License and entry strategies for an outside innovator under duopoly with combination of royalty and fixed fee

Hattori, Masahiko and Tanaka, Yasuhito

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Masahiko Hattori*
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.
and
Yasuhito Tanaka†
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.

Abstract

We consider a choice of options for an innovating firm to enter the market with or without licensing its new cost-reducing technology to the incumbent firm using a combination of a royalty per output and a fixed license fee, or to license its technology without entry. With general demand and cost functions we show the following results. When the innovating firm licenses its technology to the incumbent firm without entry, the optimal royalty rate per output for the innovating firm is zero with negative fixed fee, and when the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, the optimal royalty rate is positive with positive or negative fixed fee. Also we show that when cost functions are concave, the optimal royalty rate is one such that the incumbent firm drops out of the market and license without entry strategy and entry with license strategy are optimal for the innovator; and when cost functions are strictly convex, there is an internal solution of the optimal royalty rate under duopoly and entry with license strategy is optimal for the innovator.

Keywords: duopoly, royalty, fixed license fee.

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*mhattori@mail.doshisha.ac.jp
†yasuhito@mail.doshisha.ac.jp

1 Introduction

We consider a choice of options for an innovating firm to enter the market with or without licensing its new cost-reducing technology to the incumbent firm using a combination of a royalty per output and a fixed license fee, or to license its technology without entry also using a combination of a royalty per output and a fixed license fee.

In Proposition 4 of Kamien and Tauman (1986), assuming linear demand and cost functions and fixed license fee, it was argued that in an oligopoly when the number of firms is small (or large), entry with license strategy by the innovating firm, which is a strategy to enter the market and at the same time license its cost-reducing technology to an incumbent firm, is more profitable than license without entry strategy, which is a strategy to license its technology to an incumbent firm without entering the market. We think that their definition of license fee in the case where the innovating firm licenses its technology to an incumbent firm and does not enter the market is not appropriate. Interpreting their analysis in a duopoly model, they defined the license fee in that case by the difference between the profit of an incumbent firm in that case and its monopoly profit before entry and license by the innovating firm. However, we can think that if the negotiation between the innovating firm and an incumbent firm about the license fee breaks down, the innovating firm can enter the market without license to an incumbent firm. If the innovating firm does not enter the market nor license, its profit is zero. But, if it enters the market, its profit is positive. Therefore, such a threat is credible, and hence an incumbent firm must pay the difference between its profit in the license without entry case and its profit in the entry without license case as a license fee.

In Hattori and Tanaka (2017a), using an alternative definition of a license fee taking the above point into account, the following results about duopoly with a linear demand function and only a fixed license fee have been shown.

1. Linear cost functions (constant marginal costs):

   If the incumbent firm does not drop out when the innovating firm enters the market without license, license without entry strategy is optimal for the innovating firm. This result is converse to that in Kamien and Tauman (1986). If the incumbent firm drops out when the innovating firm enters the market without license, both license without entry strategy and entry without license strategy are optimal[1].

2. Quadratic cost functions:

   If the magnitude of the innovation is large (cost of the new technology is sufficiently low), license without strategy is optimal for the innovating firm, and if the magnitude of the innovation is small, entry with license strategy is optimal.

In this paper we consider a more general situation of duopoly with an innovating firm and an incumbent firm, in which the innovating firm imposes a combination of a royalty per output and a fixed license fee to the incumbent firm. We analyse a case of general demand and cost functions as well as a case of general demand and concave cost function and a case of general demand and strictly convex cost function. We will show the following results.

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[1] When the incumbent firm drops out of the market, the innovation is said to be drastic.

General demand and cost function case

1. When the innovating firm licenses its technology to the incumbent firm without entry, the optimal royalty rate per output for the innovating firm is zero.
2. When the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, the optimal royalty rate per output is positive.

General demand and concave cost function case

1. If the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, and the cost functions of the firms are concave, the optimal royalty rate per output for the innovating firm is one such that the output of the incumbent firm is zero.
2. The fixed license fee is negative.
3. License without entry strategy and entry with license strategy are optimal for the innovator.

General demand and strictly convex cost function case

1. If the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, and the cost functions of the firms are strictly convex, the optimal royalty rate per output for the innovating firm is positive but smaller than one such that the output of the incumbent firm is zero.
2. The equilibrium output of the innovating firm is larger than that of the incumbent firm.
3. Entry with license strategy is optimal for the innovator.

In this case the fixed license fee may be positive or negative. Please see an example in Section 5.

In the next section we review some related studies. In Section 3 we describe the model of this paper. In Section 4 we present the main results, and in Section 5 we study a case of linear demand and quadratic cost functions as an example.

2 Literature review

Various studies focus on technology adoption or R&D investment in duopoly or oligopoly. Most of them analyze the relation between the technology licensor and licensee. The difference of means of contracts, which comprise royalties, upfront fixed fees, combinations of these two, and auctions, are well discussed (Katz and Shapiro (1985)). Kamien and Tauman (2002) showed that outside innovators prefer auctions, but industry incumbents prefer royalty. This topic is discussed by Kabiraj (2004) under the Stackelberg oligopoly; here, the licensor does not have production capacity. Wang and Yang (2004) considered the case when the licensor has production capacity. Sen and Tauman (2007) compared the license system in detail, namely, when the licensor is an outsider and when it is an incumbent firm, using the combination of royalties and fixed fees. However, the existence of production capacity was externally given, and they did not analyze the choice of entry. Therefore, the optimal strategies of outside innovators, who can use the entry as a threat, require more discussion. Regarding the strategies of new entrants to the market, Duchene, Sen and Serfes (2015) focused on future entrants with old technology, and argued that a low license fee can be used to deter the entry of potential entrants. However, the firm with new technology is incumbent, and its choice of entry is not analyzed. Also, Chen (2016) analyzed the model of the endogenous market structure determined by the potential entrant with old technology and showed that the licensor uses the fixed fee and zero royalty in both the incumbent and the outside innovator cases, which are exogenously given. Creane, Chiu and Konishi (2013) examined a firm that can license its production technology to a rival when firms are heterogeneous in production costs, and showed that a complete technology transfer from one firm to another always increases joint profit under weakly concave demand when at least three firms remain in the industry.

