Robustness of subsidy in licensing under vertical differentiation: General distribution and cost functions

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Abstract

We extend the analysis of a possibility of negative royalty in licensing under oligopoly with an outside or an incumbent innovator by Liao and Sen (2005) to a case of oligopoly with vertical product differentiation under general distribution function of consumer' taste parameter and general cost functions. We consider both outside innovator case and incumbent innovator case. When the non-licensee does not drop out of the market; in the outside innovator case, if the goods of the firms are strategic substitutes (or complements), the optimal royalty rate is negative (or may be negative or positive); in the incumbent innovator case, if the goods are strategic substitutes (or complements), the optimal royalty rate may be negative or positive (is positive). When the non-licensee drops out of the market with negative royalty; in both cases, 1) If the goods are strategic substitutes, the optimal royalty rate is negative, 2) If the goods are strategic complements, the optimal royalty rate is positive.

Keywords: negative royalty, vertical differentiation, general distribution and cost functions

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1. Introduction

Liao and Sen (2005) analyzed a problem of licensing by a combination of a royalty per output and a fixed fee in an oligopoly with an outside or an incumbent innovator which has a cost reducing technology. They showed that when there are one licensee and one non-licensee, the innovator imposes a negative royalty with a positive fixed fee on the licensee. They assumed, however, linear demand and cost functions (constant marginal costs). In this paper we extend their analysis to a case of an oligopoly with vertical product differentiation in which an innovating firm has a technology for producing a high-quality good under general distribution function of consumers' taste parameter and general cost functions[1].

We consider two cases of oligopoly. The first is a case where the innovator is an outside firm, and the second is a case where it is an incumbent firm. Also about the innovation we consider two cases. The first is a case where the non-licensee continues to operate even with negative royalty, and the second is a case where the non-licensee drops out of the market. In the latter case the innovation is said to be drastic. However, we assume that the output of the non-licensee is positive when the royalty rate is zero.

We will show the following results. When the non-licensee does not drop out of the market;

1. In the outside innovator case:
   i) If the goods of the firms are strategic substitutes, the optimal royalty rate is negative.
   ii) If the goods of the firms are strategic complements, the optimal royalty rate may be negative or positive.
2. In the incumbent innovator case:
   i) If the goods are strategic substitutes, the optimal royalty rate may be negative or positive.
   ii) If the goods are strategic complements, then the optimal royalty rate is positive.

When the non-licensee drops out of the market, in both cases, if the goods are strategic substitutes, the optimal royalty rate is negative, and if the goods are strategic complements, the optimal royalty rate is positive.

In the next section we present the model of this paper, in Section 3 we analyze the outside innovator case, in Section 4 we study the incumbent innovator case, and in Section 5 we present an example of uniform distribution of consumers' taste parameter and linear cost functions. In Appendix we present analyses of demand and inverse demand functions.

In this paper we analyse only a problem of a possibility of negative royalty with one licensee and one non-license. For an outside innovator or an incumbent innovator with two potential licensees whether it sells a license to one firm, or sells licenses to two firms is an important problem. How ever, such an analysis may be complicated under general distribution function and general cost functions. It is a theme of the future research.

[1]Recently, Sen and Stamatopoulos (1980) presented an analysis of royalty and fixed fee in a duopoly under general demand and cost functions. They did not considered a possibility of negative royalty.

2. The model

Our model of vertical product differentiation is according to Mussa and Rosen (1978), Bonanno and Haworth (1998) and Tanaka (2001). There are three firms. The innovator, the licensee and the non-licensee. We call the innovator Firm I, the licensee Firm A and the non-licensee Firm B. Firm I can produce the high-quality good whose quality is k_H, Firm A produces the low-quality good whose quality is k_L, but it can produce the high-quality good buying the license, and Firm B produces the low-quality good, where $k_H > k_L > 0$. k_H and k_L are fixed. Both of the high-quality and low-quality goods are produced at the same cost. The cost function of the goods is denoted by $c(\cdot)$. It is twice continuously differentiable. The innovator imposes a royalty per output and a fixed fee on Firm A. Denote the royalty rate by r, and the fixed license fee by D.

In the market there is a continuum of consumers with the same income, denoted by y, but different values of the taste parameter ξ. Each consumer buys at most one unit of the good. If a consumer with parameter ξ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \xi k$. If a consumer does not buy any good, his utility is equal to his income y. The parameter ξ is distributed according to a smooth distribution function $\rho = F(\xi)$ in the interval $0 < \xi \leq 1$. ρ denotes the probability that the taste parameter is smaller than or equal to ξ. The size of consumers is normalized as one. The inverse function of $F(\xi)$ is denoted by $G(\rho)$. They are twice continuously differentiable, and we have $F'(\xi) > 0$ and $G'(\rho) > 0$. Note that $G(1) = 1$. Let p_L be the price of the good of quality k_L and p_H be the price of the good of quality k_H.

If Firm I is an outside innovator, the market is a duopoly with Firms A and B. If Firm I is an incumbent firm, the market is an oligopoly with three firms. Let q_A and q_B be the outputs of Firms A and B. The output of Firm I is denoted by q_I if it is an incumbent firm.

We consider two cases about the properties of the goods. A case where the goods of firms are strategic substitutes and a case where the goods of firms are strategic complements. Also about the market structure we consider two cases. The first is a case where the non-licensee continues to operate even with negative royalty, and the second is a case where the non-licensee drops out of the market. In the latter case the innovation is said to be drastic. We assume that with zero royalty the output of Firm B is positive in both of the outside innovator case and the incumbent innovator case.

3. Outside innovator

In this section we suppose that Firm I is an outside innovator.

Let ξ_L be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality good. Then,

$$\xi_L = \frac{p_L}{k_L}.$$

Let ξ_H be the value of ξ for which the corresponding consumer is indifferent between buying the low-quality good and the high-quality good. Then

$$\xi_H = \frac{p_H - p_L}{k_H - k_L}.$$

Let $q_H = q_A$ and $q_L = q_B$. The inverse demand function is described as follows.

