Vertical differentiation in oligopoly and license fees when outside innovator can enter the market: Two-step auction

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Abstract

When an outside innovating firm has a technology to produce a higher quality good than the good produced at present, it can sell licenses of its technology to incumbent firms, or enter the market and at the same time sell licenses, or enter the market without license. We examine the definitions of license fee in such a situation in an oligopoly with three firms under vertical product differentiation, one outside innovating firm and two incumbent firms, considering threat by entry of the innovating firm using a two-step auction. Also we show that in the case of uniform distribution of consumers' taste parameter and zero cost when the quality improvement (the difference between the quality of the high-quality good and the quality of the low-quality good) is small (or large), the two-step auction is (or is not) credible.

Keywords: license; entry; oligopoly; vertical differentiation; two-step auction.

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1. Introduction

In Proposition 4 of Kamien and Tauman (1986) it was argued that in an oligopoly when the number of firms is small (or very large), strategy to enter the market and at the same time license the cost-reducing technology to the incumbent firm (entry with license strategy) is more profitable than strategy to license its technology to the incumbent firm without entering the market (license without entry strategy) for the innovating firm. However, their result depends on their definition of license fee. They defined the license fee in the case of licenses without entry by the difference between the profit of an incumbent firm in that case and its profit before it buys a license without entry of the innovating firm. However, it is inappropriate from the game theoretic view point. If an incumbent firm does not buy a license, the innovating firm may punish the incumbent firm by entering the market. The innovating firm can use such a threat if and only if it is a credible threat. In a duopoly case with one incumbent firm, when the innovating firm does not enter nor sell a license, its profit is zero; on the other hand, when it enters the market without license, its profit is positive. Therefore, threat by entry without license is credible under duopoly, and then even if the innovating firm does not enter the market, the incumbent firm must pay the difference between its profit when it uses the new technology and its profit when the innovating firm enters without license as a license fee.

However, in an oligopoly with more than one incumbent firms, the credibility of threat by entry is a more subtle problem. In this paper we extend the analysis to an oligopolistic situation with three firms, one outside innovating firm and two incumbent firms under vertical product differentiation, and examine the definitions of license fee for producing a higher quality good than the good produced at present considering a two-step auction in the case of licenses without entry[1]. A two-step auction, for example, in the case of a license to one incumbent firm without entry is as follows.

(1) The first step.

The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells a license to one firm at auction *with its entry*.

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when only this firm uses the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

[1] Hattori and Tanaka (2016b) presented an analysis of license and entry choice by an innovating firm in a duopoly under vertical product differentiation.

In the first step each incumbent firm has an incentive to make a bid when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid.

We need the effective minimum bidding price because if the minimum price does not function effectively, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

A two-step auction in the case of licenses to two incumbent firms without entry is similar[2], and at the first step of the auction the incumbent firm has a will to pay the following license fee;

> the difference between its profit when both firms use the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step each incumbent firm has an incentive to make a bid when the other firm makes a bid because if it does not make a bid, the auction proceed to the next step.

Threat by such a two-step auction is credible if and only if the profit of the innovating firm when it enters the market with a license to one firm is larger than its profit when it licenses to one incumbent firm without entering the market.

It is difficult to obtain the complete results under general distribution function of consumers' taste parameter and general cost function. Therefore, we present basic formulation of general case and detailed analysis of the uniform distribution of consumers' taste parameter and zero cost case.

In the next section we present literature review. In Section 3 the model of this paper is described. In Section 4 we consider various equilibria of the oligopoly. In Section 5 we present the license fees under entry with license strategy. In Section 6 we consider a two-step auction and present the definitions of license fees under license without entry strategy. We will show that in the case of uniform distribution and zero cost when the quality improvement (the difference between the quality of the high-quality good and the quality of the low-quality good) is small (or large), the two-step auction is (or is not) credible. In Section 7 we study the optimal strategy for the innovating firm, whether it should enter or not, to how many firms it should sell licenses, in the case of uniform distribution and zero cost, and will show that when the two-step auction is credible, license to two firms without entry strategy is optimal. On the other hand, when it is not credible, entry without license strategy is optimal. Section 9 is a concluding section. In Appendix we present analyses of demand and inverse demand functions.

