, a *quantitative way to implement coopetition* in the Game Theory context.

The problem to implement the notion of coopetition in Game Theory is summarized in the following question:

- *how do, in normal form games, cooperative and non-cooperative interactions can live together simultaneously, in a Brandenburger-Nalebuff sense?*

In order to explain the above question, consider a classic two-player normal-form gain game  $G = (f, >)$  - such a game is a pair in which  $f$  is a vector valued function defined on a Cartesian product  $E \times F$  with values in the Euclidean plane  $\mathbb{R}^2$  and  $>$  is the natural strict sup-order of the Euclidean plane itself (the sup-order is indicating that the game, with payoff function  $f$ , is a gain game and not a loss game). Let  $E$  and  $F$  be the strategy sets of the two players in the game  $G$ . The two players can choose the respective strategies  $x \in E$  and  $y \in F$

- cooperatively (exchanging information and making binding agreements);
- not-cooperatively (not exchanging information or exchanging information but without possibility to make binding agreements).

The above two behavioral ways are mutually exclusive, at least in normal-form games:

- the two ways cannot be adopted simultaneously in the model of normal-form game (without using convex probability mixtures, but this is not the way suggested by Brandenburger and Nalebuff in their approach);
- there is no room, in the classic normal form game model, for a simultaneous (non-probabilistic) employment of the two behavioral extremes *cooperation* and *non-cooperation*.

### 3.1.2 Towards a possible solution.

Firstly David Carfi in [6] and then D. Carfi with D. Schilirò in [11] have proposed a manner to overcome this *impasse*, according to the idea of coepetition in the sense of Brandenburger and Nalebuff. In a Carfi's coepetitive game model,

- the players of the game have their respective strategy-sets (in which they can choose cooperatively or not cooperatively);
- there is a common strategy set  $C$  containing other strategies (possibly of different type with respect to those in the respective classic strategy sets) that *must be chosen cooperatively*;
- the strategy set  $C$  can also be structured as a Cartesian product (similarly to the profile strategy space of normal form games), but in any case the strategies belonging to this new set  $C$  must be chosen cooperatively.

## 3.2 The model for $n$ -players

We give in the following the definition of coepetitive game proposed by Carfi (in [11] and [6]).

**Definition (of  $n$ -player coepetitive game).** Let  $E = (E_i)_{i=1}^n$  be a finite  $n$ -family of non-empty sets and let  $C$  be another non-empty set. We define  **$n$ -player coepetitive gain game over the strategy support**  $(E, C)$  any pair  $G = (f, >)$ , where  $f$  is a vector function from the Cartesian product  $\times E \times C$  (here  $\times E$  denotes the classic strategy-profile space of  $n$ -player normal form games, i.e. the Cartesian product of the family  $E$ ) into the  $n$ -dimensional Euclidean space  $\mathbb{R}^n$  and  $>$  is the natural sup-order of this last Euclidean space. The element of the set  $C$  will be called **cooperative strategies of the game**.

A particular aspect of our cooperative game model is that any cooperative game  $G$  determines univocally a family of classic normal-form games and vice versa; so that any cooperative game could be defined as a family of normal-form games. In what follows we precise this very important aspect of the model.

**Definition (the family of normal-form games associated with a cooperative game).** Let  $G = (f, >)$  be a cooperative game over a strategic support  $(E, C)$ . And let

$$g = (g_z)_{z \in C}$$

be the family of classic normal-form games whose member  $g_z$  is, for any cooperative strategy  $z$  in  $C$ , the normal-form game

$$G_z := (f(\cdot, z), >),$$

where the payoff function  $f(\cdot, z)$  is the section

$$f(\cdot, z) : \times E \rightarrow \mathbb{R}^n$$

of the function  $f$ , defined (as usual) by

$$f(\cdot, z)(x) = f(x, z),$$

for every point  $x$  in the strategy profile space  $\times E$ . We call the family  $g$  (so defined) **family of normal-form games associated with (or determined by) the game  $G$**  and we call **normal section** of the game  $G$  any member of the family  $g$ .

We can prove this (obvious) theorem.

**Theorem.** *The family  $g$  of normal-form games associated with a cooperative game  $G$  uniquely determines the game. In more rigorous and complete terms, the correspondence  $G \mapsto g$  is a bijection of the space of all cooperative games - over the strategy support  $(E, C)$  - onto the space of all families of normal form games - over the strategy support  $E$  - indexed by the set  $C$ .*

*Proof.* This depends totally from the fact that we have the following natural bijection between function spaces:

$$\mathcal{F}(\times E \times C, \mathbb{R}^n) \rightarrow \mathcal{F}(C, \mathcal{F}(\times E, \mathbb{R}^n)) : f \mapsto (f(\cdot, z))_{z \in C},$$

which is a classic result of theory of sets. ■

Thus, the exam of a cooperative game should be equivalent to the exam of a whole family of normal-form games (in some sense we shall specify).

In this paper we suggest how this latter examination can be conducted and what are the solutions corresponding to the main concepts of solution which are known in the literature for the classic normal-form games, in the case of two-player cooperative games.



### 3.3 Two players coepetitive games

In this section we specify the definition and related concepts of two-player coepetitive games; sometimes (for completeness) we shall repeat some definitions of the preceding section.

**Definition (of coepetitive game).** *Let  $E$ ,  $F$  and  $C$  be three nonempty sets. We define two player coepetitive gain game carried by the strategic triple  $(E, F, C)$  any pair of the form  $G = (f, >)$ , where  $f$  is a function from the Cartesian product  $E \times F \times C$  into the real Euclidean plane  $\mathbb{R}^2$  and the binary relation  $>$  is the usual sup-order of the Cartesian plane (defined component-wise, for every couple of points  $p$  and  $q$ , by  $p > q$  iff  $p_i > q_i$ , for each index  $i$ ).*

**Remark (coepetitive games and normal form games).** The difference among a two-player normal-form (gain) game and a two player coepetitive (gain) game is the fundamental presence of the third strategy Cartesian-factor  $C$ . The presence of this third set  $C$  determines a total change of perspective with respect to the usual exam of two-player normal form games, since we now have to consider a normal form game  $G(z)$ , for every element  $z$  of the set  $C$ ; we have, then, to study an entire ordered family of normal form games in its own totality, and we have to define a new manner to study these kind of game families.

### 3.4 Terminology and notation

**Definitions.** *Let  $G = (f, >)$  be a two player coepetitive gain game carried by the strategic triple  $(E, F, C)$ . We will use the following terminologies:*

- *the function  $f$  is called the **payoff function of the game  $G$** ;*
- *the first component  $f_1$  of the payoff function  $f$  is called **payoff function of the first player** and analogously the second component  $f_2$  is called **payoff function of the second player**;*
- *the set  $E$  is said **strategy set of the first player** and the set  $F$  the **strategy set of the second player**;*
- *the set  $C$  is said the **cooperative (or common) strategy set of the two players**;*
- *the Cartesian product  $E \times F \times C$  is called the **(coepetitive) strategy space of the game  $G$** .*

**Memento.** The first component  $f_1$  of the payoff function  $f$  of a coepetitive game  $G$  is the function of the strategy space  $E \times F \times C$  of the game  $G$  into the real line  $\mathbb{R}$  defined by the first projection

$$f_1(x, y, z) := \text{pr}_1(f(x, y, z)),$$

for every strategic triple  $(x, y, z)$  in  $E \times F \times C$ ; in a similar fashion we proceed for the second component  $f_2$  of the function  $f$ .

**Interpretation.** We have:

- two players, or better an ordered pair  $(1, 2)$  of players;
- anyone of the two players has a strategy set in which to choose freely his own strategy;
- the two players can/should *cooperatively* choose strategies  $z$  in a third common strategy set  $C$ ;
- the two players will choose (after the exam of the entire game  $G$ ) their cooperative strategy  $z$  in order to maximize (in some sense we shall define) the vector gain function  $f$ .

### 3.5 Normal form games of a coepetitive game

Let  $G$  be a coepetitive game in the sense of above definitions. For any cooperative strategy  $z$  selected in the cooperative strategy space  $C$ , there is a corresponding normal form gain game

$$G_z = (p(z), >),$$

upon the strategy pair  $(E, F)$ , where the payoff function  $p(z)$  is the section

$$f(., z) : E \times F \rightarrow \mathbb{R}^2,$$

of the payoff function  $f$  of the coepetitive game - the section is defined, as usual, on the competitive strategy space  $E \times F$ , by

$$f(., z)(x, y) = f(x, y, z),$$

for every bi-strategy  $(x, y)$  in the bi-strategy space  $E \times F$ .

Let us formalize the concept of game-family associated with a coepetitive game.

**Definition (the family associated with a coepetitive game).** Let  $G = (f, >)$  be a two player coepetitive gain game carried by the strategic triple  $(E, F, C)$ . We naturally can associate with the game  $G$  a family  $g = (g_z)_{z \in C}$  of normal-form games defined by

$$g_z := G_z = (f(., z), >),$$

for every  $z$  in  $C$ , which we shall call **the family of normal-form games associated with the coepetitive game  $G$** .