A Cournot oligopoly with fixed fee under cost asymmetry was analyzed by La Manna (1993). He showed that if technologies can be replicated perfectly, a lower cost firm always has the incentive to transfer its technology; hence, while a Cournot-Nash equilibrium cannot be fully asymmetric, there exists no non-cooperative Nash equilibrium in pure strategies. On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyzed bargaining between a licensor with no production capacity and oligopolistic firms. Recent research focuses on market structure and technology improvement. Boone (2001) and Matsumura et. al. (2013) found a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) showed that technology adoption may change the market outcome. The social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may result in higher social welfare than Bertrand competition under a differentiated goods market. Hattori and Tanaka (2015) and (2016a) studied the adoption of new technology in Cournot duopoly and Stackelberg duopoly. Rebolledo and Sandonís (2012) presented an analysis of the effectiveness of research and development (R&D) subsidies in an oligopolistic model in the cases of international competition and cooperation in R&D. Hattori and Tanaka (2016b) analyzed problems about product innovation, that is, introduction of higher quality good in a duopoly with vertical product differentiation. Recently, Sen and Stamatopoulos (2016) presented an analysis of royalty and fixed fee under duopoly with general demand and cost functions. They did not considered an option of the innovator whether it enter the market or not.

In this paper we will show that a combination of non-negative royalty and negative or positive fixed fee is optimal for the innovator under duopoly with options for the innovator to enter or not to enter the market with or without license. On the other hand, Liao and Sen (2005) showed that negative royalty can be optimal under oligopoly with one innovator and two incumbent firms. When the innovator is holding relatively insignificant new technology, licensing it to only one firm with negative royalty is optimal. This strategy leads licensee more aggressive and getting more profit which is paid to licensor as a fixed license fee. This negative royalty may result in more social welfare than that where negative royalty is prohibited[2].

3 The model

There are two firms Firms A and B. Firm A is an innovating firm and Firm B is an incumbent firm. Although at present only Firm B produces a good and Firm A is an outside innovator, after entering the market Firm A also produces the same good. It has a superior new technology and can produce the good at lower cost than Firm B.

Firm A have three options. The first option is to enter the market without license to Firm B, the second option is to license its superior technology to Firm B using a combination of a royalty per output and a fixed license fee, and the third option is to enter the market with license to Firm B also using a combination of a royalty per output and a fixed license fee. If Firm A enters, the market becomes a duopoly.

Let p be the price, x_A and x_B be the outputs of Firms A and B. The inverse demand function of the good is written as

$$p(x_A + x_B).$$

We assume Cournot type behavior of the firms. The cost function of Firm B before adoption of the new technology is $c_B(x_B)$, and its cost function after adoption of the new technology is $c_A(x_B)$. The cost function of Firm A is $c_A(x_A)$. $c_A(x_A) < c_B(x_B)$ for $x_A = x_B$. We assume $c_B(0) = 0$ when Firm B drops out of the market. We analyze a case of general demand and cost functions, and also a case of general demand and concave cost function and a case of general demand and strictly convex cost function. Furthermore, as an example, we will consider a case of linear demand and quadratic cost functions.

4 The main results

4.1 Entry without license

Suppose that Firm A enters the market without license to Firm B. The inverse demand function in this case is written as $p(x_A + x_B)$. The profits of Firms A and B are

$$\pi_A = p(x_A + x_B)x_A - c_A(x_A),$$

and

$$\pi_B = p(x_A + x_B)x_B - c_B(x_B).$$

The first order conditions for profit maximization of Firms A and B are

$$p + p'x_A - c_A'(x_A) = 0,$$

and

$$p + p'x_B - c_B'(x_B) = 0.$$

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[2] They assumed linear demand and cost functions. Their analysis about outside innovator case is extended to general demand and cost functions by Hattori and Tanaka (2017b).

The second order conditions are

$$2p' + p''x_A - c_A''(x_A) < 0,$$

and

$$2p' + p''x_B - c_B''(x_B) < 0.$$

We assume that the second order conditions are satisfied in each case. Denote the equilibrium profit of Firm B in this case by π_B^e.

4.2 License without entry

Next suppose that Firm A licenses its technology to Firm B using a combination of a royalty per output and a fixed license fee, and does not enter the market. Denote the fixed license fee by L and the royalty rate per output by r. The inverse demand function is $p(x_B)$. The profit of Firm B is

$$\pi_B = px_B - c_A(x_B) - rx_B - L.$$

The first order condition for profit maximization of Firm B is

$$p + p'x_B - c_A'(x_B) - r = 0.$$

The second order condition is

$$2p'x_B + p''x_B - c_A''(x_B) < 0.$$

From these conditions we obtain

$$\frac{dx_B}{dr} = \frac{1}{2p'x_B + p''x_B - c_A''(x_B)} < 0.$$

If the negotiation between Firm A and Firm B about the license fee breaks down, Firm A can enter the market without license. When Firm A does not enter nor sell a license, its profit is zero; however, when it enters the market without license, its profit is positive. Therefore, such a threat is credible, and Firm B must pay the difference between its profit net of the royalty and its profit in the previous entry without license case as a fixed license fee. The total license fee is the sum of the royalty and the fixed license fee. L is determined so that $\pi_B = \pi_B^e$ is satisfied. Thus, it is written as

$$L = px_B - c_A(x_B) - rx_B - \pi_B^e.$$

Note that π_B^e is a constant number. Denote the total license fee, which is $L + rx_B$, by TL. Then,

$$TL = px_B - c_A(x_B) - \pi_B^e.$$

Firm A chooses r so as to maximize TL. The condition for maximization of TL with respect to r is

$$\frac{dTL}{dr} = (p + p'x_B - c_A'(x_B))\frac{dx_B}{dr} = r\frac{dx_B}{dr} = 0.$$

Since $\frac{dx_B}{dr} < 0$, we get the optimal royalty rate per output, $\tilde{r}^l$, for the innovating firm as follows.

$$\tilde{r}^l = 0.$$

We have shown the following result.

