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Growth Effects of Annuities and Government Transfers in Perpetual Youth Models*

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Abstract

We show that in overlapping generations endogenous growth models with uncertain lifetime, the introduction of government transfers always increases economic growth by crowding out the private annuity market and increasing accidental bequests. In particular, if the government imposes a flat-rate consumption tax (which is neutral to the consumption-saving margin), uses part of the tax revenue for unproductive purposes, and rebates the rest equally across agents as a lump-sum transfer, the economy grows faster and improves the welfare of future generations.

Keywords: annuity, endogenous growth, overlapping generations, redistribution

JEL classification: D58, E21, H20, H21, O41.

1 Introduction

Suppose that the government imposes some tax, uses part of the tax revenue for unproductive purposes, and rebates the rest to agents. Would this policy increase or decrease economic growth? Intuition tells us that the growth rate will decrease, since resources are wasted after all. In this paper, we show that this intuition is not generally correct: in perpetual youth models, if the tax does not directly affect growth (which is true for flat-rate consumption tax), then this redistribution policy unambiguously *increases* economic growth by crowding out the private annuity market and increasing accidental bequests.

This paper studies the effect of annuities and transfers on economic growth in perpetual youth models (Yaari, 1965; Blanchard, 1985), where agents die at a constant rate and new agents are born at the same rate. We show that perpetual youth models with annuities have three forces that modify economic growth relative to the benchmark economy with infinitely-lived agents: (i) impatience ($-$), (ii) effective risk-free rate ($+$), and (iii) accidental bequests ($-$), where ($\pm$) denote the positive or negative effect on growth. The first negative effect always dominates the second positive effect, and hence the growth rate unambiguously decreases in perpetual youth models relative to the benchmark case, which is

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similar to the well-known result that the steady state capital is lower in perpetual youth models with decreasing returns to scale (Blanchard, 1985). However, when agents receive government transfers in this economy, it reduces the third negative effect while leaving the first two effects unchanged. Consequently, the introduction of government transfers in perpetual youth models unambiguously increases economic growth.

The intuition for this somewhat surprising result is as follows. Since agents die at a constant rate $\delta > 0$, in the absence of government transfers, agents pledge their capital (wealth) to insurance companies to obtain annuities at premium δ. In the presence of transfers, part of the agents' wealth is the "government bond" (a claim to future transfers), but because the transfer is given only to agents that are alive, this government bond is not pledgeable to insurance companies. Provided that the tax instruments to finance the transfers do not directly affect growth, the introduction of government transfers crowds out the private annuity market, increases accidental bequests, and leads to higher economic growth. This is the case when we consider a flat-rate consumption tax.

This paper is related to two strands of literature. The first is the large literature on taxation and growth. In this literature, researchers typically consider dynamic models that feature some inefficiencies such as externalities, public goods, or incomplete markets, and study the effect of taxation on growth and welfare. Examples are human capital formation (Lucas, 1988; King and Rebelo, 1993), provision of productive public goods (Barro, 1990; Jones et al., 1993; Hatfield, 2015), saving behavior under uninsured idiosyncratic risk (Aiyagari, 1994; Angeletos, 2007), bequest motive (Ihori, 2001), and political economy (Jaimovich and Rebelo, 2016), among others.[1] However, sources of inefficiencies are not necessary to make the study of taxation on growth interesting. For example, Jones and Manuelli (1992) show that in an overlapping generations model with finite lives and convex technologies, (i) the consumption path in a laissez-faire economy is always bounded, but (ii) taxing the old and subsidizing the young may sustain growth. The intuition is that taxing the old makes future consumption more expensive and induces the young to save. Compared to this literature, our results are complementary since we only consider flat-rate consumption tax (with a single good), which does not directly affect growth since it is neutral to the consumption-saving margin (Stokey and Rebelo, 1995).[2]

Our paper is also related to the literature that employs perpetual youth models (Yaari, 1965; Blanchard, 1985), which are convenient for studying intergenerational issues in a tractable way and introducing stationarity in heterogeneous-agent models. Recent applications are asset pricing (Gârleanu et al., 2012; Gârleanu and Panageas, 2015), retirement (Prettner and Canning, 2014), and power laws in income and wealth distributions (Toda, 2014; Toda and Walsh, 2015; Benhabib et al., 2016; Gabaix et al., 2016), among others. Although several papers have studied the growth effects of taxation and/or annuities in perpetual youth models,[3] our mechanism that government transfers increase

[1] Although our paper is purely theoretical, for empirical evidence on the relation between taxation and growth, see for example Engen and Skinner (1996) and Lee and Gordon (2005).

[2] Other forms of taxes may mechanically affect growth by intervening in the intra- and intertemporal choice, such as differential tax rates on production factors (Easterly, 1993) or capital income taxation (Uhlig and Yanagawa, 1996).

