Capital Income Tax, Linear RD Technology, and Economic Growth

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22 April 2017

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April 21, 2017

Abstract

This paper shows that, in a R&D-based growth model in which vertical and horizontal innovations occur simultaneously, increasing the capital income tax leads to faster growth. For this result to hold, the production function for both vertical and horizontal innovations must have constant returns to scale.

Keywords: Endogenous growth, Capital income tax, Vertical innovation, Horizontal Innovation, Scale effect.
JEL Classification: O31, O40, H20, J22.

1 Introduction

The effects of capital income taxation on economic growth is an important topic for not only economists but also policymakers. A substantial body of literature concludes that taxing capital income is bad for growth [see, e.g., Judd (1985); Chamley (1986); Lucas (1990); Jones, Manuelli, and Rossi (1993); and Peretto (2003)]. However, some studies cast doubt on this view. For example, Uhilg and Yanagawa (1996), de Hek (2006), and Chen and Lu (2013) show that higher capital income taxes may lead to faster growth. Conesa, Kitao, and Kruger (2009) and Hiraguchi and Shibata (2015) have emphasized that the optimal tax rate on capital is positive. Whether a government should tax capital income remains an open question.

*This research is financially supported by Grant-in-Aid for JSPS Fellows (No. 26·3190) and Grant-in-Aid for Scientific Research (No. 16K03552).

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The present paper contributes to the literature on supporting positive capital income taxation in an endogenous growth model. The analysis is closely related to the analyses in Young (1998), Dinopoulos and Thompson (1998), and Peretto (2003), who developed R&D-based growth models with vertical and horizontal innovations. In fact, the model in the present paper is the same as that of Peretto (2003), apart from vertical innovation technology. Peretto considers that the production function for vertical innovation has decreasing returns to scale and shows that an increase in the tax rate on capital income induces a decline in the long-run growth rate. The present analysis shows that the linear production function leads to an opposite result; i.e., an increase in capital income tax has a positive effect on the growth rate.

The remainder of the present paper is organized as follows. Section 2 introduces the model. Section 3 considers the market equilibrium dynamics and derives the main result.

2 The Model

The model draws on work by Peretto (2003). It allows individuals to allocate time to labor supply and leisure, and consists of two types of innovation sector: vertical innovation and horizontal innovation. A government taxes consumption and labor, capital, and corporate incomes to provide public goods and lump-sum transfers.

2.1 Consumption and Labor Supply

I consider the closed economy populated by identical individuals who supply labor services and consumption loans in competitive labor and assets markets. The population at time t is represented as $L_t = L_0 e^{\lambda t}$, where L_0 is the initial population and λ is the rate of population growth. The lifetime utility is

$$U_t = \int_t^{\infty} e^{-(\rho-\lambda)(\tau-t)} \log u_\tau d\tau, \qquad \rho > \lambda \geq 0, \tag{1}$$

where ρ is the individual discount rate. Instantaneous utility at time t is

$$\log u_t = \log C_t + \gamma \log(1-l_t) + \mu \log G_t, \qquad \gamma, \mu > 0, \tag{2}$$

where C_t is a consumption index, l_t is the faction of time allocated to labor supply [so that $(1-l_t)$ is leisure], and G_t represents public goods supplied by the government. Constant parameters, γ and μ, are the elasticity of instantaneous utility with respect to leisure and public goods, respectively. The consumption index is symmetric over a continuum of differentiated goods,

$$C_t = \left[\int_0^{N_t} (c_{it})^{(\varepsilon-1)/\varepsilon} \, di \right]^{\varepsilon/(\varepsilon-1)}, \qquad \varepsilon > 1, \tag{3}$$

where ε is the elasticity of differentiated product substitution, c_{it} is the demand for each differentiated good, and N_t is the number of goods (firms). Individuals face the flow budget constraint

$$\dot{A}_t = [r_t(1 - t_A) - \lambda]A_t + (1 - t_L)W_t l_t - (1 + t_E)E_t + T_t. \tag{4}$$

All variables are in per capita terms. A_t is financial wealth, r_t is the rate of return on capital, W_t is the wage rate, and E_t is consumption expenditure. The wage rate is the numéraire, $W \equiv 1$. The government taxes labor income at rate t_L, capital income at rate t_A, and consumption at rate t_E, and pays lump-sum transfers T_t.

