An Empirical Theory of Pure Exchange: Individual Demand and Equilibrium

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SUMMARY: Scientists question the 'scientificity' of Neoclassical Economic Theory because microeconomics depends on an un-observable utility function, while the modern version of the theory requires that macroeconomics be built on microfoundations. The first step in remedying such an incongruous analytics is to use 'naïve' set theory to show that the utility function is indeed a misleading appendage.

KEYWORDS: Properties of Relations, Order Isomorphisms, Individual Demand, Equilibrium, Attractors' Reconstruction.

JEL Classification: B21, D11, D50.

1-INTRODUCTION

The concept of utility is a corner stone in both the classical and modern versions of the Neoclassical Theory of choice. As Ingrao and Israel (1990) have observed, " the concept is tied to the fact that it is the first implementation of the intention of the early pioneers to translate into quantitative form the other concept of the greater or lesser satisfaction', also known as utility.

Except that the quantification of the concept of utility has not ceased to receive criticisms ever since. And attempts to deflect those criticisms have led modern economists to add gradually a number of restrictions on consumers' preference sets; but, each additional restriction on preference sets seem to further complicate matters. Having finally recognized the difficulty of quantifying either utility or preference, modern economists switched to what appeared as a more appropriate notion; that is; 'ordinal ranking'. But, at the same time, they remained bent on demonstrating that each agent maximizes his or her satisfaction with regard to his or her needs. Hence, attempts to deflect criticisms are translated into a 'massage' of preference sets so as to generate a continuous utility functions with a convex hypograph without much regard to what makes sense in terms of Set Theory. In the end, this has created a utility-demand conundrum which, this author at least, believes should be addressed.

To do so, we argue in Part II that the utility-demand imbroglio arises from: a) the unreasonableness of the restrictions imposed on preference sets; b) from a misuse of order isomorphisms, and; c) from performing mathematical operations where they are undefined. We next outline the main properties of the *relations of order and equivalence,* and show how they allow the derivation of the demand curve without imposing restrictions. We also recall that mathematical physics has shown that the attractor of a non-linear dynamical (dissipative) system, no matter how complex, can be reconstructed from just one output of the said system. This is motivated by the understanding that behavior is best studied around the attractor (Mané1980; Takens, 1981; Liu, 2009). Even though a pure exchange economy is a linear-time-invariant system, the same technique may be applied to reconstruct the whole attractor; this is done in Part III. As a result, the theory of pure exchange is easily developed in terms of 'naïve' set theory based on 'observables' rather than on preconceived concepts, and obviates many of the difficulties thrown out by the modern version.

2- THE PROBLEMS OF THE MODERN VERSION OF MICROECONOMICS

When it was first presented in the 1870s, the attempt to quantify utility was immediately criticized by economic historians such Emile Levasseur, Louis Wolowski, etc. and by scientists members of the Académie des Sciences Morales et Politiques de l' Institut de France, according to Ingra and Israel. Walras himself had claimed that he did not want simply to quantify the intensity of utility but rather to put the theory of utility on a scientific footing. It is rather ironic therefore that, at first, his ideas would have received such a cold reception in a scientific forum when the same ideas were to become the cornerstone of the theory of exchange under the influence of his successors. Criticisms did not go away, however, for they are now viewed as grave violations of mathematical rules and operations (Barzilai, 2010, 2011, 2013).

To grasp the essence of the more modern objections to the quantification of an 'unobservable' utility function, it is worthwhile to begin with a brief review of the modern presentation. Modern economists rightfully argue that there exists a universal consumption set C comprising trillions of items, while $X \subseteq C$ represents baskets of goods whose elements are $x_1, x_2, x_3, \ldots, x_n$ over which the consumer expresses his or her preference. Accordingly, to arrive at a utility function that is continuous and with a convex hypograph, consumers' preference must be ordered by a relation R endowed with a number of properties. That is, R must be *complete*, *reflexive*, *continuous*, and *transitive*; and for greater generality, R must also be *monotone* and *convex*.

