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Estimating Consumers’ Willingness to Pay for the Individual Quality Attributes with DEA

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Abstract

In a highly competitive environment a product's commercial success depends increasingly more upon the ability to satisfy consumers' preferences that are highly diversified. Since a consumer product typically comprises a host of technological attributes, its market value incorporates all of the individual values of technological attributes. If the willingness-to-pay (WTP) for individual technological characteristics of a product is known, one can conjecture the overall WTP or the imputed market price for the product. The market price listed by the producer has to be equal to or lower than this WTP for the commercial survival of the product. In this paper we propose a methodology for estimating the value of individual product characteristics and thus the overall WTP of the product with DEA. Our methodology is based on a model derived from the consumer demand theory on the one hand, and the recent theoretical developments on the flexible DEA frontiers on the other hand. The paper also presents a real case study for the mobile phone market, which is characterized by its high speed of innovation. The suggested model and its empirical applications has implications for the extension of DEA methodology to the estimation of market value of a complex multi-attribute product and/or of a value of quality attribute that is not explicitly marketable in isolation. We also expect that the framework will shed some light on the successful way of product differentiation when the cost information for individual characteristics is available.

JEL Classification: D12, D46
Key words: DEA, efficient consumption, willingness to pay, multi-attribute product pricing
I. Introduction and Motivation

Estimating consumers’ willingness to pay for improvements in the quality of multi-attribute goods is of obvious importance to the producers of those goods since the latter need to know which quality attributes of their products are valued most by the customers and at how much.

However, estimating the WTP for individual attributes is not an easy matter since it is often the case that only prices for the set of attributes, i.e. the product, are observed. In principle, it is possible to use information on the prices of spare parts, e.g. batteries or LCD-s in case of the mobile phones market, but obtaining such information is costly and time-consuming. Besides, some attributes are not traded goods, such as the ability to download music or the amount of available colors on the display.

The contribution of this paper is to provide a methodology that uses only easily observable market data to derive WTP for individual attributes on a well-founded theoretical basis. Although the problem of estimating the WTP was
extensively addressed in the past, the bulk of the estimation attempts were based on conducting expensive and time-consuming questionnaire surveys in order to later derive inferences on the consumers' preferences in the space of attributes. In a typical setting, the respondents were presented with a set of economic choice questions (in the overwhelming majority of cases the contingent valuation method was used). The consensus seems to be that the key drawback of the method is the existence of a substantial gap between the hypothetical and real choices the respondents are likely to make, overestimation in the hypothetical setting being the pertinent problem (Blumenschein et al., 2001). However, given the lack or most often absence of the data on prices of individual attributes, the survey method has long been the best available methodology to estimate individual willingness to pay.

The main contribution of this paper is to offer a methodology of estimating consumers' willingness to pay for individual technological characteristics of a multi-attribute product using easily available market data. We do that by introducing the concept of consumption efficiency and estimating consumers' utility functions for individual attributes that are separable and additive in the
latter. We show that the importance consumers assign to the technological attributes is directly proportional to their shadow prices derived from the DEA estimation of price-quality frontiers. Since we estimate the explicit forms of consumers’ utility functions, we are able to derive willingness to pay at any point of price-quality space, which in turn allows one to infer the value of an improvement of existing products along one or several quality dimensions.

We apply our methodology to the case of Korean mobile phone market. We estimate the importance of eight technological attributes of the mobile phones and find that only two of them, size and weight, appear to be of significant importance for the consumers. We also find the improvements in size and weight are the two quality dimensions consumers will be most willing to pay for.

The application area of our methodology, however, does not limit itself to the valuation of mobile phone characteristics alone. It can be used for valuing any object that consumes a certain amount of investment and produces a several outputs. One important example is the valuation of Government-sponsored research projects, where the comparison between the project team’s stated
objectives can be compared to their corresponding revealed importance. Such a comparison would provide a sound basis for evaluating the success or failure of the project, possibly giving guidance to the redirection of the Government research funds.

The paper is organized as follows. The next section describes the theoretical framework. Section III presents our empirical framework and discusses the estimation results. Section IV derives policy implications and outlines the directions in which further research could proceed.
II. A theoretical framework for estimating consumers’ willingness to pay

Our key objective is to derive consumers’ marginal willingness to pay for a product’s quality attributes at every point of the observable price-attribute space. We do that by estimating consumers’ individual utility functions for each product variety.