2. Literature review

Various studies focus on technology adoption or R&D investment in duopoly or oligopoly. Most of them analyze the relation between the technology licensor and licensee. The difference of means of contracts, which comprise royalties, upfront fixed fees, combinations of these two,

[2] Please see Section 6.2.2.

and auctions, are well discussed (Katz and Shapiro (1985)). Kamien and Tauman (2002) showed that outside innovators prefer auctions, but industry incumbents prefer royalty. This topic is discussed by Kabiraj (2004) under the Stackelberg oligopoly; here, the licensor does not have production capacity. Wang and Yang (2004) considered the case when the licensor has production capacity. Sen and Tauman (2007) compared the license system in detail, namely, when the licensor is an outsider and when it is an incumbent firm, using the combination of royalties and fixed fees. However, the existence of production capacity was externally given, and they did not analyze the choice of entry. Therefore, the optimal strategies of outside innovators, who can use the entry as a threat, require more discussion. Regarding the strategies of new entrants to the market, Duchene, Sen and Serfes (2015) focused on future entrants with old technology, and argued that a low license fee can be used to deter the entry of potential entrants. However, the firm with new technology is incumbent, and its choice of entry is not analyzed. Also, Chen (2016) analyzed the model of the endogenous market structure determined by the potential entrant with old technology and showed that the licensor uses the fixed fee and zero royalty in both the incumbent and the outside innovator cases, which are exogenously given. Creane, Chiu and Konishi (2013) examined a firm that can license its production technology to a rival when firms are heterogeneous in production costs, and showed that a complete technology transfer from one firm to another always increases joint profit under weakly concave demand when at least three firms remain in the industry.

A Cournot oligopoly with fixed fee under cost asymmetry was analyzed by La Manna (1993). He showed that if technologies can be replicated perfectly, a lower cost firm always has the incentive to transfer its technology; hence, while a Cournot-Nash equilibrium cannot be fully asymmetric, there exists no non-cooperative Nash equilibrium in pure strategies. On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyzed bargaining between a licensor with no production capacity and oligopolistic firms. Recent research focuses on market structure and technology improvement. Boone (2001) and Matsumura et. al. (2013) found a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) showed that technology adoption may change the market outcome. The social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may result in higher social welfare than Bertrand competition under a differentiated goods market. Hattori and Tanaka (2015) and (2016a) studied the adoption of new technology in Cournot duopoly and Stackelberg duopoly. Rebolledo and Sandonís (2012) presented an analysis of the effectiveness of research and development (R&D) subsidies in an oligopolistic model in the cases of international competition and cooperation in R&D. Hattori and Tanaka (2016b) analyzed problems about product innovation, that is, introduction of higher quality good in a duopoly with vertical product differentiation.

3. The model

Our model of vertical product differentiation is according to Mussa and Rosen (1978), Bonanno and Haworth (1998) and Tanaka (2001). There are three firms, Firms A, B and C. Firm A can produce the high-quality good whose quality is k_H, and Firms B and C produce the low-quality good whose quality is k_L, where $k_H > k_L > 0$. k_H and k_L are fixed. Both of the high-quality and the low-quality goods are produced at the same cost.

At present only Firms B and C produce the low-quality good. Firm A is an outside innovator, and it may sell licenses to use its technology for producing the high-quality good to one or two incumbent firms (Firms B and C), and it can enter the market with the high-quality good. Call Firm A the innovating firm and Firms B and C the incumbent firms.

Firm A has five options.

(1) To enter the market, and license its technology to no incumbent firm.

(2) To enter the market, and license its technology to one incumbent firm.

(3) To enter the market, and license its technology to two incumbent firms.

