License and entry strategies for an outside innovator in duopoly with combination of royalty and fixed fee under vertical differentiation

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Abstract

We consider a choice of options for an innovating firm in duopoly under vertical differentiation to enter the market with or without licensing its technology for producing a higher quality good to the incumbent firm using a combination of a royalty per output and a fixed license fee, or to license its technology without entry. With general distribution function of consumers' taste parameter and cost function we will show that when the innovating firm licenses its technology to the incumbent firm without entry, the optimal royalty rate per output is zero with negative fixed fee, and when the innovating firm enters the market with a license to the incumbent firm, its optimal royalty rate is positive with positive or negative fixed fee. Also we show that when cost function is concave, the optimal royalty rate is one such that the incumbent firm drops out of the market; and when cost function is strictly convex, there is an internal solution of the optimal royalty rate under duopoly.

Keywords: duopoly, royalty, fixed license fee, vertical differentiation.

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1. Introduction

We consider a choice of options for an innovating firm to enter the market with or without licensing its new technology for producing a higher quality good to the incumbent firm using a combination of a royalty per output and a fixed license fee, or to license its technology without entry also using a combination of a royalty per output and a fixed license fee under vertical product differentiation.

In Proposition 4 of Kamien and Tauman (1986), assuming linear demand and cost functions, fixed license fee and cost-reducing new technology, it was argued that in an oligopoly when the number of firms is small (or large), entry with license strategy by the innovating firm, which is a strategy to enter the market and at the same time license its cost-reducing technology to an incumbent firm, is more profitable than license without entry strategy, which is a strategy to license its technology to an incumbent firm without entering the market. We think that their definition of license fee in the case where the innovating firm licenses its technology to an incumbent firm and does not enter the market is not appropriate. Interpreting their analysis in a duopoly model, they defined the license fee in that case by the difference between the profit of an incumbent firm in that case and its monopoly profit before entry and license by the innovating firm. However, we can think that if the negotiation between the innovating firm and an incumbent firm about the license fee breaks down, the innovating firm can enter the market without license to an incumbent firm. If the innovating firm does not enter the market nor license, its profit is zero. But, if it enters the market, its profit is positive. Therefore, such a threat is credible, and hence an incumbent firm must pay the difference between its profit in the license without entry case and its profit in the entry without license case as a license fee.

In Hattori and Tanaka (2016b), using an alternative definition of a license fee taking the above point into account, the following results about duopoly with uniform distribution of consumers' taste parameter, only a fixed license fee and new technology for producing a higher quality good under vertical product differentiation have been shown.

1. Linear cost functions (constant marginal costs):

   If the incumbent firm does not drop out when the innovating firm enters the market without license, license without entry strategy is optimal for the innovating firm. This result is converse to that in Kamien and Tauman (1986). If the incumbent firm drops out when the innovating firm enters the market without license, both license without entry strategy and entry without license strategy are optimal[1].

2. Quadratic cost functions:

   If the magnitude of the innovation is large (quality of the high quality good is sufficiently high), license without entry strategy is optimal for the innovating firm, and if the magnitude of the innovation is small, entry with license strategy is optimal.

In this paper we consider a more general situation of duopoly under vertical differentiation with an innovating firm and an incumbent firm, in which the innovating firm imposes a combination of a royalty per output and a fixed license fee to the incumbent firm. We analyse

[1] When the incumbent firm drops out of the market, the innovation is said to be drastic.

a case of general distribution function of consumers' taste parameter and general cost function as well as a case of general distribution and concave cost function and a case of general distribution and strictly convex cost function. We will show the following results.

General distribution and cost function case

1. When the innovating firm licenses its technology to the incumbent firm without entry, the optimal royalty rate per output for the innovating firm is zero.
2. When the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, the optimal royalty rate per output is positive.

General distribution and concave cost function case

1. If the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, and the cost function is concave, the optimal royalty rate per output for the innovating firm is one such that the output of the incumbent firm is zero.
2. The fixed license fee is negative.
3. License without entry strategy and entry with license strategy are optimal for the innovator.

General distribution and strictly convex cost function case

1. If the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, and the cost function is strictly convex, the optimal royalty rate per output for the innovating firm is positive but smaller than one such that the output of the incumbent firm is zero.
2. The equilibrium output of the innovating firm is larger than that of the incumbent firm.
3. Entry with license strategy is optimal for the innovator.

In this case the fixed license fee may be positive or negative. Please see an example in Section 5.

In the next section we review some related studies. In Section 3 we describe the model of this paper. In Section 4 we present the main results, and in Section 5 we study a case of uniform distribution of consumers' taste parameter and quadratic cost function as an example.

2. Literature review

Various studies focus on technology adoption or R&D investment in duopoly or oligopoly. Most of them analyze the relation between the technology licensor and licensee. The difference of means of contracts, which comprise royalties, upfront fixed fees, combinations of these two, and auctions, are well discussed (Katz and Shapiro (1985)). Kamien and Tauman (2002) showed that outside innovators prefer auctions, but industry incumbents prefer royalty. This topic is discussed by Kabiraj (2004) under the Stackelberg oligopoly; here, the licensor does not have production capacity. Wang and Yang (2004) considered the case when the licensor has production capacity. Sen and Tauman (2007) compared the license system in detail, namely, when the licensor is an outsider and when it is an incumbent firm, using the combination of royalties and fixed fees. However, the existence of production capacity was externally given, and they did not analyze the choice of entry. Therefore, the optimal strategies of outside innovators, who can use the entry as a threat, require more discussion. Regarding the strategies of new entrants to the market, Duchene, Sen and Serfes (2015) focused on future entrants with old technology, and argued that a low license fee can be used to deter the entry of potential entrants. However, the firm with new technology is incumbent, and its choice of entry is not analyzed. Also, Chen (2016) analyzed the model of the endogenous market structure determined by the potential entrant with old technology and showed that the licensor uses the fixed fee and zero royalty in both the incumbent and the outside innovator cases, which are exogenously given. Creane, Chiu and Konishi (2013) examined a firm that can license its production technology to a rival when firms are heterogeneous in production costs, and showed that a complete technology transfer from one firm to another always increases joint profit under weakly concave demand when at least three firms remain in the industry.

A Cournot oligopoly with fixed fee under cost asymmetry was analyzed by La Manna (1993). He showed that if technologies can be replicated perfectly, a lower cost firm always has the incentive to transfer its technology; hence, while a Cournot-Nash equilibrium cannot be fully asymmetric, there exists no non-cooperative Nash equilibrium in pure strategies. On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyzed bargaining between a licensor with no production capacity and oligopolistic firms. Recent research focuses on market structure and technology improvement. Boone (2001) and Matsumura et. al. (2013) found a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) showed that technology adoption may change the market outcome. The social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may result in higher social welfare than Bertrand competition under a differentiated goods market. Hattori and Tanaka (2015) and (2016a) studied the adoption of new technology in Cournot duopoly and Stackelberg duopoly. Rebolledo and Sandonís (2012) presented an analysis of the effectiveness of research and development (R&D) subsidies in an oligopolistic model in the cases of international competition and cooperation in R&D. Hattori and Tanaka (2016b) analyzed problems about product innovation, that is, introduction of higher quality good in a duopoly with vertical product differentiation. Recently, Sen and Stamatopoulos (1980) presented an analysis of royalty and fixed fee under duopoly with general demand and cost function. They did not considered an option of the innovator whether it enter the market or not.