1. When $q_H > 0$ and $q_L > 0$, we have $p_H = (k_H - k_L)G(1 - q_H) + k_L G(1 - q_H - q_L)$ and $p_L = k_L G(1 - q_H - q_L)$.
2. When $q_H > 0$ and $q_L = 0$, we have $p_H = k_H G(1 - q_H)$ and $p_L = k_L G(1 - q_H)$.
3. When $q_H = 0$ and $q_L > 0$, we have $p_H = k_H - k_L + k_L G(1 - q_L)$ and $p_L = k_L G(1 - q_L)$.
4. When $q_H = 0$ and $q_L = 0$, we have $p_H = k_H$ and $p_L = k_L$.

Since $G(1) = 1$, this is a continuously differentiable function with the domain $0 \leq q_H \leq 1$ and $0 \leq q_H \leq 1$. For details of derivation of the inverse demand function please see Appendix A.1.

The profits of Firm A net of the royalty and the profit of Firm B are

$$\pi_A = [(k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B)]q_A - c(q_A) - rq_A,$$

$$\pi_B = k_L G(1 - q_A - q_B)q_B - c(q_B).$$

To determine the total license fee we consider auction policy by the innovator according to Liao and Sen (2005). If Firm A refuses the payment of license fee, Firm B buys the license. Therefore, the willingness to pay of Firm A is the difference between its profit as a licensee and the profit of a non-licensee, that is, $\pi_A - \pi_B$. Thus, we have

$$L = \pi_A - \pi_B.$$

The payoff of the innovator is the sum of the royalty and the fixed license fee. Denote it by φ. Then,

$$\varphi = L + rq_A = [(k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B)]q_A - c(q_A) - [k_L G(1 - q_A - q_B)q_B - c(q_B)].$$

The first order conditions for profit maximization of Firms A and B are

$$\begin{aligned}\frac{\partial \pi_A}{\partial q_A} =& (k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B) \\ &- [(k_H - k_L)G'(1 - q_A) + k_L G'(1 - q_A - q_B)]q_A - c'(q_A) - r = 0,\end{aligned} \tag{1}$$

and

$$\frac{\partial \pi_B}{\partial q_B} = k_L G(1 - q_A - q_B) - k_L G'(1 - q_A - q_B)q_B - c'(q_B) = 0. \tag{2}$$

Let

$$\begin{aligned}\theta_A =& \frac{\partial^2 \pi_A}{\partial q_A^2} = -2[(k_H - k_L)G'(1 - q_A) + k_L G'(1 - q_A - q_B)] \\ &+ [(k_H - k_L)G''(1 - q_A) + k_L G''(1 - q_A - q_B)]q_A - c''(q_A),\end{aligned}$$

$$\theta_B = \frac{\partial^2 \pi_B}{\partial q_B^2} = -k_L 2G'(1 - q_A - q_B) + k_L G''(1 - q_A - q_B)q_B - c''(q_B),$$

$$\sigma_A = \frac{\partial^2 \pi_A}{\partial q_A q_B} = -k_L G'(1 - q_A - q_B) + k_L G''(1 - q_A - q_B)q_A,$$

and

$$\sigma_B = \frac{\partial^2 \pi_B}{\partial q_B q_A} = -k_L G'(1 - q_A - q_B) + k_L G''(1 - q_A - q_B)q_B.$$

The second order conditions are

$$\theta_A < 0,$$

and

$$\theta_B < 0.$$

Differentiating (1) and (2) with respect to r yields

$$\theta_A \frac{dq_A}{dr} + \sigma_A \frac{dq_B}{dr} = 1,$$

and

$$\sigma_B \frac{dq_A}{dr} + \theta_B \frac{dq_B}{dr} = 0.$$

From them we obtain

$$\frac{dq_A}{dr} = \frac{\theta_B}{\Delta},$$

and

$$\frac{dq_B}{dr} = -\frac{\sigma_B}{\Delta},$$

where

$$\Delta = \theta_A \theta_B - \sigma_A \sigma_B.$$

We assume

$$\Delta > 0.$$

Also we assume

$$|\theta_A| > |\sigma_A|,$$

and

$$|\theta_B| > |\sigma_B|.$$

These assumptions are derived from the stability conditions for duopoly (see Seade (1980) and Dixit (1986)). We get

$$\frac{dq_A}{dr} < 0,$$

and

$$\left|\frac{dq_A}{dr}\right| > \left|\frac{dq_B}{dr}\right|.$$

We say that the goods of the firms are strategic substitutes when $\sigma_B < 0$ and strategic complements when $\sigma_B > 0$. Then, we obtain

1. When the goods of the firms are strategic substitutes, $\frac{dq_B}{dr} > 0$.

2. When the goods of the firms are strategic complements, $\frac{dq_B}{dr} < 0$.

The condition for maximization of φ with respect to r is

$$\begin{aligned}\frac{d\varphi}{dr} =& \lambda_A \frac{dq_A}{dr} + \lambda_B \frac{dq_B}{dr} \\ =& (r + k_L G'(1 - q_A - q_B)q_B)\frac{dq_A}{dr} - k_L G'(1 - q_A - q_B)q_A \frac{dq_B}{dr} = 0,\end{aligned}$$

where

$$\lambda_A = \frac{\partial \pi_A}{\partial q_A} + r - \frac{\partial \pi_B}{\partial q_A} = r + k_L G'(1 - q_A - q_B)q_B,$$

and

$$\lambda_B = \frac{\partial \pi_A}{\partial q_B} - \frac{\partial \pi_B}{\partial q_B} = -k_L G'(1 - q_A - q_B)q_A.$$

Then, we obtain the optimal royalty rate for the innovator as follows.

$$\tilde{r} = -\frac{k_L G'(1 - q_A - q_B)}{\frac{dq_A}{dr}} \left(q_B \frac{dq_A}{dr} - q_A \frac{dq_B}{dr} \right). \tag{3}$$

Now we assume

$$G(1 - q_A) - G'(1 - q_A)q_A > 0. \tag{4}$$

The first order condition for Firm A in (1) means

$$(k_H - k_L)[G(1 - q_A) - G'(1 - q_A)q_A] + k_L[G(1 - q_A - q_B) - G'(1 - q_A - q_B)q_A] = c'(q_A) + r > 0.$$

Thus, (4) will hold.

