Mathematical Thinking Undefended on The Level of The Semester for Professional Mathematics Teacher Candidates

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Abstract
Mathematical thinking skills are very important in mathematics, both to learn math or as learning goals. Thinking skills can be seen from the description given answers in solving mathematical problems faced. Mathematical thinking skills can be seen from the types, levels, and process. Proportionally questions given to students at universities in Indonesia (semester I, III, V, and VII). These questions are a matter of description that belong to the higher-level thinking. Students choose 5 of 8 given problem. Qualitatively, the answers were analyzed by descriptive to see the tendency to think mathematically used in completing the test. The results show that students tend to choose the issues relating to the calculation. They are more use cases, examples and not an example, to evaluate the conjecture and prove to belong to the numeric argumentation. Used mathematical thinking students are very personal (intelligence, interest, and experience), and the situation (problems encountered). Thus, the level of half of the students are not guaranteed and shows the level of mathematical thinking.
Keyword: Mathematical Thingking, Personal, Situation

JEL Classification: I20 I21 I24 I29

Introduction

Thinking is very important for everyone in their daily life. Thinking is like basic learners to attend school (Marzano, 1988). Thinking is a skill that is needed in the 21st century (Darling-Hammond, 2006). The same thing was recommended by Collins (2014), teachers need to plan assessment items that allow students to use all the skills of the Taxonomy: analysis, evaluation, and creation (the "top end" of Bloom's Taxonomy); logical reasoning; judgment and critical thinking; problem-solving; and creativity and creative thinking.

Mathematics as a study containing evidence, generalization, extension, application or applied, and characterization will help the development of the nation's character. Further Disclosed by Rahmatya (2013) that the study of mathematics will develop characters like; discipline, responsibility, Independently, Recognize excellence, honest, curious, creative, caring, and hard work. Meanwhile, Suryadi (2010) highlights the role of mathematics in the national education goals to develop the potential of learners through the process to think.

The role of mathematics and mathematics education in developing the character because of mathematics and mathematics education has values in accordance with aspects of the character. The values contained in mathematics and mathematics education, among others, rationalism, openness, objective, rationalism, control, progress, mystery, accuracy, clarity, prediction, consistency, creativity, the organization of effective, efficient work, comfort, flexibility, open mind, persistence and systematic work (Dede, 2006).

How is the relationship with mathematical thinking? Mathematics as part of the subjects taught at school certainly has nothing to do with thinking. Marzano (1988) states the relationship between thinking with content knowledge. Study Aizikovitsh-Udi, E., & Cheng, D. (2015) states that learning mathematics can improve critical thinking skills. Problem-based learning mathematics can improve critical thinking skills and creative mathematical (Happy, N., Listyani, E., & Si, M., 2011). Assignments of non-routine can improve thinking ability in the lecture Calculus (Breen and O'Shea, 2011).

Relations contains knowledge and thought, known as the term to think mathematically. Thinking mathematically defined as a process, which includes; problem-solving, looking for patterns, making conjectures, testing the borders, the data inference, abstraction, explanation, justification, and others (Stein, M. K., Grover, B. W., & Henningsen, M., 1996). Mathematical thinking is a process that consists of specializing, generalizing, conjecturing, and convincing (Mason, Burton and Stacey, 2010).

Mathematical thinking into one trend research contained in the Journal of Research Mathematics Education (JRME) in the period 2009-2014 (Siswono, 2014). Similar disclosed Hannulu that topics algebraic thinking, mathematical thinking and advanced mathematical thinking are increasing with the use of qualitative research and development (Siswono, 2014).

PISA and TIMSS demands require higher level thinking skills in mathematics which adversely affects the ranking students of Indonesia in the international arena. TIMSS study in 2011 put Indonesia on 5 ranked lowest. While the PISA study in 2012 put the mathematical abilities of students are at level 1, 2 and 3 are at the level of low-level thinking.

Therefore, the duty of a teacher of mathematics, as educators, to develop the ability to think from a low level at a high level. This, of course, requires a teacher's ability to be at the level of higher-level thinking. Higher-level thinking skills teachers should be established and developed during the lectures. Similarly, a student majoring in Mathematics education graduates are predicted to become a candidate for mathematics educators.

However, the achievement of students in subjects related to thinking skills such as abstract algebra 1, algebra 2 abstract, real analysis, differential calculus, integral calculus and vector analysis is still minimal which has a value of a B +. Students are still difficulties in problems related to verification, analysis, representation in the form of graphs, and nonroutine matters.

How to improve the ability of students to think mathematically would require data on the ability to think mathematically that exist today. Therefore, researchers are interested in assessing the ability to think mathematically students majoring in mathematics education. Thinking like a mathematical proof, generalization, representation has the opportunity to study, become one of the trends of mathematics education research (Siswono, 2014). Mathematical thinking, as a process, may be identified by the answers given by the students. Studies Dissertation Hu (2014) using qualitative research to identify algebraic thinking skills for student teachers of mathematics to analyze the responses.

Based on the description, the formulation of the problem in this research are as follows.

1. The ability to think mathematically what the dominant possessed by students majoring in Mathematics education?
2. What similarities and differences in the ability to think mathematically students seen from the level of a semester?

Method

Research Design

This study used a qualitative approach to understand the phenomenon of what is experienced by the subject of the study such behavior, perception, motivation, action, and so on. holistically, and by way of description in the form of words and language, in a specific context that is naturally and by utilizing a variety of natural methods (Lexy, 2010).

Where qualitative research used is the type of Grounded Theory. The few similar studies with similar research design including; This type of research is used by Ferri (2012) to identify the mathematical thinking skills, used by Hu (2014), in his dissertation on algebraic thinking skills.