In this paper we will show that a combination of non-negative royalty and negative or positive fixed fee is optimal for the innovator under duopoly with options for the innovator to enter or not to enter the market with or without license. On the other hand, Liao and Sen (2005) showed that negative royalty can be optimal under oligopoly with one innovator and two incumbent firms. When the innovator is holding relatively insignificant new technology, licensing it to only one firm with negative royalty is optimal. This strategy leads licensee more aggressive and getting more profit which is paid to licensor as a fixed license fee. This negative royalty may result in more social welfare than that where negative royalty is prohibited[2].

3. The model

Our model of vertical product differentiation is according to Mussa and Rosen (1978), Bonanno and Haworth (1998) and Tanaka (2001). There are two firms Firms A and B. Firm A can produce the high-quality good whose quality is k_H, and Firm B produces the low-quality good whose quality is k_L, where $k_H > k_L > 0$. k_H and k_L are fixed. At present only Firm B operates as a monopolist in the market. Both of the high-quality and low-quality goods are produced at the same cost.

Now Firm A have three options. The first option is to enter the market without license to Firm B, the second option is to license its technology for producing the high-quality good to Firm B using a combination of a royalty per output and a fixed license fee, and the third option is to enter the market with license to Firm B also using a combination of a royalty per output and a fixed license fee. If Firm A enters the market, the market becomes a duopoly with or without vertical differentiation. If Firm A enters with license, both firms produce the high-quality good. If it enters without license, Firm A produces the high-quality good, but Firm B produces the low-quality good. The cost function of the high-quality and low-quality goods is denoted by $c(\cdot)$.

In the market there is a continuum of consumers with the same income, denoted by y, but different values of the taste parameter θ. Each consumer buys at most one unit of the good. If a consumer with parameter θ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \theta k$. If a consumer does not buy the good, his utility is equal to his income y. The parameter θ is distributed according to a smooth distribution function $\rho = F(\theta)$ in the interval $0 < \theta \leq 1$. ρ denotes the probability that the taste parameter is smaller than or equal to θ. The size of consumers is normalized as one. The inverse function of $F(\theta)$ is denoted by $G(\rho)$. We have $F'(\theta) > 0$ and $G'(\rho) > 0$. Note that $G(1) = 1$.

Let p_L be the price of the good of quality k_L and p_H be the price of the good of quality k_H; and let q_A and q_B be the outputs of Firms A and B.

4. The main results

4.1. Entry without license

First suppose that Firm A enters the market without license to Firm B. Then, Firm A produces the high-quality good and Firm B produces the low-quality good. Let θ_L be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the

[2] They assumed linear demand and cost functions. Their analysis about outside innovator case is extended to general demand and cost functions by Hattori and Tanaka (2017).

low-quality good. Then,

$$\theta_L = \frac{p_L}{k_L}.$$

Let θ_H be the value of θ for which the corresponding consumer is indifferent between buying the low-quality good and the high-quality good. Then

$$\theta_H = \frac{p_H - p_L}{k_H - k_L}.$$

Let $q_H = q_A$ and $q_L = q_B$. The inverse demand function is described as follows.

1. When $q_H > 0$ and $q_L > 0$, we have $p_H = (k_H - k_L)G(1 - q_H) + k_L G(1 - q_H - q_L)$ and $p_L = k_L G(1 - q_H - q_L)$.

2. When $q_H > 0$ and $q_L = 0$, we have $p_H = k_H G(1 - q_H)$ and $p_L = k_L G(1 - q_H)$.

3. When $q_H = 0$ and $q_L > 0$, we have $p_H = k_H - k_L + k_L G(1 - q_L)$ and $p_L = k_L G(1 - q_L)$.

4. When $q_H = 0$ and $q_L = 0$, we have $p_H = k_H$ and $p_L = k_L$.

Since $G(1) = 1$, this is a continuously differentiable function with the domain $0 \leq q_H \leq 1$ and $0 \leq q_H \leq 1$. About details for derivation of the inverse demand function please see A.3.

The profits of Firms A and B are written as

$$\pi_A = [(k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B)]q_A - c(q_A),$$

$$\pi_B = k_L G(1 - q_A - q_B)q_B - c(q_B).$$

The first order conditions for profit maximization of Firms A and B are

$$\begin{aligned}\frac{\partial \pi_A}{\partial q_A} =&(k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B) \\ &- [(k_H - k_L)G'(1 - q_A) + k_L G'(1 - q_A - q_B)]q_A - c'(q_A) = 0,\end{aligned}$$

$$\frac{\partial \pi_B}{\partial q_B} = k_L G(1 - q_A - q_B) - k_L G'(1 - q_A - q_B)q_B - c'(q_B) = 0.$$

The second order conditions are

$$\begin{aligned}\frac{\partial^2 \pi_A}{\partial q_A^2} =& - 2[(k_H - k_L)G'(1 - q_A) + k_L G'(1 - q_A - q_B)] \\ &+ [(k_H - k_L)G''(1 - q_A) + k_L G''(1 - q_A - q_B)]q_A - c''(q_A) < 0,\end{aligned}$$

$$\frac{\partial^2 \pi_B}{\partial q_B^2} = -k_L[2G'(1 - q_A - q_B) - G''(1 - q_A - q_B)q_B] - c''(q_B) < 0.$$

We assume that the second order conditions are satisfied in each case. Denote the equilibrium profit of Firm B by π_B^e.

4.2. License without entry

Next suppose that Firm A licenses its technology for producing the high-quality good to Firm B using a combination of a royalty per output and a fixed license fee, and does not enter the market. Then, Firm B is a monopolist. Let r be the royalty rate per output and L be the fixed license fee. Suppose that the licensor can take all of the increase in the profit of Firm B due to adoption of the new high-quality good.

Let θ_0 be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the high-quality good. Then,

$$\theta_0 = \frac{p_H}{k_H}.$$

Let $q_H = q_B$. The inverse demand function is described as follows.

1. When $q_H > 0$, we have $p_H = k_H G(1 - q_H)$.
2. When $q_H = 0$, we have $p_H = k_H$.

This is a continuously differentiable function with the domain $0 \leq q_H \leq 1$. About details for derivation of the inverse demand function please see A.1.

The profit of Firm B is

$$\pi_B = k_H G(1 - q_B)q_B - c(q_B) - rq_B - L.$$

The first order condition for profit maximization of Firm B is

$$\frac{\partial \pi_B}{\partial q_B} = k_H G(1 - q_B) - k_H G'(1 - q_B)q_B - c'(q_B) - r = 0.$$

The second order condition is

$$\frac{\partial^2 \pi_B}{\partial q_B^2} = -k_H[2G'(1 - q_B) - G''(1 - q_B)q_B] - c''(q_B) < 0.$$

From these conditions we get

$$\frac{dq_B}{dr} = \frac{1}{-k_H[2G'(1 - q_B) - G''(1 - q_B)q_B] - c''(q_B)} < 0.$$

If the negotiation between Firm A and Firm B about the license fee breaks down, Firm A can enter the market without license. When Firm A does not enter nor sell a license, its profit is zero; however, when it enters the market without license, its profit is positive. Therefore, such a threat is credible, and Firm B must pay the difference between its profit net of the royalty and its profit in the entry without license case as a fixed license fee. The fixed license fee is determined so that $\pi_B = \pi_B^e$ is satisfied, and it is written as

$$L = k_H G(1 - q_B)q_B - c(q_B) - rq_B - \pi_B^e$$

Note that π_B^e is a constant number. The total license fee is the sum of the royalty and the fixed license fee. Let TL be the total license fee. Then,

$$TL = L + rq_B = k_H G(1 - q_B)q_B - c(q_B) - \pi_B^e.$$

The condition for maximization of TL with respect to r is

$$\frac{dTL}{dr} = (k_H G(1 - q_B) - k_H G'(1 - q_B)q_B - c'(q_B))\frac{dq_B}{dr} = r\frac{dq_B}{dr} = 0.$$

Since $\frac{dq_B}{dr} < 0$, the optimal royalty rate per output, $\tilde{r}^l$, for the innovating firm is obtained as follows.

$$\tilde{r}^l = 0.$$

We have shown the following result.

