Royalty and license fee under oligopoly with or without entry of innovator: Two-step auction

Hattori, Masahiko and Tanaka, Yasuhito

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Royalty and license fee under oligopoly with or without entry of innovator: Two-step auction

Masahiko Hattori*
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.
and
Yasuhito Tanaka†
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.

Abstract

When an outside innovating firm has a cost-reducing technology, it can sell licenses of its technology to incumbent firms, or enter the market and at the same time sell licenses, or enter the market without license. We examine the definitions of license fees in such situations under oligopoly with three firms, one outside innovating firm and two incumbent firms, considering threat by entry of the innovating firm using a two-step auction. Also we suppose that the innovating firm sells its licenses using a combination of royalty per output and a fixed license fee.

Keywords: license; entry; oligopoly; innovating firm; two-step auction.

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*mhattori@mail.doshisha.ac.jp
†yasuhito@mail.doshisha.ac.jp

1 Introduction

In Proposition 4 of Kamien and Tauman (1986) it was argued that in an oligopoly when the number of firms is small (or very large), strategy to enter the market and at the same time license the cost-reducing technology to the incumbent firm (license with entry strategy) is more profitable than strategy to license its technology to the incumbent firm without entering the market (license without entry strategy) for the innovating firm. However, their result depends on their definition of license fee. They defined the license fee in the case of licenses without entry by the difference between the profit of an incumbent firm in that case and its profit before it buys a license without entry of the innovating firm. However, it is inappropriate from the game theoretic view point. If an incumbent firm does not buy a license, the innovating firm may punish the incumbent firm by entering the market. The innovating firm can use such a threat if and only if it is a credible threat. In a duopoly case with one incumbent firm, when the innovating firm does not enter nor sell a license, its profit is zero; on the other hand, when it enters the market without license, its profit is positive. Therefore, threat by entry without license is credible under duopoly, and then even if the innovating firm does not enter the market, the incumbent firm must pay the difference between its profit when it uses the new technology and its profit when the innovating firm enters without license as a license fee. For example, Hattori and Tanaka (2017a) presented analyses of license and entry choice by an innovating firm in a duopoly.

However, in an oligopoly with more than one incumbent firms, the credibility of threat by entry is a more subtle problem. In this paper we examine definitions of license fees under oligopoly with three firms, one outside innovating firm and two incumbent firms, considering a two-step auction in the case of licenses without entry. Also we suppose that the innovating firm uses a combination of royalty per output and a fixed license fee.

A two-step auction, for example, in the case of a license to one incumbent firm without entry is as follows.

1. The first step.

   The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below, and the innovating firm imposes a predetermined (positive or negative) royalty per output on the licensee. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.

2. The second step.

   The innovating firm sells a license to one firm at auction *with its entry*.

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when only this firm uses the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step each incumbent firm has an incentive to make a bid when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid.

We need the minimum bidding price because if there is no minimum price, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

A two-step auction in the case of licenses to two incumbent firms without entry is similar[1], and at the first step of the auction the incumbent firm has a will to pay the following license fee;

> the difference between its profit when both firms use the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step each incumbent firm has an incentive to make a bid when the other firm makes a bid because if it does not make a bid, the auction proceed to the next step.

Threat by such a two-step auction is credible if and only if the profit of the innovating firm when it enters the market with a license to one firm is larger than its profit when it licenses to one incumbent firm without entering the market.

In the next section we present some literature review. In Section 3 the model of this paper is described. In Section 4 we consider various equilibria of the oligopoly. In Section 5 we present an analysis of a royalty and a fixed license fee under the license with entry strategy. In Section 6 we consider a two-step auction and present an analysis of a royalty and a fixed license fee under the license without entry strategy. In Sections 5 and 6 the following results about the optimal royalty rate for the innovator will be shown (see Proposition 1).

Entry with license to one firm case The optimal royalty rate may be positive or negative.

Entry with licenses to two firms case If the goods are strategic complements, the optimal royalty rate is positive. If the goods are strategic substitutes, it may be positive or negative.

License to one firm without entry cases not using two-step auction If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it may be positive or negative.

License to one firm without entry cases using two-step auction If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it is positive.

Licenses to two firms without entry cases using or not using two-step auction The optimal royalty rate is positive.

In Section 6 also we examine the credibility of two-step auction, and will show the following results (see Proposition 2).

[1] Please see Section 6.2.2.

1. If the cost function of the new technology is linear, the profit of the innovating firm when it enters the market with a license to one firm and its profit when it licenses to one incumbent firm without entering the market are equal, that is, entry with license to one firm case and license to one firm without entry case are equivalent.

2. If the cost function of the new technology is strictly convex, two-step auction is credible.

3. If the cost function of the new technology is strictly concave, two-step auction is not credible.

In Section 7 we present an example of linear demand and quadratic cost functions. In this example two-step auction is credible. We will show that when two-step auction is credible, license to two firms without entry strategy is optimal; on the other hand, when it is not credible, entry without license strategy is optimal..

2 Literature review

Various studies focus on technology adoption or R&D investment in duopoly or oligopoly. Most of them analyze the relation between the technology licensor and licensee. The difference of means of contracts, which comprise royalties, upfront fixed fees, combinations of these two, and auctions, are well discussed (Katz and Shapiro (1985)). Kamien and Tauman (2002) show that outside innovators prefer auctions, but industry incumbents prefer royalty. This topic is discussed by Kabiraj (2004) under the Stackelberg oligopoly; here, the licensor does not have production capacity. Wang and Yang (2004) consider the case when the licensor has production capacity. Sen and Tauman (2007) compared the license system in detail, namely, when the licensor is an outsider and when it is an incumbent firm, using the combination of royalties and fixed fees. However, the existence of production capacity was externally given, and they did not analyze the choice of entry. Therefore, the optimal strategies of outside innovators, who can use the entry as a threat, require more discussion. Regarding the strategies of new entrants to the market, Duchene, Sen and Serfes (2015) focused on future entrants with old technology, and argued that while a low license fee can be used to deter the entry of potential entrants, the firm with new technology is incumbent, and its choice of entry is not analyzed. Also, Chen (2016) analyzed the model of the endogenous market structure determined by the potential entrant with old technology and showed that the licensor uses the fixed fee and zero royalty in both the incumbent and the outside innovator cases, which are exogenously given. Creane, Chiu and Konishi (2013) examined a firm that can license its production technology to a rival when firms are heterogeneous in production costs, and showed that a complete technology transfer from one firm to another always increases joint profit under weakly concave demand when at least three firms remain in the industry.