[3] See, for example, Hu (1999), Reinhart (1999), Heijdra and Ligthart (2000), Hansen and

growth by crowding out the annuity market does not seem to be known. The closest result to ours is Petrucci (2002), who shows that consumption tax and rebates increase economic growth in perpetual youth models. However, his model contains special features such as production externality, log utility, and perfect annuity market, so it is not clear whether the results are general. Most importantly, Petrucci (2002) does not identify the key mechanism that government transfers increase growth by crowding out the annuity market.

2 Growth in perpetual youth models

In this section we show how annuities and transfers affect economic growth in perpetual youth models. We first consider the benchmark economy with infinitely-lived agents, and then introduce annuities and transfers when agents enter/exit the economy.

2.1 Benchmark economy

The model is a continuous-time endogenous growth model (AK model) with a continuum of agents and a government. At time $t = 0$, there is a continuum of identical, infinitely-lived agents with mass 1, each endowed with one unit of capital.

Agents have identical, additively separable utility function with constant elasticity of substitution

$$U_t = \int_0^\infty \mathrm{e}^{-\beta s} \frac{c_{t+s}^{1-1/\varepsilon}}{1-1/\varepsilon} \,\mathrm{d}s, \tag{2.1}$$

where $\beta > 0$ is the time preference rate, $\varepsilon > 0$ is the elasticity of intertemporal substitution, and c_t is consumption at time t. As usual, the case $\varepsilon = 1$ corresponds to the log utility.

Capital can be either consumed or invested in a saving technology that yields an exogenous, risk-free return μ.[4] Alternatively, we can think of a small open economy that has access to a risk-free asset with elastic supply, whose rate is set by international investors. Thus an agent's objective is to maximize the utility (2.1) subject to the budget constraint

$$\mathrm{d}w_t = (\mu w_t - c_t)\,\mathrm{d}t, \tag{2.2}$$

where w_t is wealth. This problem is a standard Merton (1971)-type optimal consumption-saving problem. The optimal consumption rule is $c_t = m_0 w_t$,[5] where the marginal propensity to consume (m_0) is given by

$$m_0 = \varepsilon\beta + (1-\varepsilon)\mu. \tag{2.3}$$

İmrohoroğlu (2008), Heijdra and Mierau (2010, 2012), and Heijdra et al. (2014).

[4]This assumption is only for simplicity. The same results obtain with stochastic returns. If capital evolves according to a geometric Brownian motion with volatility σ, all results go through by changing μ to $\mu - \frac{\gamma\sigma^2}{2}$, where $\gamma > 0$ is the relative risk aversion of the agents. In that case one needs to consider the continuous-time analogue of the Epstein-Zin utility (Svensson, 1989; Duffie and Epstein, 1992).

[5]In order for this to be the solution, we need the parameter restriction $\varepsilon\beta + (1-\varepsilon)\mu > 0$. Otherwise, the transversality condition $\lim_{t\to\infty} \mathrm{e}^{-\beta t} U_t = 0$ is violated, and there is no solution. The transversality condition is sufficient for optimality. See, for example, the discussion of the verification theorem in Chang (2004, pp. 122–125).

The growth rate of the individual wealth α_0 (as well as the growth rate of aggregate wealth g_0 since it is a representative-agent model) is given by

$$\alpha_0 = g_0 = \mu - m_0 = \varepsilon(\mu - \beta). \tag{2.4}$$

As is well known, whether the economy grows or shrinks over time depends on whether or not the interest rate μ exceeds the time preference rate β.

2.2 Overlapping generations economy with annuities

Next, instead of assuming infinitely-lived agents, suppose that agents are born and die at constant Poisson rate $\delta > 0$, as in Yaari (1965) and Blanchard (1985). In addition to the agents, there are perfectly competitive insurance companies that offer annuities. Since agents die at a constant rate δ, the insurance premium is also δ. This means that while agents are alive, for each unit of annuity contract held, the agents receive $\delta\Delta t$ during a small time interval of length Δt. When they die, they pay 1 to the annuity company, which breaks even.

To see how the introduction of annuities affects economic growth, it is convenient to consider an economy with imperfect annuities: following Hansen and İmrohoroğlu (2008), agents can only pledge a fraction $0 \le \lambda \le 1$ of their capital for the annuity contracts. When an agent dies, his heir inherits the remaining fraction $1 - \lambda$.

The solution to the individual problem is similar to the benchmark case. Since agents die at rate $\delta > 0$, it increases the effective discount factor from β to $\beta + \delta$. Since agents can receive annuities at rate δ on fraction of wealth λ, the effective risk-free rate faced by individuals becomes $\mu + \delta\lambda$. By (2.3) and (2.4), the propensity to consume out of wealth and the individual growth rate become

$$m_1 = \varepsilon(\beta + \delta) + (1 - \varepsilon)(\mu + \delta\lambda) = m_0 + \varepsilon\delta(1 - \lambda) + \delta\lambda, \tag{2.5a}$$

$$\alpha_1 = \varepsilon(\mu + \delta\lambda - \beta - \delta) = \alpha_0 - \varepsilon\delta(1 - \lambda), \tag{2.5b}$$

respectively.