**Remark.** It is clear that with any above family of normal form games

$$g = (g_z)_{z \in C},$$

with  $g_z = (f(., z), >)$ , we can associate:

- a family of payoff spaces

$$(\text{im}f(\cdot, z))_{z \in C},$$

with members in the payoff universe  $\mathbb{R}^2$ ;

- a family of Pareto maximal boundary

$$(\partial^* G_z)_{z \in C},$$

with members contained in the payoff universe  $\mathbb{R}^2$ ;

- a family of suprema

$$(\sup G_z)_{z \in C},$$

with members belonging to the payoff universe  $\mathbb{R}^2$ ;

- a family of Nash zones

$$(\mathcal{N}(G_z))_{z \in C};$$

with members contained in the strategy space  $E \times F$ ;

- a family of conservative bi-values

$$v^\# = (v_z^\#)_{z \in C};$$

in the payoff universe  $\mathbb{R}^2$ .

And so on, for every meaningful known feature of a normal form game.

Moreover, we can interpret any of the above families as *set-valued paths* in the strategy space  $E \times F$  or in the payoff universe  $\mathbb{R}^2$ .

It is just the study of these induced families which becomes of great interest in the examination of a cooperative game  $G$  and which will enable us to define (or suggest) the various possible solutions of a cooperative game.

## 4 Solutions of a cooperative game

### 4.1 Introduction

The two players of a cooperative game  $G$  - according to the general economic principles of *monotonicity of preferences* and of non-satiation - should choose the cooperative strategy  $z$  in  $C$  in order that:

- the reasonable Nash equilibria of the game  $G_z$  are  $f$ -preferable than the reasonable Nash equilibria in each other game  $G_{z'}$ ;
- the supremum of  $G_z$  is greater (in the sense of the usual order of the Cartesian plane) than the supremum of any other game  $G_{z'}$ ;
- the Pareto maximal boundary of  $G_z$  is higher than that of any other game  $G_{z'}$ ;
- the Nash bargaining solutions in  $G_z$  are  $f$ -preferable than those in  $G_{z'}$ ;
- *in general, fixed a common kind of solution for any game  $G_z$ , say  $S(z)$  the set of these kind of solutions for the game  $G_z$ , we can consider the problem to find all the optimal solutions (in the sense of Pareto) of the set valued path  $S$ , defined on the cooperative strategy set  $C$ . Then, we should face the problem of **selection of reasonable Pareto strategies** in the set-valued path  $S$  via proper selection methods (Nash-bargaining, Kalai-Smorodinsky and so on).*

Moreover, we shall consider the maximal Pareto boundary of the payoff space  $\text{im}(f)$  as an appropriate zone for the bargaining solutions.

The payoff function of a two person cooperative game is (as in the case of normal-form game) a vector valued function with values belonging to the Cartesian plane  $\mathbb{R}^2$ . We note that in general the above criteria are multi-criteria and so they will generate multi-criteria optimization problems.

In this section we shall define rigorously some kind of solution, for two player cooperative games, based on a bargaining method, namely a Kalai-Smorodinsky bargaining type. Hence, first of all, we have to precise what kind of bargaining method we are going to use.

## 4.2 Bargaining problems

In this paper, we shall propose and use the following original *extended* (and quite general) definition of bargaining problem and, consequently, a natural generalization of Kalai-Smorodinsky solution. In the economic literature, several examples of *extended bargaining problems* and *extended Kalai-Smorodinsky solutions* are already presented. The essential root of these various extended versions of bargaining problems is the presence of *utopia points* not-directly constructed by the disagreement points and the strategy constraints. Moreover, the Kalai-type solution, of such extended bargaining problems, is always defined as a Pareto maximal point belonging to the segment joining the disagreement point with the utopia point (if any such Pareto point does exist): we shall follow the same way. In order to find suitable new win-win solutions of our realistic cooperative economic problems, we need such new kind of versatile extensions. For what concerns the existence of our new extended Kalai solutions, for the economic problems we are facing, we remark that conditions of compactness and strict convexity

will naturally hold; we remark, otherwise, that, in this paper, we are not interested in proving general or deep mathematical results, but rather to find reasonable solutions for new economic cooperative context.

**Definition (of bargaining problem).** Let  $S$  be a subset of the Cartesian plane  $\mathbb{R}^2$  and let  $a$  and  $b$  be two points of the plane with the following properties:

- they belong to the small interval containing  $S$ , if this interval is defined (indeed, it is well defined if and only if  $S$  is bounded and it is precisely the interval  $[\inf S, \sup S]_{\leq}$ );
- they are such that  $a < b$ ;
- the intersection

$$[a, b]_{\leq} \cap \partial^* S,$$

among the interval  $[a, b]_{\leq}$  with end points  $a$  and  $b$  (it is the set of points greater than  $a$  and less than  $b$ , **it is not** the segment  $[a, b]$ ) and the maximal boundary of  $S$  is non-empty.

In these conditions, we call **bargaining problem on  $S$  corresponding to the pair of extreme points  $(a, b)$** , the pair

$$P = (S, (a, b)).$$

Every point in the intersection among the interval  $[a, b]_{\leq}$  and the Pareto maximal boundary of  $S$  is called **possible solution of the problem  $P$** . Some time the first extreme point of a bargaining problem is called **the initial point of the problem** (or **disagreement point** or **threat point**) and the second extreme point of a bargaining problem is called **utopia point** of the problem.

In the above conditions, when  $S$  is convex, the problem  $P$  is said to be convex and for this case we can find in the literature many existence results for solutions of  $P$  enjoying prescribed properties (Kalai-Smorodinsky solutions, Nash bargaining solutions and so on ...).

**Remark.** Let  $S$  be a subset of the Cartesian plane  $\mathbb{R}^2$  and let  $a$  and  $b$  two points of the plane belonging to the smallest interval containing  $S$  and such that  $a \leq b$ . Assume the Pareto maximal boundary of  $S$  be non-empty. If  $a$  and  $b$  are a lower bound and an upper bound of the maximal Pareto boundary, respectively, then the intersection

$$[a, b]_{\leq} \cap \partial^* S$$

is obviously not empty. In particular, if  $a$  and  $b$  are the extrema of  $S$  (or the extrema of the Pareto boundary  $S^* = \partial^* S$ ) we can consider the following bargaining problem

$$P = (S, (a, b)), \text{ (or } P = (S^*, (a, b)))$$

and we call this particular problem a **standard bargaining problem on  $S$**  (or **standard bargaining problem on the Pareto maximal boundary  $S^*$** ).

### 4.3 Kalai solution for bargaining problems

Note the following property.

**Property.** *If  $(S, (a, b))$  is a bargaining problem with  $a < b$ , then there is at most one point in the intersection*

$$[a, b] \cap \partial^* S,$$

where  $[a, b]$  is the **segment joining the two points**  $a$  and  $b$ .

*Proof.* Since if a point  $p$  of the segment  $[a, b]$  belongs to the Pareto boundary  $\partial^* S$ , no other point of the segment itself can belong to Pareto boundary, since the segment is a totally ordered subset of the plane (remember that  $a < b$ ). ■

**Definition (Kalai-Smorodinsky).** *We call **Kalai-Smorodinsky solution** (or **best compromise solution**) of the bargaining problem  $(S, (a, b))$  the unique point of the intersection*

$$[a, b] \cap \partial^* S,$$

if this intersection is non empty.

So, in the above conditions, the Kalai-Smorodinsky solution  $k$  (if it exists) enjoys the following property: there is a real  $r$  in  $[0, 1]$  such that

$$k = a + r(b - a),$$

or

$$k - a = r(b - a),$$

hence

$$\frac{k_2 - a_2}{k_1 - a_1} = \frac{b_2 - a_2}{b_1 - a_1},$$

if the above ratios are defined; these last equality is the *characteristic property of Kalai-Smorodinsky solutions*.

We end the subsection with the following definition.

**Definition (of Pareto boundary).** *We call **Pareto boundary** every subset  $M$  of an ordered space which has only pairwise incomparable elements.*

#### 4.4 Nash (proper) solution of a coepetitive game

Let  $N := \mathcal{N}(G)$  be the union of the Nash-zone family of a coepetitive game  $G$ , that is the union of the family  $(\mathcal{N}(G_z))_{z \in C}$  of all Nash-zones of the game family  $g = (g_z)_{z \in C}$  associated to the coepetitive game  $G$ . We call *Nash path of the game  $G$*  the multi-valued path

$$z \mapsto \mathcal{N}(G_z)$$

and Nash zone of  $G$  the trajectory  $N$  of the above multi-path. Let  $N^*$  be the Pareto maximal boundary of the Nash zone  $N$ . We can consider the bargaining problem

$$P_{\mathcal{N}} = (N^*, \inf(N^*), \sup(N^*)).$$

**Definition.** *If the above bargaining problem  $P_{\mathcal{N}}$  has a Kalai-Smorodinsky solution  $k$ , we say that  $k$  is the properly coepetitive solution of the coepetitive game  $G$ .*

The term “properly coepetitive” is clear:

- *this solution  $k$  is determined by cooperation on the common strategy set  $C$  and to be selfish (competitive in the Nash sense) on the bi-strategy space  $E \times F$ .*

#### 4.5 Bargaining solutions of a coepetitive game

It is possible, for coepetitive games, to define other kind of solutions, which are not properly coepetitive, but realistic and sometime affordable. These kind of solutions are, we can say, *super-cooperative*.

Let us show some of these kind of solutions.