A Cournot oligopoly with fixed fee under cost asymmetry was analyzed by La Manna (1993). He showed that if technologies can be replicated perfectly, a lower cost firm always has the incentive to transfer its technology; hence, while a Cournot-Nash equilibrium cannot be fully asymmetric, there exists no non-cooperative Nash equilibrium in pure strategies. On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyzed bargaining between a licensor with no production capacity and oligopolistic firms. Recent research focuses on market structure and technology improvement. Boone (2001) and Matsumura et. al. (2013) found a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) showed that technology adoption may change the market outcome. The social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may result in higher social welfare than Bertrand competition under a differentiated goods market. Hattori and Tanaka (2015), Hattori and Tanaka (2016a) studied the adoption of new technology in Cournot duopoly and Stackelberg duopoly. Rebolledo and Sandonís (2012) presented an analysis of the effectiveness of research and development (R&D) subsidies in an oligopolistic model in the cases of international competition and cooperation in R&D. Hattori and Tanaka (2016) analyzed similar problems about product innovation, that is, introduction of higher quality good in a duopoly with vertical product differentiation.

3 The model

There are three firms, Firms A, B and C. At present two of them, Firms B and C, produce a homogeneous good. Firm A, which is an outside firm, has a superior cost-reducing technology and can produce the good at lower cost than Firms B and C. We call Firm A the innovating firm, and Firms B and C the incumbent firms. Firm A have the following five options.

1. To enter the market without license to incumbent firms.
2. To enter the market and license its technology to one incumbent firm.
3. To enter the market and license its technology to two incumbent firms.
4. To license its technology to one incumbent firm, but not enter the market.
5. To license its technology to two incumbent firms, but not enter the market.

Let p be the price, x_A, x_B and x_C be the outputs of Firms A, B and C. Then, the inverse demand function of the good is written as follows.

$$p = p(x_A + x_B + x_C), \text{ when Firm A enters,}$$

$$p = p(x_B + x_C), \text{ when Firm A does not enter.}$$

It is twice continuously differentiable.

The cost functions of Firms A, B and C are denoted by $c_A(x_A)$, $c_B(x_B)$ and $c_C(x_C)$. $c_B(\cdot)$ and $c_C(\cdot)$ are the same functions without license. If Firm A licenses its technology to two incumbent firms, all cost functions are the same, and if Firm A licenses its technology to one incumbent firm (for example Firm C), then the cost functions of Firms A and C are the same. They are twice continuously differentiable, and there is no fixed cost; thus $c_A(0) = c_B(0) = 0$.

In the cases with licenses the game proceeds as follows. In the first stage Firm A determines the royalty rate. In the second stage firms determine the outputs, and the fixed license fee is determined.

4 Equilibria of the oligopoly

4.1 Entry without license case

We suppose that Firm A enters the market without license to incumbent firms. Then, the market becomes a tripoly. The cost function of Firm C is $c_B(x_C)$. The profits of Firms A, B and C are written as

$$\pi_A = p(x_A + x_B + x_C)x_A - c_A(x_A),$$

$$\pi_B = p(x_A + x_B + x_C)x_B - c_B(x_B),$$

$$\pi_C = p(x_A + x_B + x_C)x_C - c_B(x_C).$$

We assume Cournot type behavior of the firms. The first order conditions for profit maximization are

$$p + p'x_A - c_A'(x_A) = 0,$$

$$p + p'x_B - c_B'(x_B) = 0,$$

$$p + p'x_C - c_B'(x_C) = 0.$$

The second order conditions are

$$2p' + p''x_A - c_A''(x_A) < 0,$$

$$2p' + p''x_B - c_B''(x_B) < 0,$$

$$2p' + p''x_C - c_B''(x_C) < 0.$$

Hereafter we assume that the second order conditions in each case are satisfied. Denote the equilibrium profits in this case by π_A^{e0}, π_B^{e0} and π_C^{e0}.

4.2 License to one firm without entry case

Suppose that Firm A licenses its technology to one firm, Firm C, but it does not enter the market. Then, the market is a duopoly. The cost function of Firm C is $c_A(x_C)$. Denote the royalty per output and the fixed license fee by r and L. The profits of the firms are written as

$$\pi_B = p(x_B + x_C)x_B - c_B(x_B),$$

$$\pi_C = p(x_B + x_C)x_C - c_A(x_C) - rx_C - L.$$

The first order conditions for profit maximization are

$$p + p'x_B - c_B'(x_B) = 0, \tag{1a}$$

$$p + p'x_C - c_A'(x_C) - r = 0. \tag{1b}$$

Denote the equilibrium profits and the license fee in this case by π_B^{l1}, π_C^{l1} and L^{l1}. Differentiating the first order conditions with respect to r, we obtain

$$\frac{dx_B}{dr} = -\frac{p' + p''x_B}{\Delta},$$

$$\frac{dx_C}{dr} = \frac{2p' + p''x_B - c_B''(x_B)}{\Delta},$$

where

$$\Delta = (2p' + p''x_B - c_B''(x_B))(2p' + p''x_C - c_A''(x_C)) - (p' + p''x_B)(p' + p''x_C).$$

From the second order conditions and the stability conditions for oligopoly (see Seade (1980) and Dixit (1986)), we have $\frac{dx_C}{dr} < 0$. When the goods of the firms are strategic substitutes, $p' + p''x_C < 0$, and when the goods are strategic complements, $p' + p''x_C > 0$. We have $\frac{dx_B}{dr} > 0$ in the former case, and $\frac{dx_B}{dr} < 0$ in the latter case.