To derive the growth rate of the aggregate economy, consider what happens to an agent with wealth w_t between a short time period Δt. If the agent survives (which occurs with probability $1 - \delta\Delta t$), then the wealth grows at rate α_1 in (2.5b), so it becomes $(1 + \alpha_1\Delta t)w_t$. If the agent dies (which occurs with probability $\delta\Delta t$), according to the annuity contract λw_t will go to the insurance company, which is then distributed among the surviving agents as annuities. The heir inherits the remaining capital $(1 - \lambda)w_t$. Hence by accounting we obtain

$$\begin{aligned} W + \Delta W &= \underbrace{(1 - \delta\Delta t)(1 + \alpha_1\Delta t)W}_{\text{Aggregate wealth of surviving agents}} + \underbrace{\delta\Delta t(1 - \lambda)W_t}_{\text{Aggregate wealth of newborn agents}} \\ &= (1 + \alpha_1\Delta t)W - \delta\lambda W\Delta t + \text{higher order terms}. \end{aligned} \tag{2.6}$$

Subtracting W from both sides and letting $\Delta t \to 0$, the aggregate wealth evolves according to

$$\mathrm{d}W = (\alpha_1 - \delta\lambda)W\,\mathrm{d}t.$$

Therefore the growth rate of aggregate wealth (and hence all aggregate variables) is

$$g_1 = \alpha_1 - \delta\lambda = \alpha_0 - \varepsilon\delta(1-\lambda) - \delta\lambda, \tag{2.7}$$

where α_0, α_1 are given by (2.4) and (2.5b), respectively.

Since $\varepsilon, \delta > 0$ and $0 \le \lambda \le 1$, we immediately obtain the following result.

Proposition 1. *In perpetual youth models, the growth rate of the economy is unambiguously lower than the benchmark economy with infinitely-lived agents.*

Although this result is well known (for example, with decreasing returns to scale, Blanchard (1985) shows that the steady state capital is lower in perpetual youth models), it is useful to highlight the forces contributing to this result. There are essentially three factors. First, in perpetual youth models, agents become more impatient ($\beta \nearrow \beta + \delta$) and increase the propensity to consume (reduce savings), which decreases growth. Second, in the presence of annuities, agents face a higher effective risk-free rate ($\mu \nearrow \mu + \delta\lambda$), which increases growth. The first effect always dominates the second, since the first is proportional to δ, while the second is proportional to $\delta\lambda$, where $0 \le \lambda \le 1$. These two effects explain the term $-\varepsilon\delta(1-\lambda) \le 0$ in (2.7).[6] Finally, there is a third factor, accidental bequests. As is clear from (2.6), the last term in (2.7) comes from the initial wealth of newborn agents, which is proportional to $\delta(1-\lambda)$. The term $-\delta\lambda \le 0$ can thus be interpreted as the reduction of accidental bequests due to annuities.

Note that while the growth rate decreases in perpetual youth models relative to the benchmark economy with infinitely-lived agents, how the growth rate changes with respect to the annuity participation rate λ is ambiguous. To see this, by (2.7) we obtain

$$g_1 = \alpha_0 - \varepsilon\delta - (1-\varepsilon)\delta\lambda,$$

so increasing annuity participation λ increases (decreases) growth if $\varepsilon > 1$ ($\varepsilon < 1$). With log utility ($\varepsilon = 1$), the annuity participation does not affect economic growth.

2.3 OLG economy with annuities and transfers

Finally, we introduce a government to the model. Suppose that the government gives a lump-sum transfer T_t to all agents that are alive at time t. For now, it does not matter how the transfer is financed. All we need is that the transfer T_t is proportional to the aggregate wealth W_t, which grows at rate $g = g_2$ in equilibrium.

Letting k_t be the capital of a typical agent, the individual budget constraint is

$$\mathrm{d}k_t = (\mu + \delta\lambda)k_t \,\mathrm{d}t - c_t \,\mathrm{d}t + T_t \,\mathrm{d}t. \tag{2.8}$$

Let $r = \mu + \delta\lambda$ be the effective risk-free rate and

$$b_t = \int_0^\infty T_{t+s} \mathrm{e}^{-rs} \,\mathrm{d}s = \int_0^\infty T_t \mathrm{e}^{gs} \mathrm{e}^{-rs} \,\mathrm{d}s = \frac{T_t}{r-g} \tag{2.9}$$

[6] Note that with perfect annuity ($\lambda = 1$), as is often assumed in the literature, these two effects exactly cancel out (Barro and Friedman, 1977).