Lemma 1. *When the innovating firm licenses its technology to the incumbent firm without entry, the optimal royalty rate per output for the innovating firm is zero.*

4.3 Entry with license

Suppose that Firm A enters the market and at the same time licenses its technology to Firm B using a combination of a royalty per output and a fixed license fee. The cost function of Firm B is $c_A(\cdot)$ in this case. Similarly to the previous case, we denote the fixed license fee by L and the royalty rate per output by r. The inverse demand function is $p(x_A + x_B)$. The profits of Firms A and B are

$$\pi_A = px_A - c_A(x_A),$$

and

$$\pi_B = px_B - c_A(x_B) - rx_B - L.$$

The first order conditions for profit maximization of Firms A and B are

$$p + p'x_A - c_A'(x_A) = 0, \tag{1}$$

and

$$p + p'x_B - c_A'(x_B) - r = 0. \tag{2}$$

The second order conditions are

$$2p' + p''x_A - c_A''(x_A) < 0,$$

and

$$2p' + p''x_B - c_A''(x_B) < 0.$$

Differentiating (1) and (2) with respect to r yields

$$(2p' + p''x_A - c_A''(x_A))\frac{dx_A}{dr} + (p' + p''x_A)\frac{dx_B}{dr} = 0,$$

$$(p' + p''x_B)\frac{dx_A}{dr} + (2p' + p''x_B - c_B''(x_B))\frac{dx_B}{dr} - 1 = 0.$$

Solving them, we obtain

$$\frac{dx_A}{dr} = -\frac{1}{\Delta}(p' + p''x_A),$$

and

$$\frac{dx_B}{dr} = \frac{1}{\Delta}(2p' + p''x_A - c_A''(x_A)) < 0,$$

where

$$\Delta = (2p' + p''x_A - c_A''(x_A))(2p' + p''x_B - c_A''(x_B)) - (p' + p''x_A)(p' + p''x_B).$$

We assume

$$\Delta > 0.$$

Also we assume

$$|2p' + p''x_A - c''_A(x_A)| > |p' + p''x_A|,$$

and

$$|2p' + p''x_B - c''_A(x_B)| > |p' + p''x_B|.$$

These assumptions are obtained from the stability conditions for the equilibrium of duopoly[3]. Then,

$$p' - c''_A(x_A) < 0,\ p' - c''_A(x_B) < 0.$$

Hence,

$$\frac{dx_A}{dr} + \frac{dx_B}{dr} = \frac{1}{\Delta}(p' - c''_A(x_A)) < 0,$$

and

$$\left|\frac{dx_B}{dr}\right| > \left|\frac{dx_A}{dr}\right|. \tag{3}$$

We have $\frac{dx_A}{dr} > 0$ when $p' + p''x_A < 0$ and $\frac{dx_A}{dr} < 0$ when $p' + p''x_A > 0$. In the former case the goods of the firms are strategic substitutes, and in the latter case they are strategic complements. These properties do not affect the main results of this paper.

Similarly to the previous case, Firm B must pay the difference between its profit net of the royalty and its profit in the entry without license case as a fixed license fee. The fixed license fee should be equal to

$$L = px_B - c_A(x_B) - rx_B - \pi^e_B.$$

The total license fee is

$$TL = px_B - c_A(x_B) - \pi^e_B.$$

The total profit of Firm A is the sum of the total license fee and its profit as a firm in the duopoly. It is equal to

$$\pi_A + TL = px_A - c_A(x_A) + px_B - c_A(x_B) - \pi^e_B.$$

π^e_B is constant. Firm A chooses r so as to maximize $\pi_A + TL$. Differentiating $\pi_A + TL$ with respect to r yields

$$\begin{aligned}\frac{d}{dr}(\pi_A + TL) =&(p + p'x_A - c'_A(x_A) + p'x_B)\frac{dx_A}{dr} + (p + p'x_B - c'_A(x_B) + p'x_A)\frac{dx_B}{dr} \\ =&p'x_B\frac{dx_A}{dr} + (r + p'x_A)\frac{dx_B}{dr}.\end{aligned} \tag{4}$$

If there is an internal solution of r which maximizes $\pi_A + TL$, it is

$$\tilde{r}^{el} = -\frac{p'}{\frac{dx_B}{dr}}\left(x_A\frac{dx_B}{dr} + x_B\frac{dx_A}{dr}\right).$$

We show that the optimal royalty rate is positive.

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[3] See Seade (1980) and Dixit (1986).

Lemma 2. *When the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, its optimal royalty rate per output is positive.*

Proof. Suppose $r = 0$. Then, (1) and (2) mean $x_A = x_B$. From (3)

$$x_A \frac{dx_B}{dr} + x_B \frac{dx_A}{dr} < 0.$$

Substituting $r = 0$ into (4), we find

$$\frac{d}{dr}(\pi_A + TL)\bigg|_{r=0} = p'\left(x_B \frac{dx_A}{dr} + x_A \frac{dx_B}{dr}\right) > 0.$$

Therefore, the optimal royalty rate is positive. □

Now we consider two specific cases.

