Is GDP more volatile in developing countries after taking the shadow economy into account? Evidence from Latin America

Solis-Garcia, Mario and Xie, Yingtong

Macalester College, University of Wisconsin-Madison

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Is GDP more volatile in developing countries after taking the shadow economy into account? Evidence from Latin America*

Mario Solis-Garcia†
Macalester College

Yingtong Xie‡
University of Wisconsin-Madison

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Abstract

Why is GDP more volatile in developing countries? In this paper we propose an explanation that can account for the substantial differences in the volatility of measured real GDP per capita between developing and developed countries. Our explanation involves the often overlooked fact that developing economies have a sizable shadow economy. We build a two-sector model that distinguishes between measured (formal) and total (formal and shadow) outputs; using data from Latin America, our model results suggest that developing and developed economies are fairly similar in terms of the volatility of total real GDP. We also document an apparent puzzle, in that the model suggests that the volatility of the size of the shadow economy should be substantially larger than what is observed in the real world. We believe that this may be indicative of frictions that prevent agents from optimally moving between the formal and shadow economies.

JEL codes: E26, E32, O17.
Keywords: shadow economy, business cycles, DSGE models, Bayesian estimation.

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† Corresponding author. Department of Economics, Macalester College, 1600 Grand Avenue, St. Paul, MN 55105. E-mail: msolisga@macalester.edu.

‡ E-mail: yingtong.xie@wisc.edu.

1 Introduction

There is a sizable difference in the volatility of *measured* real GDP per capita (hereafter, RGDP) between developing and developed countries. In particular, the volatility of measured RGDP in Latin American countries is significantly higher than that in the United States and Canada, as shown in Figure 1:

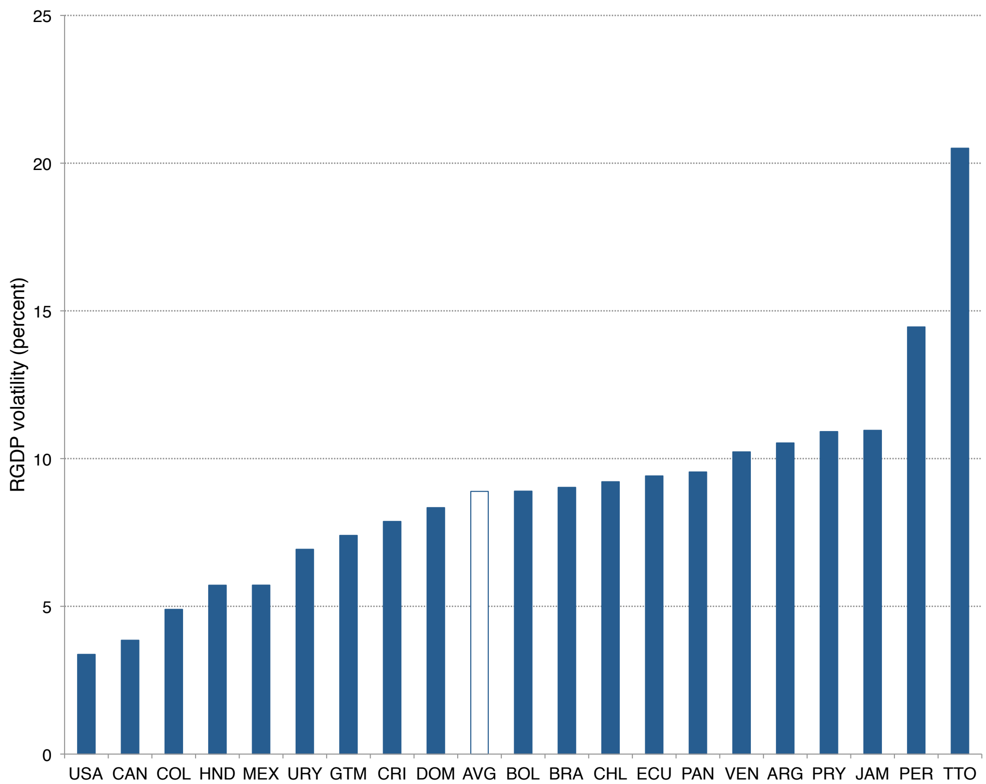

Figure 1: Measured RGDP volatility, selected countries, 1950-2011.

Notes: Measured RGDP volatility is calculated as the standard deviation of the residual of a regression of the logarithm of the variable against a linear and a quadratic time trends. The data for the regression runs from 1950 to 2011, except for Chile, Dominican Republic, and Paraguay (data starts in 1951) and Jamaica (1953). "AVG" denotes the average standard deviation across all countries. See Appendix A for a list of the country codes.

The average volatility of measured RGDP for the developing countries in the figure is 2.6 times as large as that in the United States. In particular, Colombia and Honduras are 1.5 and 1.7 times as volatile, while the corresponding values for Peru and Trinidad and Tobago are 4.3 and 6.1, respectively.

Several studies have tried to account for this empirical fact. For example, Neumeyer and Perri (2005) analyze the role of interest rates and country risk, which is amplified under the presence of working capital restrictions. Aguiar and Gopinath (2007) focus instead on the contribution of stochastic trends in developing economies, which add to the overall volatility of their business cycle.

In this paper we provide a different perspective, based on the often overlooked fact that the size of the shadow economy is larger in developing countries than it is in developed ones. Since the economic activity of the shadow sector is at best poorly measured and, consequently, is not included in the values shown in Figure 1, we claim that the actual volatility of RGDP in developing economies (or more precisely, in economies with a sizable shadow economy) is not as large as shown in the figure. In other words, this paper argues that there is a connection between the high values of measured RGDP volatility and the presence of a large shadow sector.

For our purposes, the shadow economy is the sum of all market-based production of (legal) goods and services that are not reported to government authorities (we exclude illegal and home production; see Section 2 for a detailed account). These firms choose to remain underground for a variety of reasons; arguably, the most important is to avoid payment of taxes or social security contributions, labor market regulations, or compliance with administrative procedures.

Conventional wisdom places firms in the shadow economy as an "escape valve" from the effects of recessions: the output of these productive units (relative to measured output) tends to increase in bad times and decrease in good times. A contribution of this paper is to investigate the relationship between the shadow economy—and its escape valve role—and the volatility of measured RGDP in developing economies.

Using data from a set of Latin American and Caribbean countries (we add the United States and Canada as well),[1] we find that the volatility of total RGDP is about 1.3%, substantially lower than the corresponding value for measured RGDP (5.5%). The main assumption that we require is that measured RGDP only includes the activities of the formal sector. (This key assumption is also used by Fernández and Meza 2015

[1] We analyze countries in Latin America and the Caribbean since the size of the shadow economy is relatively large in these countries; we add the United States and Canada as a control to test the predictions of the model on developed economies that have a smaller shadow sector.

in their study of the Mexican shadow employment.)

We envision the following mechanism: in less developed countries, measured RGDP fluctuations are amplified as negative shocks to total factor productivity (TFP) in the formal sector generate a movement of productive factors from the formal to the shadow sector,[2] hence lowering measured RGDP while *total* RGDP (which includes both formal and *unmeasured* shadow production) is not lowered significantly. This implies that total RGDP volatility should be smaller than measured RGDP volatility as the latter fails to consider the behavior of a large portion of economic activity.

In order to test our mechanism and assess the connection between measured RGDP volatility and the shadow sector, we build a two-sector dynamic stochastic general equilibrium (DSGE) model that includes formal (measured) and shadow (unmeasured) activities. In our model, a representative consumer/producer has access to a formal technology that employs both capital and labor but can also work with a shadow technology that requires only labor input. Both the formal and shadow technologies are subject to TFP shocks. The simulated data from our model generates a value of total RGDP volatility that is twice as large (2.6%) but fairly homogeneous among countries, irrespective of the development status of the country (using simulated data, measured RGDP volatility equals 9.6%, almost double the real-world value). These results suggest that countries have a similar level of volatility once we take into account the dynamics of the shadow economy.

Our model can also successfully reproduce two correlations found in the real world, namely, (1) a negative relationship between the size of the shadow economy and measured RGDP, and (2) a positive correlation between the volatility of the size of the shadow economy and the volatility of measured RGDP. However, our model fails to generate the low volatility of the size of the shadow sector that is observed in the data. We believe that this may be indicative of frictions that prevent agents from optimally switching between the production technologies.

[2] We do not model this particular mechanism but offer some potential causes for it: lack of unemployment benefits—particularly relevant in less developed economies, low matching efficiency, or lack of information about available jobs in the formal sector.

1.1 Connection with the literature

The use of DSGE models to understand the behavior of the shadow economy is fairly recent;[3] several papers stand out in the literature and here we discuss their connection to our work. Ihrig and Moe (2004) use the data from Schneider and Enste (2000) to document three empirical relations for a cross-section of countries in 1990. First, a negative and convex relationship between the size of the shadow economy and a country's measured RGDP; second, a positive relationship between the size of the shadow economy and tax rates, and third, a negative relationship between the size of the shadow economy and (tax) enforcement. They build a two-sector model—which is the basis of the model used in this paper—and calibrate it to the economy of Sri Lanka. Their simulations verify that the model is broadly consistent with their empirical findings. While our emphasis is not on the connection between tax rates and enforcement and the size of the shadow economy, we are able to confirm their first empirical finding using more recent data (Schneider, Buehn, and Montenegro 2010). Moreover, we improve upon their modeling strategy by adding a leisure choice to the consumer/producer so that switching between sectors is not an "either-or" option. Finally, we take the model to the data for a variety of countries and we use both calibration and Bayesian estimation procedures to derive our results.