be the present value of future transfers at time t, which also grows at rate g. b_t can be interpreted as a "government bond", which pays out dividend T_t at time t. Let $w_t = k_t + b_t$ be the effective wealth of an agent. By the individual budget constraint (2.8), we obtain

$$\begin{aligned} \mathrm{d}w_t = \mathrm{d}k_t + \mathrm{d}b_t &= rk_t\,\mathrm{d}t - c_t\,\mathrm{d}t + T_t\,\mathrm{d}t + \mathrm{d}b_t \\ &= rk_t\,\mathrm{d}t - c_t\,\mathrm{d}t + (r-g)b_t\,\mathrm{d}t + gb_t\,\mathrm{d}t \\ &= rw_t\,\mathrm{d}t - c_t\,\mathrm{d}t, \end{aligned} \tag{2.10}$$

where we have used (2.9) and $w_t = k_t + b_t$. Note that this equation is the same as (2.2) except that μ is replaced by $r = \mu + \delta\lambda$. Therefore the individual decision rule is the same as in (2.5a), and the individual growth rate α_2 equals α_1 in (2.5b).

However, the introduction of government transfers does affect aggregate growth. The reason is because the aggregate wealth is not just capital but also contains the government bond (present value of future transfers, b_t). While capital can be pledged to obtain annuities, the government bond cannot, because the transfer T_t is given only to agents that are alive at time t. Essentially, the annuity participation is λ for capital but 0 for the government bond. Thus, the aggregate wealth of newborn agents in (2.6) becomes

$$\delta\Delta t((1-\lambda)K_t + b_t) = \delta\Delta t(1-\lambda\theta)W_t, \tag{2.11}$$

where $\theta = K/W$ is the capital-wealth ratio. Therefore, repeating the argument to derive (2.7), the growth rate of aggregate wealth (and hence all aggregate variables) with transfers is

$$g_2 = \alpha_2 - \delta\lambda\theta = \alpha_0 - \varepsilon\delta(1-\lambda) - \delta\lambda\theta. \tag{2.12}$$

Comparing (2.7) and (2.12), it follows that

$$g_2 - g_1 = \delta\lambda(1-\theta) \geq 0$$

since the government bond is in net positive supply ($b_t \geq 0$ implies $\theta = K/W \leq 1$). In summary, we obtain the following result.

Proposition 2. *In perpetual youth models, other things being equal, the introduction of government transfers increases the aggregate growth rate.*

An immediate corollary is that lump-sum *taxes* in perpetual youth models *decrease* aggregate growth.

The intuition for this somewhat surprising result is as follows. As discussed before, in perpetual youth models with annuities, there are three forces that affect economic growth, (i) increased impatience ($-\varepsilon\delta$ in (2.12)), (ii) increased effective risk-free rate ($+\varepsilon\delta\lambda$ in (2.12)), and (iii) reduction of accidental bequests ($-\delta\lambda\theta$ in (2.12)). The first two effects are in total always negative, and they do not depend on government transfers since the time preference rate ($\beta + \delta$) is exogenous and the effective risk-free rate ($\mu + \delta\lambda$) depends only on the technology and the annuity participation rate, which are exogenous in our setting. However, the government transfer does affect accidental bequests. With government transfers, part of agents' wealth is in the form of government bond, which cannot be pledged for annuities. Therefore, government transfers crowd out the annuity market, which increase accidental bequests and hence economic growth.

3 Example: consumption tax and rebates

In the previous section, we showed that in perpetual youth models with annuities, the introduction of government transfers increases economic growth. However, we did not specify how the transfers were financed but took them as given. In this section, we show in a (closed economy) general equilibrium model that if the transfers are financed by a flat-rate consumption tax, all the results go through.

3.1 Model

Consider the AK model with a continuum of agents as before. The government imposes a flat-rate tax on consumption at rate $\tau \geq 0$. The government uses part of the tax revenue for unproductive purposes, which does not affect the utility of any agent. Only fraction $0 \leq \kappa \leq 1$ of the revenue will remain, and the government redistributes it as lump-sum transfers to all existing agents.

Letting k_t be the capital of a typical agent and T_t be the government transfer to the agents, the individual budget constraint is

$$\mathrm{d}k_t = (\mu + \delta\lambda)k_t \,\mathrm{d}t - (1+\tau)c_t \,\mathrm{d}t + T_t \,\mathrm{d}t. \tag{3.1}$$

Note that (3.1) is identical to (2.8), except that consumption is taxed at rate τ. Let C_t be the aggregate consumption. Since consumption is taxed at rate τ and the government rebates fraction κ of the tax revenue to the agents, the government budget constraint is

$$T_t = \kappa\tau C_t. \tag{3.2}$$

An equilibrium is defined by an individual consumption rule $\{c_t\}$ and an aggregate consumption process $\{C_t\}$ such that (i) given the transfer T_t, agents maximize utility subject to the individual budget constraint (3.1), (ii) the government budget constraint (3.2) holds, and (iii) individual consumption $\{c_t\}$ and aggregate consumption $\{C_t\}$ are consistent.