Concave cost function case

Assume that the cost functions of Firms A and B before adoption of the new technology are $c_A(x_A)$ and $c_B(x_B)$ such that $c_A(x_A) < c_B(x_B)$ for $x_A = x_B$, $c_A''(\cdot) \leq 0$ and $c_B''(\cdot) \leq 0$, or $c_A'(x) < c_A'(y)$ and $c_B'(x) < c_B'(y)$ for $x > y$. The cost function of Firm B after adoption of the new technology is $c_A(x_B)$. From (1) and (2)

$$p + p'x_A - c_A'(x_A) = 0,\ p + p'x_B - c_A'(x_B) - r = 0.$$

Suppose $x_B = 0$. Then,

$$r = p - c_A'(0).$$

Denote this value of r by $\bar{\bar{r}}$. It is a value of the royalty rate such that the output of Firm B is just zero, that is, it drops out of the market. We call such a royalty rate per output *prohibitive*. Also we have

$$\bar{\bar{r}} + p'x_A = c_A'(x_A) - c_A'(0) \leq 0.$$

Substituting this and $x_B = 0$ into (4) yields

$$\frac{d}{dr}(\pi_A + TL) = (c_A'(x_A) - c_A'(0))\frac{dx_B}{dr} \geq 0.$$

Therefore, the optimal royalty rate per output is $\bar{\bar{r}}$. In this case $\tilde{r}^{el} = -p'x_A$. Comparing $\bar{\bar{r}}$ and $\tilde{r}^{el}$,

$$\bar{\bar{r}} - \tilde{r}^{el} = c_A'(x_A) - c_A'(0) \leq 0.$$

However, it is nonsense to impose a royalty larger than $\bar{\bar{r}}$. The fixed license fee in this case is negative as the following inequality shows

$$L = px_B - c_A(x_B) - rx_B - \pi_B^e = -c_A(0) - \pi_B^e < 0.$$

It compensates the profit of Firm B in the case of entry without license.

We have shown the following result.

Theorem 1. *1. If the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, and the cost functions of the firms are concave, the optimal royalty rate per output for the innovating firm is one such that the output of the incumbent firm is zero, that is, the royalty rate per output is prohibitive.*

2. The fixed license fee in this case is negative.

Strictly convex cost function case

Assume that the cost functions of Firms A and B before adoption of the new technology are $c_A(x_A)$ and $c_B(x_B)$ such that $c_A(x_A) < c_B(x_B)$ for $x_A = x_B$, $c''_A(\cdot) > 0$ and $c''_B(\cdot) > 0$, or $c'_A(x) > c'_A(y)$ and $c'_B(x) > c'_B(y)$ for $x > y$. The cost function of Firm B after adoption of the new technology is $c_A(x_B)$. (1) and (2) are rewritten as

$$p + p'x_A - c'_A(x_A) = 0, \tag{5}$$

and

$$p + p'x_B - c'_A(x_B) - r = 0. \tag{6}$$

Suppose $x_B = 0$. Then,

$$r = p - c'_A(0) = \bar{\bar{r}},$$

and

$$\bar{\bar{r}} + p'x_A = c'_A(x_A) - c'_A(0) > 0.$$

Substituting this and $x_B = 0$ into (4) yields

$$\frac{d}{dr}(\pi_A + TL) = (c'_A(x_A) - c'_A(0))\frac{dx_B}{dr} < 0.$$

Therefore, there is an internal solution of the optimal royalty rate, $\tilde{r}^{el} = -p'x_A > 0$. It is smaller than $\bar{\bar{r}}$. From (5) and (6), x_A is larger than x_B.

We have shown the following result.

Theorem 2. *1. If the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, and the cost functions of the firms are strictly convex, the optimal royalty rate per output for the innovating firm is positive and smaller than one such that the output of the incumbent firm is zero.*

2. The equilibrium output of Firm A is larger than that of Firm B.

The fixed license fee in this case may be positive or negative. Please see an example in the next section.

4.4 The optimal strategy for the innovator

In this subsection we consider the optimal strategy for the innovating firm. The results depend on the form of cost functions.

Concave cost function case

When the cost functions of the firms are concave, entry with license strategy and license without entry strategy are equivalent. In both cases the monopolistic situation is realized. In the license without entry case the monopolist is Firm B, and in the case of entry with license it is Firm A. Because the payoff of Firm A in the monopolistic situation is larger than its profit in the duopolistic situation when it enters the market without license, license without entry strategy and entry with license strategy are optimal.

The monopoly profit including royalty revenue is maximized at zero royalty rate. Thus, the optimal royalty rate in the case of license without entry is zero. On the other hand, in the case of entry with license the market is duopolistic with small royalty rate. When the cost functions are concave, the monopolistic situation is optimal for the innovating firm. Therefore, the innovating firm gets larger profit by driving out the incumbent firm from the market with prohibitive royalty rate. Then, we need negative fixed fee to compensate the profit of the incumbent firm that it can get in the case of entry without license.

Strictly convex cost function case

In the case where Firm A enters the market with license, setting the value of r as one such that the output of Firm B is zero, the monopolistic situation which is the same as that in the case of license without entry can be realized. On the other hand, the optimal royalty rate per output is different from such a value. Therefore, entry with license strategy is optimal.

Summarizing the results in the following theorem;

Theorem 3. 1. *When the cost functions of the firms are concave, license without entry strategy and entry with license strategy are optimal for the innovating firm.*

2. *When the cost functions of the firms are strictly convex, entry with license strategy is optimal for the innovating firm.*

In the case of entry with license the market is duopolistic, and when the cost functions of the firms are strictly convex, the payoff of the innovating firm in duopolistic situation is larger than that in monopolistic situation because partition of production between two firms is more efficient than concentration of production to one firm under strictly convex cost functions. There is a positive internal solution of the optimal royalty rate which is not prohibitive.