Proposition 1. *When the innovating firm licenses its technology to the incumbent firm without entry, the optimal royalty rate per output for the innovating firm is zero.*

4.3. Entry with license

Suppose that Firm A enters the market and at the same time licenses its technology for producing the high-quality good to Firm B using a combination of a royalty per output and a fixed license fee. Suppose that the licensor can take all of the increase in the profit of Firm B due to adoption of the new high-quality good. In this case both firms produce the high-quality good.

Let $q_H = q_A + q_B$. Similarly to the previous case the inverse demand function is as follows.

1. When $q_H > 0$, we have $p_H = k_H G(1 - q_H)$.
2. When $q_H = 0$, we have $p_H = k_H$.

The profits of Firms A and B are

$$\pi_A = k_H G(1 - q_A - q_B)q_A - c(q_A),$$

$$\pi_B = k_H G(1 - q_A - q_B)q_B - c(q_B) - rq_B - L.$$

The first order conditions for profit maximization of Firms A and B are

$$\frac{\partial \pi_A}{\partial q_A} = k_H G(1 - q_A - q_B) - k_H G'(1 - q_A - q_B)q_A - c'(q_A) = 0, \tag{1}$$

$$\frac{\partial \pi_B}{\partial q_B} = k_H G(1 - q_A - q_B) - k_H G'(1 - q_A - q_B)q_B - c'(q_B) - r = 0. \tag{2}$$

The second order conditions are

$$\frac{\partial^2 \pi_A}{\partial q_A^2} = -k_H[2G'(1 - q_A - q_B) - G''(1 - q_A - q_B)q_A] - c''(q_A) < 0,$$

$$\frac{\partial^2 \pi_B}{\partial q_B^2} = -k_H[2G'(1-q_A-q_B) - G''(1-q_A-q_B)q_B] - c''(q_B) < 0.$$

Differentiating (1) and (2) with respect to r yields

$$\begin{aligned}&\{-k_H[2G'(1-q_A-q_B) - G''(1-q_A-q_B)q_A] - c''(q_A)\}\frac{dq_A}{dr}\\ &- k_H[G'(1-q_A-q_B) - G''(1-q_A-q_B)q_A]\frac{dq_B}{dr} = 0,\end{aligned}$$

$$\begin{aligned}&- k_H[G'(1-q_A-q_B) - G''(1-q_A-q_B)q_B]\frac{dq_A}{dr}\\ &+ \{-k_H[2G'(1-q_A-q_B) - G''(1-q_A-q_B)q_B] - c''(q_B)\}\frac{dq_B}{dr} - 1 = 0.\end{aligned}$$

Solving them, we obtain

$$\frac{dq_A}{dr} = \frac{1}{\Delta}\{k_H[G'(1-q_A-q_B) - G''(1-q_A-q_B)q_A]\},$$

and

$$\frac{dq_B}{dr} = \frac{1}{\Delta}\{-k_H[2G'(1-q_A-q_B) - G''(1-q_A-q_B)q_A] - c''(q_A)\} < 0,$$

where

$$\begin{aligned}\Delta =&\{k_H[2G'(1-q_A-q_B) - G''(1-q_A-q_B)q_A] + c''(q_A)\}\{k_H[2G'(1-q_A-q_B)\\ &- G''(1-q_A-q_B)q_B] + c''(q_B)\} - \{k_H[G'(1-q_A-q_B)\\ &- G''(1-q_A-q_B)q_A]\}\{k_H[G'(1-q_A-q_B) - G''(1-q_A-q_B)q_B]\}.\end{aligned}$$

We assume

$$\Delta > 0.$$

Also we assume

$$|k_H[2G'(1-q_A-q_B) - G''(1-q_A-q_B)q_A] + c''(q_A)| > |k_H[G'(1-q_A-q_B) - G''(1-q_A-q_B)q_A|,$$

and

$$|k_H[2G'(1-q_A-q_B) - G''(1-q_A-q_B)q_B] + c''(q_B)| > |k_H[G'(1-q_A-q_B) - G''(1-q_A-q_B)q_B|.$$

These assumptions are obtained from the stability conditions for the equilibrium of duopoly[3]. Then,

$$-k_H G'(1-q_A-q_B) - c''(q_A) < 0,\ -k_H G'(1-q_A-q_B) - c''(q_B) < 0.$$

Hence, we have

$$\frac{dq_A}{dr} + \frac{dq_B}{dr} = \frac{1}{\Delta}[-k_H G'(1-q_A-q_B) - c''(q_A)] < 0,$$

[3]See Seade (1980) and Dixit (1986)

and

$$\left|\frac{dq_B}{dr}\right| > \left|\frac{dq_A}{dr}\right|. \tag{3}$$

We have $\frac{dq_A}{dr} > 0$ when $-k_H[G'(1-q_A-q_B) - G''(1-q_A-q_B)q_A] < 0$ and $\frac{dq_A}{dr} < 0$ when $-k_H[G'(1-q_A-q_B) - G''(1-q_A-q_B)q_A] > 0$. In the former case the goods of the firms are strategic substitutes, and in the latter case they are strategic complements. These properties do not affect the main results of this paper.

Similarly to the previous case, Firm B must pay the difference between its profit net of the royalty and its profit in the entry without license case as a fixed license fee. The fixed license fee, L, is determine so that $\pi_B = \pi_B^e$ is satisfied. It is written as

$$L = k_H G(1-q_A-q_B)q_B - c(q_B) - rq_B - \pi_B^e.$$

The total license fee, TL, is

$$TL = L + rq_B = k_H G(1-q_A-q_B)q_B - c(q_B) - \pi_B^e.$$

The total profit of Firm A is the sum of the total license fee and its profit as a firm in the duopoly. It is equal to

$$\pi_A + TL = \pi_A + L + rq_B = k_H G(1-q_A-q_B)q_A - c(q_A) + k_H G(1-q_A-q_B)q_B - c(q_B) - \pi_B^e.$$

π_B^e is constant. Firm A chooses r so as to maximize $\pi_A + TL$. Differentiating $\pi_A + TL$ with respect to r yields

$$\begin{aligned}\frac{d}{dr}(\pi_A + TL) =& [k_H G(1-q_A-q_B) - k_H G'(1-q_A-q_B)q_A - c'(q_A) \\ & - k_H G'(1-q_A-q_B)q_B]\frac{dq_A}{dr} + [k_H G(1-q_A-q_B) \\ & - k_H G'(1-q_A-q_B)q_B - c'(q_B) - k_H G'(1-q_A-q_B)q_A]\frac{dq_B}{dr} \\ =& - k_H G'(1-q_A-q_B)q_B\frac{dq_A}{dr} + (r - k_H G'(1-q_A-q_B)q_A)\frac{dq_B}{dr}.\end{aligned} \tag{4}$$

If there is an internal solution of r which maximizes $\pi_A + TL$, it is

$$\tilde{r}^{el} = \frac{k_H G'(1-q_A-q_B)}{\frac{dq_B}{dr}}\left(q_A\frac{dq_B}{dr} + q_B\frac{dq_A}{dr}\right).$$

We show that the optimal royalty rate is positive.