4.3 Licenses to two firms without entry case

Suppose that Firm A licenses its technology to two firms, Firms B and C, but it does not enter the market. The cost functions of Firms B and C are $c_A(\cdot)$. The profits of the firms are written as

$$\pi_B = p(x_B + x_C)x_B - c_A(x_B) - rx_B - L,$$

$$\pi_C = p(x_B + x_C)x_C - c_A(x_C) - rx_C - L.$$

The first order conditions for profit maximization are

$$p + p'x_B - c_A'(x_B) - r = 0,$$

$$p + p'x_C - c_A'(x_C) - r = 0.$$

Denote the equilibrium profits and the license fee in this case by π_B^{l2}, π_C^{l2} and L^{l2}. In this case we have $x_B = x_C$ and $\pi_B^{l2} = \pi_C^{l2}$, Differentiating the first order conditions with respect to r, we obtain

$$\frac{dx_B}{dr} = \frac{dx_C}{dr} = \frac{p' - c_A''(x_B)}{\Delta}.$$

where

$$\Delta = (2p' + p''x_B - c_A''(x_B))(2p' + p''x_C - c_A''(x_C)) - (p' + p''x_B)(p' + p''x_C).$$

From the stability conditions we can assume $p' - c_A''(x_B) < 0$. Then, similarly to the previous case we have $\frac{dx_B}{dr} < 0$ and $\frac{dx_C}{dr} < 0$.

4.4 Entry with a license to one firm case

Next suppose that Firm A enters the market and sells a license to one firm, Firm C. The cost function of Firm C is $c_A(x_C)$. The profits of Firms A, B and C are written as

$$\pi_A = p(x_A + x_B + x_C)x_A - c_A(x_A),$$

$$\pi_B = p(x_A + x_B + x_C)x_B - c_B(x_B),$$

$$\pi_C = p(x_A + x_B + x_C)x_C - c_A(x_C) - rx_C - L.$$

The first order conditions for profit maximization are

$$p + p'x_A - c_A'(x_A) = 0, \tag{2a}$$

$$p + p'x_B - c_B'(x_B) = 0, \tag{2b}$$

$$p + p'x_C - c_A'(x_C) - r = 0. \tag{2c}$$

Denote the equilibrium profits and the license fee in this case by π_A^{e1}, π_B^{e1}, π_C^{e1} and L^{e1}. Differentiating the first order conditions with respect to r, we obtain

$$\frac{dx_A}{dr} = \frac{\sigma_A(\sigma_B - \theta_B)}{\Gamma},$$

$$\frac{dx_B}{dr} = \frac{\sigma_B(\sigma_A - \theta_A)}{\Gamma},$$

$$\frac{dx_C}{dr} = \frac{\theta_A\theta_B - \sigma_A\sigma_B}{\Gamma},$$

where

$$\theta_A = 2p' + p''x_A - c_A''(x_A),\ \theta_B = 2p' + p''x_B - c_B''(x_B),\ \theta_C = 2p' + p''x_C - c_A''(x_C),$$

$$\sigma_A = p' + p''x_A,\ \sigma_B = p' + p''x_B,\ \sigma_C = p' + p''x_C,$$

and

$$\Gamma = \theta_A\theta_B\theta_C - \theta_A\sigma_B\sigma_C - \theta_B\sigma_A\sigma_C - \theta_C\sigma_A\sigma_B + 2\sigma_A\sigma_B\sigma_C.$$

From the second order conditions and the stability conditions, θ's are negative, $\Gamma < 0$, and the absolute values of θ's are larger than those of σ's. Then, we have $\frac{dx_C}{dr} < 0$. When the goods of the firms are strategic substitutes, $\sigma_A < 0$, $\sigma_B < 0$, $\sigma_C < 0$, and when the goods are strategic complements, $\sigma_A > 0$, $\sigma_B > 0$, $\sigma_C > 0$. We have $\frac{dx_A}{dr} > 0$, $\frac{dx_B}{dr} > 0$ in the former case, and $\frac{dx_A}{dr} < 0$, $\frac{dx_B}{dr} < 0$ in the latter case.

4.5 Entry with licenses to two firms case

Next suppose that Firm A enters the market and sells licenses to Firms B and C. The cost functions of Firms B and C are $c_A(\cdot)$. The profits of Firms A, B and C are written as

$$\pi_A = p(x_A + x_B + x_C)x_A - c_A(x_A),$$

$$\pi_B = p(x_A + x_B + x_C)x_B - c_A(x_B) - rx_B - L,$$

$$\pi_C = p(x_A + x_B + x_C)x_C - c_A(x_C) - rx_C - L.$$

The first order conditions for profit maximization are

$$p + p'x_A - c_A'(x_A) = 0,$$

$$p + p'x_B - c_A'(x_B) - r = 0,$$

$$p + p'x_C - c_A'(x_C) - r = 0.$$

Denote the equilibrium profits and the license fee by π_A^{e2}, π_B^{e2}, π_C^{e2} and L^{e2}. In this case $x_B = x_C$ and $\pi_B^{e2} = \pi_C^{e2}$.

Differentiating the first order conditions with respect to r, we obtain

$$\frac{dx_A}{dr} = \frac{2\sigma_A(\sigma_B - \theta_B)}{\Gamma},$$

$$\frac{dx_B}{dr} = \frac{dx_C}{dr} = \frac{\theta_A(\theta_B - \sigma_B)}{\Gamma}.$$

where

$$\theta_A = 2p' + p''x_A - c_A''(x_A),\ \theta_B = 2p' + p''x_B - c_A''(x_B),\ \theta_C = 2p' + p''x_C - c_A''(x_C),$$

$$\sigma_A = p' + p''x_A,\ \sigma_B = p' + p''x_B,\ \sigma_C = p' + p''x_C,$$

and

$$\Gamma = \theta_A\theta_B\theta_C - \theta_A\sigma_B\sigma_C - \theta_B\sigma_A\sigma_C - \theta_C\sigma_A\sigma_B + 2\sigma_A\sigma_B\sigma_C.$$

We have $\theta_B = \theta_C$ and $\sigma_B = \sigma_C$. Similarly to the previous case we get $\frac{dx_B}{dr} < 0$ and $\frac{dx_C}{dr} < 0$. $\frac{dx_A}{dr} > 0$ if $\sigma_A < 0$, and $\frac{dx_A}{dr} < 0$ if $\sigma_A > 0$.

5 Royalty and license fees in the cases of licenses with entry

In the case of licenses with entry the license fee is equal to the usual willingness to pay for the incumbent firms. We follow the arguments by Kamien and Tauman (1986) and Sen and Tauman (2007) about license fee by auction.

5.1 License to one firm

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when only this firm uses the new technology with entry of the innovating firm and its profit when only the rival firm buys the license with entry of the innovating firm.

