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Abstract. *Discounted Cash Flow techniques are the generally accepted methods for valuing firms. Such methods do not provide explicit acknowledgment of the value determinants and overlook their interrelations. This paper proposes a different method of firm valuation based on fuzzy logic and expert systems. It does represent a conceptual transposition of Discounted Cash Flow techniques but, unlike the latter, it takes explicit account of quantitative and qualitative variables and their mutual integration. Financial, strategic and business aspects are considered by focusing on twenty-nine value drivers that are combined together via "if-then" rules. The output of the system is a real number in the interval [0,1], which represents the value-creation power of the firm. To corroborate the model a sensitivity analysis is conducted. The system may be used for rating and ranking firms as well as for assessing the impact of managers' decisions on value creation and as a tool of corporate governance.*

Keywords. Firms' evaluation, fuzzy logic, expert system, rating, acquisition, sensitivity analysis.

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Introduction

Firm valuation is of preeminent importance in both corporate finance and business economics. Corporate finance literature provides us with numerous methods of company evaluation. There is an almost unanimous consensus that the cash flow discounting (DCF) methods represent the theoretically sound methods to be used for appraisal aims. Yet, such methods fail to take explicit account of important variables concerning strategy, quality of products and services, managers' abilities and skills, commitment, market knowledge, synergies and joint costs, etc. Also, an analysis of the segment where the firm operates is disregarded, but interrelations among suppliers, customers, distributors, entry and exit barriers, intensity of rivalry are fundamental for evaluating firms. Discounted cash flow methods essentially neglect business and strategic factors which turn to be vitally important in determining a firm's value. As Damodaran (2001) puts it,

> The emphasis on quantitative elements – return on capital and reinvestment rates for profitable firms, and margins, revenue growth and sales to capital ratios for unprofitable firms – may strike some as skewed. After all, growth is determined by a number of subjective factors: the quality of management, the strengths of a firm's marketing, its capacity to form partnership with other firms, the management's strategic vision, among many others. Where, you might ask, is the room in the growth equations that have been presented in this chapter for these factors? The answer is that qualitative factors matter and they all ultimately have to show up in one or more of the quantitative inputs. (p. 178)

In a nutshell, DCF techniques just rely on two numbers: cash flow and cost of capital and a ratio is taken to compute the firm value. In such a way, valuation is a *black box*, one never knows how those numbers are created and justification is not given as to how those numbers derive from fundamental determinants and how they are integrated. One of the most important consequence of this lack of transparency is data's manipulability by financial advisors of companies (e.g. in an IPO).

On the other side, business economics and strategic management literature are more concerned with qualitative analysis. Strategic management literature suggests that the firm can derive a superior capability to create value from both the structure of the industry where the company operates (or intends to invest) and from its internal resources and core competencies. According to the Structure-Conduct-Performance paradigm the sources for creating a sustainable competitive advantage have to be found in the industry structure, which determines the intensity of rivalry inside the competitive arena. These studies focus on how the structure of the sector influences firms' strategic behavior and performance, starting from the idea that the supply and demand characteristics determine the nature of the competition. The Resource-based theory changes the focus of the analysis, postulating that firms can derive a superior capability to create value principally from its ability to develop and exploit superior competencies and skills: the endowment of resources and capabilities are the primary sources of the firm's profitability. Contributions in strategic management are aimed at supplying more sophisticated analyses than the DCF technique, in order to achieve higher reliability in firm valuation. Yet, strategic management literature abstains from providing formal models. Real life is considered to be too complex to be described in a simplistic formal model and the idea that reality is not always "quantitative" should imply that nonquantitative schemas might be more reliable for decision making. In other terms, the "intuition" of the entrepreneur cannot be replaced by aseptic analytic methods (Isenberg, 1984).[1]

This paper aims at providing a model for firm valuation while taking into account contributions of

[1] This approach traces back to Simon (1955, 1956) who claims that individuals often act in contexts of incomplete (and unclear) information and are not able to precisely formulate all possible solutions of a decision-making process and the corresponding consequences. They have a bounded rationality so that maximizing tools are unsuited for solving investment evaluation's problems. In these cases, the use of heuristics may be of some help to the decision maker (Gigerenzer, Todd and the ABC Research Group, 1999).

corporate finance as well as strategic management studies. In particular, we construct a formal model which takes account of the experience of the decision makers and combines logic and intuition to assess a firm's ability of creating value. The approach followed results in a method of rating and ranking firms. The model presented may also be used to inform about the impact of a particular management's decision on value creation or to compensate managers on the basis of their performance.

The approach followed makes use of expert systems and fuzzy logic. An expert system is a tool meant for replicating the way of reasoning of one or more experts. Fuzzy logic is a cognitive framework that adequately replicates the natural way human beings cognize the world and think about problems and situations and enables us to formalize qualitative and vague concepts. We think that the integration of expert systems and fuzzy logic for company evaluation and, in general, for decision-making purposes represents a reliable methodology that could be appealing for managers, practitioners, analysts: our model here proposed does not rest on simplistic assumptions (as often financial models do for mathematical tractability), it does not excessively simplify description of reality, it does not engage in complicated formalization and does not require advanced knowledge of mathematics, it is intuitive and comprehensible by any evaluator, it is extremely flexible (it can be changed by the evaluator), it is able to handle both quantitative and qualitative variables, it is not restricted to a small number of variables (twenty-nine inputs are considered, but many more can be added). At the same time it does not renounce to formalization and gives a proper numerical value for the firm at hand.

As the reader will see, the evaluation derives from logical implications ("if-then" rules). Implications are our natural cognitive tools so anyone can understand them and construct them. At the same time we have a methodology which is actually a formal model and automatically gives us a numerical value. Fuzzy logic is in this sense a superb tool since the complexity of real-life situations is handled through "vague" variables and "vague" interactions, which better replicate human mind in describing phenomena. The mental processes of human beings are actually imperfect and imprecise, since individuals often act in contexts of incomplete (and unclear) information. A fuzzy expert system seems to adequately replicate such imprecision and imperfectness, and the sensitivity analysis we have conducted seem to corroborate the model.

Our approach is just a first attempt to develop a new methodology for appraising firms and business units. We think that this path is fruitful for dealing with complex situations where a great number of value drivers must be taken into account, both qualitative and quantitative, and/or where explicit account of their interrelations must be taken for a better description of the evaluation process.

The paper is organized as follows. The first section briefly describes fuzzy expert systems. The second section present a fuzzy expert system for rating firms and the rules governing the relations among variables. The third section is devoted to the corroboration of the model via sensitivity analysis. The fourth section discusses the methodological features of the model. Some remarks conclude the paper.

1. Fuzzy Logic and expert systems

The way we cognize the world is vague and multivalued. The more we aim at realizing coherence with the world, the more we encounter fuzziness. Think of the sentence "this man is old": is it true? Aristotelian logic would answer yes or no, true or false, but this is a simplification and a rude truth-functional evaluation. Fuzzy mathematics rests on the realistic assumption that all things belong to a set at a certain degree, so that a man always belongs to the set of old men at a certain degree, as well as to the set of young men at a certain degree. The set of old men is then a fuzzy set with no sharp boundary, as well as the set of large companies, small cars, beautiful flowers, interesting pictures etc. Equivalently, in a business context, a sentence such as "the quality of this firm's products is high" is always true at a certain degree (possibly a zero degree) as well as the sentence "the quality of this firm's products is low" is always true at a certain degree (possibly zero).

In this paper we will deal with both certain and uncertain events. Our model makes use of sentences of both types. Sentences such as "Resources and skills are high" or "Suppliers' concentration is low" has to do with certainty and are intrinsically fuzzy ("height" of a firm's resources and skills or "lowness" of suppliers' concentration in the sector are matters of degree); sentences such as "The financial flexibility needed in the future will be high" or "Processes efficiency of the target firm will be very good" has to do with uncertainty (future events) and are intrinsically fuzzy (how high will be the need for financial flexibility? At what degree will the processes efficiency be very good?).

Fuzzy logic enables us to formalize linguistic attributes such as 'low', 'high', 'good', 'excellent', 'positive', 'interesting', 'fruitful', 'adequate' and so on. For a single variable, more attributes may be used and graphically represented in the same graph. As an example, we describe the coverage ratio[2] by using six

[2] Coverage ratio is to be considered as a random variable in a forward-looking perspective (obviously, to guess an expected value of it one may usefully ground on the past periods, if one regards the future as partially determined by the past).

linguistic attributes and the corresponding degrees: A coverage ratio is then at one time VeryLow, Low, MediumLow, MediumHigh, High, VeryHigh. Graphically, we may represent these attributes through fuzzy numbers in the following way:

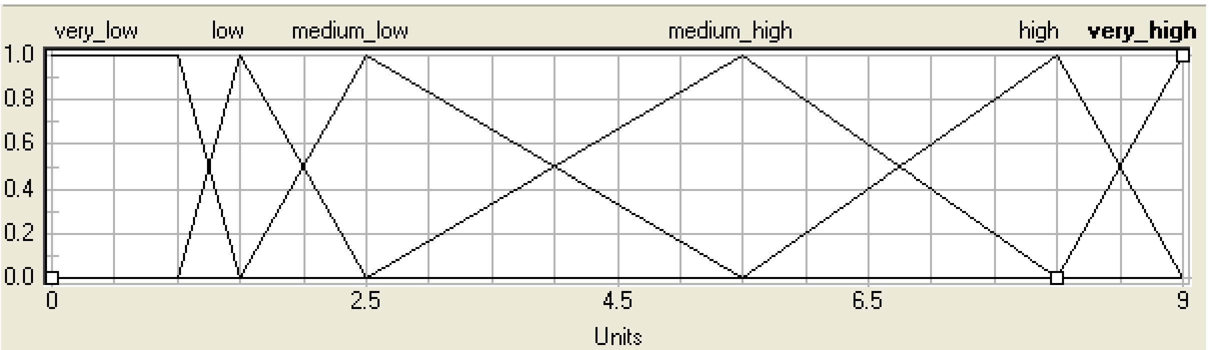

Figure 1. *Coverage ratio.*

The *x*-axis collects all possible numerical values for the coverage ratio, whose unit of measure is given by EBIT/Interest. The *y*-axis collects the degrees at which a linguistic attribute is activated. The VeryLow attribute is represented by a trapezium (its basis ranges from 0 to 1.5) and the others are depicted as triangles (their bases range, respectively, from 1 to 2.5, from 1.5 to 5.5, from 2.5 to 8, from 8 to 9). For example, a coverage ratio of 1.1 is VeryLow at a degree of 80%, Low at a degree of 20%, MediumLow at a zero degree, MediumHigh at a zero degree, High at a zero degree, VeryHigh at a zero degree. A coverage ratio of 6.5 is VeryLow at a zero degree, Low at a zero degree, MediumLow at a zero degree, MediumHigh at a degree of 60%, High at a degree of 40%, VeryHigh at a zero degree. [3]