Differentiating (1) and (2) with respect to k_H, we obtain

$$\frac{dq_A}{dk_H} = -\frac{\theta_B(G(1 - q_A) - G'(1 - q_A)q_A)}{\Delta} > 0,$$

and

$$\frac{dq_B}{dk_H} = \frac{\sigma_B(G(1 - q_A) - G'(1 - q_A)q_A)}{\Delta}.$$

$\frac{dq_B}{dk_H}$ has the same sign as that of σ_B. Since $|\theta_B| > |\sigma_B|$, we have $\left|\frac{dq_A}{dk_H}\right| > \left|\frac{dq_B}{dk_H}\right|$. The larger the value of k_H is, the larger the value of $q_A - q_B$ is.

We show the following two propositions.

Proposition 1. *In the case where the non-licensee continues to operate we obtain the following results.*

1. *If the goods of the firms are strategic substitutes, the optimal royalty rate is negative.*

2. *If the goods of the firms are strategic complements, the optimal royalty rate may be negative or positive.*

Proof. 1. If the goods of the firms are strategic substitutes, we have $\frac{dq_B}{dr} > 0$. Then, $\tilde{r} < 0$ because $-\frac{k_L G'(1-q_A-q_B)}{\frac{dq_A}{dr}} > 0$.

2. If the goods of the firms are strategic complements, we have $\frac{dq_B}{dr} < 0$. Then, $\tilde{r} < 0$ or $\tilde{r} > 0$ depending on $q_B \frac{dq_A}{dr} - q_A \frac{dq_B}{dr} < 0$ or $q_B \frac{dq_A}{dr} - q_A \frac{dq_B}{dr} > 0$.

   If q_B is sufficiently smaller than q_A although Firm B does not drop out, it is likely that $q_B \frac{dq_A}{dr} - q_A \frac{dq_B}{dr} > 0$ and $\tilde{r} > 0$.

□

Proposition 2. *In the case where the non-licensee drops out of the market we obtain the following results.*

1. *If the goods of the firms are strategic substitutes, the optimal royalty rate is negative.*

2. *If the goods of the firms are strategic complements, the optimal royalty rate is positive.*

Proof. 1. If

$$\left.\frac{d\varphi}{dr}\right|_{q_B=0} = r\frac{dq_A}{dr} - k_L G'(1-q_A)q_A \frac{dq_B}{dr} > 0,$$

then, $q_B > 0$ at the optimal state for the innovator and we have the previous case.

On the other hand, if $\frac{d\varphi}{dr} \leq 0$ when $q_B = 0$, the licensee is a monopolist and the optimal royalty rate for the innovator is one such that $q_B = 0$. It is negative because $q_B > 0$ with zero royalty and $\frac{dq_B}{dr} > 0$.

   If Firm A is the monopolist, the payoff of Firm I is equal to the profit of Firm A including the royalty. It is maximized by zero royalty rate. However, since $q_B > 0$ when $r = 0$, the optimal royalty rate is one at which Firm B just drops out. Please see an example in Section 5.

2. If $\frac{d\varphi}{dr} < 0$ at $q_B = 0$, then $q_B > 0$ at the optimal state for the innovator and we have the previous case.

   On the other hand, if $\frac{d\varphi}{dr} \geq 0$ at $q_B = 0$, then the licensee is a monopolist and the optimal royalty rate for the innovator is one such that $q_B = 0$. It is positive because $q_B > 0$ with zero royalty and $\frac{dq_B}{dr} < 0$.

□

4. Incumbent innovator

In this section we suppose that the innovator is an incumbent firm. Firm I as well as Firm A produce the high-quality good. Only Firm B produces the low-quality good. Let $q_H = q_I + q_A$ and $q_L = q_B$. Similarly to the previous case the inverse demand function is described as follows.

1. When $q_H > 0$ and $q_L > 0$, we have $p_H = (k_H - k_L)G(1 - q_H) + k_L G(1 - q_H - q_L)$ and $p_L = k_L G(1 - q_H - q_L)$.
2. When $q_H > 0$ and $q_L = 0$, we have $p_H = k_H G(1 - q_H)$ and $p_L = k_L G(1 - q_H)$.
3. When $q_H = 0$ and $q_L > 0$, we have $p_H = k_H - k_L + k_L G(1 - q_L)$ and $p_L = k_L G(1 - q_L)$.
4. When $q_H = 0$ and $q_L = 0$, we have $p_H = k_H$ and $p_L = k_L$.

The profit of Firm I, that of Firm A net of the royalty and that of Firm B are

$$\pi_I = [(k_H - k_L)G(1 - q_I - q_A) + k_L G(1 - q_I - q_A - q_B)]q_I - c(q_I),$$

$$\pi_A = [(k_H - k_L)G(1 - q_I - q_A) + k_L G(1 - q_I - q_A - q_B)]q_A - c(q_A) - rq_A,$$

and

$$\pi_B = k_L G(1 - q_I - q_A - q_B)q_B - c(q_B).$$

The fixed license fee, L, satisfies the following relation.

$$L = \pi_A - \pi_B.$$

The first order conditions for profit maximization of Firms I, A and B are

$$\begin{aligned}\frac{\partial \pi_I}{\partial q_I} =&(k_H - k_L)G(1 - q_I - q_A) + k_L G(1 - q_I - q_A - q_B) \\ &- [(k_H - k_L)G'(1 - q_I - q_A) + k_L G'(1 - q_I - q_A - q_B)]q_I - c'(q_I) = 0,\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial \pi_A}{\partial q_A} =&(k_H - k_L)G(1 - q_I - q_A) + k_L G(1 - q_I - q_A - q_B) \\ &- [(k_H - k_L)G'(1 - q_I - q_A) + k_L G'(1 - q_I - q_A - q_B)]q_A - c'(q_A) - r = 0,\end{aligned} \tag{6}$$

and

$$\frac{\partial \pi_B}{\partial q_B} = k_L G(1 - q_I - q_A - q_B) - k_L G'(1 - q_I - q_A - q_B)q_B - c'(q_B) = 0. \tag{7}$$