We assume each consumer is characterized by her multi-attribute utility function defined on the set of attributes that comprise the product:

\[
U^k(z^k) = \sum_{i=1}^{N} w_i^k u_i^k(z_i^k)
\]

where \( z^k = (z_1^k, z_2^k, \ldots, z_N^k) \) is a vector of \( N \) attribute values for product \( k=1..K \), \( u_i^k(z_i^k) \) are product \( k \)'s utility functions (to be specified later) for the \( l-th \) attribute that add up to the product \( k \)'s utility function \( U^k(z^k) \) with weights \( w_i^k \).

\footnote{We assume additivity of utility function (1) and its separability in the individual attributes. Although more general functional forms can be adopted, we expect that the}
be shown (see Kirkwood, 1997) that a utility function is additive if and only if the single attribute utility functions $u^k_i(z_i, \rho^k_i)$ are of the following form:

$$u^k_i(z^k_i, \rho^k_i) = \frac{1 - e^{-\frac{z^k_i - u^k_i}{\rho^k_i}}}{1 - e^{-\frac{u^k_i}{\rho^k_i}}}$$  \hspace{1cm} (2)

Assuming attributes $z^k_i$ are valued in such a way that the least values correspond to the least preferred quality attribute levels, for positive $\rho - s$ (2) is an increasing function of attribute $z^k_i$ with decreasing marginal utility, which is a textbook utility function. This function monotonically increases over the range of attribute quality measures. It is equal to zero in the least preferred value of attribute $I$ and is equal to one in the most preferred attribute value. Parameter $\rho$ is most often interpreted as a measure of risk tolerance. More generally, however, lower absolute values of $\rho$ imply higher extent of the utility function’s curvature, while higher absolute values of $\rho$ make (2) more of a straight line.

The main argument of our theoretical discussion may not change but it only makes the algebraic manipulation more demanding. Besides satisfying the most essential conditions a well-behaved utility function must meet, specifications (1) and (2) greatly simplify the ensuing theoretical discussion. For a more detailed discussion of the functional form in (1) and (2), see Kirkwood (1997).
Willingness to pay for individual attributes using DEA

We assume there are $k$ consumers in the market, each one consuming a single product, so that the number of products in the market is also equal to $k^2$.

Consumer $k$ maximizes the following money-metric utility function (see Weymark, 1985, and Alcantud and Manrique, 2001):

$$Max V^k (\tilde{z}^k, p^k) = Max \left[ Y^k - p^k + \sum_{i=1}^{N} w_i u_i^k (z_i) \right]$$

(3)

where $Y^k$ is consumer $k$’s income, which is fixed for every consumer $k$ and $p^k$ is product $k$’s price.

The first-order conditions for (3) yield the following:

$$\frac{\partial p^k}{\partial z_i^k} = w_i^k \frac{\partial u_i^k (z_i)}{\partial z_i} \bigg|_{z_i^k}$$

(4)

We show that relationship (4) holds not only in the actually observed point of consumers’ choice $(z_i^k, p^k)$, but also in any other point of price-quality attribute

\footnote{For the case of the mobile phone market in Korea we are analyzing in the empirical part of this paper, this assumption seems rather plausible, Korea’s penetration rate of the mobile phones being one of the world’s highest. Intense competition in that market results in small and roughly equal market shares for each type of mobile phone.}
space, defining a family of consumer $k$'s indifference curves in terms of utility function $V^k(z^k, p^k)$. Indeed, consider an arbitrary increase in the attribute vector $\Delta z^k$. We define willingness to pay for that improvement in quality as a change in price $\Delta p^k$ associated with the change in quality $\Delta z^k$, that would leave consumer $k$ exactly on the same indifference curve as she were before the change in quality took place (see Smith, 1997, and Wertenbroch and Skiera, 2001, for a discussion on and the definition of the concept). One can formalize this definition as follows:

$$V^k(z^k + \Delta z^k, p^k + \Delta p^k) = V^k(z^k, p^k)$$  \hspace{1cm} (5)$$

For any set of quality attributes $z^k$ and associated price $p^k$ relationship (5) defines an indifference surface in the price-quality space.