It is worthwhile noting that the use of linguistic attributes and vague predicates to provide information and to help decision makers take decisions is a commonplace in both business economics and corporate finance. As for the latter, an enlightening example is the "riskiness" of a bond. Rating agencies rate bonds by using letters such as AAA, B, CC etc. This kind of scoring is just a symbolization of linguistic attributes such as "very low risk", "low risk", "medium-low risk", "medium-high", "high risk" etc or vague predicates such as "very high capacity of repaying the debt", "high capacity of repaying the debt", "adequate capacity", "low capacity (speculative debt)" etc.[4] It is also quite common to associate (in an informal way) real numbers to linguistic attributes and vice versa. For example, a coverage ratio is often deemed as low or high according to its numerical value, an association is stipulated with the bond rating (Damodaran, 1999, p. 278) and hence with the probability of default. As for business economics, the literature is well grounded on qualitative analysis using vague predicates describing situations, variables, value drivers etc. (e.g., see qualitative analyses of economic sectors and firms' strategies in Porter, 1980, 1985).

An expert system is a software addressed to achievements usually performed by a human expert. It consists of a knowledge base and an inferential engine. If a question is asked, the system will try to infer the answer from the knowledge base, using logic and the heuristics of the inferential engine. The knowledge must be represented in symbolic forms so as to be stocked and used by a computer. The most common method to this end is to use rule blocks. For example, a simple rule based on conditional ("if-then") implications is the following:

IF market forecasts are favorable
AND the quality of the products is very high
AND the intensity of rivalry is low,
THEN prospective profits are high.

Fuzzy expert systems use fuzzy data, fuzzy rules, and fuzzy inference, in addition to the standard ones implemented in the ordinary expert systems. Referring to the above mentioned implication it is necessary to

[3] As for any value greater than 9, the system considers it VeryHigh at a degree of 100% and the other linguistic attributes are activated at a zero degree. The fact that some attributes are activated at a zero degree is due to the specific shape we have chosen to represent this variable. Nothing prevents us to shape the variable with a different graphical representation.
[4] The number of attributes varies from agency to agency (e.g. Standard & Poor's has 8 attributes, Moody's has 9 attributes).

specify at what degree the market forecasts are favorable, at what degree the quality of the products is very high and at what degree the intensity of rivalry is low. If the system receives the information provided by the antecedent (along with the corresponding degrees), the system will infer, using its inferential engine, the sentence "prospective profits are high" (along with the corresponding degree).

The main steps of a fuzzy system design are: (1) Selection of the type of fuzzy system. We have chosen a modular system consisting of several fuzzy modules linked together, (2) definition of input and output variables, definition of linguistic attributes (fuzzy values) for each variable along with the corresponding membership function (fuzzification of input and output), (3) definition of the set of fuzzy rules ("if-then" rules), (4) choice of the fuzzy inference method (selection of aggregation operators for antecedent and consequent), (5) translation of the fuzzy output in a "crisp" value (defuzzification), (6) test of the fuzzy system: change of membership functions and fuzzy rules if necessary, tuning of the fuzzy system, validation of results via sensitivity analysis. In building fuzzy expert systems, the fuzzification of the inputs and the construction of blocks of fuzzy rules are crucial. A usual method is to rest on information obtained through interviews to the experts of the problem or on generally agreed upon knowledge. To give a simple formalization, let x_i be the i-th input variable, i=1,2,…, n, and let $k_i \in N$ be the number of linguistic attributes of x_i. Denote with y the output variable and with $h \in N$ the number of its linguistic attributes. Once selected the units of measure and the domains X_i and Y for x_i and y respectively, the membership function $\mu_{L_i^j}(x_i): X_i \to [0,1]$ is defined for each L_i^j, which represents the j-th linguistic attribute of x_i, $1 \le j \le \max_i k_i$. For each variable, its fuzzy number is graphically determined by the linguistic attributes along with their membership degrees $\mu_{L_i^j}(x_i)$, plotted in an xy-plane (see for example Figure 1 above). The graphs of the various fuzzy numbers overlap in some zone, so that x_i is L_i^j at a certain degree $\mu_{L_i^j}(x_i)$ for each j (analogously for y). The second step is the block-rules construction. This consists of fuzzy rules, which are implications of the following type:

IF

$$[x_1 \text{ is } L_1^{a_1} \text{ at } \mu_{L_1^{a_1}}(x_1)] \otimes [x_2 \text{ is } L_2^{a_2} \text{ at } \mu_{L_2^{a_2}}(x_2)] \otimes \ldots \otimes [x_n \text{ is } L_n^{a_n} \text{ at } \mu_{L_n^{a_n}}(x_n)]$$

THEN

$$[y \text{ is } L_y^r \text{ at } \mu_{L_y^r}(y)]$$

where $1 \le a_i \le k_i$. The IF part is called "antecedent" and the symbol $\otimes$ represents one of the possible aggregation operators. In practice, the MIN operator is used for the logical connective AND while the MAX operator is used for the logical connective OR (a convex combination of MIN and MAX is also common). The THEN part is called "consequent". The aggregation of antecedent and consequent can be made in several ways. The most used are the MAX and the BSUM methods and the choice between the two depends on the type of application. In either case, the two methods produce a fuzzy set that represents the output. This fuzzy set is then "defuzzified" and turned to a real number in [0,1].

The number of scientific contributions using fuzzy logic in business and finance has sharply increased in the recent past. Sugeno (1985), Tanaka (1997), Bojadziev and Bojadziev (1997) and Von Altrock (1997) show that fuzzy logic may be safely and usefully applied to business, financial, industrial applications. Zebda (1989, 1991) deals with vagueness and accounting. Abdel-Kader, Dugdale and Taylor (1998) cite a large number of nonquantifiable factors that firms consider important for investments' decisions (the authors use the term "intangible benefits"): variables such as quality of outputs, reputation, company image, employee morale, experience with new technology, consistency with corporate strategy etc. may not be treated with the classic financial criteria and often are integrated in the decision process in a nonfinancial way or simply neglected. Some other drivers have a direct financial impact but are not suited for mathematical tractability (at least not directly), e.g. financial flexibility, bargaining power, customers' loyalty, synergies. In all theses case fuzzy logic may be used (see also Buckley, Eslami and Feuring, 2002, for economic applications; Craiger, Coovert and Teachout, 2003, for a fuzzy rule-based system predicting job performance; Chen, Tzeng and Tang, 2005, for an example of fuzzy multiple criteria decision making). Several fuzzy expert systems dealing with business and financial applications have been recently presented. Following suggestions in Magni (1998) an expert system has been created evaluating a real option (Magni, Mastroleo and Facchinetti, 2002). A real-life case has been also studied and replicated regarding a firm's acquisition (Magni et al., 2004). Other applications have been proposed in other economic fields such as credit scoring (Facchinetti et al., 2001), insurance (Facchinetti and Mastroleo, 2001), industrial districts (Facchinetti, Mastroleo and Paba, 2000).

2. The model

In our model, the final output is determined by three main factors, which depend on other variables which in turn depend on others variables and so on until, proceeding backward, a set of initial independent variables (the inputs) is reached. The three main factors are: the stand-alone value of the firm (*Equity Value*), the additional value derived by the optimization of the capital structure (*Additional Firm Value*), the synergies realizable (*Synergies*). The final output measures the value-creation power of the firm and may be used as a rating. The analysis may be accomplished for various purposes: financial analysts may adopt it for ranking firms belonging to a particular industry, or shareholders may use it for rewarding managers or as an incentive tool. Managers themselves may perform the analysis to understand whether a particular decision increases or decreases the value-creation power of the firm. In all these cases, the synergies do not play any role, so this variable should be not considered (it should be "disactivated") and the output obtained is therefore an *objective* rating. If a merger or an acquisition is under examination, all three main factors should be taken into account and the output represents a *subjective* rating (because synergies change from buyer to buyer) and is a measure of the opportunity of the acquisition.

The expert system here presented processes a number of convenient input variables (*value drivers*) for a given firm which first affect the three main factors above mentioned and, via combination of the latter, produce an output in the interval [0,1]. The output is here named *Rating* and is univocally determined once a specific value for each value driver is fixed. The way value drivers determine the *Rating* is modular: they affect some intermediate variables which in turn affect other intermediate variables and so on until the three main factors are determined, which in turn determine the final *Rating*. The modular approach therefore consists of grouping the value drivers in modules which are in turn grouped in higher-level modules and so on progressively narrowing the number of variables involved until only one variable (the output) is left. Our model incorporates 29 value drivers (16 of which are considered qualitative, 13 are quantitative), 22 intermediate variables, 23 rule blocks and 730 fuzzy rules.[5] In Appendix a description for each (input and intermediate) variable is given (the symbols given in brackets are used in Figures 2 and 3 for convenience).