The Subject Of Research

Participants in this study were students in the Department of Mathematics Education Faculty of Education and Teaching at the State Islamic Institute Nurjati Sheikh Cirebon - Indonesia Semester I-VII. Students of each half Elected by purposive snowball sampling

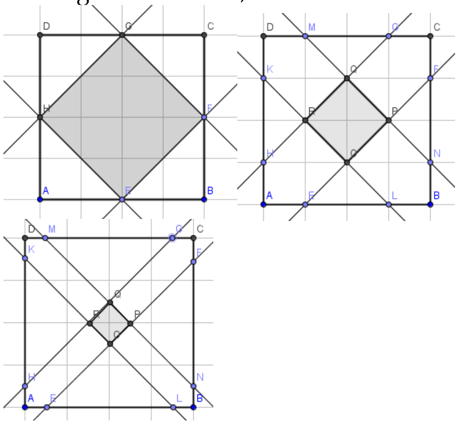
(Suri, 2011). This is done, because of the student Believed to Be Able to complete a given test. In addition, they can choose issues According to ability. Therefore, the respondents identified as having adequate skills and knowledge to get it done. As for the of participants in this study, as many as 34 students of mathematics education consisting of: (semesters 7 = 5 students, semester 5 = 20 students, half 3 = 6 students, and half of 1 = 3 students).

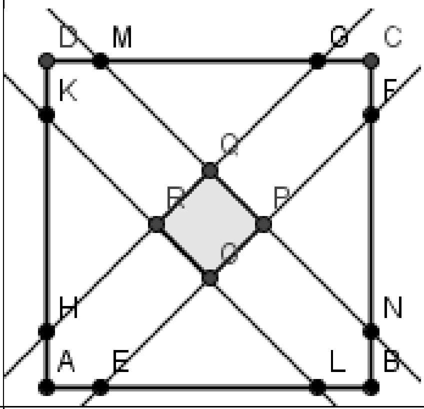
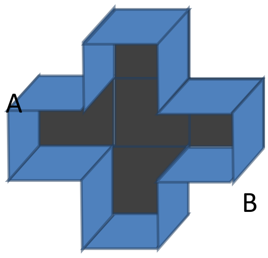
Data Collection

Instruments used in the form of eight questions focused on the level or type of thinking such as make and test conjectures, evaluate statements, prove, solve problems, represent a problem in graphs, generalize, analyze, logically creation statement, the types of visual or symbolic thought. Data collection techniques based on the principle of phenomenology, which seeks to understand in depth answers to the five questions that have to be solved by the students. The answers given will be interpreted based on the existing framework. A template used for the characteristics of the existing mathematical thinking. The data collected will be identified in order to build and develop the theory, especially concerning to think mathematically (Merriam, 1998).

The instruments used can be seen below

Table 1 Research Instruments

No Item	Question	Indicator
1a	Misalkan a,b, c adalah 3 bilangan asli berurutan. Apakah jumlah ketiga bilangan tersebut kelipatan 3? Jelaskan	Investigate and prove the allegations
1b	Apakah jumlah 5 bilangan asli berurutan merupakan kelipatan 5? Jelaskan	Investigate and prove the allegations
1c	Apakah jumlah k bilangan asli berurutan merupakan kelipatan dari k? Jelaskan	Make conjectures and prove
2a	Amati gambar berikut;  Tentukanlah perbandingan luas daerah yang diarsir dengan luas perseginya? Jelaskan	Identifying patterns in the problem area of comparative non-routine
2b	Jika DM=DK=AH=AE=BL=BN=CF=CG, tentukanlah perbandingan luas daerah yang diarsir OPQR dengan persegi ABCD?	Determine the general pattern comparison area of square section on non-routine matter

		
3	Diketahui sebuah fungsi $f(n) = n^2 - 8n + 18$, apakah *f*(*n*) > *f*(*n*-1), untuk n bilangan asli, selalu benar, kadang-kadang, atau tidak benar? Jelaskan!	Evaluate the correctness of a statement
4	Tentukanlah nilai a dan b agar bilangan 32a1b4 habis dibagi 4? Apakah ada kemungkinan untuk nilai a dan b yang lain? Jelaskan	Specifies an alternative solution
5	Seorang pelari tercepat dapat menempuh 100 m dengan waktu 9,27 detik. a) Sketsalah grafik posisi pelari terhadap waktu pelari tersebut? Mengapa demikian b) Sketsalah grafik kecepatan pelaripada saat t detik? Mengapa demikian c) Apakah ada kemungkinan grafik yang lain? Jelaskan	The creation of a representation of a problem in graph form logically
6	sebuah kotak tempat semut disusun seperti gambar dibawah. Apabila setiap rusuknya sama panjang, tentukanlah jarak minimum yang dapt ditempuh oleh semut dari A ke B? Mengapa demikian! 	Skills in resolving problems related to optimization
7	Diketahui sebuah fungsi, $f(x) = x^3 - x^2 - x$ a) Tentukanlah persamaan garis singgung pada kurva di titik x = -1 b) Tentukanlah persamaan garis singgung lain pada kurva yang sejajar dengan persamaan di a)	procedural capability in determining the equation of a tangent analyzing the characteristics of the functions and tangents
8	Diketahui sin (*A* + *B*) = sin*A* cos*B* + cos*A* sin*B* dan cos (*A* + *B*) = cos*A* cos*B* – sin*A* sin*B* a. Apabila *B* = *A*, bagaimana persamaan keduanya sekarang? b. Bagaimana dengan sin(*A* + *B*) / cos(*A* + *B*)?apa yang dapat diperoleh c. Susunlah persamaan baru yang bisa diperoleh dengan memanipulasi cara yang serupa dengan cara a) atau dengan cara b)? Tunjukkan	Evaluate statement Justify a statement The creation of a new statement and justify

Validity and Data Analysis

The validity of the data is determined through the instrument validation by experts. While the data analysis conducted during the study and after the data collection process. Data processing and analysis are done through a descriptive-analytic to get a good interpretation and comprehensive (Morse, 2002).

Results and Discussion

Problem number 3 seems to be a favorite for students because it is done by all existing participants. For each half of the participants also apparent, from half of 1 to 7, this matter is dominated by the participants. The material on this matter concerning the function of the domain of natural numbers. This problem is stated as follows;

> **Problem Number 3**
> " Diketahui sebuah fungsi $f(n) = n^2 - 8n + 18$, apakah $f(n) > f(n\text{-}1)$, untuk n bilangan asli, selalu benar, kadang-kadang, atau tidak benar? Jelaskan!"