Using a two-sector DSGE model, Busato and Chiarini (2004) conclude that adding a shadow economy to the model generates three main findings: first, a better fit to Italian data (relative to the indivisible labor model of Hansen 1985); second, a stronger propagation mechanism of TFP shocks, and third, a greater degree of risk sharing due to the presence of two different labor alternatives. To calibrate the model, they use the formal/shadow output decomposition of Bovi (1999) and they calibrate the share of shadow labor using the values found in Schneider and Enste (2000). Other parameters are set to match some target values of the Italian economy. Relative to their work, we go one step ahead both in the scope (many Latin American

[3] The following papers share a DSGE approach to the shadow economy but do not center their analysis on its relation to business cycle fluctuations: Ciccarone, Giuli, and Marchetti (2015) focus on labor market dynamics and undeclared work; Fernández and Meza (2015) analyze on the behavior of shadow economy employment in Mexico; and Gomis-Porqueras, Peralta-Alva, and Waller (2014) estimate the size of the shadow economy using a money search model.

economies as opposed to Italy) and in the parametrization of the model (by adding a Bayesian estimation component). We decide not to incorporate the particular functional form for the utility function, which is based on Cho and Cooley (1994).

Finally, we need to mention the contributions of Conesa, Díaz-Moreno, and Galdón-Sánchez (2001, 2002) and Restrepo-Echavarría (2014) as their work is closest to ours. First consider the work of Conesa et al. Their goal is to account for the differences in volatility of output between countries by using a model that includes a shadow economy. That said, their mechanism is based on the relationship between the wage premium (defined as the wage difference between working in the formal and shadow sectors) and the participation rate. Other things the same, a lower wage premium reduces the participation rate, since the opportunity cost of working in the shadow sector goes down. In turn, shocks to the productivity of the registered firms will shift a larger proportion of agents between both sectors. Hence, countries with different values for the wage premium will experience different fluctuations in response to TFP shocks.[4] At a deeper level, their results rely on the shadow economy as an external production alternative but do not use shadow economy size estimates to derive the main results.[5] In contrast, our model does not make any assumptions about the connection between the wage premium and the participation rate; as mentioned above, we use the estimates from Schneider et al. (2010) as our starting point and then build our model from the bottom up while taking these values into account.

The work of Restrepo-Echavarría shares our interest in determining the effects of the shadow economy over the business cycle but is considerably different to what we do below. She looks at how the relative volatility of consumption to GDP is affected by the presence of a shadow economy and quantifies this

[4] Their mechanism assumes a link between the participation rate and the shadow economy, in that a low participation rate suggests a high activity in the shadow sector. Schneider and Enste (2000) discuss this mechanism as the basis of a measurement approach (see Section 3) and characterize it as a weak indicator of the size of the shadow economy for two reasons. First, a low participation rate may have other causes different from an active shadow economy and second, people may opt to work in the formal and shadow economy at the same time. We choose to remain agnostic about the role of the participation rate and focus on the direct effect of TFP shocks instead.

[5] For example, in Conesa et al. (2002) the authors use a simulated grid of participation rates and then use these values as inputs to the model in order to obtain a value for the volatility of measured RGDP. Hence, the influence of the shadow economy is indirect—it matters insofar the participation rate allows for larger fluctuations between the level of employment in both sectors.

in a two-sector DSGE model with formal and shadow consumption goods; she finds that including the shadow economy can better account for the relative volatility of consumption. As shown below, our model is interested in the volatility of GDP on its own; in addition, while we keep a two-sector model in the background we don't distinguish between different consumption goods. Overall, we believe that her work and ours are analyzing different implications of the shadow economy.

1.2 Roadmap

The rest of the paper is structured as follows. A brief discussion of the measurement issues of the shadow economy are presented in Section 2. The empirical facts that motivate the paper are found in Section 3 while Section 4 presents a dynamic model of the shadow sector. Section 5 discusses the parametrization of our model and our results are presented in Section 6. Section 7 concludes.

2 Characterizing and measuring the shadow economy

The literature on the shadow economy shows that the terminology can sometimes be loosely interchanged for other concepts that are not necessarily equivalent. The Organisation for Economic Co-operation and Development (see OECD 2002) has defined the *non-observed economy* as a term that stands in for the following categories of production: (1) underground production (goods and services that are kept off the market in order to avoid taxes or regulations), (2) illegal production (goods and services that are prohibited by law), (3) informal sector production (goods and services that are produced by firms that are either unregistered or below a threshold of employment), (4) production of households for own-final use (goods and services produced within the household for self-consumption), and (5) statistical underground (goods and services that should be accounted for but are not because they are overlooked by statistical agencies).

Our notion of the shadow economy consists of the sum of underground (1) and informal sector (3) production; the remaining three categories are left out of our unit of analysis. (Schneider et al. 2010 clearly state that illegal production is left out. In addition, the statistical underground is hard to quantify, and production of households for own-final use—often called home production—considers a different set of productive activities that are not meant to be traded in the market.[6])

Measuring the size of the shadow economy is far from an easy task. Schneider and Enste (2000) discuss three main methodologies to calculate this value; here we discuss them briefly and refer the reader to their paper for additional details. By "direct approaches," Schneider and Enste refer to direct surveys and samples that attempt to quantify the number of productive entities that belong in the shadow economy. However, by their very nature, these methods are prone to providing biased estimates as respondents may be inclined to lie about their formal/shadow status. Moreover, the cost of implementing methods of this kind makes it unlikely to be used in a frequent basis.

The second methodology relies on macroeconomic indicators to infer the size of the shadow economy over time. Schneider and Enste denote this broad methodology as "indirect approaches" and list five techniques. First, we can look at the *discrepancy between the expenditure and income measures in national accounts*: since these (by construction) need to be the same, any difference between expenditures and income values of GDP could provide a measure of the shadow economy. Similarly, we can look at the *discrepancy between the official and actual labor force*: a fall in the participation rate could point to an active shadow economy. In the *transactions approach*, the researcher conjectures a stable relation between the total volume of transactions and GDP and uses this as a base to quantify the size of the shadow economy. The *currency demand approach* assumes that all shadow economy transactions are carried out in cash, so that an increase in shadow economy activity will result in an increase in the demand for currency. Finally, the *physical input method* uses the (near unit) elasticity between electricity and GDP as well as the growth of electricity consumption to infer the growth of the shadow economy.

The third methodology (known as the "model approach") uses structural econometric models to back out

[6] Ingram, Kocherlakota, and Savin (1997) use economic theory to quantify the production of households sector.

the size of the shadow economy. In a nutshell, the main idea behind this class of models is that the shadow economy does not have a single cause and does not exhibit a single effect when it operates over time. Hence, a structural econometric framework can be used to infer the size of the shadow sector by looking (simultaneously) both at the hypothesized causes of the shadow economy (e.g., tax rates and regulation) as well as the hypothesized effects (e.g., participation rates and currency demand). This methodology is often called the multiple-indicators multiple-causes model (MIMIC) and is the technique used by Schneider et al. (2010) to derive the values that we use in this study.[7]

3 Some empirical facts

In what follows, we first compare the the standard deviation of measured RGDP (as shown in Figure 1) with that of total RGDP as implied by the presence of a shadow sector. For each country/year pair, we set total RGDP to equal measured RGDP times $1+Y_{S/F}$, where $Y_{S/F}$ denotes the shadow economy size (relative to measured RGDP; in what follows we use the estimates of Schneider et al. 2010).

We first show that the standard deviation of measured RGDP is larger that of total RGDP. We then document a negative relationship between measured RGDP and the size of the shadow economy as well as a positive correlation between the volatility of measured RGDP and the volatility of the size of the shadow economy. While we're not the first to document the negative relationship between RGDP and shadow sector size, to the best of our knowledge we are the first to document the positive correlation between the volatilities of these variables. (See Appendix A for the data sources, the list of countries included in the analysis, as well as for a detailed explanation regarding variable construction for each of the figures below.)

[7] We are aware of the work of Elgin and Öztunalı (2012), who build longer time series of the size of the shadow economy (60 years, relative to the 9 observations found in Schneider et al. 2010). We have decided against using this dataset because of the methodology used by the authors: they use a DSGE model to infer the size of the shadow economy, but their model is structurally different from the one we use here as it assumes inelastic labor supply. While they document a substantial correlation to the results found in Schneider et al., we believe that using these values as an input to our Bayesian procedure will generate a sizable bias in estimates. As a partial robustness check, we verify that the empirical facts derived from their estimates are not qualitatively different from the ones shown in Figures 2, 3, and 4 below; see the Technical Appendix (Solis-Garcia and Xie 2016) for details.