As anticipated, the value drivers do not affect the final output directly. They are gathered into groups forming intermediate variables with specific economic and financial meaning (e.g. the business risk, the specific risk, the strategic risk, the competitive power, the quality of the firm's product). Such variables are in turn grouped to form second-level intermediate variables (e.g. the operating risk, the revenues, the operating costs) and then a third-level set of intermediate variables are created and so on until the final *Rating* is reached. The approach is then modular and gives rise to a conceptual map, an evaluation tree that is run from branches to trunk (see Figure 2). From a mathematical point of view, the connection between the set of the value drivers and the output may be represented by a function of 29 independent variables x_i, $i = 1, 2, ..., 29$, affecting the dependent variable y (*Rating*), so that $y = f(x_1, x_2, ..., x_{29})$. Our model is aimed at giving a formal expression to $f(\cdot)$ so that for any 29-tuple the expert system automatically provides us with a specified value of y. The modular approach depicted in Figure 2 may be alternatively represented by a composed function. In particular we have seven composing functions:

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[5] As will be reminded for some drivers, to construct the fuzzy numbers for the quantitative variables (and thus to associate linguistic attributes to numerical values) the evaluator can rest on the statistical distribution of the variable around the industry average.

$$x \to f_1(x) \to f_2(f_1(x)) \to f_3(f_2(f_1(x))) \to \cdots \to f_7(f_6(...(f_2(f_1(x)))))) = y \,.$$

In the modular approach we follow, each vector is transformed into a vector having fewer components: This means that some of the variables have been grouped to generate an intermediate variable. Looking at Figure 3, we see that starting from 29 we get to 24 components (via f_1), then to 21 (via f_2), whence to 15 (via f_3), to 12 (via f_4), to 8 (via f_5), to 3 (via f_6), and finally to 1. Let us denote, respectively, with v_i the *i*-th vector *i*=1,2…,7 in Figure 3. We have $v_i = f_i(x)$ for every *i*. We now describe how Figure 2 and Figure 3 work, showing that they correspond to the same model by means of a different graphical representation: actually, Figure 2 is the representation of our expert system depicted as a conceptual map, Figure 3 is the same expert system described as a composed function.

In Figure 3, you may see that the *Rating* y of the firm depends on three variables: *AddFirmValue*, *EquityValue*, *Synergies* (the latter will be activated only in case of acquisition). In Figure 2 the same concept is illustrated by sketching three arrows leaving from these three variables (depicted with rectangular blocks) and converging to the block of *Rating*. Conceptually, the rating of a firm then depends on the combination of these three variables. The three variables are fuzzified with fuzzy numbers. For example, take *Synergies*. Its formal fuzzy representation is given in Figure 4. We have selected 5 linguistic attributes to describe the value of the synergies: VeryLow, Low, Medium, High, VeryHigh. The *x*-axis ranges from 0 to 1, the *y*-axis measures the degree of membership of the fuzzy numbers describing the linguistic attributes. That is, a level of *Synergies* of 0.85 in the *x*-axis means that *Synergies* is High at a degree of 60%, VeryHigh at a degree of 40%, Medium at a zero degree, Low at a zero degree, VeryLow at a zero degree. A level of 0.1 means that *Synergies* is VeryHigh at a zero degree, High at a zero degree, Medium at a degree of zero, Low at a degree of 20%, VeryLow at a degree of 80%. Similar fuzzy numbers describe the other variables.

To determine the *Rating*, starting from the three variables, the expert system rests on a rule block containing "if-then" implications. Table 1 is an extract of such a rule block. The rule block is self-explaining. For example, row 16 says that if *AddFirmValue* is High, *EquityValue* is Low, and *Synergies* is VeryHigh, then *Rating* is High. Row 6 (where a blank space is left in the first column) is to be read as follows: Whatever the value of *AddFirmValue*, if *EquityValue* is VeryHigh and *Synergies* is High, then *Rating* is VeryHigh. As the reader may easily note, *Rating*, as a function of the three variables, is, so to say, increasing with respect to each of them: The greater one of the three, the greater the final output. The rule block is composed by 113 rules and exhausts all possible cases, that is for each possible (linguistic) value of the three variables we have a particular (linguistic) value of *Rating*. In other terms, each of the three variables will have a linguistic value along with its membership degree and will determine a linguistic value of *Rating* along with its membership degree. The output is then "defuzzified" and produces a real number in the interval [0,1]: The greater the number, the greater the firm's capacity of creating value. Each of the three aforementioned variables (which constitute the sixth vector in Figure 3), is in turn function of other 8 variables specified in the fifth vector of Figure 3. Take for example *EquityValue*: it depends on *FirmValue* and *OutstDebt* (as in standard analysis, the equity value is the difference between the firm value and the outstanding debt), whereas the other six variables of the vector are uninfluential (they play a role in affecting *AddFirmValue* and *Synergies*). In Figure 3, *EquityValue* is seen as a function of all 8 variables, but 6 of them are, so to say, inactive. In Figure 2 this fact is represented by connecting only *FirmValue* and *OutsDebt* to *EquityValue* by drawing two converging arrows (and disregarding the other variables). The rule block *EquityValue* is represented in Table 2. As an example, row 7 is to be read as follows: Whatever the value of the other variables, if *FirmValue* is Medium and *OutstDebt* is VeryLow, then *EquityValue* is Medium. Row 1 should be read as follows: whatever the value of the other 6 variables, whatever

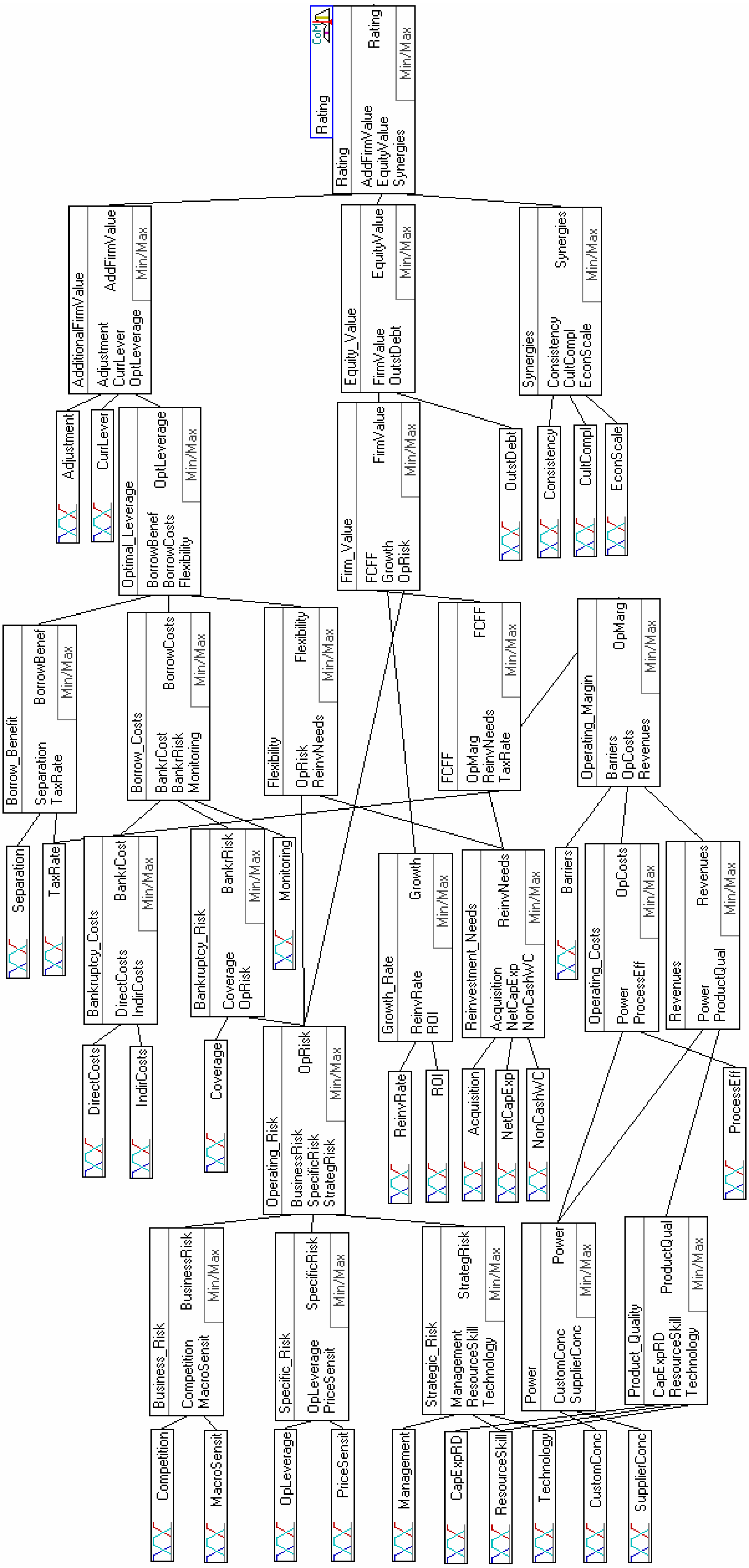
Rating
AdditionalFirmValue
Equity_Value
Synergies
Optimal_Leverage
Firm_Value
Borrow_Benefit
Borrow_Costs
Flexibility
FCFF
Operating_Margin
Bankruptcy_Costs
Bankruptcy_Risk
Growth_Rate
Reinvestment_Needs
Operating_Costs
Revenues
Operating_Risk
Business_Risk
Specific_Risk
Strategic_Risk
Power
Product_Quality
Adjustment
CurrLever
OutstDebt
Consistency
CultCompl
EconScale
Separation
TaxRate
Monitoring
Barriers
DirectCosts
IndirCosts
Coverage
ReinvRate
ROI
Acquisition
NetCapExp
NonCashWC
ProcessEff
Competition
MacroSensit
OpLeverage
PriceSensit
Management
CapExpRD
ResourceSkill
Technology
CustomConc
SupplierConc