This is because the incumbent firms know that there will be one licensee regardless of whether or not it buys a license. Then, the fixed license fee is

$$L^{e1} = (\pi_C^{e1} + L^{e1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{e1} = \pi_B^{e1}$. The total payoff of Firm A in this case is written as

$$\varphi^{e1} = \pi_A^{e1} + rx_C + L^{e1} = px_A - c_A(x_A) + px_C - c_A(x_C) - (px_B - c_B(x_B)).$$

Using the first order conditions, the condition for maximization of φ with respect to r is written as follows.

$$\frac{d\varphi^{e1}}{dr} = r\frac{dx_C}{dr} + p'(x_C - x_B)\frac{dx_A}{dr} + p'(x_A - x_B)\frac{dx_C}{dr} + p'(x_A + x_C)\frac{dx_B}{dr} = 0,$$

Then, we get the optimal royalty rate for the innovator as follows.

$$\tilde{r}^{e1} = -\frac{p'}{\frac{dx_C}{dr}}\left[(x_C - x_B)\frac{dx_A}{dr} + (x_A - x_B)\frac{dx_C}{dr} + (x_A + x_C)\frac{dx_B}{dr}\right]. \tag{3}$$

This may be positive or negative,

5.2 Licenses to two firms

The willingness to pay for each incumbent firm in this case is equal to

> the difference between its profit when two firms use the new technology with entry of the innovating firm and its profit when only the rival firm buys the license with entry of the innovating firm.

This is because the incumbent firms know that there will be one licensee when it does not buy a license. In this case there is a minimum bidding price which is equal to the willingness to pay for the incumbents because without the minimum bidding price no firm makes a positive bid. The fixed license fee is

$$L^{e2} = (\pi_C^{e2} + L^{e2}) - \pi_B^{e1}.$$

This means $\pi_C^{e2} = \pi_B^{e1}$. The total payoff of Firm B is written as

$$\varphi^{e2} = \pi_A^{e1} + rx_B + rx_C + 2L^{e2} = px_A - c_A(x_A) + px_B - c_A(x_B) + px_C - c_A(x_C) - 2\pi_B^{e1}.$$

Note that π_B^{e1} is constant and irrelevant to determination of the royalty rate in this case. Using the first order conditions, the condition for maximization of φ with respect to r is written as follows.

$$\frac{d\varphi^{e2}}{dr} = r\left(\frac{dx_B}{dr} + \frac{dx_C}{dr}\right) + p'(x_B + x_C)\frac{dx_A}{dr} + p'(x_A + x_B)\frac{dx_C}{dr} + p'(x_A + x_C)\frac{dx_B}{dr}.$$

The optimal royalty rate is

$$\tilde{r}^{e2} = -\frac{p'}{\frac{dx_B}{dr} + \frac{dx_C}{dr}}\left[(x_B + x_C)\frac{dx_A}{dr} + (x_A + x_B)\frac{dx_C}{dr} + (x_A + x_C)\frac{dx_B}{dr}\right].$$

If the goods are strategic complements, $\tilde{r}^{e2} > 0$ because $\frac{dx_A}{dr} < 0$, $\frac{dx_B}{dr} < 0$ and $\frac{dx_C}{dr} < 0$. If the goods are strategic substitutes, it may be positive or negative.

6 Royalty and license fees in the case of licenses without entry: two-step auction

6.1 One-step auction

If the licenses are auctioned off to the incumbent firms by one-step auction, the license fee is determined by the usual willingness to pay for the incumbent firms described in Kamien and Tauman (1986) and Sen and Tauman (2007).

6.1.1 License to one firm

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when only this firm uses the new technology without entry of the innovating firm and its profit when only the rival firm buys the license without entry of the innovating firm.

Then, the fixed license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{l1}.$$

This equation means $\pi_C^{l1} = \pi_B^{l1}$. Denote L in this case by $\tilde{L}^{l1}$, and denote the total payoff of the innovator by $\tilde{\varphi}^{l1}$ to distinguish it from the total payoff in the two-step auction case, which is denoted by $\hat{\varphi}^{l1}$. $\tilde{\varphi}^{l1}$ is

$$\tilde{\varphi}^{l1} = rx_C + \tilde{L}^{l1} = px_C - c_A(x_C) - (px_B - c_B(x_B)).$$

Using the first order conditions, the condition for maximization of $\tilde{\varphi}^{l1}$ with respect to r is written as

$$\frac{d\tilde{\varphi}^{l1}}{dr} = r\frac{dx_C}{dr} + p'x_C\frac{dx_B}{dr} - p'x_B\frac{dx_C}{dr} = 0.$$

Then, we obtain the optimal royalty rate for the innovator as follows.

$$r^{l1} = \frac{p'}{\frac{dx_C}{dr}}\left(x_B\frac{dx_C}{dr} - x_C\frac{dx_B}{dr}\right). \tag{4}$$

Denote it by $\tilde{r}^{l1}$. If the goods are strategic substitutes, $\tilde{r}^{l1} < 0$ because $\frac{dx_B}{dr} > 0$; if the goods are strategic complements, it may be positive or negative because $\frac{dx_C}{dr} < 0$ and $\frac{dx_B}{dr} < 0$.

6.1.2 Licenses to two firms

The willingness to pay for each incumbent firm in this case is equal to

> the difference between its profit when two firms use the new technology without entry of the innovating firm and its profit when only the rival firm buys the license without entry of the innovating firm.

There is a minimum bidding price which is equal to the willingness to pay for the incumbents. The license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{l1}.$$

This means $\pi_C^{l2} = \pi_B^{l1}$. Denote L in this case by $\tilde{L}^{l2}$, and denote the total payoff of the innovator by $\tilde{\varphi}^{l2}$. It is

$$\tilde{\varphi}^{l2} = r(x_B + x_C) + 2\tilde{L}^{l2} = px_B - c_A(x_B) + px_C - c_A(x_C) - 2\pi_B^{l1}.$$

Note that π_B^{l1} is constant and irrelevant to determination of the royalty rate. The condition for maximization of $\tilde{\varphi}^{l2}$ with respect to r is

$$\frac{d\tilde{\varphi}^{l2}}{dr} = r\left(\frac{dx_B}{dr} + \frac{dx_C}{dr}\right) + p'x_C\frac{dx_B}{dr} + p'x_B\frac{dx_C}{dr} = 0.$$

The optimal royalty rate is

$$\tilde{r}^{l2} = -\frac{p'}{\frac{dx_B}{dr} + \frac{dx_C}{dr}}\left(x_C\frac{dx_B}{dr} + x_B\frac{dx_C}{dr}\right).$$

This is positive.