3.1 Measured and total RGDP volatility

We first verify that for our sample of countries, total RGDP volatility is smaller than measured RGDP volatility. Although the values we use are restricted to the period 1999-2007 (which corresponds to the range of estimates in the work of Schneider et al.), this result can be clearly seen in Figure 2 below (the diagonal represents the 45° line).

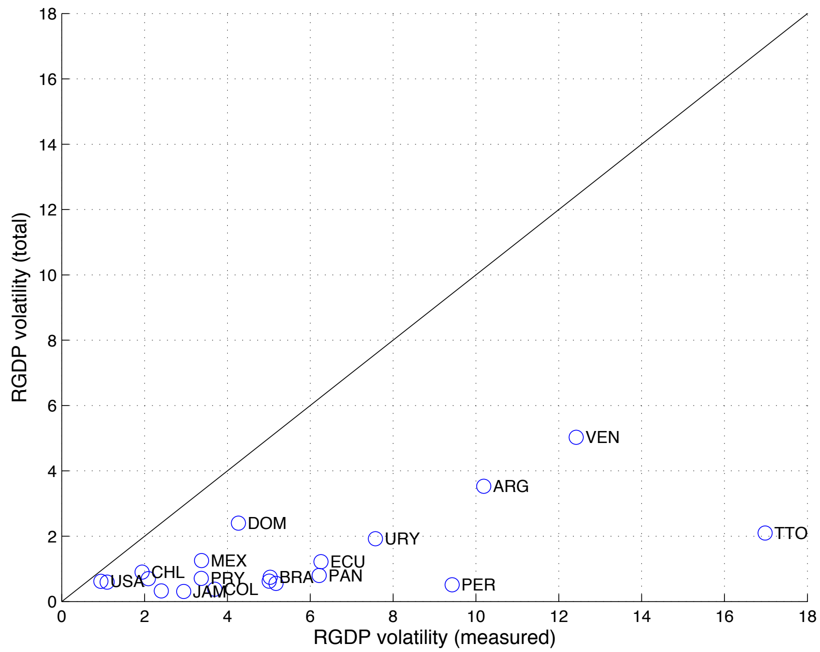

Figure 2: Measured and total RGDP volatility.

Measured RGDP volatility averages 5.5% while total RGDP volatility averages 1.3% (the value for measured RGDP volatility does not correspond to the one shown in Figure 1 since the periods involved in the calculations are different). An interesting feature of Figure 2 is that for the United States and Canada (see the marker nearest to the United States), measured and total RGDP volatility are nearly identical—a direct consequence of the small size of their shadow economies. Moreover, a larger shadow sector does not necessarily imply that total volatility is lower: for example, Venezuela's shadow sector averages 33.8% while Argentina's averages 25.3%.

3.2 Measured RGDP and shadow economy size

We now show that measured RGDP and the size of the shadow economy are negatively correlated. To help visualize the relationship, we use the average ratio of measured RGDP in a particular country relative to measured RGDP in the United States.

Figure 3 plots the logarithm of this variable together with the size of the shadow economy for the countries in our sample (we add the best-fitting trendline as well). The correlation coefficient between both series is -0.68; the main message derived from Figure 3 is that, on average, countries with a higher measured RGDP tend to have smaller shadow economies.

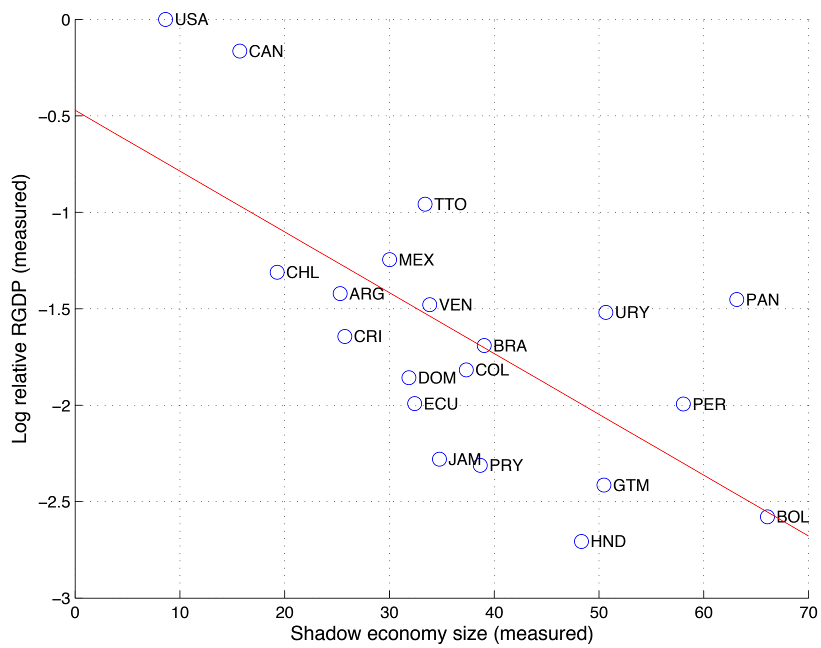

Figure 3: Log (relative) measured RGDP and size of the shadow economy.

We also calculate the percent deviations from trend for measured RGDP and the size of the shadow economy over time after using a linear-quadratic detrending procedure. We find that the shadow economy is a countercyclical variable: the percent deviations from trend for that variable are negatively correlated with those of measured RGDP (the average correlation is fairly large at -0.79). This is shown in Table 1 below.

Country	Correlation	Country	Correlation
Argentina	-0.99	Honduras	-0.88
Bolivia	-0.71	Jamaica	-0.98
Brazil	-0.89	Mexico	-0.97
Canada	-0.84	Panama	-0.24
Chile	-0.81	Paraguay	-0.79
Colombia	-0.55	Peru	-0.65
Costa Rica	-0.31	Trinidad and Tobago	-0.78
Dominican Republic	-0.93	United States	-0.91
Ecuador	-0.83	Uruguay	-0.94
Guatemala	-0.79	Venezuela	-0.99
		Average	-0.79

Table 1: Correlation coefficient, percent deviation from trend in {RGDP, shadow economy}.

3.3 Measured RGDP and shadow economy volatilities

The last empirical finding that we document is the positive correlation between the volatilities of measured RGDP and the size of the shadow economy. Figure 4 shows a scatterplot for these two variables. The figure suggests that, on average, countries that exhibit a high volatility in measured RGDP also exhibit a high volatility in the size of the shadow economy (the correlation coefficient between both series is 0.34).

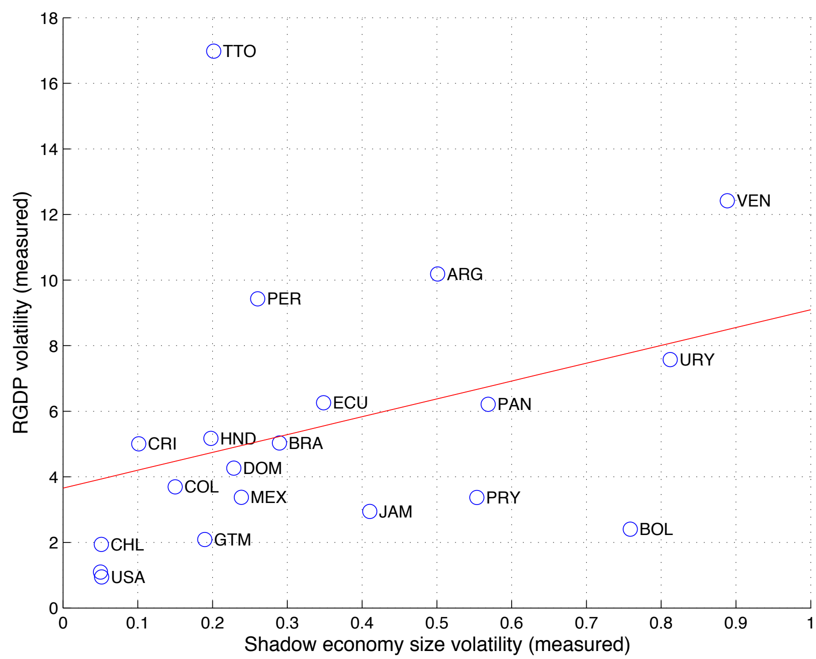

Figure 4: Volatilities of measured RGDP and size of the shadow economy.

4 Model

We first present the model and then characterize its equilibrium in detail. Our model follows Ihrig and Moe (2004) with some modifications. The economy has a representative consumer/producer who has access to a primary technology that produces output using capital and labor according to a Cobb-Douglas production function (associated with the formal sector) but can also work with a second technology that operates in the shadow sector and only requires labor input. Both formal and shadow technologies are subject to productivity shocks. There is a government that levies taxes on formal sector output and uses these resources to fund a sequence of expenditure.

The consumer's problem corresponds to choosing sequences of consumption C_t, leisure L_t, investment X_t, formal labor N_{Ft}, and shadow labor N_{St} to solve

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t (\log C_t + \phi \log L_t) \\
\text{s.t.} \quad & C_t + X_t = (1-\tau_t) z_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha} + B z_{St} N_{St}^{\gamma} \\
& K_{t+1} = (1-\delta) K_t + X_t \\
& T = L_t + N_{Ft} + N_{St}.
\end{aligned}$$

In the above, $\phi \geq 0$ denotes the weight of leisure in the utility function; $\tau_t \in [0,1]$ stands for the formal sector tax rate; z_{Ft} is TFP for the formal sector; K_t denotes the formal sector capital stock; $\alpha \in (0,1)$ is the share of capital in aggregate output; $B > 0$ is a normalizing constant (which will be important in our calibration exercise to set the right shadow/formal output ratio); z_{St} is TFP for the shadow sector; $\gamma \in (0,1]$ is the labor share of shadow output; and $T > 0$ is the total amount of hours available per period.

