Non-Sterilized Interventions May Yield Perverse Effects on Spot Foreign Exchange Rates

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Abstract

We study the effects of non-sterilized intervention on a spot foreign exchange rate using a multi-period game-theoretical model which involves an unspecified number of competitive traders, a finite number of strategic traders (forex dealers), and the central bank of the home country. Simulating the subgame-perfect Nash equilibrium of the two-stage game played by the strategic traders in each period, we show that the non-sterilized intervention of the central bank may lead to a perverse result. This result may arise when the intervention becomes strong enough to unintentionally induce some of the strategic traders -who have previously traded in the direction desired by the monetary authority- to optimally switch to the opposite trade direction.

Keywords: Exchange rate; central bank intervention; foreign exchange dealers; imperfect competition

JEL Classification Numbers: D43; F31; G20

1 Introduction

Exchange rate intervention has been a frequently used monetary policy option throughout the world since the 1985 Plaza Meeting of G5 industrialized countries, resulting in an agreement on the need for coordinated intervention to stabilize the U.S. dollar against the other major currencies. While some central banks always use either sterilized or non-sterilized interventions, some others alternatively use these policy options.[1]

[1]According to a survey conducted by Neely (2000) among the monetary authorities of 22 countries, involving Belgium, Brazil, Canada, Chile, Czech Republic, Denmark, France, Germany, Hong Kong, Indonesia, Ireland, Italy, Japan, Mexico, New Zealand, Poland, South Korea, Spain, Sweden, Switzerland, Taiwan, and United States, the share of the respondents who always use sterilized interventions is 40% and who always use non-sterilized interventions is 30%, while the remaining 30% uses sterilized and non-sterilized interventions alternatively.

Under sterilized intervention, a central bank takes an action to offset the effects of its intervention on the monetary base, so as to leave the liquidity supply in the country unchanged. Therefore, a pure monetary approach to exchange rate determination leaves open the questions whether and why sterilized intervention could be effective. While some empirical evidence for the effectiveness of sterilized intervention was provided in the early 1990s (Dominguez, 1990; Dominguez and Frankel, 1993), analytical answers as to why it could be effective had been much earlier offered by two competing models in international finance. Of these, the portfolio channel model (Black, 1973; Kouri, 1976; Branson 1977; and Girton and Henderson, 1977) predicts that in financial markets where investors diversify their domestic and foreign asset holdings with respect to risk-return tradeoffs, a sterilized intervention that changes the composition of domestic assets must inevitably change the return of these assets relative to foreign assets, leading to a change in the exchange rate.

The second model, known as the signalling channel (Ross, 1977; Mussa, 1981; Dominguez, 1992), suggests that a central bank can use the sterilized intervention as a means of signalling its private information about future fundamentals. When the investors in a financial market find the signalling of the intervening central bank credible and accordingly revise their expectations about future fundamentals, they would necessarily change their expectations about the future spot exchange rate, leading to a change in the current spot exchange rate. While both of these two channels implicitly assumes that the induced response of the exchange rate to sterilized intervention is in the direction desirable for the central bank, this is not always supported by the historical data. For example, the sterilized intervention of the Federal Reserve during the period after the Louvre Meeting in 1987 is known to have a perverse effect on the exchange rate, as reported by Dominguez and Frankel (1993). A theoretical explanation for this puzzle was offered by Bhattacharya and Weller (1997) with the help of an asymmetric information model of sterilized intervention where the central bank has private information about the targeted foreign exchange rate whereas risk-averse speculators who can engage in both spot and forward exchanges have private information about future spot rates. For this model, perverse responses to sterilized interventions are associated with an upward sloping speculative demand curve that can be observed when the effect of lowering the spot exchange rate on the expected value of the future spot rate dominates its effect on the current forward rate.

Unlike sterilized intervention, non-sterilized intervention is believed to have an indisputable effect on the exchange rate. In fact, there is a consensus among the majority of economists that non-sterilized purchases (sales) of the home currency by the central bank must lead to a subsequent appreciation (depreciation) of the home currency. As to why this prediction must be true, the literature offers various reasons. One of them is the "interest-rate channel" (also known as the "liquidity channel") that is present in all standard macroeconomic models. When the central bank purchases (sells) a foreign currency without sterilizing it, the liquidity in the home country increases, exerting downward pressure on the short-term nominal interest rate and consequently weakening the home currency. A second reason is the "inventory adjustment channel" (Lyons, 1997; 2001), according to which the foreign exchange dealers always adjust the prices of their trade orders to ensure that their inventories of foreign currencies are not undesirably large or small at the end of any trading day. Since this channel assumes that each foreign exchange dealer perceives the trade order of the central bank just like the trade order of any other foreign exchange dealer, a purchase (sell) order of the central bank of a non-negligible amount would induce the foreign exchange dealers in the market to increase (decrease) their prices. As another reason, the "signalling channel" -that we have discussed above for the case of sterilized intervention- can also explain why non-sterilized intervention works its effects in the direction desired by the central bank. In this paper, we suggest that the common prediction shared by all these channels as to the effectiveness of non-sterilized intervention needs not be always true. That is, we argue that non-sterilized intervention may, too, have a perverse effect on the exchange rate. Moreover, these perverse effects can arise due to a new channel which we call 'strategic trade switching'.

We obtain our findings with the help of a multi-period game-theoretical model of foreign exchange. This model involves an unspecified number of competitive traders, a finite number of strategic traders (forex dealers), and the central bank of the home country, all of whom can buy and sell in a spot foreign exchange market in each period. All competitive traders in our model are atomistic price takers: they always take the exchange rate given and conventionally trade with respect to an upward sloping supply function and a downward sloping demand function. Strategic traders on the other hand have some degree of power to influence the exchange rate, enabling them to always maximize their monetary profits from trading by optimally choosing their trade orders. The remaining trader in our model, the central bank of the home country, has no intention to make money through foreign exchange trade; in fact it can even lose money to the strategic traders.[2] The central bank intervenes to the foreign exchange market in order to limit the short-run variability of the exchange rate around a prespecified target. We

[2]The assumption that money making is not the priority of the central bank in foreign exchange market interventions was empirically supported by LeBaron (1999).

assume that the central bank's intervention is direct and non-sterilized, i.e. the central bank intervenes by either buying or selling the foreign currency, while allowing its trades to influence the monetary base in the home country. Since the aim of our paper is to show the possibility of a new channel through which the non-sterilized intervention could generate undesirable effects on the exchange rate, our model is constructed to be as simple as possible to eliminate the presence of the aforementioned three channels of affection. Thus, we exclude interest rates and forward currency exchanges, isolating ourselves from the interest-rate channel and the signalling channel, respectively. Additionally, in order to make the inventory adjustment channel non-functional, we also assume away -on the part of the strategic traders- any motive other than profit maximization. That is to say, in any period a strategic trader in our model chooses to be a buyer or a seller independently from the size of her existing inventory (cash holding) of the foreign currency.

An important feature of our multi-period model is that whenever a strategic trader buys a particular currency in any period, the average acquirement price of the cash she has been holding in that currency changes, too. Computing the average acquirement prices of her home and foreign currency holdings and conjecturing a market clearing exchange rate in each period, each strategic trader can calculate her unit profits from buying and selling the foreign currency. We assume that using these calculations, each strategic trader first decides whether to buy or sell the foreign currency and observing the simultaneously made decisions of all others, she next decides how much to trade. Absolutely, these decisions cannot be made trivially. The market clearing exchange rate conjectured by any strategic trader must depend on the decisions (trade orders) of all other traders. Therefore, each strategic trader, while solving her optimization problems, has to take into account her conjectures about the decisions of all other strategic traders (along with her knowledge about the actions of the competitive traders and the central bank). Here we should note that the conjectures of any two strategic traders about the decision of a third strategic trader will always be the same because the strategic traders will not be allowed to have and process private information like in the information revelations models.

Definitely, the exclusion of private information from our model will simplify the task of solving the strategic traders' interdependent optimization problems to a great extent. As a matter of fact, we will handle this task by formulating the decision making process of the strategic traders in each period as a two-stage extensive form game with complete and perfect information and then solving this game using the concept of subgame perfect Nash equilibrium due to Selten (1965). Evidently, the players of this game are the strategic traders in our model. In stage 1 the players non-cooperatively decide whether to buy or sell the foreign currency, and in stage 2 -after observing all of the decisions made in stage 1- the players non-cooperatively determine their trade quantities. So, each player's complete strategy before the game starts must include her trade direction (the plan whether to buy or sell the foreign currency) to be revealed in stage 1 along with how much she will trade at each subgame in stage 2. Given all possible strategies of all players, each player can then calculate her terminal payoff at each strategy profile taking into account the associated market clearing level of the exchange rate.