6.2 Two-step auction

We consider a two-step auction for each case.

6.2.1 License to one firm

In this case the two-step auction is practiced as follows.

1. The first step.

   The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below, and the innovating firm imposes a predetermined (positive or negative) royalty per output on the licensee. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.

2. The second step.

   The innovating firm sells a license to one firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

   $$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

the difference between its profit when only this firm uses the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

Then, the license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{l1} = \pi_B^{e1}$. Denote L^{l1} in this case by $\hat{L}^{l1}$.

In the first step each incumbent firm has an incentive to make a bid with the license fee L^{l1} when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid.

We need the minimum bidding price L^{l1} because the profit of a non-licensee is π_B^{l1} which is larger than π_B^{e1}. If there is no minimum price, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

Denote the total payoff of the innovator in this case by $\hat{\varphi}^{l1}$. Then,

$$\hat{\varphi}^{l1} = rx_C + \hat{L}^{l1} = px_C - c_A(x_C) - \pi_B^{e1}.$$

Note that π_B^{e1} is a constant number in this case which is determined in the entry with a license to one firm case. The condition for maximization of φ with respect to r is

$$\frac{d\hat{\varphi}^{l1}}{dr} = r\frac{dx_C}{dr} + p'x_C\frac{dx_B}{dr} = 0.$$

Then, we obtain the optimal royalty rate for the innovator as follows.

$$r^{l1} = -\frac{p'}{\frac{dx_C}{dr}}x_C\frac{dx_B}{dr}.$$

Denote it by $\hat{r}^{l1}$. If the goods are strategic substitutes, $\hat{r}^{l1} < 0$ because $\frac{dx_B}{dr} > 0$, and if the goods are strategic complements, $\hat{r}^{l1} > 0$ because $\frac{dx_B}{dr} < 0$.

6.2.2 Licenses to two firms

We consider the following two-step auction

1. The first step.

   The innovating firm sells licenses to two firms at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below and both firms make bids, and the innovating firm imposes a predetermined (positive or negative) royalty per output on the licensee. If both firms make bids, they get licenses. If at least one of the firms does not make a bid, then the auction proceeds to the next step.

2. The second step.

   The innovating firm sells a license to one firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when two firms use the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

The minimum bidding price should be equal to this willingness to pay. Then, the license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{e1}.$$

This means $\pi_C^{l2} = \pi_B^{e1}$. Denote L^{l2} in this case by $\hat{L}^{l2}$.

In the first step each incumbent firm has an incentive to make a bid when the other firm makes a bid because if it does not make a bid, the auction proceeds to the next step.

Denote the total payoff of the innovator in this case by $\hat{\varphi}^{l2}$. It is

$$\hat{\varphi}^{l2} = r(x_B + x_C) + 2\hat{L}^{l2} = px_B - c_A(x_B) + px_C - c_A(x_C) - 2\pi_B^{e1}.$$

Note that π_B^{e1} is constant and irrelevant to determination of the royalty rate in this case. The condition for maximization of $\hat{\varphi}^{l2}$ with respect to r is

$$\frac{d\hat{\varphi}^{l2}}{dr} = r\left(\frac{dx_B}{dr} + \frac{dx_C}{dr}\right) + p'x_C\frac{dx_B}{dr} + p'x_B\frac{dx_C}{dr} = 0.$$

The optimal royalty rate is

$$r^{l2} = -\frac{p'}{\frac{dx_B}{dr} + \frac{dx_C}{dr}}\left(x_C\frac{dx_B}{dr} + x_B\frac{dx_C}{dr}\right).$$

Denote it by $\hat{r}^{l2}$. We see $\hat{r}^{l2} = \tilde{r}^{l2}$, but the total payoff of the innovator with two-step auction and that without two-step auction are different because the fixed license fees in two cases are different.

We summarize the results about the optimal royalty rates for the innovator in the following proposition.

Proposition 1. ***Entry with license to one firm case*** *The optimal royalty rate may be positive or negative.*

Entry with licenses to two firms case *If the goods are strategic complements, the optimal royalty rate is positive. If the goods are strategic substitutes, it may be positive or negative.*

License to one firm without entry case not using two-step auction *If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it may be positive or negative.*

License to one firm without entry case using two-step auction *If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it is positive.*

Licenses to two firms without entry case using or not using two-step auction *The optimal royalty rate is positive.*

6.3 Credibility of two-step auction

In this subsection we will prove our main results. The innovating firm uses a two-step auction if and only if the threat by the existence of the second step of the auction is credible, and it is credible if and only if the total payoff of the innovating firm when it enters the market with a license to one firm is larger than its payoff when it does not enter and sells a license to one firm not using a two-step auction. Therefore, if

$$\pi_A^{e1} + \tilde{r}^{e1} x_C + L^{e1} \geq \tilde{r}^{l1} x_C + \tilde{L}^{l1},$$

two-step auction is credible. On the other hand, if

$$\tilde{r}^{l1} x_C + \tilde{L}^{l1} > \pi_A^{e1} + \tilde{r}^{e1} x_C + L^{e1},$$

two-step auction is not credible.

We show the following proposition. Note that $c_A(0) = 0$, that is, the fixed cost of the new technology is zero.

Proposition 2. *1. If the marginal cost of the new technology is constant, that is, the cost function is linear, entry with a license to one firm case and license to one firm without entry case are equivalent. The marginal cost of the old technology (technology of the non-licensee) need not be constant.*

2. If the cost function of the firms is strictly convex, two-step auction is credible.

3. If the cost function of the firms is strictly concave, two-step auction is not credible.

Proof. 1. First consider the case of entry with a license to one firm. Let $\bar{x} = x_A + x_C$. Denote the constant marginal cost of the new technology by c, and denote the total payoff of the innovator by φ^{e1}. It is written as

$$\varphi^{e1} = p\bar{x} - c\bar{x} - (px_B - c_B(x_B)).$$

If the marginal cost of the new technology is constant, $c_A'' = 0$. Thus, $\frac{d\bar{x}}{dr} = \frac{dx_A}{dr} + \frac{dx_C}{dr}$ and $\frac{dx_B}{dr}$ in Section 4.4 are written as

$$\frac{d\bar{x}}{dr} = \frac{p'(2p' + p''x_B - c_B''(x_B))}{\Gamma} = \frac{p'\theta_B}{\Gamma}, \ \frac{dx_B}{dr} = \frac{-p'(p' + p''x_B)}{\Gamma} = -\frac{p'\sigma_B}{\Gamma}.$$