We include a government sector to account for the potential effect that tax rates have over the size of the informal sector (e.g., Ihrig and Moe 2004). We assume that the government uses tax revenue to fund a stream of non-productive expenditure G_t and that it satisfies its period-by-period budget constraint, following

$$G_t = \tau_t z_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha}.$$

4.1 Characterization of equilibrium

Setting $Y_{Ft} \equiv z_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha}$, $Y_{St} \equiv B z_{St} N_{St}^{\gamma}$, and $Y_t \equiv Y_{Ft} + Y_{St}$, it is a simple task to show that the model's equilibrium is characterized by[8]

$$\begin{aligned}
C_t^{-1} &= \alpha\beta E_t C_{t+1}^{-1}(1-\tau_{t+1}) Y_{F,t+1} K_{t+1}^{-1} + \beta(1-\delta) E_t C_{t+1}^{-1} && (4.1)\\
\phi C_t &= (1-\alpha)(1-\tau_t) Y_{Ft} N_{Ft}^{-1} L_t && (4.2)\\
\phi C_t &= \gamma Y_{St} N_{St}^{-1} L_t && (4.3)\\
K_{t+1} &= (1-\delta) K_t + X_t && (4.4)\\
T &= L_t + N_{Ft} + N_{St} && (4.5)\\
Y_{Ft} &= z_{Ft} K_t^{\alpha} N_{Ft}^{1-\alpha} && (4.6)\\
Y_{St} &= B z_{St} N_{St}^{\gamma} && (4.7)\\
Y_t &= Y_{Ft} + Y_{St} && (4.8)\\
Y_t &= C_t + X_t + G_t && (4.9)\\
G_t &= \tau_t Y_{Ft}. && (4.10)
\end{aligned}$$

Equation (4.1) is the usual intertemporal condition found in standard dynamic models. Equations (4.2) and (4.3) represent the model's intratemporal conditions; we have two of these as the marginal utility of consumption should be equated with the marginal utilities of working for the formal and shadow sectors. The rest of the equilibrium conditions are standard: equation (4.4) is the law of motion of (formal sec-

[8] See the Technical Appendix (Solis-Garcia and Xie 2016) for a detailed exposition of the characterization of the model.

tor) capital; (4.5) is the consumer's time constraint; (4.6) and (4.7) are the formal and shadow production technologies; (4.8) is our measure of total output; (4.9) is the aggregate feasibility condition; and (4.10) represents equilibrium government spending.

We also add the laws of motion for the tax rate and formal and shadow TFP:[9]

$$\begin{aligned}
\tau_t &= (1-\rho_\tau)\tau + \rho_\tau \tau_{t-1} + \varepsilon_{\tau t} \\
z_{Ft} &= (1-\rho_F)z_F + \rho_F z_{F,t-1} + \varepsilon_{Ft} \\
z_{St} &= (1-\rho_S)z_S + \rho_S z_{S,t-1} + \varepsilon_{St}.
\end{aligned}$$

In the above, $\{\tau, z_F, z_S\}$ denote steady-state values. In addition, for $j \in \{\tau, F, S\}$, $\rho_j \in (-1,1)$, $E(\varepsilon_{jt}) = 0$, and $\text{var}(\varepsilon_{jt}) = \sigma_j^2$. We also assume a potential correlation between the innovations to formal and shadow TFP. Hence, we infer $E(\varepsilon_{Ft}\varepsilon_{St}) = \varphi_{FS}\sigma_F\sigma_S$.

4.2 Steady state

(In the sections that follow, variables without time subscripts denote steady state values.) We now find the closed-form solutions for the system's steady state values; these will be relevant to the calibration exercise we perform in Section 5.2. We start by setting $T = z_F = z_S = 1$; after some algebra, we obtain a set of closed-form solutions. First, the formal sector's capital stock K as a function of formal sector labor:

$$K = \left[\frac{\alpha\beta(1-\tau)}{1-\beta(1-\delta)}\right]^{1/(1-\alpha)} N_F \equiv \omega_K N_F. \tag{4.11}$$

Second, shadow economy labor N_S as a function of the normalizing constant B:

$$N_S = \left[\frac{\gamma}{(1-\alpha)(1-\tau)\omega_K^{\alpha}}\right]^{1/(1-\gamma)} B^{1/(1-\gamma)} \equiv \omega_S B^{1/(1-\gamma)}. \tag{4.12}$$

[9] In the Technical Appendix (Solis-Garcia and Xie 2016) we modify the process for shadow TFP so that it is a deterministic function of formal TFP. The results we obtain are qualitatively similar to what we document below.

Third, consumption C as a function of B and formal sector labor:

$$C = \frac{(1-\alpha)(1-\tau)\omega_K^{\alpha}[1-\omega_S B^{1/(1-\gamma)} - N_F]}{\phi}.$$

Finally, formal sector labor N_F as a function of B, ϕ, and other parameter values:

$$N_F = \frac{(1-\alpha)(1-\tau)\omega_K^{\alpha} - [(1-\alpha)(1-\tau)\omega_K^{\alpha}\omega_S + \phi\omega_S^{\gamma}]B^{1/(1-\gamma)}}{\phi[(1-\tau)\omega_K^{\alpha} - \delta\omega_K] + (1-\alpha)(1-\tau)\omega_K^{\alpha}}. \tag{4.13}$$

4.3 Impulse-response analysis

Before proceeding to the parametrization, it's useful to see how the model behaves in response to shocks to TFP. Since we speculate that productivity shocks shift resources between sectors, we would like to see the magnitude of these changes under reasonable parameter values.[10] We present impulse-response functions for formal and total output in response to a 1% *negative* innovation to formal and shadow TFP; by doing this, we want to compare the behavior of measured output (formal) and how it compares to total output given the same innovation.

We first consider the response of formal sector output. Figure 5 shows that a negative innovation to formal sector TFP decreases formal output by almost 9% (relative to its steady state) on impact. While this is expected, the figure also shows the influence of a negative innovation to shadow sector TFP: shadow output rises on impact by roughly 1% and quickly returns to the steady state.

Figure 6 shows the behavior of total output as the economy receives shocks to formal and shadow TFP. The impulse-response function shows that a negative shock to formal TFP reduces total output by nearly 5% on impact. A negative shock to shadow TFP decreases total output a bit over half a percent point.

These results are consistent with our conjecture: a negative shock to formal-sector TFP reduces mea-

[10] For this exercise we set the discount rate β to 0.96 and the capital share α to 0.33. All the remaining values correspond to the sample averages contained in Tables 5, 6, and 7. For steady state values, we fix $\tau = 0.15$, $N_F = 0.15$, and $Y_{S/F} = 0.37$. We set the depreciation rate δ to 0.11; the shadow labor share γ to 0.19; the persistence parameters for TFP processes ρ_F and ρ_S to 0.87 and 0.50 and the one for tax rates ρ_τ to 0.84. Finally, we set the correlation between the formal and shadow TFP innovations to 0.17.

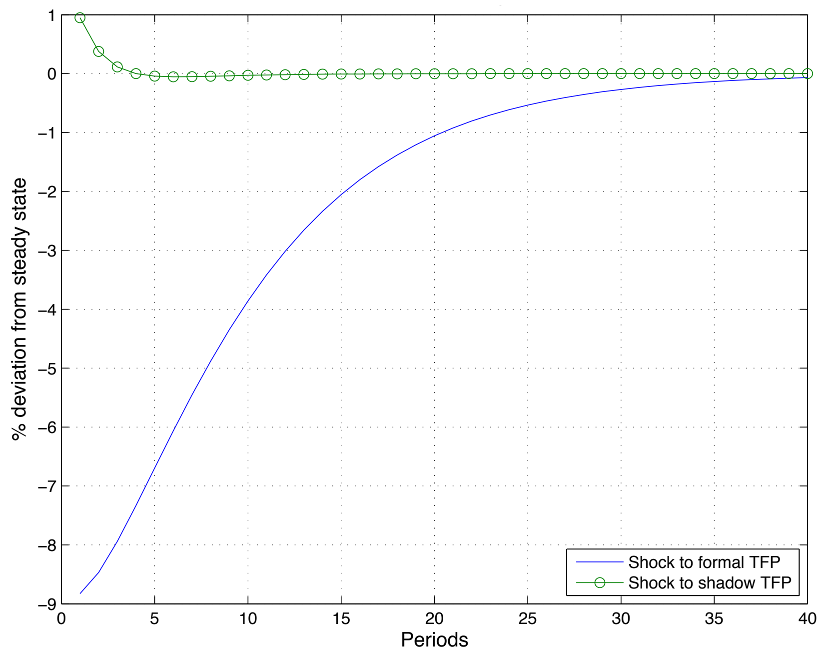

Figure 5: Impulse-response of formal sector output.

sured RGDP by 9% yet total RGDP falls by 5% only. This suggests that the importance of shocks to formal and shadow TFP is not trivial.