Proposition 2. *When the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, its optimal royalty rate per output is positive.*

Proof. Suppose $r = 0$. Then, (1) and (2) mean $q_A = q_B$. From (3)

$$q_A\frac{dq_B}{dr} + q_B\frac{dq_A}{dr} < 0.$$

Substituting $r = 0$ into (4), we find

$$\frac{d}{dr}(\pi_A + TL)\bigg|_{r=0} = -k_H G'(1 - q_A - q_B)\left(q_A \frac{dq_B}{dr} + q_B \frac{dq_A}{dr}\right) > 0.$$

Therefore, the optimal royalty rate is positive. □

Now we consider two specific cases.

Concave cost function case

Assume that the cost functions of Firms A and B before adoption of the new technology are $c(q_A)$ and $c(q_B)$ such that $c''(\cdot) \leq 0$ or $c'(x) < c'(y)$ for $x > y$. From (1) and (2)

$$k_H G(1 - q_A - q_B) - k_H G'(1 - q_A - q_B)q_A - c'(q_A) = 0,$$

$$k_H G(1 - q_A - q_B) - k_H G'(1 - q_A - q_B)q_B - c'(q_B) - r = 0.$$

Suppose $q_B = 0$. Then,

$$r = k_H G(1 - q_A - q_B) - c'(0) = p_H - c'(0),$$

Denote this value of the royalty rate by $\bar{\bar{r}}$. It is a value of the royalty rate such that the output of Firm B is just zero, that is, it drops out of the market. We call such a royalty rate *prohibitive*. Also we have

$$\bar{\bar{r}} - k_H G'(1 - q_A - q_B)q_A = c'(q_A) - c'(0) \leq 0.$$

Substituting this and $q_B = 0$ into (4) yields

$$\frac{d}{dr}(\pi_A + TL) = (c'(q_A) - c'(0))\frac{dq_B}{dr} \geq 0.$$

Therefore, the optimal royalty rate per output is $\bar{\bar{r}}$. In this case $\tilde{r}^{el} = k_H G'(1 - q_A - q_B)q_A$. Comparing $\bar{\bar{r}}$ and $\tilde{r}^{el}$,

$$\bar{\bar{r}} - \tilde{r}^{el} = c'(q_A) - c'(0) \leq 0.$$

However, it is nonsense to impose a royalty larger than $\bar{\bar{r}}$. The fixed license fee is negative as the following inequality shows

$$L = k_H G(1 - q_A - q_B)q_B - c(q_B) - rq_B - \pi_B^e = -c(0) - \pi_B^e < 0.$$

It compensates the profit of Firm B in the case of entry without license.

We have shown the following result.

Proposition 3. 1. *If the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, and the cost function is concave, the optimal royalty rate per output for the innovating firm is one such that the output of the incumbent firm is zero, that is, the royalty rate per output is prohibitive.*

2. *In this case the fixed license fee is negative.*

Strictly convex cost function case

Assume that the cost functions of Firms A and B before adoption of the new technology are $c(q_A)$ and $c(q_B)$ such that $c''(\cdot) > 0$ or $c'(x) > c'(y)$ for $x > y$. (1) and (2) are rewritten as

$$k_H G(1 - q_A - q_B) - k_H G'(1 - q_A - q_B)q_A - c'(q_A) = 0, \tag{5}$$

and

$$k_H G(1 - q_A - q_B) - k_H G'(1 - q_A - q_B)q_B - c'(q_B) - r = 0. \tag{6}$$

Assume $q_B = 0$. Then,

$$\bar{\bar{r}} = k_H G(1 - q_A - q_B) - c'(0) = p_H - c'(0),$$

and

$$\bar{\bar{r}} - k_H G'(1 - q_A - q_B)q_A = c'(q_A) - c'(0) > 0.$$

Substituting this and $q_B = 0$ into (4) yields

$$\frac{d}{dr}(\pi_A + TL) = (c'(q_A) - c'(0))\frac{dq_B}{dr} < 0.$$

Therefore, there is an internal solution of the optimal royalty rate, $\tilde{r}^{el} = k_H G'(1-q_A-q_B)q_A > 0$. It is smaller than $\bar{\bar{r}}$. From (5) and (6), q_A is larger than q_B.

We have shown the following result.

Proposition 4. *1. If the innovating firm enters the market and at the same time licenses its technology to the incumbent firm, and the cost function is strictly convex, the optimal royalty rate per output for the innovating firm is positive and smaller than one such that the output of the incumbent firm is zero.*

2. The equilibrium output of Firm A is larger than that of Firm B.

The fixed license fee in this case may be positive or negative. Please see an example in the next section.

4.4. The optimal strategy for the innovator

In this subsection we consider the optimal strategy for the innovating firm. The results depend on the form of cost function.

Concave cost function case

When the cost function is concave, entry with license strategy and license without entry strategy are equivalent. In both cases the monopolistic situation is realized. In the license without entry case the monopolist is Firm B, and in the case of entry with license it is Firm A. Because the payoff of Firm A in the monopolistic situation is larger than its profit in the duopolistic situation when it enters the market without license, license without entry strategy and entry with license strategy are optimal.

The monopoly profit including royalty revenue is maximized at zero royalty rate. Thus, the optimal royalty rate in the case of license without entry is zero. On the other hand, in the case of entry with license the market is duopolistic with small royalty rate. When the cost function is concave, the monopolistic situation is optimal for the innovating firm. Therefore, the innovating firm gets larger profit by driving out the incumbent firm from the market with prohibitive royalty rate. Then, we need negative fixed fee to compensate the profit of the incumbent firm that it can get in the case of entry without license.

Strictly convex cost function case

In the case where Firm A enters the market with license, setting the value of r as one such that the output of Firm B is zero, the monopolistic situation which is the same as that in the case of license without entry can be realized. On the other hand, the optimal royalty rate per output is different from such a value. Therefore, entry with license strategy is optimal.

Summarizing the results in the following proposition;

Proposition 5. *1. When the cost function is concave, license without entry strategy and entry with license strategy are optimal for the innovating firm.*

2. When the cost function is strictly convex, entry with license strategy is optimal for the innovating firm.

In the case of entry with license the market is duopolistic, and when the cost function is strictly convex, the payoff of the innovating firm in duopolistic situation is larger than that in monopolistic situation because partition of production between two firms is more efficient than concentration of production to one firm under strictly convex cost function. There is a positive internal solution of the optimal royalty rate which is not prohibitive.

5. An example of uniform distribution and quadratic cost function case

Assume that $\rho = F(\theta)$ has a uniform distribution, the (common) cost function is quadratic and there is no fixed cost. Then, $\rho = \theta$, $\theta = G(\rho) = \rho$, $F'(\theta) = G'(\rho) = 1$ and $F''(\theta) = G''(\rho) = 0$. The cost functions of Firms A and B are cq_A^2 and cq_B^2 with $c > 0$.