The condition for maximization of φ^{e1} with respect to r is

$$(p + p'\bar{x} - c - p'x_B)\frac{d\bar{x}}{dr} - (p + p'x_B - c_B'(x_B) - p'\bar{x})\frac{dx_B}{dr} = 0. \tag{5}$$

From (2a) and (2c) we have

$$p + p'\bar{x} - c = r - p + c.$$

From this and (2b), (5) is rewritten as

$$(r - p + c - p'x_B)\frac{d\bar{x}}{dr} + p'\bar{x}\frac{dx_B}{dr} = 0.$$

Then, the optimal royalty rate is written as

$$\tilde{r}^{e1} = p - c + p'x_B + p'\bar{x}\frac{\sigma_B}{\theta_B}.$$

The first order condition for Firm C, (2c), with $r = \tilde{r}^{e1}$ is rewritten as

$$p + p'x_C - c - \left(p - c + p'x_B + p'\bar{x}\frac{\sigma_B}{\theta_B}\right) = p'(x_C - x_B) - p'\bar{x}\frac{\sigma_B}{\theta_B} = 0.$$

With $x_A + x_C = \bar{x}$, this and the first order condition for Firm A, (2a),

$$p + p'x_A - c = 0$$

imply

$$p + p'\bar{x} - c - p'x_B - p'\bar{x}\frac{\sigma_B}{\theta_B} = 0. \tag{6}$$

Next consider the case of license to one firm without entry not using a two-step auction. Let $\bar{x} = x_C$. Denote the total payoff of the innovator in this case by $\tilde{\varphi}^{l1}$. It is written as

$$\tilde{\varphi}^{l1} = p\bar{x} - c\bar{x} - (px_B - c_B(x_B)).$$

This is the same as φ^{e1}. If $c_A'' = 0$, $\frac{d\bar{x}}{dr} = \frac{dx_C}{dr}$ and $\frac{dx_B}{dr}$ in Section 4.2 are written as

$$\frac{d\bar{x}}{dr} = \frac{\theta_B}{\Delta},\ \frac{dx_B}{dr} = -\frac{\sigma_B}{\Delta},$$

θ_B and σ_B in this case are the same as those in the previous case. The condition for maximization of $\tilde{\varphi}^{l1}$ with respect to r is

$$(p + p'\bar{x} - c - p'x_B)\frac{d\bar{x}}{dr} - (p + p'x_B - c_B'(x_B) - p'\bar{x})\frac{dx_B}{dr} = 0. \tag{7}$$

From (1a) and (1b), (7) is rewritten as

$$(r - p'x_B)\frac{d\bar{x}}{dr} + p'\bar{x}\frac{dx_B}{dr} = 0.$$

Then, the optimal royalty rate is

$$\tilde{r}^{l1} = p' x_B + p' \bar{x} \frac{\sigma_B}{\theta_B}.$$

The first order condition for Firm C, (1a), with $x_C = \bar{x}$ and $r = \tilde{r}^{l1}$ is rewritten as

$$p + p'\bar{x} - c - p' x_B - p' \bar{x} \frac{\sigma_B}{\theta_B} = 0. \tag{8}$$

(6) and (8) are the same. Therefore, two cases are equivalent.

2. φ^{e1} with $\bar{x} = x_A + x_C$ is

$$\varphi^{e1} = p\bar{x} - c_A(x_A) - c_A(x_C) - (p x_B - c_B(x_B)).$$

$\tilde{\varphi}^{l1}$ with $\bar{x} = x_C$ is written as

$$\tilde{\varphi}^{l1} = p\bar{x} - c_A(\bar{x}) - (p x_B - c_B(x_B)).$$

If the cost function of the new technology, $c_A(\cdot)$, is strictly convex,

$$c_A(x_C) < \frac{x_C}{x_A + x_C} c_A(x_A + x_C) + \left(1 - \frac{x_C}{x_A + x_C}\right) c_A(0) = \frac{x_C}{x_A + x_C} c_A(x_A + x_C),$$

$$c_A(x_A) < \frac{x_A}{x_A + x_C} c_A(x_A + x_C) + \left(1 - \frac{x_A}{x_A + x_C}\right) c_A(0) = \frac{x_A}{x_A + x_C} c_A(x_A + x_C).$$

Then,

$$c_A(x_A) + c_A(x_C) < c_A(x_A + x_C).$$

Separation of production between two firms is more efficient than concentration to one firm. Thus, φ^{e1} is larger than $\tilde{\varphi}^{l1}$ when $x_A + x_C$ in the case of entry with a license and x_C in the case of license without entry are equal, and the maximum value of φ^{e1} is larger than the maximum value of $\tilde{\varphi}^{l1}$. Hence, two-step auction is credible.

3. Similarly to the case of strictly convex cost function, if the cost function of the new technology, $c_A(\cdot)$, is strictly concave, we find

$$c_A(x_A) + c_A(x_C) > c_A(x_A + x_C).$$

Concentration of production to one firm is more efficient than separation between two firms. Thus, $\tilde{\varphi}^{l1}$ is larger than φ^{e1} when $x_A + x_C$ in the case of entry with a license and x_C in the case of license without entry are equal, and the maximum value of $\tilde{\varphi}^{l1}$ is larger than the maximum value of φ^{e1}. Hence, two-step auction is not credible.