Setting $dV^k=0$, we derive the following expression for the gradient of the indifference surface:

$$\frac{dp^k}{d\bar{z}^k} = \frac{\partial V^k(\bar{z}^k, Y^k) / \partial \bar{z}^k}{\partial V^k(\bar{z}^k, Y^k) / \partial p^k}$$  \hspace{1cm} (6)$$
Since the derivative in the denominator of (6) is equal to unity, (6) is equivalent to saying that for each quality attribute $i$, the slope of the indifference curve in the subspace of a single attribute and price:

$$\frac{dp^k}{dz_i} = w^k_i \frac{\partial u^k_i(z_i)}{\partial z_i}$$

which is equivalent to (4) at the actually observed consumer choice $z^k_i$, but is true for any level of quality attribute $z_i$.

(7) says that consumer $k$'s marginal willingness to pay for an additional unit of attribute $z_i$ is proportional to her marginal utility for that attribute with coefficient $w$. Weights $w$ can be thought of as measures of attributes’ importance for the consumers and can be defined in the context of hedonic price function theory as the product of the observed attribute level and consumers’ marginal willingness to pay at that point. To formalize this idea, the relationship between price $p$ and the set of attributes $z$ can be rewritten as $p=h(z)$. Function $p=h(z)$ is commonly referred to as a hedonic function. In his
seminal contribution Rosen (1974) developed a model for pricing a multi-attribute product, defining hedonic price functions as loci of equilibrium outcomes resulting from interactions of individual attributes’ producers and consumers. They can be thought of as a sort of “regression line” in the space of consumer choices where the observed deviations of actual choices from the line are considered to be random and not related to consumers’ behavior. We believe, however, that consumers tend to differ in the efficiency with which they make their choices, or that they differ in their consumption efficiency.

At the intuitive level the inefficient consumption means some consumers pay higher prices relative to the others for combinations of attributes that set these consumers at the same product utility level $V^k(\tilde{z}^k, p^k)$ . Alternatively, consumers can be viewed as “production units” where prices they pay for the multi-attribute products are the sole input, while the set of attributes they derive from the purchase are their outputs. For the same price, a relatively inefficient consumer will enjoy a relatively smaller utility level from the type of product she buys relatively to the consumer who is more efficient. We call efficiency thus defined “consumption efficiency”.
Assuming the presence of consumption inefficiencies, we can rewrite the conventional hedonic price function as \( p = h(\theta z) \), where \( \theta > 1 \) is the uniform factor by which the actually accrued set of attributes \( z \) should be increased in order to render the observed consumption pattern \((p,z)\) efficient. Obviously, \( \theta = 1 \) corresponds to an efficient consumption case.

It can be shown that derivatives of hedonic price functions with respect to the attributes are equal to consumers’ marginal willingness to pay for the attributes in equilibrium (see Markandya, 1992, on the proof of this and the discussion of hedonic price functions in general). It then follows from (7) that consumer \( k \)'s marginal willingness to pay for attribute \( z_i \) at the point of her choice is equal to:

\[
\frac{\partial h(\theta z)}{\partial z_i} = w_i^k \frac{\partial u^k(z_i^*)}{\partial z_i} \tag{8}
\]
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\( \frac{\partial h(\theta_k)}{\partial (z_i)} \) can be thought of as attribute \( z_i \)'s price in equilibrium since the right hand side of (8) is the marginal rate of substitution of money for the quality attribute \( z_i \), which is by definition the latter’s price. In this way, \( z_i \frac{\partial h(\theta_k)}{\partial (z_i)} \) represents consumer \( k \)'s valuation of the level of attribute \( z_i^k \) she chose. We thus suggest using the observed valuations of quality attribute levels for each consumer as the weights in her multi-attribute utility function (1):

\[ w_i^k = z_i^k \left. \frac{\partial h(\theta_k)}{\partial (z_i)} \right|_{z_i^k} \]

(9)

Since for any quality level \( z_i \) consumer \( k \)'s valuation of the latter is equal to \( z_i \frac{dp^k}{dz_i} \), consumers’ willingness to pay for a change of an attribute’s level from level \( z_a \) to level \( z_b \) is given by integrating the valuation of an attribute along the indifference curve corresponding to level \( z_a \). In other words, denoting \( R^k(z_a, z_b) \) consumer \( k \)'s willingness to pay for a ‘swing’ from \( z_a \) to \( z_b \) it follows from (7) that\(^3\):