5 Parametrization

We now describe the parametrization details of our model. First, we list the data sources behind our choice of observable variables and specify the targets between steady state values and real-world averages. We then explain our calibration strategy, which depends on whether the country under analysis has formal labor data available or not. Finally, we present the Bayesian priors we use in our econometric exercise.

We group the model's parameters in the vector Θ:

$$\Theta = \{\tau, z_F, z_S, N_F, Y_{S/F}; \alpha, \beta, \phi, \delta, B, \gamma, \rho_F, \rho_S, \rho_\tau, \sigma_F, \sigma_S, \sigma_\tau, \varphi_{FS}\},$$

where the first five elements correspond to steady state values. (Note that each country in the sample will have a different Θ vector; see Appendix B for the calibrated and estimated values of all countries.)

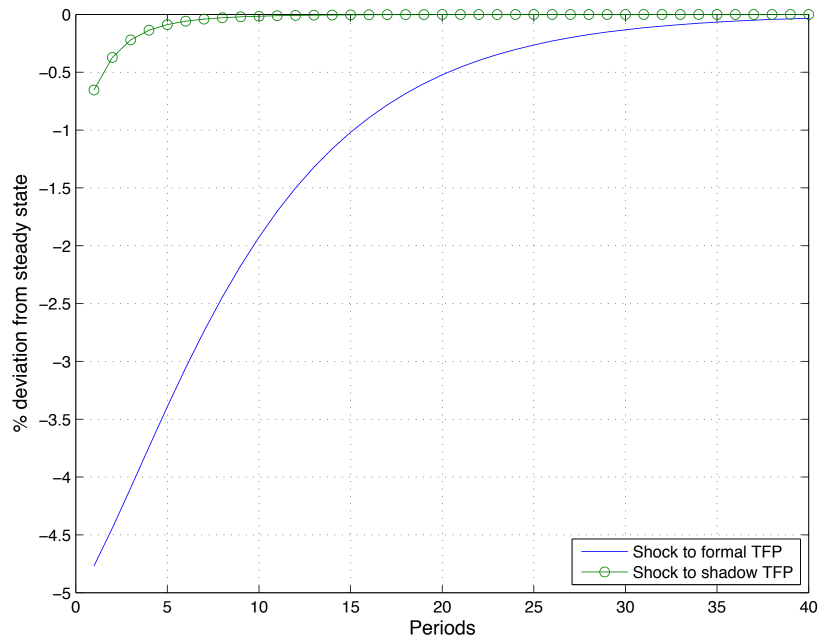

Figure 6: Impulse-response of total output.

5.1 Observable variables and data sources

We use five main variables as observables.[11] First, we obtain measured RGDP (Y_{Ft}^{obs}) and TFP (z_{Ft}^{obs}) from the Penn World Table 8.1 (Feenstra, Inklaar, and Timmer 2015); we take these series to correspond to model variables Y_{Ft} and z_{Ft}. From the same source, we also obtain the ratio of government expenditure to GDP (τ_t^{obs}) that, given the period-by-period budget balance assumption, is equivalent to the model's tax rate τ_t.

From the Total Economy Database (TED), we get total hours worked and create a measure of formal labor input (N_{Ft}^{obs}), where hours are expressed relative to 5000 hours per year. Finally, from Schneider et al. (2010) we obtain the value of the shadow economy ($Y_{St/Ft}^{\text{obs}}$), expressed as a fraction of measured output; we use this value to back out shadow sector GDP (Y_{St}^{obs}), which is the actual variable we use as an observable.

In taking the model to the data we face the following problem: a subset of countries in the TED database do not have statistics on hours worked.[12] This creates some issues with the estimation procedure since we cannot use this observable variable for all the countries in the sample. To resolve this difficulty, we design a

[11] See Appendix A for additional details on the observable variables.

[12] These countries are Bolivia, the Dominican Republic, Guatemala, Honduras, Panama, and Paraguay. See the Technical Appendix (Solis-Garcia and Xie 2016) for details on how our results change when we exclude these countries from the analysis.

joint calibration and estimation strategy that allows us to maximize the use of information that the variable provides.

5.2 Calibration

The set of calibrated parameters is given by $\Theta_C \equiv \{\tau, z_F, z_S, N_F, Y_{S/F}; \alpha, \beta, \phi, B, \rho_F, \rho_\tau, \sigma_F, \sigma_\tau\}$. First consider the steady state values $\{\tau, z_F, z_S, N_F, Y_{S/F}\}$. We set $z_F = z_S = 1$ and map the remaining values to the sample averages of the variables defined in Section 5.1: for each country, we calculate the steady state tax rate τ, the steady state formal labor share N_F, and the steady state shadow-to-formal GDP ratio $Y_{S/F}$.

For the remaining elements in Θ_C, we fix the share of capital income α to 0.33 and the household's discount factor β to 0.96. (These values are standard in the literature and are consistent with an annual frequency as well.) We then obtain a long sample of values for TFP and tax rates (from 1950 to 2011, using the same sources as detailed in Appendix A) and calculate correlation and volatility coefficients (i.e., ρ_j and σ_j for $j \in \{\tau, F\}$) for each series. Finally, to obtain the values for ϕ and B, we need to consider whether data for formal labor is available or not. This is discussed below.

5.2.1 Formal labor data available

When formal labor data are available, our strategy follows a two-step approach. First, we calibrate parameter B to be consistent with the steady state values N_F and $Y_{S/F}$; second, we use the value of B along with the steady state N_F value to calibrate parameter ϕ.

To obtain the calibrated value of B, note that by construction

$$Y_{S/F} = \frac{Y_S}{Y_F} = \frac{\omega_S^\gamma B^{1/(1-\gamma)}}{\omega_K^\alpha N_F}. \tag{5.1}$$

Hence, we can solve for B directly; we get

$$B = \left[\frac{Y_{S/F}\,\omega_K^{\alpha} N_F}{\omega_S^{\gamma}}\right]^{1-\gamma}. \tag{5.2}$$

With B at hand, we can solve for ϕ from (4.13):

$$\phi = \frac{(1-\alpha)(1-\tau)\omega_K^{\alpha}[1-\omega_S B^{1/(1-\gamma)} - N_F]}{N_F[(1-\tau)\omega_K^{\alpha} - \delta\omega_K] + \omega_S^{\gamma} B^{1/(1-\gamma)}}. \tag{5.3}$$

By inspection, B and ϕ depend both on steady state values $\{\tau, N_F, Y_{S/F}\}$ as well as parameters $\{\gamma, \delta\}$ (both directly and via ω_K and ω_S). Since the latter are estimated via Bayesian methods, we put B and ϕ as direct functions of γ and δ so that they are updated at every iteration following (5.2) and (5.3).

5.2.2 No formal labor data available

When there is no available formal labor data, our approach changes slightly relative to the one described above. We start by rewriting (4.13) as

$$N_F = \omega_{F1} - \omega_{F2} B^{1/(1-\gamma)} \tag{5.4}$$

where

$$\begin{aligned}
\omega_{F0} &\equiv \phi[(1-\tau)\omega_K^{\alpha} - \delta\omega_K] + (1-\alpha)(1-\tau)\omega_K^{\alpha} \\
\omega_{F1} &\equiv (1-\alpha)(1-\tau)\omega_K^{\alpha}/\omega_{F0} \\
\omega_{F2} &\equiv [(1-\alpha)(1-\tau)\omega_K^{\alpha}\omega_S + \phi\,\omega_S^{\gamma}]/\omega_{F0}.
\end{aligned}$$

We combine (5.4) and (5.1) to get

$$Y_{S/F} = \frac{\omega_S^{\gamma} B^{1/(1-\gamma)}}{\omega_K^{\alpha} N_F} = \frac{\omega_S^{\gamma} B^{1/(1-\gamma)}}{\omega_K^{\alpha} [\omega_{F1} - \omega_{F2} B^{1/(1-\gamma)}]}$$

and from this equation we can solve for B:

$$B = \left[\frac{Y_{S/F}\, \omega_K^{\alpha} \omega_{F1}}{\omega_S^{\gamma} + Y_{S/F}\, \omega_K^{\alpha} \omega_{F2}} \right]^{1-\gamma}. \tag{5.5}$$

Equation (5.5) shows that B depends on steady state values $\{\tau, Y_{S/F}\}$ and parameters $\{\gamma, \delta, \phi\}$. Hence, we put B as a direct function of $\{\gamma, \delta, \phi\}$ so that its value is updated at every iteration following (5.5). The details on how we handle parameter ϕ are discussed in Section 5.3.

5.3 Bayesian priors

For all countries, we estimate parameters in the vector $\Theta_E \equiv \{\delta, \gamma, \rho_S, \sigma_S, \varphi_{FS}\}$ using the prior distributions shown in Table 2. We add measurement errors on all of our observable variables, which we set to Gamma distributions with mean and standard deviations equal to 0.125 and 0.0722 of the empirical standard deviations from each observable variable.