5.1. Entry without license

Suppose that Firm A enters the market without license. The equilibrium values of the variables are as follows.

$$q_A = \frac{2k_Hk_L + 2ck_H - k_L^2}{4k_Hk_L + 4ck_L + 4ck_H + 4c^2 - k_L^2},\ q_B = \frac{k_L(k_H + 2c)}{4k_Hk_L + 4ck_L + 4ck_H + 4c^2 - k_L^2},$$

$$p_H = \frac{(k_H + 2c)(2k_Hk_L - k_L^2 + 2ck_H)}{4k_Hk_L + 4ck_L + 4ck_H + 4c^2 - k_L^2},\ p_L = \frac{k_L(k_H + 2c)(k_L + 2c)}{4k_Hk_L + 4ck_L + 4ck_H + 4c^2 - k_L^2},$$
$$\pi_A = \frac{(k_H + c)(2k_Hk_L + 2ck_H - k_L^2)^2}{(4k_Hk_L + 4ck_L + 4ck_H + 4c^2 - k_L^2)^2},\ \pi_B^e = \frac{k_L^2(k_H + 2c)^2(k_L + c)}{(4k_Hk_L + 4ck_L + 4ck_H + 4c^2 - k_L^2)^2}.$$

5.2. License without entry

Suppose that Firm A licenses its technology to Firm B using a combination of a royalty per output and a fixed license fee without entering the market. The equilibrium values of the variables are as follows.

$$q_B = \frac{k_H - r}{2(k_H + c)},\ p_H = \frac{k_H(k_H + 2c + r)}{2(k_H + c)},\ \pi_B = \frac{(k_H - r)^2}{4(k_H + c)} - L.$$

Firm B must pay the difference between its profit net of the royalty and its profit in the entry without license case as a fixed license fee. The fixed fee, L, is determined so that $\pi_B^l = \pi_B^e$ is satisfied. Then, we get

$$\begin{aligned} L =& \frac{(k_H - r)^2}{4(k_H + c)} - \frac{k_L^2(k_H + 2c)^2(k_L + c)}{(4k_Hk_L + 4ck_L + 4ck_H + 4c^2 - k_L^2)^2} \\ =& \frac{A}{4(k_H + c)(k_L^2 - 4k_Hk_L - 4ck_L - 4ck_H - 4c^2)^2}, \end{aligned}$$

where

$$\begin{aligned} A =& k_L^4r^2 - 8k_Hk_L^3r^2 - 8ck_L^3r^2 + 16k_H^2k_L^2r^2 + 24ck_Hk_L^2r^2 + 8c^2k_L^2r^2 + 32ck_H^2k_Lr^2 \\ &+ 64c^2k_Hk_Lr^2 + 32c^3k_Lr^2 + 16c^2k_H^2r^2 + 32c^3k_Hr^2 + 16c^4r^2 - 2k_Hk_L^4r + 16k_H^2k_L^3r \\ &+ 16ck_Hk_L^3r - 32k_H^3k_L^2r - 48ck_H^2k_L^2r - 16c^2k_Hk_L^2r - 64ck_H^3k_Lr - 128c^2k_H^2k_Lr - 64c^3k_Hk_Lr \\ &- 32c^2k_H^3r - 64c^3k_H^2r - 32c^4k_Hr + k_H^2k_L^4 - 12k_H^3k_L^3 - 28ck_H^2k_L^3 - 32c^2k_Hk_L^3 - 16c^3k_L^3 \\ &+ 16k_H^4k_L^2 + 20ck_H^3k_L^2 - 12c^2k_H^2k_L^2 - 32c^3k_Hk_L^2 - 16c^4k_L^2 + 32ck_H^4k_L + 64c^2k_H^3k_L \\ &+ 32c^3k_H^2k_L + 16c^2k_H^4 + 32c^3k_H^3 + 16c^4k_H^2. \end{aligned}$$

Denote the total license fee in this case by TL^l. It is written as

$$TL^l = L + rq_B = \frac{B}{4(k_H + c)(k_L^2 - 4k_Hk_L - 4ck_L - 4ck_H - 4c^2)^2},$$

where

$$\begin{aligned} B =& 8k_Hk_L^3r^2 - k_L^4r^2 + 8ck_L^3r^2 - 16k_H^2k_L^2r^2 - 24ck_Hk_L^2r^2 - 8c^2k_L^2r^2 - 32ck_H^2k_Lr^2 \\ &- 64c^2k_Hk_Lr^2 - 32c^3k_Lr^2 - 16c^2k_H^2r^2 - 32c^3k_Hr^2 - 16c^4r^2 + k_H^2k_L^4 - 12k_H^3k_L^3 \\ &- 28ck_H^2k_L^3 - 32c^2k_Hk_L^3 - 16c^3k_L^3 + 16k_H^4k_L^2 + 20ck_H^3k_L^2 - 12c^2k_H^2k_L^2 - 32c^3k_Hk_L^2 \\ &- 16c^4k_L^2 + 32ck_H^4k_L + 64c^2k_H^3k_L + 32c^3k_H^2k_L + 16c^2k_H^4 + 32c^3k_H^3 + 16c^4k_H^2. \end{aligned}$$

Maximizing TL^l with respect to r, the optimal royalty rate is obtained as follows.

$$\tilde{r}^l = 0.$$

Then, the fixed fee and the total license fee are equal to

$$L = TL^l = \frac{C}{4(k_H + c)(k_L^2 - 4k_H k_L - 4ck_L - 4ck_H - 4c^2)^2},$$

where

$$\begin{aligned} C =& k_H^2 k_L^4 - 12k_H^3 k_L^3 - 28ck_H^2 k_L^3 - 32c^2 k_H k_L^3 - 16c^3 k_L^3 + 16k_H^4 k_L^2 + 20ck_H^3 k_L^2 \\ &- 12c^2 k_H^2 k_L^2 - 32c^3 k_H k_L^2 - 16c^4 k_L^2 + 32ck_H^4 k_L + 64c^2 k_H^3 k_L + 32c^3 k_H^2 k_L + 16c^2 k_H^4 \\ &+ 32c^3 k_H^3 + 16c^4 k_H^2. \end{aligned}$$

5.3. Entry with license

Suppose that Firm A enters the market with license to Firm B using a combination of a royalty per output and a fixed license fee. The equilibrium values of the variables are as follows.

$$q_A = \frac{k_H(k_H + 2c + r)}{(k_H + 2c)(3k_H + 2c)},\ q_B = \frac{k_H^2 + 2ck_H - 2k_H r - 2cr}{(k_H + 2c)(3k_H + 2c)},\ p_H = \frac{k_H(k_H + 2c + r)}{3k_H + 2c},$$

$$\pi_A = \frac{k_H^2(k_H + c)(r + k_H + 2c)^2}{(k_H + 2c)^2(3k_H + 2c)^2},\ \pi_B = \frac{(k_H + c)(k_H^2 + 2ck_H - 2k_H r - 2cr)^2}{(k_H + 2c)^2(3k_H + 2c)^2} - L,$$

Also in this case Firm B must pay the difference between its profit net of the royalty and its profit in the entry without license case as a fixed license fee. The fixed license fee should be equal to

$$\begin{aligned} L =& \frac{(k_H + c)(k_H^2 + 2ck_H - 2k_H r - 2cr)^2}{(k_H + 2c)^2(3k_H + 2c)^2} - \frac{k_L^2(k_H + 2c)^2(k_L + c)}{(4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2)^2} \\ =& \frac{D}{(k_H + 2c)^2(3k_H + 2c)^2(4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2)^2}. \end{aligned}$$

The total license fee is

$$TL = L + rq_B = \frac{E}{(k_H + 2c)^2(3k_H + 2c)^2(4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2)^2}.$$

The total profit of Firm A is the sum of the total license fee and its profit as a firm in the duopoly. It is equal to

$$\pi_A + TL = \frac{F}{(k_H + 2c)^2(3k_H + 2c)^2(4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2)^2}.$$

About details of D, E and F please see B. Firm A chooses r so as to maximize $\pi_A + TL$. We get the optimal royalty rate as follows;

$$\tilde{r}^{el} = \frac{k_H^4 + 4ck_H^3 + 4c^2k_H^2}{2k_H^3 + 18ck_H^2 + 24c^2k_H + 8c^3} > 0.$$

With this royalty rate the outputs of the firms are

$$q_A = \frac{k_H(k_H^2 + 6ck_H + 4c^2)}{2(k_H + c)(k_H^2 + 8ck_H + 4c^2)} > 0,$$

$$q_B = \frac{2ck_H}{k_H^2 + 8ck_H + 4c^2} > 0.$$

q_B is positive and smaller than q_A because

$$q_B - q_A = -\frac{k_H^2(k_H + 2c)}{2(k_H + c)(k_H^2 + 8ck_H + 4c^2)} < 0.$$

The price of the high-quality good is

$$p_H = \frac{k_H(k_H + 2c)(k_H^2 + 6ck_H + 4c^2)}{2(k_H + c)(k_H^2 + 8ck_H + 4c^2)}.$$

Comparing p_H with $\tilde{r}^{el}$ yields

$$p_H - \tilde{r}^{el} = \frac{2ck_H(k_H + 2c)}{k_H^2 + 8ck_H + 4c^2} > 0.$$

Thus, $0 < \tilde{r}^{el} < p_H$.