Problem number 5 was not done by a student yet. The matter concerning the representation of the position of a sprinter in graphical form. This problem is stated as follows;

> **Problem Number 5**
> "Seorang pelari tercepat dapat menempuh 100 m dengan waktu 9,27 detik. "
> a) Sketsalah grafik posisi pelari terhadap waktu pelari tersebut? Mengapa demikian
> b) Sketsalah grafik kecepatan pelari pada saat t detik? Mengapa demikian
> c) Apakah ada kemungkinan grafik yang lain? Jelaskan"

The more detailed and clear to the research data, results and discussion to answer the problem described in this study.

1. ***Enjoy with numerical, visual avoid***

Problems related to numbers and calculations seem to be the interest of students. Selection matters relating dominated by numbers and calculations. Question 3 relating to the nature of the function of the natural numbers, the issue number 1 relating to the nature of numbers sequentially, Question 4 relating to the characteristics of divisibility, the issues that dominate accomplished by the participants. Further successively about number 8, number 2, number 6 and number 7.

Problem Number 1a

"Misalkan a,b, c adalah 3 bilangan asli berurutan. Apakah jumlah ketiga bilangan tersebut kelipatan 3? Jelaskan"

Answer :

Type of Description Answers	**Semester - The number of students who answered**	**The coding's kind of thinking**
Yes, of course. Because if we let a, b, c it's 1, 2 and 3, the three that add up 1 + 2 + 3 = 6, is a multiple of 3, it is evident that the result of the sum of three consecutive natural numbers are multiples of 3	Third semester = 2 student fifth semester = 5 student seventh semester = 6 student	Concluded from the examples given
True, as has been proved by summing the three original numbers sequentially result multiples of 3, for example 1,2, 3 = 6; 2,3,4 = 9; 3,4,5 = 12	half to one = 2 students third semester = 1 student fifth semester = 5 student seventh semester = 2 student	Conclude from a few examples
3 sequence of natural numbers = multiples of 3. Let k + k + 1 + k + 2 = 3k + 3. Because each segment 3k / 3 and 3/3 is divisible by 3, the number of three successive natural numbers are multiples of 3	third semester = 3 student fifth semester = 7 student	Investigate and prove the statement

Answers are included in the pattern 1 and 2, still has not led to the proof of the statement in question. Giving examples given a number of new leads coming into effect of a given statement or supporting a statement. This argument can be classified in the numeric argument (Tabach, et al, 2010). However, to prove it should be applied to all natural numbers (Mathematical Association & Boys' school's committee, 1965). Symbolization of three consecutive natural numbers with k, k + 1, k + 2 or a, a + 1, a + 2 is needed to prove the statement known as the symbolic argument (Tabach, et al, 2010). Deductive proof (Mathematical Association & Boys' school's committee, 1965). deductive reasoning (Karadag, 2009; Katagiri, 2004) *convincing* (Stacey, 2006).

Problem Number 1b

"Apakah jumlah 5 bilangan asli berurutan merupakan kelipatan 5? Jelaskan ?"

Answer :

Type of Description Answers	Semester - The number of students who answered	The coding's kind of thinking
5 natural number sequence (in addition) 10,11,12,13,14 = 60 → multiples of 5	third semester = 2 student fifth semester = 6 student seventh semester = 2 student	Conclude from the examples given
Right, because every natural number sequence, the fifth number of the number sequence, eg 1 + 2 + 3 + 4 + 5 = 15; 2 + 3 + 4 + 5 + 6 = 20; 3 + 4 + 5 + 6 + 7 = 25 So 15,20,25 are multiples of 5	half to one = 2 students third semester = 1 student fifth semester = 4 student seventh semester = 2 student	Conclude from a few examples
Eg a, b, c, d, e ∈ N. If a = n, n≥0, then a = n .... (1); b = n + 1 .... (2); c = n + 2 .... (3); d = n + 3 .... (4); e = n + 1 .... (5) if (1), (2), (3), (4) and (5) are summed then a + b + c + d + e = n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10 eg, n = 2, 5n + 10 = 20 is a multiple of 5. So, it is evident that the number of five numbers is a multiple of 5	third semester = 2 student fifth semester = 7 student	Investigate and prove the statement

Answers are included in the pattern 1 and 2, still has not led to the proof of the statement in question. Giving examples given a number of new leads coming into effect of a given statement or supporting the statement. This argument can be classified in the numeric argument (Tabach, et al, 2010). However, to prove it should be applied to all natural numbers (Mathematical Association & Boys' school's committee, 1965). Symbolization of three consecutive natural numbers with k, k + 1, k + 2 or a, a + 1, a + 2 is needed to prove the statement known as the symbolic argument (Tabach, et al, 2010). Deductive proof (Mathematical Association & Boys' school's committee, 1965). deductive reasoning (Karadag, 2009; Katagiri, 2004) *convincing* (Stacey, 2006).

Problem Number 1c

"jumlah k bilangan asli berurutan merupakan kelipatan k?jelaskan"

Answer :

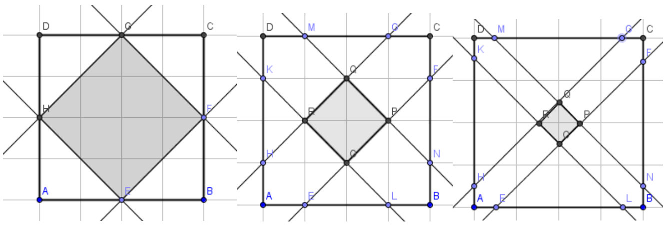
Type of Description Answers	Semester - The number of students who answered	The coding's kind of thinking
No, because it only applies to odd multiples. Examples of negation. 6 natural numbers which add up the result is not a multiple of 6, eg a = 1, b = 2, c = 3, d = 4 if the sum result is 10 and not a multiple of 6 Yes! Because the number of consecutive k natural numbers is a multiple k, where k is an odd number	third semester = 1 student fifth semester = 2 student	Provision of evaluating an alleged counterexample.
Right, because every number c is odd natural number sequence will generate multiplier k	half to one = 2 students fifth semester = 4 student	Analogy than previously thought

itself. (If we look at the results of a and b) True, the reason is the same as the previous pattern	seventh semester = 1 student	for k odd
For example; k = 4 Download bialngan original sequence 1,2,3,4 add up 1 + 2 + 3 + 4 = 10 is not a multiple of 4 Example: k = 5 berurutan1,2,3,4,5 Download original numbers add up 1 + 2 + 3 + 4 + 5 = 15 is a multiple of 5 So, if k is odd multiples. Even if k is not a multiple	fifth semester = 2 student seventh semester = 7 student	Giving an example and not an example, be creative new allegations

