Royalty and license fee under vertical differentiation in oligopoly with or without entry of innovator: Two-step auction

Hattori, Masahiko and Tanaka, Yasuhito

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Royalty and license fee under vertical differentiation in oligopoly with or without entry of innovator: Two-step auction

Masahiko Hattori*
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.
and
Yasuhito Tanaka†
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.

Abstract

When an outside innovating firm has a technology to produce a higher quality good than the good produced at present, it can sell licenses of its technology to incumbent firms, or enter the market and at the same time sell licenses, or enter the market without license. We examine the definitions of license fee in such a situation in an oligopoly with three firms under vertical product differentiation, one outside innovating firm and two incumbent firms, considering threat by entry of the innovating firm using a two-step auction. We also present an example of the optimal strategy for the innovating firm under the assumption of uniform distribution of consumers' taste parameter and zero cost. Also we suppose that the innovating firm sells its licenses using a combination of royalty per output and a fixed license fee.

Keywords: royalty, license fee; entry; oligopoly; vertical differentiation; two-step auction.

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*M. Hattori, mhattori@mail.doshisha.ac.jp
†Y. Tanaka, yasuhito@mail.doshisha.ac.jp

1. Introduction

In Proposition 4 of Kamien and Tauman (1986) it was argued that in an oligopoly when the number of firms is small (or very large), strategy to enter the market and at the same time license the cost-reducing technology to the incumbent firm (entry with license strategy) is more profitable than strategy to license its technology to the incumbent firm without entering the market (license without entry strategy) for the innovating firm. However, their result depends on their definition of license fee. They defined the license fee in the case of licenses without entry by the difference between the profit of an incumbent firm in that case and its profit before it buys a license without entry of the innovating firm. However, it is inappropriate from the game theoretic view point. If an incumbent firm does not buy a license, the innovating firm may punish the incumbent firm by entering the market. The innovating firm can use such a threat if and only if it is a credible threat. In a duopoly case with one incumbent firm, when the innovating firm does not enter nor sell a license, its profit is zero; on the other hand, when it enters the market without license, its profit is positive. Therefore, threat by entry without license is credible under duopoly, and then even if the innovating firm does not enter the market, the incumbent firm must pay the difference between its profit when it uses the new technology and its profit when the innovating firm enters without license as a license fee. However, in an oligopoly with more than one incumbent firms, the credibility of threat by entry is a more subtle problem.

In this paper we extend this analysis to an oligopolistic situation with three firms, one outside innovating firm and two incumbent firms under vertical product differentiation, and examine the definitions of license fee for producing a higher quality good than the good produced at present considering a two-step auction in the case of licenses without entry[1]. Also we suppose that the innovating firm uses a combination of royalty per output and a fixed license fee. A two-step auction, for example, in the case of a license to one incumbent firm without entry is as follows.

(1) The first step.

The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below, and the innovating firm imposes a predetermined (positive or negative) royalty per output on the licensee. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells a license to one firm at auction *with its entry*.

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

[1] Hattori and Tanaka (2016b) presented an analysis of license and entry choice by an innovating firm in a duopoly under vertical product differentiation.

the difference between its profit when only this firm uses the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step each incumbent firm has an incentive to make a bid when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid.

We need the effective minimum bidding price because if the minimum price does not function effectively, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

A two-step auction in the case of licenses to two incumbent firms without entry is similar[2], and at the first step of the auction the incumbent firm has a will to pay the following license fee;

> the difference between its profit when both firms use the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step each incumbent firm has an incentive to make a bid when the other firm makes a bid because if it does not make a bid, the auction proceed to the next step.

Threat by such a two-step auction is credible if and only if the profit of the innovating firm when it enters the market with a license to one firm is larger than its profit when it licenses to one incumbent firm without entering the market.

In the next section we present literature review. In Section 3 the model of this paper is described. In Section 4 we consider various equilibria of the oligopoly. In Section 5 we present the license fees under entry with license strategy. In Section 6 we consider a two-step auction and present the definitions of license fees under license without entry strategy. In Sections 5 and 6 the following results about the optimal royalty rate for the innovator will be shown (see Proposition 1).

Entry with license to one firm case The optimal royalty rate may be positive or negative.

Entry with licenses to two firms case If the goods are strategic complements, the optimal royalty rate is positive. If the goods are strategic substitutes, it may be positive or negative.

License to one firm without entry cases not using two-step auction case If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it may be positive or negative.

License to one firm without entry cases using two-step auction case If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it is positive.

Licenses to two firms without entry cases using or not using two-step auction case The optimal royalty rate is positive.

[2]Please see Section 6.2.2.

Also in Section 6 we examine the credibility of two-step auction, and will show the following results (see Proposition 2).

(1) If the cost function of the new technology is linear, the profit of the innovating firm when it enters the market with a license to one firm and its profit when it licenses to one incumbent firm without entering the market are equal, that is, entry with license to one firm case and license to one firm without entry case are equivalent.

(2) If the cost function of the new technology is strictly convex, two-step auction is credible.

(3) If the cost function of the new technology is strictly concave, two-step auction is not credible.

In Section 7 we present an example with uniform distribution of consumers' taste parameter and zero cost. We will show that when two-step auction is credible, license to two firms without entry strategy is optimal; on the other hand, when it is not credible, entry without license strategy is optimal. In Appendix we present analyses of demand and inverse demand functions.

2. Literature review

Various studies focus on technology adoption or R&D investment in duopoly or oligopoly. Most of them analyze the relation between the technology licensor and licensee. The difference of means of contracts, which comprise royalties, upfront fixed fees, combinations of these two, and auctions, are well discussed (Katz and Shapiro (1985)). Kamien and Tauman (2002) showed that outside innovators prefer auctions, but industry incumbents prefer royalty. This topic is discussed by Kabiraj (2004) under the Stackelberg oligopoly; here, the licensor does not have production capacity. Wang and Yang (2004) considered the case when the licensor has production capacity. Sen and Tauman (2007) compared the license system in detail, namely, when the licensor is an outsider and when it is an incumbent firm, using the combination of royalties and fixed fees. However, the existence of production capacity was externally given, and they did not analyze the choice of entry. Therefore, the optimal strategies of outside innovators, who can use the entry as a threat, require more discussion. Regarding the strategies of new entrants to the market, Duchene, Sen and Serfes (2015) focused on future entrants with old technology, and argued that a low license fee can be used to deter the entry of potential entrants. However, the firm with new technology is incumbent, and its choice of entry is not analyzed. Also, Chen (2016) analyzed the model of the endogenous market structure determined by the potential entrant with old technology and showed that the licensor uses the fixed fee and zero royalty in both the incumbent and the outside innovator cases, which are exogenously given. Creane, Chiu and Konishi (2013) examined a firm that can license its production technology to a rival when firms are heterogeneous in production costs, and showed that a complete technology transfer from one firm to another always increases joint profit under weakly concave demand when at least three firms remain in the industry.

A Cournot oligopoly with fixed fee under cost asymmetry was analyzed by La Manna (1993). He showed that if technologies can be replicated perfectly, a lower cost firm always has the incentive to transfer its technology; hence, while a Cournot-Nash equilibrium cannot be fully asymmetric, there exists no non-cooperative Nash equilibrium in pure strategies. On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyzed bargaining between a licensor with no production capacity and oligopolistic firms. Recent research focuses on market structure and technology improvement. Boone (2001) and Matsumura et. al. (2013) found a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) showed that technology adoption may change the market outcome. The social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may result in higher social welfare than Bertrand competition under a differentiated goods market. Hattori and Tanaka (2015) and (2016a) studied the adoption of new technology in Cournot duopoly and Stackelberg duopoly. Rebolledo and Sandonís (2012) presented an analysis of the effectiveness of research and development (R&D) subsidies in an oligopolistic model in the cases of international competition and cooperation in R&D. Hattori and Tanaka (2016b) analyzed problems about product innovation, that is, introduction of higher quality good in a duopoly with vertical product differentiation.

3. The model

Our model of vertical product differentiation is according to Mussa and Rosen (1978), Bonanno and Haworth (1998) and Tanaka (2001). There are three firms, Firms A, B and C. Firm A can produce the high-quality good whose quality is k_H, and Firms B and C produce the low-quality good whose quality is k_L, where $k_H > k_L > 0$. k_H and k_L are fixed. Both of the high-quality and the low-quality goods are produced at the same cost.