A strategy profile in an extensive-form game is said to be a subgame-perfect Nash equilibrium (Selten, 1965) if it is an equilibrium á la Nash (1950) on every proper subgame of the original game. As we have assumed perfect information in our two-stage game, we can solve it starting from each subgame in stage 2. That is, we can first find -for each possible partition of strategic traders into non-exclusive sets of buyers and sellers- a profile of trade quantities (stage 2 strategies) constituting a Nash equilibrium, where none of the strategic traders has a strong incentive to unilaterally deviate from her strategy. After replacing each subgame in stage 2 with the payoffs generated by a Nash equilibrium play, we can then move back to stage 1 to check whether any partition of strategic traders can be in Nash equilibrium. We show in Section 3 that the subgames in stage 2 can always be solved in pure strategies: For any period and for any partition of strategic traders a pure-strategy Nash equilibrium profile of trade quantities exists, and it can be uniquely characterized (Proposition 1). But, unfortunately, due to the finiteness of the game played in stage 1, a pure-strategy equilibrium of trade directions, leading to an equilibrium partition of strategic traders, does not always exist, implying in turn the possibility of non-existence of a pure-strategy subgame-perfect Nash equilibrium of our two-stage game (Proposition 2). Besides, in situations an equilibrium partition of the strategic traders exists, it can be found only through extensive calculations, checking the 'no unilateral deviation' condition for all strategic traders at all possible partitions of strategic traders using their corresponding equilibrium trade quantities at these partitions. Definitely, the lack of a characterization for the equilibrium partition of strategic traders renders it impossible to study the equilibrium effects of the central bank's interventions theoretically. Nevertheless, we are still able to pursue this comparative statics exercise numerically in Section 4.

Our computer simulations show that the non-sterilized direct intervention of the central bank may lead to a perverse effect on the exchange rate. Since the central bank in our model aims to stabilize the exchange rate around a prespecified short-run target, the desirable direction of trade from the viewpoint of the central bank requires -under a backward-looking adjustment rule- buying (selling) foreign currency in any period if the equilibrium exchange rate was below (above) the target in the previous period. We also know that the central bank pushes the exchange rate upwards when it buys foreign currency. Oppositely, the central bank pulls the exchange rate downwards when it sells foreign currency. Thus, the intervention always creates a negative effect on the equilibrium profits of any strategic trader who trades in the direction desired by the central bank, whereas it creates a positive effect on the equilibrium profits of any strategic trader who trades in the opposite direction. In addition, both of these effects become stronger when the scale of the intervention is larger. Given these facts, consider a situation where some of the strategic traders in our model find it optimal to trade in some period in the direction desired by the central bank while its intervention is at some particular level. Further suppose that the contingent profits of these traders from trading in the other direction are only slightly lower than their current profits, while for all the remaining strategic traders in the market the gap between profits from buying and selling the foreign currency is sufficiently large. For this situation, it is obvious that a slight increase in the intervention of the central bank in the next period may induce the set of strategic traders in our consideration to optimally switch from the trade direction desired by the central bank to the opposite direction where the positive effect of intervention has just become stronger, while the assumed limited change in intervention could only yield negligible impacts on the trade orders of the remaining strategic traders due to their assumed large profit gaps. Definitely, the effect of trade reversals by the strategic traders in our consideration and the effect of the central bank's slightly increased intervention on the aggregate excess demand for foreign currency would work in opposite directions. In situations where the former effect dominates the latter, the equilibrium exchange rate would move in the direction undesired by the central bank, creating a perverse result of intervention. Since this result arises in our model when some strategic traders switch their trade direction, we call the underlying mechanism as 'strategic trade switching' channel, accordingly.

We believe that our study can be positioned within a strand of literature on market microstructure, dealing with the process of trade and price determination under the imperfect markets hypothesis. While some pioneering works of this literature are due Kyle (1985), Lyons (1997), and Evans and Lyons (2002a, 2002b), the closest work to ours is due Basu (2012), who also considers an oligopolistic model of competition involving both strategic traders and competitive traders. His objective is entirely different though, for he studies whether a central bank can devalue its currency without building up foreign reserves. Apart from this difference, the oligopolistic exchange model of Basu (2012) is unilateral, i.e., all strategic traders are assumed to be on the demand side of the market, whereas our model is bilateral -allowing the strategic traders to optimally place themselves at any side of the market. Actually, a bilateral oligopolistic exchange model was also addressed in the technical appendix of Basu (2009), an earlier version of Basu (2012). However, in that model the set of buying dealers and the set of selling dealers are exogenously given for the game played by the dealers, whereas in our model these two sets are also determined in equilibrium.

The rest of our paper is organized as follows: Section 2 presents our model and Section 3 presents our theoretical results. Section 4 involves the results of computer simulations illustrating the possibility of a perverse result of non-sterilized direct intervention. Finally, Section 5 concludes.

2 Model

We consider a multi-period model for a spot foreign exchange market in a two-country world, involving the home country (H) and the foreign country (F). This foreign exchange market contains an unspecified number of competitive traders -who take all prices as given in their exchanges- and a total of $n \geq 2$ strategic traders of home or foreign origin, buying or selling the foreign currency (in exchange of the home currency). We assume that the central bank of the home country (hereafter, simply the central bank) also trades in the same market to limit the variability of the spot exchange rate around a prespecified short-run target. The set of strategic traders is denoted by $N = \{1, 2, \ldots, n\}$, and -for convenience- the central bank is denoted by $n + 1$.

Let p_t be the exchange rate in period t; implying that one unit of the foreign currency is bought and sold at p_t units of the home currency. For simplicity, let the spread between buy and sell prices of currencies be zero in each period. Also, let the non-negative real numbers $q^B_{i,t}$ and $q^S_{i,t}$ respectively denote the quantity of the foreign currency bought and the quantity of the foreign currency sold by trader i in period t.

The strategic traders and the central bank in the market have non-negative cash holdings in both home and foreign currencies.[3] For any trader $i \in N \cup \{n + 1\}$, let $M^H_{i,t}$ and $M^F_{i,t}$ respectively denote her cash holdings in the home and the foreign currency. Assuming that initial cash holdings $M^H_{i,0}$ and $M^F_{i,0}$ of trader i are given, we can calculate her cash holdings in each period $t \geq 1$ as follows:

$$M^H_{i,t} = M^H_{i,t-1} - (q^B_{i,t} - q^S_{i,t})\, p_t, \tag{1}$$

[3]The same is true for the competitive traders, as well. We omit their cash holdings for they will not be relevant for our results.

and

$$M^F_{i,t} = M^F_{i,t-1} + q^B_{i,t} - q^S_{i,t}. \tag{2}$$

While the strategic traders and the central bank exchange currencies and change their cash balances in each period, the average acquirement prices of these balances also change. Let $p^H_{i,t}$ denote for trader i the average acquirement price of the home currency (in terms of the home currency) between periods 0 and t, implying that for each unit of home currency agent i is holding, he or she has sold until the end of period t exactly $1/p^H_{i,t}$ units of the foreign currency on average. Likewise, let $p^F_{i,t}$ denote for trader i the average acquirement price of the foreign currency (in terms of the home currency) between periods 0 and t, implying that for each unit of the foreign currency agent i is holding, he or she has sold until the end of period t exactly $p^F_{i,t}$ units of the home currency on average. Assuming that $p^H_{i,0}$ and $p^F_{i,0}$ are given at the beginning of period 1, the average acquirement price of home (foreign) currency holdings of trader $i \in N \cup \{n+1\}$ can be obtained in each period t by calculating the quantity-weighted average of the average acquirement price of her home (foreign) currency holdings in period $t-1$ and the purchase price of her home (foreign) currency acquired in period t as follows:

$$p^H_{i,t} = \frac{M^H_{i,t-1}p^H_{i,t-1} + \max(0, M^H_{i,t} - M^H_{i,t-1})p_t}{M^H_{i,t-1} + \max(0, M^H_{i,t} - M^H_{i,t-1})}, \tag{3}$$

and

$$p^F_{i,t} = \frac{M^F_{i,t-1}p^F_{i,t-1} + \max(0, M^F_{i,t} - M^F_{i,t-1})p_t}{M^F_{i,t} + \max(0, M^F_{i,t} - M^F_{i,t-1})}. \tag{4}$$

Using these average acquirement price calculations, each strategic trader can decide whether to buy or sell the foreign currency in any period, as we will later show. (Since the central bank, player $n+1$, in our model will have no intention to earn profit through its interventions, we will not need to calculate $p^H_{n+1,t}$ or $p^F_{n+1,t}$. On the other hand, we will need to calculate the cash holdings of the central bank in the home and foreign currencies, $M^H_{n+1,t}$ and $M^F_{n+1,t}$ respectively, to check in our simulations whether its currency trades are feasible.)

For the competitive traders as a whole, the supply and demand relationships for the foreign currency are respectively given by the following two functions:

$$s(p_t) = a + cp_t, \tag{5}$$

$$d(p_t) = b - p_t, \tag{6}$$

where a, b, c are positive real numbers. We assume $b > a$ to ensure that the equilibrium exchange rate would be positive even when the strategic traders and the central bank did not trade. Since, we consider a short-run model in our paper, we exclude the effect of any fundamentals (other than the central bank's intervention) in the above supply and demand functions as well as in the decisions of the strategic traders.