3.2 Equilibrium

To characterize the equilibrium, we first solve the individual problem. Solving a Merton (1971)-type optimal consumption-saving problem as before, the marginal propensity to consume and the individual growth rate are given by

$$m = \frac{1}{1+\tau}(\varepsilon(\beta+\delta) + (1-\varepsilon)(\mu+\delta\lambda)) = \frac{m_2}{1+\tau}, \tag{3.3a}$$

$$\alpha = \varepsilon(\mu + \delta\lambda - \beta - \delta) = \alpha_2. \tag{3.3b}$$

Note from (3.3a) that in the presence of consumption tax, agents cut consumption proportionally. As a result, consumption expenditures are unchanged, and so is the wealth growth rate (3.3b). As is well-known, consumption tax is neutral to the intertemporal choice. This point is important because it shows that our results are not driven by interventions in the consumption-saving margin.

Next, we characterize the growth rate g of aggregate variables by calculating the tax transfer in two ways. By the optimal consumption rule (3.3a), the aggregate consumption is given by $C_t = \frac{m_2}{1+\tau} W_t$, where

$$m_2 = \varepsilon(\beta+\delta) + (1-\varepsilon)(\mu+\delta\lambda).$$

By the government budget constraint (3.2), the transfer is

$$T_t = \kappa\tau C_t = \frac{\tau}{1+\tau}\kappa m_2 W_t.$$

On the other hand, by the definition of the present value of future transfers (2.9) and the capital-wealth ratio, we have

$$T_t = (r-g)b_t = (r-g)(1-\theta)W_t,$$

where $r = \mu + \delta\lambda$ is the effective risk-free rate and $g = \alpha_2 - \delta\lambda\theta$ is given by (2.12). Since by the individual budget constraint we have $\alpha = r - m_2$, it follows that

$$\begin{aligned} &(r - (r - m_2 - \delta\lambda\theta))(1-\theta) = \frac{\tau}{1+\tau}\kappa m_2 \\ \iff &\delta\lambda\theta(1-\theta) + m_2\left(1 - \frac{\tau}{1+\tau}\kappa - \theta\right) = 0. \end{aligned} \tag{3.4}$$

Since $\delta, \lambda > 0$ are exogenous, the aggregate growth rate g in (2.12) entirely depends on the capital-wealth ratio θ. Furthermore, g is decreasing in θ. Our main result is as follows.

Proposition 3. *Suppose that $\lambda, \kappa > 0$, so agents have access to annuities and the government redistributes at least some of the tax revenue. Assume $m_2 := \varepsilon(\beta+\delta) + (1-\varepsilon)(\mu+\delta\lambda) > 0$, so a solution to the optimal consumption-saving problem exists. Then the followings are true.*

1. *The equilibrium capital-wealth ratio θ is the unique solution in $(0,1)$ to the quadratic equation (3.4).*

2. *The equilibrium capital-wealth ratio θ is decreasing in the tax rate τ; the economic growth rate (2.12) is increasing in the tax rate τ.*

Proof. To show that $0 < \theta < 1$ is unique, let

$$f(x,q) = \delta\lambda x(1-x) + m_2(1 - \kappa q - x),$$

where $q = \frac{\tau}{1+\tau} < 1$. Since $\kappa \le 1$, we have $f(0,q) = m_2(1-\kappa q) > 0$. Since f is quadratic and concave in x, it follows that $f(x,q) = 0$ has one positive solution θ and one negative solution. Since $f(1,q) = -m_2\kappa q < 0$, it follows that $\theta \in (0,1)$ is unique.

Since q is increasing in τ, to show that θ is decreasing in τ, it suffices to show that θ is decreasing in q. Since f is quadratic in x, $f(0,q) > 0$, $f(1,q) < 0$, and $f(\theta,q) = 0$, it follows that $f(x,q)$ changes sign from positive to negative at $x = \theta$. Therefore $\frac{\partial f}{\partial x}(\theta,q) < 0$. Since $\partial f/\partial q = -m_2\kappa < 0$, by the implicit function theorem we have $\mathrm{d}\theta/\mathrm{d}q = -(\partial f/\partial q)/(\partial f/\partial x) < 0$. ☐

The mechanism for this "paradoxical economic growth" is as follows. By Proposition 2, in perpetual youth models with annuities, the introduction of government transfers increases economic growth by crowding out the annuity market and increasing accidental bequests. Since consumption tax does not affect the intertemporal choice, government transfers financed by a consumption tax unambiguously increases economic growth.