Figure 2

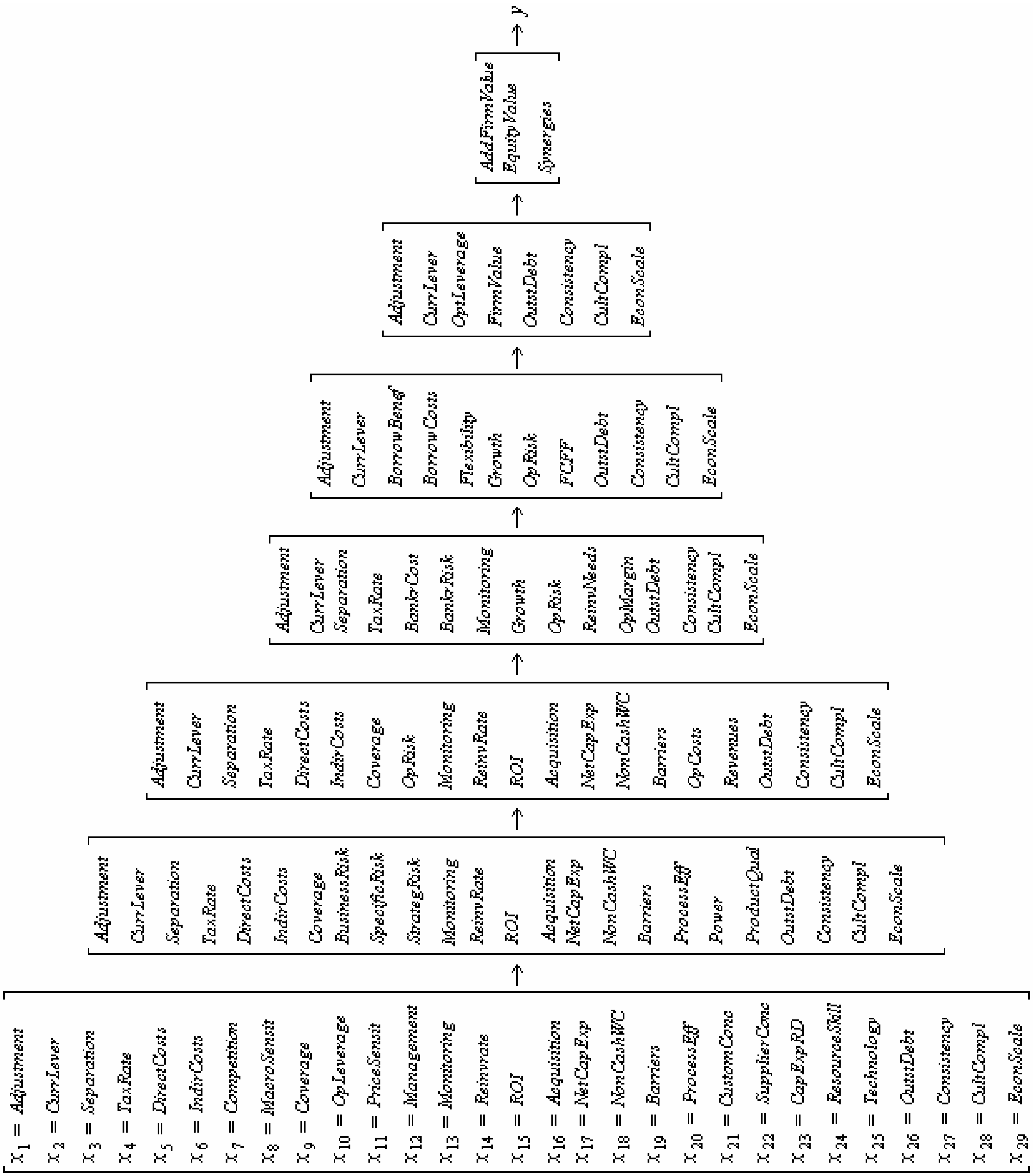
X 1 = Adjustment
X 2 = CurrLever
X 3 = Separation
X 4 = TaxRate
X 5 = DirectCosts
X 6 = IndirCosts
X 7 = Competition
X 8 = MacroSensit
X 9 = Coverage
X 10 = OpLeverage
X 11 = PriceSensit
X 12 = Management
X 13 = Monitoring
X 14 = Reinvrate
X 15 = ROI
X 16 = Acquisition
X 17 = NetCapExp
X 18 = NonCashWC
X 19 = Barriers
X 20 = ProcessEff
X 21 = CustomConc
X 22 = SupplierConc
X 23 = CapExpRD
X 24 = ResourceSkill
X 25 = Technology
X 26 = OutstDebt
X 27 = Consistency
X 28 = CultCompl
X 29 = EconScale
Adjustment
CurrLever
Separation
TaxRate
DirectCosts
IndirCosts
Coverage
BusinessRisk
SpecificRisk
StrategRisk
Monitoring
ReinvRate
ROI
Acquisition
NetCapExp
NonCashWC
Barriers
ProcessEff
Power
ProductQual
OutstDebt
Consistency
CultCompl
EconScale
Adjustment
CurrLever
Separation
TaxRate
DirectCosts
IndirCosts
Coverage
OpRisk
Monitoring
ReinvRate
ROI
Acquisition
NetCapExp
NonCashWC
Barriers
OpCosts
Revenues
OutstDebt
Consistency
CultCompl
EconScale
Adjustment
CurrLever
Separation
TaxRate
BankrCost
BankrRisk
Monitoring
Growth
OpRisk
ReinvNeeds
OpMargin
OutstDebt
Consistency
CultCompl
EconScale
Adjustment
CurrLever
BorrowBenef
BorrowCosts
Flexibility
Growth
OpRisk
FCFF
OutstDebt
Consistency
CultCompl
EconScale
Adjustment
CurrLever
OptLeverage
FirmValue
OutstDebt
Consistency
CultCompl
EconScale
AddFirmValue
EquityValue
Synergies
y

Figure 3

the value of *OutsDebt*, if *FirmValue* is VeryLow, then *EquityValue* is VeryLow. Note that, mathematically, *EquityValue* may be seen as a function increasing with respect to *FirmValue* (the greater *FirmValue*, the greater *EquityValue*), decreasing with respect to *OutstDebt* (the greater the debt the smaller the equity value), constant with respect to the other six variables (any variation in the latter is uninfluential). It is also worthwhile noting that the rules do not simply determine monotonicity, but affect the readiness at which the dependent variables reacts to changes in the independent variables. In Table 2 the reader may see that when *FirmValue* is low, *EquityValue* is not very sensitive to changes in the *OutsDebt* (it only changes from Low to VeryLow when *OutstDebt* varies from VeryLow to VeryHigh). This means that when the firm value is low it does not matter much the value of the debt: the equity value of the firm may not be High at a nonzero degree (that is, if you have a small amount of money, you do not hold much even if that money is not borrowed but is

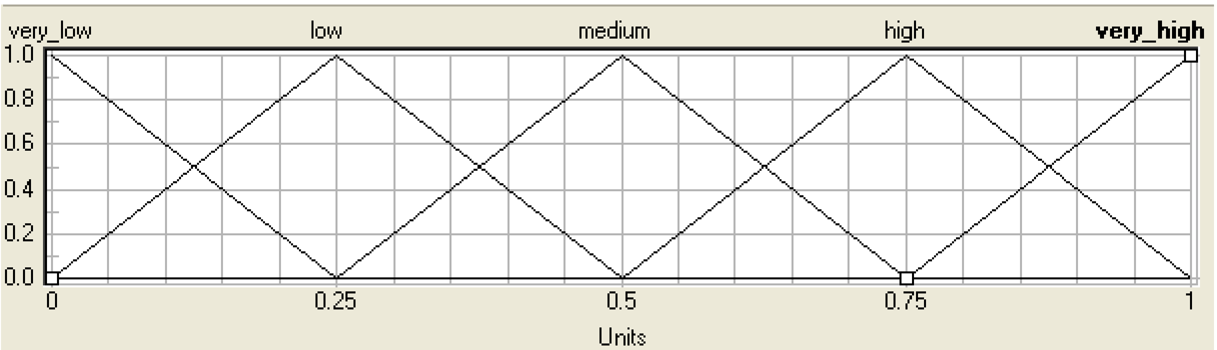

Figure 4. *Synergies*

Table 1. *Extract from the Rule Block "Rating"*

IF			THEN
AddFirmValue	**EquityValue**	**Synergies**	**Rating**
Zero	High	High	High
Low	High	High	High
Medium_low	High	High	High
Medium_high	High	High	Very_high
High	High	High	Very_high
	Very_high	High	Very_high
Zero	Very_low	Very_high	Medium_low
Low	Very_low	Very_high	Medium_low
Medium_low	Very_low	Very_high	Medium
Medium_high	Very_low	Very_high	Medium_high
High	Very_low	Very_high	Medium_high
Zero	Low	Very_high	Medium_low
Low	Low	Very_high	Medium
Medium_low	Low	Very_high	Medium_high
Medium_high	Low	Very_high	Medium_high
High	Low	Very_high	High
Zero	Medium	Very_high	Medium_high
Low	Medium	Very_high	Medium_high
Medium_low	Medium	Very_high	High
Medium_high	Medium	Very_high	High
High	Medium	Very_high	Very_high
	High	Very_high	Very_high
	Very_high	Very_high	Very_high

your own net worth). More responsive is *EquityValue* to changes in *OutsDebt* when *FirmValue* is High. In this case, *OutsDebt* plays a role: *EquityValue* decreases from High to Low as *OutstDebt* increases from VeryLow to VeryHigh.

Proceeding backward in Figure 3, we find that the 8 variables are in turn functions of 12 variables (fifth vector). Some components of the fifth vector are active, some others are inactive. For example, *OptLeverage* is increasing with respect to *BorrowBenef*, decreasing with respect to *BorrowCosts*, decreasing with respect to *Flexibility*, while it is constant with respect to the other variables of the fifth vector (Figure 2 shows then only the arrows converging from the three active variables to *OptLeverage*). Iterating backwards one gets to the very inputs of the system. For example, *StrategRisk* (second vector) may be seen as a function of all 29 value drivers: It is decreasing with respect to *Management*, *ResourceSkill* and *Technology*, and it is constant with respect to the other 26 value drivers (i.e. they are inactive). As seen, the value drivers are given specific crisp values and then defuzzified, and the system automatically produces a fuzzy output which is defuzzified so that a real number in [0,1] is brought out.