At present only Firms B and C produce the low-quality good. Firm A is an outside innovator, and it may sell licenses to use its technology for producing the high-quality good to one or two incumbent firms (Firms B and C), and it can enter the market with the high-quality good. Call Firm A the innovating firm and Firms B and C the incumbent firms.

Firm A has five options.

(1) To enter the market, and license its technology to no incumbent firm.

(2) To enter the market, and license its technology to one incumbent firm.

(3) To enter the market, and license its technology to two incumbent firms.

(4) To license its technology to one incumbent firm, but does not enter the market.

(5) To license its technology to two incumbent firms, but does not enter the market.

The cost function of the firms is $c(\cdot)$, which is twice continuously differentiable. There is no fixed cost; thus $c(0) = 0$.

In the market there is a continuum of consumers with the same income, denoted by y, but different values of the taste parameter ξ. Each consumer buys at most one unit of the good. If a consumer with parameter ξ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \xi k$. If a consumer does not buy the good, his utility is equal to his income y. The parameter ξ is distributed according to a twice continuously differentiable distribution function $\rho = F(\xi)$ in the interval $0 < \xi \leq 1$. We assume that there is no atom. ρ denotes the probability that the taste parameter is smaller than or equal to ξ. The size of consumers is normalized as one. The inverse function of $F(\xi)$ is denoted by $G(\rho)$. Note that $G(1) = 1$.

Let p_L and q_L be the price and supply of the good of quality k_L; p_H and q_H be the price and supply of the good of quality k_H; and let q_A, q_B and q_C be the outputs of Firms A, B and C.

In the cases with licenses the game proceeds as follows. In the first stage Firm A determines the royalty rate. In the second stage firms determine the outputs, and the fixed license fee is determined.

4. General analysis

4.1. Entry without license case

Suppose that Firm A (the innovating firm) enters into the market without license to Firm B nor C. Then, Firm A supplies the high-quality good and Firms B and C supply the low-quality good. Let ξ_L be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality good. Then,

$$\xi_L = \frac{p_L}{k_L}.$$

Let ξ_H be the value of ξ for which the corresponding consumer is indifferent between buying the low-quality good and buying the high-quality good. Then

$$\xi_H = \frac{p_H - p_L}{k_H - k_L}.$$

Let $q_H = q_A$ and $q_L = q_B + q_C$. The inverse demand function is described as follows.

(1) When $q_H > 0$ and $q_L > 0$, we have $p_H = (k_H - k_L)G(1 - q_H) + k_L G(1 - q_H - q_L)$ and $p_L = k_L G(1 - q_H - q_L)$.

(2) When $q_H > 0$ and $q_L = 0$, we have $p_H = k_H G(1 - q_H)$ and $p_L = k_L G(1 - q_H)$.

(3) When $q_H = 0$ and $q_L > 0$, we have $p_H = k_H - k_L + k_L G(1 - q_L)$ and $p_L = k_L G(1 - q_L)$.

(4) When $q_H = 0$ and $q_L = 0$, we have $p_H = k_H$ and $p_L = k_L$.

Since $G(1) = 1$, this is a continuously differentiable function with the domain $0 \leq q_H \leq 1$ and $0 \leq q_H \leq 1$. For details of derivation of the inverse demand function please see Appendix A.3.

The profits of Firms A, B and C are written as

$$\pi_A = [(k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B - q_C)]q_A - c(q_A),$$

$$\pi_B = k_L G(1 - q_A - q_B - q_C)q_B - c(q_B),$$

$$\pi_C = k_L G(1 - q_A - q_B - q_C)q_B - c(q_C).$$

The first order conditions for profit maximization of Firms A, B and C are

$$\begin{aligned}\frac{\partial \pi_A}{\partial q_A} =&(k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B - q_C)\\ &- [(k_H - k_L)G'(1 - q_A) + k_L G'(1 - q_A - q_B - q_C)]q_A - c'(q_A) = 0,\end{aligned}$$

$$\frac{\partial \pi_B}{\partial q_B} = k_L G(1 - q_A - q_B - q_C) - k_L G'(1 - q_A - q_B - q_C)q_B - c'(q_B) = 0,$$

$$\frac{\partial \pi_C}{\partial q_C} = k_L G(1 - q_A - q_B - q_C) - k_L G'(1 - q_A - q_B - q_C)q_C - c'(q_C) = 0.$$

The second order conditions are

$$\begin{aligned}\frac{\partial^2 \pi_A}{\partial q_A^2} =& - 2[(k_H - k_L)G'(1 - q_A) + k_L G'(1 - q_A - q_B - q_C)]\\ &+ [(k_H - k_L)G''(1 - q_A) + k_L G''(1 - q_A - q_B - q_C)]q_A - c''(q_A) < 0,\end{aligned}$$

$$\frac{\partial^2 \pi_B}{\partial q_B^2} = -k_L[2G'(1 - q_A - q_B - q_C) - G''(1 - q_A - q_B - q_C)q_B] - c''(q_B) < 0,$$

$$\frac{\partial^2 \pi_C}{\partial q_C^2} = -k_L[2G'(1 - q_A - q_B - q_C) - G''(1 - q_A - q_B - q_C)q_C] - c''(q_C) < 0.$$

Hereafter we assume that the second order conditions in each case are satisfied.

Denote the equilibrium profits of Firms A, B and C in this case by π_A^{e0}, π_B^{e0} and π_C^{e0}.

4.2. Entry with license to one firm case

Suppose that Firm A enters into the market and licenses its technology for producing the high-quality good to one of the incumbent firms. We assume that it is Firm C. Then, Firms A and C produce the high-quality good, and Firm B produces the low-quality good. Let $q_H = q_A + q_C$ and $q_L = q_B$. The inverse demand function is the same as that in the previous case.

Denote the royalty per output and the fixed license fee by r and L. The profits of Firms A, B and C are

$$\pi_A = [(k_H - k_L)G(1 - q_A - q_C) + k_L G(1 - q_A - q_B - q_C)]q_A - c(q_A),$$

$$\pi_B = k_L G(1 - q_A - q_B - q_C)q_B - c(q_B),$$

$$\pi_C = [(k_H - k_L)G(1 - q_A - q_C) + k_L G(1 - q_A - q_B - q_C)]q_C - c(q_C) - rq_C - L.$$

The first order conditions are

$$\begin{aligned}&(k_H - k_L)G(1 - q_A - q_C) + k_L G(1 - q_A - q_B - q_C)\\ &- [(k_H - k_L)G'(1 - q_A - q_C) + k_L G'(1 - q_A - q_B - q_C)]q_A - c'(q_A) = 0,\end{aligned} \tag{1a}$$

$$k_L G(1 - q_A - q_B - q_C) - k_L G'(1 - q_A - q_B - q_C)q_B - c'(q_B) = 0, \tag{1b}$$

$$(k_H - k_L)G(1 - q_A - q_C) + k_L G(1 - q_A - q_B - q_C) \\ - [(k_H - k_L)G'(1 - q_A - q_C) + k_L G'(1 - q_A - q_B - q_C)]q_C - c'(q_C) - r = 0. \tag{1c}$$

Denote the equilibrium profits of Firms A, B and C by π_A^{e1}, π_B^{e1} and π_C^{e1}. Differentiating (1a), (1b) and (1c) with respect to r, we obtain $\frac{dq_A}{dr}$, $\frac{dq_A}{dr}$ and $\frac{dq_A}{dr}$. About details of them see Appendix B. We have $\frac{dq_C}{dr} < 0$. If the goods are strategic substitutes, σ's are negative, and $\frac{dq_A}{dr}$ and $\frac{dq_B}{dr}$ are positive. If the goods are strategic complements, σ's are positive, and $\frac{dq_A}{dr}$ and $\frac{dq_B}{dr}$ are negative.

4.3. Entry with licenses to two firms case

Suppose that Firm A enters into the market and licenses its technology for producing the high-quality good to both incumbent firms. Then, all firms produce the high-quality good. Let ξ_0 be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the high-quality good. Then

$$\xi_0 = \frac{p_H}{k_H}.$$

Let $q_H = q_A + q_B + q_C$. The inverse demand function is described as follows.