(4) To license its technology to one incumbent firm, but does not enter the market.

(5) To license its technology to two incumbent firms, but does not enter the market.

The cost function of the goods is $c(\cdot)$, which is twice continuously differentiable.

There is a continuum of consumers with the same income, denoted by y, but different values of the taste parameter θ. Each consumer buys at most one unit of the good. If a consumer with parameter θ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \theta k$. If a consumer does not buy any good, his utility is equal to his income y. The parameter θ is distributed according to a twice continuously differentiable distribution function $\rho = F(\theta)$ in the interval $0 < \theta \leq 1$. We assume that there is no atom. ρ denotes the probability that the taste parameter is smaller than or equal to θ. The size of consumers is normalized as one. The inverse function of $F(\theta)$ is denoted by $G(\rho)$. Note that $G(1) = 1$.

Let p_L and q_L be the price and supply of the good of quality k_L; p_H and q_H be the price and supply of the good of quality k_H; and let q_A, q_B and q_C be the outputs of Firms A, B and C.

4. Equilibria of oligopoly

4.1. Entry without license

Suppose that Firm A (the innovating firm) enters into the market without license to Firm B nor C. Then, Firm A supplies the high-quality good and Firms B and C supply the low-quality good. Let θ_L be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality good. Then,

$$\theta_L = \frac{p_L}{k_L}.$$

Let θ_H be the value of θ for which the corresponding consumer is indifferent between buying the low-quality good and buying the high-quality good. Then

$$\theta_H = \frac{p_H - p_L}{k_H - k_L}.$$

Let $q_H = q_A$ and $q_L = q_B + q_C$. The inverse demand function is described as follows.

(1) When $q_H > 0$ and $q_L > 0$, we have $p_H = (k_H - k_L)G(1 - q_H) + k_L G(1 - q_H - q_L)$ and $p_L = k_L G(1 - q_H - q_L)$.

(2) When $q_H > 0$ and $q_L = 0$, we have $p_H = k_H G(1 - q_H)$ and $p_L = k_L G(1 - q_H)$.

(3) When $q_H = 0$ and $q_L > 0$, we have $p_H = k_H - k_L + k_L G(1 - q_L)$ and $p_L = k_L G(1 - q_L)$.

(4) When $q_H = 0$ and $q_L = 0$, we have $p_H = k_H$ and $p_L = k_L$.

Since $G(1) = 1$, this is a continuously differentiable function with the domain $0 \le q_H \le 1$ and $0 \le q_H \le 1$. For details of derivation of the inverse demand function please see Appendix A.3.

The profits of Firms A, B and C are written as

$$\pi_A = [(k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B - q_C)]q_A - c(q_A),$$

$$\pi_B = k_L G(1 - q_A - q_B - q_C)q_B - c(q_B),$$
$$\pi_C = k_L G(1 - q_A - q_B - q_C)q_B - c(q_C).$$

Uniform distribution and zero cost case

Specifically we assume that $\rho = F(\theta)$ has a uniform distribution. Then, $\rho = \theta, \theta = G(\rho) = \rho$, $F'(\theta) = G'(\rho) = 1$ and $F''(\theta) = G''(\rho) = 0$. Moreover, we assume that the high-quality and the low-quality goods are produced at zero cost. Denote $k_H = t k_L,\ t > 1$. The profits of Firms A, B and C are written as

$$\pi_A = [(k_H - k_L)(1 - q_A) + k_L(1 - q_A - q_B - q_C)]q_A,$$

$$\pi_B = k_L(1 - q_A - q_B - q_C)q_B,\ \pi_C = k_L(1 - q_A - q_B - q_C)q_B.$$

The first order conditions for the firms, the equilibrium prices, the equilibrium outputs and profits of Firms A, B and C are