Consider a coepetitive game  $G$  and

- its Pareto maximal boundary  $M$  and the corresponding pair of extrema  $(a_M, b_M)$ ;
- the Nash zone  $\mathcal{N}(G)$  of the game in the payoff space and its extrema  $(a_N, b_N)$ ;
- the conservative set-value  $G^\#$  (the set of all conservative values of the family  $g$  associated with the coepetitive game  $G$ ) and its extrema  $(a^\#, b^\#)$ .

We call:

- **Pareto compromise solution of the game  $G$**  the best compromise solution (K-S solution) of the problem

$$(M, (a_M, b_M)),$$

*if this solution exists;*

- **Nash-Pareto compromise solution of the game  $G$**  the best compromise solution of the problem

$$(M, (b_N, b_M))$$

if this solution exists;

- **conservative-Pareto compromise solution of the game  $G$**  the best compromise of the problem

$$(M, (b^\#, b_M))$$

if this solution exists.

## 4.6 Transferable utility solutions

Other possible compromises we suggest are the following.

Consider the transferable utility Pareto boundary  $M$  of the cooperative game  $G$ , that is the set of all points  $p$  in the Euclidean plane (universe of payoffs), between the extrema of  $G$ , such that their sum

$$+(p) := p_1 + p_2$$

is equal to the maximum value of the addition  $+$  of the real line  $\mathbb{R}$  over the payoff space  $f(E \times F \times C)$  of the game  $G$ .

**Definition (TU Pareto solution).** We call **transferable utility compromise solution of the cooperative game  $G$**  the solution of any bargaining problem  $(M, (a, b))$ , where

- $a$  and  $b$  are points of the smallest interval containing the payoff space of  $G$
- $b$  is a point strongly greater than  $a$ ;
- $M$  is the transferable utility Pareto boundary of the game  $G$ ;
- the points  $a$  and  $b$  belong to different half-planes determined by  $M$ .

Note that the above fourth axiom is equivalent to require that the segment joining the points  $a$  and  $b$  intersect  $M$ .



## 4.7 Win-win solutions

In the applications, if the game  $G$  has a member  $G_0$  of its family which can be considered as an “initial game” - in the sense that the pre-coopetitive situation is represented by this normal form game  $G_0$  - the aims of our study (following the standard ideas on coopetitive interactions) are

- to “enlarge the pie”;
- to obtain a win-win solution with respect to the initial situation.

So that we will choose as a threat point  $a$  in TU problem  $(M, (a, b))$  the supremum of the initial game  $G_0$ .

**Definition (of win-win solution).** *Let  $(G, z_0)$  be a coopetitive game with an initial point, that is a coopetitive game  $G$  with a fixed common strategy  $z_0$  (of its common strategy set  $C$ ). We call the game  $G_{z_0}$  as the initial game of  $(G, z_0)$ . We call win-win solution of the game  $(G, z_0)$  any strategy profile  $s = (x, y, z)$  such that the payoff of  $G$  at  $s$  is strictly greater than the supremum  $L$  of the payoff core of the initial game  $G(z_0)$ .*

**Remark 1.** The payoff core of a normal form gain game  $G$  is the portion of the Pareto maximal boundary  $G^*$  of the game which is greater than the conservative bi-value of  $G$ .

**Remark 2.** From an applicative point of view, the above requirement (to be strictly greater than  $L$ ) is very strong. More realistically, we can consider as win-win solutions those strategy profiles which are strictly greater than any reasonable solution of the initial game  $G_{z_0}$ .

**Remark 3.** Strictly speaking, a win-win solution could be not Pareto efficient: it is a situation in which the players both gain with respect to an initial condition (and this is exactly the idea we follow in the rigorous definition given above).

**Remark 4.** In particular, observe that, if the collective payoff function

$$^+(f) = f_1 + f_2$$

has a maximum (on the strategy profile space  $S$ ) strictly greater than the collective payoff  $L_1 + L_2$  at the supremum  $L$  of the payoff core of the game  $G_{z_0}$ , the portion  $M(> L)$  of Transferable Utility Pareto boundary  $M$  which is greater than  $L$  is non-void and it is a segment. So that we can choose as a threat point  $a$  in our problem  $(M, (a, b))$  the supremum  $L$  of the payoff core of the initial game  $G_0$  to obtain some compromise solution.

#### 4.7.1 Standard win-win solution.

A natural choice for the utopia point  $b$  is the supremum of the portion  $M_{\geq a}$  of the transferable utility Pareto boundary  $M$  which is upon (greater than) this point  $a$ :

$$M_{\geq a} = \{m \in M : m \geq a\}.$$

#### 4.7.2 Non standard win-win solution.

Another kind of solution can be obtained by choosing  $b$  as the supremum of the portion of  $M$  that is bounded between the minimum and maximum value of that player  $i$  that gains more in the cooperative interaction, in the sense that

$$\max(\text{pr}_i(\text{im}f)) - \max(\text{pr}_i(\text{im}f_0)) > \max(\text{pr}_{3-i}(\text{im}f)) - \max(\text{pr}_{3-i}(\text{im}f_0)).$$

#### 4.7.3 Final general remarks.

In the development of a cooperative game, we consider:

- a first virtual phase, in which the two players make a binding agreement on what cooperative strategy  $z$  should be selected from the cooperative set  $C$ , in order to respect their own rationality.
- then, a second virtual phase, in which the two players choose their strategies forming the profile  $(x, y)$  to implement in the game  $G(z)$ .

Now, in the second phase of our cooperative game  $G$  we consider the following 4 possibilities:

1. the two players are non-cooperative in the second phase and they do or do not exchange info, but the players choose (in any case) Nash equilibrium strategies for the game  $G(z)$ ; in this case, for some rationality reason, the two players have devised that the chosen equilibrium is the “better equilibrium choice” in the entire game  $G$ ; we have here only one binding agreement in the entire development of the game;
2. the two players are cooperative also in the second phase and they make a binding agreement in order to choose a Pareto payoff on the cooperative Pareto boundary; in this case we need two binding agreements in the entire development of the game;
3. the two players are cooperative also in the second phase and they make two binding agreements, in order to reach the Pareto payoff (on the cooperative Pareto boundary) with maximum collective gain (first agreement) and to share the collective gain according to a certain subdivision (second agreement); in this case we need three binding agreements in the entire development of the game;

4. the two players are non-cooperative in the second phase (and they do or do not exchange information), the player choose (in any case) Nash equilibrium strategies; the two players have devised that the chosen equilibrium is the equilibrium with maximum collective gain and they make only one binding agreement to share the collective gain according to a certain subdivision; in this case we need two binding agreements in the entire development of the game.

## 5 Coepetitive games for Greek crisis

Our first hypothesis is that Germany must stimulate the domestic demand and to re-balance its trade surplus in favor of Greece. The second hypothesis is that Greece, a country with a declining competitiveness of its products and a small export share, aims at growth by undertaking innovative investments and by increasing its exports primarily towards Germany and also towards the other euro countries.

The coepetitive models that we propose hereunder must be interpreted as normative models, in the sense that:

- *they impose some clear a priori conditions to be respected, by binding contracts, in order to enlarge the possible outcomes of both countries;*
- *consequently, they show appropriate win-win strategy solutions, chosen by considering both competitive and cooperative behaviors, simultaneously;*
- *finally, they propose appropriate fair divisions of the win-win payoff solutions.*

The strategy spaces of the two models are:

- the strategy set of Germany  $E$ , set of all possible consumptions of Germany, in our model, given in a conventional monetary unit; we shall assume that the strategies of Germany directly influence only Germany pay-off;
- the strategy set of Greece  $F$ , set of all possible investments of Greece, in our model, given in a conventional monetary unit (different from the above Germany monetary unit); we shall assume that the strategies of Greece directly influence only Greece pay-off;
- a shared strategy set  $C$ , whose elements are determined together by the two countries, when they choose their own respective strategies  $x$  and  $y$ , Germany and Greece. Every strategy  $z$  in  $C$  represents an amount - given in a third conventional monetary unit - of Greek exports imported into Germany, by respecting a binding contract.

Therefore, in the two models, we assume that Germany and Greece define the set of coepetitive strategies.

## 6 The first model

**Main Strategic assumptions.** We assume that:

- any real number  $x$ , belonging to the unit interval  $U := [0, 1]$ , can represent a consumption of Germany (given in an appropriate conventional monetary unit);
- any real number  $y$ , in the same unit interval  $U$ , can represent an investment of Greece (given in another appropriate conventional monetary unit);
- any real number  $z$ , again in  $U$ , can be the amount of Greek exports which is imported by Germany (given in conventional monetary unit).

### 6.1 Payoff function of Germany

We assume that the payoff function of Germany  $f_1$  is its *gross domestic demand*:

- $f_1$  is equal to the private consumption function  $C_1$  plus the gross investment function  $I_1$  plus government spending (that we shall assume equal 0, or constant) plus export function  $X_1$  minus the import function  $M_1$ , that is

$$f_1 = C_1 + I_1 + X_1 - M_1.$$

We assume that:

- the private consumption function  $C_1$  is the first projection of the strategic Cartesian cube  $U^3$ , that is defined by

$$C_1(x, y, z) = x,$$

for every consumption  $x$  in  $U$ , since we assume the private consumption of Germany the first strategic component of strategy profiles in  $U^3$ ;

- the gross investment function  $I_1$  is constant on the cube  $U^3$ , and by translation we can suppose  $I_1$  equal zero;
- the export function  $X_1$  is defined by

$$X_1(x, y, z) = (1 + x)^{-1},$$

for every consumption  $x$  of Germany; so we assume that the export function  $X_1$  is a strictly decreasing function with respect to the first argument;

- the import function  $M_1$  is the third projection of the strategic cube, namely

$$M_1(x, y, z) = z,$$

for every  $z \in U$ , because we assume the import function  $M_1$  depending only upon the cooperative strategy  $z$  of the cooperative game  $G$ , our third strategic component of the strategy profiles in  $U^3$ .