\(^3\) We assume that weights \( w_i^k \) in consumer’s utility function \( U^k(z^k) \) in (1) remain fixed as the quality of one or several of the product’s attributes changes.
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\[ R^k(z_a, z_b) = \int_{z_a}^{z_b} z_i \frac{dp^k}{dz_i} dz_i = w_i^k \int_{z_a}^{z_b} z_i \frac{\partial u^k_i(z_i)}{\partial z_i} dz_i \quad (10) \]

In the general framework of cost-benefit analysis \( R^k(z_a, z_b) \) can be used for comparison of the consequences of several alternative actions, making it a useful tool in decision making.

As follows from (9) and (10), estimating WTP requires estimating consumption inefficiency factors \( \theta \), the derivatives of the generalized hedonic price functions \( h(\theta z) \) and individual attribute utility functions \( u^k_i(z_i) \).

We employ the DEA framework in order to estimate consumption inefficiency levels. The latter can be thought of as the distances between the actually observed consumption choices \( (\tilde{z}^k, p^k) \) and the ‘best practice’ ones, given by \( h(\tilde{z}) \), which in our context we call a consumption efficiency frontier. There are two reasons why we opted for the DEA framework against the parametric estimation techniques for that purpose. First, the DEA methodology allows for multiple outputs. Second, it does not impose any specific restrictions on the functional form of the consumption efficiency frontier.
Among the variety of DEA models we opted for the output-oriented one pioneered by Banker, Charnes and Cooper (1984). The key advantage of this model with respect to the alternatives is that it produces convex feasible price-attribute sets, the property, which is a necessary condition for the existence and uniqueness of equilibrium in the Rosen-type multi-attribute product market (see Moulton, 1998, and Liegey, 2000, for the relevant discussion).

Denote \( p^k \) the price of product \( k \) and \( \theta \) the factor by which the set of attributes \( z^k \) has to be multiplied in order to make its consumption at price \( p^k \) efficient. In order to describe the generalized hedonic price-quality frontier and to measure consumption efficiency, we formulate and solve the following linear programming problem for each product \( k \):
Willingness to pay for individual attributes using DEA

\[
\begin{align*}
&\text{Max } \theta \\
&\quad p^k \geq \sum_{j=1}^{N} p^j \lambda^j \\
&\quad \theta c_i^k \leq \sum_{j=1}^{N} z_i^j \lambda^j, i = 1, \ldots, M \\
&\quad \sum_{j=1}^{N} \lambda^j = 1 \\
&\quad \lambda^j \geq 0, j = 1, \ldots, N
\end{align*}
\]

where \( N \) is the number of products and \( M \) is the number of a product’s attributes.

The above problem has the following dual form:

\[
\begin{align*}
&\text{Min} \left[ \mu p^k - \mu_0 \right] \\
&\quad \sum_{i=1}^{M} \nu_i z_i = 1 \\
&\quad \sum_{i=1}^{M} \nu_i z_i^j \leq \mu p^j - \mu_0, j = 1, \ldots, N \\
&\quad \mu \geq 0 \\
&\quad \nu_i \geq 0, i = 1, \ldots, M
\end{align*}
\]

The solutions for problems (11) and (12) consist of consumption efficiency level \( \theta^* \), intensity coefficients \( (\lambda^j)^* \) and dual variables \( (\nu_i)^*, \mu^* \) and \( \mu_0^* \). \( (\lambda^j)^* \) are the normalized dual prices of attribute \( j \).
Multiplying the first constraint in the dual problem (12) by the product’s price \( p^k \) produces decomposition of the latter into the sum of the prices of individual attributes: 
\[
p^k = \sum_{i=1}^{M} \left( p^k (v_i)^* \right) z_i^k.
\]
In this setting, \( p^k (v_i)^* \) is attribute \( i \)'s shadow price for product \( k \). Note that these shadow prices are calculated on the estimated price-quality frontier, so that for the estimated hedonic price frontier function, \( p = h(z) \), the shadow price of the \( i \)-th attribute is equal to \( \frac{\partial h(\theta z)}{\partial (\theta z_i)} \).