Individuals maximize (1) subject to equations (2)–(4). The optimal condition for the problem is obtained as follows.

$$\frac{\dot{E}_t}{E_t} = r_t(1 - t_A) - \rho \tag{5}$$

$$L_t l_t = L_t \left[1 - \frac{1 + t_E}{1 - t_L} \gamma E_t \right] \tag{6}$$

Equation (5) is a Euler equation, and equation (6) is the aggregate labor supply.

Furthermore, at each time, individuals decide how they consume each differentiated good to maximize (3), given the expenditure E_t. Solving the well-known static problem yields the aggregate consumption of good i,

$$X_{it} = L_t c_{it} = L_t E_t \frac{P_{it}^{-\varepsilon}}{\int_0^{N_t} P_{jt}^{1-\varepsilon} dj}, \tag{7}$$

where P_{it} is good i's price.

2.2 Production

The firm with a patent supplies its differentiated good exclusively with the technology

$$X_{it} = Z_{it}^{\theta}(L_{X_{it}} - \phi), \qquad 0 < \theta < 1, \quad \phi > 0, \tag{8}$$

where X_{it} is output, $L_{X_{it}}$ is labor employment, and ϕ is a fixed management cost. Z_{it}^{θ} is labor productivity, which is a function of the firm's accumulated stock of innovations, Z_{it}, with elasticity θ.

2.3 Vertical Innovation: Corporate R&D

The firm can increase its productivity by innovation, which occurs according to

$$\dot{Z}_{it} = \alpha K_t L_{Z_{it}}, \qquad \alpha > 0, \tag{9}$$

where $\dot{Z}_{it}$ is the flow of innovations generated by employing $L_{Z_{it}}$ units of labor in R&D for an interval of time dt, and αK is the productivity of labor in R&D, as determined by the exogenous parameter α and the stock of public knowledge, $K_t = Z_t \equiv \frac{1}{N_t}\int_0^{N_t} Z_{it} di$. The level of public knowledge is determined by the average productivity among each firm; thus, (9) is rewritten as

$$\dot{Z}_{it} = \alpha Z_t L_{Z_{it}}. \tag{10}$$

The function is linear with respect to labor. However, Peretto (2003) assumes the decreasing returns to scale function. This change generates the opposite effect of capital income tax on the growth rate, as discussed later.

The present discounted value of after-tax profit for the firm that has a patent on the differentiated good i is

$$V_{it} = \int_t^{\infty} e^{-\int_t^{\tau} r_s ds}(1 - t_{\pi})\Pi_{i\tau} d\tau,$$

where t_{π} is the tax rate on profit, and pre-tax profit is $\Pi_{it} = P_{it}X_{it} - L_{X_{it}} - L_{Z_{it}}$.

At any time, t, the firm chooses price to maximize the pre-tax profit subject to the demand (7), the technology (8), and the given Z_{it}. The optimal price for good i is

$$P_{it} = \frac{\varepsilon}{\varepsilon - 1} Z_{it}^{-\theta}. \tag{11}$$

Given the price, the demand for each good i is obtained as follows:

$$X_{it} = \frac{\varepsilon - 1}{\varepsilon} \frac{Z_{it}^{\theta\varepsilon}}{\int_0^{N_t} Z_{jt}^{\theta(\varepsilon-1)} dj} E_t.$$

Substituting these into pre-tax profit yields the maximized profit

$$\Pi_{it} = \frac{Z_{it}^{\theta(\varepsilon-1)}}{\int_0^{N_t} Z_{jt}^{\theta(\varepsilon-1)} dj} \frac{E_t L_t}{\varepsilon} - \phi - L_{Z_{it}}.$$

Before proceeding to the dynamic problem, I impose the following two assumptions for analytical simplicity.

Assumption 1. *Previous corporate R&D generates an external effect that causes present R&D productivity to increase.*

All firms choose their R&D strategies without recognizing that present R&D has a positive spillover to future R&D technology. The next assumption guarantees that the second order condition of the R&D decision problem discussed below is satisfied.

Assumption 2. $\theta(\varepsilon - 1) < 1$.

Each firm chooses R&D strategies to maximize the present discounted value of after-tax profit, into which the maximized profit is substituted, subject to the innovation technology (10) and rival firms' strategies.