2.1- More on the properties of R

In this case, the relation R is a binary relation on X or a subset of the Cartesian product of two sets X and U, where U stands for utility. Here X is an ordinal space (since modern economists have rejected cardinal utility), and $U \in \Re$, where $\Re$ is the set of reals. In other words,

$$R \subseteq X \text{ x } U \in \Re.$$

It must immediately be reemphasized that X is an ordinal space or a set of baskets of consumption goods over which the relations of order and equivalence are *defined,* but over which mathematical operations are not defined. We will return to the Cartesian product in a moment. Let us first examine the demand of completeness. What does it mean then to say that R is complete? It means that,

$$\forall x_1, x_2 \in X: \text{either } x_1 \preccurlyeq x_2 \vee x_2 \preccurlyeq x_1.$$

Economists claim that the consumer thinks that x_2, say, is at least as good as x_1. An order relation is total if every (x_1, and x_2) $\in$ X, ordered by R, are comparable. That is, a relation $\preccurlyeq$ on X is:

a) $\preccurlyeq$ is a partial order, and;
b) any two elements in X are comparable;

if the relation satisfies both a) and b), then $(X, \preccurlyeq)$ is a chain as it is for every subset of $(\Re, \leq)$. A linear order has a higher quality than a partial ordering because every two elements are comparable; for example, the pair $(X, \prec)$ is well-ordered if every subset $V \subseteq X$ has a smallest element with respect to $\prec$. Thus, a linear order is irreflexive $(x_1 \nprec x_1)$, antisymmetric (because $(x_1 \prec x_2) \wedge (x_2 \prec x_1)$ cannot both be true), and transitive. Now imagine a universal set C that contains trillions of elements, then X must have elements in the hundreds. To demand completeness (in the sense of the modern version) from consumers with asymmetric market information sets is too a strong or even an impossible demand.

Next, consider the property of convexity. By definition, X is convex if and only if every linear combination of its elements is in X. Sometime, it is expressed as follows: $\forall(x_1, x_2, x_3) \in X$, and $x_1 \neq x_2$: if $x_1 \succcurlyeq x_3 \preccurlyeq x_2$, then $x_3 < [\lambda x_1 + (1 - \lambda) x_2]$, where $\lambda \varepsilon (0,1)$. Put more simply: If $x_1 \gtrsim x_3$, then $x_3 \lesssim [\lambda x_1 + (1 - \lambda) x_3$. This is to say that the consumer will always prefers more to less; something that can be considered as a repetition of monotonicity? Anyway, it is supposed to guarantee well-behaved demand and smooth indifference curves. Now applying such an average to indifference utility contours gives a misleading answer. Two distant points A and B on an indifference contour give the same level of utility. Then, for any $\lambda \in (0, 1)$, the linear combination $(\lambda A + (1 - \lambda) B)$ put the consumer on a higher contour, according to economists. However, a contour is a level curve or a segment of the perimeter of a surface. All points on the linear combination of A and B lie on a surface orthogonal to the contour. Hence, the linear combination would leave the consumer at the same utility level, even if economists had the power to rotate the surface 90^0.

The same remark applies to the concept of continuity. It means, for example, $x_1, x_2, x_3 \varepsilon X$. If $x_2 \gtrsim x_1 \wedge x_3 \gtrsim x_1$, then $x_1 \lesssim [\lambda x_2 + (1 - \lambda) x_3]$. This requirement aims at a continuous utility function, saying again that consumers prefer averages without specifying how one goes about measuring fractions of units. It is also a trivium that suffers the same defect as the one above, because it involves mathematical operations in ordinal space.

Further, economists argue that if any two sets ordered by R satisfying these requirements, then a mapping: $f: X \rightarrow U \in \Re$, or simply $f: X \rightarrow \Re$, is a bijection. Then: $f \subseteq (X \times U)$ such that $\forall x \in X$, $\exists$ exactly one $(x, u) \in f$, where X is the domain of f and $U \in \Re$ is the codomain, implying $|X| = |U|$. If f exists, then one may conclude from the 'onto' map that $f(x) \leq f(u)$ if and only if $x \leq u$, etc.

To understand the objection of mathematicians to this setup, we put X on the abscissa and U on the ordinate. Then the slope of the one-to-one mapping f, $\Delta\Re/\Delta X$, is not defined because that would entail performing a mathematical operation on an ordinal space; hence, R cannot be defined for the lack of definition of the Cartesian product as already indicated above.