The fixed fee and the total profit of Firm A are

$$L = \frac{G}{(k_H^2 + 8ck_H + 4c^2)^2(k_L^2 - 4k_Hk_L - 4ck_L - 4ck_H - 4c^2)^2},$$

and

$$\pi_A + TL = \frac{H}{4(k_H + c)(k_H^2 + 8ck_H + 4c^2)(k_L^2 - 4k_Hk_L - 4ck_L - 4ck_H - 4c^2)^2},$$

where

$$\begin{aligned} G =& 4c^2k_H^3k_L^4 + 4c^3k_H^2k_L^4 - k_H^6k_L^3 - 20ck_H^5k_L^3 - 172c^2k_H^4k_L^3 - 480c^3k_H^3k_L^3 \\ & - 592c^4k_H^2k_L^3 - 320c^5k_Hk_L^3 - 64c^6k_L^3 - ck_H^6k_L^2 + 44c^2k_H^5k_L^2 + 20c^3k_H^4k_L^2 \\ & - 288c^4k_H^3k_L^2 - 528c^5k_H^2k_L^2 - 320c^6k_Hk_L^2 - 64c^7k_L^2 + 128c^3k_H^5k_L + 384c^4k_H^4k_L \\ & + 384c^5k_H^3k_L + 128c^6k_H^2k_L + 64c^4k_H^5 + 192c^5k_H^4 + 192c^6k_H^3 + 64c^7k_H^2, \end{aligned}$$

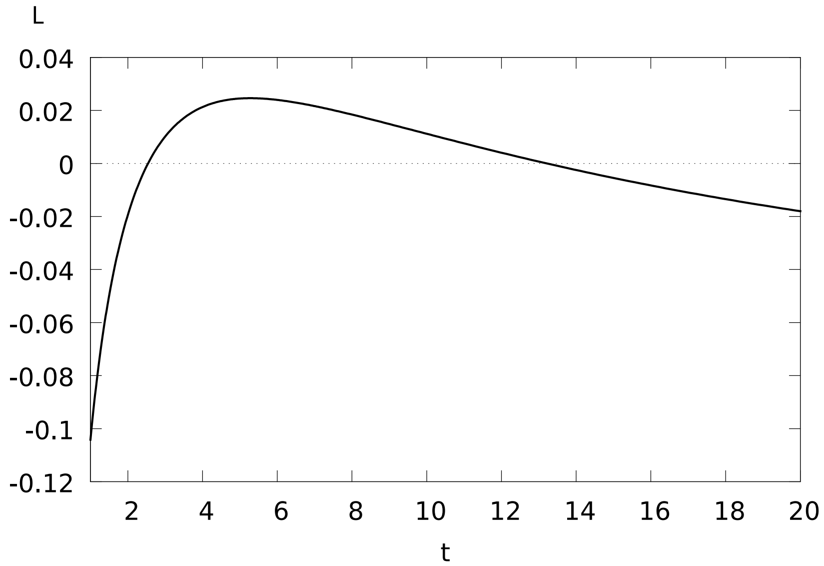

Figure 1: Relation between t and L

and

$$\begin{aligned}H =&k_H^4 k_L^4 + 8ck_H^3 k_L^4 + 8c^2 k_H^2 k_L^4 - 12k_H^5 k_L^3 - 124ck_H^4 k_L^3 - 336c^2 k_H^3 k_L^3 \\ &- 416c^3 k_H^2 k_L^3 - 256c^4 k_H k_L^3 - 64c^5 k_L^3 + 16k_H^6 k_L^2 + 148ck_H^5 k_L^2 + 276c^2 k_H^4 k_L^2 \\ &+ 48c^3 k_H^3 k_L^2 - 288c^4 k_H^2 k_L^2 - 256c^5 k_H k_L^2 - 64c^6 k_L^2 + 32ck_H^6 k_L + 320c^2 k_H^5 k_L + 800c^3 k_H^4 k_L \\ &+ 768c^4 k_H^3 k_L + 256c^5 k_H^2 k_L + 16c^2 k_H^6 + 160c^3 k_H^5 + 400c^4 k_H^4 + 384c^5 k_H^3 + 128c^6 k_H^2.\end{aligned}$$

The fixed license fee, L, in this case may be negative. Assume $c = 1$, $k_L = 2$, and denote $k_H = tk_L$, $t > 1$. Then, we obtain the relation between t and L as depicted in Figure 1. L is negative when $1 < t < \frac{85256799}{33554432} \approx 2.54$ or $t > \frac{443556587}{33554432} \approx 13.22$. Thus, when the magnitude of the innovation is small or is very large, the fixed license fee is negative. Denote the profit of Firm A and the total license fee in this case by π_A^{el} and TL^{el}.

5.4. The optimal strategies for the innovator

Let us compare $\pi_A^{el} + TL^{el}$ and TL^l;

$$(\pi_A^{el} + TL^{el}) - TL^l = \frac{c^2 k_H^2}{(k_H + c)(k_H^2 + 8ck_H + 4c^2)} > 0.$$

Compare $\pi_A^{el} + TL^{el}$ and π_A^e;

$$\pi_A^{el} + TL^{el} - \pi_A^e = \frac{I}{4(k_H + c)(k_H^2 + 8ck_H + 4c^2)(k_L^2 - 4k_H k_L - 4ck_L - 4ck_H - 4c^2)^2} > 0,$$

where

$$
\begin{aligned}
I =&128c^5k_H^3 + 128c^5k_H^2k_L - 192c^5k_Hk_L^2 - 64c^5k_L^3 + 64c^4k_H^4 + 256c^4k_H^3k_L - 192c^4k_Hk_L^3 \\
&- 96c^4k_H^2k_L^2 - 16c^4k_L^4 + 128c^3k_H^3k_L^2 + 128c^3k_H^4k_L - 64c^3k_Hk_L^4 - 160c^3k_H^2k_L^3 \\
&+ 4k_H^5k_L^3 - 3k_H^4k_L^4 + 36ck_H^4k_L^3 - 32ck_H^3k_L^4 - 76c^2k_H^2k_L^4 + 4ck_H^5k_L^2 + 100c^2k_H^4k_L^2 \\
&- 64c^6k_L^2 + 64c^6k_H^2 \\
=&64c^5(2k_H^3 + 2k_H^2k_L - 3k_Hk_L^2 - k_L^3) + 16c^4(4k_H^4 + 16k_H^3k_L - 12k_Hk_L^3 - 6k_H^2k_L^2 - k_L^4) \\
&+ 32c^3k_Hk_L(4k_H^2k_L + 4k_H^3 - 2k_L^3 - 5k_Hk_L^2) + 64c^6(k_H^2 - k_L^2) + 4c^2k_H^2k_L^2(25k_H^2 - 19k_L^2) \\
&+ 4ck_H^3k_L^2(9k_Hk_L + k_H^2 - 8k_L^2) + k_H^4k_L^3(4k_H - 3k_L) > 0.
\end{aligned}
$$

Therefore entry with license strategy is the optimal strategy for the innovating firm.