Is this paradoxical growth quantitatively important? To answer this question note that since $0 < \theta < 1$ and $\delta, \lambda, m_2 > 0$, in order for (3.4) to hold we need $\theta > 1 - \frac{\tau}{1+\tau}\kappa$. Since from (3.4) we have $\theta \to 1$ as $\tau \to 0$, the range of θ spans an interval of at most length $\frac{\tau}{1+\tau}\kappa$. Since the aggregate growth rate g in (2.12) contains the term $\delta\lambda\theta$, it follows that the economic growth rate increases by no more than

$$\delta \frac{\tau}{1+\tau} \lambda\kappa$$

when the tax rate changes from 0 to τ, independent of the preference parameters. Thus even with overly optimistic estimates of $\delta = 1/50$ (average life expectancy of 50 years), $\tau = 1/4$ (25% consumption tax), $\lambda = 1/2$ (50% participation in the annuity market), and $\kappa = 1$ (100% tax rebate), the upper bound of the growth effect is only 0.2 percentage points. With more realistic numbers like $\delta = 1/80$, $\tau = 1/10$, $\lambda = 1/4$, and $\kappa = 1/4$, the upper bound is 0.007 percentage points, which is quite small.

3.3 Welfare

Finally, in this section we address the welfare implications of the above tax and transfer policy. Because agents receive annuities (sell life insurance), each time an agent dies, his heir inherits only part of the parent's wealth. Therefore dynasties that experienced fewer turnovers become relatively richer, which complicates the welfare analysis. To avoid this issue, in this section we assume that the government imposes a 100% estate tax to all agents and distributes this tax revenue across all newborn agents.[7] By doing so, the initial wealth of a newborn agent depends only on the aggregate wealth at that point and not on the parent's wealth.

Let w_t be the wealth of a typical agent alive at time t. By solving the optimal consumption-saving problem, we can show that the (undiscounted) value function of the agent is

$$U_t = \frac{1}{1 - 1/\varepsilon} \left(\frac{a}{1+\tau} w_t \right)^{1-1/\varepsilon},$$

where

$$a = (\beta + \delta)^{\frac{1}{1-1/\varepsilon}} (\varepsilon(\beta + \delta) + (1 - \varepsilon)(\mu + \delta\lambda))^{\frac{1}{1-\varepsilon}}.$$

Since a depends only on exogenous parameters, the welfare of an agent can be evaluated by $\frac{w_t}{1+\tau}$ in consumption equivalent.

Let K_t, W_t be the aggregate capital and wealth at time t. Without loss of generality, we may normalized $K_0 = 1$. Since $K/W = \theta$, the welfare of agents alive at $t = 0$ is

$$V_0 = \frac{W_0}{1+\tau} = \frac{1}{(1+\tau)\theta}. \tag{3.5}$$

By (2.11), the initial wealth of an agent born at time $t > 0$ is $(1 - \lambda\theta)W_t$. Therefore the welfare of this agent is

$$V_t = \frac{(1 - \lambda\theta)W_t}{1+\tau} = \frac{1 - \lambda\theta}{(1+\tau)\theta} \mathrm{e}^{gt}, \tag{3.6}$$

[7] This assumption is maintained in other papers, for example Hu (1999).

where g is the growth rate of the economy given by g_2 in (2.12). By Proposition 3, the growth rate g is increasing in the consumption tax rate τ. Since e^{gt} grows exponentially in t while $\frac{1-\lambda\theta}{(1+\tau)\theta}$ is constant, it follows that the tax/transfer policy unambiguously benefits agents that are born in the sufficiently distant future. However, the following proposition shows that the initial generation is always hurt from the tax/transfer policy.

Proposition 4. *The denominator in* (3.5)*,* $(1+\tau)\theta$*, is increasing in* τ*, so raising the consumption tax rate decreases the welfare of the initial generation.*

Proof. Let $y = (1+\tau)\theta$. Multiplying (3.4) by $(1+\tau)^2$, y is the solution in $(0, 1+\tau)$ to the quadratic equation

$$F(y,\tau) = \delta\lambda y(1+\tau-y) + m_2(1+\tau)(1+\tau-\tau\kappa-y) = 0. \tag{3.7}$$

As in the proof of Proposition 3, we obtain $\partial F/\partial y < 0$. Differentiating (3.7) with respect to τ, we obtain

$$\begin{aligned}\frac{\partial F}{\partial \tau} &= \delta\lambda y + m_2((1+\tau-\tau\kappa-y) + (1+\tau)(1-\kappa)) \\ &= \delta\lambda y - \delta\lambda y\frac{1+\tau-y}{1+\tau} + m_2(1+\tau)(1-\kappa) \\ &= \frac{\delta\lambda y^2}{1+\tau} + m_2(1+\tau)(1-\kappa) > 0,\end{aligned}$$

where we have used (3.7) in the second equality. By the implicit function theorem, we obtain $\mathrm{d}y/\,\mathrm{d}\tau = -(\partial F/\partial\tau)/(\partial F/\partial y) > 0$. □