More formally, a) economists began by rejecting cardinal preferences and utility and in the end they just 'cardinalize' the ordinal space and call it a set of utils; then f is nothing but $id_{(x)}: X \rightarrow X$, or $id_{(x)}$ is an equivalence mapping; and b) one cannot tacitly use utility to define preference and then use preference to define utility; this would be in contradiction to a consequence of Gödel's (1931) Incompleteness Theorem, which asserts that a mapping can only quantify something other than itself. In other words, for f to satisfy the technical requirements of the Second Incompleteness Theorem, it cannot prove the consistency of any system U that in turn proves the consistency of X. If X exists, it must prove the existence of another set of reals, $\mathcal{X}$, ordered antisymmetrically, to which X is isomorphic, and in which mathematical operations are defined. Moreover, what would it mean to map X into a set that has nothing to do with the problem; imagine mapping X into a temperature scale, then what would be the meaning of the temperature of the consumer? The next section will be devoted to an effort to eliminate these sorts of incongruities.

2.2- More on Relations of Order and Equivalence

In naïve set theory, terminology and symbols may vary from an author to another, but concepts and properties do not give rise to controversies. For tractability, therefore, we first define the main terms used in this study. And since properties of relations of order and equivalence are standard, we state them without proof:

Ordinal Space: An ordinal space is a set of distinct items such as X, on which the relations of order and equivalence are defined, but mathematical operations such as addition, subtraction, multiplication, differentiation, etc. are not defined.

Diagonal of X: the diagonal of X is denoted: $\Delta_x = \{x_1, x_2)|x \in X\}$.

The inverse of R is denoted: $R^{-1} = \{(x_1, x_2)|(x_2, x_1) \in R$.

Total Preorder: A total preorder on X, denoted (X, $\preccurlyeq$), is a preorder such that if $(x_1, x_2) \in X$, then:

$$(x_1 \preccurlyeq x_2) \vee (x_2 \prec x_1) \vee x_1 \sim x_2;$$
$$\text{if } (x_1 \prec x_2) \Leftrightarrow (x_1 \preccurlyeq x_2) \wedge \neg (x_2 \preccurlyeq x_1);$$
$$\text{if } (x_1 \sim x_2) \Leftrightarrow (x_1 \preccurlyeq x_2) \vee (x_2 \preccurlyeq x_1);$$
$$\text{if } (x_1 \preccurlyeq x_2) \wedge (x_2 \preccurlyeq x_1) \Leftrightarrow x_1 = x_2.$$

As already indicated, it is too demanding to require that X be linearly ordered. It is also a little confusing when the modern version demands completeness, reflexivity, transitivity, etc. because completeness in this sense includes symmetry, antisymmetry, reflexivity, and transitivity. At any rate, it is not good science to impose restrictions on preferences so as to attain a desired result. The scientist is supposed to restrict himself or herself to observables. If some of the properties demanded by the modern version cannot be observed or demonstrated in a formal system of logic, it is more reasonable to assume that consumers with asymmetric market information sets would order X only partially. If X is partially ordered, the *order relation* satisfies:

i) $\Delta_x \subseteq R$;

ii) $R \cap R^{-1} \subseteq \Delta_x$

iii) $R \circ R \subseteq R$.

If X is quasi-ordered, then i) is replaced by $\Delta_x \cap R = \varnothing$. Otherwise, properties i)- iii) define a partially ordered set or a *poset*[1]. The *equivalence relation* satisfies:

iv) $\Delta_x \subseteq R$;

v) $R = R^{-1}$

vi) $R \circ R \subseteq R$

That is, the order relation is reflexive, antisymmetric, and transitive, and the equivalence relation is reflexive, symmetric and transitive.

Function: A function f (or mapping) is a link between the elements of at least two posets, X and $\mathcal{X}$. The function is also a subset of the Cartesian product of X and $\mathcal{X}$. f is a well behaved transformation describing how one element of X is mapped into an element of $\mathcal{X}$. Thus for a relation to be a function, it must satisfy two conditions:

a) Every element of X has an association with an element of $\mathcal{X}$ and;

b) each element of X can only be associated to one and only one element of $\mathcal{X}$.