5 An example of linear demand and quadratic cost function case

The cost functions of Firms A and B before adoption of the new technology are $c_A x_A^2$ and $c_B x_B^2$ with $0 < c_A < c_B$. The cost function of Firm B after adoption of the new technology is $c_A x_B^2$.

5.1 Entry without license

Suppose that Firm A enters the market without license to Firm B. The inverse demand function is assumed to be

$$p = a - x_A - x_B.$$

The profits of Firms A and B are

$$\pi_A = (a - x_A - x_B)x_A - c_A x_A^2,\ \pi_B = (a - x_A - x_B)x_B - c_B x_B^2.$$

The conditions for profit maximization of Firms A and B are

$$a - 2x_A - x_B - 2c_A x_A = 0,\ a - x_A - 2x_B - 2c_B x_B = 0.$$

The equilibrium outputs, price and profits are

$$x_A = \frac{a(2c_B + 1)}{4c_A c_B + 4c_B + 4c_A + 3},\ x_B = \frac{a(2c_A + 1)}{4c_A c_B + 4c_B + 4c_A + 3},$$

$$p = \frac{a(2c_A + 1)(2c_B + 1)}{4c_A c_B + 4c_B + 4c_A + 3},$$

$$\pi_A = \frac{a^2(c_A + 1)(2c_B + 1)^2}{(4c_A c_B + 4c_B + 4c_A + 3)^2},\ \pi_B = \frac{a^2(2c_A + 1)^2(c_B + 1)}{(4c_A c_B + 4c_B + 4c_A + 3)^2}.$$

Denote π_A and π_B in this case by π_A^e and π_B^e.

5.2 License without entry

Next suppose that Firm A licenses its technology to Firm B using a combination of a royalty per output and a fixed license fee, and does not enter the market. The inverse demand function is

$$p = a - x_B.$$

The profit of Firm B is

$$\pi_B = (a - x_B)x_B - c_A x_B^2 - r x_B - L.$$

The equilibrium output, price and profit are

$$x_B = \frac{a - r}{2(c_A + 1)},\ p = \frac{a + r + 2ac_A}{2(c_A + 1)},\ \pi_B = \frac{(a - r)^2}{4(c_A + 1)} - L.$$

Firm B must pay the difference between its profit net of the royalty and its profit in the previous entry without license case as a fixed license fee. The fixed license fee, L, is determined so that $\pi_B = \pi_B^e$ is satisfied. Thus,

$$L = \frac{(a - r)^2}{4(c_A + 1)} - \frac{a^2(2c_A + 1)^2(c_B + 1)}{(4c_A c_B + 4c_B + 4c_A + 3)^2} = \frac{A}{4(c_A + 1)(4c_A c_B + 4c_B + 4c_A + 3)^2}.$$

Denote the total license fee by TL^l. Then,

$$TL^l = L + rx_B = \frac{B}{4(c_A+1)(4c_Ac_B+4c_B+4c_A+3)^2)}.$$

About details of A and B please see Appendix. Maximizing TL^l with respect to r, the optimal royalty rate is obtained as follows.

$$\tilde{r}^l = 0.$$

The fixed fee and the total license fee are equal to

$$L = TL^l = \frac{a^2(16c_A^2c_B^2+32c_Ac_B^2+16c_B^2-16c_A^3c_B+36c_Ac_B+20c_B-16c_A^3-16c_A^2+4c_A+5)}{4(c_A+1)(4c_Ac_B+4c_B+4c_A+3)^2}.$$

5.3 Entry with license

Suppose that Firm A enters the market and at the same time licenses its technology to Firm B using a combination of a royalty per output and a fixed license fee. The inverse demand function is

$$p = a - x_A - x_B.$$

The profits of Firms A and B are

$$\pi_A = (a - x_A - x_B)x_A - c_Ax_A^2,\ \pi_B = (a - x_A - x_B)x_B - c_Ax_B^2 - rx_B - L.$$

The conditions for profit maximization of Firms A and B are

$$a - 2x_A - x_B - 2c_Ax_A = 0,\ a - x_A - 2x_B - 2c_Ax_B - r = 0.$$

The equilibrium outputs, price and profits are

$$x_A = \frac{a+r+2ac_A}{(2c_A+1)(2c_A+3)},\ x_B = \frac{a-2c_Ar-2r+2ac_A}{(2c_A+1)(2c_A+3)},\ p = \frac{a+r+2ac_A}{2c_A+3},$$

$$\pi_A = \frac{(c_A+1)(r+2ac_A+a)^2}{(2c_A+1)^2(2c_A+3)^2},\ \pi_B = \frac{(c_A+1)(2c_Ar+2r-2ac_A-a)^2}{(2c_A+1)^2(2c_A+3)^2} - L.$$

Also in this case Firm B must pay the difference between its profit net of the royalty and its profit in the entry without license case as a fixed license fee. The fixed license fee should be equal to

$$\begin{aligned} L =& \frac{(c_A+1)(2c_Ar+2r-2ac_A-a)^2}{(2c_A+1)^2(2c_A+3)^2} - \frac{a^2(2c_A+1)^2(c_B+1)}{(4c_Ac_B+4c_B+4c_A+3)^2} \\ =& \frac{C}{(2c_A+1)^2(2c_A+3)^2(4c_Ac_B+4c_B+4c_A+3)^2}. \end{aligned}$$