(1) When $q_H > 0$, we have $p_H = k_H G(1 - q_H)$.

(2) When $q_H = 0$, we have $p_H = k_H$.

Since $G(1) = 1$, this is a continuously differentiable function. About details for derivation of the inverse demand function please see Appendix A.1.

The profits of the firms are

$$\pi_A = k_H G(1 - q_A - q_B - q_C)q_A - c(q_A).$$

$$\pi_B = k_H G(1 - q_A - q_B - q_C)q_B - c(q_B) - rq_B - L.$$

$$\pi_C = k_H G(1 - q_A - q_B - q_C)q_C - c(q_C) - rq_C - L.$$

The first order conditions are

$$k_H G(1 - q_A - q_B - q_C) - k_H G'(1 - q_A - q_B - q_C)q_A - c'(q_A) = 0. \tag{2a}$$

$$k_H G(1 - q_A - q_B - q_C) - k_H G'(1 - q_A - q_B - q_C)q_B - r - c'(q_B) = 0, \tag{2b}$$

$$k_H G(1 - q_A - q_B - q_C) - k_H G'(1 - q_A - q_B - q_C)q_C - r - c'(q_C) = 0. \tag{2c}$$

Denote the equilibrium profits of Firms A, B and C by π_A^{e2}, π_B^{e2} and π_C^{e2}. Differentiating (2a), (2b) and (2c) with respect to r, we obtain $\frac{dq_A}{dr}$, $\frac{dq_B}{dr}$ and $\frac{dq_C}{dr}$. About details of them see Appendix C. We have $\frac{dq_B}{dr} < 0$ and $\frac{dq_C}{dr} < 0$. If the goods are strategic substitutes, σ's are negative and $\frac{dq_A}{dr}$ is positive. If the goods are strategic complements, σ's are positive and $\frac{dq_A}{dr}$ is negative.

4.4. License to one firm without entry case

Suppose that Firm A sells a license of its technology to one of the incumbent firms and does not enter the market. We assume that it is Firm C. Firm B still produces the low-quality good. Let $q_H = q_H$ and $q_L = q_B$. The inverse demand function is the same as that in the entry without license case.

The profits of Firms B and C are

$$\pi_B = k_L G(1 - q_B - q_C)q_B - c(q_B),$$

$$\pi_C = [(k_H - k_L)G(1 - q_C) + k_L G(1 - q_B - q_C)]q_C - c(q_C) - rq_C - L.$$

The first order conditions are

$$k_L G(1 - q_B - q_C) - k_L G'(1 - q_B - q_C)q_B - c'(q_B) = 0, \tag{3a}$$

$$\begin{aligned}(k_H - k_L)G(1 - q_C) + k_L G(1 - q_B - q_C) - [(k_H - k_L)G'(1 - q_C)\\ + k_L G'(1 - q_B - q_C)]q_C - r - c'(q_C) = 0.\end{aligned} \tag{3b}$$

Denote the equilibrium profits of Firms B and C by π_B^{l1} and π_C^{l1}. Differentiating (3a) and (3b) with respect to r, we obtain

$$\frac{dq_B}{dr} = -\frac{-k_L G'(1 - q_B - q_C) + k_L G''(1 - q_B - q_C)q_B}{\Gamma},$$

and

$$\frac{dq_C}{dr} = \frac{-2k_L G'(1 - q_B - q_C) + k_L G''(1 - q_B - q_C)q_B - c''(q_B)}{\Gamma} < 0,$$

where

$$\begin{aligned}\theta_C =& -2[(k_H - k_L)G'(1 - q_C) + k_L G'(1 - q_B - q_C)] + [(k_H - k_L)G''(1 - q_C)\\ &+ k_L G''(1 - q_B - q_C)]q_B - c''(q_C),\\ \Gamma =& [-2k_L G'(1 - q_B - q_C) + k_L G''(1 - q_B - q_C)q_B - c''(q_B)]\theta_C.\end{aligned}$$

4.5. Licenses to two firms without entry case

Suppose that Firm A sells licenses of its technology to two incumbent firms and does not enter the market. Then, Firms B and C produce the high-quality good. Let $q_H = q_B + q_C$. The inverse demand function is the same as that in the entry with licenses to two firms case.

The profits of the firms are

$$\pi_B = k_H G(1 - q_B - q_C)q_B - c(q_B) - rq_B - L,$$

$$\pi_C = k_H G(1 - q_B - q_C)q_C - c(q_C) - rq_C - L.$$

The first order conditions are

$$k_H G(1 - q_B - q_C) - k_H G'(1 - q_B - q_C)q_B - r - c'(q_B) = 0, \tag{4a}$$

$$k_H G(1-q_B-q_C) - k_H G'(1-q_B-q_C)q_C - r - c'(q_C) = 0. \tag{4b}$$

Denote the equilibrium profits of Firms B and C by π_B^{l2} and π_C^{l2}. In this case we have $q_B = q_C$. Differentiating (4a) and (4b) with respect to r, we obtain

$$\frac{dq_B}{dr} = \frac{dq_C}{dr} = \frac{-k_H G'(1-q_B-q_C) - c''(q_B)}{\Gamma'} < 0,$$

where

$$\begin{aligned}\Gamma' =& [-2k_H G'(1-q_B-q_C) + k_H G''(1-q_B-q_C)q_B - c''(q_B)]\times \\ & [-2k_H G'(1-q_B-q_C) + k_H G''(1-q_B-q_C)q_C - c''(q_C)].\end{aligned}$$

5. Royalty and license fees in the cases of licenses with entry

In the cases of licenses with entry the fixed license fee is equal to the usual willingness to pay for the incumbent firms. We follow the arguments by Kamien and Tauman (1986) and Sen and Tauman (2007) about license fees by auction.

5.1. License to one firm

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when only this firm uses the technology for producing the high-quality good with entry of the innovating firm and its profit when only the rival firm buys the license with entry of the innovating firm.

This is because each incumbent firm knows that there will be one licensee regardless of whether or not it buys a license. The incumbent firms B and C have the same willingness to pay, so even when one of them does not make a bid, the rival firm gets the license. The fixed license fee is

$$L^{e1} = (\pi_C^{e1} + L^{e1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{e1} = \pi_B^{e1}$. The total payoff of Firm B in this case is written as

$$\begin{aligned}\varphi^{e1} = \pi_A^{e1} + rq_C + L^{e1} =& [(k_H-k_L)G(1-q_A-q_C) + k_L G(1-q_A-q_B-q_C)]q_A - c(q_A) \\ &+ [(k_H-k_L)G(1-q_A-q_C) + k_L G(1-q_A-q_B-q_C)]q_C \\ &- c(q_C) - (k_L G(1-q_A-q_B-q_C)q_B - c(q_B)).\end{aligned}$$

Using the first order conditions, the condition for maximization of φ with respect to r is written as follows.

$$\begin{aligned}\frac{d\varphi^{e1}}{dr} =& r\frac{dq_C}{dr} - (k_H-k_L)G'(1-q_A-q_C)\left(q_C\frac{dq_A}{dr} + q_A\frac{dq_C}{dr}\right) \\ &- k_L G'(1-q_A-q_B-q_C)\left[(q_C-q_B)\frac{dq_A}{dr} + (q_A-q_B)\frac{dq_C}{dr} + (q_A+q_C)\frac{dq_B}{dr}\right] = 0,\end{aligned}$$

Then, we get the optimal royalty rate for the innovator as follows.

$$\begin{aligned}\tilde{r}^{e1} =& \frac{(k_H - k_L)G'(1 - q_A - q_C)}{\frac{dq_C}{dr}}\left(q_C\frac{dq_A}{dr} + q_A\frac{dq_C}{dr}\right) \\ &+ \frac{k_L G'(1 - q_A - q_B - q_C)}{\frac{dq_C}{dr}}\left[(q_C - q_B)\frac{dq_A}{dr} + (q_A - q_B)\frac{dq_C}{dr} + (q_A + q_C)\frac{dq_B}{dr}\right].\end{aligned} \tag{5}$$

This may be positive or negative.