Thus, we have completed to describe the basic structures of our model. We can now consider the clearing of the foreign exchange market. Given equations (5) and (6), we can define the period-t excess supply of the competitive traders as

$$\psi(p_t) = s(p_t) - d(p_t) = a - b + (c+1)p_t. \tag{7}$$

Also, we can denote by $q^{ED}_{n+1,t} = q^B_{n+1,t} - q^S_{n+1,t}$ the central bank's period-t excess demand for the foreign currency. Then, in any period t the clearing of the foreign exchange market implies that the excess demand for the foreign currency by the non-competitive traders (the central bank and the strategic traders) must be equal to the excess supply of the foreign currency by the competitive traders:

$$q^{ED}_{n+1,t} + \sum_{i=1}^{n}(q^B_{i,t} - q^S_{i,t}) = \psi(p_t). \tag{8}$$

To solve for the exchange rate p_t in the above equality, we define the function

$$\xi(x) = \psi^{-1}(x) = \frac{b - a + x}{c+1} \tag{9}$$

for every $x \geq 0$, leading to the solution

$$p_t = \frac{b - a + q^{ED}_{n+1,t} + \sum_{i=1}^{n}(q^B_{i,t} - q^S_{i,t})}{c+1}. \tag{10}$$

In the above equation the parameters a, b, and c are always fixed. In each period t, the central bank, i.e. trader $n+1$, can control the excess demand variable $q^{ED}_{n+1,t}$. On the other hand, each strategic trader $i \in N$ can control her foreign currency purchase and sale, $q^B_{i,t}$ and $q^S_{i,t}$, respectively.

At this stage we will not be interested in how the central bank will vary its control variable $q^{ED}_{n+1,t}$. Moreover, we will assume that the variable $q^{ED}_{n+1,t}$ and the parameters a, b, and c are known by, and exogenously given to, all strategic traders. Having said this, we are ready to describe the decision problem faced by each strategic trader. Let N^B_t denote the set of strategic traders who buy the foreign currency in period t. Similarly, let N^S_t denote the set of strategic traders who sell the foreign currency in period t. Obviously,

$N_t^B \cup N_t^S = N$. Moreover, $N_t^B \cap N_t^S = \emptyset$, each strategic trader finds either selling or buying more profitable. This implies for all $i \in N_t^B$ we have $q_{i,t}^S = 0$ (no buying dealer sells the foreign currency in the same period) and for all $i \in N_t^S$ we have $q_{i,t}^B = 0$ (no selling dealer buys the foreign currency in the same period). In the zero-probability event that buying and selling the foreign currency are equally profitable for a strategic trader, we will allow this trader to arbitrarily determine whether she will buy or sell.

Now we will define the profit of each buyer and each seller, by treating the sets N_t^B and N_t^S as already determined. Eventually, these two sets will be determined in the equilibrium of our game, as will be shown later. So, given the sets of strategic buyers and sellers N_t^B and N_t^S, and their purchases and sales of the foreign currency, $(q_{j,t}^B)_{j\in N_t^B}$, and $(q_{l,t}^S)_{l\in N_t^S}$, the profit (measured in the home currency) of each strategic buyer $i \in N_t^B$ expected from buying the foreign currency can be calculated as

$$\begin{aligned}
&\pi_{i,t}^B((q_{j,t}^B)_{j\in N_t^B}, (q_{l,t}^S)_{l\in N_t^S}) \\
&= p_{i,t-1}^H\, q_{i,t}^B - p_t\, q_{i,t}^B \\
&= \left(\frac{(c+1)p_{i,t-1}^H - b + a - q_{n+1,t}^{ED} - \sum_{j\in N_t^B} q_{j,t}^B + \sum_{l\in N_t^S} q_{l,t}^S}{c+1} \right) q_{i,t}^B. \qquad (11)
\end{aligned}$$

Likewise, for each strategic seller $k \in N_t^S$, the profit (measured in the home currency) expected from selling the foreign currency can be calculated as

$$\begin{aligned}
&\pi_k^S((q_{j,t}^B)_{j\in N_t^B}, (q_{l,t}^S)_{l\in N_t^S}) \\
&= p_t q_{k,t}^S - p_{k,t-1}^F q_{k,t}^S \\
&= \left(\frac{-(c+1)p_{k,t-1}^F + b - a + q_{n+1,t}^{ED} + \sum_{j\in N_t^B} q_{j,t}^B - \sum_{l\in N_t^S} q_{l,t}^S}{c+1} \right) q_{k,t}^S. \qquad (12)
\end{aligned}$$

Note that each strategic buyer $i \in N_t^B$ in period t seeks a non-negative quantity of purchase $q_{i,t}^B$ that is maximizing (11) and also feasible, i.e., that can be bought using her beginning-of-period cash holdings in the home currency $M_{i,t-1}^H$. Similarly, each strategic seller $k \in N_t^S$ in period t seeks a non-negative quantity of sale $q_{k,t}^S$ that is maximizing (12) and also feasible, i.e., that does not exceed her beginning-of-period cash holdings in the foreign currency $M_{i,t-1}^F$. These two sets of objectives are interdependent because of a common variable, p_t, entering both (11) and (12). That is to say, the choice of $q_{i,t}^B$ affects, through its influence on p_t, the choice of $q_{i,t}^S$, and vice versa. This implies that each strategic trader, when determining her choice of trade order, has to take into account the choices of all other strategic traders.

For convenience, we represent the decision problems of the strategic traders in each period using a two-stage extensive form game with complete and perfect information. Evidently, the players of this game are the strategic traders in our model. In stage 1, each player non-cooperatively decides whether to buy or sell the foreign currency. The decisions of all players defines a partition of them into non-exclusive sets of buyers and sellers. In stage 2, after observing this partition, each player determines her trade quantity. So, each player's complete strategy before the game starts involves her trade direction in stage 1, i.e., the plan whether to buy or sell foreign currency, along with how much she will trade at any subgame played in stage 2. It is clear that using her profits from buying and selling the foreign currency each player can then calculate her terminal payoffs at each strategy profile of the players using equations (11) and (12), taking into account the corresponding market clearing level of the exchange rate.

A strategy profile in an extensive-form game is said to be a subgame-perfect Nash equilibrium when it is a Nash equilibrium on every proper subgame of the original game. Thanks to our perfect information assumption, implying that the players observe at the beginning of stage 2 all decisions made in stage 1, we can solve our game starting from each subgame in stage 2.

Stage 2: In this stage, we seek -for each possible partition of strategic traders into non-exclusive sets of buyers and sellers- a strategy profile of trade quantities constituting a Nash equilibrium, where none of the strategic traders has a strong incentive to unilaterally deviate from her strategy. Formally, we say that given any partition $\{N_t^B, N_t^S\}$ of N, the strategy profile $\left((q_{j,t}^{*B})_{j\in N_t^B}, (q_{l,t}^{*S})_{l\in N_t^S}\right)$ is a Nash equilibrium if the following two conditions hold:

i) For all $i \in N_t^B$ and for all $q_{i,t}^B \in [0, M_{i,t-1}^H]$

$$\pi_{i,t}^B\left((q_{j,t}^{*B})_{j\in N_t^B}, (q_{l,t}^{*S})_{l\in N_t^S}\right) \geq \pi_{i,t}^B\left(q_{i,t}^B, (q_{j,t}^{*B})_{j\in N_t^B\setminus\{i\}}, (q_{l,t}^{*S})_{l\in N_t^S}\right). \tag{13}$$

ii) For all $k \in N_t^S$ and for all $q_{k,t}^S \in [0, M_{i,t-1}^F]$

$$\pi_{k,t}^S\left((q_{j,t}^{*B})_{j\in N_t^B}, (q_{l,t}^{*S})_{l\in N_t^S}\right) \geq \pi_{k,t}^S\left((q_{j,t}^{*B})_{j\in N_t^B}, q_{k,t}^S, (q_{l,t}^{*S})_{l\in N_t^S\setminus\{k\}}\right). \tag{14}$$

Above, condition (i) states that each strategic buyer i finds the purchase strategy $q_{i,t}^{*B} \in [0, M_{i,t-1}^H]$ profit maximizing if all other buyers and all sellers stick to their strategies in the profile $\left((q_{j,t}^{*B})_{j\in N_t^B\setminus\{i\}}, (q_{l,t}^{*S})_{l\in N_t^S}\right)$. Likewise, condition (ii) states that each strategic seller k finds the sale strategy $q_{k,t}^{*S} \in [0, M_{k,t-1}^F]$ profit maximizing if all other sellers and all buyers stick to their strategies in the profile $\left((q^{*B}_{j,t})_{j\in N^B_t},(q^{*S}_{l,t})_{l\in N^S_t\setminus\{k\}}\right)$. In short, a strategy profile of trade quantities is a Nash equilibrium if no buyer or seller has a strict incentive for unilateral deviation (to another trade quantity). Since the sets N^B_t and N^S_t partition the set of strategic traders N and since we have assumed that N is constant, we can denote the Nash equilibrium purchase of each buyer $i \in N^B_t$ by $q^{*B}_{i,t}(N^B_t)$ and the Nash equilibrium sale of each seller $k \in N^S_t$ by $q^{*S}_{i,t}(N^S_t)$, for simplicity. Now, we can go back to stage 1.