Parameter	Description	Distribution	Mean	SD
δ	Depreciation rate	Beta	0.1	0.025
γ	Shadow labor share	Beta	0.5	0.2
ρ_S	Autocorrelation	Beta	0.5	0.2
σ_S	Standard deviation	Inverse Gamma	0.05	0.02
φ_{FS}	Correlation	Modified Beta*	0.0	0.3

* The modified Beta distribution is defined over the interval $[-1, 1]$.

Table 2: Prior distributions.

As mentioned above, a subsample of countries does not have data for formal labor. In this case, we have to determine a value for ϕ; we estimate the parameter alongside those in Table 2. First, we calculate the average and standard deviation of the (calibrated) value of ϕ for the countries that do have formal labor data. Once this is done, we impose a Gamma prior distribution and use these values for the remaining countries. Table 3 shows the details on the prior for ϕ.[13,14]

ϕ_j, $j=$	Mean estimate
Argentina	3.7875
Brazil	2.7318
Chile	4.0691
Colombia	3.4545
Costa Rica	2.7952
Ecuador	3.1978
Jamaica	3.1187
Mexico	3.9772
Peru	2.8556
Trinidad and Tobago	3.9768
Uruguay	2.8076
Venezuela	4.0570
Average	3.4024
Standard deviation	0.5459

Table 3: Obtaining the prior distribution for ϕ.

6 Results and analysis

As a first step, we use the model to generate time series for formal output and the size of the shadow economy. We use the simulated data to verify the claim that total RGDP volatility is lower than measured RGDP volatility given that we now consider the shadow economy's output. We then show that the model data are able to replicate the negative relationship between (relative) measured RGDP and the size of the shadow economy, and the positive relationship between the volatilities of measured RGDP and the size of the shadow economy.

[13] Since this is not a problem for the two developed countries in the sample—Canada and the United States—we take the average over the sample of Latin American countries that have labor data available. We have checked whether including Canada and the U.S. makes a difference in terms of our results, but it doesn't seem that this is the case.

[14] To obtain the estimates in Table 3, we run one chain of 1 million draws, discarding the first 750 thousand draws. In all cases, the acceptance rate falls in the range $[0.225, 0.275]$.

6.1 Measured and total RGDP volatility

As mentioned in the introduction, our conjecture is that the difference in RGDP volatility between developing and developed countries can be accounted for by the mismeasurement of RGDP. To test that conjecture, we obtain the measure of measured and total RGDP for all countries in the sample and derive a volatility measure for both series. Figure 7 replicates Figure 2 using artificial data from the model.

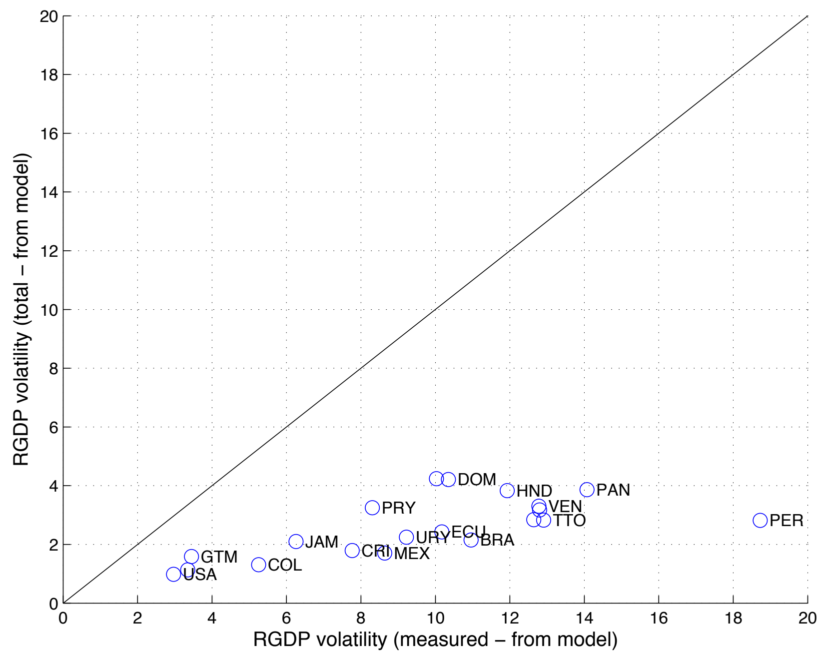

Figure 7: Measured and total RGDP volatility (model data).

Visual inspection shows that for all the countries in the sample, total RGDP volatility estimates are lower than measured RGDP volatility estimates. In particular, most values are clustered between 2 and 4%: the average value for the total RGDP volatility is 2.6% with a standard deviation of 1.0%, while the corresponding values for measured RGDP volatility are 9.6% and 4.0%. As is the case in the real world, measured and total RGDP volatility for the United States and Canada are fairly similar.

6.2 Consistency checks

We now put our model through a couple of consistency checks. We verify that the simulated data are able to replicate the empirical regularities outlined in Section 3, namely, a positive relationship between formal RGDP and the size of the shadow economy, and a negative relationship between the volatility of both variables.[15]

6.2.1 Measured RGDP and shadow economy size

Figure 8 relates formal sector relative RGDP and shadow sector size using the simulated data from the model. Our results support a negative relationship between relative RGDP and the size of the shadow sector; to allow for a quick comparison, Figure 9 contains the same values as Figure 3.

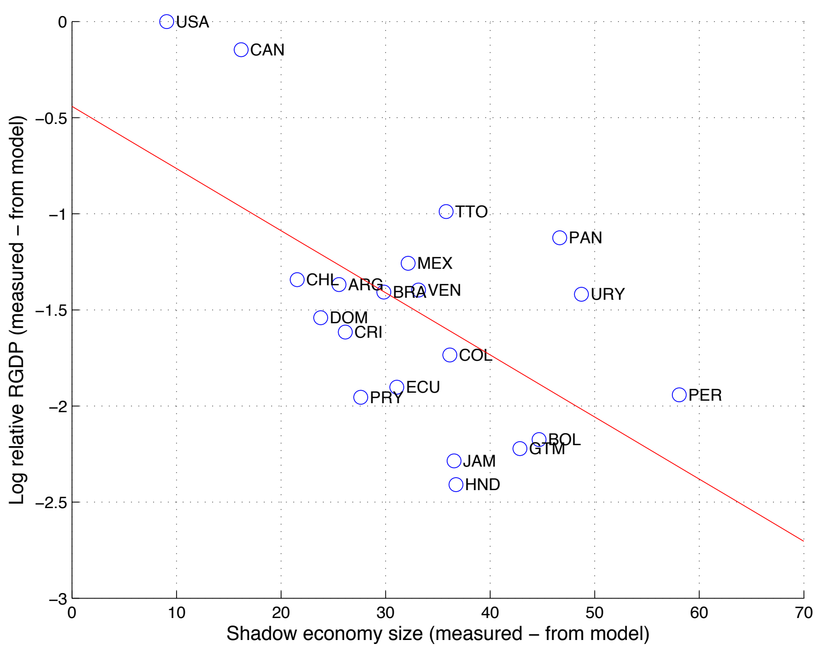

Figure 8: (Log) Relative RGDP and size of the shadow economy (model data).

By inspection, both figures are very similar, which suggests that the model does a good job at capturing the features of the real world. (The correlation coefficient using simulated data is -0.59; real-world data yields a coefficient of -0.68.) That said, if we exclude the countries that have no formal labor data available

[15] We use formal RGDP as it is the model equivalent to measured RGDP in the real world; which is what we use in Section 3.

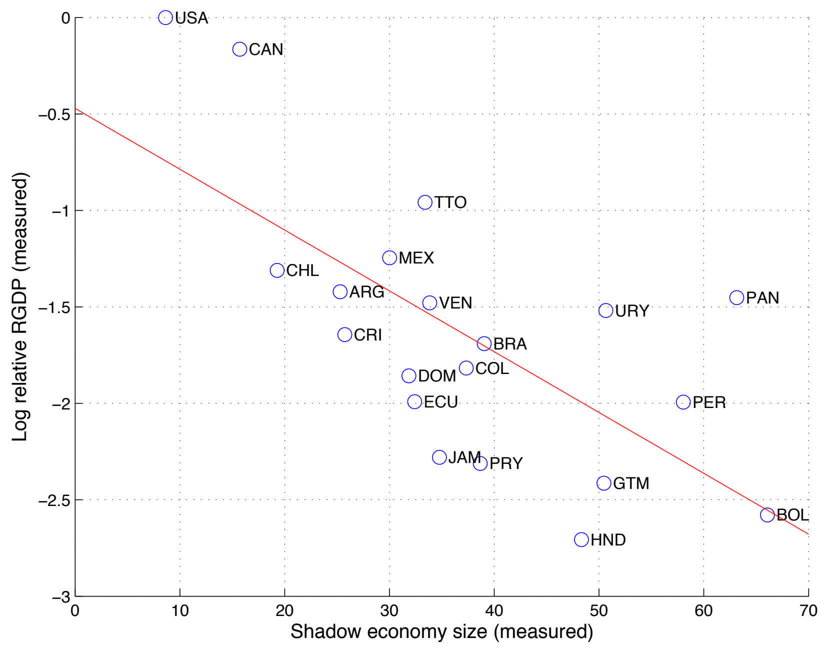

Figure 9: (Log) Relative RGDP and size of the shadow economy (real world).