**Recap.** We then assume as payoff function of Germany its gross domestic demand  $f_1$ , which in our model is equal, at every triple  $(x, y, z)$  in the strategic cube  $U^3$ , to the sum of the strategies  $x, -z$  with the export function  $X_1$ , viewed as a reaction function with respect to the German domestic consumption (so that  $f_1$  is the difference of the first and third projection of the Cartesian product  $U^3$  plus the function export function  $X_1$ ).

Concluding, the payoff function of Germany is the function  $f_1$  of the cube  $U^3$  into the real line  $\mathbb{R}$ , defined by

$$f_1(x, y, z) = x + 1/(x + 1) - z,$$

for every triple  $(x, y, z)$  in the cube  $U^3$ ; where the reaction function  $X_1$ , defined from the unit interval  $U$  into the real line  $\mathbb{R}$  by

$$X_1(x) = 1/(x + 1),$$

for every consumption  $x$  of Germany in the interval  $U$ , is the export function of Germany mapping the level  $x$  of consumption into the level  $X_1(x)$  of German export corresponding to that consumption level  $x$ .

The function  $X_1$  is a strictly decreasing function, and only this monotonicity is the relevant property of  $X_1$  for our cooperative model.

## 6.2 Payoff function of Greece

We assume that the payoff function of Greece  $f_2$  is again its gross domestic demand - private consumption  $C_2$  plus gross investment  $I_2$  plus government spending (assumed to be 0) plus exports  $X_2$  minus imports  $M_2$ ),

$$f_2 = C_2 + I_2 + X_2 - M_2.$$

We assume that:

- the function  $C_2$  is irrelevant in our analysis, since we assume the private consumption independent from the choice of the strategic triple  $(x, y, z)$  in the cube  $U^3$ , in other terms we assume the function  $C_2$  constant on the cube  $U^3$  and by translation we can suppose  $C_2$  itself equal zero;
- the function  $I_2$  is defined by

$$I_2(x, y, z) = y + nz,$$

for every  $(x, y, z)$  in  $U^3$  (see later for the justification);

- the export function  $X_2$  is the linear function defined by

$$X_2(x, y, z) = z + my,$$

for every  $(x, y, z)$  in  $U^3$  (see later for the justification);

- the function  $M_2$  is irrelevant in our analysis, since we assume the import function independent from the choice of the triple  $(x, y, z)$  in  $U^3$ , in other terms we assume the import function  $M_2$  constant on the cube  $U^3$  and, by translation, we can suppose the import  $M_2$  equal zero.

So the payoff function of Greece is the linear function  $f_2$  of the cube  $U^3$  into the real line  $\mathbb{R}$ , defined by

$$f_2(x, y, z) = (y + nz) + (z + my) = (1 + m)y + (1 + n)z,$$

for every pair  $(x, y, z)$  in the strategic Cartesian cube  $U^3$ .

We note that the function  $f_2$  does not depend upon the strategies  $x$  in  $U$  chosen by Germany and that  $f_2$  is a linear function.

The definition of the functions investment  $I_2$  and export  $X_2$  must be studied deeply and carefully, and are fundamental to find the win-win solution.

- For every investment strategy  $y$  in  $U$ , the term  $my$  represents the quantity (monetary) effect of the Greek investment  $y$  on the Greek exports. In fact, the investments, specially innovative investments, contribute at improving the competitiveness of Greek goods, favoring the exports.
- For every cooperative strategy  $z$  in  $U$ , the term  $nz$  is the cross-effect of the cooperative variable  $z$  representing the additive level of investment required to support the production of the production  $z$  itself.
- We assume the factors  $m$  and  $n$  strictly positive.

### 6.3 Payoff function of the game

We so have build up a cooperative gain game with payoff function given by

$$\begin{aligned} f(x, y, z) &= (x + 1/(x + 1) - z, (1 + m)y + (1 + n)z) = \\ &= (x + 1/(x + 1), (1 + m)y) + z(-1, 1 + n) \end{aligned}$$

for every  $x, y, z$  in  $[0, 1]$ .

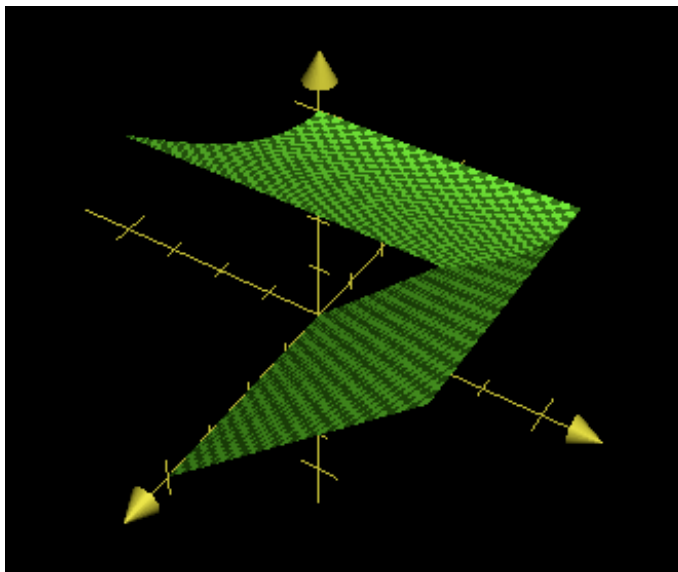


Figure 1: 3D representation of the initial game  $(f(., 0), <)$ .

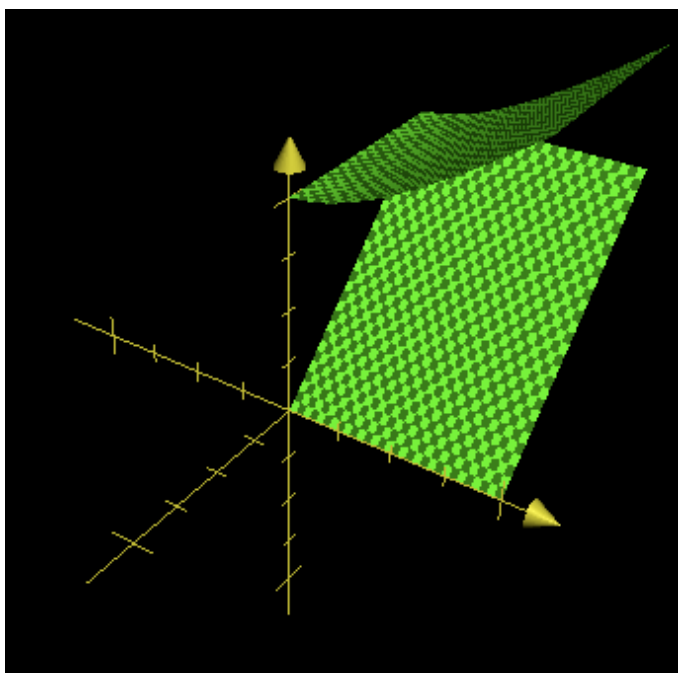


Figure 2: 3D representation of the initial game  $(f(., 0), <)$ .

## 6.4 Study of the game $G = (f, >)$

Note that, fixed a cooperative strategy  $z$  in  $U$ , the section game  $G(z) = (p(z), >)$  with payoff function  $p(z)$ , defined on the square  $U \times U$  by

$$p(z)(x, y) = f(x, y, z),$$

is the translation of the game  $G(0)$  by the “cooperative” vector

$$v(z) = z(-1, 1 + n),$$

so that we can study the initial game  $G(0)$  and then we can translate the various informations of the game  $G(0)$  by the vector  $v(z)$ .

So, let us consider the initial game  $G(0)$ . The strategy square  $S = U^2$  of  $G(0)$  has vertices  $0_2$ ,  $e_1$ ,  $1_2$  and  $e_2$ , where  $0_2$  is the origin,  $e_1$  is the first canonical vector  $(1, 0)$ ,  $1_2$  is the sum of the two canonical vectors  $(1, 1)$  and  $e_2$  is the second canonical vector  $(0, 1)$ .

## 6.5 Topological Boundary of the payoff space of $G_0$

In order to determine the Pareto boundary of the payoff space, we shall use the technics introduced by D. Carfi in [5] and used in [2],[7], [8], [9] and [10]. We have

$$p_0(x, y) = (x + 1/(x + 1), (1 + m)y),$$

for every  $x, y$  in  $[0, 1]$ . The transformation of the side  $[0, e_1]$  is the trace of the (parametric) curve  $c : U \rightarrow \mathbb{R}^2$  defined by

$$c(x) = f(x, 0, 0) = (x + 1/(x + 1), 0),$$

that is the segment

$$[f(0), f(e_1)] = [(1, 0), (3/2, 0)].$$

The transformation of the segment  $[0, e_2]$  is the trace of the curve  $c : U \rightarrow \mathbb{R}^2$  defined by

$$c(y) = f(0, y, 0) = (1, (1 + m)y),$$

that is the segment

$$[f(0), f(e_2)] = [(1, 0), (1, 1 + m)].$$

The transformation of the segment  $[e_1, 1_2]$  is the trace of the curve  $c : U \rightarrow \mathbb{R}^2$  defined by

$$c(y) = f(1, y, 0) = (1 + 1/2, (1 + m)y),$$

that is the segment

$$[f(e_1), f(1_2)] = [(3/2, 0), (3/2, 1 + m)].$$



**Critical zone of  $G(0)$ .** The Critical zone of the game  $G(0)$  is empty. Indeed the Jacobian matrix is

$$J_f(x, y) = \begin{pmatrix} 1 + (1 + x)^{-2} & 0 \\ 0 & 1 + m \end{pmatrix},$$

which is invertible for every  $x, y$  in  $U$ .