Stage 1: We can replace each subgame in stage 2 with the payoffs generated by a Nash equilibrium play, and check whether any partition of the strategic traders, generated by the strategy profile of the players in stage 1, can be in Nash equilibrium. Formally, we say that the partition $\{N^{*B}_t, N^{*S}_t\}$ is a Nash equilibrium partition if the following two conditions hold:

i) For all $i \in N^{*B}_t$

$$\begin{aligned}&\pi^B_{i,t}\left((q^{*B}_{j,t}(N^{*B}_t))_{j\in N^{*B}_t},(q^{*S}_{l,t}(N^{*S}_t))_{l\in N^{*S}_t}\right)\geq\\&\pi^S_{i,t}\left((q^{*B}_{j,t}(N^{*B}_t\setminus\{i\}))_{j\in N^{*B}_t\setminus\{i\}},(q^{*S}_{l,t}(N^{*S}_t\cup\{i\}))_{l\in N^{*S}_t\cup\{i\}}\right).\end{aligned}\tag{15}$$

ii) For all $k \in N^{*S}_t$

$$\begin{aligned}&\pi^S_{k,t}\left((q^{*B}_{j,t}(N^{*B}_t))_{j\in N^{*B}_t},(q^{*S}_{l,t}(N^{*S}_t))_{l\in N^{*S}_t}\right)\geq\\&\pi^B_{k,t}\left((q^{*B}_{j,t}(N^{*B}_t\cup\{i\}))_{j\in N^{*B}_t\cup\{i\}},(q^{*S}_{l,t}(N^{*S}_t\setminus\{i\}))_{l\in N^{*S}_t\setminus\{i\}}\right).\end{aligned}\tag{16}$$

If a partition is in Nash equilibrium in any period, every strategic trader must be satisfied with her decision regarding whether to become a buyer or seller in that period, given the decisions of the others. Accordingly, the first condition above requires that no strategic buyer can be strictly better off by acting like a strategic seller and choosing the optimal quantity to sell. Conversely, the second condition requires that no strategic seller can be strictly better off by acting like a strategic buyer and choosing the optimal quantity to buy.

Having described the equilibrium in each stage of our extensive-form game, we can say that a profile of buying/selling decisions yielding the partition $\{N^{*B}_t, N^{*S}_t\}$ and a profile of trade quantities $\left(q^{*B}_{j,t}(N^B_t))_{j\in N^B_t},(q^{*S}_{l,t}(N^S_t))_{l\in N^S_t}\right)$ defined for each possible partition $\{N^B_t, N^S_t\}$ together constitute a subgame-perfect Nash equilibrium of our two-stage game if:

(i) for each partition $\{N_t^B, N_t^S\}$, the strategy profile $\left(q_{j,t}^{*B}(N_t^B)\right)_{j\in N_t^B}, (q_{l,t}^{*S}(N_t^S))_{l\in N_t^S}\big)$ is in Nash equilibrium, and

ii) the partition $\{N_t^{*B}, N_t^{*S}\}$ is in Nash equilibrium.

In the next section, we will investigate whether our game is solvable by the notion of subgame-perfect Nash equilibrium in pure strategies.

3 Theoretical Results

Below we first characterize the equilibrium of each subgame played in the second stage of our foreign exchange game.

Proposition 1. *For any partition of strategic traders, $\{N_t^B, N_t^S\}$, a pure strategy Nash equilibrium profile of trade quantities always exists and this equilibrium along with the corresponding market clearing price is uniquely characterized by (17-(19):*

$$q_{i,t}^{*B} = \frac{-b+a-q_{n+1,t}^{ED}}{n+1} + \frac{c+1}{n+1}\left(np_{i,t-1}^H - \sum_{j\in N_t^B\setminus\{i\}} p_{j,t-1}^H - \sum_{l\in N_t^S} p_{l,t-1}^F\right), \forall i \in N_t^B \quad (17)$$

$$q_{k,t}^{*S} = \frac{b-a+q_{n+1,t}^{ED}}{n+1} - \frac{c+1}{n+1}\left(np_{k,t-1}^F - \sum_{j\in N_t^B} p_{j,t-1}^H - \sum_{l\in N_t^S\setminus\{k\}} p_{l,t-1}^F\right), \forall k \in N_t^S \quad (18)$$

$$p_t^* = \frac{1}{n+1}\left(\frac{b-a+q_{n+1,t}^{ED}}{c+1}\right) + \frac{1}{n+1}\left(\sum_{i\in N_t^B} p_{i,t-1}^H + \sum_{k\in N_t^S} p_{k,t-1}^F\right). \quad (19)$$

Proof. See Appendix.

Note that by replacing the arbitrary partition $\{N_t^B, N_t^S\}$ in equations (17)-(19) of Proposition 1 with the equilibrium partition $\{N_t^{*B}, N_t^{*S}\}$, whenever it exists, we can calculate the subgame-perfect Nash equilibrium of our two-stage game and the corresponding market clearing exchange rate. Also recall that we can search for the equilibrium partition $\{N_t^{*B}, N_t^{*S}\}$ in stage 1 of our game, by checking for each partition $\{N_t^B, N_t^S\}$ the inequality conditions (15) and (16) using the Nash equilibrium profile of trade quantities wherever necessary. However, as it is well known, a pure strategy Nash equilibrium may not exist in games with a finite number of players and strategies. Apparently, this raises a red flag for the game played in stage 1, where the number of players is finite and each player has a finite number of strategies, namely the two strategies of 'committing to buy the foreign currency in stage 2' and 'committing to sell the foreign currency in stage 2'. Our next result shows that our concern in that regard is not unfounded; for some specifications of our model no partition of strategic traders can be an equilibrium, directly implying that for these cases no subgame-perfect Nash equilibrium can exist in pure strategies.[4]

Proposition 2. *For some settings of our model, the corresponding two-stage foreign exchange game has no subgame-perfect Nash equilibrium in pure strategies.*

Proof. See Appendix.

Unfortunately, we are facing at this point not only the possibility of non-existence of a pure-strategy equilibrium partition of strategic traders but also the impossibility of characterizing a closed-form solution for the pure-strategy equilibrium partition whenever it exists. Thus, we are unable to make comparative statics on our theoretical results. To shed more light on this matter, consider the following thought experiment where we change the foreign currency purchase of the central bank to see its impact on the equilibrium exchange rate. For convenience, suppose that a subgame-perfect Nash equilibrium exists before this experiment, allowing us to change the partition $\{N_t^B, N_t^S\}$ with the equilibrium partition $\{N_t^{*B}, N_t^{*S}\}$ in equations (17)-(19). Now, consider an increase in the central bank's period-t excess demand for the foreign currency $\bar{q}_{n+1,t}^{ED}$ by one unit. This would increase up the value of the first parenthesis in equation (19) by $1/[(n+1)(c+1)]$ units. But, we are unable to predict whether or how the assumed unit change in $\bar{q}_{n+1,t}^{ED}$

[4] The negative result about the existence of equilibrium partitions only concerns the equilibrium in pure strategies where each player is restricted to choose to be either a buyer or a seller with probability one. If we had allowed mixed (non-pure) strategies we would always have an equilibrium thanks to the existence result of Nash (1950). In such a game, each strategic player $i \in N$ would believe that every other player would be a buyer (of the foreign currency) with some probability $v_i \in [0, 1]$ and a seller with probability $1 - v_i$. Since we can easily find, as illustrated in the next section, some model settings which will lead to the existence of a pure-strategy equilibrium partition of the strategic traders, we have abstained in this section from characterizing the mixed strategy equilibrium partition(s).

would impact on the equilibrium partition $\{N_t^{*B}, N_t^{*S}\}$ (or on its very existence), and consequently on the terms inside the second parenthesis in equation (19). In short, our theoretical results do not allow us to analytically study the effects of non-sterilized direct interventions of the central bank on the exchange rate. Nevertheless, we will be able to conduct this analysis numerically in Section 4.

4 Simulation Results

Here, we simulate our model to study the response of its equilibrium to non-sterilized direct interventions of the central bank. (We conduct our computations using GAUSS Software Version 3.2.34 [Aptech Systems, 1998]. The source code of our simulation program is available upon request.)

For our simulations, we consider a market with three strategic traders, i.e., $n = 3$. Accordingly, the set of non-competitive traders becomes $N = \{1, 2, 3, 4\}$, with trader 4 denoting the central bank. We also assume that all non-competitive traders have the same initial cash holdings, satisfying $M_{i,0}^H = 100$ and $M_{i,0}^F = 20$ for $i = 1, 2, 3, 4$. The central bank, i.e., trader 4, intervenes to the market only directly, by buying or selling the foreign currency. However, unlike the strategic traders, the central bank has no intention to achieve any monetary gain through currency trade. It trades the foreign currency only to limit the variability of the exchange rate around a prespecified short-run target. Formally, the central bank trades in each period $t \geq 1$ (or controls its excess demand $q_{4,t}^{*ED}$) according to a backward-looking rule given by

$$q_{4,t}^{*ED} = (\bar{P} - p_{t-1}^*)\bar{Q}, \tag{20}$$

where p_{t-1}^* is the equilibrium exchange rate in period $t-1$, $\bar{P}$ is a prespecified (short-run) target for the equilibrium exchange rate, and $\bar{Q} > 0$ is a parameter affecting the size of intervention. This rule implies that the central bank buys (sells) the foreign currency in period t if and only if the equilibrium exchange rate observed in period $t - 1$ is below (above) the target.