5.2. Licenses to two firms

The willingness to pay for each incumbent firm in this case is equal to

> the difference between its profit when two firms use the technology for producing the high-quality good with entry of the innovating firm and its profit when only the rival firm buys the license with entry of the innovating firm.

This is because each incumbent firm knows that there will be one licensee when it does not buy a license. In this case there is a minimum bidding price which is equal to the willingness to pay for the incumbents because without the minimum bidding price no firm makes a positive bid. The fixed license fee is

$$L^{e2} = (\pi_C^{e2} + L^{e2}) - \pi_B^{e1}.$$

This means $\pi_C^{e2} = \pi_B^{e1}$. The total payoff of the innovator is

$$\begin{aligned}\varphi^{e2} =& \pi_A^{e2} + r(q_B + q_C) + 2L^{e2} = k_H G(1 - q_A - q_B - q_C)(q_A + q_B + q_C) \\ &- c(q_A) - c(q_B) - c(q_C) - 2\pi_B^{e1}.\end{aligned}$$

Note that π_B^{e1} is constant and irrelevant to determination of the royalty rate in this case. Using the first order conditions, the condition for maximization of φ^{e2} with respect to r is written as follows.

$$\begin{aligned}&\frac{d\varphi^{e2}}{dr} = r\left(\frac{dq_B}{dr} + \frac{dq_C}{dr}\right) - k_H G'(1 - q_A - q_B - q_C)(q_B + q_C)\frac{dq_A}{dr} \\ &- k_H G'(1 - q_A - q_B - q_C)(q_A + q_C)\frac{dq_B}{dr} - k_H G'(1 - q_A - q_B - q_C)(q_A + q_B)\frac{dq_C}{dr} = 0.\end{aligned}$$

The optimal royalty rate is

$$\tilde{r}^{e2} = \frac{k_H G'(1 - q_A - q_B - q_C)}{\frac{dq_B}{dr} + \frac{dq_B}{dr}}\left[(q_B + q_C)\frac{dq_A}{dr} + (q_A + q_C)\frac{dq_B}{dr} + (q_A + q_B)\frac{dq_C}{dr}\right].$$

If the goods are strategic complements, $\tilde{r}^{e2} > 0$. If the goods are strategic substitutes, $\tilde{r}^{e2}$ may be positive or negative.

6. Royalty and license fees in the case of licenses without entry: two-step auction

6.1. One-step auction

If the licenses are auctioned off to the incumbent firms by one-step auction, the fixed license fee is determined by the usual willingness to pay for the incumbent firms described in Kamien and Tauman (1986) and Sen and Tauman (2007).

6.1.1. License to one firm

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when only this firm uses the technology for producing the high-quality good without entry of the innovating firm and its profit when only the rival firm buys the license without entry of the innovating firm.

Then, the fixed license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{l1}.$$

This equation means $\pi_C^{l1} = \pi_B^{l1}$. Denote L in this case by $\tilde{L}^{l1}$, and denote the total payoff of the innovator in this case by $\tilde{\varphi}^{l1}$ to distinguish it from the total payoff in the two-step auction case. It is

$$\begin{aligned}\tilde{\varphi}^{l1} =& rq_C + \tilde{L}^{l1} = [(k_H - k_L)G(1 - q_C) + k_L G(1 - q_B - q_C)]q_C - c(q_C)\\ &- (k_L G(1 - q_B - q_C)q_B - c(q_B)).\end{aligned}$$

Using the first order conditions, the condition for maximization of $\tilde{\varphi}^{l1}$ with respect to r is written as

$$\frac{d\tilde{\varphi}^{l1}}{dr} = (r + k_L G'(1 - q_B - q_C)q_B)\frac{dq_C}{dr} - k_L G'(1 - q_B - q_C)q_C\frac{dq_B}{dr} = 0,$$

Then, we obtain the optimal royalty rate for the innovator as follows.

$$r^{l1} = -\frac{k_L G'(1 - q_B - q_C)}{\frac{dq_C}{dr}}\left(q_B\frac{dq_C}{dr} - q_C\frac{dq_B}{dr}\right). \tag{6}$$

Denote it by $\tilde{r}^{l1}$. If the goods are strategic substitutes, $\tilde{r}^{l1} < 0$. If the goods are strategic complements, $\tilde{r}^{l1}$ may be positive or negative.

6.1.2. Licenses to two firms

The willingness to pay for each incumbent firm in this case is equal to

> the difference between its profit when two firms use the technology for producing the high-quality good without entry of the innovating firm and its profit when only the rival firm buys the license without entry of the innovating firm.

There is a minimum bidding price which is equal to the willingness to pay for the incumbents. The fixed license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{l1}.$$

This means $\pi_C^{l2} = \pi_B^{l1}$. Denote L in this case by $\tilde{L}^{l2}$, and denote the total payoff of the innovator by $\tilde{\varphi}^{l2}$. It is

$$\tilde{\varphi}^{l2} = r(q_B + q_C) + 2\tilde{L}^{l2} = k_H G(1 - q_B - q_C)(q_B + q_C) - c(q_B) - c(q_C) - 2\pi_B^{l1}.$$

Note that π_B^{l1} is constant and irrelevant to determination of the royalty rate. The condition for maximization of $\tilde{\varphi}^{l2}$ with respect to r is

$$\frac{d\tilde{\varphi}^{l2}}{dr} = r\left(\frac{dq_B}{dr} + \frac{dq_C}{dr}\right) - k_H G'(1 - q_B - q_C)q_B\frac{dq_C}{dr} - k_H G'(1 - q_B - q_C)q_C\frac{dq_B}{dr} = 0.$$

The optimal royalty rate is

$$r^{l2} = \frac{k_H G'(1 - q_B - q_C)}{\frac{dq_B}{dr} + \frac{dq_C}{dr}}\left(q_B\frac{dq_C}{dr} + q_C\frac{dq_B}{dr}\right).$$

Denote it by $\tilde{r}^{l2}$. This is positive.

6.2. Two-step auction

We consider a two-step auction for each case.

6.2.1. License to one firm

In this case the two-step auction is practiced as follows.

(1) The first step.

The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below, and the innovating firm imposes a predetermined (positive or negative) royalty per output on the licensee. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells a license to one firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

the difference between its profit when only this firm uses the technology for producing the high-quality good *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

Then, the license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{l1} = \pi_B^{e1}$. Denote L in this case by $\hat{L}^{l1}$.

In the first step each incumbent firm has an incentive to make a bid with the license fee $\hat{L}^{l1}$ when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid. We need the effective minimum bidding price $\hat{L}^{l1}$ because the profit of a non-licensee is π_B^{l1} which is larger than π_B^{e1}. If the minimum price does not function effectively, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

Denote the total payoff of the innovator in this case by $\hat{\varphi}^{l1}$. Then,

$$\hat{\varphi}^{l1} = rq_C + \hat{L}^{l1} = [(k_H - k_L)G(1 - q_C) + k_L G(1 - q_B - q_C)]q_C - c(q_C) - \pi_B^{e1}.$$

Note that π_B^{e1} is a constant number which is determined in the entry with a license to one firm case. The condition for maximization of φ with respect to r is

$$\frac{d\hat{\varphi}^{l1}}{dr} = r\frac{dq_C}{dr} - k_L G'(1 - q_B - q_C)q_C\frac{dq_B}{dr} = 0.$$

Then, we obtain the optimal royalty rate for the innovator as follows.

$$r^{l1} = \frac{k_L G'(1 - q_B - q_C)q_C}{\frac{dq_C}{dr}}\frac{dq_B}{dr}. \tag{7}$$

Denote it by $\hat{r}^{l1}$.

6.2.2. Licenses to two firms

We consider the following two-step auction

(1) The first step.

The innovating firm sells licenses to two firms at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below and both firms make bids, and the innovating firm imposes a predetermined (positive or negative) royalty per output on the licensee . If both firms make bids, they get licenses. If at least one of the firms does not make a bid, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells a license to *one* firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when two firms use the technology for producing the high-quality good *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

The minimum bidding price should be equal to this willingness to pay. Then, the license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{e1}.$$

This means $\pi_C^{l2} = \pi_B^{e1}$. Denote L in this case by $\hat{L}^{l2}$.