A proof counter example is an alternative in evaluating allegations or statements. This evaluation could lead to new allegations. Giving counter example is part of the process specializing (Stacey, 2004). Description more answers using the analogy thinking, from the previous issue. The analogy is part of inductive thinking (Pólya, 1990). They use about similarities 1a and 1b are applicable to an odd number. Description Other answers included the use of examples and not an example that produces a new allegation. Giving is not an example (counterexamples) is used to reject a statement or allegation (Zazkis, et all, 2008).
One important aspect of mathematical thinking is a skill in making a conjecture and prove it. In this study, which revealed the skills associated with skills in making conjectures relating to area ratio of a square. The problem is selected in order to identify the type of mathematical thinking that is used by the student. As for results obtained can be seen in the table given below;

Problem Number 2a

"Amati gambar berikut;



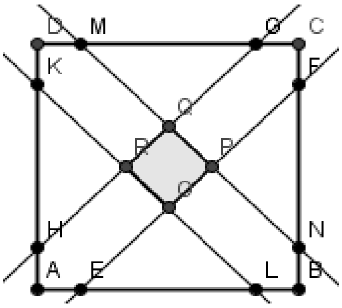
Tentukanlah perbandingan luas daerah yang diarsir dengan luas perseginya? Jelaskan

Answer :

Type of Description Answers	Semester - The number of students who answered	The coding's kind of thinking
▪ Counting the number of boxes that identify immediate visual and numerical thinking Figure 1, 8; 16 or 8:16 = 1: 2; Figure 2, 2:16 or 2:16 = 1: 8; Figure 3 ¼ ▪ The first image = ½, the second image = 1/8, the third image = 1/16, because every box has the same value	seventh semester = 6 student half to one = 2 students fifth semester = 11 student	Counting the number of boxes that indicate direct visual and numerical thinking
Square with s = 4, Figure 1, the box rhombus with d1 = d2 = 4 box area of the square = 4 x 4 = 16 Size rhombus = ½ ½ d1.d2 = 4.4 = 8, so the ratio of the square and rhombus = 2: 1, so also with figures 2 and 3,	fifth semester = 1 student	Use a formula to determine the extent of indicating in visual thinking and procedural

Problem Number 2b

"Jika DM=DK=AH=AE=BL=BN=CF=CG, tentukanlah perbandingan luas daerah yang diarsir OPQR dengan persegi ABCD?"



Answer :

Type of Description Answers	Semester - The number of students who answered	The coding's kind of thinking
Because CG = CF, then QR=√(2CG²)= CG√2. Because the same as Figure 2 then the comparison 2:16	third semester = 3 student	Trying to formulate a common or conjecture, but the image is more influence in determining the decision
So it can be inferred that the larger the area that in hatching the more spacious small square ABCD, and vice versa	seventh semester = 2 student fifth semester = 11 student	Making conclusions still less supported by the facts right.
Observed from the first image of the second part of this almost the same just sketches is slightly different, so, if it was concluded that comparison is equal to the first section	half to one = 2 students	Visual skills are very dominant in determining the decision taken

Students in determining the comparison more by direct counting the number of small square shading. This is evident from the answers given, either half to one, half to three, five semesters or seven semesters. They write directly, for example, first image = ½, second image = 1/8, third image = 1/16, because every box has the same value. Only one student, semester 5, which is trying to use the formula for the area of the rhombus. It can be seen that dominates the visual skills of students in solving the problems of this area (Ferri, 2012). Indications towards visual skills can also be seen from the answers given in the next issue by saying, "Because the same as the comparison figure 2 2/16", "Observed from the first part of the second part of this picture is almost the same sketch only slightly different, so, if concluded the same comparison with the first part, "" it can be concluded that the greater the area of the shaded region, the smaller the area of the square ABCD, and vice versa (although this is not the right answer).

There is one student that indicate analytic skills (Ferri, 2012). It uses the formula for the area of a rhombus, ½ d1, d2, which in turn determines the outcome procedurally. However, no student shaded views that shape can also be seen as a square shape.

About the second part of this number, a matter made to see the skill in making generalizations about common forms of the comprehensive comparison of the shaded square with a square of magnitude. The answers are made tend to be imprecise. They would make a statement that is less supported by facts or data. This indicates the skills to use the data to make the conjecture is still weak (Scusa, 2008). Weak make conjectures show the weakness in the skills of abstraction (Karadag, 2009). Stacey (2006) describes as the weak students, in general, see from the examples that exist (specialization-generalization).

High-level thinking skills other than the skills in evaluating statements or allegations. This research attempts to reveal the students' ability to evaluate the claims that can be shown or demonstrated in various ways. The result can be seen in the following table.

Problem Number 3

"Diketahui sebuah fungsi $f(n) = n^2 - 8n + 18$, apakah *f*(*n*) > *f*(*n*-1), untuk n bilangan asli, selalu benar, kadang-kadang, atau tidak benar? Jelaskan!"

Answer :

Type of Description Answers	Semester - The number of students who answered	The coding's kind of thinking
▪ Not true, because once proved the result f(n) < f(n-1), for example, f (1) = 1^2 - 8 (1) + 6 = 1-8-18 = 11 f (1-1) = (1-1), 2-8 (1-1) + 6 = 1 + 1 – 1-8 + 8 + 18 = 19, f (n) < f (n-1) ▪ f (n) = n^2 - 8n + 6, then n = 2, f (2) = 22-8 (2) + 6 = 6; f (n-1) = n2-10n + 27, then n = 2 = 22-10 (2) + 27 11 = F (n) < f (n-1) so, untrue	half to one = 2 students third semester = 1 student third semester = 2 student	Taking an example to counter statements
I think, sometimes, when n >1, then f (n)> f (n-1), but when n = 1, f (n) <f (n-1). Pembuktiann n = 1, F(n) >f(n-1), 1-8+18>0-8(0)+18→11>18 This is a contradiction, for it must be f (n) <f (n -1) for n = 1	third semester = 3 student fifth semester = 4 student	One example to evaluate the claims
▪ Seventh semester students gave the following answer; For n = 2, f (2) = 4-16 + 18 = 6 <f (1) = 1-8 + 18 = 11 For n = 5, f (5) = 25-40 + 18 = 3> f (4) = 16-32 +18 = 2 So, sometimes correctly Another student took the case n = 1 and n = 10, n = 2 and n = 11 (3 org, n = 1 and n = 11 (2 org), ▪ The fifth semester students take the example n = 3 and n = 2, n = 6 and n = 5	seventh semester = 7 student fifth semester = 8 student	Giving some cases to evaluate the claims