$$(k_H - k_L)(1 - q_A) + k_L(1 - q_A - q_B - q_C) - k_H q_A = 0,$$

$$k_L(1 - q_A - q_B - q_C) - k_L q_B = 0,\ k_L(1 - q_A - q_B - q_C) - k_L q_C = 0,$$
$$p_H = \frac{k_L(3t - 2)}{2(3t - 1)},\ p_L = \frac{k_L}{2(3t - 1)},\ q_A = \frac{3t - 2}{2(3t - 1)},\ q_B = q_C = \frac{t}{2(3t - 1)},$$
$$\pi_A = \frac{k_L t(3t - 2)^2}{4(3t - 1)^2},\ \pi_B = \pi_C = \frac{k_L t^2}{4(3t - 1)^2}.$$

Denote the equilibrium profits of Firms A, B and C in this case by π_A^{e0}, π_B^{e0} and π_C^{e0}.

4.2. Entry with license to one firm

Suppose that Firm A enters into the market and licenses its technology for producing the high-quality good to one of the incumbent firms. We assume that it is Firm C. Then, Firms A and C produce the high-quality good, and Firm B produces the low-quality good. The inverse demand function is the same as that in the previous case with $q_H = q_A + q_C$ and $q_L = q_B$. Denote the license fee in this case by L^{e1}. The profits of Firms A, B and C are

$$\pi_A = [(k_H - k_L)G(1 - q_A - q_C) + k_L G(1 - q_A - q_B - q_C)]q_A - c(q_A),$$

$$\pi_B = k_L G(1 - q_A - q_B - q_C)q_B - c(q_B),$$

$$\pi_C = [(k_H - k_L)G(1 - q_A - q_C) + k_L G(1 - q_A - q_B - q_C)]q_C - c(q_C) - L^{e1}.$$

Uniform distribution and zero cost case

In the case of uniform distribution and zero cost the profits of Firms A, B and C are written as

$$\pi_A = [(k_H - k_L)(1 - q_A - q_C) + k_L(1 - q_A - q_B - q_C)]q_A,$$

$$\pi_B = k_L(1 - q_A - q_B - q_C)q_B,$$

$$\pi_C = [(k_H - k_L)(1 - q_A - q_C) + k_L(1 - q_A - q_B - q_C)]q_C - L^{e1}.$$

The first order conditions for the firms, the equilibrium prices, the equilibrium outputs and profits of Firms A, B and C are

$$(k_H - k_L)(1 - q_A - q_C) + k_L(1 - q_A - q_B - q_C) - k_H q_A = 0,\ k_L(1 - q_A - q_B - q_C) - k_L q_B = 0,$$

$$(k_H - k_L)(1 - q_A - q_C) + k_L(1 - q_A - q_B - q_C) - k_H q_C = 0,\ p_H = \frac{k_L t(2t-1)}{2(3t-1)},\ p_L = \frac{k_L t}{2(3t-1)},$$

$$q_A = \frac{2t-1}{2(3t-1)},\ q_B = \frac{t}{2(3t-1)},\ q_C = \frac{2t-1}{2(3t-1)},$$

$$\pi_A = \frac{k_L t(2t-1)^2}{4(3t-1)^2},\ \pi_B = \frac{k_L t^2}{4(3t-1)^2},\ \pi_C = \frac{k_L t(2t-1)^2}{4(3t-1)^2} - L^{e1}.$$

Denote the equilibrium profits of Firms A, B and C by π_A^{e1}, π_B^{e1} and π_C^{e1}.

4.3. Entry with licenses to two firms

Suppose that Firm A enters into the market and licenses its technology for producing the high-quality good to both incumbent firms. Then, all firms produce the high-quality good. Let θ_0 be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the high-quality good. Then

$$\theta_0 = \frac{p_H}{k_H}.$$

Let $q_H = q_A + q_B + q_C$. The inverse demand function is described as follows.

(1) When $q_H > 0$, we have $p_H = k_H G(1 - q_H)$.

(2) When $q_H = 0$, we have $p_H = k_H$.