The total license fee is

$$TL = L + rx_B = \frac{D}{(2c_A+1)^2(2c_A+3)^2(4c_Ac_B+4c_B+4c_A+3)^2}.$$

The total profit of Firm A is equal to

$$\pi_A + TL = \frac{E}{(2c_A+1)^2(2c_A+3)^2(4c_Ac_B+4c_B+4c_A+3)^2}.$$

About details of C, D and E please see Appendix. Firm A chooses r so as to maximize $\pi_A + TL$. We get the optimal royalty rate as follows;

$$\tilde{r}^{el} = \frac{4ac_A^2+4ac_A+a}{8c_A^3+24c_A^2+18c_A+2} > 0.$$

With this royalty rate the outputs of Firms A and B are

$$x_A = \frac{a(4c_A^2+6c_A+1)}{2(c_A+1)(4c_A^2+8c_A+1)} > 0,$$

$$x_B = \frac{2ac_A}{4c_A^2+8c_A+1} > 0.$$

x_B is positive and smaller than x_A because

$$x_A - x_B = \frac{a(2c_A+1)}{2(c_A+1)(4c_A^2+8c_A+1)} > 0.$$

The price of the good is

$$p = \frac{a(2c_A+1)(4c_A^2+6c_A+1)}{2(c_A+1)(4c_A^2+8c_A+1)}.$$

Comparing p with $\tilde{r}^{el}$ yields

$$p - \tilde{r}^{el} = \frac{2ac_A(2c_A+1)}{4c_A^2+8c_A+1} > 0.$$

Thus, $0 < \tilde{r}^{el} < p$.

The fixed license fee and the total profit of Firm A are

$$L = \frac{a^2F}{(4c_A^2+8c_A+1)^2(4c_Ac_B+4c_B+4c_A+3)^2},$$

and

$$\pi_A + TL = \frac{a^2G}{4(c_A+1)(4c_A^2+8c_A+1)(4c_Ac_B+4c_B+4c_A+3)^2}.$$

About details of F and G please see Appendix. The fixed license fee, L, in this case may be negative. Assume $c_B = 10$, and denote $c_A = tc_B$, $0 < t < 1$. Then, we obtain the relation between t and L as depicted in Figure 1.

L is negative when $0 < t < \frac{96475}{33554432} \approx 0.00586$ or $1 > t > \frac{30113483}{33554432} \approx 0.89745$. Thus, when the magnitude of the innovation is small or is very large, the fixed license fee is negative.

Denote the profit of Firm A in the market and the total license fee in this case by π_A^{el} and TL^{el}.

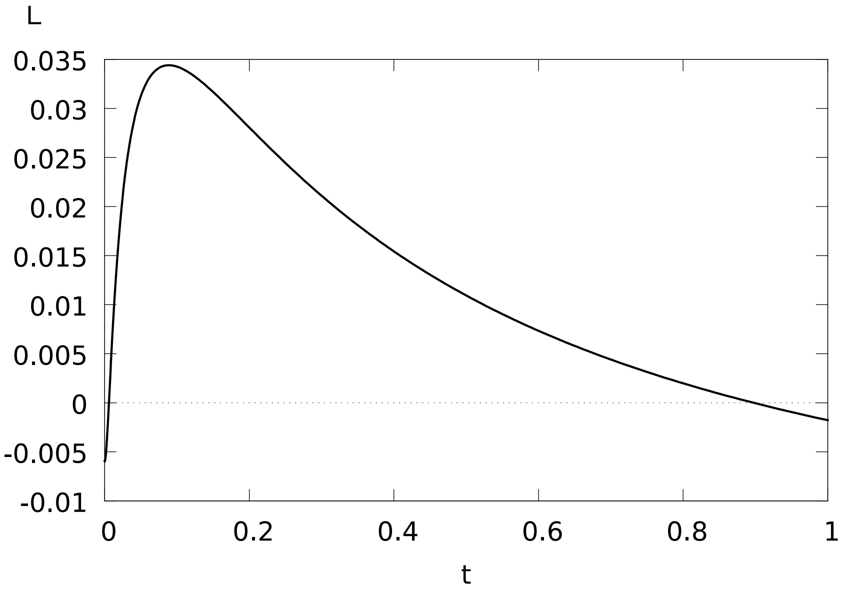

Figure 1: Relation between t and L

5.4 The optimal strategy for the innovator

Let us compare $\pi_A^{el} + TL^{el}$ and TL^l;

$$(\pi_A^{el} + TL^{el}) - TL^l = \frac{a^2 c_A^2}{(c_A + 1)(4c_A^2 + 8c_A + 1)} > 0.$$

Compare TL^l and π_A^e;

$$TL^l - \pi_A^e = \frac{a^2 H}{4(c_A + 1)(4c_A^2 + 8c_A + 1)(4c_A c_B + 4c_B + 4c_A + 3)^2} > 0.$$

About details of H please see Appendix. Therefore, entry with license strategy is the optimal strategy for the innovating firm.

6 Concluding Remark

We have analyzed the choice of options for the innovating firm under duopoly to enter the market with or without licensing its cost-reducing technology to the incumbent firm, or to license without entry, using a combination of a royalty per output and a fixed license fee. We have shown that the results depend on the form of cost functions of the firms. In the future research we want to extend the analysis in this paper to an oligopolistic situation.

Appendix: Details of calculation

$$\begin{aligned}A =&16c_A^2c_B^2r^2 + 32c_Ac_B^2r^2 + 16c_B^2r^2 + 32c_A^2c_Br^2 + 56c_Ac_Br^2 + 24c_Br^2 + 16c_A^2r^2 + 24c_Ar^2\\&+ 9r^2 - 32ac_A^2c_B^2r - 64ac_Ac_B^2r - 32ac_B^2r - 64ac_A^2c_Br - 112ac_Ac_Br - 48ac_Br - 32ac_A^2r\\&- 48ac_Ar - 18ar + 16a^2c_A^2c_B^2 + 32a^2c_Ac_B^2 + 16a^2c_B^2 - 16a^2c_A^3c_B + 36a^2c_Ac_B + 20a^2c_B\\&- 16a^2c_A^3 - 16a^2c_A^2 + 4a^2c_A + 5a^2,\end{aligned}$$