Table 2. *Rule Block "EquityValue"*

IF		**THEN**
FirmValue	**OutstDebt**	**EquityValue**
Very_low		Very_low
Low	Very_low	Low
Low	Low	Low
Low	Medium	Very_low
Low	High	Very_low
Low	Very_high	Very_low
Medium	Very_low	Medium
Medium	Low	Medium
Medium	Medium	Medium
Medium	High	Low
Medium	Very_high	Very_low
High	Very_low	High
High	Low	High
High	Medium	High
High	High	Medium
High	Very_high	Low
Very_high	Very_low	Very_high
Very_high	Low	Very_high
Very_high	Medium	High
Very_high	High	High
Very_high	Very_high	Medium

3. Ilustrative examples and corroboration of the model

The model we have constructed needs corroboration. To this end, we have tested its reliability through a series of simulations, varying the value of one or more value drivers simultaneously, while leaving the others fixed. The greater the number of simulations, the higher the degree of corroboration of the model. For reasons of space, only two of the simulations are described in the present section (the reader will find it useful to keep attention on Figure 2 in order to better comprehend the explanation).

The first simulation shows different values of *Rating* for different values of *EconScale*, with the other value drivers held fixed. The value of *EconScale* is made to rise from 0 to 1, while the other variables are kept fixed at the following levels: *Adjustment=0.1, CurrLever=0.1, Separation=1, TaxRate=0.6, DirectCosts=0.1, IndirectCosts=0.4, Competition=0.05, MacroSensit=0.1, Coverage=9, OpLeverage=0.1, PriceSensit=0.1, Management=0.8, Monitoring=0, ReinvRate=0.1, ROI=0.16, Acquisition=0, NetCapExp=0, NonCashWC=0, Barriers=0, ProcessEff=0.8, CustomConc=0.1, SupplierConc=0.1, CapExpRD=0.8, ResourceSkill=1, Technology=1, OutstDebt=0.1, Consistency=0.4, CultCompl=0.4.* In Table 3 you see 11 columns corresponding to 11 different cases. Each column shows the selected value of EconScale and the consequent values of *Synergies*, *AddFirmValue*, *EquityValue* and the final *Rating*.

Table 3. *Changes in EconScale*

Case	1	2	3	4	5	6	7	8	9	10	11
EconScale	*0*	*0,1*	*0,2*	*0,3*	*0,4*	*0,5*	*0,6*	*0,7*	*0,8*	*0,9*	*1*
AddFirmValue	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
EquityValue	0,500	0,500	0,500	0,500	0,500	0,500	0,500	0,500	0,500	0,500	0,500
Synergies	0,200	0,250	0,292	0,333	0,375	0,450	0,500	0,542	0,583	0,625	0,700
Rating	**0,467**	**0,500**	**0,528**	**0,556**	**0,583**	**0,633**	**0,667**	**0,694**	**0,722**	**0,750**	**0,800**

As one expects, as *EconScale* increases *Synergies* increases, and this in turn causes *Rating* to increase (note that changes in *EconScale* do not produce any variation in *AddFirmValue* and *EquityValue*, as it is obvious). With a minimum value of *EconScale* (case 1) *Synergies* is low, but not minimum, since the value of *CultCompl* and *Consistency* partially compensate. *Rating* is still greater (a medium 0.467) since the low value of *Synergies* is offset, to a certain extent, by a very high *AddFirmValue* and a medium *EquityValue*. The extremely favorable value of *AddFirmValue* is due in turn to exceptionally favorable values of the relevant value drivers (e.g. *MacroSensit*, *PriceSensit*, *Adjustment*, *Direct Costs* etc.); as for the medium value of *EquityValue*, some drivers describe a positive situation: *Competition* and *MacroSensit* determine very low values for *BusinessRisk*, which positively reflects on *OpRisk* and hence on *FirmValue* and then on *EquityValue*. Others are less favorable: for Example, *TaxRate* is quite high and determines a low value of *FCFF*, which partially counterbalances the just mentioned positive effects. This simulation is graphically represented in Figure 5, where the abscissa collects the eleven cases, the *y*-axis shows *Rating*, *EquityValue* and *AddFirmValue* (we have interpolated the eleven points in order to have continuous functions). As the reader can see, *Rating* increases, though less readily than *Synergies*: as noted, the medium value of *EquityValue* makes the slope of *Rating* smaller than that of *Synergies*.

The first simulation confirms that the final output reacts in a correct way (both in terms of correlation and in terms of magnitude) as one of the value drivers varies. The second simulation (Table 4 and Figure 6) aims at testing a less simplified relation by changing two value drivers (*Separation* and *TaxRate*) simultaneosuly while leaving the others fixed at the following levels: *Adjustment=1, CurrLever=0, DirectCosts=0.1, IndirectCosts=0.4, Competition=1, MacroSensit=0.5, Coverage=9, OpLeverage=1, PriceSensit=0.9, Management=0.8, Monitoring=0, ReinvRate=0, ROI=0, Acquisition=0, NetCapExp=0.8, NonCashWC=0, Barriers=0, ProcessEff=0, CustomConc=0, SupplierConc=0, CapExpRD=0, ResourceSkill=1, Technology=0.5, OutstDebt=0, Consistency=0.7, CultCompl=0.7, EconScale=0.7.* In Table 4 *Separation* and *Tax Rate* are exogenously fixed to the various levels, and the corresponding values of *Rating*, *Synergies*, *AddFirmValue* and *EquityValue* are shown. In this case we also show the (defuzzified) values of other intermediate variables for the sake of a better understanding.

As the reader may glean from the graph or from the table, increases of both *Separation* and *Tax* do not lead to a monotonic variation of *Rating*. *Rating* keeps constant from case 1 to case 3, then decreases until case 6, then slightly increases in the last five cases. This behavior is due to the fact that *Separation* and *Tax* positively affect *AddFirmValue*, but *Tax* negatively affects *EquityValue*. Actually, the increase in *Separation* and *Tax* causes *BorrowBenef* to increase (from 0.15 to 0.9). This in turn generates, other things being fixed, a growth in *OptLeverage* (from 0.417 to 0.667) which in turn reverberates positively on *AddFirmValue* which rises from 0.312 to 0.438. The latter's growth is not pronounced (and in the first four cases it keeps constant) because *Adjustment* is maximum: with a maximum cost of adjustment, the (increasing) benefit from an increasing optimal leverage is partially offset by the expenses (and time) required to reach the optimal level, so that the function increases slowly. On the other side, *EquityValue* decreases, for the increase in *Tax* causes *FCFF* to decrease (not sharply because the rise in *Tax* is partially counterbalanced by a medium level of *OpMarg* and a minimum level of *ReinvNeeds*).[6] A reduction in *FCFF* results in a drop of *FirmValue*, other things being fixed, from 0.312 to 0.062 (the starting level of *FirmValue* is a little bit low, owing to the fact that *OpRisk* is medium high and *Growth* is minimum). Note that *EquityValue* perfectly reflects *FirmValue*, since *OutsDebt* is minimum.

[6] The values of *OpMarg* and *ReinvNeeds* are not shown in Table 4.

Rating is therefore the result of two opposing effects: initially *Rating* is at a medium level and keeps constant as long as *AddFirmValue* and *EquityValue* keeps constant. From case 3 to 6 *EquityValue* decreases and *AddFirmValue* rises. This two conflicting effects results in a decrease of *Rating*, since *EquityValue* has been given a greater weight in determining the final output (and *AddFirmValue*'s rise is too slow to offset this path). From case 7 to case 10 *EquityValue* keeps constant and the increase in *AddFirmValue* enables *Rating* to slightly increase as well.