The ability to evaluate the statement included in the higher-level thinking skills (Krathwohl, 2002). Students tend to use examples and are not examples to evaluate a statement. One example is given for rejecting such a statement delivered semester students to one and a half to three. They immediately took the decision not true, regardless of the case for the other n. They can criticize but still weak in check as part of evaluating skills.

A case was given to evaluate the claims. They, semester students to three and a half to five, taking the example of n = 1 such that f (n) <f (n-1) and concluded that the statement is sometimes true. In contrast to the previously stated is not true. Intuition is the basis for the emergence of allegations that the statement is sometimes true. They have not shown a logical argument about suspicions. Analysis of the cases n others do not do so they do not know exactly when the statement is true and when the statement is wrong. This indicates that the student is using intuition rather than analysis in his thinking.

Several other students examine some cases to decide. They seek to analyze the case of n given to conclude that the statement is sometimes true. However, the selection of these cases can not be evaluated accurately and logically.

See pattern description of the answers given, the students tend to use the numeric reference to resolve this matter (Sevimli, et al, 2012). No student who tried to use the graphical or visual reference to complete. No students who try to evaluate directly (direct evidence such as the following)

Note $f(n) = n^2 - 8n + 18$,

then $f(n-1) = (n-1)^2 - 8(n-1) + 18$

$$= n^2 - 2n + 1 - 8n + 8 + 18$$

$$= n^2 - 10n + 27$$

$$f(n) > f(n-1), \text{then } n^2 - 8n + 18 > n^2 - 10n + 27$$

$$\text{so } -8n + 18 > -10n + 27$$

$$\Rightarrow 2n > 9, n > 4{,}5$$

so, f(n) > f(n-1) for n > 4

So we can conclude that f(n) > f(n-1) is always true for n > 4

Problem Number 4

"Tentukanlah nilai a dan b agar bilangan 32a1b4 habis dibagi 4? Apakah ada kemungkinan untuk nilai a dan b yang lain? Jelaskan "

Answer :

Type of Description Answers	**Semester - The number of students who answered**	**The coding's kind of thinking**
▪ there are, because there are other possibilities as evidenced eg a=0, b=0→32a1b4=320104:4=80276 proven Suppose a=4, b=4 →32a1b4=324144:4=81036 proven ▪ For example a = 1 and b = 2 then 321124. Numbers are divisible by 4, the view from the second last number is 6. So then 24/4 proven	half to one = 2 students third semester = 2 student	Analogy to make a conjecture
▪ Suppose (A = 1, b = 2 then 321 124: 4 = 80241) (a = 2, b = 3 then 322 134: 4 = 80533.5) (a = 3, b = 4 then 323 144: 4 = 80 786) from the above description we take a = 1 or 3 and the value of b = 2 or 4 proven divisible by 4. Then there are numbers which allow a and b with a value notes must be odd and b value should be even. in addition also that a number is divisible by 4 if the last two digits of numbers divisible by 4 ▪ if a = 4, b = 8 then 32a1b4 divisible by 4 if a = 1, b = 0 then 32a1b4 divisible by 4 so, it is possible for other values of a and b, if a and b to a valid whole number and b count even	third semester = 1 student fifth semester = 2 student seventh semester = 2 student fifth semester = 2 student	Analyze the case to evaluate the alleged (analogy)
For example 32a1b4 divisible by 4 300,000: 4 = 75,000 20000: 4 = 5000 a (10000): 4 = a (250) 100: 4 = 25 b (10): 4 = 5/2 b 4: 4 = 1 So, 32a1b4 be divisible by 4 if a and b are multiples of 4	seventh semester = 1 student fifth semester = 11 student	Parse constituent patterns of numbers to make allegations

This problem is made to identify the skills of creative thinking that are focused on determining the logical alternative dispute resolution. This problem is most completed by students. Three broad categories of mathematical thinking can be identified. First, use an analogy to make a conjecture, analyzing some cases to make a conjecture, and parse the numbers constituent patterns to make assumptions. The third type of response is still not lead to the right answer and logical. The conclusions they make like, "Numbers are divisible by 4, the view of the last two numbers then 24/4 is 6", "there is still a number that allows a and b with a value notes must be odd and b values must even. other than that any way that a number is divisible by 4 if the last two digits of numbers divisible by 4 "," 32a1b4 be divisible by 4 if a and b are multiples of 4 ".

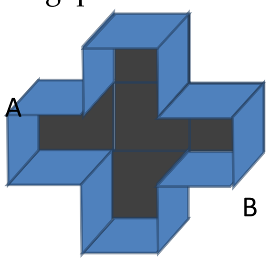
The answers given still not indicate the expected answer, especially in determining whether there is an alternative solution. Making a = 1 and b = 2, is the first solution. This section can be completed by all participants there. However, determining alternative other settlement is still not solved properly by semester students 1, 3, and 7.

The fifth-semester students tend to lead to the desired answer. They use the thinking skills of analysis to decipher the pattern of numbers constituent (Krathwohl, 2002). However, it failed to conclude a solution. This failure is due to the use of analogy thinking (Katagiri, 2004) or generalizations (Karadag, 2009) which are less precise.

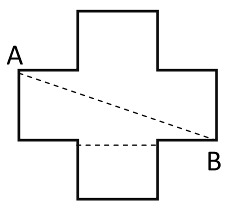
Problem-solving related to geometrical very difficult for students. Understanding within a context of everyday life become an obstacle to translating it into a language or mathematical models. Nevertheless, the students understand the Pythagoras procedure to measure the distance. These results can be seen answers to the following.