Model Settings: We will simulate our model for 50 consecutive periods. We assume that the foreign currency supply and demand functions of the competitive traders are parameterized by $a = 4$, $b = 13.9$, and $c = 1.9$. On the other hand, the average acquirement price of cash holdings of the strategic traders are as in Table 1.

Table 1.

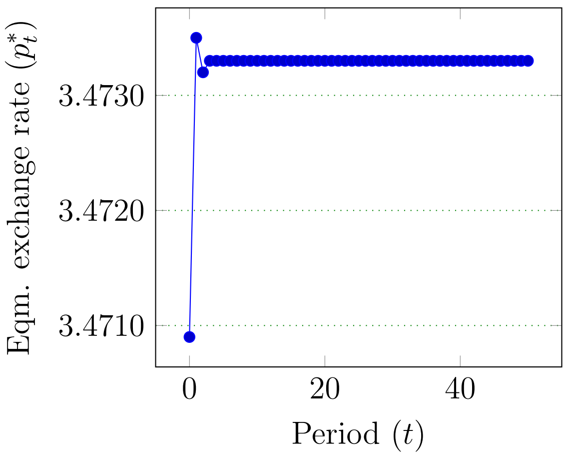
$p^H_{1,0}$	$p^H_{2,0}$	$p^H_{3,0}$	$p^F_{1,0}$	$p^F_{2,0}$	$p^F_{3,0}$
3.5500	3.4500	3.5000	3.5000	3.4200	3.4400

We set the central bank's target exchange rate $\bar{P}$ to 3.5000 and vary the intervention parameter $\bar{Q}$ inside the set $\{1, 4, 7, 10, 13, 16\}$. Note that each value of $\bar{Q}$ will define along with all other parameters a distinct simulation of our model lasting for 50 periods.

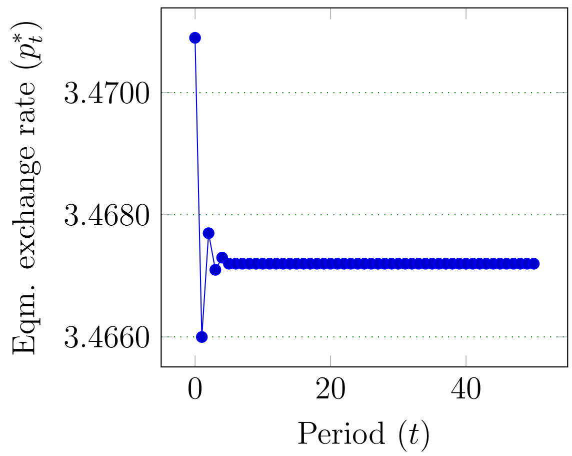
For all six values of $\bar{Q}$, we have found that an equilibrium partition of strategic traders $\{N_t^{B^*}, N_t^{S^*}\}$ exists in all 50 periods. Inserting the calculated equilibrium partition into equations (17)-(19), we have computed in each period the market clearing exchange rate and the equilibrium trades of the strategic traders. (In any period t, where there were two or more equilibrium partitions, we selected the one that minimized the difference between the market clearing exchange rates in period t and $t-1$.) Hereafter, we will call the market clearing exchange rate, calculated at the subgame-perfect equilibrium of our game, as the equilibrium exchange rate simply.

In Figure 1 below, we plot the equilibrium exchange rate p_t^* for the six values of $\bar{Q}$. We should note that in all simulations, the central bank starts to intervene in period 1. Therefore, for period 0, we set the parameter $\bar{Q}$ to 0, leading to an equilibrium exchange rate of $p_0^* = 3.4709$. Accordingly, in all six panels of Figure 1 the graph of p_t^* starts at the point $(0, 3.4709)$. Additional observations about Figure 1 are in order. First, when the size of $\bar{Q}$ is sufficiently low as in panels (a)-(d), the equilibrium exchange rate fluctuates converging to a steady-state level, and otherwise the equilibrium exchange rate fluctuates without convergence as in panels (e)-(f). Also, as we note from panels (a)-(d), the speed of convergence is decreasing in the size of $\bar{Q}$. Our second observation, obtained from panels (a)-(d), is that the steady-state level of the equilibrium exchange rate may differ from the central bank's target rate, which could in fact never be attained. On the other hand, whenever $\bar{Q}$ becomes quite high as in panel (f) of Figure 1, the central bank can achieve its target, but only temporarily while the equilibrium exchange rate moves on a non-converging oscillatory path. The first and second observations might simply imply that the central bank, while planning to conduct a non-sterilized intervention, may face a dilemma between never achieving its target and creating huge exchange rate fluctuations around its target.

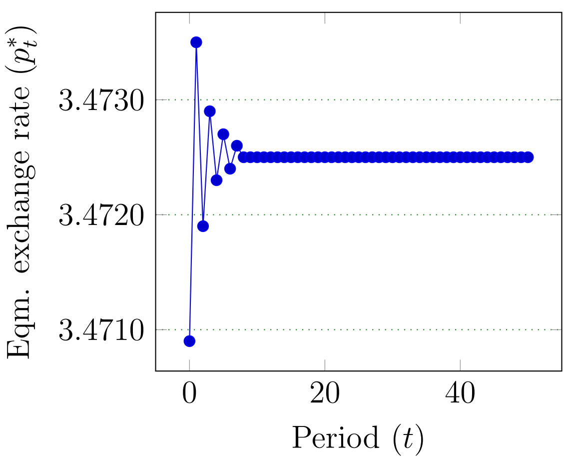
Figure 1. The Time Path of the Equilibrium Exchange Rate
for Different Intervention Strengths