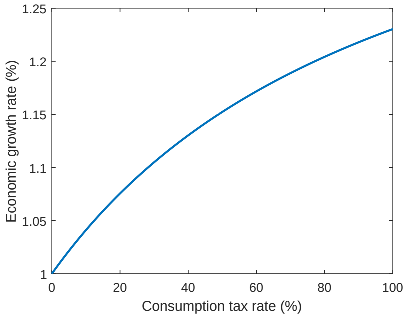
As a numerical example, suppose that $\beta = 0.05$ (5% discounting), $\varepsilon = 0.5$, $\mu = 0.1$ (10% investment return), $\delta = 1/50$ (average lifespan 50 years), $\lambda = 0.5$ (50% participation in annuity), and $\kappa = 0.5$ (50% tax rebate). Figure 1 shows the results when the consumption tax rate changes from 0% to 100%.

Figure 1a shows the aggregate growth rate of the economy. Consistent with Proposition 3, the growth rate is monotonically increasing in the tax rate, though the magnitudes are quite small. Figure 1b shows the welfare criterion (3.5) of the initial generation. Consistent with Proposition 4, the initial generation is hurt by the tax/transfer policy. The welfare loss is quite large: with 20% consumption tax, the initial generation loses about 10% in consumption equivalent. Because the economy grows faster under the tax/transfer policy, the generations in the distant enough future gain in terms of welfare. Figure 1c shows the generation that is indifferent between the tax/transfer policy and no tax, which is computed by equating (3.6) to the value corresponding to $\tau = 0$. With 20% consumption tax, it takes about 40 years to break even.

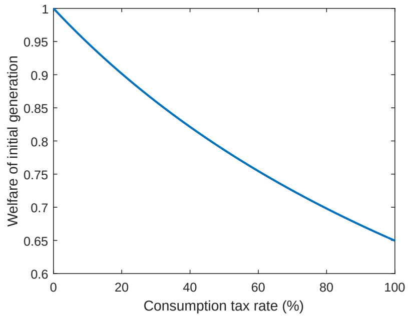
References

S. Rao Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics*, 109(3):659–684, 1994. doi:10.2307/2118417.

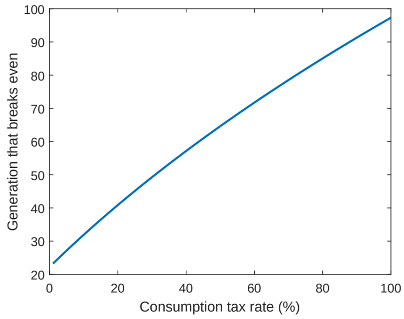
George-Marios Angeletos. Uninsured idiosyncratic investment risk and aggregate saving. *Review of Economic Dynamics*, 10(1):1–30, January 2007. doi:10.1016/j.red.2006.11.001.



(a) Aggregate growth rate.



(b) Welfare of initial generation.



(c) Generation that breaks even.

Figure 1: Growth and welfare implications of consumption tax.

Robert J. Barro. Government spending in a simple model of endogeneous growth. *Journal of Political Economy*, 98(5):103–125, October 1990. doi:10.1086/261726.

Robert J. Barro and James W. Friedman. On uncertain lifetimes. *Journal of Political Economy*, 85(4):843–849, August 1977. doi:10.1086/260603.

Jess Benhabib, Alberto Bisin, and Shenghao Zhu. The distribution of wealth in the Blanchard-Yaari model. *Macroeconomic Dynamics*, 20:466–481, March 2016. doi:10.1017/S1365100514000066.

Olivier J. Blanchard. Debt, deficits, and finite horizons. *Journal of Political Economy*, 93(2):223–247, April 1985. doi:10.1086/261297.

Fwu-Ranq Chang. *Stochastic Optimization in Continuous Time*. Cambridge University Press, 2004.

Darrell Duffie and Larry G. Epstein. Stochastic differential utility. *Econometrica*, 60(2):353–394, March 1992. doi:10.2307/2951600.

William Easterly. How much do distortions affect growth? *Journal of Monetary Economics*, 32(2):187–212, November 1993. doi:10.1016/0304-3932(93)90002-W.

Eric Engen and Jonathan Skinner. Taxation and economic growth. *National Tax Journal*, 49(4):617–642, December 1996.

Xavier Gabaix, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. The dynamics of inequality. *Econometrica*, 84(6):2071–2111, November 2016. doi:10.3982/ECTA13569.

Nicolae Gârleanu and Stavros Panageas. Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing. *Journal of Political Economy*, 123(3):670–685, June 2015. doi:10.1086/680996.

Nicolae Gârleanu, Leonid Kogan, and Stavros Panageas. Displacement risk and asset returns. *Journal of Financial Economics*, 105(3):491–510, September 2012. doi:10.1016/j.jfineco.2012.04.002.