Since R&D follows constant returns to scale technology, the equilibrium condition for finite R&D to occur is

$$q_{it} = \frac{1 - t_\pi}{\alpha Z_t}, \tag{12}$$

where q_{it} is the co-state variable, which is the marginal value of productivity Z_{it}. Equation (12) implies that the marginal value is equal to its marginal cost. An optimal R&D level is not yet determined. As discussed below, it is determined by such as no arbitrage condition in the capital market.

The return to innovation must satisfy the following.

$$r_t = (1 - t_\pi)\theta(\varepsilon - 1)\frac{Z_{it}^{\theta(\varepsilon-1)-1}}{\int_0^{N_t} Z_{jt}^{\theta(\varepsilon-1)} dj}\frac{E_t L_t}{\varepsilon q_{it}} + \frac{\dot{q}_{it}}{q_{it}}. \tag{13}$$

The transversality condition is $\lim_{\tau\to\infty} e^{-\int_t^\tau r_s ds} q_{i\tau} Z_{i\tau} = 0$.

2.4 Horizontal Innovation: Entrepreneurial R&D

The main objective of entrepreneurial R&D is the creation of new goods. Entrepreneurs can create new goods and enter the industry by using only labor inputs .

$$\dot{N}_t = \beta L_{Nt}, \qquad \beta > 0, \tag{14}$$

where β is the productivity of labor in entry, and L_{Nt} is the amount of employment required to create $\dot{N}_t$ new firms for an interval of time dt. The productivity of entrepreneurs is equal to the average productivity among incumbent firms, $\frac{1}{N_t}\int_0^{N_t} Z_{jt} dj$, and incumbent firms are symmetric. This implies that entrant firms are also symmetric with respect to productivity. Therefore, the values for new firms are always the same as those for symmetric incumbent firms.

Entrepreneurs may enter freely into variety-expanding R&D. They finance the product development costs by issuing equity. The after-tax profit for entrepreneurs is $(1-t_\pi)\pi_t^{R\&D}dt = (1-t_\pi)(V_t dN_t - W_t L_{N_t} dt)$. Imposing the free entry condition on this implies

$$V_t = \frac{1}{\beta} \quad \Leftrightarrow \quad L_{N_t} > 0. \tag{15}$$

Entry is positive if the value of the firm is equal to its start-up cost. The profit that accrues to an entrepreneur is given by the expression derived for incumbents. Thus, the market value of a firm's shares satisfies the arbitrage condition: $r_t = (1-t_\pi)\frac{\Pi_{it}}{V_t} + \frac{\dot{V}_t}{V_t}$. Note that the second term in the right-hand side is always zero, because V_t is constant over time. Imposing symmetry on the pre-tax profit for production firm i, I obtain the following:

$$\Pi_t = \Pi_{it} = \Pi_{jt} = \frac{E_t L_t}{\varepsilon N_t} - \phi - L_{Z_t}, \quad \text{for all } j \neq i. \tag{16}$$

Substituting this and (15) into the arbitrage condition yields the rate of return on entrepreneurial R&D

$$r_t = (1-t_\pi)\beta \left[\frac{E_t L_t}{\varepsilon N_t} - \phi - L_{Z_t} \right]. \tag{17}$$

2.5 The Government

The government taxes consumption, labor income, capital income, and corporate profit. These tax rates are constant over time. The government produces public goods, hiring labor at $W_t \equiv 1$. The production function is $G_t = L_{G_t}$, where L_{G_t} is public employment at time t. The government cannot borrow and allocates fraction g of tax revenues to the provision of public goods and fraction $1-g$ to lump-sum transfers to individuals. This satisfies the budget constraint: $t_L L_t + t_\pi \int_0^{N_t} \Pi_{it} di + t_E E_t L_t + t_A r_t A_t L_t = L_{G_t} + T_t L_t$.

2.6 The Labor Market

There are four sources of labor demand. First, the production sector employs $\int_0^{N_t} L_{X_{it}} di$ units of labor to produce differentiated goods. Second, in the corporate R&D sector, $\int_0^{N_t} L_{Z_{it}} di$ units of labor are employed. Third, employment in the entrepreneurial R&D sector is L_{Nt}. Fourth, L_{Gt} units of labor are employed to provide public goods. Equating units of labor to the aggregate labor supply L_t gives the labor market clearing condition: $L_t = \int_0^{N_t} (L_{X_{it}} + L_{Z_{it}}) di + L_{Nt} + L_{G_t}$.