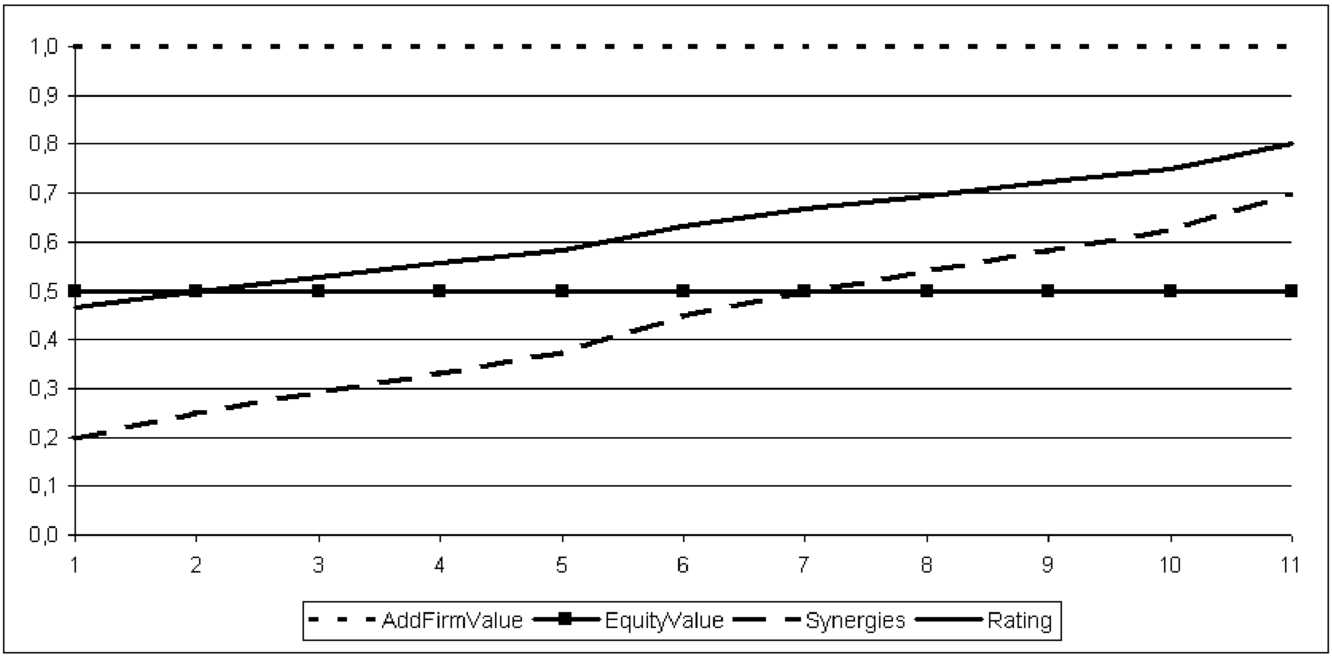

Figure 5

Table 4. *Changes in Separation and TaxRate*

Case	1	2	3	4	5	6	7	8	9	10	11
Separation	*0,3*	*0,35*	*0,4*	*0,45*	*0,5*	*0,55*	*0,6*	*0,65*	*0,7*	*0,75*	*0,8*
TaxRate	*0,15*	*0,2*	*0,25*	*0,3*	*0,35*	*0,4*	*0,45*	*0,5*	*0,55*	*0,6*	*0,65*
BorrowBenef	0,150	0,175	0,312	0,583	0,750	0,775	0,800	0,825	0,850	0,875	0,900
OptLeverage	0,417	0,417	0,417	0,517	0,583	0,597	0,611	0,623	0,636	0,651	0,667
AddFirmValue	0,312	0,312	0,312	0,312	0,375	0,387	0,400	0,408	0,417	0,426	0,438
FCFF	0,500	0,500	0,500	0,458	0,354	0,250	0,250	0,250	0,250	0,250	0,250
FirmValue	0,312	0,312	0,312	0,259	0,170	0,062	0,062	0,062	0,062	0,062	0,062
EquityValue	0,312	0,312	0,312	0,259	0,170	0,062	0,062	0,062	0,062	0,062	0,062
Synergies	0,714	0,714	0,714	0,714	0,714	0,714	0,714	0,714	0,714	0,714	0,714
Rating	**0,477**	**0,477**	**0,477**	**0,439**	**0,360**	**0,307**	**0,313**	**0,318**	**0,322**	**0,327**	**0,333**

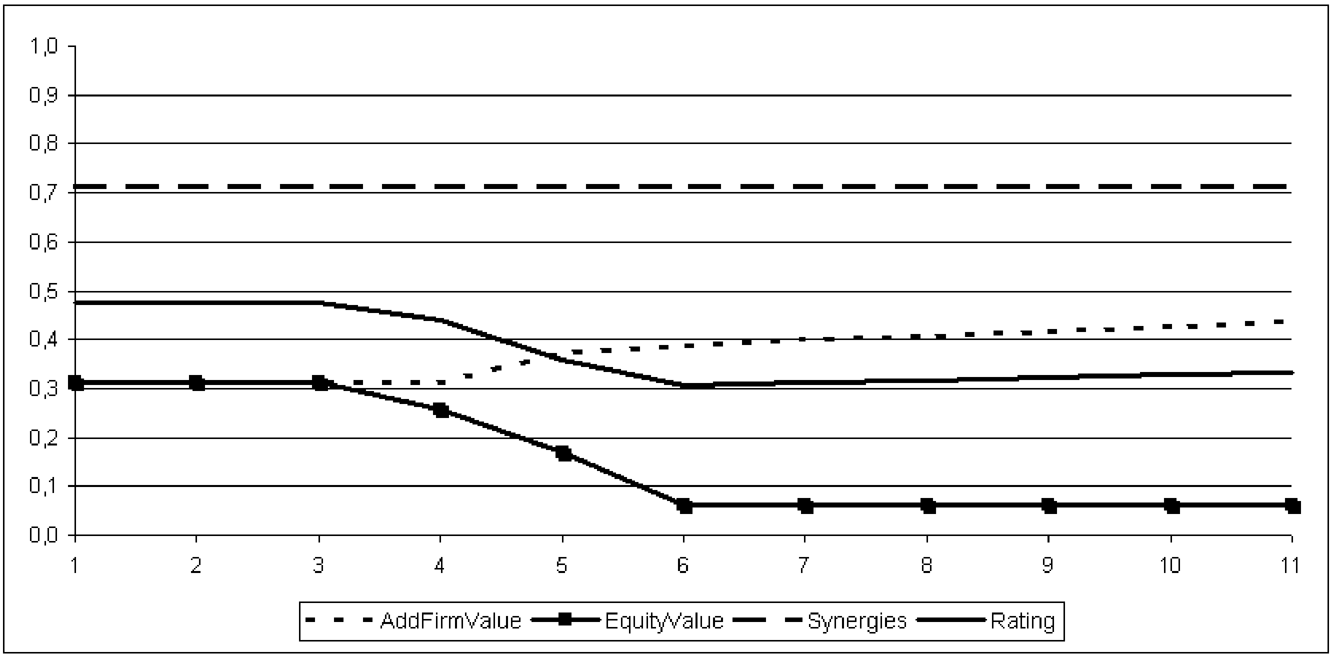

Figure 6

4. Methodological analysis

The structure of our model is consistent with corporate finance techniques and theories (DCF techniques and ownership structure theory) as well as strategic management conceptual framework. As for DCF techniques, the standard computation of the equity value is given by the difference $E = V - D$ where *V* is the firm value and *D* is the outstanding debt (see Fernández, 2002, for a detailed review of DCF techniques and their relations). The firm value is in turn computed as a present value,:

$$V = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+WACC)^t} = \frac{FCF}{WACC - g}$$

where $FCF_t = FCF_{t-1}(1+g)$ and $FCF_0 := FCF$, with *FCF*=Free Cash Flow, *g*=growth rate, *WACC*=Weighted Average Cost of Capital. The latter is an opportunity cost, i.e. the foregone return of alternatives businesses in the same class of risk (usually in the same sector) and is an increasing function of risk. The equity is then an increasing function of *V* and a decreasing function of *D*, and the firm value is in turn increasing with respect to both FCF and *g*, and is decreasing with risk. Consistently, our equity-value rule block depends on both firm value (positive correlation) and outstanding debt (negative correlation), while the firm value depends on free cash flow and growth (positive correlation) as well as directly on operating risk (negative correlation). But while DCF technique is a black box where the determinants of FCF and *g* are not highlighted and where risk appears only implicitly in the *wacc*, our model explicitly takes account of these variables, as Figure 3 shows. In addition, DCF methodology does not explicitly take into consideration problems of capital structure optimization, apparently in accordance with Modigliani and Miller's principles of debt irrelevance. When analysts try to consider this aspect, they simply modify the *wacc* in a rather unspecified way. In contrast, we make use of important results of the Ownership structure theory (Jensen and Meckling, 1976; Jensen, 1986) which claims that an optimal capital structure exists and it is a function of several determinants. We have then added the variable Additional Firm Value which depends, one way or another, on fifteen value drivers, whose interrelations determine borrowing costs and borrowing benefits for the firm, as well as the need for flexibility (see Figure 3). Further, DCF techniques are unsuited for quantification of synergies and, if a quantification is ever attempted, the analyst just raises discretionally the FCF. Conversely, we consider them explicitly as a third important determinant of rating, and explicit relations between synergies' determinants are stated via the specific rule block.

The rigorous way of discounting cash flows adopted by the DCF approach is therefore offset by a nontransparent way of fixing the value for FCF, *g* and *wacc*. The value of these three inputs are left to the analyst's or managers' arbitrariness, so causing possible manipulations of data. Our model then does retain the conceptual frame of the DCF approach, but integrates it with strategic management theories. For example, the presence of Resources and Skills as a value driver is coherent with the findings of the Resource-based view according to which a variety of peculiar skills and intangible abilities can greatly affect a firm's performance (Barney, 1986, 1991, 2001; Grant, 1991; Levinthal, 1995). Likewise, the variables Barriers, Customer Concentration and Supplier Concentration give expression to Porter's (1980, 1985) industry analysis.

The integrated view we propose is also consistent with bounded rationality in that preeminent importance is given to recognizing limited computational capabilities and presence of constraints. For example, the presence in our model of the value driver Management is consistent with the literature on Top Management Teams (TMTs) which claims that managers and employees differ in expertise so that the quality of the TMT's decisions substantially influence the performance of the firm (Bromiley, 2005, p. 78. See also Simons, Pelled and Smith, 1999). Likewise, the presence of Technology conforms to the finding of most studies of organizational learning according to which owning a specific technology is valuable because transferring technologies is difficult and takes time even in the same sector (Bromiley, 2005, p. 84).

In addition, the very formal structure of our model is consistent with a behavioral perspective in strategic management: By creating nontrivial connections among variables and expressing relationships via 'if-then' implications we try to give formal clothing to the idea that a firm is a complex system made of physical, intellectual and financial assets, knowledge and skills of management and employees, which is grounded on complex interrelations (see Bromiley, 2005, for relationships among bounded rationality, Resource-based view, Porter's industry analysis, and for a behavioral foundation of strategic management).

The output produced by all the interrelations is the result of processing absolute as well as relative variables. The former are defined relative to the sector in which the firm operates (e.g., Operating Leverage and ROI), the latter are firm-specific and do not relate to the other firms in the sector (e.g., Management, Resources and Skills). The rating is then a mix of absolute/relative indexes and should be interpreted in connection with the purposes for which it is used. In particular, there are at least six uses:

(a) Rating listed or unlisted companies and decomposing the rating into three driving factors (rating of equity value, rating of synergies, rating of additional firm value)
(b) Rewarding and compensating managers
(c) Evaluating and comparing business units of a firm
(d) Measuring the impact of the firm's possible policies and strategies on value creation
(e) Evaluating the impact of particular decisions taken by managers on value creation
(f) Helping decision makers in strategic decisions

Rating companies has to do with public information about firms in an industry and their relative value-creation powers: a score of 1 individuates excellent firms in that sector, a score of 0 refer to very poorly managed firms in the sector. Managers' rewarding is usually based on relative performance so that the score is confronted with other firms' scores or past scores of the firm. Comparing business units of the same firm is evidently a relative judgment so that the business unit with the higher score realizes the greater value. The other three uses of the model have to do with decisions, so that it is the change of the score that matters: If rating increases decisions create value, otherwise they destroy value.