Let

$$\begin{aligned}\theta_I =&\frac{\partial^2 \pi_I}{\partial q_I^2} = -2[(k_H - k_L)G'(1 - q_I - q_A) + k_L G'(1 - q_I - q_A - q_B)] \\ &+ [(k_H - k_L)G''(1 - q_I - q_A) + k_L G''(1 - q_I - q_A - q_B)]q_I - c''(q_I), \\ \theta_A =&\frac{\partial^2 \pi_A}{\partial q_A^2} = -2[(k_H - k_L)G'(1 - q_I - q_A) + k_L G'(1 - q_I - q_A - q_B)] \\ &+ [(k_H - k_L)G''(1 - q_I - q_A) + k_L G''(1 - q_I - q_A - q_B)]q_A - c''(q_A),\end{aligned}$$

$$\theta_B = \frac{\partial^2 \pi_B}{\partial q_B^2} = -2k_L G'(1-q_I-q_A-q_B) + k_L G''(1-q_I-q_A-q_B)q_B - c''(q_B),$$

$$\begin{aligned}\sigma_{IA} =& \frac{\partial^2 \pi_I}{\partial q_I q_A} = -(k_H-k_L)G'(1-q_I-q_A) - k_L G'(1-q_I-q_A-q_B) \\ &+ [(k_H-k_L)G''(1-q_I-q_A) + k_L G''(1-q_I-q_A-q_B)]q_I,\end{aligned}$$

$$\sigma_{IB} = \frac{\partial^2 \pi_I}{\partial q_I q_B} = -k_L G'(1-q_I-q_A-q_B) + k_L G''(1-q_I-q_A-q_B)q_I,$$

$$\begin{aligned}\sigma_{AI} =& \frac{\partial^2 \pi_A}{\partial q_A q_I} = -(k_H-k_L)G'(1-q_I-q_A) - k_L G'(1-q_I-q_A-q_B) \\ &+ [(k_H-k_L)G''(1-q_I-q_A) + k_L G''(1-q_I-q_A-q_B)]q_A,\end{aligned}$$

$$\sigma_{AB} = \frac{\partial^2 \pi_A}{\partial q_A q_B} = -k_L G'(1-q_I-q_A-q_B) + k_L G''(1-q_I-q_A-q_B)q_A,$$

and

$$\sigma_B = \frac{\partial^2 \pi_B}{\partial q_B q_A} = \frac{\partial^2 \pi_B}{\partial q_B q_I} = -k_L G'(1-q_I-q_A-q_B) + k_L G''(1-q_I-q_A-q_B)q_B.$$

The second order conditions are

$$\theta_I < 0,$$

$$\theta_A < 0,$$

and

$$\theta_B < 0.$$

Differentiating (5) , (6) and (7) with respect to r yields

$$\theta_I \frac{dq_I}{dr} + \sigma_{IA}\frac{dq_A}{dr} + \sigma_{IB}\frac{dq_B}{dr} = 0,$$

$$\sigma_{AI}\frac{dq_I}{dr} + \theta_A\frac{dq_A}{dr} + \sigma_{AB}\frac{dq_B}{dr} = 1,$$

$$\sigma_B\frac{dq_I}{dr} + \sigma_B\frac{dq_A}{dr} + \theta_B\frac{dq_B}{dr} = 0.$$

From them we obtain

$$\frac{dq_I}{dr} = -\frac{\theta_B\sigma_{IA} - \sigma_{IB}\sigma_B}{\Gamma},$$

$$\frac{dq_A}{dr} = \frac{\theta_B\theta_I - \sigma_{IB}\sigma_B}{\Gamma},$$

$$\frac{dq_B}{dr} = -\frac{(\theta_I - \sigma_{IA})\sigma_B}{\Gamma},$$

where

$$\Gamma = \theta_I\theta_A\theta_B - \sigma_{AB}\sigma_B\theta_I - \sigma_{IB}\sigma_B\theta_A - \sigma_{IA}\sigma_{AI}\theta_B + \sigma_{IA}\sigma_{AB}\sigma_B + \sigma_{IB}\sigma_{AI}\sigma_B.$$

We assume

$$\Gamma < 0.$$

Also we assume

$$\theta_I\theta_B - \sigma_{IB}\sigma_B > 0,$$

$$|\theta_I| > |\sigma_{IA}|,\ |\theta_I| > |\sigma_{IB}|,\ |\theta_A| > |\sigma_{AI}|,\ |\theta_A| > |\sigma_{AB}|,\ |\theta_B| > |\sigma_B|.$$

These assumptions are derived from the stability conditions for oligopoly (see Seade (1980) and Dixit (1986)). We have

$$\frac{dq_A}{dr} < 0.$$

Further, we assume that θ_I, θ_A and θ_B have larger absolute values than those of σ_B, σ_{IB}, σ_{IA}, σ_{AB} and σ_{AI}. Then, we can think that the following relations are satisfied

$$\left|\frac{dq_A}{dr}\right| > \left|\frac{dq_I}{dr}\right|,\ \left|\frac{dq_A}{dr}\right| > \left|\frac{dq_B}{dr}\right|.$$

We say that the goods of the firms are strategic substitutes when σ_{IA}, σ_{AI}, σ_{IB}, σ_{BI} and σ_B are negative, and strategic complements when σ_{IA}, σ_{AI}, σ_{IB}, σ_{BI} and σ_B are positive. Then, we obtain