$$\begin{aligned}B =&16a^2c_A^2c_B^2 + 32a^2c_Ac_B^2 + 16a^2c_B^2 - 16a^2c_A^3c_B + 36a^2c_Ac_B + 20a^2c_B - 16a^2c_A^3\\&- 16a^2c_A^2 + 4a^2c_A + 5a^2 - 16c_A^2c_B^2r^2 - 32c_Ac_B^2r^2 - 16c_B^2r^2 - 32c_A^2c_Br^2 - 56c_Ac_Br^2\\&- 24c_Br^2 - 16c_A^2r^2 - 24c_Ar^2 - 9r^2,\end{aligned}$$

$$\begin{aligned}C =&64c_A^5c_B^2r^2 + 320c_A^4c_B^2r^2 + 640c_A^3c_B^2r^2 + 640c_A^2c_B^2r^2 + 320c_Ac_B^2r^2 + 64c_B^2r^2\\&+ 128c_A^5c_Br^2 + 608c_A^4c_Br^2 + 1152c_A^3c_Br^2 + 1088c_A^2c_Br^2 + 512c_Ac_Br^2 + 96c_Br^2 + 64c_A^5r^2\\&+ 288c_A^4r^2 + 516c_A^3r^2 + 460c_A^2r^2 + 204c_Ar^2 + 36r^2 - 128ac_A^5c_B^2r - 576ac_A^4c_B^2r\\&- 1024ac_A^3c_B^2r - 896ac_A^2c_B^2r - 384ac_Ac_B^2r - 64ac_B^2r - 256ac_A^5c_Br - 1088ac_A^4c_Br\\&- 1824ac_A^3c_Br - 1504ac_A^2c_Br - 608ac_Ac_Br - 96ac_Br - 128ac_A^5r - 512ac_A^4r - 808ac_A^3r\\&- 628ac_A^2r - 240ac_Ar - 36ar + 64a^2c_A^5c_B^2 + 256a^2c_A^4c_B^2 + 400a^2c_A^3c_B^2 + 304a^2c_A^2c_B^2\\&+ 112a^2c_Ac_B^2 + 16a^2c_B^2 - 64a^2c_A^6c_B - 192a^2c_A^5c_B - 144a^2c_A^4c_B + 96a^2c_A^3c_B + 188a^2c_A^2c_B\\&+ 92a^2c_Ac_B + 15a^2c_B - 64a^2c_A^6 - 256a^2c_A^5 - 400a^2c_A^4 - 300a^2c_A^3 - 108a^2c_A^2 - 15a^2c_A,\end{aligned}$$

$$\begin{aligned}D =&96a^2c_A^3c_B + 188a^2c_A^2c_B + 92a^2c_Ac_B + 15a^2c_B - 64a^2c_A^6 - 256a^2c_A^5 - 400a^2c_A^4\\&- 300a^2c_A^3 - 108a^2c_A^2 - 15a^2c_A - 64c_A^5c_B^2r^2 - 320c_A^4c_B^2r^2 - 608c_A^3c_B^2r^2\\&- 544c_A^2c_B^2r^2 - 224c_Ac_B^2r^2 - 32c_B^2r^2 - 128c_A^5c_Br^2 - 608c_A^4c_Br^2 - 1088c_A^3c_Br^2\\&- 912c_A^2c_Br^2 - 352c_Ac_Br^2 - 48c_Br^2 - 64c_A^5r^2 - 288c_A^4r^2 - 484c_A^3r^2 - 380c_A^2r^2 - 138c_Ar^2\\&- 18r^2 - 32ac_A^3c_B^2r - 80ac_A^2c_B^2r - 64ac_Ac_B^2r - 16ac_B^2r - 64ac_A^3c_Br - 144ac_A^2c_Br\\&- 104ac_Ac_Br - 24ac_Br - 32ac_A^3r - 64ac_A^2r - 42ac_Ar - 9ar + 64a^2c_A^5c_B^2 + 256a^2c_A^4c_B^2\\&+ 400a^2c_A^3c_B^2 + 304a^2c_A^2c_B^2 + 112a^2c_Ac_B^2 + 16a^2c_B^2 - 64a^2c_A^6c_B - 192a^2c_A^5c_B\\&- 144a^2c_A^4c_B,\end{aligned}$$

$$\begin{aligned} E =& 336a^2c_A^4c_B + 800a^2c_A^3c_B + 692a^2c_A^2c_B + 268a^2c_Ac_B + 39a^2c_B - 64a^2c_A^6 \\ & - 192a^2c_A^5 - 176a^2c_A^4 + 8a^2c_A^3 + 100a^2c_A^2 + 54a^2c_A + 9a^2 - 64c_A^5c_B^2r^2 - 320c_A^4c_B^2r^2 \\ & - 592c_A^3c_B^2r^2 - 496c_A^2c_B^2r^2 - 176c_Ac_B^2r^2 - 16c_B^2r^2 - 128c_A^5c_Br^2 - 608c_A^4c_Br^2 \\ & - 1056c_A^3c_Br^2 - 824c_A^2c_Br^2 - 272c_Ac_Br^2 - 24c_Br^2 - 64c_A^5r^2 - 288c_A^4r^2 - 468c_A^3r^2 \\ & - 340c_A^2r^2 - 105c_Ar^2 - 9r^2 + 64ac_A^4c_B^2r + 192ac_A^3c_B^2r + 208ac_A^2c_B^2r + 96ac_Ac_B^2r \\ & + 16ac_B^2r + 128ac_A^4c_Br + 352ac_A^3c_Br + 352ac_A^2c_Br + 152ac_Ac_Br + 24ac_Br + 64ac_A^4r \\ & + 160ac_A^3r + 148ac_A^2r + 60ac_Ar + 9ar + 128a^2c_A^5c_B^2 + 512a^2c_A^4c_B^2 + 800a^2c_A^3c_B^2 \\ & + 608a^2c_A^2c_B^2 + 224a^2c_Ac_B^2 + 32a^2c_B^2 - 64a^2c_A^6c_B - 64a^2c_A^5c_B, \end{aligned}$$