We thus obtain the following expression for the weight:
\[
w_i^k = z_i^k \left. \frac{\partial h(\theta z)}{\partial z_i} \right|_{z_i = z_i^k} = z_i^k \theta^k \left. \frac{\partial h(\theta z)}{\partial (\theta z_i)} \right|_{z_i = z_i^k} = z_i^k \theta^k (p^k v_i^*). \]
III. Empirical framework and estimation results

The Data

We collected prices and quality attributes of mobile phones for September 2001 in the Korean domestic market. Prices and basic quality attributes were obtained at the website of two Internet sites for price comparisons, the Best Buyer (http://www.bestbuyer.co.kr) and Enuri.Com (http://www.enuri.com/).

Although information on a substantial variety of mobile phone attributes was available, we selected eight variables, the information on which was available for most of the mobile phone models. Three of these variables are continuous and consist of the calling time (minutes), volume of the phone box (cubic mm) and weight (kg). Four additional attributes are represented by the following dummy variables: the external LCD, the third generation dummy, sound harmony and animation/music download capability. Finally, the color variable is essentially discrete, only assuming three values and measured in the number of
bits to represent colors. We re-scaled the size and the weight variables so as to make sure the lower limit for both represents the least desirable value of attribute for the consumer. For that reason we took the reciprocal of the volume to represent the size, and we used the reciprocal of weight instead of the weight proper. Each mobile phone model is thus characterized by eight attributes in our dataset, measured in such a way that higher attribute levels correspond to the higher quality of the attribute. The number of observations is 118. Table 1 presents summary statistics for our dataset:
Table 1: Summary statistics for the mobile phones’ quality attributes

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Calling Time (min)</th>
<th>Size (mm³)</th>
<th>Weight (kg)</th>
<th>External LCD (dummy)</th>
<th>Average Number of Bits Representing Color</th>
<th>Share of the 3rd Generation Phones (dummy)</th>
<th>Share of Phones with 16 Harmony Sounds (dummy)</th>
<th>Share of Phones with Downloadable Animation (dummy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>215452</td>
<td>137</td>
<td>65606</td>
<td>75.37</td>
<td>0.54</td>
<td>2</td>
<td>31.36%</td>
<td>9.32%</td>
<td>65.25%</td>
</tr>
<tr>
<td>SD</td>
<td>93877</td>
<td>36</td>
<td>449444</td>
<td>658.90</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Min</td>
<td>40000</td>
<td>75</td>
<td>47476</td>
<td>60</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Max</td>
<td>459900</td>
<td>220</td>
<td>123970</td>
<td>132</td>
<td>1</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
The distributions for size, weight and calling time are skewed towards the best observable value of attribute in the market, possibly reflecting the ongoing process of improvement of mobile phone sets along these directions. About a half of all mobile phones have external display and allow for animation downloads. About a third of the phones belong to the third generation, reflecting high extent of mobile phones penetration in the country, while only one out of ten phones allows for a 16-sound music. Finally, while potentially the modern mobile phones’ LCD is capable of representing 256 colors (corresponding to 8 color bits), the average phone only has four of them (two bits).
Estimation results

Below we provide our estimates of weights $w^t_i$ in (1) on all eight attributes:

Table 2: Weights on the Individual Attribute Utility Functions, Korean Won

<table>
<thead>
<tr>
<th></th>
<th>Calling Time</th>
<th>Volume</th>
<th>Weight</th>
<th>Exterior LCD display</th>
<th>The Number of Possible Colors</th>
<th>Third Generation</th>
<th>Sixteen Harmony Sounds</th>
<th>Animation download possibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>3.15</td>
<td>2053</td>
<td>23167</td>
<td>663</td>
<td>8638</td>
<td>3206</td>
<td>1065</td>
<td>5301</td>
</tr>
<tr>
<td>SD</td>
<td>10.54</td>
<td>4535</td>
<td>42345</td>
<td>3156</td>
<td>19460</td>
<td>7341</td>
<td>7191</td>
<td>19365</td>
</tr>
<tr>
<td>Min</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Max</td>
<td>56.70</td>
<td>22524</td>
<td>177818</td>
<td>20856</td>
<td>78571</td>
<td>39250</td>
<td>55250</td>
<td>100869</td>
</tr>
<tr>
<td>% of zero weights</td>
<td>88.98%</td>
<td>69.49%</td>
<td>67.80%</td>
<td>94.07%</td>
<td>73.73%</td>
<td>71.19%</td>
<td>95.76%</td>
<td>91.53%</td>
</tr>
</tbody>
</table>

Table 2 allows one to assess relative importance of the mobile phones’ attributes for the consumers. For each attribute there is a group of consumers (whose share relative to the total number of consumers is reported in the last line of Table 2) who do not assign any importance to that attribute. Thus, almost 75% of consumers are indifferent with respect to how many different colors their
display has and less than 5% are taking heed of whether the music produced by their terminals consists of sixteen harmony sounds. On the other hand, more than one-third of the consumers care about the volume and the weight of their mobile phones.