6. Concluding Remark

We have analyzed the choice of options for the innovating firm under duopoly with vertical product differentiation to enter the market with or without licensing its technology for producing a higher quality good to the incumbent firm, or to license without entry using a combination of a royalty per output and a fixed license fee. We have shown that the results depend on the form of cost function. In the future research we want to extend the analysis in this paper to an oligopolistic situation.

A. Detailed analysis of demand functions

If a consumer with taste parameter θ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \theta k$. Let θ_0 be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the high-quality good. Then,

$$\theta_0 = \frac{p_H}{k_H}.$$

Let θ_L be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality good. Then,

$$\theta_L = \frac{p_L}{k_L}.$$

Let θ_H be the value of θ for which the corresponding consumer is indifferent between buying the low-quality good and buying the high-quality good. Then

$$\theta_H = \frac{p_H - p_L}{k_H - k_L}.$$

We find

$$\theta_0 = \frac{(k_H - k_L)\theta_H + k_L\theta_L}{k_H}.$$

Therefore, $\theta_L \geq \theta_0 \geq \theta_H$ or $\theta_H > \theta_0 > \theta_L$.

For $\theta > (<)\theta_L$,

$$y - p_L + \theta k_L > (<)y.$$

For $\theta > (<)\theta_0$,

$$y - p_H + \theta k_H > (<)y.$$

For $\theta > (<)\theta_H$,

$$y - p_H + \theta k_H > (<)y - p_L + \theta k_L.$$

A.1. Licenses without entry case

In this case only Firm B produces the high-quality good. Let q_H be the demand for the high-quality good. Then, we get

1. When $p_H \geq k_H$ ($\theta_0 \geq 1$), we have $q_H = 0$.
2. When $p_H < k_H$ ($\theta_0 < 1$), we have $q_H = 1 - F(\theta_0)$.

The inverse demand function is described as follows.

1. When $q_H > 0$, we have $p_H = k_H G(1 - q_H)$.
2. When $q_H = 0$, we have $p_H = k_H$.

This is a continuously differentiable function with the domain $0 \leq q_H \leq 1$. We have $q_H = q_B$.

A.2. Entry with license case

In this case both Firms A and B produce the high-quality good. Let $q_H = q_A + q_B$. The inverse demand function is the same as that in the previous case.

A.3. Entry without license case

In this case Firm A produces the high-quality good, and Firm B produces the low-quality good. Let q_H be the demand for the high-quality good and q_L be the demand for the low-quality good. Then, we get

1. When $p_H \geq k_H$ ($\theta_0 \geq 1$) and $p_L \geq k_L$ ($\theta_L \geq 1$), we have $q_H = 0$ and $q_L = 0$.
2. When $p_H < k_H$ ($\theta_0 < 1$) and $p_L \geq \frac{p_H}{k_H} k_L$ ($\theta_L \geq \theta_0 \geq \theta_H$), we have $q_H = 1 - F(\theta_0)$ and $q_L = 0$.
3. When $p_L < k_L$ ($\theta_L < 1$), $p_H > \frac{p_L}{k_L} k_H$ ($\theta_H > \theta_0 > \theta_L$) and $p_H - p_L \geq k_H - k_L$ ($\theta_H \geq 1$), we have $q_H = 0$ and $q_L = 1 - F(\theta_L)$.
4. When $p_L < k_L$ ($\theta_L < 1$), $p_H > \frac{k_H}{k_L} p_L$ ($\theta_H > \theta_0 > \theta_L$) and $p_H - p_L < k_H - k_L$ ($\theta_H < 1$), we have $q_L = F(\theta_H) - F(\theta_L)$ and $q_H = 1 - F(\theta_H)$.

From this demand function we obtain the inverse demand function as follows.

1. When $q_H > 0$ and $q_L > 0$, we have $p_H = (k_H - k_L)G(1 - q_H) + k_L G(1 - q_H - q_L)$ and $p_L = k_L G(1 - q_H - q_L)$.
2. When $q_H > 0$ and $q_L = 0$, we have $p_H = k_H G(1 - q_H)$ and $p_L = k_L G(1 - q_H)$.
3. When $q_H = 0$ and $q_L > 0$, we have $p_H = k_H - k_L + k_L G(1 - q_L)$ and $p_L = k_L G(1 - q_L)$.
4. When $q_H = 0$ and $q_L = 0$, we have $p_H = k_H$ and $p_L = k_L$.

This is a continuously differentiable inverse demand function with the domain $0 \leq q_H \leq 1$ and $0 \leq q_L \leq 1$. We have $q_H = q_A$ and $q_L = q_B$.

B. Details of calculation

$$
\begin{aligned}
D =& 4k_H^3 k_L^4 r^2 + 12ck_H^2 k_L^4 r^2 + 12c^2 k_H k_L^4 r^2 + 4c^3 k_L^4 r^2 - 32k_H^4 k_L^3 r^2 \\
&- 128ck_H^3 k_L^3 r^2 - 192c^2 k_H^2 k_L^3 r^2 - 128c^3 k_H k_L^3 r^2 - 32c^4 k_L^3 r^2 + 64k_H^5 k_L^2 r^2 \\
&+ 288ck_H^4 k_L^2 r^2 + 512c^2 k_H^3 k_L^2 r^2 + 448c^3 k_H^2 k_L^2 r^2 + 192c^4 k_H k_L^2 r^2 + 32c^5 k_L^2 r^2 \\
&+ 128ck_H^5 k_L r^2 + 640c^2 k_H^4 k_L r^2 + 1280c^3 k_H^3 k_L r^2 + 1280c^4 k_H^2 k_L r^2 + 640c^5 k_H k_L r^2 \\
&+ 128c^6 k_L r^2 + 64c^2 k_H^5 r^2 + 320c^3 k_H^4 r^2 + 640c^4 k_H^3 r^2 + 640c^5 k_H^2 r^2 + 320c^6 k_H r^2 \\
&+ 64c^7 r^2 - 4k_H^4 k_L^4 r - 16ck_H^3 k_L^4 r - 20c^2 k_H^2 k_L^4 r - 8c^3 k_H k_L^4 r + 32k_H^5 k_L^3 r \\
&+ 160ck_H^4 k_L^3 r + 288c^2 k_H^3 k_L^3 r + 224c^3 k_H^2 k_L^3 r + 64c^4 k_H k_L^3 r - 64k_H^6 k_L^2 r \\
&- 352ck_H^5 k_L^2 r - 736c^2 k_H^4 k_L^2 r - 736c^3 k_H^3 k_L^2 r - 352c^4 k_H^2 k_L^2 r - 64c^5 k_H k_L^2 r \\
&- 128ck_H^6 k_L r - 768c^2 k_H^5 k_L r - 1792c^3 k_H^4 k_L r - 2048c^4 k_H^3 k_L r - 1152c^5 k_H^2 k_L r \\
&- 256c^6 k_H k_L r - 64c^2 k_H^6 r - 384c^3 k_H^5 r - 896c^4 k_H^4 r - 1024c^5 k_H^3 r - 576c^6 k_H^2 r - 128c^7 k_H r \\
&+ k_H^5 k_L^4 + 5ck_H^4 k_L^4 + 8c^2 k_H^3 k_L^4 + 4c^3 k_H^2 k_L^4 - 17k_H^6 k_L^3 - 132ck_H^5 k_L^3 \\
&- 420c^2 k_H^4 k_L^3 - 704c^3 k_H^3 k_L^3 - 656c^4 k_H^2 k_L^3 - 320c^5 k_H k_L^3 - 64c^6 k_L^3 + 16k_H^7 k_L^2 \\
&+ 95ck_H^6 k_L^2 + 172c^2 k_H^5 k_L^2 - 20c^3 k_H^4 k_L^2 - 448c^4 k_H^3 k_L^2 - 592c^5 k_H^2 k_L^2 \\
&- 320c^6 k_H k_L^2 - 64c^7 k_L^2 + 32ck_H^7 k_L + 224c^2 k_H^6 k_L + 608c^3 k_H^5 k_L + 800c^4 k_H^4 k_L + 512c^5 k_H^3 k_L \\
&+ 128c^6 k_H^2 k_L + 16c^2 k_H^7 + 112c^3 k_H^6 + 304c^4 k_H^5 + 400c^5 k_H^4 + 256c^6 k_H^3 + 64c^7 k_H^2.
\end{aligned}
$$