**Payoff space of the game  $G(0)$ .** So, the payoff space of the game  $G(0)$  is the transformation of topological boundary of the strategic square, that is the rectangle with vertices  $f(0, 0)$ ,  $f(e_1)$ ,  $f(1, 1)$  and  $f(e_2)$ .

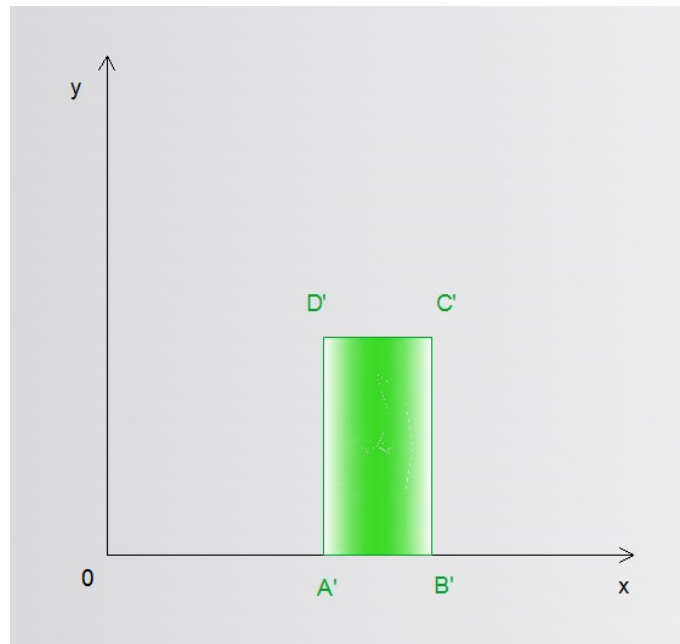


Figure 3: Initial payoff space of the game  $(f, <)$ .

**Nash equilibria.** The unique Nash equilibrium is the bistrategy  $(1, 1)$ . Indeed,

$$1 + (1 + x)^{-2} > 0$$

so the function  $f_1$  is increasing with respect to the first argument and analogously

$$1 + m > 0$$

so that the Nash equilibrium is  $(1, 1)$ .

## 6.6 The payoff space of the cooperative game $G$

The image of the payoff function  $f$ , is the union of the family of payoff spaces

$$(\text{imp}_z)_{z \in C},$$

that is the convex envelope of the union of the image  $p_0(S)$  ( $S$  is the square  $U \times U$ ) and of its translation by the vector  $v(1)$ , namely the payoff space  $p_1(S)$ : the image of  $f$  is an hexagon with vertices  $f(0,0)$ ,  $f(e_1)$ ,  $f(1,1)$  and their translations by  $v(1)$ .

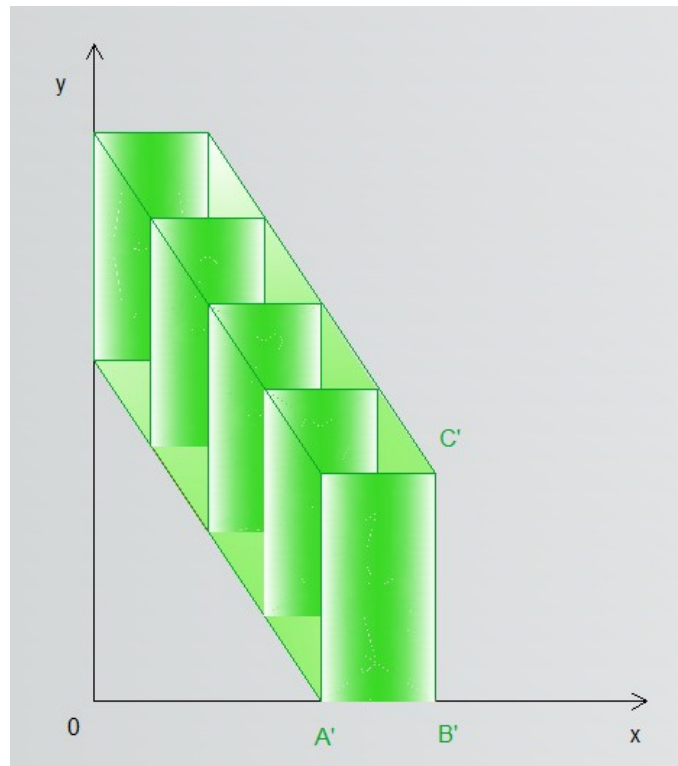


Figure 4: Payoff space of the game  $(f, <)$ .

## 6.7 Pareto maximal boundary of payoff space of $G$

The Pareto sup-boundary of the cooperative payoff space  $f(S)$  is the segment  $[P', Q']$ , where  $P' = f(1, 1)$  and

$$Q' = P' + v(1).$$

**Possibility of global growth.** It is important to note that the absolute slope of the Pareto (cooperative) boundary is  $1 + n$ . Thus the collective payoff  $f_1 + f_2$  of the game is not constant on the Pareto boundary and, therefore, the game implies the possibility of a global growth.

**Trivial bargaining solutions.** The Nash bargaining solution on the segment  $[P', Q']$  with respect to the infimum of the Pareto boundary and the Kalai-Smorodinsky bargaining solution on the segment  $[P', Q']$ , with respect to the infimum and the supremum of the Pareto boundary, coincide with the medium point of the segment  $[P', Q']$ . This solution is not acceptable from Germany point of view, it is collectively better than the supremum of  $G_0$  but it is disadvantageous for Germany (it suffers a loss!): this solution can be thought as a rebalancing solution but it is not realistically implementable.

## 6.8 Transferable utility solution

In this cooperative context it is more convenient to adopt a transferable utility solution, indeed:

- the point of maximum collective gain on the whole of the cooperative payoff space is the point

$$Q' = (1/2, 2 + m + n).$$

### 6.8.1 Rebalancing win-win best compromise solution.

Thus we propose a rebalancing win-win kind of cooperative solution, as it follows (in the case  $m = 0$ ):

1. we consider the portion  $s$  of transferable utility Pareto boundary

$$M := (0, 5/2 + n) + \mathbb{R}(1, -1),$$

obtained by intersecting  $M$  itself with the strip determined (spanned by convexifying) by the straight lines  $e_2 + \mathbb{R}e_1$  and

$$(2 + n)e_2 + \mathbb{R}e_1,$$

*these are the straight lines of maximum gain for Greece in games  $G(0)$  and  $G$  respectively.*

2. we consider the Kalai-Smorodinsky segment  $s'$  of vertices  $(3/2, 1)$  - supremum of the game  $G(0)$  - and the supremum of the segment  $s$ .
3. our best payoff cooperative compromise is the unique point  $K$  in the intersection of segments  $s$  and  $s'$ , that is the best compromise solution of the bargaining problem

$$(s, (\sup G_0, \sup s)).$$

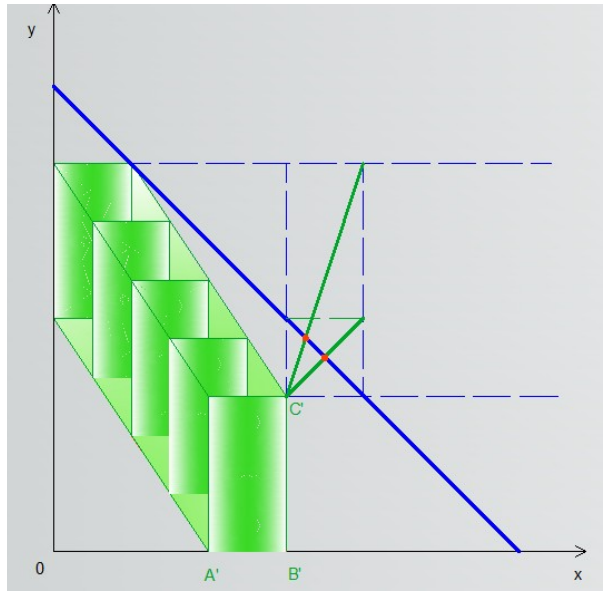


Figure 5: Two Kalai win-win solutions of the game  $(f, <)$ , represented with  $n = 1/2$ .

## 6.9 Win-win solution

This best payoff cooperative compromise  $K$  represents a win-win solution with respect to the initial supremum  $(3/2, 1)$ . So that, as we repeatedly said, *also Germany can increase its initial profit from cooperation.*

**Win-win strategy procedure.** The win-win payoff  $K$  can be obtained (by chance) in a properly cooperative fashion in the following way:

- 1) the two players agree on the cooperative strategy 1 of the common set  $C$ ;
- 2) the two players implement their respective Nash strategies of game  $G(1)$ ; the unique Nash equilibrium of  $G(1)$  is the bistrategy  $(1, 1)$ ;
- 3) finally, they share the “social pie”

$$5/2 + n = (f_1 + f_2)(1, 1, 1),$$

in a **cooperative fashion** (by contract) according to the decomposition  $K$ .