Problem Number 6

sebuah kotak tempat semut disusun seperti gambar dibawah. Apabila setiap rusuknya sama panjang, tentukanlah jarak minimum yang dapat ditempuh oleh semut dari A ke B? Mengapa demikian?



Answer :

Type of Description Answers	**Semester - The number of students who answered**	**The coding's kind of thinking**
▪ example; each edge = 1 cm, then the closest distance = $\sqrt{1^2+3^2} = \sqrt{1+9} = \sqrt{10}$because every edge at the closest way to use the formula of Pythagoras.  ▪ Case and calculation of long edge = 1 unit, the minimum distance the ants taken from A to B is a 6 unit	half to one = 2 students seventh semester = 6 student	Taking a case, the calculation procedure
because each rib at the smallest distance from A to B is, for example edge = x, AB = AC + CD + DB $=\sqrt{x^2+x^2}$+ x + x = x√2 +2x = x(√2 +2)	third semester = 3 student	The general case, procedural

There are three types of answers that can be identified. First, the students tried to take the case of a certain length and calculate the formula of Pythagoras. They are wrong in breaking down the form of geometry into their webs. This shows the weakness of thinking skills representation of visual and symbolic (Karadag, 2009; Marzano, et al, 1988), and the weakness of symbolic thinking (Katagiri, 2004), the low skills of visual thinking (Ferri, 2012) and thought generalizations (Stacey, 2004). Second, the students use the symbol x to declare the long side/rib of the box provided. However, they use a combination of calculations with Pythagoras and the length of two sides. They could be said to have skills in making an abstraction or a symbol of the long edge. Third, seventh-semester students take a long case rib = 1 and calculate the shortest distance by summing the six sides that connect two points A and B.

In general, students have not been able to determine the shortest distance between two points located on the geometry. They have difficulty in framing/nets of geometry that are

not routine. This led, student wrong in determining the distance of the shortest as experienced by students of semester one and a half to three. Nonetheless, they are procedurally know how to calculate the distance of two points.
The use of derivatives to determine the gradient of a tangent identified in this study. A given problem can be solved pursued by each student in the different semester. This problem consists of two parts, the first question is more on skill in using concept search procedure tangent at a point located on the curve. In the descriptive description of the answer can be seen in the answers to the following.

Problem Number 7
Diketahui sebuah fungsi, $f(x) = x^3 - x^2 - x$

a. Tentukanlah persamaan garis singgung pada kurva di titik x = -1
b. Tentukan pula persamaan garis singgung lain pada kurva yang sejajar dengan persamaan tersebut.

Answer :

Type of Description Answers	Semester - The number of students who answered	The coding's kind of thinking
$f(x) = y, y = x^3 - x^2 - x$ example x = -1, y=(-1)^3- (-1)^2- (-1)= -1 -1 +1 = -1, $m = y' = 3x^2 - 2x$ = 3 + 2 = 5 $y - y1 = m(x - x1)$➔$y + 1 = 5(x + 1)$➔$y + 1 = 5x + 5$➔ y = 5x-4	third semester = 3 student	Using standard procedures
$f(x) = y,\ y = x^3 - x^2 - x,$ example x = -1, y=(-1)^3- (-1)^2- (-1)= -1 -1 +1 = -1, $m = y' = 3x^2 - 2x - 1$ = 3 + 2 -1 = 4; $y - y1 = m(x - x1)$➔$y + 1 = 4(x + 1)$➔$y + 1 = 4x + 4$➔y = 4x+3	seventh semester = 2 student	
From equation 5x-y+4 = 0 and generate x = -4/5 and y = 4, then the equation is obtained, for example c = -x-4 = 4/5 – 4 = -16/5. So,$\frac{-4}{5}x^2 + 4y + c = 0$➔ $\frac{-4}{5}x^2 + 4y - \frac{16}{5} = 0$➔$-4x^2 + 20y - 16 = 0$(sem 3 = 3 org)	third semester = 3 student	Trial and error

Description student answers are indicated on the type of procedural. They use the first derivative to determine the gradient and uses equations known gradients gradient and a given point. This indicates the level of thinking application (Krathwohl, 2002). However, a third-semester student made a mistake in determining its derivatives which have implications for the determination of the gradient magnitudes are also wrong. No fifth-semester students work on this matter, and only two students of the seventh semester were answered accurately and correctly. The second part of the problem is simply solved by students of the third semester even though they are wrong. They use trial and error to get it done. According to Stacey (2004) is the ability to think spatialization. No student who tries to describe in graphic form.
Problem section b) included in the category of analytic skills. These skills are characterized by the ability to parse and determine the relationship between a tangent to the function, and other tangents. However, none of them work on this matter properly. The third-semester students are trying to do did not understand any other tangents. They produce an equation of a tangent in the form of a quadratic equation. This indicates that the student has not been able to diagnose problems with either the low or analytic thinking skills (Krathwohl, 2002) owned by third-semester students.
Trigonometry is one of the topics to be taught to students. Trigonometry many have representation from one form to another. Skills symbolic representation at the level of students is needed. This study sought to uncover these skills. Description of student answer can be seen in the following table.

Problem Number

Diketahui sin $(A + B)$ = sin$A$ cos$B$ + cos$A$ sin$B$ dan cos $(A + B)$ = cos$A$ cos$B$ – sin$A$ sin$B$

a. Apabila $B = A$, bagaimana persamaan keduanya sekarang?

b. Bagaimana dengan sin$(A + B)$ / cos$(A + B)$?apa yang dapat diperoleh

c. Susunlah persamaan baru yang bisa diperoleh dengan memanipulasi cara yang serupa dengan cara a) atau dengan cara b)? Tunjukkan

Answer 8a :

Type of Description Answers	**Semester - The number of students who answered**	**The coding's kind of thinking**
B=A,Sin(A+A)=sinAcosA+cosAsinA, Cos(A+A)=cosAcosA-sinAsinA	third semester = 2 student	Symbol manipulation
= 2sinAcosA	third semester = 1 student	
B=A, Sin(A+A)=sinAcosA+cosAsinA, Sin2A= 2sinA cos A Cos(A+A)=cosAcosA-sinAsinA, Cos2A = cos2A – sin2A,	fifth semester = 14 student seventh semester = 3 student	Manipulation to make a new statement

Two types of descriptions answers may be indicated to manipulate symbols and manipulate symbols to create a new statement. The third-semester student's manipulation simply by changing B to A. They get an answer, Sin(A+A) = sinA. cosA + cosA. sinA, dan Cos (A+A) = cosA.cosA – sinA.sinA. Description This answer indicates that the third-semester students have not skilled in transforming one form to another. Changes in one form of the equation to another equation form is part of the thinking skills of representation (Karadag, 2009; Marzano, 1988). This capability can be classed in the knowledge of the procedure, but not yet on metacognitive knowledge (Marzano, 1988).