$$\begin{aligned} F =& 64c_A^5c_B^2 + 192c_A^4c_B^2 + 192c_A^3c_B^2 + 64c_A^2c_B^2 - 64c_A^6c_B - 192c_A^5c_B - 208c_A^4c_B \\ & - 96c_A^3c_B - 44c_A^2c_B - 20c_Ac_B - c_B - 64c_A^6 - 256c_A^5 - 400c_A^4 - 284c_A^3 - 104c_A^2 - 20c_A - 1, \end{aligned}$$

$$\begin{aligned} G =& 128c_A^4c_B^2 + 384c_A^3c_B^2 + 400c_A^2c_B^2 + 160c_Ac_B^2 + 16c_B^2 - 64c_A^5c_B + 352c_A^3c_B \\ & + 464c_A^2c_B + 196c_Ac_B + 20c_B - 64c_A^5 - 128c_A^4 - 32c_A^3 + 72c_A^2 + 44c_A + 5, \end{aligned}$$

$$\begin{aligned} H =& 64c_A^4c_B^2 + 128c_A^3c_B^2 + 64c_A^2c_B^2 - 64c_A^5c_B - 64c_A^4c_B + 96c_A^3c_B + 128c_A^2c_B \\ & + 36c_Ac_B + 4c_B - 64c_A^5 - 144c_A^4 - 96c_A^3 - 12c_A^2 + 4c_A + 1 \\ =& 64c_A^4c_B(c_B - c_A) + 64c_A^3(2c_B^2 - c_Ac_B - c_A^2) + 16c_A^2(4c_B^2 + 6c_Ac_B - 9c_A^2) \\ & + 32c_A^2(4c_B - 3c_A) + 12c_A(3c_B - c_A) + 4(c_B - c_A) + 1 > 0. \end{aligned}$$

References

Boone, J. (2001), “Intensity of competition and the incentive to innovate,” *International Journal of Industrial Organization*, **19**, pp.705-726.

Chen C.-S. (2016), “Endogenous market structure and technology licensing,” *Japanese Economic Review*, forthcoming.

Creane, A., Chiu, Y. K. and Konishi, H. (2013), “Choosing a licensee from heterogeneous rivals,” *Games and Economic Behavior*, **82**, pp. 254-268.

Dixit, A. (1986), “Comparative statics for oligopoly, ”, *International Economic Review*, **27**, pp. 107-122

Duchene, A., Sen, D. and Serfes, K. (2015), “Patent licensing and entry deterrence: The role of low royalties,” *Economica*, **82**, pp. 1324-1348.

Hattori, M. and Tanaka, Y. (2015), “Subsidy or tax policy for new technology adoption in duopoly with quadratic and linear cost functions,” *Economics Bulletin*, **35**, pp. 1423-1433.

Hattori, M. and Tanaka, Y. (2016a), “Subsidizing new technology adoption in a Stackelberg duopoly: cases of substitutes and complements,” *Italian Economic Journal*, **2**, pp. 197-215.

Hattori, M. and Tanaka, Y. (2016b), “License or entry with vertical differentiation in duopoly,” *Economics and Business Letters*, **5**, pp. 17-29.

Hattori, M. and Tanaka, Y. (2017a), “License and entry strategies for an outside innovator under duopoly,” *Italian Economic Journal*, forthcoming.

Hattori, M. and Tanaka, Y. (2017b), “Robustness of subsidy in licensing with outside innovator: General demand and cost functions,” mimeograph.

Kabiraj, T. (2004), “Patent licensing in a leadership structure,” *The Manchester School*, **72**, pp.188-205.

Kamien, T. and Tauman, Y. (1986), “Fees versus royalties and the private value of a patent,” *Quarterly Journal of Economics*, **101**, pp.471-492.

Kamien, T. and Tauman, Y. (2002), “Patent licensing: the inside story,” *The Manchester School*, **70**, pp.7-15.

Katz, M. and Shapiro, C. (1985), “On the licensing of innovations,” *Rand Journal of Economics*, **16**, pp. 504-520.

La Manna M. (1993), “Asymmetric Oligopoly and Technology Transfers,” *Economic Journal*, **103**, pp.436-443.

Liao, C. and Sen, D. (2005), “Subsidy in licensing: optimality and welfare implications,” *The Manchester School*, **73**, pp.281-299.

Matsumura, T., Matsushima, N. and Cato, S. (2013), “Competitiveness and R&D competition revisited,” *Economic modelling*, **31**, pp.541-547.

Pal, R. (2010), “Technology adoption in a differentiated duopoly: Cournot versus Bertrand,” *Research in Economics*, **64**, pp. 128-136.

Rebolledo, M. and J. Sandonís (2012), “The effectiveness of R&D subsidies,” *Economics of Innovation and New Technology*, **21**, pp. 815-825.

Seade, J. K., “The stability of Cournot revisited,” *Journal of Economic Theory*, **15**, pp. 15-27.

Sen, D. and Stamatopoulos, G. (2016) “Licensing under general demand and cost functions,” *European Journal of Operations Research*, **253**, pp. 673-680.

Sen, D. and Tauman, Y. (2007), "General licensing schemes for a cost-reducing Innovation," *Games and Economic Behavior*, **59**, pp.163-186.

Watanabe N. and Muto, S. (2008), "Stable profit sharing in a patent licensing game: general bargaining outcomes," *International Journal of Game Theory*, **37**, pp.505-523.

Wang, X. H. and Yang, B. Z. (2004), "On technology licensing in a Stackelberg duopoly," *Australian Economic Papers*, **43**, pp. 448-458.