In the first step each incumbent firm has an incentive to make a bid when the other firm makes a bid because if it does not make a bid, the auction proceeds to the next step.

Denote the total payoff of the innovator in this case by $\hat{\varphi}^{l2}$. It is

$$\hat{\varphi}^{l2} = r(q_B + q_C) + 2\hat{L}^{l2} = k_H G(1 - q_B - q_C)(q_B + q_C) - c(q_B) - c(q_C) - 2\pi_B^{e1}.$$

Note that π_B^{e1} is constant and irrelevant to determination of the royalty rate in this case. The condition for maximization of $\hat{\varphi}^{l2}$ with respect to r is

$$\frac{d\hat{\varphi}^{l2}}{dr} = r\left(\frac{dq_B}{dr} + \frac{dq_C}{dr}\right) - k_H G'(1 - q_B - q_C)\left(q_B\frac{dq_C}{dr} + q_C\frac{dq_B}{dr}\right) = 0.$$

$$\tilde{r} = \frac{k_H G'(1 - q_B - q_C)}{\frac{dq_B}{dr} + \frac{dq_C}{dr}}\left(q_B\frac{dq_C}{dr} + q_C\frac{dq_B}{dr}\right).$$

Denote it by $\hat{r}^{l2}$. We see $\hat{r}^{l2} = \tilde{r}^{l2}$, but the total payoff of the innovator with two-step auction and that without two-step auction are different.

Summarizing the results about the optimal royalty rates for the innovator.

Proposition 1. **Entry with license to one firm case** *The optimal royalty rate may be positive or negative.*

Entry with licenses to two firms case *If the goods are strategic complements, the optimal royalty rate is positive. If the goods are strategic substitutes, it may be positive or negative.*

License to one firm without entry case not using two-step auction case *If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it may be positive or negative.*

License to one firm without entry case using two-step auction case *If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it is positive.*

Licenses to two firms without entry case using or not using two-step auction case *The optimal royalty rate is positive.*

6.3. Credibility of two-step auction

In this subsection we will prove our main results. The innovating firm uses a two-step auction if and only if the threat by the existence of the second step of the auction is credible, and it is credible if and only if the total payoff of the innovating firm when it enters the market with a license to one firm is larger than its payoff when it does not enter and sells a license to one firm not using a two-step auction. Therefore, if

$$\pi_A^{e1} + \tilde{r}^{e1} q_C + L^{e1} \geq \tilde{r}^{l1} q_C + \tilde{L}^{l1},$$

two-step auction is credible. On the other hand, if

$$\tilde{r}^{l1} q_C + \tilde{L}^{l1} > \pi_A^{e1} + \tilde{r}^{e1} q_C + L^{e1},$$

two-step auction is not credible.

We show the following proposition. Note that we assume $c(0) = 0$, that is, the fixed cost is zero.

Proposition 2. (1) *If the marginal cost is constant, that is the cost function is linear, entry with license to one firm case and license to one firm without entry case are equivalent.*

(2) *If the cost function of the firms is strictly convex, two-step auction is credible.*

(3) *If the cost function of the firms is strictly concave, two-step auction is not credible.*

Proof. (1) First consider the case of entry with a license to one firm. Let $\bar{q} = q_A + q_C$. Denote the constant marginal cost by c, and denote the total payoff of the innovator by φ^{e1}. It is written as

$$\begin{aligned}
\varphi^{e1} =& \pi_A^{e1} + r q_C + L^{e1} = [(k_H - k_L)G(1 - q_A - q_C) + k_L G(1 - q_A - q_B - q_C)]q_A \\
&- c q_A + [(k_H - k_L)G(1 - q_A - q_C) + k_L G(1 - q_A - q_B - q_C)]q_C - c q_C \\
&- (k_L G(1 - q_A - q_B - q_C) q_B - c q_B) \\
=& [(k_H - k_L)G(1 - \bar{q}) + k_L G(1 - \bar{q} - q_B)]\bar{q} - c\bar{q} - (k_L G(1 - \bar{q} - q_B) q_B - c q_B).
\end{aligned}$$

If the marginal cost is constant, $c'' = 0$. Thus, $\frac{d\bar{q}}{dr} = \frac{dq_A}{dr} + \frac{dq_C}{dr}$ and $\frac{dq_B}{dr}$ in Section 4.2 are written as (see also Appendix B)

$$\frac{d\bar{q}}{dr} = \frac{[-(k_H - k_L)G'(1 - \bar{q}) + k_L G'(1 - \bar{q} - q_B)]\theta_B}{\Delta'},$$

$$\frac{dq_B}{dr} = -\frac{[-(k_H - k_L)G'(1 - \bar{q}) + k_L G'(1 - \bar{q} - q_B)]\sigma_B}{\Delta'}.$$

where

$$\theta_B = -k_L[2G'(1 - \bar{q} - q_B) - G''(1 - \bar{q} - q_B)q_B],$$

$$\sigma_B = -k_H G'(1 - \bar{q} - q_B) + k_H G''(1 - \bar{q} - q_B)]q_B.$$

The condition for maximization of φ^{e1} with respect to r is

$$\lambda_1 \frac{d\bar{q}}{dr} - \lambda_2 \frac{dq_B}{dr} = 0, \tag{8}$$

where

$$\begin{aligned}\lambda_1 =&(k_H - k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B)\\ &-[(k_H-k_L)G'(1-\bar{q}) + k_L G'(1-\bar{q}-q_B)]\bar{q} - c + k_L G'(1-\bar{q}-q_B)q_B,\end{aligned}$$

$$\lambda_2 = k_L G(1-\bar{q}-q_B) - k_L G'(1-\bar{q}-q_B)q_B - c + k_L G'(1-\bar{q}-q_B)\bar{q}.$$

From (1a) and (1c) we have

$$\begin{aligned}&(k_H - k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B) - [(k_H-k_L)G'(1-\bar{q})\\ &+ k_L G'(1-\bar{q}-q_B)]\bar{q} - c = r - [(k_H-k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B)] + c.\end{aligned}$$

From this and (1b), (8) is rewritten as

$$\begin{aligned}&\{r - [(k_H-k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B)] + c + k_L G'(1-\bar{q}-q_B)q_B\}\frac{d\bar{q}}{dr}\\ &- k_L G'(1-\bar{q}-q_B)\bar{q}\frac{dq_B}{dr} = 0.\end{aligned}$$

Then, the optimal royalty rate is written as

$$\begin{aligned}\tilde{r}^{e1} =&(k_H-k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B) - c - k_L G'(1-\bar{q}-q_B)q_B\\ &- k_L G'(1-\bar{q}-q_B)\bar{q}\frac{\sigma_B}{\theta_B}.\end{aligned}$$

The first order condition for Firm C, (1c), with $r = \tilde{r}^{e1}$ is rewritten as

$$\begin{aligned}&(k_H-k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B)\\ &-[(k_H-k_L)G'(1-\bar{q}) + k_L G'(1-\bar{q}-q_B)]q_C - c\\ &-(k_H-k_L)G(1-\bar{q}) - k_L G(1-\bar{q}-q_B) + c + k_L G'(1-\bar{q}-q_B)q_B\\ &+ k_L G'(1-\bar{q}-q_B)\bar{q}\frac{\sigma_B}{\theta_B}\\ =&-[(k_H-k_L)G'(1-\bar{q}) + k_L G'(1-\bar{q}-q_B)]q_C + k_L G'(1-\bar{q}-q_B)q_B\\ &+ k_L G'(1-\bar{q}-q_B)\bar{q}\frac{\sigma_B}{\theta_B} = 0.\end{aligned}$$

With $q_A + q_C = \bar{q}$, this and the first order condition for Firm A, (1a),

$$\begin{aligned}&(k_H-k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B) - [(k_H-k_L)G'(1-\bar{q})\\ &+ k_L G'(1-\bar{q}-q_B)]q_A - c = 0\end{aligned}$$

imply

$$
\begin{aligned}
&(k_H - k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B) - [(k_H - k_L)G'(1-\bar{q}) \\
&+ k_L G'(1-\bar{q}-q_B)]\bar{q} - c + k_L G'(1-\bar{q}-q_B)q_B \\
&+ k_L G'(1-\bar{q}-q_B)\bar{q}\frac{\sigma_B}{\theta_B} = 0.
\end{aligned} \tag{9}
$$