Thus, a mapping from X to $\mathcal{X}$ is said to be a poset isomorphism if f: $X \to \mathcal{X}$ is a bijection and if both f and f^{-1}: $\mathcal{X} \to X$ are monotone; that is, f is an injective function and f^{-1} is a surjective function. If $\mathcal{X}$ is a set of real numbers that is isomorphic to X, then various pairs satisfy the properties shown in Table 1 below.

Table 1: Pairs of Relations on $\mathcal{X} \varepsilon \mathfrak{R}$.

Properties of Pairs	$(\mathcal{X}, \leq)$	$(\mathcal{X}, <)$	$(\mathcal{X}, =)$
$\Delta_x \subseteq R$	yes		yes
$\Delta_x \cap R = \varnothing$		yes	
$R = R^{-1}$			yes
$R \cap R^{-1} \subseteq \Delta_x$	yes	yes	yes
$R \circ R \subseteq R$	yes	yes	yes

2.3 Individual Demand from the real Set $\mathcal{X}$.

The next step is to find a set of reals such as $\mathcal{X}$ thrown- off by the system of exchange that is isomorphic to the ordinal space X. The most obvious one that can be found is the set of agents' budget shares $\boldsymbol{\alpha} \varepsilon (0, 1)$. We will return to that set below. In the meantime, let us first consider the most likely form of a pure exchange economy. That is,

(1) $$\dot{\mathbf{p}} = \mathbf{M}\,\mathbf{p};$$

[1] On a poset $(\preccurlyeq)$ is the inverse of $(\succsim)$; $(\prec)$ is the inverse of $(\succ)$. Further, if $(x_1 \preccurlyeq x_2) \vee x_1 \prec x_2$, then $x_2 \succsim x_1$, $x_2 \succ x_1$.

if price **p** is the variable of concern, then (1) describes an invariant time linear system, where **M** is a real square positive linear matrix converging to an observable sink. But this remains to be demonstrated.

The best approach to a greater understanding of the dynamic process of exchange would be to begin with the nature of the equilibrium point and work backward to discover the rich patterns that **M** may contain, in particular the nature individual and community demands. This could be done on the assumption that the state of equilibrium is consistent with an equilibrium state for each agent; that is, at a market clearing position for each and every participant. Put differently, in an equilibrium state:

The market valuation of good j ε n to consumer i ε m must be equal to a share α^i_j *of the consumer's budget* B^i.

We now let x^i_j represent good j brought by i; ω^i_j represents the initial endowments j ε n of i ε m, and B^i is i's budget. If initial endowment j is transformed into goods. The equilibrium equation is:

(2)
$$p_j x^i_j = \alpha^i_j B^i = \alpha^i_j [\Sigma^n{}_j p_j \omega^i_j] = \alpha^i_j [\omega^i_j p_j + \Sigma^n{}_{k \neq j} p_k \omega^i_k].$$

Hence, the individual demand equation is:

(3)
$$x^i_j = \alpha^i_j [\omega^i_j + (\Sigma^n{}_{k \neq j} p_k \omega^i_k) / p_j], \; p_j > 0, \forall j \,\varepsilon\, n.$$

Equation (3) asserts that the demand curve of i for good j has the form of a rectangular hyperbola with a lower bound determined by input ω^i_j. This finding only confirms what observations have shown; that is: There exists

An inverse relationship between the price of good j (p_j) *and the quantity* (x^i_j) *traded.*

Nonetheless, one should be careful in interpreting equation (3), for it is only an observable equilibrium point on the demand curve of consumer i. As already stressed, the demand curve can be reconstructed by working backward, but to do so we must first compute the point price elasticity. For ease of exposition, however, we will not stared (*) equilibrium values and we compute the point elasticity of, say good 1:

(4)
$$(\partial x^i_1 p_1 / \partial p_1 x^i_1) = - \alpha^i_1 \textstyle\sum_{j=2} (p_j \omega^i_j)/p_1 x^i_1) = - \alpha^i_1 \{[B^i - (p_1 \omega^i_1) / B^i) B^i]\}/p_1 x^i_1 = - (1 - \alpha^i_1);$$

making use of (2), it is shown that the point elasticity of good 1 for agent i is a function of a share of his budget devoted to good 1. As $p_1 > 0$, $\alpha^i_1 > 0$, and $\Sigma\alpha^i_1 = 1$, then point-price elasticity of good 1 lies between zero and one. *We can conclude that the point price elasticity of any good j is always inelastic.*