## 7 The second model

In this second model, we consider a linear affine mutual interaction between Germany and Greece, more adherent to the real state of the Euro-area.

Specifically, in opposition to the above first model:

- we consider an interaction between the two countries also *at the level of their non-cooperative strategies*;
- we assume that *Greece also should import (by contract) some German production*;
- we assume, that the German revenue, given by the exportations in Greece of the above production, *is absorbed by the Germany bank system* - in order to pay the Greece debts with the German bank system - so that this money does not appear in the payoff function of Germany (as possible gain) but only in the payoff function of Greece (as a loss).

### 7.0.1 Main Strategic assumptions.

We assume that:

- any real number  $x$ , belonging to the interval  $E := [0, 3]$ , represents a possible consumption of Germany (given in an appropriate conventional monetary unit);
- any real number  $y$ , in the same interval  $F := E$ , represents a possible investment of Greece (given in another appropriate conventional monetary unit);
- any real number  $z$ , again in the interval  $C = [0, 2]$ , can be the amount of Greek exports which is imported by Germany (given in conventional monetary unit).

## 7.1 Payoff function of Germany

We assume that the payoff function of Germany  $f_1$  is its Keynesian *gross domestic demand*:

- $f_1$  is equal to the private consumption function  $C_1$  plus the gross investment function  $I_1$  plus government spending (that we shall assume equal 2, constant in our interaction) plus export function  $X_1$  minus the import function  $M_1$ , that is

$$f_1 = 2 + C_1 + I_1 + X_1 - M_1.$$

We assume that:

- the German private consumption function  $C_1$  is the first projection of the strategic cooperative space  $S := E^2 \times C$ , that is defined by

$$C_1(x, y, z) = x,$$

for every possible german consumption  $x$  in  $E$ , this because we assumed the private consumption of Germany to be the first strategic component of strategy profiles in  $S$ ;

- the gross investment function  $I_1$  is constant on the space  $S$ , and by translation we can suppose  $I_1$  equal zero;
- the export function  $X_1$  is defined by

$$X_1(x, y, z) = -y/3,$$

for every Greek possible investment  $y$  in innovative technology; so we assume that the export function  $X_1$  is a strictly decreasing function with respect to the second argument;

- the import function  $M_1$  is the third projection of the strategic space, namely

$$M_1(x, y, z) = z,$$

for every cooperative strategy  $z \in 2U$ , because we assume the import function  $M_1$  depending only upon the cooperative strategy  $z$  of the cooperative game  $G$ , our third strategic component of the strategy profiles in  $S$ .

**Recap.** We then assume as payoff function of Germany its Keynesian gross domestic demand  $f_1$ , which in our model is equal, at every triple  $(x, y, z)$  in the profile strategy set  $S$ , to the sum of the strategies  $x, -z$  with the export function  $X_1$ , viewed as a reaction function to the Greece investments (so that  $f_1$  is the difference of the first and third projection of the strategy profile space  $S$  plus the function export function  $X_1$ ).

Concluding, the payoff function of Germany is the function  $f_1$  of the set  $S$  into the real line  $\mathbb{R}$ , defined by

$$f_1(x, y, z) = 2 + x - y/3 - z,$$

for every triple  $(x, y, z)$  in the space  $S$ ; where the reaction function  $X_1$ , defined from the space  $S$  into the real line  $\mathbb{R}$  by

$$X_1(x, y, z) = -y/3,$$

for every possible investment  $y$  of Greece in the interval  $3U$ , is the export function of Germany mapping the level  $y$  of Greece investment into the level  $X_1(x, y, z)$  of German export, corresponding to the Greece investment level  $y$ .

The function  $X_1$  is a strictly decreasing function in the second argument, and this monotonicity is a relevant property of  $X_1$  for our cooperative model.

## 7.2 Payoff function of Greece

We assume that the payoff function of Greece  $f_2$  is again its Keynesian gross domestic demand - private consumption  $C_2$  plus gross investment  $I_2$  plus government spending (assumed to be 2) plus exports  $X_2$  minus imports  $M_2$ ), so that

$$f_2 = 2 + C_2 + I_2 + X_2 - M_2.$$

We assume that:

- the function  $C_2$  is irrelevant in our analysis, since we assume the Greek private consumptions independent from the choice of the strategic triple  $(x, y, z)$  in the space  $S$ ; in other terms, we assume the function  $C_2$  constant on the space  $S$  and by translation we can suppose  $C_2$  itself equal zero;

- the function  $I_2 : S \rightarrow \mathbb{R}$  is defined by

$$I_2(x, y, z) = y + nz,$$

for every  $(x, y, z)$  in  $S$  (see above for the justification);

- the export function  $X_2$  is the linear function defined by

$$X_2(x, y, z) = z + my,$$

for every  $(x, y, z)$  in  $S$  (see above for the justification);

- the function  $M_2$  is now relevant in our analysis, since we assume the import function, by *coopetitive contract with Germany*, dependent on the choice of the triple  $(x, y, z)$  in  $S$ , specifically, we assume the import function  $M_2$  defined on the space  $S$  by

$$M_2(x, y, z) := -2x/3,$$

so, Greece too now, must import some German product, with value  $-2x/3$  for each possible German consumption  $x$ .

So, the payoff function of Greece is the linear function  $f_2$  of the space  $S$  into the real line  $\mathbb{R}$ , defined by

$$\begin{aligned} f_2(x, y, z) &= 2 - 2x/3 + (y + nz) + (z + my) = \\ &= 2 - 2x/3 + (1 + m)y + (1 + n)z, \end{aligned}$$

for every pair  $(x, y, z)$  in the strategic Cartesian space  $S$ .

We note that the function  $f_2$  depends now significantly upon the strategies  $x$  in  $E$ , chosen by Germany, and that  $f_2$  is again a linear function.

We shall assume the factors  $m$  and  $n$  non-negative and equal respectively (only for simplicity) to 0 and 1/2.

### 7.3 Payoff function of the game

We so have build up a cooperative gain game with payoff function  $f : S \rightarrow \mathbb{R}^2$ , given by

$$\begin{aligned} f(x, y, z) &= (2 + x - y/3 - z, 2 - 2x/3 + (1 + m)y + (1 + n)z) = \\ &= (2, 2) + (x - y/3, -2x/3 + (1 + m)y) + z(-1, 1 + n), \end{aligned}$$

for every  $(x, y, z)$  in  $[0, 3]^2 \times [0, 2]$ .

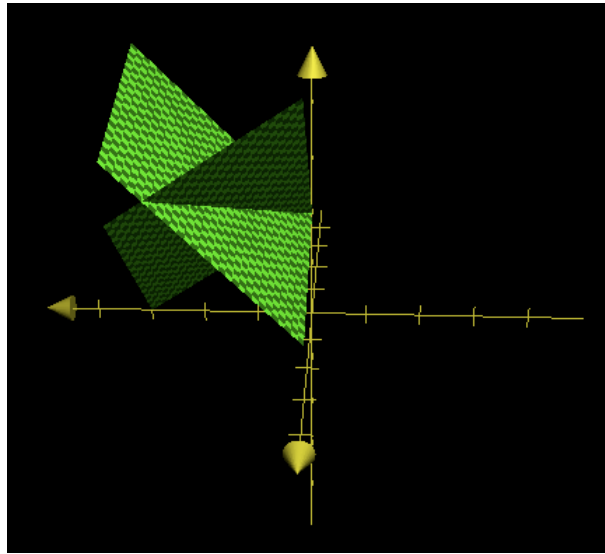


Figure 6: 3D representation of  $(f, <)$ .

### 7.4 Study of the second game $G = (f, >)$

Note that, fixed a cooperative strategy  $z$  in  $2U$ , the section game  $G(z) = (p(z), >)$  with payoff function  $p(z)$ , defined on the square  $E^2$  by

$$p(z)(x, y) := f(x, y, z),$$

is the translation of the game  $G(0)$  by the “cooperative” vector

$$v(z) = z(-1, 1 + n),$$

so that, we can study the initial game  $G(0)$  and then we can translate the various informations of the game  $G(0)$  by the vectors  $v(z)$ , to obtain the corresponding information for the game  $G(z)$ .



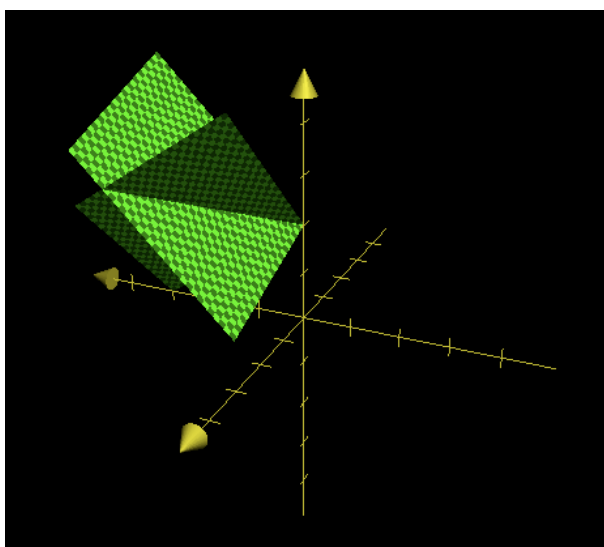


Figure 7: 3D representation of  $(f, <)$ .