(Bolivia, Dominican Republic, Guatemala, Honduras, Panama, and Paraguay) then we can see that Figure 8 and Figure 9 are nearly identical.[16]

6.2.2 Measured RGDP and shadow economy volatilities

We now take a look at the relationship between the volatilities of formal RGDP and the size of the shadow economy. We follow the same logic as above and present two graphs to contrast our results. Figure 10 shows the relationship that results from model data, while Figure 11 is identical to Figure 4. We use the same scale in both axes to facilitate comparison between graphs.

The positive relation between both variables is evident from both figures (the correlation coefficient using model data is 0.36 while the one using real-world data is 0.34), yet there is a clear difference between them: the volatilities of measured RGDP and shadow economy size are larger in the model than in the data, as Table 4 confirms:

[16] See the Technical Appendix (Solis-Garcia and Xie 2016) for results excluding these countries.

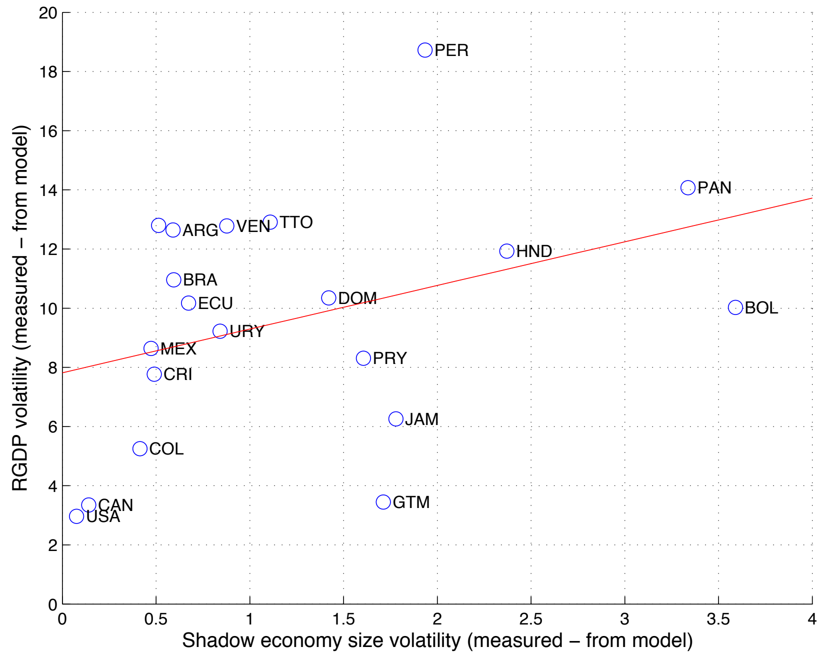

Figure 10: Volatilities of RGDP and size of the shadow economy (model data).

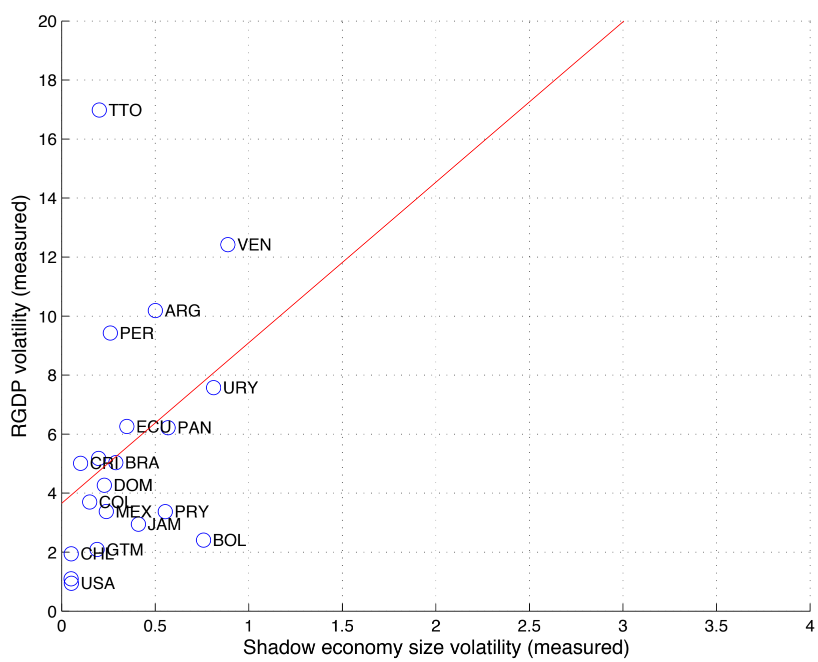

Figure 11: Volatilities of RGDP and size of the shadow economy (real world).

Variable	Observed	Simulated data
Measured RGDP	5.5205	9.6285
Shadow economy size	0.3426	1.2272

Table 4: Volatility (sample average).

6.3 A volatility puzzle?

The results above suggest that our mechanism holds promise in accounting for the higher volatility of measured RGDP in Latin American countries. However, the evidence also shows a disconnection from the theoretical predictions of the model and the real world.

From our results, theory suggests that we should observe a higher volatility both in measured RGDP and in the size of the shadow economy. Using the values contained in Table 4, the model's measured RGDP volatility is 1.7 times the real-world measured RGDP volatility, yet the model's shadow economy size volatility is 3.6 times as high as the observed value. At face value, this result implies that agents in the real world do not switch back and forth between the formal and shadow technologies. For some reason, the shadow economy's "escape valve" role does not seem to be used to its maximum.[17]

7 Conclusion

In this paper, we propose a mechanism to account for the substantial difference in the volatility of measured RGDP between developing and developed countries; this mechanism involves the fairly overlooked fact that developing economies have a sizable shadow economy. We build a model that includes a shadow economy and distinguishes between measured (formal) and total (shadow and formal) output. The results of our model show that economies in North and South America, including the Caribbean, are fairly similar in terms of total RGDP volatility (2.6%, compared to 1.3% using real-world data). We document an apparent puzzle in

[17] A seminar participant suggested thinking about the shadow economy as an absorbent state: once the agent uses the shadow technology it is impossible to go back and use the formal one. The puzzle discussed above could be consistent with a slightly different version of this idea, where the agent is not "locked-in" the shadow technology but needs to wait some periods before going back to the formal technology. A somewhat similar mechanism is implemented in Fiess, Fugazza, and Maloney (2010).

that the model suggests that the volatility of the size of the shadow economy should be substantially larger than what is observed in the real world. We believe that this may be indicative of frictions that prevent agents from optimally moving between production technology.

This paper adds to our understanding of the shadow economy and its relationship with business cycles in general. Understanding the connection between these two concepts (loosely speaking, how the shadow economy actually operates as an escape valve) should prove useful to characterize the properties of the business cycle in developing economies. Its conclusions may also serve as a handy tool for policymakers as oftentimes their efforts are directed at affecting the size of the shadow economy (either by attempting to formalize producers or changing the rules of the game by altering the country's tax policy).[18]

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[18] Our results suggest that policy actions that try to discourage agents' participation in the shadow economy may be Pareto-worsening, as they possibly hamper the ability of agents to switch towards more efficient production technologies.

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A Data appendix

A.1 Country list

The analysis includes Argentina (ARG), Bolivia (BOL), Brazil (BRA), Canada (CAN), Chile (CHL), Colombia (COL), Costa Rica (CRI), the Dominican Republic (DOM), Ecuador (ECU), Guatemala (GTM), Honduras (HND), Jamaica (JAM), Mexico (MEX), Panama (PAN), Paraguay (PRY), Peru (PER), Trinidad and Tobago (TTO), the United States (USA), Uruguay (URY), and Venezuela (VEN).

A.2 Section 3 details

Data sources We obtain (measured) RGDP from the Penn World Table 8.1 (Feenstra et al. 2015, calculated as the ratio of series `rgdpna` and `pop`) and shadow economy size (relative to measured RGDP) from Schneider et al. (2010).

Figure 2 We calculate total RGDP as follows: for each country i in the sample and for each $t \in \{1999, 2007\}$, define

$$\text{RGDP}_{i,t}^{\text{TOT}} = \text{RGDP}_{i,t} \times \left(1 + Y_{i,t}^{S/F}\right)$$

where $Y_{i,t}^{S/F}$ denotes the shadow economy size (relative to measured RGDP) for country i in year t. We then transform measured and total RGDP levels into percent deviations from trend. Given that several of the countries under consideration have experienced recurrent crises, a linear trend may not be the best choice to detrend the data; hence, we allow for a quadratic term along the usual linear trend. In this sense, we run the following regressions for each country in our sample:

$$\begin{aligned}
\log(\text{RGDP}_{i,t}) &= \beta_{0,i} + \beta_{1,i}\, t + \beta_{2,i}\, t^2 + \varepsilon_{i,t} \\
\log(\text{RGDP}_{i,t}^{\text{TOT}}) &= \beta_{0,i}^{\text{TOT}} + \beta_{1,i}^{\text{TOT}}\, t + \beta_{2,i}^{\text{TOT}}\, t^2 + \varepsilon_{i,t}^{\text{TOT}},
\end{aligned}$$

where the residuals $\varepsilon_{i,t}$ and $\varepsilon_{i,t}^{\text{TOT}}$ are our measure of detrended RGDP, on which we calculate the standard deviation to obtain the values shown in the graph.