3 The Market Equilibrium Dynamics

3.1 Equilibrium Values and Dynamic Equations

The assumption that firm's productivity Z_{it} is symmetric causes price P_{it} and output X_{it} to be symmetric. That is, for all i, $P_t = P_{it} = \frac{\varepsilon}{\varepsilon-1} Z_t^{-\theta}$, and $X_t = X_{it} = \frac{\varepsilon-1}{\varepsilon}\frac{E_t L_t}{N_t} Z_t^{\theta}$. Substituting the latter into (8) yields

$$L_{X_t} = \frac{\varepsilon - 1}{\varepsilon}\frac{E_t L_t}{N_t} + \phi. \tag{18}$$

In what follows, I focus on an internal equilibrium, where both corporate and entrepreneurial R&D occur.[1] In this situation, equalization of the returns to vertical innovation and horizontal innovation is required. In the capital market, this is called no arbitrage condition. Since, under the homogeneous productivity Z_t, equation (13) can be rewritten as

$$r_t = \alpha\theta(\varepsilon - 1)\frac{E_t L_t}{\varepsilon N_t} - \alpha L_{Z_t}, \tag{19}$$

no arbitrage condition is as follows.

$$\alpha \left[\theta(\varepsilon - 1)\frac{E_t L_t}{\varepsilon N_t} - L_{Z_t}\right] = (1 - t_\pi)\beta \left[\frac{E_t L_t}{\varepsilon N_t} - \phi - L_{Z_t}\right]. \tag{20}$$

This equation holds at all moments in time and characterizes equilibrium.

Before proceeding to analysis of economic dynamics, I impose the following assumption. It guarantees the stability of an internal equilibrium, in which two kinds of R&D are implemented.

Assumption 3. $\alpha\theta(\varepsilon - 1) > (1 - t_\pi)\beta$.

Under Assumptions 1–3, the level of corporate R&D is determined so that equation (20) can be satisfied all times. Solving (20) for L_{Z_t} in the corporate R&D sector yields

$$L_{Z_t} = \frac{\alpha\theta(\varepsilon - 1) - (1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta}\frac{E_t L_t}{\varepsilon N_t} + \frac{(1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta}\phi. \tag{21}$$

The interest rate is simultaneously determined,

$$r_t = \frac{\alpha(1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta}\left\{[1 - \theta(\varepsilon - 1)]\frac{E_t L_t}{\varepsilon N_t} - \phi\right\}. \tag{22}$$

[1] In the present model, since R&D functions (9) and (14) are linear functions of labor input, it is possible that one of the two R&Ds is not implemented. In other words, a corner solution may occur. For the aim of this paper, however, the internal solution is assumed.

These are illustrated as the following figure.[2]

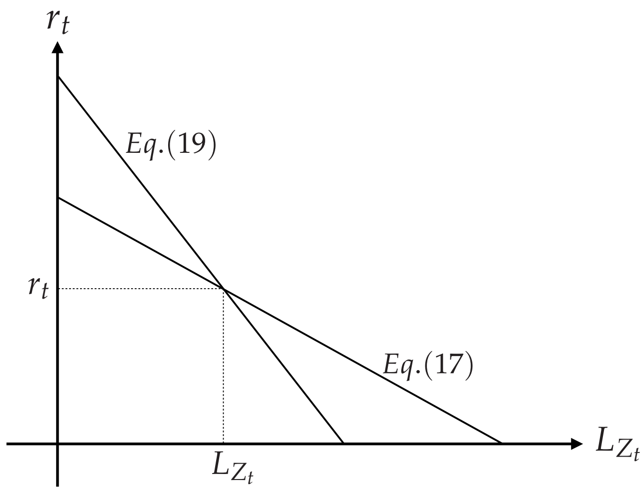

Figure 1: Equilibrium on vertical and horizontal R&D

The after-tax rate of return to investment is indeed the rate of return to saving since, in this economy, the only financial asset available to individuals is ownership shares of firms (stocks). In particular, the capital market clears when $A_t L_t = N_t V_t$. Using this condition, the arbitrage condition, $r_t = (1 - t_\pi)\frac{\Pi_t}{V_t}$, and equation (16), one can rewrite public employment as

$$L_{G_t} = g\left\{t_L L_t + [t_\pi + t_A(1 - t_\pi)]\left(\frac{E_t L_t}{\varepsilon N_t} - \phi - L_{Z_t}\right) N_t + t_E E_t L_t\right\}. \quad (23)$$