The value of weights for the five attributes whose quality is measured by a discrete variable represents consumers’ valuation of a unit increase in quality of the respective attribute. Thus, it follows from Table 2 that an average consumer would be willing to pay about 9000 Won for each additional color in the display, 5000 Won for the possibility to download animation, 3200 Won for the third generation upgrade, 1000 Won for the addition of sixteen harmony sounds and less than half of that for the exterior display.

Among the four attributes that are measured by a continuous variable, ‘Volume’ and ‘Weight’ appear to be more important to the consumers than the ‘Number of Possible Colors’ and the ‘Third Generation Technology’ in terms of the share of consumers who assign non-zero importance to these attributes. However, ‘Weight’ is by far the only attribute that is most important both in terms
of the share of non-zero weights and in terms of the average weight.

Now that we know the weights consumers ascribe to the individual quality attributes, we need a way to calculate partial derivatives of the individual attributes’ utility functions in order to arrive at the calculable version of the expression (10) for marginal willingness to pay. That, in turn, requires estimating parameter $\rho$ in the individual attribute utility function (2).

Empirical literature on multi-attribute utility functions applications abounds with the estimation accounts of the exponential one-parameter utility functions (see Kirkwood, 1997). The overwhelming majority of the empirical work aimed at estimating the type of utility functions as in (2) is based on conducting detailed questionnaires with a few respondents, whose answers are later used for inferring information on the respondents’ preferences, ultimately resulting in a specific $\rho$ for each attribute’s utility function.

This paper contributes to the multi-attribute utility literature in that it suggests a methodology of estimating the $\rho$-s that does not require conducting surveys
that are prone to being subjective and are also costly. Our methodology solely relies on the market data such as the products’ prices and basic characteristics. This kind of data is readily available, is cheap to collect and employs information on the choices of much greater a number of decision makers than the more conventional survey methodology does.

We find $\rho$ by substituting expression (9) for the individual attribute utility function weights into (8) and solving the resulting equation:

$$F(\rho) = z_i \left. \frac{\partial u_i(z_i)}{\partial z_i} \right|_{z_i} - 1 = 0$$  \hspace{1cm} (13)

In order to solve (13) for $\rho$, we apply the following iteration procedure widely known as the Newton iteration method. The method starts at an arbitrary value $\rho_0$ and computes a sequence of iterations for the parameter according to the following:

$$\rho_j = \rho_{j-1} - \frac{F(\rho_{j-1})}{[\partial F(\rho_{j-1})/\partial \rho_{j-1}]}$$  \hspace{1cm} (14)
Although in the majority of cases the solution for (14) exists, it is often not unique. When the solution for $\rho$ does not exist, we interpret that as evidence of the functional form misspecification for that particular case and stick to the linear utility normalized to vary between zero and one over the attribute’s observed range as the simplest possible functional form.

All estimated $\rho$-s in our sample are positive, implying decreasing marginal utilities for individual attributes for all products in our dataset, which accords well with the basic microeconomic utility theory. In case there are two solutions, we take the lowest value of $\rho$ as the true solution, allowing for the maximum extent to which marginal utility of the consumers diminishes with the attributes’ quality$^4$. We only estimated parameter $\rho$ for three out of eight attributes since these three attributes are the only continuous ones in our dataset.

$^4$ As follows from (10), the more constant marginal utility implied by the overwhelming majority of our estimates in case of the higher $\rho - s$ revises down the estimates of consumers’ willingness to pay for a given quality change. Thus, our estimates of (10) represent the upper boundary thereof.
Table 3 below presents our estimates of the ‘curvature’ parameter $\rho$.