Since $G(1) = 1$, this is a continuously differentiable function. About details for derivation of the inverse demand function please see Appendix A.1.

Denote the license fee in this case by L^{e2}. The profits of the firms are

$$\pi_A = k_H G(1 - q_A - q_B - q_C)q_A - c(q_A),$$

$$\pi_B = k_H G(1 - q_A - q_B - q_C)q_B - c(q_B) - L^{e2},$$
$$\pi_C = k_H G(1 - q_A - q_B - q_C)q_C - c(q_C) - L^{e2}.$$

Uniform distribution and zero cost case

In the case of uniform distribution and zero cost the profits of Firms A, B and C are written as

$$\pi_A = k_H(1 - q_A - q_B - q_C)q_A,$$

$$\pi_B = k_H(1 - q_A - q_B - q_C)q_B - L^{e2},$$
$$\pi_C = k_H(1 - q_A - q_B - q_C)q_C - L^{e2}.$$

The first order conditions for the firms, the equilibrium prices, the equilibrium outputs and profits of Firms A, B and C are

$$k_H(1-q_A-q_B-q_C)-k_H q_A = 0,\ k_H(1-q_A-q_B-q_C)-k_H q_B = 0,\ k_H(1-q_A-q_B-q_C)-k_H q_C = 0,$$

$$p_H = \frac{k_L t}{4},\ q_A = q_B = q_C = \frac{1}{4},$$
$$\pi_A = \frac{k_L t}{16},\ \pi_B = \pi_C = \frac{k_L t}{16} - L^{e2}.$$

Denote the equilibrium profits of Firms A, B and C by π_A^{e2}, π_B^{e2} and π_C^{e2}.

4.4. License to one firm without entry

Suppose that Firm A sells a license of its technology to one of the incumbent firms and does not enter the market. We assume that it is Firm C. Firm B still produces the low-quality good. The inverse demand function is the same as that in the entry without license case with $q_H = q_H$ and $q_L = q_B$. Denote the license fee in this case by L^{l1}. The profits of Firms B and C are

$$\pi_B = k_L G(1 - q_B - q_C)q_B - c(q_B),$$

$$\pi_C = [(k_H - k_L)G(1 - q_C) + k_L G(1 - q_B - q_C)]q_C - c(q_C) - L^{l1}.$$

Uniform distribution and zero cost case

In the case of uniform distribution and zero cost the profits of Firms A, B and C are written as

$$\pi_B = k_L(1 - q_B - q_C)q_B,$$

$$\pi_C = [(k_H - k_L)(1 - q_C) + k_L(1 - q_B - q_C)]q_C - L^{l1}.$$

The first order conditions for the firms, the equilibrium prices, the equilibrium outputs and profits of Firms B and C are

$$k_L(1 - q_B - q_C) - k_L q_B = 0,\ (k_H - k_L)(1 - q_C) + k_L(1 - q_B - q_C) - k_H q_C = 0,$$

$$p_H = \frac{k_L t(2t-1)}{4t-1},\ p_L = \frac{k_L t}{4t-1},\ q_B = \frac{t}{4t-1},\ q_C = \frac{2t-1}{4t-1},$$

$$\pi_B = \frac{k_L t^2}{(4t-1)^2},\ \pi_C = \frac{k_L t(2t-1)^2}{(4t-1)^2} - L^{l1}.$$

Denote the equilibrium profits of Firms B and C by π_B^{l1} and π_C^{l1}.