The market equilibrium dynamics can be described by the Euler equation and the growth rate of the number of goods per capita, $n_t \equiv \frac{N_t}{L_t}$. Using (22), the Euler equation can be written as

$$\frac{\dot{E}_t}{E_t} = \frac{\alpha(1 - t_A)(1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta}\left\{[1 - \theta(\varepsilon - 1)]\frac{E_t}{\varepsilon n_t} - \phi\right\} - \rho.$$

The labor market clearing condition in the symmetric situation reads $L_t = N_t(L_{X_t} + L_{Z_t}) + L_{N_t} + L_{G_t}$. Using (14), (18), and (23), this can be rewritten

$$\frac{\dot{n}_t}{n_t} = \beta\left(\frac{1}{n_t} - \frac{1}{\alpha - (1 - t_\pi)\beta}\left\{[\alpha(1 + \theta)(\varepsilon - 1) - \varepsilon(1 - t_\pi)\beta]\frac{E_t}{\varepsilon n_t} + \alpha\phi\right\} - L_{G_t}\right) - \lambda.$$

As shown below this system has a unique steady state that can be shown to be saddle stable under Assumptions 2 and 3.

[2]If Assumption 2 is not satisfied, the interest rate is always negative.

3.2 Steady State Analysis

Set $\dot{E}_t = 0$ to obtain

$$E_t = \frac{\varepsilon}{1-\theta(\varepsilon-1)}\left[\frac{\alpha-(1-t_\pi)\beta}{\alpha(1-t_A)(1-t_\pi)\beta}\rho+\phi\right]n_t, \tag{24}$$

and set $\dot{n}_t = 0$ to obtain

$$E_t = -\frac{[\alpha-(1-t_\pi)\beta]\varepsilon}{\alpha(1+\theta)(\varepsilon-1)-\varepsilon(1-t_\pi)}\left[\left(\frac{\lambda}{\beta}+L_{G_t}+\frac{\alpha\phi}{\alpha-(1-t_\pi)\beta}\right)n_t-1\right]. \tag{25}$$

Under Assumptions 2 and 3, the slope of the $\dot{E}_t = 0$ line is positive, and that of the $\dot{n}_t = 0$ line is negative and its intercept is positive.

The intersection in (n_t, E_t) space of equations (24) and (25) determines the steady state values of consumption expenditure and the number of goods per capita, as illustrated in Figure 2. The steady state values are represented as n^* and E^*.

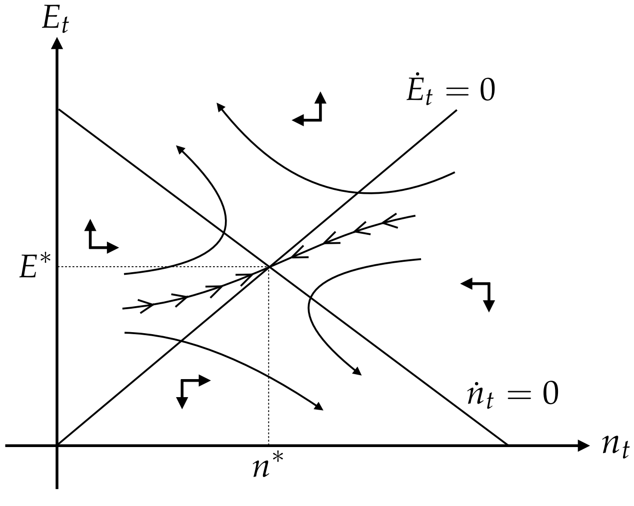

Figure 2: The Phase Diagram on E_t and n_t

Figure 2 states that, in the case where the initial number of goods per capita, n_0, is relative low, specifically $n_0 < n^*$, the number of goods per capita, n_t, and the consumption expenditure, E_t, both increase toward the steady state. In addition, one can confirm that the ratio $\frac{E_t}{n_t}$ gradually decreases. The amount of the input into corporate R&D, L_{Z_t}, decreases as the economy approaches the steady state.