$$
\begin{aligned}
E =& 16k_H^4k_L^3r^2 - 2k_H^3k_L^4r^2 - 10ck_H^2k_L^4r^2 - 12c^2k_Hk_L^4r^2 - 4c^3k_L^4r^2 \\
&+ 96ck_H^3k_L^3r^2 + 176c^2k_H^2k_L^3r^2 + 128c^3k_Hk_L^3r^2 + 32c^4k_L^3r^2 - 32k_H^5k_L^2r^2 \\
&- 208ck_H^4k_L^2r^2 - 448c^2k_H^3k_L^2r^2 - 432c^3k_H^2k_L^2r^2 - 192c^4k_Hk_L^2r^2 - 32c^5k_L^2r^2 \\
&- 64ck_H^5k_Lr^2 - 448c^2k_H^4k_Lr^2 - 1088c^3k_H^3k_Lr^2 - 1216c^4k_H^2k_Lr^2 - 640c^5k_Hk_Lr^2 \\
&- 128c^6k_Lr^2 - 32c^2k_H^5r^2 - 224c^3k_H^4r^2 - 544c^4k_H^3r^2 - 608c^5k_H^2r^2 - 320c^6k_Hr^2 \\
&- 64c^7r^2 - k_H^4k_L^4r - 2ck_H^3k_L^4r + 8k_H^5k_L^3r + 24ck_H^4k_L^3r + 16c^2k_H^3k_L^3r - 16k_H^6k_L^2r \\
&- 56ck_H^5k_L^2r - 56c^2k_H^4k_L^2r - 16c^3k_H^3k_L^2r - 32ck_H^6k_Lr - 128c^2k_H^5k_Lr - 160c^3k_H^4k_Lr \\
&- 64c^4k_H^3k_Lr - 16c^2k_H^6r - 64c^3k_H^5r - 80c^4k_H^4r - 32c^5k_H^3r + k_H^5k_L^4 + 5ck_H^4k_L^4 \\
&+ 8c^2k_H^3k_L^4 + 4c^3k_H^2k_L^4 - 17k_H^6k_L^3 - 132ck_H^5k_L^3 - 420c^2k_H^4k_L^3 - 704c^3k_H^3k_L^3 \\
&- 656c^4k_H^2k_L^3 - 320c^5k_Hk_L^3 - 64c^6k_L^3 + 16k_H^7k_L^2 + 95ck_H^6k_L^2 + 172c^2k_H^5k_L^2 \\
&- 20c^3k_H^4k_L^2 - 448c^4k_H^3k_L^2 - 592c^5k_H^2k_L^2 - 320c^6k_Hk_L^2 - 64c^7k_L^2 + 32ck_H^7k_L \\
&+ 224c^2k_H^6k_L + 608c^3k_H^5k_L + 800c^4k_H^4k_L + 512c^5k_H^3k_L + 128c^6k_H^2k_L + 16c^2k_H^7 \\
&+ 112c^3k_H^6 + 304c^4k_H^5 + 400c^5k_H^4 + 256c^6k_H^3 + 64c^7k_H^2.
\end{aligned}
$$

$$
\begin{aligned}
F =& 8k_H^4k_L^3r^2 - k_H^3k_L^4r^2 - 9ck_H^2k_L^4r^2 - 12c^2k_Hk_L^4r^2 - 4c^3k_L^4r^2 \\
&+ 80ck_H^3k_L^3r^2 + 168c^2k_H^2k_L^3r^2 + 128c^3k_Hk_L^3r^2 + 32c^4k_L^3r^2 - 16k_H^5k_L^2r^2 \\
&- 168ck_H^4k_L^2r^2 - 416c^2k_H^3k_L^2r^2 - 424c^3k_H^2k_L^2r^2 - 192c^4k_Hk_L^2r^2 - 32c^5k_L^2r^2 \\
&- 32ck_H^5k_Lr^2 - 352c^2k_H^4k_Lr^2 - 992c^3k_H^3k_Lr^2 - 1184c^4k_H^2k_Lr^2 - 640c^5k_Hk_Lr^2 \\
&- 128c^6k_Lr^2 - 16c^2k_H^5r^2 - 176c^3k_H^4r^2 - 496c^4k_H^3r^2 - 592c^5k_H^2r^2 - 320c^6k_Hr^2 \\
&- 64c^7r^2 + k_H^4k_L^4r + 4ck_H^3k_L^4r + 4c^2k_H^2k_L^4r - 8k_H^5k_L^3r - 40ck_H^4k_L^3r - 64c^2k_H^3k_L^3r \\
&- 32c^3k_H^2k_L^3r + 16k_H^6k_L^2r + 88ck_H^5k_L^2r + 168c^2k_H^4k_L^2r + 128c^3k_H^3k_L^2r + 32c^4k_H^2k_L^2r \\
&+ 32ck_H^6k_Lr + 192c^2k_H^5k_Lr + 416c^3k_H^4k_Lr + 384c^4k_H^3k_Lr + 128c^5k_H^2k_Lr + 16c^2k_H^6r \\
&+ 96c^3k_H^5r + 208c^4k_H^4r + 192c^5k_H^3r + 64c^6k_H^2r + 2k_H^5k_L^4 + 10ck_H^4k_L^4 + 16c^2k_H^3k_L^4 \\
&+ 8c^3k_H^2k_L^4 - 25k_H^6k_L^3 - 180ck_H^5k_L^3 - 524c^2k_H^4k_L^3 - 800c^3k_H^3k_L^3 - 688c^4k_H^2k_L^3 \\
&- 320c^5k_Hk_L^3 - 64c^6k_L^3 + 32k_H^7k_L^2 + 199ck_H^6k_L^2 + 428c^2k_H^5k_L^2 + 276c^3k_H^4k_L^2 \\
&- 288c^4k_H^3k_L^2 - 560c^5k_H^2k_L^2 - 320c^6k_Hk_L^2 - 64c^7k_L^2 + 64ck_H^7k_L + 448c^2k_H^6k_L \\
&+ 1216c^3k_H^5k_L + 1600c^4k_H^4k_L + 1024c^5k_H^3k_L + 256c^6k_H^2k_L + 32c^2k_H^7 + 224c^3k_H^6 \\
&+ 608c^4k_H^5 + 800c^5k_H^4 + 512c^6k_H^3 + 128c^7k_H^2.
\end{aligned}
$$

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