Gary D. Hansen and Selahattin İmrohoroğlu. Consumption over the life cycle: The role of annuities. *Review of Economic Dynamics*, 11(3):566–583, July 2008. doi:10.1016/j.red.2007.12.004.

John William Hatfield. Federalism, taxation, and economic growth. *Journal of Urban Economics*, 87:114–125, May 2015. doi:10.1016/j.jue.2015.01.003.

Ben J. Heijdra and Jenny E. Ligthart. The dynamic macroeconomic effects of tax policy in an overlapping generations model. *Oxford Economic Papers*, 52 (4):677–701, 2000. doi:10.1093/oep/52.4.677.

Ben J. Heijdra and Jochen O. Mierau. Growth effects of consumption and labor-income taxation in an overlapping-generations life-cycle model. *Macroeconomic Dynamics*, 14(S2):151–175, November 2010. doi:10.1017/S1365100510000337.

Ben J. Heijdra and Jochen O. Mierau. The individual life-cycle, annuity market imperfections and economic growth. *Journal of Economic Dynamics and Control*, 36(6):876–890, June 2012. doi:10.1016/j.jedc.2012.01.001.

Ben J. Heijdra, Jochen O. Mierau, and Laurie S. M. Reijnders. A tragedy of annuitization? longevity insurance in general equilibrium. *Macroeconomic Dynamics*, 18(7):1607–1634, October 2014. doi:10.1017/S1365100513000072.

Sheng-Cheng Hu. Economic growth in the perpetual-youth model: Implications of the annuity market and demographics. *Journal of Macroeconomics*, 21(1): 107–124, Winter 1999. doi:10.1016/S0164-0704(99)00093-2.

Toshihiro Ihori. Wealth taxation and economic growth. *Journal of Public Economics*, 79(1):129–148, January 2001. doi:10.1016/S0047-2727(00)00098-0.

Nir Jaimovich and Sergio Rebelo. Nonlinear effects of taxation on growth. *Journal of Political Economy*, 2016. doi:10.1086/689607.

Larry E. Jones and Rodolfo E. Manuelli. Finite lifetime and growth. *Journal of Economic Theory*, 58(2):171–197, December 1992. doi:10.1016/0022-0531(92)90052-J.

Larry E. Jones, Rodolfo E. Manuelli, and Peter Rossi. Optimal taxation in models of endogenous growth. *Journal of Political Economy*, 101(3):485–517, June 1993. doi:10.1086/261884.

Robert G. King and Sergio Rebelo. Public policy and economic growth: Developing neoclassical implications. *Journal of Political Economy*, 98(5):126–150, October 1993. doi:10.1086/261727.

Young Lee and Roger H. Gordon. Tax structure and economic growth. *Journal of Public Economics*, 89(5-6):1027–1043, June 2005. doi:10.1016/j.jpubeco.2004.07.002.

Robert E. Lucas, Jr. On the mechanics of economic development. *Journal of Monetary Economics*, 22(1):3–42, July 1988. doi:10.1016/0304-3932(88)90168-7.

Robert C. Merton. Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory*, 3(4):373–413, December 1971. doi:10.1016/0022-0531(71)90038-X.

Alberto Petrucci. Consumption taxation and endogenous growth in a model with new generations. *International Tax and Public Finance*, 9(5):553–566, September 2002. doi:10.1023/A:1020917620238.

Klaus Prettner and David Canning. Increasing life expectancy and optimal retirement in general equilibrium. *Economic Theory*, 56(1):191–217, May 2014. doi:10.1007/s00199-013-0776-9.

Vincent Raymond Reinhart. Death and taxes: Their implications for endogenous growth. *Economics Letters*, 62(3):339–345, March 1999. doi:10.1016/S0165-1765(98)00250-X.

Nancy L. Stokey and Sergio Rebelo. Growth effects of flat-rate taxes. *Journal of Political Economy*, 103(3):519–550, June 1995. doi:10.1086/261993.

Lars E. O. Svensson. Portfolio choice with non-expected utility in continuous time. *Economics Letters*, 30(4):313–317, October 1989. doi:10.1016/0165-1765(89)90084-0.

Alexis Akira Toda. Incomplete market dynamics and cross-sectional distributions. *Journal of Economic Theory*, 154:310–348, November 2014. doi:10.1016/j.jet.2014.09.015.

Alexis Akira Toda and Kieran Walsh. The double power law in consumption and implications for testing Euler equations. *Journal of Political Economy*, 123(5):1177–1200, October 2015. doi:10.1086/682729.

Harald Uhlig and Noriyuki Yanagawa. Increasing the capital income tax may lead to faster growth. *European Economic Review*, 40(8):1521–1540, November 1996. doi:10.1016/0014-2921(96)00032-3.

Menahem E. Yaari. Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies*, 32(2):137–150, April 1965. doi:10.2307/2296058.