The growth rate of productivity, L_Z^*, and the interest rate, r^*, in the long-run are determined to equalize the rate of returns on two kinds of R&D.[3]

$$L_Z^* = \frac{\alpha\theta(\varepsilon-1)-(1-t_\pi)\beta}{\alpha-(1-t_\pi)\beta}\frac{E^*}{\varepsilon n^*} + \frac{(1-t_\pi)\beta}{\alpha-(1-t_\pi)\beta}\phi \tag{26}$$

$$r^* = \frac{\alpha(1-t_\pi)\beta}{\alpha-(1-t_\pi)\beta}\left\{[1-\theta(\varepsilon-1)]\frac{E^*}{\varepsilon n^*} - \phi\right\} \tag{27}$$

The steady state rate of return, r^*, is dependent on the consumption expenditure per good, $\frac{E^*}{n^*}$. On the balanced growth path, however, consumption expenditure, E_t, is constant[4] and, hence, r^* is determined to satisfy the condition that the after-tax interest rate is equal to the discount rate:

$$(1-t_A)r^* = \rho. \tag{28}$$

From this condition, one can confirm that the vales of $\frac{E^*}{n^*}$ and L_Z^* are determined irrespective of the labor market equilibrium. This means that the growth rate of productivity is independent of the population L_t and, specifically, that there is no scale effect in the present model.[5]

These three equations (26)–(28) yield the following important result of this paper.

Proposition 1. *Capital income tax has a positive effect on the growth rate of productivity,* L_Z^*.

Proof. See Appendix B. □

The intuition of the proposition is as follows. The introduction of and/or increase in capital income tax leads to a higher rate of return on R&D [see equation (28)]. The higher rate of return stimulates the consumption expenditure per good [see equation (27)], which increases the growth rate of productivity [see equation (26)]. By contrast, in Peretto (2003), a rise in the consumption expenditure per good is relatively low, with the result that firms must reduce the number of employees for the higher rate of return to hold. This leads to a decline in the productivity growth.

Effects of other fiscal variables on the productivity growth are the same as those obtained in Peretto (2003). Corporate income tax has a positive

[3] In the present model, the growth rate of productivity is given as $\frac{\dot{Z}_t}{Z_t} = \alpha L_{Z_t}$, which depends on the labor employment in corporate R&D. Thus, one can express the productivity growth as L_{Z_t}.

[4] See the Euler equation (5).

[5] The labor market equilibrium is achieved by the adjustment of the number of firms per capita, n^*.

effect on productivity growth, but labor income and consumption taxes have no effect.[6]

The growth rate of an individual's utility is derived as follows.[7]

$$\frac{\dot{u}_t}{u_t} = \theta \frac{\dot{Z}_t}{Z_t} + \left(\frac{1}{\varepsilon - 1} + \mu \right) \lambda.$$

One can confirm that the growth rate of an individual's utility is independent of the population scale and increases as capital income taxes increase.

[6]See Appendix C for the proof of this.

[7]See Appendix D for a detailed derivation.

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Appendix

A Sign of $\alpha(1+\theta)(\varepsilon-1)-\varepsilon(1-t_\pi)\beta$

In this section, it is confirmed that the expression $\alpha(1+\theta)(\varepsilon-1)-\varepsilon(1-t_\pi)\beta$ is positive. This expression can be rewritten as follows.

$$\begin{aligned}
&\alpha(1+\theta)(\varepsilon-1)-\varepsilon(1-t_\pi)\beta\\
&\qquad=\alpha(1+\theta)(\varepsilon-1)-(1-t_\pi)\beta+(1-t_\pi)\beta-\varepsilon(1-t_\pi)\beta\\
&\qquad=\alpha\theta(\varepsilon-1)-(1-t_\pi)\beta+\alpha(\varepsilon-1)-(\varepsilon-1)(1-t_\pi)\beta\\
&\qquad=\alpha\theta(\varepsilon-1)-(1-t_\pi)\beta+(\varepsilon-1)[\alpha-(1-t_\pi)\beta]
\end{aligned}$$

The sum of the first two terms is positive, due to Assumption 3, and the expression in square brackets of third term is also positive under Assumptions 2 and 3. Therefore, $\alpha(1+\theta)(\varepsilon-1)-\varepsilon(1-t_\pi)\beta$ is positive.