□

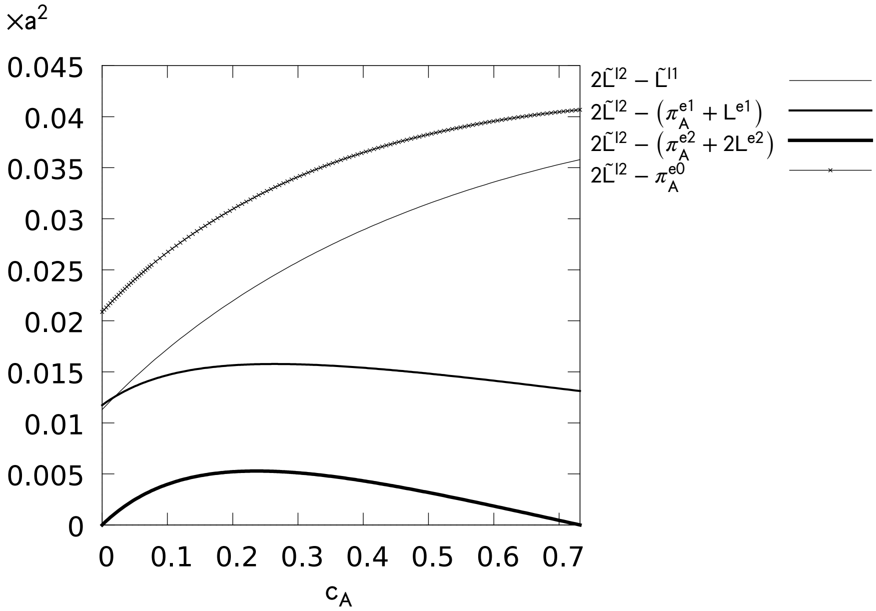

Figure 1: Optimal strategy for the innovator when $0 < c_A < \sqrt{3} - 1$

7 An example

As an example we assume that the inverse demand function is

$$p = a - x_A - x_B - x_C,$$

when Firm A enters. When it does not enter, $p = a - x_B - x_C$. a is a positive constant. The cost functions of the firms are quadratic. They are $\frac{1}{2}c_A x_A^2$ for Firm A. For Firm B and C with the old technology they are $\frac{1}{2}c_B x_B^2$ and $\frac{1}{2}c_B x_C^2$. With the new technology they are $\frac{1}{2}c_A x_B^2$ and $\frac{1}{2}c_A x_C^2$. We present summaries of the calculation results. About details of λ_A, λ_B, λ_C and λ_D please see Appendix.

License to one firm without entry not using two-step auction case The optimal royalty rate and the total payoff of the innovator are

$$\tilde{r}^{l1} = -\frac{a}{c_B + 2} < 0,$$

$$\tilde{r}^{l1} x_C + \tilde{L}^{l1} = \frac{a^2(c_B^2 - c_A c_B + 2c_B - 2c_A + 1)}{2(c_B + 2)(c_A c_B + 2c_B + 2c_A + 3)}.$$

Entry without license case The profit of the innovator is

$$\pi_A^{e0} = \frac{a^2(c_A + 2)(c_B + 1)^2}{2(c_A c_B + 2c_B + 3c_A + 4)^2}.$$

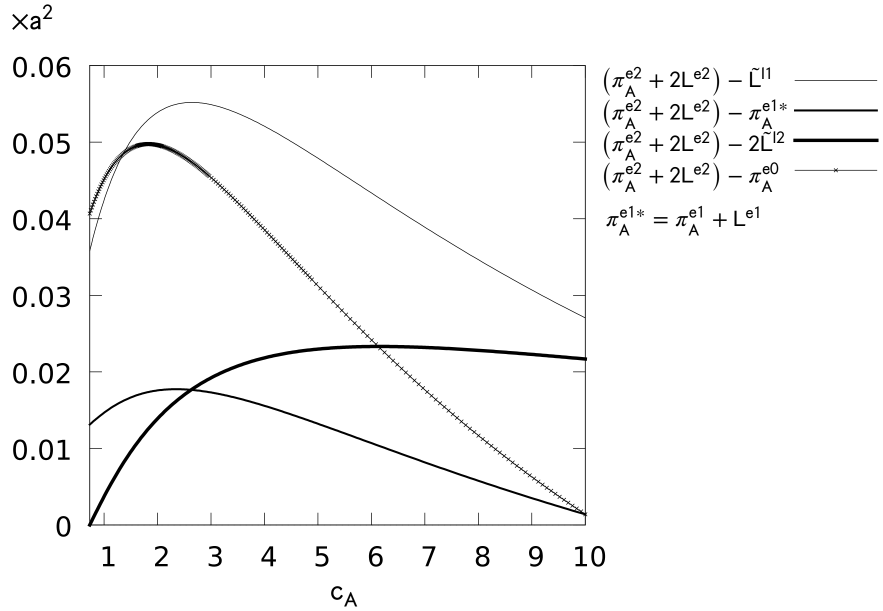

Figure 2: Optimal strategy for the innovator when $c_A > \sqrt{3} - 1$

Entry with a license to one firm case The optimal royalty rate and the total payoff of the innovator are

$$\tilde{r}^{e1} = \frac{a(c_A+1)^2(c_B^2 - c_Ac_B - 2c_A - 2)}{(c_Ac_B + 2c_B + 2c_A + 3)(c_A^2c_B + 4c_Ac_B + c_B + 2c_A^2 + 6c_A + 2)},$$

$$\begin{aligned}&\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1}\\ =&\frac{a^2(2c_A^2c_B^2 + 4c_Ac_B^2 + c_B^2 - c_A^3c_B + 2c_A^2c_B + 7c_Ac_B + 2c_B - 2c_A^3 - 2c_A^2 + 2c_A + 1)}{2(c_Ac_B + 2c_B + 2c_A + 3)(c_A^2c_B + 4c_Ac_B + c_B + 2c_A^2 + 6c_A + 2)}.\end{aligned}$$

Entry with licenses to two firms case The optimal royalty rate and the total payoff of the innovator are

$$\tilde{r}^{e2} = \frac{2a(c_A+1)^2}{(c_A+2)(c_A^2 + 6c_A + 2)} > 0,$$

$$\begin{aligned}&\pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2}\\ =&\frac{a^2\lambda_A}{2(c_A+2)(c_A^2+6c_A+2)(c_Ac_B+2c_B+2c_A+3)^2(c_A^2c_B+4c_Ac_B+c_B+2c_A^2+6c_A+2)^2}.\end{aligned}$$

License to one firm without entry case using two-step auction case The optimal royalty rate and the total payoff of the innovator are

$$\hat{r}^{l1} = -\frac{a(c_B+1)}{(c_B+2)(c_Ac_B + 2c_B + 2c_A + 2)} > 0,$$

$$\hat{r}^{l1}x_C+\hat{L}^{l1} = \frac{a^2\lambda_B}{\lambda_C}.$$

Licenses to two firms without entry case using two-step auction case The optimal royalty rate and the total payoff of the innovator are

$$\hat{r}^{l2} = \frac{a}{c_A+4} > 0,$$

$$\begin{aligned}&\hat{r}^{l2}(x_B+x_C)+2\hat{L}^{l2}\\ &\quad= \frac{a^2\lambda_D}{(c_A+4)(c_Ac_B+2c_B+2c_A+3)^2(c_A^2c_B+4c_Ac_B+c_B+2c_A^2+6c_A+2)^2}.\end{aligned}$$