**Table 3: Summary Statistics for the Individual Utility Functions**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calling Time</th>
<th>Ratio to 'Calling Time' Range</th>
<th>$10^6$/Size</th>
<th>Ratio to 'Size' Range</th>
<th>1/Weight</th>
<th>Ratio to 'Weight' Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>111.97</td>
<td>0.77</td>
<td>28.69</td>
<td>2.21</td>
<td>6.69</td>
<td>0.74</td>
</tr>
<tr>
<td>SD</td>
<td>102.63</td>
<td>0.71</td>
<td>191.18</td>
<td>14.71</td>
<td>11.85</td>
<td>1.30</td>
</tr>
<tr>
<td>Min</td>
<td>2.02</td>
<td>0.01</td>
<td>1.66</td>
<td>0.13</td>
<td>1.36</td>
<td>0.15</td>
</tr>
<tr>
<td>Max</td>
<td>349.92</td>
<td>2.41</td>
<td>2017.13</td>
<td>155.20</td>
<td>123.76</td>
<td>13.61</td>
</tr>
<tr>
<td>% of suspected misspecifications</td>
<td>22.88%</td>
<td>5.93%</td>
<td>8.47%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In case of all three continuous attributes we estimate parameter $\rho$ to vary significantly across different types of mobile phone. However, in order to make sound judgment about the extent of the utility function’s curvature it represents, it is necessary to compare the value of the parameter to the range of the attribute it is calculated for. The greater the ratio of the former to the latter, the less variation there is in consumers’ marginal utilities over the attribute’s range. ‘Weight’ and ‘Calling time’ feature similar values of this ratio, which are much
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lower than the one for the ‘Size’, implying more curvature of the utility functions for the former two attributes.

Below we plot the individual utility functions for the three attributes with their average $\rho$ parameters.

**Figure 1: Individual utility functions for ‘Calling Time’, ‘Size’ and ‘Weight’**

The upper two lines on Figure 1 are ‘Weight’ and ‘Calling Time’. As there is
some curvature to the utility function for ‘Size’ (the lowest line), the latter is essentially a straight line, implying the uniformity of consumers’ valuation of marginal quality increases over the attribute’s range. In contrast, consumers value reductions in weight and size of their mobile phones more when their phones are heavier and bigger than they do when the latter are lighter and smaller.

We are now ready to derive consumers’ willingness to pay for an increase in quality for any one of the three continuous attributes from the current and unto the best observable level, using relationship (10). Computing (10) for every attribute considered for quality enhancement and comparing it with the associated costs makes a sound basis for assessing the profits or losses associated with such an enhancement. Table 4 presents our estimates of additional revenue (10) for the ‘Calling Time’, ‘Size’ and ‘Weight’, as these appear to be the most important attributes for the consumers (see our discussion of the attributes’ weights presented by Table 2).
Table 4: Consumers’ Willingness to Pay for a Change to Best Values of Calling Time, Size and Weight, Korean Won.

<table>
<thead>
<tr>
<th></th>
<th>WTP for Calling Time</th>
<th>Ratio of WTP to Observed Price</th>
<th>WTP for Size</th>
<th>Ratio of WTP to Observed Price</th>
<th>WTP for Weight</th>
<th>Ratio of WTP to Observed Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>184</td>
<td>0.09%</td>
<td>9104</td>
<td>4.23%</td>
<td>50960</td>
<td>23.65%</td>
</tr>
<tr>
<td>SD</td>
<td>926</td>
<td>21876</td>
<td>121868</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>0.00</td>
<td>0.00%</td>
<td>0.00</td>
<td>0.00%</td>
<td>0.00</td>
<td>0.00%</td>
</tr>
<tr>
<td>Max</td>
<td>7490</td>
<td>1.63%</td>
<td>118471</td>
<td>25.76%</td>
<td>839742</td>
<td>182.59%</td>
</tr>
</tbody>
</table>

Note: Since utility is decreasing in size and weight, the reported values for those attributes correspond to the decrease of the observed values to the minimum observed values.

According to our estimates, weight reduction appears to be the most powerful revenue generator in the domain of Korean mobile phone development. Indeed, additional revenue from decreasing the weight of the existing mobile phone models to the lowest weight observable in the market could potentially increase the existing revenues by about 20%. The increase in revenues associated with feasible size improvements would not exceed 5%, while increasing the calling time would not result in any essential rewards.
IV. Conclusions and Policy Implications

In this paper we combined the revealed preference approach to the hedonic price theory and the multi-attribute utility functions theory in order to infer individual attributes’ values of a complex product. To create the link between the two theories, we employed the DEA framework, thus broadening the area of the application of the latter.