Figure 3 We calculate relative RGDP (with respect to the United States) as follows: for each country i in the sample and for each $t \in \{1999, \ldots, 2007\}$, we define the variable

$$\text{RELGDP}_{i,t} = \frac{\text{RGDP}_{i,t}}{\text{RGDP}_{\text{USA},t}},$$

so that $\text{RELGDP}_{i,t}$ denotes the RGDP ratio between country i and the United States in year t. Once this variable is calculated, we derive the measure of relative RGDP by taking the simple average over the sample of T periods available:

$$\text{RELGDP}_i = \frac{1}{9} \sum_{t=1999}^{2007} \text{RELGDP}_{i,t}.$$

The variable that is plotted in the figure is the logarithm of the value above. Finally, the shadow economy size is the simple average of the values obtained from Schneider et al. (2010).

Figure 4 We transform (measured) RGDP and shadow economy size levels into percent deviations from trend following the same guidelines as in Figure 2, though we make some small changes to the way we calculate RGDP volatility in this case. For each country i in the sample and for each $t \in \{1950, \ldots, 2011\}$, we run the following regression:

$$\log(\text{RGDP}_{i,t}) = \gamma_{0,i} + \gamma_{1,i}\, t + \gamma_{2,i}\, t^2 + \eta_{i,t}, \tag{A.1}$$

where the residuals $\eta_{i,t}$ are our measure of detrended RGDP, on which we calculate the standard deviation to obtain the values shown in the graph. Since the size of the shadow economy is expressed as a percentage relative to measured RGDP, we do not need to log the dependent variable as in equation (A.1). Hence, we run the following regression for each country i in our sample and each $t \in \{1999, 2007\}$:

$$Y_{i,t}^{S/F} = \theta_{0,i} + \theta_{1,i}\, t + \theta_{2,i}\, t^2 + \mu_{i,t}. \tag{A.2}$$

As above, the residuals $\mu_{i,t}$ become the observables for the size of the shadow economy.

A.3 Section 5 details

Data sources We obtain TFP (series `rtfpna`) and the share of government expenditure (series `csh_g`) from the Penn World Table 8.1 (Feenstra et al. 2015). We take formal hours from the series "Total hours worked" from The Conference Board *Total Economy Database*™, May 2016, `http://www.conference-board.org/data/economydatabase/`. Data sources on RGDP and the size of the shadow economy are detailed in Section A.2.

Observable variables We use percent deviations from trend as observable variables. Measured and shadow RGDP, formal TFP, and formal hours are detrended following (A.1); tax rates follow (A.2).

B Parameter appendix

B.1 Steady state values

We map the steady state values in our model to the sample averages of the variables defined in Section 5.1. The values for the steady state triple $\{\tau, N_F, Y_{S/F}\}$ are shown below.

B.2 Calibration (first stage)

The set of calibrated parameters is given by $\Theta_C \equiv \{\alpha, \beta, \phi, B, \rho_F, \rho_\tau, \sigma_F, \sigma_\tau\}$. We set $\alpha = 0.33$ and $\beta = 0.96$ for all countries. The values for $\{\rho_F, \rho_\tau, \sigma_F, \sigma_\tau\}$ are derived from sample averages for each country and presented in Table 6 below.

As mentioned in Section 5.2, parameters $\{B, \phi\}$ are determined at the same time as the Bayesian estimation procedure takes place. We present the joint values in the following section.

Country	τ	N_F	$Y_{S/F}$
Argentina	0.1008	0.1412	0.2530
Bolivia	0.1751	*N/A*	0.6607
Brazil	0.1510	0.1640	0.3904
Canada	0.1387	0.1620	0.1571
Chile	0.1802	0.1414	0.1928
Colombia	0.0988	0.1396	0.3733
Costa Rica	0.1451	0.1806	0.2574
Dominican Republic	0.1254	*N/A*	0.3186
Ecuador	0.2064	0.1490	0.3240
Guatemala	0.1334	*N/A*	0.5047
Honduras	0.133	*N/A*	0.4832
Jamaica	0.1816	0.1502	0.3477
Mexico	0.0926	0.1329	0.3001
Panama	0.1816	*N/A*	0.6314
Paraguay	0.1258	*N/A*	0.3867
Peru	0.1683	0.1350	0.5804
Trinidad and Tobago	0.1174	0.1273	0.3340
United States	0.1233	0.1608	0.0863
Uruguay	0.1619	0.1455	0.5064
Venezuela	0.2432	0.1198	0.3384
Average	0.1492	0.1464	0.3713

N/A: no data available for formal hours.

Table 5: Steady state values.

Country	ρ_F	ρ_τ	σ_F	σ_τ
Argentina	0.9246	0.7586	0.0430	0.0140
Bolivia	0.9055	0.8007	0.0408	0.0128
Brazil	0.9753	0.9317	0.0327	0.0190
Canada	0.7691	0.8304	0.0178	0.0063
Chile	0.8718	0.8621	0.0477	0.0122
Colombia	0.8336	0.8470	0.0212	0.0056
Costa Rica	0.9263	0.9558	0.0261	0.0068
Dominican Republic	0.8310	0.8083	0.0426	0.0146
Ecuador	0.8922	0.8774	0.0366	0.0146
Guatemala	0.9769	0.8210	0.0192	0.0063
Honduras	0.7491	0.9076	0.0367	0.0110
Jamaica	0.8755	0.8659	0.0330	0.0210
Mexico	0.9184	0.8676	0.0294	0.0052
Panama	0.6282	0.8919	0.0356	0.0148
Paraguay	0.8433	0.7804	0.0367	0.0104
Peru	0.9540	0.6920	0.0479	0.0118
Trinidad and Tobago	0.9063	0.9292	0.0428	0.0131
United States	0.8385	0.6778	0.0141	0.0060
Uruguay	0.8132	0.8292	0.0366	0.0105
Venezuela	0.8926	0.7859	0.0457	0.0249
Average	0.8663	0.8360	0.0343	0.0120

Table 6: Calibrated values (first stage).

B.3 Estimation and calibration (second stage)

The set of estimated parameters is given by $\Theta_E \equiv \{\delta, \gamma, \rho_S, \sigma_S, \varphi_{FS}\}$; the priors for each parameter are listed in Table 2. We now present the mean estimates for these parameters, as well as the implied values for the pair $\{B, \phi\}$.[19,20]

[19] Recall that for the set of countries without formal labor data, parameter B is backed out from steady state and estimated values, and parameter ϕ is estimated using the average and standard deviation found in Table 3. We use an asterisk to distinguish these countries from the other ones.

[20] See the Technical Appendix (Solis-Garcia and Xie 2016) for the posterior distribution of all the parameters.

Country	Estimated					Calibrated	
	δ	γ	ρ_S	σ_S	φ_{FS}	B	ϕ
Argentina	0.0900	0.2708	0.4007	0.0314	0.3726	0.1632	3.7875
Bolivia	0.1037	0.1824	0.4578	0.0287	-0.0108	0.1971	3.3290
Brazil	0.1071	0.1661	0.4845	0.0278	0.0966	0.1694	2.7318
Canada	0.1245	0.1749	0.5025	0.0276	0.1927	0.0776	3.6720
Chile	0.1213	0.1776	0.5098	0.0283	0.1133	0.0811	4.0691
Colombia	0.1129	0.1850	0.5043	0.0278	0.1170	0.1546	3.4545
Costa Rica	0.1047	0.2089	0.5054	0.0282	0.0806	0.1505	2.7952
Dominican Republic	0.1083	0.1917	0.5070	0.0307	0.3750	0.1466	3.3031
Ecuador	0.1044	0.1898	0.4888	0.0283	0.2051	0.1391	3.1978
Guatemala	0.1037	0.1804	0.4729	0.0276	0.1175	0.1814	3.3375
Honduras	0.1073	0.1806	0.5032	0.0276	0.1789	0.1770	3.3037
Jamaica	0.1031	0.1884	0.4640	0.0276	-0.1417	0.1514	3.1187
Mexico	0.1201	0.1610	0.5288	0.0279	0.3201	0.1514	3.9772
Panama	0.1060	0.1885	0.4422	0.0291	0.1132	0.1949	3.2909
Paraguay	0.1060	0.1904	0.4803	0.0284	0.0469	0.1633	3.3209
Peru	0.1080	0.1786	0.4827	0.0279	0.1417	0.2040	2.8556
Trinidad and Tobago	0.1157	0.2015	0.5389	0.0309	0.1991	0.1351	3.9768
United States	0.0981	0.1850	0.4645	0.0275	0.0403	0.0543	4.0129
Uruguay	0.1132	0.1788	0.5274	0.0288	0.2899	0.1918	2.8076
Venezuela	0.1364	0.1729	0.7228	0.0356	0.5920	0.1004	4.0570
Average	0.1097	0.1877	0.4994	0.0289	0.1720	0.1492	3.4199

* No formal labor data are available for this country.

Table 7: Mean estimates and calibrated values (second stage).