1. When the goods of the firms are strategic substitutes, $\frac{dq_I}{dr} > 0$ and $\frac{dq_B}{dr} > 0$.

2. When the goods of the firms are strategic complements, $\frac{dq_I}{dr} < 0$ and $\frac{dq_B}{dr} < 0$.

The payoff of the innovator is the sum of the royalty, the fixed license fee and its profit as a firm in the oligopoly. Denote it by φ. Then,

$$\begin{aligned}\varphi =&\pi_I + L + rq_A = [(k_H - k_L)G(1 - q_I - q_A) + k_LG(1 - q_I - q_A - q_B)]q_I - c(q_I)\\ &+ [(k_H - k_L)G(1 - q_I - q_A) + k_LG(1 - q_I - q_A - q_B)]q_A - c(q_A)\\ &- [k_LG(1 - q_I - q_A - q_B)q_B - c(q_B)].\end{aligned}$$

The condition for maximization of φ with respect to r is

$$\begin{aligned}\frac{d\varphi}{dr} =&\lambda_I\frac{dq_I}{dr} + \lambda_A\frac{dq_A}{dr} - \lambda_B\frac{dq_B}{dr}\\ =&r\frac{dq_A}{dr} - (k_H - k_L)G'(1 - q_I - q_A)\left(q_A\frac{dq_I}{dr} + q_I\frac{dq_A}{dr}\right)\\ &- k_LG'(1 - q_I - q_A - q_B)\left[(q_A - q_B)\frac{dq_I}{dr} + (q_I - q_B)\frac{dq_A}{dr} + (q_I + q_A)\frac{dq_B}{dr}\right] = 0,\end{aligned}$$

where, using the first order conditions,

$$\begin{aligned}\lambda_I =& \frac{\partial \pi_I}{\partial q_I} + \frac{\partial \pi_A}{\partial q_I} - \frac{\partial \pi_B}{\partial q_I} \\ =& -[(k_H - k_L)G'(1 - q_I - q_A)q_A + k_L G'(1 - q_I - q_A - q_B)q_A] \\ & + k_L G'(1 - q_I - q_A - q_B)q_B,\end{aligned}$$

$$\begin{aligned}\lambda_A =& \frac{\partial \pi_I}{\partial q_A} + \frac{\partial \pi_A}{\partial q_A} + r - \frac{\partial \pi_B}{\partial q_A} \\ =& r - [(k_H - k_L)G'(1 - q_I - q_A)q_I + k_L G'(1 - q_I - q_A - q_B)q_I] \\ & + k_L G'(1 - q_I - q_A - q_B)q_B,\end{aligned}$$

and

$$\lambda_B = \frac{\partial \pi_I}{\partial q_B} + \frac{\partial \pi_A}{\partial q_B} - \frac{\partial \pi_B}{\partial q_B} = -k_L G'(1 - q_I - q_A - q_B)q_I - k_L G'(1 - q_I - q_A - q_B)q_A.$$

Then, we get the optimal royalty rate for the innovator as follows.

$$\begin{aligned}\tilde{r} =& \frac{(k_H - k_L)G'(1 - q_I - q_A)}{\frac{dq_A}{dr}} \left(q_A \frac{dq_I}{dr} + q_I \frac{dq_A}{dr} \right) \\ & + \frac{k_L G'(1 - q_I - q_A - q_B)}{\frac{dq_A}{dr}} \left[(q_A - q_B)\frac{dq_I}{dr} + (q_I - q_B)\frac{dq_A}{dr} + (q_I + q_A)\frac{dq_B}{dr} \right].\end{aligned} \tag{8}$$

Now we assume

$$\begin{cases} G(1 - q_I - q_A) - G'(1 - q_I - q_A)q_I > 0, \\ G(1 - q_I - q_A) - G'(1 - q_I - q_A)q_A > 0. \end{cases} \tag{9}$$

The first order conditions for Firm I and Firm A, (5) and (6), mean

$$\begin{aligned}&(k_H - k_L)[G(1 - q_I - q_A) - G'(1 - q_I - q_A)q_I] \\ &+ k_L[G(1 - q_I - q_A - q_B) - G'(1 - q_I - q_A - q_B)q_I] = c'(q_I) > 0,\end{aligned}$$

and

$$\begin{aligned}&(k_H - k_L)[G(1 - q_I - q_A) - G'(1 - q_I - q_A)q_A] \\ &+ k_L[G(1 - q_I - q_A - q_B) - G'(1 - q_I - q_A - q_B)q_A] = c'(q_A) + r > 0.\end{aligned}$$

Thus, (9) will hold.

Differentiating (5) , (6) and (7) with respect to k_H, we obtain

$$\frac{dq_I}{dk_H} = -\frac{\eta_I(\theta_A\theta_B - \sigma_{AB}\sigma_B) - \eta_A(\theta_B\sigma_{IA} - \sigma_{IB}\sigma_B)}{\Gamma},$$

$$\frac{dq_A}{dk_H} = -\frac{\eta_A(\theta_B\theta_I - \sigma_B\sigma_{IB}) - \eta_I(\theta_B\sigma_{AI} - \sigma_{AB}\sigma_B)}{\Gamma},$$

$$\frac{dq_B}{dk_H} = \frac{\sigma_B[\eta_A(\theta_I - \sigma_{IA}) + \eta_I(\theta_A - \sigma_{AI})]}{\Gamma}.$$

where

$$\eta_I = G(1 - q_I - q_A) - G'(1 - q_I - q_A)q_I > 0,$$

$$\eta_A = G(1 - q_I - q_A) - G'(1 - q_I - q_A)q_A > 0.$$

We assume

1. θ_I, θ_A and θ_B have larger absolute values than those of σ_B, σ_{IB}, σ_{IA}, σ_{AB} and σ_{AI}.

2. η_I and η_A have similar values.

Then, we can think that the following relations are satisfied

$$\frac{dq_I}{dk_H} > 0, \ \frac{dq_A}{dk_H} > 0,$$

$$\frac{dq_I}{dk_H} - \frac{dq_B}{dk_H} > 0,$$

$$\frac{dq_A}{dk_H} - \frac{dq_B}{dk_H} > 0.$$

The larger the value of k_H is, the larger the values of $q_I - q_B$ and $q_A - q_B$ are.

We show the following two propositions.