A seventh very important use of the model is possible: Pricing firms. To this end, we need again relative as well as absolute determinants, because value depends on variables expressing absolute performance as well as variables expressing relative performance. The DCF approach consider these two salient features only implicitly by focusing on a ratio between an absolute measure (FCF) and a relative measure (*wacc*), the former representing the cash flow the firm will generate, the latter reflecting alternative rates of return available in the market. In contrast, our model acknowledges both absolute and relative variables in an explicit way. On the basis of a mix of relative/absolute variables it is possible to provide the potential buyer a price of the target firm, which should represent the (fair) value for a specific buyer, i.e. the highest price payable for the firm (for reasons of space, we do not focus here on this problem. See Malagoli, Magni and Mastroleo, 2004, for some hints). [7]

[7] This model has been applied to a real-life case, namely the acquisition of Camuzzi by Enel, the ex Italian monopolist in the electric energy production, and a study is in progress for obtaining the price of Camuzzi.

Conclusions

This paper presents a fuzzy expert system replicating the reasoning of a human expert (or experts' panel). Subjects interested in this kind of tool include rating agencies, financial analysts, investors (shareholders, bondholders), banks and managers. The field of application of such an expert system is manifold: it is an evaluation technique as well as a corporate governance tool and a device for assessing the increase in value associated to particular decisions.

The fuzzy expert system proposed in this work is an alternative to the decision models and evaluation models existing in the literature. Financial economics offers us elegant models such as the discounted cash flow methods which, though widely used, do not rely on an explicit analysis of all drivers at play. The evaluation of a firm is then grounded on computations of cash flows whose magnitude is often arbitrary (especially when business and strategic considerations are involved). In a sense, the DCF methodology just helps us in the final step (discounting cash flows with a risk-adjusted rate of return), but does not inform us how many and which value drivers are taken into account nor how they have been aggregated nor is it specified their direct or indirect financial impact. When other formally impeccable models are used, e.g. options theory (Trigeorgis, 1996) or dynamic programming (Dixit and Pindyck, 1994), they seem to be mathematically complex and not intuitive, and, admittedly, "*managers do not have the necessary mathematical skills to implement or even understand it*" (Lander and Pinches, 1998). Further they are capable of dealing only with a very limited number of variables (usually one) and require unrealistic assumptions, which are highly simplified for mathematical tractability, so that the result is that of shunting aside reality. On the other side, business economics seems to follow an opposite point of view: reality is so complex that it is impossible to formalize or even rationalize the situation at hand. When an attempt is made to search for some drivers influencing the value of an economic activity (and consequently, the solution of the decision process) this is accomplished in an informal way by trying to guess the drivers but omitting to offer a model that connects them.

Looking at the positive side of these two disciplines we may see that finance suggests us that we do need formal models for a better description and rationalization of the evaluation process, while business economics suggests us that reality cannot be described by merely resting on mathematical models, complex in their application and simplified in their assumptions. This paper proposes a model which seems to meet both requirements. Expert systems and fuzzy logic, combined together, seem to be an interesting tool for valuing firms (as well as coping with other kinds of business and financial decision processes). The approach we offer is easy to understand and easy to implement, it does not require advanced knowledge of mathematics and does not make any particular assumption on the variables affecting the value of the option. The solution derives from logical implications (the "if-then" rules), so anyone can understand them and construct them. At the same time we have a formal model, which rationalizes the evaluation process and automatically gives us the final value. Fuzzy logic is in this sense a good tool for describing the value of a firm, since the complexity of real-life situations is handled through "vague" variables and "vague" interactions, which better replicate human mind as well as economic phenomena. The mental processes of human beings are actually imperfect and imprecise, and individuals often act in contexts of incomplete (and unclear) information. Any individual is not able to precisely formulate all possible solutions of a decision-making process and the corresponding consequences. They have a bounded rationality (Simon, 1955, 1956) so that formal models, as far as classical logic is concerned, are unsuited for solving investment evaluation's problems. We think we have showed that the "vague" connections we have accomplished by constructing the fuzzy expert system adequately replicate such imprecision and imperfectness. Also, a fuzzy approach, unlike classical ones, seems to be capable of integrating qualitative and quantitative analysis, so that the model is not forced to limit its scope to numerical variables with well-specified units of measures but can handle any type of qualitative drivers (which is an impossible task for classical mathematics). Furthermore, we can handle a high number of value drivers, simplifying the design of the whole system, dramatically reducing its complexity and intelligibility: the system is modular, therefore not explosive, since we run it from branches to trunk. As a result, we are able to shape the problem so as to take explicit consideration of business, strategic, organizational, financial aspects. In particular, the expert system presented in this paper is an integrated methodology that contemplates diverse theories and models: The DCF technique, the Ownership Structure theory, the Resource-based view, the industry analysis. Finally, the system is extremely flexible, one can introduce many more value drivers and change in any moment the rules connecting drivers and intermediate variables.

As a final remark, we would like to briefly point out the cognitive implications of our system. It combines the intuition and experience of experts and the formal rigor of a logic system. Therefore, the model presented testifies that bounded and unbounded rationality are not opposite, contrary to what is commonly maintained in cognitive psychology and in economics (see Gilovich, Griffin and Kahneman, 2002; Gigerenzer and Selten, 2001). The two systems of reasoning (heuristic-based and rule-based) on which the two kinds of rationality seem to reside (Sloman, 1996, 2002) is implicitly highlighted in the very essence of this model. But the heuristic-based system and the rule-based system do not conflict here; they do not compete to produce their own solutions (as often happens), but concurrently cooperate to provide decision makers with a unified output. Intuition, experience and creativity supply the system with the knowledge base; deduction, (fuzzy) mathematics and finance theory furnish the tools for the inferences.

APPENDIX

VALUE DRIVERS

Acquisition *(Acquisition)*. The portion of capital expenditures represented by the target firm's prospective external investments. Since firms seldom make acquisitions every year and since it's difficult to set a normalized monetary value for expected acquisitions, this variable has been treated as a qualitative variable.
Cost of Adjustment (*Adjustment*). This variable has a qualitative expression in the model and represents the costs that the target firm has to sustain to pass from the current capital structure to the optimal one.
Entry Barriers (*Barriers*). The entry barriers which prevent potential competitors from entering the business. It is a qualitative variable.
Capital Expenditures in R&D (*CapExpRD*). Research and development expenses represent a capital expenditure. To determine the value of capital expenditures in research and development one may use their average value (calculated over the last five years) as a percentage of revenues.
Competitive Rivalry *(Competition)*. The pressure determined by the rivalry among competitors inside the competitive arena. Due to the difficulties in finding a quantitative expression to the competitive rivalry, this variable is considered as a typical qualitative variable.
Strategic Resources Consistency (*Consistency*). This variable is activated only if a merger is under examination and represents the consistency between resources and skills owned by the acquiring firm and resources and skills needed to compete in the specific sector in which the target firm operates.
Coverage ratio (*Coverage*). The coverage ratio is the ratio EBIT/Financial expenses and should represent a quick measure of financial rating of the target firm. The intervals of this variable are determined on the same ranges set by the most important rating agencies, which associate different rating to different values of financial ratios (coverage ratio is probably the most significant financial ratio).
Culture and Complementary Resources (*CultCompl*). The presence of complementary resources is one of the most significant reasons for mergers (if no merger is under examination, this value driver must be disactivated). We use the term complementarities to identify resources and skills complementarities and market complementarities, considering the diseconomies derived from any kind of overlapping and cannibalization. Even if an acquisition or a merger is viable and it makes sense in terms of complementary resources, nevertheless it can fail because of a cultural mismatch between the buyer and the target company. So, this variable expresses both the degree of complementarity and the degree of potential cultural match between the two enterprises (if the cultural match is too difficult to be taken into account, one may rely on the former only).
Current Leverage (*CurrLever*). The current debt/equity ratio of the company. In the general definition of our model presented here we have defined the intervals of this variable using the same ranges employed by rating agencies. As for all quantitative variables, it will be significant to make use of the statistical distribution of the financial leverage of a sample of firms for any industry.
Customer Concentration (*CustomConc*) .To measure customer concentration we use the ratio
(average sales per client)/(total sales).
Direct Costs of Distress (*DirectCosts*). Direct costs of bankruptcy (a qualitative variable in our model) are the procedure's costs that have to be sustained in case of distress.
Economies of Scale (*EconScale*). This variable has a qualitative expression in the model (ranging from low to high), and represents the economies of scale that a merger can grant (if no merger is under examination, this value driver must be disactivated). Like *CultCompl* this variable takes on a subjective meaning and depends on the unique match between a specific buyer and the target firm.
Indirect Costs of Distress (*IndirCosts*). Indirect costs of bankruptcy depend on the specific characteristics of the firm. There are some types of firms whose indirect costs of bankruptcy are typically high: firms whose products have a long life cycle, manufacturers whose products' value for the customers is strictly related to complementary services and products supplied by others companies, and firms that sell products which require frequent assistance services. This variable is qualitative.

Sensitivity to Macroeconomic Variables (*MacroSensit*). To express the sensitivity to macroeconomic factors that affect the business risk, we consider the unlevered beta of the industry as significant, picking 0.85 as a medium value, which is regarded to be the "barycenter" value of unlevered beta by many researchers and study centers (adaptations for the specific industrial sector under examination are required).

Quality of Management (*Management*). The quality of management affects strategic risk and depends on the experience of the firm's management. It is a qualitative variable with a range from zero (low) to one (high) and with 0.5 as a medium value.

Monitoring Costs (*Monitoring*). The costs of monitoring represent the costs that banks and bondholders have to sustain in order to control management's activity. Due to the difficulties in quantifying these costs we consider this variable qualitative.