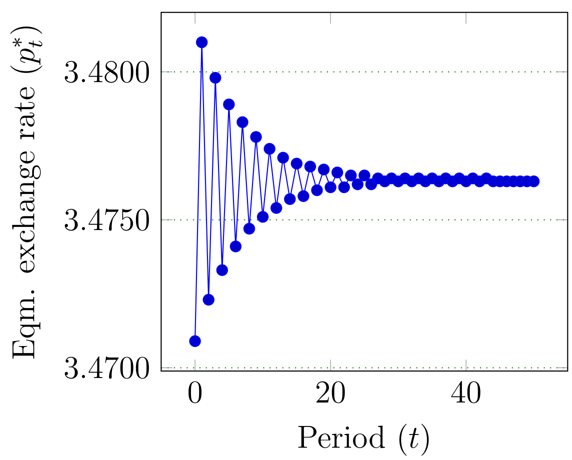
(a) $\bar{Q}$=1



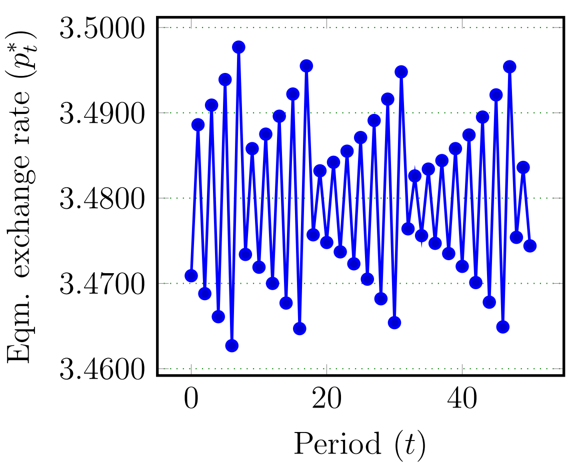
(b) $\bar{Q}$=4



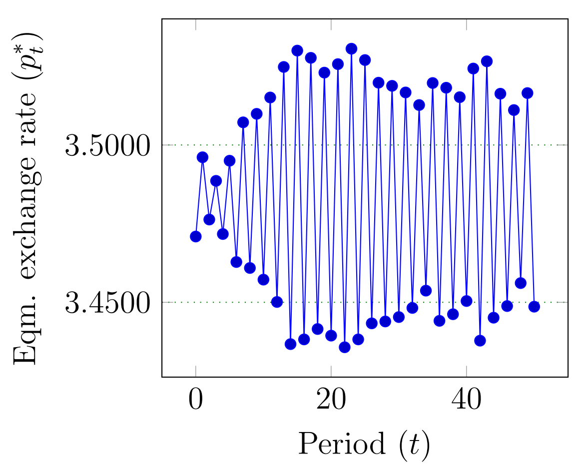
(c) $\bar{Q}$=7



(d) $\bar{Q}$=10



(e) $\bar{Q}$=13



(f) $\bar{Q}$=16

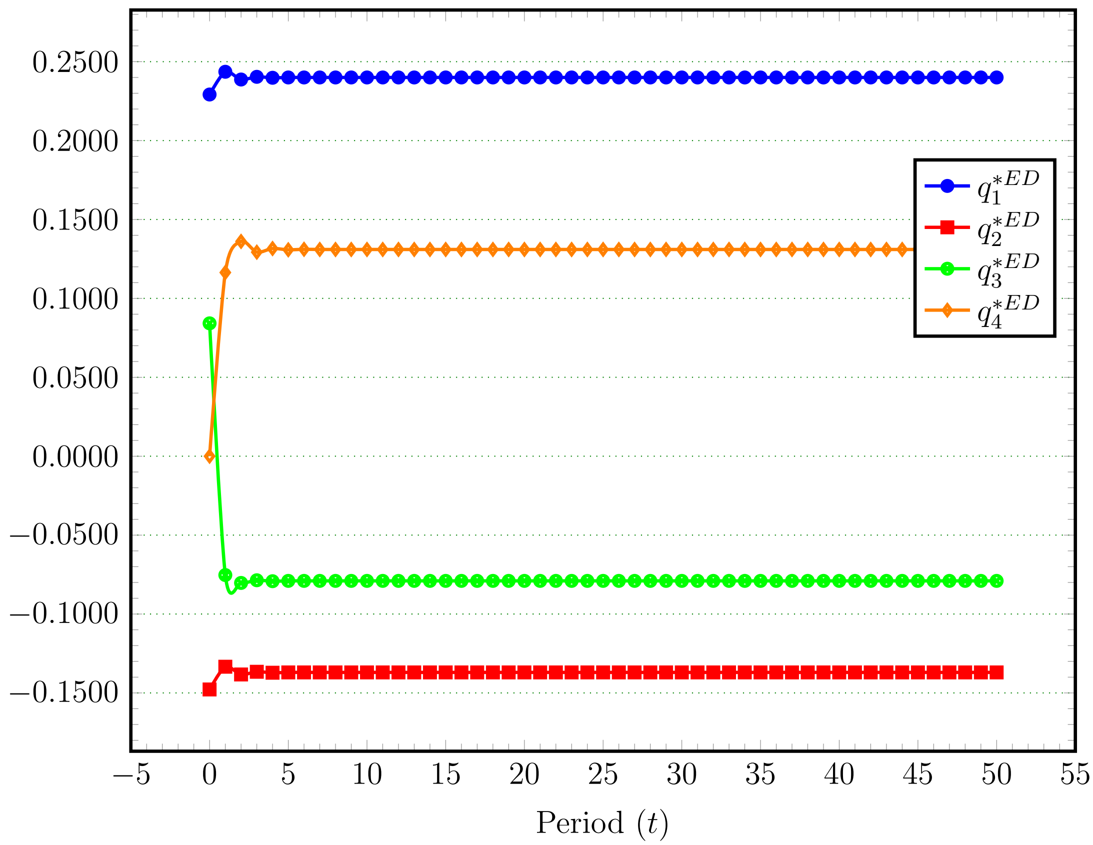
We should also note that in panels (a)-(e) of Figure 1 the equilibrium exchange rate p_t^* never exceeds the target rate, $\bar{P} = 3.5000$. For these panels, the right hand side of equation (20) would always become positive, requiring the central bank to buy the foreign currency in each of the 50 simulation periods. Despite that, the equilibrium exchange rate p_t^* in panels (b) and (e) of Figure 1 is found to fall below its starting level, $p_0^* = 3.4710$, in some periods. To explain the reason of this perverse result, we will focus on panel (b) and report in Table 2 the equilibrium exchange rate and the central bank's excess demand for the foreign currency in some selected periods. As is evident from the first two rows of this table, the foreign currency purchase of the central bank has increased by 0.1164 units in period 1, leading to a decrease in the equilibrium exchange rate by 50 pips (from 3.4710 to 3.4660). In all subsequent periods the equilibrium exchange rate positively responds to the central bank's purchases, in line with equation (20). Apparently, the perverse result observed in period 1 is so large that the steady state level of the equilibrium exchange rate, which is reached around period 7 at a value of 3.4672, is not only lower than the target exchange rate of 3.5000 but also the pre-intervention level of $p_0^* = 3.4710$.

Table 2.

t	$q_{4,t+1}^{*ED}$	p_t^*
0	0	3.4710
1	0.1164	3.4660
2	0.1361	3.4677
3	0.1293	3.4671
4	0.1316	3.4673
5	0.1308	3.4672
6	0.1311	3.4672
7	0.1310	3.4672
...	...	...
50	0.1310	3.4672

What we have just illustrated above points to a more general puzzle where the central bank, aiming to stabilize the equilibrium exchange rate around a target by non-sterilized direct intervention, may instead unintentionally move it away from the target like in panel (b) of Figure 1, if not leading to an unstable fluctuation as in panels (e) and (f). Surely, such perverse results can not arise in a perfectly competitive market defined by conventional supply and demand functions. But, the exchange market we model in our paper is not perfectly competitive, and neither are the actual foreign exchange markets to the best of our observation. It is entirely the imperfection of our exchange market, i.e., the existence of foreign exchange dealers that can strategically act against each other and against the central bank, that drives the illustrated perverse response. To shed more light upon this, we will plot in Figure 2 the time paths of the four non-competitive traders' equilibrium excess demands for the foreign currency (i.e., $q_{i,t}^{*ED} = q_{i,t}^{*B} - q_{i,t}^{*S}$ for $i = 1, \ldots, 4$ and $t = 0, \ldots, 50$) corresponding to the simulation in panel (b).

Figure 2. Non-Competitive Traders' Equilibrium Excess Demands for the Foreign Currency



In the above figure we can immediately observe that player 1 (on the blue curve) is always a buyer of the foreign currency while player 2 (on the red curve) is always a seller. On the other hand, the third player (on the green curve), who buys the foreign currency in period 0 where there exists no intervention ($q_{4,0}^{*ED} = 0$), decides to be a seller from period 1 onwards, where the central bank (on the orange curve) starts to directly intervene. To have a closer look at the changes in these four curves in period 1, we report in Table 3 the data drawn from Figure 2 for periods 0 and 1, along with the total excess demand for the foreign currency by the non-competitive traders. Here, we observe that when the central bank starts to intervene in period 1 by purchasing 0.1164 units of the foreign currency, the total excess demand of the non-competitive traders falls down from the pre-intervention level of 0.1657 units to 0.1513 units, entirely because of player 3's switching from buying to selling. On the other hand, the central bank's intervention has no direct influence on the excess supply of the competitive traders, i.e., the right hand-side of the market clearing condition (8). Thus, the fall in the total excess demand of the non-competitive traders for the foreign currency caused by the central bank's intervention leads to a reduction in the aggregate excess demand before the adjustment of exchange rate occurs, and consequently this reduction pushes down the market clearing exchange rate of period 1, p_1^*, to a level lower than the pre-intervention rate of p_0^*.

Table 3.

t	$q_{1,t}^{*ED}$	$q_{2,t}^{*ED}$	$q_{3,t}^{*ED}$	$q_{4,t}^{*ED}$	$\sum_{i=1}^{4} q_{i,t}^{*ED}$
0	0.2292	-0.1478	0.0842	0.0000	0.1657
1	0.2436	-0.1334	-0.0753	0.1164	0.1513

The reason why in Figure 2 the third player -and only this player- changes the direction of trade can be understood by re-inspecting Table 1, where we observe that the average acquirement prices of both home and foreign currency holdings are initially higher for player 1, and lower for player 2, than the target exchange rate of 3.5000 and also the period-0 equilibrium exchange rate of $p_0^* = 3.4709$. This table suggests that as long as the equilibrium exchange rate does not exceed the target rate, player 1 would prefer to be a buyer of the foreign currency whereas player 2 would prefer to be a seller. That is why the trade directions of these two players never change in any simulation periods. For player 3, however, the situation is not the same. The average acquirement price of the home currency $p_{3,0}^H$ for player 3 is just equal to the central bank's target rate of 3.5000, while the average acquirement price of the foreign currency $p_{3,0}^F = 3.4400$ is below the pre-intervention rate of $p_0^* = 3.4709$. One can check that in period 0 the unit profits from buying and selling the foreign currency are very close for player 3, and therefore in period 1 the profits of this player and the equilibrium exchange rate become extremely sensitive to the intervention of the central bank. In period 0, where there is no intervention, player 3 finds it optimal to buy the foreign currency, because if she chooses to become a seller instead, the equilibrium price would not be sufficiently high to warrant her switching from buying to selling. However, when the foreign currency purchase of the central bank drastically rises by $q_{4,1}^{*ED} = 0.1164$ units in period 1, there arises an opportunity for player 3 to switch from buying to selling the foreign currency without reducing the equilibrium exchange rate, and the profitability of selling relative to buying, too much. To put it in a different way, the central bank's purchases of the foreign currency in period 1 substitute for player 3's purchases of the foreign currency in period 0. So, the central bank's trading in period 1 almost like player 3 of the previous period, makes it possible for player 3 to act in period 1 as if she is a new player. Resultingly, she optimally switches from buying to selling the foreign currency since she finds that the repercussions of this switching on the equilibrium price and consequently on her profits would be unintentionally compensated by the central bank's purchases of the foreign currency.