Semester students of fifth and seventh semesters have a better knowledge of the procedure so that they could declare Sin(2A) = 2.sinA. cosA. It shows also that they have a better symbolic representation than half of 3. In addition, they also indicated that the skills to check the statement. They are at the level of thinking of evaluation (Krathwohl, 2002).

Problem 8b intended fatherly see the skills of students in justifying the new statement will be made. The description of the answers given student can be seen in the following table.

Answer 8b:

Type of Description Answers	**Semester - The number of students who answered**	**The coding's kind of thinking**
$\frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB} = \frac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB} : cosAcosB$ $= \frac{\frac{sinA}{cosA} + \frac{sinB}{cosB}}{1 - \frac{sinA}{cosA} \cdot \frac{sinB}{cosB}} = \frac{tgA + tgB}{1 - tgA . tgB} = tg(A + B)$	third semester = 2 student seventh semester = 2 student	Manipulation procedures appropriately
$\frac{\sin(A+B)}{\cos(A+B)} = \frac{2sinAcosA}{cosAcosA - sinAsinA} = \frac{2sinAcosA}{cosAcosA} - \frac{2sinAcosA}{sinAsinA} = \frac{2sinA}{cosA} - \frac{2cosA}{sinA}$ =2tgA-2 cotgA, tgA = cotg A ➔ $tg^2A=1$	fifth semester = 10 student	An error manipulation

The third-semester students and seven students can compose a new statement with a logical argument and systematic (answer 1).They can use the previously valid statement. A good procedure is also done to get a new statement. They can also connect multiple concepts in trigonometry. They can use deductive reasoning (Katagiri, 2004), symbolic thinking (Karadag, 2009), and creating a statement and justify it (Krathwold, 2002), classified in the thought process convincing (Stacey, 2004).

The fifth semester students having an error in resolving this issue. They decipher the answer 2, $\frac{\sin(A+B)}{\cos(A+B)} = \frac{2sinAcosA}{cosAcosA - sinAsinA}$ on this line they assume that sin (A + B) = sin (A + A) as in the previous exercise.

$=\frac{2sinAcosA}{cosAcosA}-\frac{2sinAcosA}{sinAsinA}=\frac{2sinA}{cosA}-\frac{2cosA}{sinA}$ on this line, they made a mistake in simplifying fractions in the form of trigonometry. 0 =2tgA-2 cotgA in this section students assume that it is equal to 0.
Tg A = cotg A so tg2A = 1. Mistakes were made indicating that students have difficulty in representing the shape of a similarity trigonometric other similarities.
On the next question, which is about 8c, given to seeing the mathematical thinking skills of students, especially related to making and justify the conjecture. The description given student can be seen in the following table.

Answer 8c:

Type of Description Answers	Semester - The number of students who answered	The coding's kind of thinking
if (A+B) = C and (A-B)=D, then A= ½ (C+D) and B= ½ (C-D) Sin(A+B)+sin(A-B)=2sinAcosB, so SinC + sinD =2 sin ½ (C+D) cos½ (C-D)	seventh semester = 2 student fifth semester = 5 student	Abstraction, make a statement and show

Students first semester and third semester is not working on this. Two students of the fifth semester of the seventh semester and work the same way. They dare to create a new symbol, C = A + B and D = A-B. They can also express the equation in another form ie, A = ½ (C + D) and B = ½ (C-D). It shows can think symbolism and symbolic representation (Katagiri, 2004; Karadag, 2009). They also seamlessly use procedural and factual knowledge to get a new equation logically.

2. Thinking Mathematically Very Personal and Situational

Marzano, et al (1988) suggest that forward- thinking is the foundation of education. Mathematical thinking is very important in learning mathematics. Problem-solving and mathematical tasks require mathematical thinking skills. Someone will use his mind when faced with problems. The level of emotional intelligence and will affect a person's ability to think. Katagiri (2010), delivered three things related to mathematical thinking; 1) content, 2) the mathematical way, and 3) the mathematical attitude.
This section is arranged in order to answer the problem formulation, What similarities and differences in the ability to think mathematically students seen from the level of a semester? Detailed descriptions can be presented below;
About the first part of a and b is done by the students of each semester. Giving an example and not an example of a strategy or method that is widely used students from half of 1 to 7. The example given vary, there is the first example, there are some examples. They find it sufficient to demonstrate or prove the statement by one and some examples. They still use the numeric argument (Tabach, et al, 2008), the calculation process is speasilisasi (Stacey, 2004). What they do is basically provide support for the statement.
There are two semesters students 3 and 7 5th semester student who can prove symbolically (Tabach, et al, 2008), shows the generalizability of three natural numbers sequentially by k, k + 1, k + 2 or a, a + 1, a + 2 (Stacey, 2004), using deductive reasoning (Karadag, 2009; Katagiri, 2010), at the level of mathematical reasoning according to Mullis, et al (Suryadi, 2011). More formally, they write about the evidence in one section b, which means it can communicate ideas (Sumarmo, et al, 2010)
1c problems are problems related to the skill level of evaluation (Krathwohl, 2002). Unity semester students using reasoning analogous to see the pattern of the previous problem (Katagiri, 2010; Sumarmo, et al, 2010). Semester three students use a counter example to reject the statement (Zazkis and Chernoff, 2008). The fifth-semester students tend to use a variety of thinking skills. The seventh-semester students tend to perform analysis of various cases (Krathwohl, 2002;, an example and not the example, do the checking for evaluating

statement (Krathwohl, 2002), building through the analytical-evaluative statement (Fonkert, 2012).