To reconstruct the demand curve of i, we first put it in a price-quantity diagram, including the lower bound. To take an example, suppose we have a value of – 0.8 in (4). Then from the equilibrium point we go horizontally left the value of $(\Delta x/x) = 0.8$ and move vertically $(\Delta p/p) = 1$, and reverse the whole procedure on the right of the observed point. Once the demand curve is reconstructed, we may conclude the following:

a) The demand curve of i for good j is inelastic; and since it depends on α^i_j, each agent faces a different demand curve for each good j;

b) the higher is α^i_j, the more inelastic is the demand curve of i;

c) from (4), one may conclude that there is a tendency for prices to increase in the presence of fixed supply; it follows that prices must fall in the presence of increased supply, except of course that of the numéraire;

d) the cross price elasticity of, say, goods 1 and 2, say, is;

$$(\partial x^i_1 p_2) / (\partial p_2 x^i_1) = (1 - \alpha^i_2) > 0;$$

hence, goods 1 and 2 are substitutes, and no good is a complement to any other, as shown in (5) below. However, should the budget share of good 2 approaches unity, then goods 1 and 2 may be considered independent;

e) another interesting case is where consumer i possesses only one endowment, say, ω^i_3 and wishes to buy x^i_1. Clearly, i cannot buy x^i_1 if $p_3 \omega^i_3 < p_1 x^i_1$. On the other hand, if for some reason $p_3 > p_1$, then the price-elasticity of i's demand curve is:

$$(\partial x^i_1 p_1) / \partial p_1 x^i_1 = - \alpha^i_1 p_3 \omega^i_3 (1 - \alpha^i_1) / \alpha^i_1 p_1 x^i_1$$
$$= - p_3 \omega^i_3 (1 - \alpha^i_1) / p_1 x^i_1.$$

Thus, one obtains all the basic results about individual demand without putting restrictions on consumers' preferences. Moreover, the set $\mathcal{X}$ allows all mathematical operations to be performed on observables without any reference to an unobservable set of utils or to an undefined utility function. Therefore,

> *if both X and $\mathcal{X}$ are ordered antisymmetrically, then they are isomorphically related. Hence, f: X → $\mathcal{X}$ (see Table 1). The mapping f from a poset X to $\mathcal{X}$ is* ***monotone*** *and order preserving. In this case f: X →$\Re$, then the pair ($\mathcal{X}$, ≤) is of no interest. In $\Re$, f is strictly increasing for f is one-to-one, then either $x_j > x_k \vee x_j < x_k$. So by monotonicity, $f(\alpha_j) > f(\alpha_k) \vee f(\alpha_j) < f(\alpha_k)$. Hence, the dynamic process of exchange reduces to consumers' search for f, which is observable only at equilibrium.*

3- A GENERAL MARKET EQUILIBRIUM

In this set up, with fixed supply, the demand function and market equilibrium follow immediately from (2). The whole system is developed in Dominique (2008), but many readers have requested a more extended version. For tractability, we keep the same symbols so that interested readers can easily verify our assertions.

Recalling that consumers are indexed i (i ε m), and goods are indexed by j (j ε n). Consumer i enters the market with initial endowments ω^i_j. Instead of Walras' tatônnement, *it is proper to view the process of exchange as consumers'* *effort to map their preferences X (ordered as a poset) into the set of budget shares* $\boldsymbol{\alpha}^i = \{\alpha^i_1, \alpha^i_2, \ldots, \alpha^i_n)$, keeping in mind that ($\mathfrak{X}^i$ ε $\Re$). Initial endowments are transformed into goods, x_j. The general equilibrium exists when each i satisfies (2) above. Thus, $x^i_j = \alpha^i_j\,(p_1\,\omega^i_1 + p_2\,\omega_2 + \ldots + p_n\,\omega^i_n)/p_j$. The total purchase of good j by m agents is $(x^1_j + x^2_j + \ldots + x^m_j)$. Collecting all components of p_j, $\forall j$ ε n and for $\forall i$ ε m, we have the system:

(5) $$\dot{\mathbf{p}} = \mathbf{dg}\,(1/\,p_j)\,[\mathbf{M} - \mathbf{dg}\,(\Sigma^m_j\,\omega^i_j)]\,\mathbf{p}.$$

For tractability, we expand the square matrix **M**:

$$\mathbf{M} = \begin{bmatrix} (\alpha^1_1\,\omega^1_1 + \alpha^2_1\,\omega^2_1 +,\ldots,+ \alpha^m_1\,\omega^m_1) & (\alpha^1_1\,\omega^1_2 +,\ldots,+ \alpha^m_2\,\omega^m_2) & \ldots & (\alpha^1_1\,\omega^1_n +,\ldots,+ \alpha^m_1\,\omega^m_n) \\ (\alpha^1_2\,\omega^1_1 +,\ldots, + \alpha^m_2\,\omega^m_1) \;\ldots & (\alpha^1_2\,\omega^1_2 +, \ldots, + \alpha^m_2\,\omega^m_2) & \ldots & (\alpha^1_2\,\omega^1_n +,\ldots, + \alpha^m_2\,\omega^m_n) \\ . & . & . & . \\ \ldots & \ldots & \ldots & \ldots \\ . & . & . & . \\ . & . & . & . \\ (\alpha^1_n\,\omega^1_1 +,\ldots, + \alpha^m_n\,\omega^m_1) \;\ldots & (\alpha^1_n\,\omega^1_2 +,\ldots, + \alpha^m_n\,\omega^m_2) & \ldots & (\alpha^1_n\,\omega^1_n +,\ldots, + \alpha^m_n\,\omega^m_n) \end{bmatrix}.$$

As is, **M** is not invertible since its rank is (n -1). We can then write down the augmented matrix **M** and next use the Gauss-Jordan elimination to find the reduced row echelon form of the augmented matrix of **M**. One would find a free variable. If the free variable is price and the rank is (n -1), then the values of the (n -1) prices would depend on the value assigned to the free variable, which would be the numéraire. Assuming that the rank of M is (n-1), we next observe that element $m_{11} = \Sigma^m_1\,\alpha^i_1\,\omega^i_1 < \Sigma\,\omega_1$, and so it is for all $m_{22}, \ldots, m_{nn}$. Hence Tr $(\mathbf{M}_{(n-1)\,x\,)n-1)}) < 0$ and the determinant $|\mathbf{M}| > 0$. Put differently, $\mathbf{M}_{(n-1)\,x}\,(n\text{-}1)$ is a Metzler matrix, defined as $m_{ii} < 0$, and $m_{ij} > 0$, $\forall\ i \neq j$. Then starting with any nonnegative initial price, the system will preserve the nonnegativity of the state vector, **p**. Equation (5) describes a system that will converge to the equilibrium while preserving the nonnegativity of prices. Therefore, the initial assumption made in (1) to the effect that a pure exchange economy is a linear time invariant system is justified.

It can also be asserted from (5) that the excess demand vector, $\dot{\mathbf{p}}$, is observable only at **0**. It can then be seen that the difficulties alluded to in Sonnenschein (1972, 1973) do not arise. Moreover, with regard to the concern of Debreu (1970), it can be said that $\dot{\mathbf{p}}$**,** whether positive or negative, approaches the equilibrium tangentially.

Observe also that the individual demand function (2) is homogeneous of degree zero in prices. And $\dot{\mathbf{p}}$ is also homogeneous of degree zero in prices. If we do not follow the above procedure, we could presume that normalization is an alternative, for prices represent a trading ratio between two goods $(x_j\,/\,x_k) = (p_k\,/p_j)$, $\forall j,k$ ε n. All prices can be normalized by setting a $p_j = 1$ to arrive at absolute values for prices. **M** is n-dimensional, but there will be only n -1 prices to determine after normalization. This is resolved by deleting one line and one column of **M** to arrive at the solution of an (n -1) x (n – 1) real matrix.