5 Conclusion

In this paper, we have considered a multi-period model for a spot foreign exchange market that involves a finite number of strategic traders, an unspecified number of competitive traders, and a central bank with the goal to stabilize the exchange rate around a prespecified (short-run) target. The key feature of this market is that prices and quantities are determined together, unlike in rational expectations models where the quantity decisions are conditional on prices. Each period of our model involves a two-stage game (with complete and perfect information) played by the strategic traders. In stage 1 of this game, each strategic trader non-cooperatively commits to whether to buy or sell the foreign currency, and in stage 2, after observing the stage 1 commitments of all strategic traders, each strategic trader non-cooperatively decides how much to trade in the direction she determined in stage 1. We have showed that a meaningful solution (a pure-strategy subgame-perfect Nash equilibrium) of this game may not always exist and whenever it exists it cannot be characterized in a closed form. Thus, we have made some numerical settings for the variables and parameters in our model to make our game solvable, and calculated the equilibrium solution for different strengths of interventions using a computer program. Our calculations have showed that non-sterilized direct interventions of the central bank to this market may yield perverse effects on the equilibrium (spot) exchange rate. The underlying reason for this puzzle is entirely strategic: As the intervention of the central bank moves the equilibrium exchange rate towards the target, the profits of strategic traders from buying and selling the foreign currency change. As a matter of fact, an increase in the intervention of the central bank, through its effect on the equilibrium exchange rate, decreases the profits of the strategic traders who trade in the direction targeted by the central bank, while increasing the profits of those trading in the other direction. If, at some level of intervention, there are some strategic traders for whom these two profits are sufficiently close to each other and if these traders have found it optimal to trade in the direction targeted by the central bank, then even a slight increase in the intervention of the central bank may unintentionally lead these traders to optimally switch their trading to the opposite direction. These trade reversals would destabilize the aggregate excess demand for foreign currency and resultingly move the equilibrium exchange rate away from the targeted level. This perverse result along with the previous results of Dominguez and Frankel (1993) and Bhattacharya and Weller (1997) implies that interventions may yield perverse responses regardless whether they are sterilized or non-sterilized.

Besides its simplicity, our model has several limitations, as well. First of all, we have assumed that each strategic trader has complete information about the acquirement prices of the home and foreign currency balances of all other strategic traders. This assumption may require all strategic traders to observe or guess the currency transactions of all other traders, which may be impossible or very difficult since these transactions are officially anonymous. However, we should note that it is possible to alleviate this drawback of our model by introducing incomplete information on the part of strategic traders, though at the expense of complicating the computations we should make to solve our two-stage game.

Another drawback of our model is that the strategic traders are assumed to trade in each period their equilibrium orders. While this assumption also necessitates common knowledge about the conjectures of each strategic trader about all other strategic traders' strategies, in reality the formation of common knowledge, and consequently the formation of an equilibrium, may take long periods of time. Thus, the strategic traders may actually trade in some periods non-equilibrium quantities and even trade in directions unsupported by any equilibria.

In addition, the two-stage extensive form game played by the spot market dealers in each period may be an inadequate representation of the actual trading process taking place in the foreign exchange markets. That is, each dealer, instead of determining her trade direction and trade quantity sequentially like in our model, might determine these two variables simultaneously like in reality, leading to a single-stage game. However, we should also note that the normal-form representation of our extensive-form game would actually allow us to study the Nash equilibrium of a such a single-stage game at the expense of some additional computational costs.

Furthermore, our model limits the definition of profits from trading foreign currency by disregarding the strategic traders' expectations about the profitability of future trades. For example, a strategic trader's unit profits from buying the foreign currency in our model is for simplicity defined to be the average acquirement price of home currency holdings used for the transaction net of the price (exchange rate) paid for a unit foreign currency. We could have alternatively defined this unit profit as the expected future worth of a unit foreign currency net of its current price. Clearly, this definition would require -under some rationality assumptions- the strategic traders to solve their future optimization problems in advance in order to make predictions about the future exchange rates. While we admit that extending our model in this direction may be fruitful, whether this forward-looking alternative definition or our current definition of unit profits is more realistic is entirely a behavioral question which is beyond the scope of this paper.

It should be apparent that neither of the limitations discussed above is responsible for the perverse result of intervention. Irrespective of whether strategic traders play their complete or incomplete information equilibrium or non-equilibrium strategies or play a two-stage game or single-stage game, and irrespective of the definition of unit profits from buying and selling the foreign currency, perverse responses to the central bank's interventions may arise if at the prevailing exchange rate some sufficiently large strategic traders in the market find themselves, with respect to their profit calculations, right at the edge of switching their trades to the direction untargeted by the central bank.

Future research may extend our work to study whether perverse responses may also arise when the central bank intervenes not (only) directly (through exchanges of the foreign currency) but (also) indirectly, say, by controlling the nominal interest rate of the domestic financial assets, so as to influence the excess demand for the foreign currency.

Finally, we believe that the findings in our paper not only add to our understanding of how the imperfect exchange markets operate in the presence of non-sterilized interventions but also may provide some guidance for monetary authorities. Needless to say, correctly anticipating when a perverse result of intervention would arise might not be an easy or even achievable task as it would require the central bank to always closely watch and be aware of officially anonymous and also extremely frequent transactions of strategic traders in the market, to estimate their contingent profits from buying or selling the foreign currency so as to correctly guess their trade orders. Thus, central banks might not be always successful in conducting a non-sterilized intervention without creating some perverse effects on the exchange rate. Since similar perverse results are known to arise under a sterilized intervention as well, our findings may unintentionally contribute to a debate whether it would not be better -in most cases- for the central bank not to conduct interventions in any form and just leave the determination of the foreign exchange rate to market makers.

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Appendix

Proof of Proposition 1. For any trader $i \in N_t^B$ the profit from buying the foreign currency can be written as

$$\begin{aligned}\pi_{i,t}^B(N_t^B, N_t^S) &= p_{i,t-1}^H\, q_{i,t}^B - p_t q_{i,t}^B && (21)\\ &= \left(\frac{(c+1)p_{i,t-1}^H - b + a - q_{n+1,t}^{ED} - \sum_{j\in N_t^B} q_{j,t}^B + \sum_{l\in N_t^S} q_{l,t}^S}{c+1}\right) q_{i,t}^B.\end{aligned}$$

The first order condition for profit maximization implies

$$\begin{aligned}&\frac{\partial}{\partial q_{i,t}^B}\pi_{i,t}^B(N_t^B, N_t^S) =\\ &\frac{(c+1)p_{i,t-1}^H - b + a - q_{n+1,t}^{ED} - \sum_{j\in N_t^B\setminus\{i\}} q_{j,t}^B + \sum_{l\in N_t^S} q_{l,t}^S}{c+1} - \frac{2q_{i,t}^B}{c+1} = 0. \qquad (22)\end{aligned}$$

It follows that for any $i, j \in N_t^B$ we have

$$(c+1)[p_{i,t-1}^H - p_{j,t-1}^H] - (q_{j,t}^B - q_{i,t}^B) = 2(q_{i,t}^B - q_{j,t}^B) \qquad (23)$$

or

$$q_{j,t}^B = q_{i,t}^B - (c+1)[p_{i,t-1}^H - p_{j,t-1}^H]. \qquad (24)$$

Thus, we have

$$\sum_{j\in N_t^B\setminus\{i\}} q_{j,t}^B = (|N_t^B| - 1)q_{i,t}^B - (|N_t^B| - 1)(c+1)p_{i,t-1}^H + (c+1)\sum_{j\in N_t^B\setminus\{i\}} p_{j,t-1}^H \qquad (25)$$

implying

$$\sum_{j\in N_t^B} q_{j,t}^B = |N_t^B| q_{i,t}^B - (|N_t^B| - 1)(c+1)p_{i,t-1}^H + (c+1)\sum_{j\in N_t^B\setminus\{i\}} p_{j,t-1}^H. \qquad (26)$$

Likewise, for any trader $k \in N_t^S$, the profit from selling the foreign currency equals

$$\begin{aligned}\pi_k^S(N_t^B, N_t^S) &= p_t q_{k,t}^S - p_{k,t-1}^F q_{k,t}^S && (27)\\ &= \left(\frac{-(c+1)p_{k,t-1}^F + b - a + q_{n+1,t}^{ED} + \sum_{j\in N_t^B} q_{j,t}^B - \sum_{l\in N_t^S} q_{l,t}^S}{c+1}\right) q_{k,t}^S.\end{aligned}$$

The first order condition for profit maximization implies

$$\begin{aligned}&\frac{\partial}{\partial q_{k,t}^S}\pi_k^S(N_t^B, N_t^S) =\\ &\frac{-(c+1)p_{k,t-1}^F + b - a + q_{n+1,t}^{ED} + \sum_{j\in N_t^B} q_{j,t}^B - \sum_{l\in N_t^S\setminus\{k\}} q_{l,t}^S}{c+1} - \frac{2q_{k,t}^S}{c+1} = 0. \qquad (28)\end{aligned}$$