We developed a methodology of inferring the unobservable prices of individual technological attributes when the only observables are the multi-attribute product’s price and its technological characteristics. The literature on hedonic prices gave a partial solution to this problem, but it only allowed for price inference at the point of observed consumers’ choice. We extend this framework to estimating the individual attributes’ prices by developing a methodology for estimating the money-metric multi-attribute utility function, which allows us to construct indifference maps in the two-dimensional income-attribute spaces for each attribute and product and hence derive marginal
willingness to pay for an attribute at any quality level of the latter.

We applied our methodology to the case of Korean mobile phone market and find that out of eight observable attributes, only two appear to be of concern to the consumers, the mobile phone’s weight and size. The former seems to be by far the most acute issue of concern, however, both in terms of the weight consumers place on this attribute in their utility functions and in terms of the potential revenue that could be accrued by the firms that would attempt to bring quality improvement to the existing models by decreasing their weight.

Our theoretical framework allows for a novel interpretation of the inefficiency measure yielded by the DEA methodology. Namely, we interpret the inefficiency factor as the rate by which consumers overestimate the value of technological attributes. We then prove that the more inefficient the consumers are, the lower quality of the mobile phones they choose. Consequently, we derive an expression for the upper limit of the additional revenue Korean mobile phone producers might accrue should they increase the quality level of any one of the phones’ technological attributes to the level of best practice. It follows directly
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from that expression (15) that the more efficient the consumers are, the higher the revenue the producers will be likely to accrue from improving the technological quality of mobile phones. This bears direct policy implications both for the producers and for the Government, since taking measures for improving consumption efficiency will result not only in the increased sales revenue, but also in the consumer welfare who, by definition of consumption efficiency, will acquire more quality for a lower price, resulting in the Pareto-improvement of wealth in the economy. The measures aimed at increasing consumption efficiency may include informing consumers about providing consumers with a cheap and easy access to information about the technological attributes, which may include advertising, maintaining the web-sites that allow for a quick and easy comparison between various models and the like.

The methodology presented in this paper has much wider a spectrum of applications than the analysis of mobile phone market since it allows to price any set of outputs associated with a single input that can be measured in monetary units, such as price. We thus suggest the R&D evaluation by the Government would be one area of application. The inefficiency coefficients for
each project would help estimate the allocative efficiency of the Government’s research and development funds, with the attributes for each project representing the key Government objectives in the society’s welfare function. Estimating the R&D project-specific weights for each Government-specified objective would thus allow one to estimate the extent to which project participants’ objectives are aligned with those of the Government.

This paper also suggests a several directions of future research. On the theoretical side, it is important to introduce the link between consumption inefficiency $\theta$ and the single attribute utility function parameter $\rho$ into our framework. One can think about parameter $\rho$ representing the extent to which the respective attribute is a necessary good in the microeconomic sense. That is, low (absolute) values of $\rho$ imply consumers become fairly indifferent towards quality improvements of an attribute once a certain threshold of it has been reached. That in turn might imply that consumers become less concerned about the price of this specific attribute and hence consume less efficiently on the range of attribute beyond the ‘threshold quality’, while being efficient on the quality values inferior to the latter. On the empirical side, the dataset used in this paper can be turned into a panel by adding the observations on quality
attributes and mobile phone prices for a several months. The panel data analysis is interesting since it allows one to trace changes in consumers’ preferences over the attributes and link them to the producers’ policies such as advertising, promotions and the like. Finally, as we mentioned before, our methodology can be applied to the evaluation of performance of research and development projects according to multiple performance criteria. Performance evaluation of that kind bears important implications for the Government policy qua research funds (re-)allocation and efficient usage, but to our knowledge has solely relied on the qualitative analysis that heavily relies on the subjective, costly and time-consuming questionnaires. The methodology developed in this paper allows one to avoid this type of problems and would thus contribute to improving efficiency of the Government policy.
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References


Ryan, M., and Fernando, S. M., “Testing for Consistency in Willingness to Pay
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