Next consider the case of license to one firm without entry not using a two-step auction. Let $\bar{q} = q_C$. Denote the total payoff of the innovator in this case by $\tilde{\varphi}^{l1}$. It is written as

$$\tilde{\varphi}^{l1} = [(k_H - k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B)]\bar{q} - c\bar{q} - (k_L G(1-\bar{q}-q_B)q_B - cq_B).$$

This is the same as φ^{e1}. If $c'' = 0$, $\frac{d\bar{q}}{dr} = \frac{dq_C}{dr}$ and $\frac{dq_B}{dr}$ in Section 4.4 are written as

$$\frac{d\bar{q}}{dr} = \frac{\theta_B}{\Delta},\ \frac{dq_B}{dr} = -\frac{\sigma_B}{\Delta},$$

θ_B and σ_B in this case are the same as those in the previous case. The condition for maximization of $\tilde{\varphi}^{l1}$ with respect to r is

$$
\begin{aligned}
&\{[(k_H - k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B)] - [(k_H - k_L)G'(1-\bar{q}) \\
&+ k_L G'(1-\bar{q}-q_B)] - c + k_L G'(1-\bar{q}-q_B)q_B\}\frac{d\bar{q}}{dr} \\
&- [k_L G(1-\bar{q}-q_B) - k_L G'(1-\bar{q}-q_B)q_B - c + k_L G'(1-\bar{q}-q_B)\bar{q}]\frac{dq_B}{dr} = 0.
\end{aligned} \tag{10}
$$

From (3a) and (3b), (10) is rewritten as

$$(r + k_L G'(1-\bar{q}-q_B)q_B)\frac{d\bar{q}}{dr} - k_L G'(1-\bar{q}-q_B)\bar{q}\frac{dq_B}{dr} = 0.$$

Then, the optimal royalty rate is

$$\tilde{r}^{l1} = -k_L G'(1-\bar{q}-q_B)q_B - k_L G'(1-\bar{q}-q_B)\bar{q}\frac{\sigma_B}{\theta_B}.$$

The first order condition for Firm C, (3b), with $q_C = \bar{q}$ and $r = \tilde{r}^{l1}$ is rewritten as

$$
\begin{aligned}
&(k_H - k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B) - [(k_H - k_L)G'(1-\bar{q}) \\
&+ k_L G'(1-\bar{q}-q_B)]q_C - c + k_L G'(1-\bar{q}-q_B)q_B \\
&+ k_L G'(1-\bar{q}-q_B)\bar{q}\frac{\sigma_B}{\theta_B} = 0.
\end{aligned} \tag{11}
$$

(9) and (11) are the same. Therefore, two cases are equivalent.

(2) φ^{e1} with $\bar{q} = q_A + q_C$ is

$$
\begin{aligned}
\varphi^{e1} =& [(k_H - k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B)]\bar{q} - c(q_A) - c(q_C) \\
&- (k_L G(1-\bar{q}-q_B-)q_B - c(q_B)).
\end{aligned}
$$

$\tilde{\varphi}^{l1}$ with $\bar{q} = q_C$ is written as

$$\begin{aligned}\tilde{\varphi}^{l1} =&[(k_H - k_L)G(1-\bar{q}) + k_L G(1-\bar{q}-q_B)]\bar{q} - c(\bar{q}) - (k_L G(1-\bar{q}-q_B-)q_B - c(q_B))\\ =&\varphi^{e1} + c(q_A) + c(q_C) - c(q_A+q_C).\end{aligned}$$

If the cost function is strictly convex,

$$c(q_C) < \frac{q_C}{q_A+q_C}c(q_A+q_C) + \left(1 - \frac{q_C}{q_A+q_C}\right)c(0) = \frac{q_C}{q_A+q_C}c(q_A+q_C),$$

$$c(q_A) < \frac{q_A}{q_A+q_C}c(q_A+q_C) + \left(1 - \frac{q_A}{q_A+q_C}\right)c(0) = \frac{q_A}{q_A+q_C}c(q_A+q_C).$$

Then,

$$c(q_A) + c(q_C) < c(q_A+q_C).$$

Separation of production between two firms is more efficient than concentration to one firm. Thus, φ^{e1} is larger than $\tilde{\varphi}^{l1}$ when $q_A + q_C$ in the case of entry with a license and q_C in the case of license without entry are equal, and the maximum value of φ^{e1} is larger than the maximum vale of $\tilde{\varphi}^{l1}$. Hence, two-step auction is credible.

(3) Similarly to the case of strictly convex cost function, if the cost function is strictly concave, we find

$$c(q_A) + c(q_C) > c(q_A+q_C).$$

Concentration of production to one firm is more efficient than separation between two firms. Thus, $\tilde{\varphi}^{l1}$ is larger than φ^{e1} when $q_A + q_C$ in the case of entry with a license and q_C in the case of license without entry are equal, and the maximum value of $\tilde{\varphi}^{l1}$ is larger than the maximum vale of φ^{e1}. Hence, two-step auction is not credible.

□

7. An example

As an example we assume that $\rho = F(\theta)$ has a uniform distribution. Then, $\rho = \theta, \theta = G(\rho) = \rho$, $F'(\theta) = G'(\rho) = 1$ and $F''(\theta) = G''(\rho) = 0$. We consider a case of convex cost function. The cost function of the firms is $c(q_i) = \frac{1}{2}k_L q_i^2, i = A, B, C$. Denote $k_H = qk_L,\ q > 1$. We present summaries of the calculation results.

License to one firm without entry not using two-step auction case The optimal royalty rate and the total payoff of the innovator are

$$\tilde{r}^{l1} = -\frac{k_L}{3} < 0,$$

$$\tilde{r}^{l1}q_C + \tilde{L}^{l1} = \frac{k_L(9q^2 - 6q - 2)}{12(3q+1)}.$$

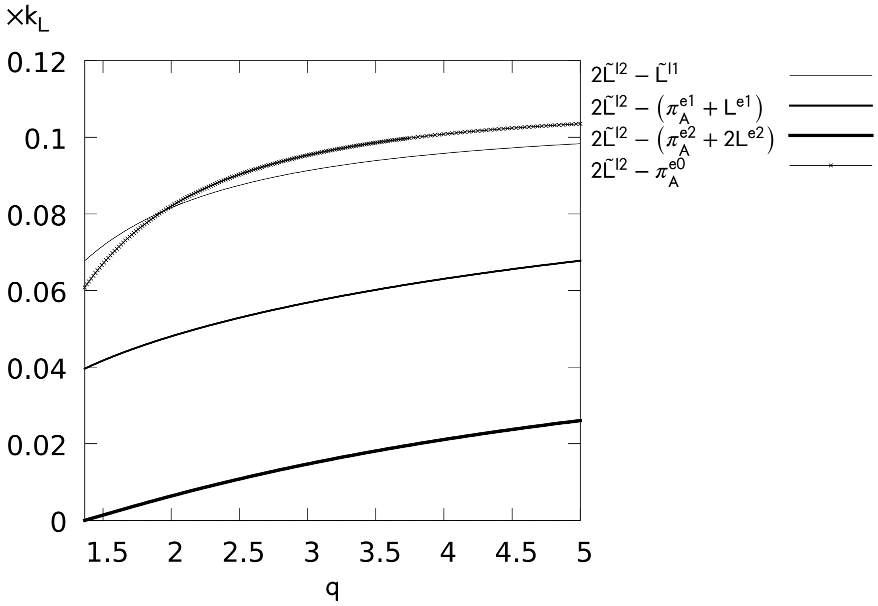

Figure 1: Optimal strategy for the innovator when $0 < q < \frac{\sqrt{3}+1}{2}$

Entry without license case The profit of the innovator is

$$\pi_A^{e0} = \frac{k_L(2q-1)^2(2q+1)}{2(4q+1)^2}.$$

Entry with license to one firm case The optimal royalty rate and the total payoff of the innovator are

$$\tilde{r}^{e1} = \frac{k_L(q+1)^2(9q^2-12q-1)}{2(3q+1)(3q^2+12q+1)},$$

$$\pi_A^{e1} + \tilde{r}^{e1}q_C + \tilde{L}^{e1} = \frac{k_L(9q^4+30q^3-8q^2-14q-1)}{4(3q+1)(3q^2+12q+1)}.$$

If $1 < q < \frac{\sqrt{5}+2}{3}$, $\tilde{r}^{e1} < 0$.