It follows that for any $k, l \in N_t^S$ we have

$$-(c+1)(p_{k,t-1}^F - p_{l,t-1}^F) - (q_{l,t}^S - q_{k,t}^S) = 2(q_{k,t}^S - q_{l,t}^S) \tag{29}$$

or

$$q_{l,t}^S = q_{k,t}^S + (c+1)(p_{k,t-1}^F - p_{l,t-1}^F). \tag{30}$$

Thus, we have

$$\sum_{l \in N_t^S \setminus \{k\}} q_{l,t}^S = (|N_t^S| - 1)q_{k,t}^S + (|N_t^S| - 1)(c+1)p_{k,t-1}^F - (c+1) \sum_{l \in N_t^S \setminus \{k\}} p_{l,t-1}^F, \tag{31}$$

implying

$$\sum_{l \in N_t^S} q_{l,t}^S = |N_t^S| q_{k,t}^S + (|N_t^S| - 1)(c+1)p_{k,t-1}^F - (c+1) \sum_{l \in N_t^S \setminus \{k\}} p_{l,t-1}^F. \tag{32}$$

Then, the first-order conditions

$$(c+1)p_{i,t-1}^H - b + a - q_{n+1,t}^{ED} - \sum_{j \in N_t^B \setminus \{i\}} q_{j,t}^B + \sum_{l \in N_t^S} q_{l,t}^S - 2q_{i,t}^B = 0 \tag{33}$$

and

$$-(c+1)p_{k,t-1}^F + b - a + q_{n+1,t}^{ED} + \sum_{j \in N_t^B} q_{j,t}^B - \sum_{l \in N_t^S \setminus \{k\}} q_{l,t}^S - 2q_{k,t}^S = 0 \tag{34}$$

imply

$$\begin{aligned} 2q_{i,t}^B &= (c+1)p_{i,t-1}^H - b + a - q_{n+1,t}^{ED} \\ &\quad - \left((|N_t^B| - 1)q_{i,t}^B - (|N_t^B| - 1)(c+1)p_{i,t-1}^H + (c+1) \sum_{j \in N_t^B \setminus \{i\}} p_{j,t-1}^H \right) \\ &\quad + |N_t^S| q_{k,t}^S + (|N_t^S| - 1)(c+1)p_{k,t-1}^F - (c+1) \sum_{l \in N_t^S \setminus \{k\}} p_{l,t-1}^F \end{aligned} \tag{35}$$

and

$$\begin{aligned} 2q_{k,t}^S &= -(c+1)p_{k,t-1}^F + b - a + q_{n+1,t}^{ED} + \\ &\quad + \left(|N_t^B| q_{i,t}^B - (|N_t^B| - 1)(c+1)p_{i,t-1}^H + (c+1) \sum_{j \in N_t^B \setminus \{i\}} p_{j,t-1}^H \right) \\ &\quad - \left((|N_t^S| - 1)q_{k,t}^S + (|N_t^S| - 1)(c+1)p_{k,t-1}^F - (c+1) \sum_{l \in N_t^S \setminus \{k\}} p_{l,t-1}^F \right) \end{aligned} \tag{36}$$

further implying

$$\begin{aligned}
q_{i,t}^{B} &= \frac{1}{|N_t^B|+1}\left((c+1)p_{i,t-1}^{H} - b + a - q_{n+1,t}^{ED}\right) \\
&\quad - \frac{c+1}{|N_t^B|+1}\left(-(|N_t^B|-1)p_{i,t-1}^{H} - (|N_t^S|-1)p_{k,t-1}^{F}\right) \\
&\quad - \frac{c+1}{|N_t^B|+1}\left(\sum_{j\in N_t^B\setminus\{i\}} p_{j,t-1}^{H} + \sum_{l\in N_t^S\setminus\{k\}} p_{l,t-1}^{F}\right) + \frac{|N_t^S| q_{k,t}^{S}}{|N_t^B|+1} \qquad (37)
\end{aligned}$$

and

$$\begin{aligned}
q_{k,t}^{S} &= \frac{1}{|N_t^S|+1}\left(-(c+1)p_{k,t-1}^{F} + b - a + q_{n+1,t}^{ED}\right) \\
&\quad - \frac{c+1}{|N_t^S|+1}\left((|N_t^B|-1)p_{i,t-1}^{H} + (|N_t^S|-1)p_{k,t-1}^{F}\right) \\
&\quad - \frac{c+1}{|N_t^S|+1}\left(-\sum_{j\in N_t^B\setminus\{i\}} p_{j,t-1}^{H} - \sum_{l\in N_t^S\setminus\{k\}} p_{l,t-1}^{F}\right) + \frac{|N_t^B| q_{i,t}^{B}}{|N_t^S|+1}. \qquad (38)
\end{aligned}$$

Solving the above two equations together yields

$$\begin{aligned}
q_{i,t}^{*B} &= \frac{1}{|N_t^B|+1}\left((c+1)p_{i,t-1}^{H} - b + a - q_{n+1,t}^{ED}\right) \\
&\quad - \frac{c+1}{|N_t^B|+1}\left(-(|N_t^B|-1)p_{i,t-1}^{H} - (|N_t^S|-1)p_{k,t}^{F} + \sum_{j\in N_t^B\setminus\{i\}} p_{j,t-1}^{H} + \sum_{l\in N_t^S\setminus\{k\}} p_{l,t-1}^{F}\right) \\
&\quad + \frac{|N_t^S|}{|N_t^B|+1}\frac{1}{|N_t^S|+1}\left(-(c+1)p_{k,t-1}^{F} + b - a + q_{n+1,t}^{ED}\right) \\
&\quad - \frac{|N_t^S|}{|N_t^B|+1}\frac{c+1}{|N_t^S|+1}\left((|N_t^B|-1)p_{i,t-1}^{H} + (|N_t^S|-1)p_{k,t-1}^{F}\right) \\
&\quad - \frac{|N_t^S|}{|N_t^B|+1}\frac{c+1}{|N_t^S|+1}\left(-\sum_{j\in N_t^B\setminus\{i\}} p_{j,t-1}^{H} - \sum_{l\in N_t^S\setminus\{k\}} p_{l,t-1}^{F}\right) \\
&\quad + \frac{|N_t^S|}{|N_t^B|+1}\frac{|N_t^B| q_{i,t}^{B}}{|N_t^S|+1} \qquad (39)
\end{aligned}$$

implying

$$\begin{aligned}
q_{i,t}^{*B} &= \frac{1}{n+1}\left(-b + a - q_{n+1,t}^{ED}\right) \\
&\quad + \frac{c+1}{n+1}\left(n p_{i,t-1}^{H} - \sum_{j\in N_t^B\setminus\{i\}} p_{j,t-1}^{H} - \sum_{l\in N_t^S} p_{l,t-1}^{F}\right). \qquad (40)
\end{aligned}$$

Also, inserting (40) into (38) yields

$$
\begin{aligned}
q_{k,t}^{*S} &= \frac{1}{n+1}\left(b-a+q_{n+1,t}^{ED}\right) \\
&\quad -\frac{c+1}{n+1}\left(np_{k,t-1}^{F}-\sum_{j\in N_t^B} p_{j,t-1}^{H}-\sum_{l\in N_t^S\setminus\{k\}} p_{l,t-1}^{F}\right). \qquad (41)
\end{aligned}
$$

Thus, we can calculate

$$
\begin{aligned}
&\sum_{j\in N_t^B} q_{j,t}^{B}-\sum_{l\in N_t^S} q_{l,t}^{S} \\
&= \frac{n}{n+1}\left(-b+a-q_{n+1,t}^{ED}\right)+\frac{(c+1)}{n+1}\left(\sum_{j\in N_t^B} p_{j,t}^{H}+\sum_{l\in N_t^S} p_{l,t}^{F}\right). \qquad (42)
\end{aligned}
$$

Finally, inserting (42) into (10) yields

$$
\begin{aligned}
p^{*} &= \frac{b-a}{c+1}+\frac{q_{n+1,t}^{ED}+\sum_{j\in N_t^B} q_{j,t}^{*B}-\sum_{l\in N_t^S} q_{l,t}^{*S}}{c+1} \\
&= \frac{1}{n+1}\left(\frac{b-a+q_{n+1,t}^{ED}}{c+1}\right)+\frac{1}{n+1}\left(\sum_{j\in N_t^B} p_{j,t-1}^{H}+\sum_{l\in N_t^S} p_{l,t-1}^{F}\right) \qquad (43)
\end{aligned}
$$

which completes the proof. □

Proof of Proposition 2. Consider the spot foreign exchange market with the following specifications:

$n=3$	$a=1$	$b=12$	$c=0.2$
$p_0=3.8907$	$\bar{P}=3.8$	$\bar{Q}=-11$	$T=100$
$M_{1,0}^{H}=15$	$M_{1,0}^{F}=5$	$M_{2,0}^{H}=15$	$M_{2,0}^{F}=5$
$M_{3,0}^{H}=15$	$M_{3,0}^{F}=5$	$p_{1,0}^{H}=3.9500$	$p_{1,0}^{F}=3.8500$
$p_{2,0}^{H}=3.9500$	$p_{2,0}^{F}=3.8500$	$p_{3,0}^{H}=3.9400$	$p_{3,0}^{F}=3.8550$

One can check with the aid of a computer program that for $t=10$ no partition $\{N_t^{*B}, N_t^{*S}\}$ of strategic traders $\{1,2,3\}$ can be an equilibrium. □