Comparing $\pi_A^{e1}+\tilde{r}^{e1}x_C+\tilde{L}^{e1}$ and $\tilde{r}^{l1}x_C+\tilde{L}^{l1}$,

$$\begin{aligned}&\pi_A^{e1}+\tilde{r}^{e1}x_C+\tilde{L}^{e1}-(\tilde{r}^{l1}x_C+\tilde{L}^{l1})\\ &\quad=\frac{a^2c_A(c_B+1)(c_Ac_B^2+5c_Ac_B+2c_B+6c_A+2)}{2(c_B+2)(c_Ac_B+2c_B+2c_A+3)(c_A^2c_B+4c_Ac_B+c_B+2c_A^2+6c_A+2)}.\end{aligned}$$

Therefore, two-step auction is credible. About this example we get the following results.

1. If $0 < c_A < \sqrt{3}-1$, licenses to two firms without entry strategy is optimal for the innovator. Please see Figure 1.
2. If $c_A > \sqrt{3}-1$, entry with licenses to two firms strategy is optimal for the innovator. Please see Figure 2.

8 Concluding remarks and the future research

Appendix: Details of calculations

$$\begin{aligned}\lambda_A =&3c_A^8c_B^4+42c_A^7c_B^4+236c_A^6c_B^4+684c_A^5c_B^4+1095c_A^4c_B^4+962c_A^3c_B^4+438c_A^2c_B^4\\ &+96c_Ac_B^4+8c_B^4-2c_A^9c_B^3-8c_A^8c_B^3+114c_A^7c_B^3+1012c_A^6c_B^3+3364c_A^5c_B^3+5696c_A^4c_B^3\\ &+5160c_A^3c_B^3+2424c_A^2c_B^3+552c_Ac_B^3+48c_B^3-12c_A^9c_B^2-112c_A^8c_B^2-216c_A^7c_B^2\\ &+995c_A^6c_B^2+5454c_A^5c_B^2+10628c_A^4c_B^2+10296c_A^3c_B^2+5103c_A^2c_B^2+1230c_Ac_B^2+114c_B^2\\ &-24c_A^9c_B-256c_A^8c_B-896c_A^7c_B-700c_A^6c_B+2970c_A^5c_B+8444c_A^4c_B+9262c_A^3c_B\\ &+4964c_A^2c_B+1284c_Ac_B+128c_B-16c_A^9-176c_A^8-688c_A^7-1060c_A^6+92c_A^5+2436c_A^4\\ &+3252c_A^3+1908c_A^2+528c_A+56,\end{aligned}$$

$$
\begin{aligned}
\lambda_B =& c_A^6 c_B^6 + 12c_A^5 c_B^6 + 54c_A^4 c_B^6 + 112c_A^3 c_B^6 + 105c_A^2 c_B^6 + 36c_A c_B^6 + 4c_B^6 - c_A^7 c_B^5 + 74c_A^5 c_B^5 \\
&+ 406c_A^4 c_B^5 + 876c_A^3 c_B^5 + 826c_A^2 c_B^5 + 293c_A c_B^5 + 34c_B^5 - 10c_A^7 c_B^4 - 53c_A^6 c_B^4 + 90c_A^5 c_B^4 \\
&+ 1125c_A^4 c_B^4 + 2716c_A^3 c_B^4 + 2641c_A^2 c_B^4 + 976c_A c_B^4 + 119c_B^4 - 40c_A^7 c_B^3 - 264c_A^6 c_B^3 \\
&- 338c_A^5 c_B^3 + 1258c_A^4 c_B^3 + 4179c_A^3 c_B^3 + 4372c_A^2 c_B^3 + 1705c_A c_B^3 + 220c_B^3 - 80c_A^7 c_B^2 \\
&- 552c_A^6 c_B^2 - 1148c_A^5 c_B^2 + 89c_A^4 c_B^2 + 3210c_A^3 c_B^2 + 3928c_A^2 c_B^2 + 1650c_A c_B^2 + 227c_B^2 \\
&- 80c_A^7 c_B - 544c_A^6 c_B - 1272c_A^5 c_B - 852c_A^4 c_B + 1016c_A^3 c_B + 1800c_A^2 c_B + 840c_A c_B \\
&+ 124c_B - 32c_A^7 - 208c_A^6 - 496c_A^5 - 460c_A^4 + 32c_A^3 + 324c_A^2 + 176c_A + 28,
\end{aligned}
$$

$$
\begin{aligned}
\lambda_C =& 2(c_B + 2)(c_A c_B + 2c_B + 2c_A + 2)(c_A c_B + 2c_B + 2c_A + 3)^2(c_A^2 c_B + 4c_A c_B + c_B \\
&+ 2c_A^2 + 6c_A + 2)^2,
\end{aligned}
$$

$$
\begin{aligned}
\lambda_D =& c_A^6 c_B^4 + 12c_A^5 c_B^4 + 54c_A^4 c_B^4 + 112c_A^3 c_B^4 + 105c_A^2 c_B^4 + 36c_A c_B^4 + 4c_B^4 - c_A^7 c_B^3 - 4c_A^6 c_B^3 \\
&+ 34c_A^5 c_B^3 + 250c_A^4 c_B^3 + 584c_A^3 c_B^3 + 568c_A^2 c_B^3 + 205c_A c_B^3 + 24c_B^3 - 6c_A^7 c_B^2 - 44c_A^6 c_B^2 \\
&- 46c_A^5 c_B^2 + 347c_A^4 c_B^2 + 1092c_A^3 c_B^2 + 1152c_A^2 c_B^2 + 448c_A c_B^2 + 57c_B^2 - 12c_A^7 c_B - 96c_A^6 c_B \\
&- 212c_A^5 c_B + 88c_A^4 c_B + 871c_A^3 c_B + 1062c_A^2 c_B + 455c_A c_B + 64c_B - 8c_A^7 - 64c_A^6 \\
&- 160c_A^5 - 76c_A^4 + 254c_A^3 + 384c_A^2 + 182c_A + 28,
\end{aligned}
$$

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