Entry with licenses to two firms case The optimal royalty rate and the total payoff of the innovator are

$$\tilde{r}^{e2} = \frac{2k_Lq^2(q+1)^2}{(2q+1)(2q^2+6q+1)} > 0,$$

$$\begin{aligned}&\pi_A^{e2} + \tilde{r}^{e2}(q_B+q_C) + 2\tilde{L}^{e2}\\ &= \frac{k_L}{4(2q+1)(3q+1)^2(2q^2+6q+1)(3q^2+12q+1)^2}(324q^{10}+3672q^9+14904q^8\\ &+25368q^7+14805q^6-3318q^5-7781q^4-3660q^3-777q^2-78q-3).\end{aligned}$$

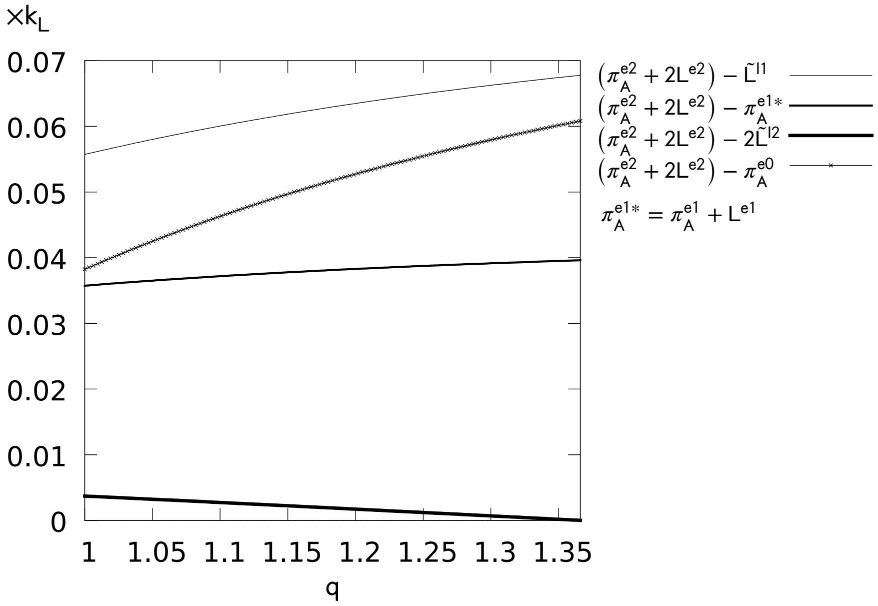

Figure 2: Optimal strategy for the innovator when $q > \frac{\sqrt{3}+1}{2}$

License to one firm without entry case using two-step auction case The optimal royalty rate and the total payoff of the innovator are

$$\hat{r}^{l1} = -\frac{k_L(3q-1)}{3(6q+1)} < 0,$$

$$\hat{r}^{l1} q_C + \hat{L}^{l1} = \frac{k_L}{24(3q+1)^2(6q+1)(3q^2+12q+1)^2}(2916q^8 + 22842q^7 + 44307q^6 - 7452q^5 - 23031q^4 - 8838q^3 - 1371q^2 - 120q - 5).$$

Licenses to two firms without entry case using two-step auction case The optimal royalty rate and the total payoff of the innovator are

$$\hat{r}^{l2} = \frac{k_L q^2}{4q+1} > 0,$$

$$\hat{r}^{l2}(q_B + q_C) + 2\hat{L}^{l2} = \frac{k_L}{4(3q+1)^2(4q+1)(3q^2+12q+1)^2}(324q^8 + 2700q^7 + 6345q^6 + 2454q^5 - 1761q^4 - 1728q^3 - 521q^2 - 66q - 3).$$

Comparing $\pi_A^{e1} + \tilde{r}^{e1} q_C + \tilde{L}^{e1}$ and $\tilde{r}^{l1} q_C + \tilde{L}^{l1}$,

$$\pi_A^{e1} + \tilde{r}^{e1} q_C + \tilde{L}^{e1} - (\tilde{r}^{l1} q_C + \tilde{L}^{l1}) = \frac{k_L(3q-1)(15q+1)}{12(3q+1)(3q^2+12q+1)} > 0.$$

Therefore, two-step auction is credible. About this example we get the following results.

(1) If $0 < q < \frac{\sqrt{3}+1}{2}(\approx= 1.366)$, licenses to two firms without entry strategy is optimal for the innovator. Please see Figure 1.

(2) If $q > \frac{\sqrt{3}+1}{2}$, entry with licenses to two firms strategy is optimal for the innovator. Please see Figure 2.

Appendices

A. Detailed analysis of demand functions

If a consumer with taste parameter ξ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \xi k$. Let ξ_0 be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the high-quality good. Then,

$$\xi_0 = \frac{p_H}{k_H}.$$

Let ξ_L be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality good. Then,

$$\xi_L = \frac{p_L}{k_L}.$$

Let ξ_H be the value of ξ for which the corresponding consumer is indifferent between buying the low-quality good and buying the high-quality good. Then

$$\xi_H = \frac{p_H - p_L}{k_H - k_L}.$$

We find

$$\xi_0 = \frac{(k_H - k_L)\xi_H + k_L\xi_L}{k_H}.$$

Therefore, $\xi_L \geq \xi_0 \geq \xi_H$ or $\xi_H > \xi_0 > \xi_L$.

For $\xi > (<)\xi_L$,

$$y - p_L + \xi k_L > (<)y.$$

For $\xi > (<)\xi_0$,

$$y - p_H + \xi k_H > (<)y.$$

For $\xi > (<)\xi_H$,

$$y - p_H + \xi k_H > (<)y - p_L + \xi k_L.$$

A.1. Licenses to two firms without entry case

In this case Firms B and C produce the high-quality good. Let q_H be the demand for the high-quality good. Then, we get

(1) When $p_H \geq k_H$ ($\xi_0 \geq 1$), we have $q_H = 0$.

(2) When $p_H < k_H$ ($\xi_0 < 1$), we have $q_H = 1 - F(\xi_0)$.

Then, the inverse demand function is described as follows.

(1) When $q_H > 0$, we have $p_H = k_H G(1 - q_H)$.

(2) When $q_H = 0$, we have $p_H = k_H$.

This is a continuously differentiable inverse demand function with the domain $0 \leq q_H \leq 1$. We have $q_H = q_B + q_C$.

A.2. Licenses to two firms with entry case

In this case all firms produce the high-quality good. Let $q_H = q_A + q_B + q_C$. The inverse demand function is the same as that in Case A.1.

A.3. License to one firm without entry case

In this case Firm C produces the high-quality good, and Firm B produces the low-quality good. Let q_H be the demand for the high-quality good and q_L be the demand for the low-quality good. Then, we get[3]

(1) When $p_H \geq k_H$ ($\xi_0 \geq 1$) and $p_L \geq k_L$ ($\xi_L \geq 1$), we have $q_H = 0$ and $q_L = 0$.

(2) When $p_H < k_H$ ($\xi_0 < 1$) and $p_L \geq \frac{p_H}{k_H} k_L$ ($\xi_L \geq \xi_0 \geq \xi_H$), we have $q_H = 1 - F(\xi_0)$ and $q_L = 0$.

(3) When $p_L < k_L$ ($\xi_L < 1$), $p_H > \frac{p_L}{k_L} k_H$ ($\xi_H > \xi_0 > \xi_L$) and $p_H - p_L \geq k_H - k_L$ ($\xi_H \geq 1$), we have $q_H = 0$ and $q_L = 1 - F(\xi_L)$.

(4) When $p_L < k_L$ ($\xi_L < 1$), $p_H > \frac{k_H}{k_L} p_L$ ($\xi_H > \xi_0 > \xi_L$) and $p_H - p_L < k_H - k_L$ ($\xi_H < 1$), we have $q_L = F(\xi_H) - F(\xi_L)$ and $q_H = 1 - F(\xi_H)$.