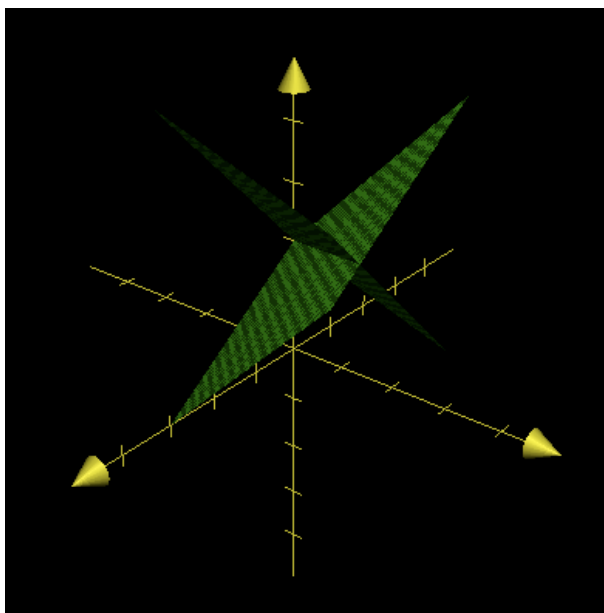


Figure 8: 3D representation of  $(f, <)$ .

So, let us consider the initial game  $G(0)$ . The strategy square  $E^2$  of  $G(0)$  has vertices  $0_2$ ,  $3e_1$ ,  $3_2$  and  $3e_2$ , where  $0_2$  is the origin of the plane  $\mathbb{R}^2$ ,  $e_1$  is the first canonical vector  $(1, 0)$ ,  $3_2$  is the vectors  $(3, 3)$  and  $e_2$  is the second canonical vector.

## 7.5 Topological Boundary of the payoff space of $G_0$

In order to determine the the payoff space of the linear game it is sufficient to transform the four vertices of the strategy square (the game is an affine invertible game), the critical zone is empty.

### 7.5.1 Payoff space of the game $G(0)$ .

So, the payoff space of the game  $G(0)$  is the transformation of the topological boundary of the strategy square, that is the parallelogram with vertices  $f(0, 0)$ ,  $f(3e_1)$ ,  $f(3, 3)$  and  $f(3e_2)$ . As we show in the below figure 9.

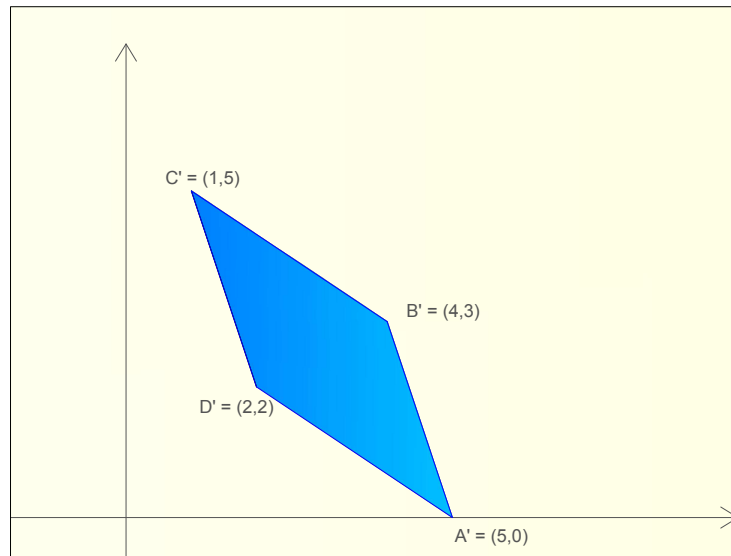


Figure 9: Initial payoff space of the game  $(f, <)$ .

### 7.5.2 Nash equilibria.

The unique Nash equilibrium is the bistrategy  $(3, 3)$ . Indeed, the function  $f_1$  is linear increasing with respect to the first argument and analogously the function  $f_2$  is linear and increasing with respect to the second argument.

## 7.6 The payoff space of the cooperative game $G$

The image of the payoff function  $f$ , is the union of the family of payoff spaces

$$(\text{imp}_z)_{z \in C},$$

that is the convex envelope of the union of the image  $p_0(E^2)$  and of its translation by the vector  $v(2)$ , namely the payoff space  $p_2(E^2)$ : the image of  $f$  is an hexagon with vertices  $f(0, 0)$ ,  $f(3e_1)$ ,  $f(3, 3)$  and their translations by  $v(2)$ . As we show below.

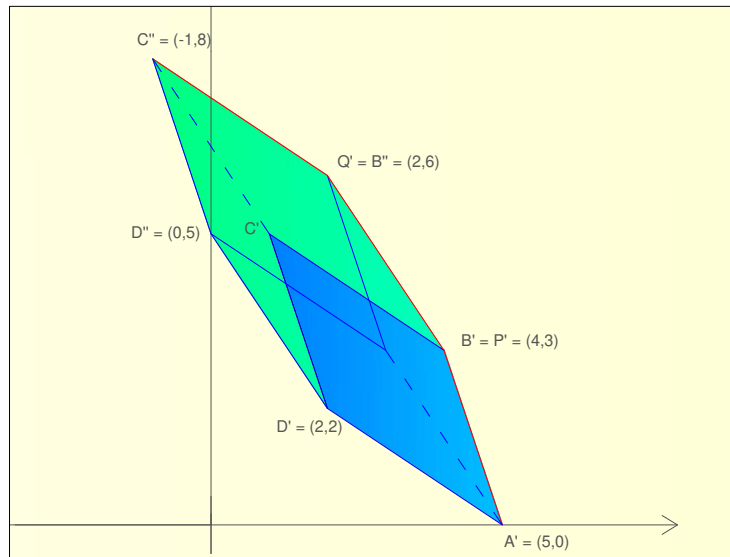


Figure 10: Payoff space of the game  $(f, <)$ .

## 7.7 Pareto maximal boundary of the payoff space of $G$

The Pareto sup-boundary of the cooperative payoff space  $f(S)$  is the union of the segments  $[A', B']$ ,  $[P', Q']$  and  $[Q', C''']$ , where  $P' = f(3, 3, 0)$  and

$$Q' = P' + v(2).$$

### 7.7.1 Possibility of global growth.

It is important to note that the absolute slopes of the segments  $[A', B']$ ,  $[P', Q']$  of the Pareto (cooperative) boundary are strictly greater than 1. Thus the collective payoff  $f_1 + f_2$  of the game is not constant on the Pareto boundary and, therefore, the game implies the possibility of a transferable utility global growth.

### 7.7.2 Trivial bargaining solutions.

The Nash bargaining solution on the entire payoff space, with respect to the infimum of the Pareto boundary and the Kalai-Smorodinsky bargaining solution, with respect to the infimum and the supremum of the Pareto boundary, are not acceptable for Germany: they are collectively (TU) better than the Nash payoff of  $G_0$  but they are disadvantageous for Germany (it suffers a loss!): these solutions could be thought as rebalancing solutions, but they are not realistically implementable.

## 7.8 Transferable utility solutions

In this cooperative context it is more convenient to adopt a transferable utility solution, indeed:

- the point of maximum collective gain on the whole of the cooperative payoff space is the point  $Q' = (2, 6)$ .

### 7.8.1 Rebalancing win-win solution relative to maximum gain for Greece in $G$ .

Thus we propose a rebalancing win-win cooperative solution relative to maximum gain for Greece in  $G$ , as it follows (in the case  $m = 0$ ):

1. we consider the portion  $s$  of transferable utility Pareto boundary

$$M := Q' + \mathbb{R}(1, -1),$$

obtained by intersecting  $M$  itself with the strip determined (spanned by convexifying) by the straight lines  $P' + \mathbb{R}e_1$  and  $C''' + \mathbb{R}e_1$ , *these are the straight lines of Nash gain for Greece in the initial game  $G(0)$  and of maximum gain for Greece in  $G$ , respectively.*

2. we consider the Kalai-Smorodinsky segment  $s'$  of vertices  $B'$  - Nash payoff of the game  $G(0)$  - and the supremum of the segment  $s$ .
3. our best payoff rebalancing cooperative compromise is the unique point  $K$  in the intersection of segments  $s$  and  $s'$ , that is the best compromise solution of the bargaining problem  $(s, (B', \sup s))$ .

Figure 11 below shows the above extended Kalai-Smorodinsky solution  $K$  and the Kalai-Smorodinsky solution  $K'$  of the classic bargaining problem  $(M, B')$ . It is evident that the distribution  $K$  is a rebalancing solution in favor of Greece with respect to the classic solution  $K'$ .