B Proof of Proposition 1

I investigate how capital income tax affects the growth rate of productivity, L_Z^*. The interest rate in the steady state can be written as

$$r^*=\frac{\rho}{1-t_A}.$$

Differentiating this with respect to t_A yields

$$\frac{\partial r^*}{\partial t_A}=\frac{\rho}{(1-t_A)^2}>0.$$

Thus, capital income tax increases the rate of return on R&D. The interest rate affects the consumption expenditure per good, $\frac{E^*}{n^*}$. To investigate this effect, I rearrange equation (27) as follows.

$$\frac{E^*}{n^*}=\frac{\varepsilon}{1-\theta(\varepsilon-1)}\left[\frac{\alpha-(1-t_\pi)\beta}{\alpha(1-t_\pi)\beta}r^*+\phi\right]. \tag{29}$$

Differentiating this with respect to r^* yields

$$\frac{d\,(E^*/n^*)}{dr^*}=\frac{\varepsilon}{1-\theta(\varepsilon-1)}\left[\frac{\alpha-(1-t_\pi)\beta}{\alpha(1-t_\pi)\beta}\right]>0.$$

This implies that the consumption expenditure per good is increasing along with the interest rate. Differentiating equation (26) with respect to $\frac{E^*}{n^*}$, one can easily confirm that the growth rate of productivity, L_Z^*, is an increasing function of the consumption expenditure per good.

$$\frac{dL_Z^*}{d\left(E^*/n^*\right)} = \frac{\alpha\theta(\varepsilon-1)-(1-t_\pi)\beta}{\alpha-(1-t_\pi)\beta}\frac{1}{\varepsilon} > 0.$$

Therefore, the capital income tax has a positive effect on the growth rate of productivity, $(dL_Z^*/dt_A) > 0$.

C Effects of Corporate Income, Labor Income, and Consumption Taxes

Firstly, I investigate how corporate income tax affects the growth rate of productivity, L_Z^*. There are two effects of the tax on L_Z^*: a direct effect and an indirect effect. The indirect effect is through a change in the consumption expenditure per good. Since this tax has no effect on the interest rate, I differentiate (29) with respect to t_π, which yields

$$\frac{d\left(E^*/n^*\right)}{dt_\pi} = \frac{\varepsilon}{1-\theta(\varepsilon-1)}\frac{\alpha^2\beta}{\{\alpha(1-t_\pi)\beta\}^2}r^* > 0.$$

Thus, corporate income tax increases the consumption expenditure per good. Considering the indirect effect, I differentiate (26) with respect to t_π.

$$\frac{\partial L_Z^*}{\partial t_\pi} = \frac{\alpha\beta}{\{\alpha-(1-t_\pi)\beta\}^2}\left[\{1-\theta(\varepsilon-1)\}\frac{E^*}{\varepsilon n^*}-\phi\right] + \frac{\alpha\theta(\varepsilon-1)-(1-t_\pi)\beta}{\{\alpha-(1-t_\pi)\beta\}\varepsilon}\frac{\partial\left(E^*/n^*\right)}{\partial t_\pi} > 0.$$

Therefore, the corporate income tax has a positive effect on the growth rate of productivity.

Secondly, it is clear that labor income tax and consumption tax have no effect on the growth rate of productivity, L_Z^*, because equations (26)–(28) are independent of these tax parameters.

D Derivation of the Growth Rate of an Individual's Utility

In the symmetric case, the equilibrium consumption index can be written as follows.

$$C_t = \left[\int_0^{N_t}(c_t)^{(\varepsilon-1)/\varepsilon}di\right]^{\varepsilon/(\varepsilon-1)} = \left[N_t(c_t)^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} = N_t^{\varepsilon/(\varepsilon-1)}c_t,$$

where $c_t = c_{it} = c_{jt}$, for all $j \neq i$. Using the aggregate consumption of each differentiated good, obtained in equation (7), and the production function, (8), one can rearrange this expression:

$$C_t = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} X_t = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} Z_t^{\theta} (L_{X_t} - \phi).$$

In the steady state, the labor employment in the production sector is constant and, thus,

$$C_t = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} Z_t^{\theta} (L_X^* - \phi). \tag{30}$$