From this demand function we obtain the inverse demand function as follows.

(1) When $q_H > 0$ and $q_L > 0$, we have $p_H = (k_H - k_L)G(1 - q_H) + k_L G(1 - q_H - q_L)$ and $p_L = k_L G(1 - q_H - q_L)$.

(2) When $q_H > 0$ and $q_L = 0$, we have $p_H = k_H G(1 - q_H)$ and $p_L = k_L G(1 - q_H)$.

(3) When $q_H = 0$ and $q_L > 0$, we have $p_H = k_H - k_L + k_L G(1 - q_L)$ and $p_L = k_L G(1 - q_L)$.

(4) When $q_H = 0$ and $q_L = 0$, we have $p_H = k_H$ and $p_L = k_L$.

This is a continuously differentiable inverse demand function with the domain $0 \leq q_H \leq 1$ and $0 \leq q_L \leq 1$. We have $q_H = q_C$ and $q_L = q_B$.

[3] We owe this formulation to an anonymous referee.

A.4. Entry with license to one firm case

In this case Firms A and C produce the high-quality good, and Firm B produces the low-quality good. The inverse demand function is the same as that in Case A.3 with $q_H = q_A + q_C$ and $q_L = q_B$.

A.5. Entry without license case

In this case Firm A produces the high-quality good, and Firms B and C produce the low-quality good. The inverse demand function is the same as that in Case A.3 with $q_H = q_A$ and $q_L = q_B + q_C$.

B. Details about $\frac{dq_A}{dr}$, $\frac{dq_B}{dr}$ and $\frac{dq_C}{dr}$ in Section 4.2.

Differentiating (1a), (1b) and (1c) with respect to r, we obtain

$$\frac{dq_A}{dr} = \frac{\sigma_{AB}\sigma_B - \sigma_{AC}\theta_B}{\Delta'},\ \frac{dq_B}{dr} = \frac{\sigma_{AC}\sigma_B - \sigma_B\theta_A}{\Delta'},\ \frac{dq_C}{dr} = \frac{\theta_A\theta_B - \sigma_{AB}\sigma_B}{\Delta'},$$

where

$$\begin{aligned}\theta_A = \frac{\partial^2\pi_A}{\partial q_A^2} =& -2[(k_H - k_L)G'(1 - q_A - q_C) + k_L G'(1 - q_A - q_B - q_C)] \\ &+ [(k_H - k_L)G''(1 - q_A - q_C) + k_L G''(1 - q_A - q_B - q_C)]q_A - c''(q_A),\end{aligned}$$

$$\theta_B = \frac{\partial^2\pi_B}{\partial q_B^2} = -k_L[2G'(1 - q_A - q_B - q_C) - G''(1 - q_A - q_B - q_C)q_B] - c''(q_B),$$

$$\begin{aligned}\theta_C = \frac{\partial^2\pi_C}{\partial q_C^2} =& -2[(k_H - k_L)G'(1 - q_A - q_C) + k_L G'(1 - q_A - q_B - q_C)] \\ &+ [(k_H - k_L)G''(1 - q_A - q_C) + k_L G''(1 - q_A - q_B - q_C)]q_C - c''(q_C),\end{aligned}$$

$$\sigma_{AB} = \frac{\partial^2\pi_A}{\partial q_A q_B} = -k_L G'(1 - q_A - q_B - q_C) + k_L G''(1 - q_A - q_B - q_C)]q_A,$$

$$\begin{aligned}\sigma_{AC} =& \frac{\partial^2\pi_A}{\partial q_A q_C} = -(k_H - k_L)G'(1 - q_A - q_C) + k_L G'(1 - q_A - q_B - q_C) \\ &+ [(k_H - k_L)G''(1 - q_A - q_C) + k_L G''(1 - q_A - q_B - q_C)]q_A,\end{aligned}$$

$$\sigma_B = \frac{\partial^2\pi_B}{\partial q_B q_A} = \frac{\partial^2\pi_B}{\partial q_B q_C} = -k_L G'(1 - q_A - q_B - q_C) + k_L G''(1 - q_A - q_B - q_C)]q_B,$$

$$\sigma_{CA} = \frac{\partial^2 \pi_C}{\partial q_C q_A} = -(k_H - k_L)G'(1 - q_A - q_C) + k_L G'(1 - q_A - q_B - q_C)$$
$$+ [(k_H - k_L)G''(1 - q_A - q_C) + k_L G''(1 - q_A - q_B - q_C)]q_C,$$

$$\sigma_{CB} = \frac{\partial^2 \pi_C}{\partial q_C q_B} = -k_L G'(1 - q_A - q_B - q_C) + k_L G''(1 - q_A - q_B - q_C)]q_C,$$
$$\Delta' = \theta_A\theta_B\theta_C - \theta_A\sigma_B\sigma_{CB} - \theta_B\sigma_{AC}\sigma_{CA} - \theta_C\sigma_{AB}\sigma_B + \sigma_{AC}\sigma_B\sigma_{CB} + \sigma_{AB}\sigma_B\sigma_{CA}.$$

By the second order conditions, $\theta_A < 0$, $\theta_B < 0$, $\theta_C < 0$. From the stability conditions for oligopoly (Seade (1980) and Dixit (1986)) $\Delta' < 0$ and we can assume that the absolute values of θ_A, θ_B and θ_C are larger than those of σ's. We have $\frac{dq_C}{dr} < 0$. If the goods are strategic substitutes, σ's are negative and $\frac{dq_A}{dr}$ and $\frac{dq_B}{dr}$ are positive. If the goods are strategic complements, σ's are positive and $\frac{dq_A}{dr}$ and $\frac{dq_B}{dr}$ are positive.

C. Details about $\frac{dq_A}{dr}$, $\frac{dq_B}{dr}$ and $\frac{dq_C}{dr}$ in Section 4.3.

Differentiating (2a), (2b) and (2c) with respect to r, we obtain

$$\frac{dq_A}{dr} = \frac{\sigma_A(\sigma_B - \theta_B + \sigma_C - \theta_C)}{\Delta},$$
$$\frac{dq_B}{dr} = \frac{\theta_A\theta_C - \theta_A\sigma_B + \sigma_A\sigma_B - \sigma_A\sigma_C}{\Delta},$$
$$\frac{dq_C}{dr} = \frac{\theta_A\theta_B - \theta_A\sigma_C + \sigma_A\sigma_C - \sigma_A\sigma_B}{\Delta},$$

where

$$\theta_A = -2k_H G'(1 - q_A - q_B - q_C) + k_H G''(1 - q_A - q_B - q_C)]q_A - c''(q_A),$$
$$\theta_B = -2k_H G'(1 - q_A - q_B - q_C) + k_H G''(1 - q_A - q_B - q_C)]q_B - c''(q_B),$$
$$\theta_C = -2k_H G'(1 - q_A - q_B - q_C) + k_H G''(1 - q_A - q_B - q_C)]q_C - c''(q_C),$$
$$\sigma_A = -k_H G'(1 - q_A - q_B - q_C) + k_H G''(1 - q_A - q_B - q_C)]q_A,$$
$$\sigma_B = -k_H G'(1 - q_A - q_B - q_C) + k_H G''(1 - q_A - q_B - q_C)]q_B,$$
$$\sigma_C = -k_H G'(1 - q_A - q_B - q_C) + k_H G''(1 - q_A - q_B - q_C)]q_C,$$
$$\Delta = \theta_A\theta_B\theta_C - \theta_A\sigma_B\sigma_C - \theta_B\sigma_A\sigma_C - \theta_C\sigma_A\sigma_B + \sigma_A\sigma_B\sigma_C.$$

By the second order conditions, $\theta_A < 0$, $\theta_B < 0$, $\theta_C < 0$. From the stability conditions for oligopoly (Seade (1980) and Dixit (1986)) $\Delta > 0$ and we can assume that the absolute values of θ_A, θ_B and θ_C are larger than those of σ_A, σ_B and σ_C. We have $\frac{dq_B}{dr} < 0$ and $\frac{dq_C}{dr} < 0$. If the goods are strategic substitutes, σ's are negative and $\frac{dq_A}{dr}$ is positive. If the goods are strategic complements, σ's are positive and $\frac{dq_A}{dr}$ is negative.

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