Proposition 3. *In the case where the non-licensee continues to operate we obtain the following results.*

1. *If the goods are strategic substitutes, then the optimal royalty rate may be negative or positive.*

2. *If the goods are strategic complements, then the optimal royalty rate is positive.*

Proof.

1. If the goods are strategic substitutes, $\frac{dq_I}{dr} > 0$, $\frac{dq_A}{dr} < 0$ and $\frac{dq_B}{dr} > 0$. Then, since $q_A\frac{dq_I}{dr} > 0$, $q_I\frac{dq_A}{dr} < 0$, $(q_A - q_B)\frac{dq_I}{dr} > 0$, $(q_I - q_B)\frac{dq_A}{dr} < 0$ and $(q_I + q_A)\frac{dq_B}{dr} > 0$, the optimal royalty rate in (8) may be negative or positive.

   An example in the next section demonstrates that the optimal royalty rate is likely to be positive when k_H is large.

2. If the goods are strategic complements, $\frac{dq_I}{dr}$, $\frac{dq_A}{dr}$ and $\frac{dq_B}{dr}$ are all negative. Then, $\frac{d\varphi}{dr}$ when $r = 0$ is positive because $q_I - q_B > 0$, $q_A - q_B > 0$ and $q_I + q_A > 0$. Thus, the optimal royalty rate is positive.

□

Proposition 4. *In the case where the non-licensee drops out of the market we obtain the following results.*

1. *If the goods of the firms are strategic substitutes, the optimal royalty rate is negative.*
2. *If the goods of the firms are strategic complements, the optimal royalty rate is positive.*

The proof is similar to the proof of Proposition 2. Note

$$\left.\frac{d\varphi}{dr}\right|_{q_B=0} = r\frac{dq_A}{dr} - k_H G'(1-q_I-q_A)\left(q_A\frac{dq_I}{dr} + q_I\frac{dq_A}{dr}\right) - k_L G'(1-q_I-q_A)(q_I+q_A)\frac{dq_B}{dr}.$$

Proof. 1. If

$$\left.\frac{d\varphi}{dr}\right|_{q_B=0} > 0,$$

then, $q_B > 0$ at the optimal state for the innovator and we have the previous case.

On the other hand, if $\frac{d\varphi}{dr} \leq 0$ when $q_B = 0$, the market is a duopoly with the innovator and the licensee, and the optimal royalty rate for the innovator is one such that $q_B = 0$. It is negative because $q_B > 0$ with zero royalty and $\frac{dq_B}{dr} > 0$.

In the example below we will see that if consumers' taste parameter has a uniform distribution and the cost functions are linear, there exists no case where Firm B drops out under the assumption that its output when $r = 0$ is positive.

In another research (Hattori and Tanaka (2017)) we have shown that in the duopolistic situation with the innovator and the licensee without non-licensee the optimal royalty rate is positive and depends on the form of cost functions; whether they are concave or convex.

2. If $\frac{d\varphi}{dr} < 0$ at $q_B = 0$, then $q_B > 0$ at the optimal state for the innovator and we have the previous case.

   On the other hand, if $\frac{d\varphi}{dr} \geq 0$ at $q_B = 0$, then the market is a duopoly with the innovator and the licensee, and the optimal royalty rate for the innovator is one such that $q_B = 0$. It is positive because $q_B > 0$ with zero royalty and $\frac{dq_B}{dr} < 0$.

□

5. An example of uniform distribution and constant marginal costs

Assume that $\rho = F(\xi)$ has a uniform distribution, the (common) cost function is linear and there is no fixed cost. Then, $\rho = \xi$, $\xi = G(\rho) = \rho$, $F'(\xi) = G'(\rho) = 1$ and $F''(\xi) = G''(\rho) = 0$. The marginal cost is denoted by c. Assume $0 < c < k_L$. In this example the goods of the firms are strategic substitutes because $G'' = 0$.

5.1. Outside innovator

When the innovator is an outside firm and the non-licensee does not drop out of the market, the equilibrium values of the variables are obtained as follows.

$$q_A = \frac{2k_H - k_L - c - 2r}{4k_H - k_L},\ q_B = \frac{k_L r + k_H k_L + ck_L - 2ck_H}{k_L(4k_H - k_L)},$$

$$p_H = \frac{2k_H^2 + 3ck_H - k_L r + 2k_H r - k_H k_L - ck_L}{4k_H - k_L},\ p_L = \frac{k_L r + k_H k_L + 2ck_H}{4k_H - k_L}.$$

The total license fee which is the sum of the royalty and the fixed fee is

$$\varphi = \frac{k_H^2 k_L + c^2 k_L - c^2 k_H - k_L r^2 - k_L^2 r + ck_L r - k_H k_L^2}{k_L(4k_H - k_L)}.$$

The optimal royalty rate for the innovator is

$$\tilde{r} = -\frac{k_L - c}{2} < 0.$$

A case where non-licensee drops out In this example there may exist a case where Firm B drops out under the assumption that its output when $r = 0$ is positive if $c < k_L < 2c$. Since $q_B|_{r=0} = \frac{k_H k_L + ck_L - 2ck_L}{k_L(4k_H - k_L)} > 0$, we need

$$k_H k_L + ck_L - 2ck_H > 0.$$

When $q_B = 0$,

$$r = -\frac{k_H k_L + ck_L - 2ck_H}{k_L}.$$

Then,

$$\left.\frac{d\varphi}{dr}\right|_{q_B=0} = \frac{2k_H k_L + 3ck_L - k_L^2 - 4ck_H}{k_L(4k_H - k_L)} = \frac{(k_L - 2c)(k_H - k_L) + k_H k_L + ck_L - 2ck_H}{k_L(4k_H - k_L)}.$$