Substituting (18) into this, I obtain

$$C_t = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} Z_t^{\theta} \left(\frac{\varepsilon - 1}{\varepsilon} \frac{E^*}{n^*} + \phi - \phi \right) = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} Z_t^{\theta} \left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{E^*}{n^*}.$$

Differentiating this with respect to t yields

$$\dot{C}_t = \frac{N_t^{\varepsilon/(\varepsilon-1)}}{L_t} Z_t^{\theta} \left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{E^*}{n^*} \left[\frac{\varepsilon}{\varepsilon - 1} \frac{\dot{N}_t}{N_t} - \frac{\dot{L}_t}{L_t} + \theta \frac{\dot{Z}_t}{Z_t} \right]. \tag{31}$$

The growth rate of the consumption index is, therefore,

$$\frac{\dot{C}_t}{C_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\dot{N}_t}{N_t} - \frac{\dot{L}_t}{L_t} + \theta \frac{\dot{Z}_t}{Z_t}.$$

Now, I consider the relationship between the growth rates of N_t and L_t. The definition of the number of firms per capita is $n_t \equiv \frac{N_t}{L_t}$. Differentiating this with respect to t yields

$$\frac{\dot{n}_t}{n_t} = \frac{\dot{N}_t}{N_t} - \frac{\dot{L}_t}{L_t}.$$

In the steady state, $\dot{n}_t$ is zero, which means that the growth rate of the number of goods is equal to that of the population, $\frac{\dot{N}_t}{N_t} = \frac{\dot{L}_t}{L_t} = \lambda$. Hence,

$$\frac{\dot{C}_t}{C_t} = \theta \frac{\dot{Z}_t}{Z_t} + \frac{1}{\varepsilon - 1} \lambda, \tag{32}$$

where the growth rate of productivity is

$$\frac{\dot{Z}_t}{Z_t} = \alpha L_Z^* = \alpha \left[\frac{\alpha\theta(\varepsilon - 1) - (1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta} \frac{E^*}{\varepsilon n^*} + \frac{(1 - t_\pi)\beta}{\alpha - (1 - t_\pi)\beta} \phi \right]. \tag{33}$$

I derive the growth rate of an individual's utility. The instantaneous utility function is defined as

$$\log u_t = \log C_t + \gamma \log(1 - l_t) + \mu \log G_t, \qquad \gamma, \mu > 0.$$

Firstly, C_t is the consumption index; its long-run growth rate is calculated in equation (32). Secondly, the fraction of time allocated to labor supply is represented as l_t; its optimal value is obtained in equation (6). The time-dependent variable for this is only the consumption expenditure, E_t. This variable converges to E^* in the long-run, which implies that l_t is constant in the long-run,

$$l^* = 1 - \frac{1 + t_E}{1 - t_L} \gamma E^*.$$

Thus, the growth rate of the fraction of time allocated to labor supply is zero. Thirdly, G_t represents public goods supplied by the government. The production function is $G_t = L_{G_t}$. The steady state value of L_{G_t} is

$$L_{G_t} = g \left\{ t_L L_t + [t_\pi + t_A(1 - t_\pi)] \left(\frac{E^*}{\varepsilon n^*} - \phi - L_Z^* \right) n^* L_t + t_E E^* L_t \right\}.$$

The growth rate of public goods is, therefore,

$$\begin{aligned} \frac{\dot{G}_t}{G_t} = \frac{\dot{L}_{G_t}}{L_{G_t}} &= \frac{g \dot{L}_t \left\{ t_L + [t_\pi + t_A(1 - t_\pi)] \left(\frac{E^*}{\varepsilon n^*} - \phi - L_Z^* \right) n^* + t_E E^* \right\}}{g L_t \left\{ t_L + [t_\pi + t_A(1 - t_\pi)] \left(\frac{E^*}{\varepsilon n^*} - \phi - L_Z^* \right) n^* + t_E E^* \right\}} \\ &= \lambda. \end{aligned}$$

Combining these results yields the growth rate of an individual's utility.

$$\frac{\dot{u}_t}{u_t} = \frac{\dot{C}_t}{C_t} + \mu \frac{\dot{G}_t}{G_t} = \theta \frac{\dot{Z}_t}{Z_t} + \left(\frac{1}{\varepsilon - 1} + \mu \right) \lambda.$$

This is not affected by the population scale but is endogenously determined by parameters such as preference and fiscal variables.