The normalization throws-of another important conclusion. Entry c) above applies to all goods, including the numéraire. Thus an increase in the quantity of the numéraire brings a fall in the price of the numéraire, which in turn will cause all other prices to increase; this fact is the foundation of the so-called *quantity theory of money.*

The matrix **M** is rich in terms of information. For example, suppose total supply is consumed over the market period, then (5) may represent a sequential market. All i may supply, say, good1.Thus, $p_1 \sum^m x^i_1 = c\ m\ \omega_1 = c\ \Sigma^m \omega^i_1$, where c is the average cost of ω_1. If $\Sigma^m x^i_1 \geq \Sigma^m \omega^i_1$, then we have: $p_1 \geq c$. That is, if the industry were able to produce good1at average cost c and sell at price p_1, it would show a small profit. Therefore, *there is an incentive to reduce the average costs of inputs.* To repeat: If the industry is able to buy an input below its market price, the matrix **M** will undergo a change and will converge to another equilibrium, which of course would not be Pareto optimal.

Finally, we note in passing that economists are often criticized for their attachment to the equilibrium concept. That particular criticism is not warranted since the equilibrium point is the correct place to study behavior. On the other hand, the affirmation that consumers enter the market to maximize some unobservable utility function is nothing but a theoretical sprain. A consumer enters the market not to maximize some unknown utility function but to bring into reality his or her *wants*, which may include needs, pleasure, etc. Anyway: T*he claim that the consumer maximizes his or her utility function cannot be substantiated in the present set-up.*

CONCLUDING REMARKS

As it can be seen, even the simplest of models of exchange is able to reveal a number of fundamental tendencies of the exchange process. They are: 1) Without appealing on the concept of utility, it shows that prices and quantities are inversely related; 2) consumers' demand and community demand are observable at the equilibrium point. The individual demand curve can be reconstructed. If so, it has the form of a rectangular hyperbola whose price elasticity lies between zero and one; 3) there is a tendency for product prices to increase, and for input costs to decrease, and 4) if the process is sequential, its attractor can be reconstructed and studied.

We do not pretend that this simple structure is a fair representation of **actual** and more modern markets. The latter is nonlinear and very complex. However, we believe that the fundamental tendencies are carried over to actual markets, and that attractors' reconstruction is a first step toward a greater understanding of complex nonlinear markets.

REFERENCES

Barzilai, J. (2010), "Preference Function Modelling: The Mathematical Foundation of Decision Theory." In Ehrgott, M., Figueira, J. R. and Greco, S, (eds.) *Trends in Multiple Criteria Analysis*, Springer, 57-86: Boston.

Barzilai, J. (2011), “On Ordinal, Cardinal, and Expected Utility.” Technical Report, Dept. of Industrial Engineering, Dalhousie University, Halifax, N. S. Canada, 4-7.

Barzilai, J, (2013). “Inapplicable Operations on Ordinal, Cardinal, and Expected Utility.” *Real-World Economics Review*, 63, 98-102.

Debreu, G. (1970). “Economies with a Finite Set of Equilibria.” *Econometrica*, 38 (3), 387-392.

Dominique, C-R. (2008), “Walrasian Solutions Without Utility Functions.” Economics and Econometrics Research Institute, EERI Research Papers, Brussels, October.

Gödel, K. (1931). “Uber formal unentscheid bare Satze der Principia Mathematica und verwandter Systeme I.” Monatshefte fur *Mathematik und Physik*, v, 38, 173-198.

Ingrao, B. and Israel, G. (1990). *The Invisible Hand*. MIT Press, 154-161: Cambridge.

Liu, Z. (2009). “Chaotic Time Series Analysis.” *Mathematical Problems in Engineering*, vol. 2010, Art.ID 720190.

Mané, R. (1980). “On the Dimension of the Compact Invariant Sets of Certain Non-Linear Maps.” In Rand, D. A. and Young, L. S. (eds) *Volume 898 of Springer Lecture Notes in Mathematics*, Berlin: Springer Verlag, 320.

Sonnenschein, H. (1972), “Market Excess Demand Functions.” *Econometrica*, 40 (3), 549-563.

Sonnenschein, H. (1973). “Do Walras’ Law and Continuity Characterize the Class of Community Excess Demand Functions.” *Journal of Economics Theory*, 6, 345-354.

Takens, F. (1981). “Detecting Strange Attractors in Turbulence.” In Rand, D. A. anYoung, L. S. (eds) *Dynamical Systems and Turbulence*. New York: Springer Verlag, d 366-381.