If $c < k_L < 2c$, this may be negative. If it is so, by 1 of Proposition 4 the optimal royalty rate is negative. Then, calculating the equilibrium values of the variables assuming the monopoly of Firm A, the total license fee is

$$\varphi|_{q_B=0} = \frac{(k_H - c - r)(k_H - c + r)}{4k_H}.$$

It is maximized by $r = 0$. However, by the assumption $q_B > 0$ when $r = 0$. Therefore, the optimal royalty rate is

$$-\frac{k_H k_L + ck_L - 2ck_H}{k_L}.$$

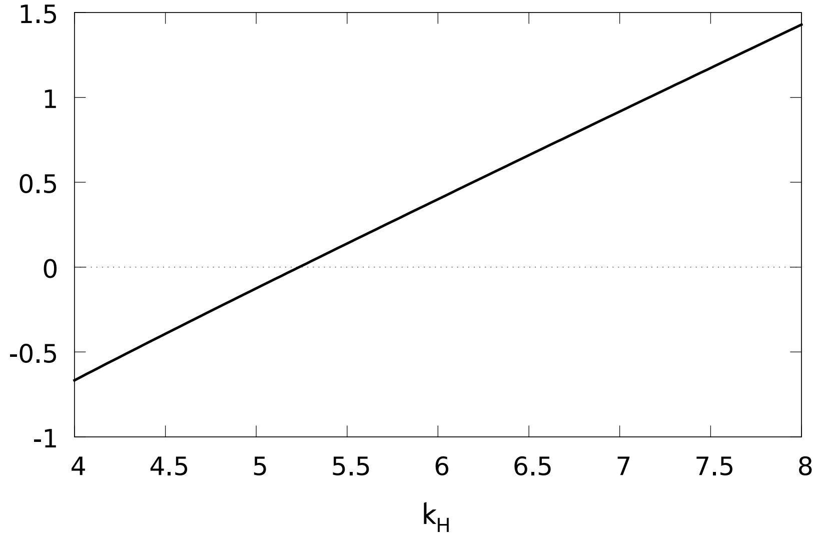

Figure 1: Relation between k_H and the optimal royalty rate in the incumbent innovator case

5.2. Incumbent innovator

When the innovator is an incumbent firm, the equilibrium values of the variables are obtained as follows.

$$q_I = \frac{2k_H^2 - ck_H - k_L r + 2k_H r - k_H k_L}{2k_H(3k_H - k_L)},\ q_A = \frac{k_L r - 4k_H r - k_H k_L + 2k_H^2 - ck_H}{2k_H(3k_H - k_L)},$$

$$q_B = \frac{k_L r + k_H k_L + 2ck_L - 3ck_H}{2k_L(3k_H - k_L)},$$

$$p_H = \frac{2k_H^2 + 5ck_H - k_L r + 2k_H r - k_H k_L - 2ck_L}{2(3k_H - k_L)},\ p_L = \frac{k_L r + k_H k_L + 3ck_H}{2(3k_H - k_L)}.$$

The total payoff of the innovator which is the sum of the royalty, the fixed fee and the profit of the innovator as a firm in the oligopoly is

$$\frac{A}{4k_L(3k_H - k_L)^2)},$$

where

$$\begin{aligned} A =& k_L^2 r^2 - 4k_H k_L r^2 + 2k_L^3 r - 8k_H k_L^2 r - 2ck_L^2 r + 4k_H^2 k_L r + 4ck_H k_L r + 2k_H k_L^3 \\ &- 9k_H^2 k_L^2 - 4c^2 k_L^2 + 8k_H^3 k_L - 2ck_H^2 k_L + 14c^2 k_H k_L - 9c^2 k_H^2. \end{aligned}$$

The optimal royalty rate for the innovator is

$$r = \frac{k_L^2 - 4k_H k_L - ck_L + 2k_H^2 + 2ck_H}{4k_H - k_L}.$$

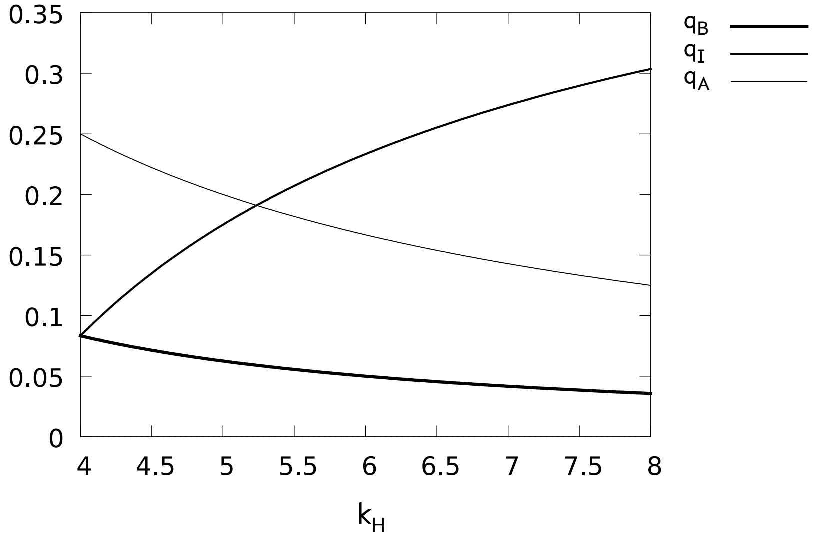

Figure 2: Relations among k_H and the outputs of the firms in the incumbent innovator case

This may be positive or negative. An example, assuming $c = 2$, $k_L = 4$, $k_L < k_H < 8$, is depicted in Figure 1. $q_B > 0$ when $r = 0$ and $k_H < 8$. This figure demonstrates that the optimal royalty rate is likely to be positive when k_H is large. The outputs of the firms in this example are positive as shown in Figure 2. In Figure 3 the relations among k_H, the royalty, the fixed fee and the profit of the innovator are depicted.

A case where non-licensee drops out In this example of uniform distribution and linear cost functions we can show that there exists no case where Firm B drops out under the assumption that its output when $r = 0$ is positive.