Problem number 2 is a matter relating to the area ratio of the square field. The first part of this question can be used as a guide to work on the problems of both parts. However, more students use the direct calculation of many small square shading and compare it with the total square. They use a numeric argument (Barkai, et al, 2008). Only one student who tried to use the formula for the area as a knowledge of the procedures (Marzano, 1988).

The second part of Question 2 is provided in order to assess the skills of generalization and conjecture students to find a general pattern comparison breadth. Unfortunately, the answers are made tend to be imprecise. They would make a statement that is less supported by facts or data. This indicates the skills to use the data to make the conjecture is still weak (Scusa, 2008). Weak make conjectures show the weakness in the skills of abstraction (Karadag, 2009). Stacey (2004) refer to as the weak students, in general, see from the examples that exist (specialization-generalization). They tend to use visualization skills (Ferri, 2012).

The third matter relates to the ability to evaluate the allegation/statement. Characteristics used to think mathematically inclined students using case examples of specific natural numbers. Semester students beginning to use the example of the case to counter allegations (Klymchuk, 2008) and evaluate the claims. students tend to use the numeric reference to resolve this matter (sevimli and Delice, 2011). The same is done by the students in the upper half by giving some cases to check and evaluate the alleged (Krathwohl, 2002). Students do not attempt to use graphics or visual skills (Ferri, 2012), or direct evidence deductively (Karadag, 2009; Katagiri, 2010).

Problem number four is a matter that is used to reveal the creative thinking skills, especially finding alternative solutions logically. Thinking analogies tend to be used by the student (Katagiri, 2004; Sumarmo and Nishitani, 2012) to take the conclusion that in some cases the numbers a and b that they provide. Others use the skills of analysis (Krathwohl, 2002) described the elements of numbers into the hundreds of thousands, tens of thousand, Riggan, hundreds, tens, and units, although the final conclusions have not been right.

Problem number 6 is concerned with determining the shortest distance between two points located on the geometry. The answers were given no right. Description answer students showed the weakness of thinking skills representation of visual and symbolic (Karadag, 2009; Marzano, 1988), and the weakness of symbolic thinking (Katagiri, 2004), the low skills of visual thinking (Ferri, 2012) and thought specialties (Stacey, 2004).

Question 7 relating to the functions and tangents. Only two students from the 7th semester were able to answer correctly apart. They were at the application level (Krathwohl, 2002) uses derivatives to determine the gradient. There are students who produce a shaped tangent with the equation quadratic equation. This indicates that the student has not been able to diagnose problems with either the low or analytic thinking skills (Krathwohl, 2002) owned by third-semester students.

Problem number 8 is given to uncover symbol manipulation skills and make a new statement. Description This answer indicates that the third-semester students have not skilled in transforming one form to another. Changes in one form of the equation to another equation form is part of the thinking skills of representation (Karadag, 2009; Marzano, 1988). This capability can be classed in the knowledge of the procedure, but not yet on metacognitive knowledge (Marzano, 1988)

Semester students of fifth and seventh semesters have a better knowledge of the procedure so that they could declare Sin (2A) = 2.sinA. CosA. It shows also that they have a better symbolic representation than half of 3. In addition, they also indicated that the skills to check the statement. They are at the level of thinking of evaluation (Krathwohl, 2002).

The third-semester students and the seventh semester can compose a new statement to the argument to a logical and systematic, at about 8b. They could use an earlier statement invalid. A good procedure is also done to get a new statement. They can also connect multiple concepts in trigonometry. They can use deductive reasoning (Katagiri, 2004), symbolic thinking (Karadag, 2009), and creating a statement and justify it (Krathwold, 2002), classified in the thought process convincing (Stacey, 2004).
Students semesters one and a half to three does not work on the problems 8c. Two students from the 7th semester and semester student 5 5 work in the same way. They dare to create a new symbol, C = A + B and D = A-B. They can also express the equation in another form ie, A = ½ (C + D) and B = ½ (C-D). It shows can think symbolism and symbolic representation (Katagiri, 2004; Karadag, 2009). They also seamlessly use procedural and factual knowledge to get a new equation logically.
Students semesters one and a half to three does not work on the problems 8c. Two-seventh and fifth-semester students of the fifth-semester students work in the same way. They dare to create a new symbol, C = A + B and D = A-B. They can also express the equation in another form ie, A = ½ (C + D) and B = ½ (C-D). It shows can think symbolism and symbolic representation (Katagiri, 2004; Karadag, 2009). They also seamlessly use procedural and factual knowledge to get a new equation logically.
The above description leads to mathematical thinking does not depend on the level of a semester. The higher the semester does not guarantee higher mathematical thinking skills. This type of thinking mathematical certainty can not be guaranteed for everyone. Students use this type of thinking tailored to the type of questions or problems. The problem of comprehensive comparison of the number 2 is done mostly with visual thinking, but would be right if using analytic thinking. Question 3 will be easier to use visual thinking through a graphical representation of f (n). So to think mathematically very personal and depends on the situation of the problems encountered.

Conclusion and Recommendation

Based on the research result, it can be concluded that; (1) Students had to argue tendency Numerically numeric or argumentation, using examples and are not examples to demonstrate and Evaluate the conjecture, have the skills to think analogy, using special process. However, it is still weak in deductive reasoning, the visual representation of the geometry and graphics, symbolic representation in trigonometry, direct evidence, generalizing from patterns of relationship variables. (2) The depth of the semester, students can not be used as a measure of the level and type of mathematical thinking of students. The higher the half does not guarantee the higher mathematical thinking, conversely the lower half of the lower level does not guarantee mathematical thinking. However, the higher the better half indicated resources in the analysis and evaluation of an allegation or conjecture. Types of mathematical thinking and mathematical thought processes used to depend on the content of math problems. Problems related to the geometry of the dominant use of visual thinking. Students to think mathematically very personal (intelligence, interests, experience), and Depending on the situation at hand (mathematics content, type of problem).
Based on the conclusion above, it is recommended that; Skills mathematical thinking student still needs to be improved, especially related to proving mathematical statements, complete the contextual issues and open-ended, represents the mathematical expression in various forms through the tasks or problems involving thinking skills high level in every subject there.

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