Net Capital Expenditures (*NetCapExp*). Net capital expenditures is one of the three determinants of the intermediate variable "reinvestment needs" and include the fair adjustments for the capitalizations of R&D and of SG&A (selling, general and administrative expenses), net of depreciations and amortizations. In order to grant more transparency to the model we do not include here the acquisitions (the latter are considered as a different value driver). The ranges of the linguistic attributes of this variable depend on the peculiarities of the different industries. To determine them, one can use the average (e.g. last five fiscal years) of the firm's ratio *NetCapExp/Revenues* compared to the industry average of the last year.

Investment in Non Cash Working Capital (*NonCashWC*). The third variable affecting the reinvestment needs is non cash working capital. Short term investments in inventories and accounts receivable represent a negative cash flow like any other long term investment in plant and equipment. As for the ranges of the linguistic attributes, they depend on the industry considered. We consider the average (e.g. last three fiscal years) of the firm's ratio *NonCashWC/Revenues*, compared to the industry average of the same ratio calculated on the last year.

Operating Leverage (*OpLeverage*). The operating leverage expresses the costs' structure of the firm: the proportion of fixed costs on total costs (fixed+variable). It is a significant driver of specific risk, because a high operating leverage (high proportion of fixed costs) magnifies the effect of sales fluctuations on profit. An average over the last three fiscal years of this ratio can be used to express this variable. The ranges of the linguistic attributes of this driver (as of all non-qualitative variables in the model) can be easily defined through a statistical calculation of the distribution of values around the industry average.

Outstanding Debt (*OutstDebt*). The outstanding debt of the target firm. It is a qualitative variable in our model. Alternatively one may use the market value of the outstanding debt, a quantitative measurement, using the statistical distribution of debts' values of the firms of the industry to define the linguistic attributes.

Consumer Price Sensitivity (*PriceSensit*). A qualitative variable expressing customers' price sensitivity. It is one of the two determinants of specific risk in our model (the other one is the operating leverage).

Processes Efficency (*ProcessEff*). The current efficiency of the firm's processes. This is a qualitative variable in the model, even though in some specific industry it is actually possible to find a quantitative measure identifying efficiency. Even if an acquisition is under examination and the process' efficiency of the target firm can be improved by the buyer, the potential improvement should not be included in the definition of this variable. This potential improvement is already considered in the model: investment in R&D (which affects product quality) and economies of scales (a determinant of synergies) are two examples of potential future improvement in efficiency.

Reinvestment Rate (*ReinvRate*). The reinvestment rate is defined as

(capital expenditures – depreciation + Δ *non cash working capital)/(EBIT(1-t)).*

An average on the last five years can be used, if representative of a forward-looking perspective. Again, the numerical intervals for the attributes of this variable can be defined through a statistical calculation of the distribution of values around the industry average.

Resources and Skills (*ResourceSkill*). Resources and skill owned by the firm. This variable has a double correlation in the tree: it is one of the two determinants of the product quality and it is one of the three determinants of the strategic risk. It is a qualitative variable.

Return on Investment (*ROI*). The return on investment of the firm is equal to the ratio *EBIT(1-t)/(capital invested)*. Like most financial analysts and economists we agree that the evaluator should subtract average noninterest-bearing liabilities (accounts payable, salaries payable) from average total assets in the denominator of ROI. Its value should be fixed in a forward-looking perspective (the geometric average in recent years may be meaningful).

Separation between Management and Shareholders *(Separation)*. It is a qualitative variable indicating the separation between the stockholders and the management. It is positively related to borrow benefits: the higher the separation, the higher the convenience to increase the debt (other things equal). It is used as a disciplinary device by the stockholders to control the management.

Supplier Concentration (*SupplierConc*). To express the supplier concentration one may use the ratio

(average purchase cost of raw materials per supplier)/(total cost of raw materials' purchases).

Tax Rate (*TaxRate*). The marginal corporate tax rate.
Technology (*Technology*). The quality and degree of technology owned by the firm. We consider technology as a qualitative variable, even if in some cases and in some specific sector, some alternative measure may be used (e.g. value of patents recorded among the intangible assets).

INTERMEDIATE VARIABLES

Additional Firm Value (*AddFirmValue*). The Additional Firm Value expresses the value that can be created by modifying the current capital structure of the firm to an optimal one. The value of this variable can be equal or higher than zero (it is zero if the outstanding debt is equal to the suggested optimal level of debt). The three determinants of *AddFirmValue* are: the current financial leverage, the optimal financial leverage, and the cost of adjustment which a firm incurs when it tries to reach an optimal capital structure.
Bankruptcy Costs (*BankrCost*). The costs of bankruptcy depend on two drivers: *DirectCosts* and *IndirectCosts*. We attribute a higher weight to the *IndirectCosts,* because of the bigger importance which is usually attributed to these costs by commercial banks and rating agencies in determining the cost of debt.
Bankruptcy Risk (*BankrRisk*). In our model the risk of bankruptcy is determined by two variables: *Coverage* and *OpRisk.* If the firm already has a rating, such a rating will capture its probability of bankruptcy and will express the entire bankruptcy risk. In this case, *BankrRisk* itself may be treated as an input variable (the flexibility of the system allows to adapt the model to the specific needs of different cases).
Borrowing Benefits (*BorrowBenef*). Borrowing benefits are determined by two drivers: *TaxRate* and *Separation*, both positively correlated with *BorrowBenef.*
Borrowing Costs *(BorrowCosts)*. We have seleceted three determinants for this variable: *BankrRisk*, *BankrCost* and *Monitoring*. These three variables are positively correlated with the intermediate variable *BorrowCosts*.
Business Risk (*BusinessRisk*). *BusinessRisk* identifies the risk of the industry in which the firm competes. It is one of the three determinants of operating risk and depends on *Competition* and *MacroSensit*.
Equity Value (*EquityValue*). *EquityValue* represents the stand-alone value of the firm and is one of the three key determinants of the model. It depends on two determinants: the firm value, positively correlated with *EquityValue*, and the outstanding debt, an input variable negatively correlated with *EquityValue*.
Free Cash Flow to Firm (*FCFF*). By Free Cash Flow to Firm we mean the cash flows available to shareholders and bondholders.[8] This variable is one of the three determinants of *FirmValue* and, in turn, depends on three drivers: two intermediate variables (the operating margin, i.e. EBIT, and the reinvestment needs), and one input variable (the tax rate).
Firm Value (*FirmValue*). The same basic variables usually used for calculating the firm value in a DCF analysis have been here employed as determinants of the firm value: Free Cash Flow to Firm, operating risk and growth rate.
Financial Flexibility (*Flexibility*). The future financial flexibility that will be necessary to the firm to be competitive on the market and to fulfill its strategy depends on two variables: the reinvestment needs deliberated in the strategic plan and the operating risk, both positively correlated with *Flexibility*.
Growth Rate (*Growth*). The growth rate must be an expected growth rate (forward-looking perspective). In our model it depends on two input variables: *ReinvRate* and *ROI*.
Operating Costs (*OpCosts*). In our model the magnitude of the operating costs depends on *ProcessEff* and on an intermediate variable expressing the bargaining power towards suppliers and clients (*Power*).
Operating Margin (*OpMarg*). The operating margin results from a composition of three variables: operating costs, revenues and entry barriers.
Operating Risk (*OpRisk*). We have assumed the point of view of a shareholder without a diversified portfolio, thus considering the whole operating risk (not only the systematic component). We therefore isolate three different determinants for the operating risk: strategic risk, business risk and specific risk. Instead of using the standard deviation of the operating returns as a measure of the total operating risk, we opt for the definition of some more accurate variables which could specifically identify the three risk determinants. Even though the use of a single statistical variable could simplify the model, we believe that a finer decomposition of risk drivers helps to get to a meticulous and faultless valuation. In order to stress the most specific risk determinants of the firm, in the operating risk's rule block we give a higher weight to the strategic and specific risk than to the business risk. Of course the flexibility of the system enables one to change and adapt the model to the specific case at hand (for example assuming the point of view of a diversified investor and therefore considering only the systematic component of risk).
Optimal Financial Leverage (*OptLeverage*). The optimal capital structure is defined by three antecedents: *BorrowBenef*, *BorrowCosts* and *Flexibility*. The former is positively correlated with *OptLeverage*, the latter two are negatively correlated.
Bargaining Power (*Power*). This variable identifies the bargaining power of the target firm towards customers and suppliers and depends on two input variables: *CustomConc* and *SupplierConc*.

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[8] A more rigorous term for what we mean is Capital Cash Flow (CCF). For the difference about FCFF and CCF see Fernández (2002, ch.2) and Ruback (2002). In our fuzzy perspective to use either term is a matter of convention.

Product Quality (*ProductQual*). The three determinants of the target firm's product quality are: *CapExpRD*, *ResourceSkill*, *Technology*. These inputs, all positively correlated with *ProductQual*, have been given the same relative importance in our model (same weight).
Reinvestment Needs (*ReinvNeeds*). In order to grant the model the greatest transparency we distinguish investments in acquisitions from capital expenditures in PPE (Property, Plant and Equipment). Consequently the reinvestment needs depend on three determinants: *NetCapExp*, *Acquisition* and *NonCashWC*, all in a forward-looking perspective.
Revenues (*Revenues*). *ProductQual* and *Power* are the antecedents of the intermediate variable *Revenues*.
Specific Risk (*SpecificRisk*). In our model the specific risk depends on two decisive factors of the firm: *OpLeverage* and *PriceSensit*.
Strategic Risk (*StrategRisk*). The strategic risk depends on three antecedents: *ResourceSkill*, *Technology* (these two input variables affect *ProductQual* as well) and *Management.*
Synergies *(Synergies)*. *Synergies* represents the subjective component of our model and are activated only if an acquisition is under examination. We have decided to limit its determinants to three input variables: the economies of scale (*EconScale*), the presence in the target firm of resources and skills fundamental to compete in the specific industry (*Consistency*), and the complementary resources (*CultCompl*). We have not incorporated diversification as a determinant of synergies, because too often the supposed purpose of curbing the risk of the business by investing in some anticyclical activity conceals some personal goals of the management not aligned with the shareholders' interests (building empires).

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