Since $q_B|_{r=0} = \frac{k_H k_L + 2ck_L - 3ck_H}{2k_L(3k_H - k_L)} > 0$, we need

$$k_H k_L + 2ck_L - 3ck_H > 0.$$

When $q_B = 0$,

$$r = -\frac{k_H k_L + 2ck_L - 3ck_H}{k_L}.$$

Then,

$$\left.\frac{d\varphi}{dr}\right|_{q_B=0} = \frac{2k_H k_L - k_L^2 + 3ck_L - 4ck_H}{2k_L(3k_H - k_L)} = \frac{(k_H - k_L)(k_L - c) + k_H k_L + 2ck_L - 3ck_H}{2k_L(3k_H - k_L)} > 0.$$

By 1 of Proposition 4 $q_B > 0$ at the optimal royalty rate because the goods are strategic substitutes.

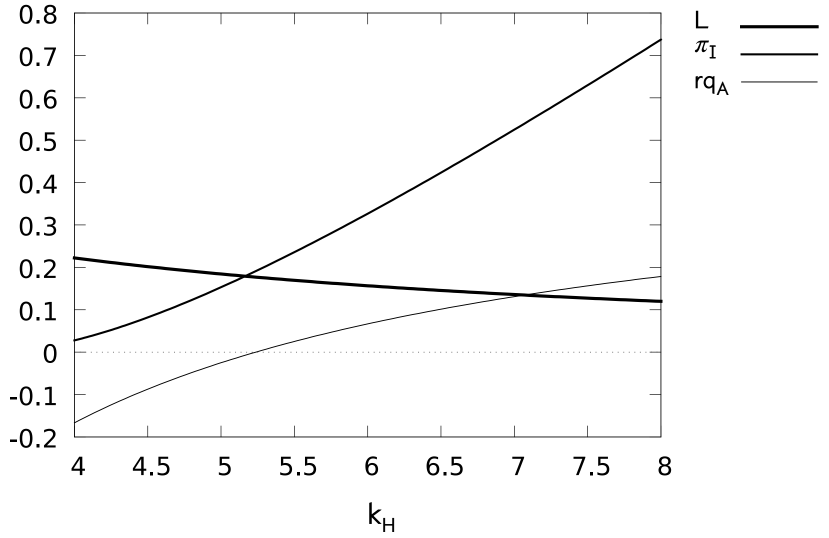

Figure 3: Relations among k_H, the royalty, the fixed fee and the profit of the innovator in the incumbent innovator case

6. Concluding Remark

We have examined a possibility of negative royalty under vertical product differentiation with an outside or an incumbent innovator, and have shown that the results depend on the property of the goods, whether they are strategic substitutes or complements.

In the future research we want to extend the analysis in this paper to, for example, an oligopoly with endogenous quality choice.

A. Appendix: Detailed analysis of demand functions

If a consumer with taste parameter ξ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \xi k$. Let ξ_0 be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the high-quality good. Then,

$$\xi_0 = \frac{p_H}{k_H}.$$

Let ξ_L be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality good. Then,

$$\xi_L = \frac{p_L}{k_L}.$$

Let ξ_H be the value of ξ for which the corresponding consumer is indifferent between buying the low-quality good and buying the high-quality good. Then

$$\xi_H = \frac{p_H - p_L}{k_H - k_L}.$$

We find

$$\xi_0 = \frac{(k_H - k_L)\xi_H + k_L\xi_L}{k_H}.$$

Therefore, $\xi_L \geq \xi_0 \geq \xi_H$ or $\xi_H > \xi_0 > \xi_L$.

For $\xi > (<)\xi_L$,

$$y - p_L + \xi k_L > (<)y.$$

For $\xi > (<)\xi_0$,

$$y - p_H + \xi k_H > (<)y.$$

For $\xi > (<)\xi_H$,

$$y - p_H + \xi k_H > (<)y - p_L + \xi k_L.$$

A.1. Outside innovator case

In this case Firm A produces the high-quality good, and Firm B produces the low-quality good. Demand for the high quality good, q_H, and demand for the low-quality good, q_L, are as follows.

1. When $p_H \geq k_H$ ($\xi_0 \geq 1$) and $p_L \geq k_L$ ($\xi_L \geq 1$), we have $q_H = 0$ and $q_L = 0$.

2. When $p_H < k_H$ ($\xi_0 < 1$) and $p_L \geq \frac{p_H}{k_H}k_L$ ($\xi_L \geq \xi_0 \geq \xi_H$), we have $q_H = 1 - F(\xi_0)$ and $q_L = 0$.

3. When $p_L < k_L$ ($\xi_L < 1$), $p_H > \frac{p_L}{k_L}k_H$ ($\xi_H > \xi_0 > \xi_L$) and $p_H - p_L \geq k_H - k_L$ ($\xi_H \geq 1$), we have $q_H = 0$ and $q_L = 1 - F(\xi_L)$.

4. When $p_L < k_L$ ($\xi_L < 1$), $p_H > \frac{k_H}{k_L}p_L$ ($\xi_H > \xi_0 > \xi_L$) and $p_H - p_L < k_H - k_L$ ($\xi_H < 1$), we have $q_L = F(\xi_H) - F(\xi_L)$ and $q_H = 1 - F(\xi_H)$.

From this demand function we obtain the inverse demand function as follows.

1. When $q_H > 0$ and $q_L > 0$, we have $p_H = (k_H - k_L)G(1 - q_H) + k_LG(1 - q_H - q_L)$ and $p_L = k_LG(1 - q_H - q_L)$.

2. When $q_H > 0$ and $q_L = 0$, we have $p_H = k_HG(1 - q_H)$ and $p_L = k_LG(1 - q_H)$.

3. When $q_H = 0$ and $q_L > 0$, we have $p_H = k_H - k_L + k_LG(1 - q_L)$ and $p_L = k_LG(1 - q_L)$.

4. When $q_H = 0$ and $q_L = 0$, we have $p_H = k_H$ and $p_L = k_L$.

This is a continuously differentiable function with the domain $0 \leq q_H \leq 1$ and $0 \leq q_L \leq 1$. We have $q_H = q_A$ and $q_L = q_B$.

A.2. Incumbent innovator case

In this case Firms I and A produce the high-quality good, and Firm B produces the low-quality good. The inverse demand function is the same as that in the previous case with $q_H = q_I + q_A$ and $q_L = q_B$.

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