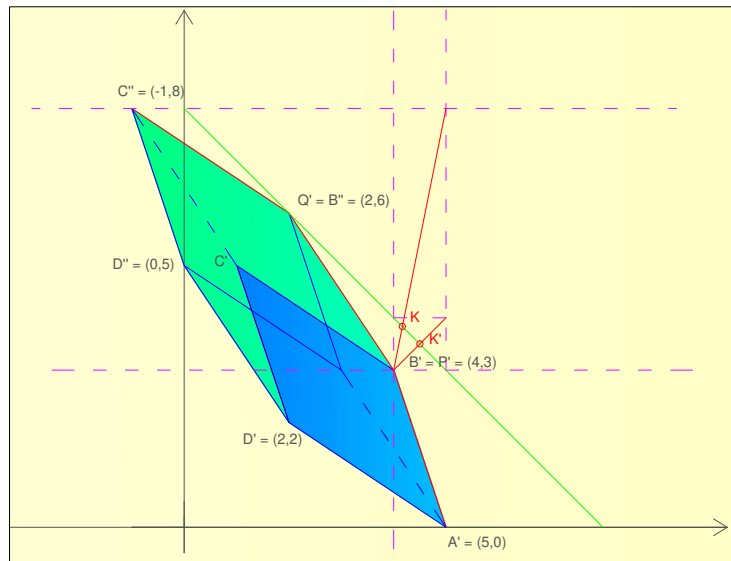


Figure 11: Two Kalai win-win solutions of the game  $(f, <)$ , represented with  $n = 1/2$ .

### 7.8.2 Rebalancing win-win solution relative to maximum Nash gain for Greece.

We propose here a more realistic rebalancing win-win cooperative solution relative to maximum Nash gain for Greece in  $G$ , as it follows (again in the case  $m = 0$ ):

1. we consider the portion  $s$  of transferable utility Pareto boundary

$$M := Q' + \mathbb{R}(1, -1),$$

obtained by intersecting  $M$  itself with the strip determined (spanned by convexifying) by the straight lines  $P' + \mathbb{R}e_1$  and  $Q' + \mathbb{R}e_1$ , these are the straight lines of Nash gain for Greece in the initial game  $G(0)$  and of maximum Nash gain for Greece in  $G$ , respectively.

2. we consider the Kalai-Smorodinsky segment  $s'$  of vertices  $B'$  - Nash payoff of the game  $G(0)$  - and the supremum of the segment  $s$ .
3. our best payoff rebalancing cooperative compromise is the unique point  $K$  in the intersection of segments  $s$  and  $s'$ , that is the best compromise solution of the bargaining problem  $(s, (B', \sup s))$ .

Figure 12 below shows the above extended Kalai-Smorodinsky solution  $K$  and the Kalai-Smorodinsky solution  $K'$  of the classic bargaining problem  $(M, B')$ . The new distribution  $K$  is a rebalancing solution in favor of Greece, more realistic than the previous rebalancing solution.

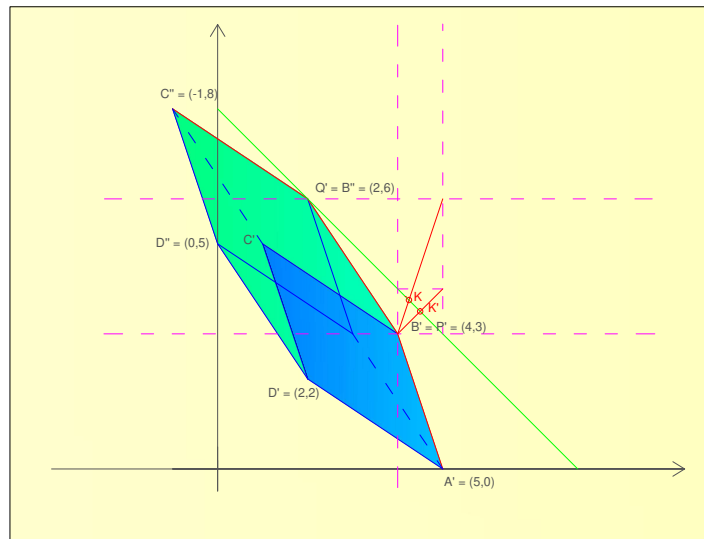


Figure 12: Two Kalai win-win solutions of the game  $(f, <)$ , represented with  $n = 1/2$ .

## 7.9 Win-win solution

The payoff extended Kalai-Smorodinsky solutions  $K$  represent win-win solutions, with respect to the initial Nash gain  $B'$ . So that, as we repeatedly said, *also Germany can increase its initial profit from cooperation.*

### 7.9.1 Win-win strategy procedure.

The win-win payoff  $K$  can be obtained in a **properly transferable utility cooperative fashion**, as it follows:

- 1) the two players agree on the cooperative strategy 2 of the common set  $C$ ;
- 2) the two players implement their respective Nash strategies in the game  $G(2)$ , so competing *à la Nash*; the unique Nash equilibrium of the game  $G(2)$  is the bistrategy  $(3, 3)$ ;
- 3) finally, they share the “social pie”

$$(f_1 + f_2)(3, 3, 2),$$

in a **transferable utility cooperative fashion** (by binding contract) according to the decomposition  $K$ .

## 8 Conclusions

In conclusion, we desire to stress that:

- the two samples of cooperative game, provided in the present contribution, are essentially *normative models*.
- our samples of cooperation have pointed out the strategies that could bring to win-win solutions, in a **transferable utility and properly cooperative perspective**, for Greece and Germany.

In the paper, we propose:

- a properly cooperative solution, which is not convenient for Germany, which is the Kalai-Smorodinsky bargaining solution on the *cooperative Nash path*, set of all possible Nash equilibria of the cooperative interaction.
- transferable utility and properly cooperative solutions, which are convenient also for Germany, which are also rebalancing for the Euro area.

- a new extended Kalai-Smorodinsky method, appropriate to determine rebalancing partitions, for *win-win solutions*, on the transferable utility Pareto boundary of the cooperative game.

The solutions offered by our cooperative model:

- aim at “enlarging the pie and sharing it fairly”;
- show win-win and rebalancing outcomes, for the two countries, within a cooperative and positive non constant-sum game path.
- allow us to find “fair” amounts of Greek exports which Germany must cooperatively import as well as the optimal Greek investments necessary to improve the Greek economy in this context, thus contributing to growth and to the stability of both the Greek and Germany economies.

## Acknowledgments

The authors wish to thank *Dr. Eng. Alessia Donato* for her valuable help in the preparation of the figures. Moreover the authors wish to thank Albert E. Steenge, E. Perrone and three anonymous referees for their helpful comments and suggestions.

## References

- [1] C. Abadi, 2010, “Germany Debates Euro Instability Fault”, *Global Post*, March 19.
- [2] S. Agreste , D. Carfi, A. Ricciardello, 2012, “Algorithms for differentiable parametric games”, *Applied Sciences (APPS)* **14**, pp. 1-14.
- [3] O. Blanchard, G. Dell’Ariccia, P. Mauro, 2010, “Rethinking Macroeconomic Policy”, *IMF SPN Working Paper*, February, pp. 1-18.
- [4] A. M. Brandenburger, B. J. Nalebuff, 1995, “The Right Game: Use Game Theory to Shape Strategy”, *Harvard Business Review* **64**, pp. 57-71, July-August.
- [5] D. Carfi, 2009, “Payoff space in  $C^1$ -games”, *Applied Sciences (APPS)* **11**, pp. 1-16.
- [6] D. Carfi, 2012, “A Model for Cooperative Games”, *MPRA working paper*, presented at Sing6, Palermo, July 6-9 (2010).
- [7] D. Carfi, F. Musolino, 2011, “Fair Redistribution in Financial Markets: a Game Theory Complete Analysis”, *Journal of Advanced Studies in Finance*, Association for Sustainable Education, Research and Science, **2**, pp. 74-100, December.



- [8] D. Carfi, E. Perrone, 2011, "Game Complete Analysis of Bertrand Duopoly", *Theoretical and Practical Research in Economic Fields*, Association for Sustainable Education, Research and Science, **2** (1), pages 5-22, June.
- [9] D. Carfi, A. Ricciardello, 2010, "An Algorithm for Payoff Space in  $C^1$ -Games", *Atti della Accademia Peloritana dei Pericolanti, Classe di Scienze Fisiche Matematiche e Naturali*, **88** (1), pp. 1-19.
- [10] D. Carfi, A. Ricciardello, 2012, "Topics in Game Theory", *APPS Monograph*, **9**.
- [11] D. Carfi, D. Schilirò, 2011, "Crisis in the Euro area: cooperative game solutions as new policy tools", *TPREF-Theoretical and Practical Research in Economic Fields* **2** (1), pp. 23-36, June.
- [12] Eurostat, 2011, News Release, Euro Indicators, Bruxelles, pp. 1-14, April.
- [13] R. Gulati, N. Nohria, A. Zaheer, 2000, "Strategic Networks", *Strategic Management Journal* **21**, pp. 203-215.
- [14] M. Mussa, 2010, "Beware of Greeks Bearing Debts", *Working Paper, Peterson Institute for International Economics*, **May 17**, <http://www.piie.com>.
- [15] M. Porter, 1985, "Competitive Advantage: Creating and Sustaining Superior Performance", *Free Press*, New York.
- [16] A. S. Posen, 2010, "Having a Large Euro Area Is an Advantage for Germany", *Die Welt*, February 19. <http://www.piie.com>.
- [17] D. Schilirò, 2010, "Investing in Knowledge: Knowledge, Human Capital and Institutions for the Long Run Growth", *Governance Of Innovation*, edited by M.J. Arentsen, W. van Rossum, A.E. Steenge, Edward Elgar, Cheltenham, pp. 33-50.
- [18] D. Schilirò, 2011, "A new governance for the EMU and the economic policy framework", MPRA Paper No. 30237, April, pp. 1-18.
- [19] J. Stiles, 2001, "Strategic Alliances", *Rethinking Strategy*, edited by H. W. Volberda, T. Elfring, Sage Publications, London.