4.5. Licenses to two firms without entry

Suppose that Firm A sells licenses of its technology to two incumbent firms and does not enter the market. Then, Firms B and C produce the high-quality good. The inverse demand function is the same as that in the entry with licenses to two firms case with $q_H = q_B + q_C$. Denote the license fee in this case by L^{l2}. The profits of the firms are

$$\pi_B = k_H G(1 - q_B - q_C)q_B - c(q_B) - L^{l2},$$

$$\pi_C = k_H G(1 - q_B - q_C)q_C - c(q_C) - L^{l2}.$$

Uniform distribution and zero cost case

In the case of uniform distribution and zero cost the profits of Firms A, B and C are written as

$$\pi_B = k_H(1 - q_B - q_C)q_B - L^{l2},$$

$$\pi_C = k_H(1 - q_B - q_C)q_C - L^{l2}.$$

The first order conditions for the firms, the equilibrium prices, the equilibrium outputs and profits of Firms B and C are

$$k_H(1 - q_B - q_C) - k_H q_B = 0,\ k_H(1 - q_B - q_C) - k_H q_C = 0,$$

$$p_H = \frac{k_L t}{3},\ q_B = q_C = \frac{1}{3},\ \pi_B^{l2} = \pi_C^{l2} = \frac{k_L t}{9} - L^{l2}.$$

Denote the equilibrium profits of Firms B and C by π_B^{l2} and π_C^{l2}.

5. License fees in the cases of licenses with entry

In the cases of licenses with entry the license fees are equal to the usual willingness to pay for the incumbent firms. We follow the arguments by Kamien and Tauman (1986) and Sen and Tauman (2007) about license fees by auction.

5.1. License to one firm

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when only this firm uses the technology for producing the high-quality good with entry of the innovating firm and its profit when only the rival firm buys the license with entry of the innovating firm.

This is because each incumbent firm knows that there will be one licensee regardless of whether or not it buys a license. The incumbent firms B and C have the same willingness to pay, so even when one of them does not make a bid, the rival firm gets the license. The license fee is

$$L^{e1} = (\pi_C^{e1} + L^{e1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{e1} = \pi_B^{e1}$. In the case of uniform distribution and zero cost we have

$$L^{e1} = \frac{k_L t(2t-1)^2}{4(3t-1)^2} - \frac{k_L t^2}{4(3t-1)^2} = \frac{k_L(t-1)t(4t-1)}{4(3t-1)^2}.$$

5.2. Licenses to two firms

The willingness to pay for each incumbent firm in this case is equal to

> the difference between its profit when two firms use the technology for producing the high-quality good with entry of the innovating firm and its profit when only the rival firm buys the license with entry of the innovating firm.

This is because each incumbent firm knows that there will be one licensee when it does not buy a license. In this case there is a minimum bidding price which is equal to the willingness to pay for the incumbents because without the minimum bidding price no firm makes a positive bid. The license fee is

$$L^{e2} = (\pi_C^{e2} + L^{e2}) - \pi_B^{e1}.$$

This means $\pi_C^{e2} = \pi_B^{e1}$. In the case of uniform distribution and zero cost we have

$$L^{e2} = \frac{k_L t}{16} - \frac{k_L t^2}{4(3t-1)^2} = \frac{k_L(t-1)t(9t-1)}{16(3t-1)^2}.$$

6. License fees in the case of licenses without entry: two-step auction

6.1. One-step auction

If the licenses are auctioned off to the incumbent firms by one-step auction, the license fee is determined by the usual willingness to pay for the incumbent firms described in Kamien and Tauman (1986) and Sen and Tauman (2007).

6.1.1. License to one firm

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when only this firm uses the technology for producing the high-quality good without entry of the innovating firm and its profit when only the rival firm buys the license without entry of the innovating firm.

Then, the license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{l1}.$$

This equation means $\pi_C^{l1} = \pi_B^{l1}$. Denote L^{l1} in this case by $\tilde{L}^{l1}$. In the case of uniform distribution and zero cost we have

$$\tilde{L}^{l1} = \frac{k_L t(2t-1)^2}{(4t-1)^2} - \frac{k_L t^2}{(4t-1)^2} = \frac{k_L(t-1)t}{4t-1}.$$

6.1.2. Licenses to two firms

The willingness to pay for each incumbent firm in this case is equal to

> the difference between its profit when two firms use the technology for producing the high-quality good without entry of the innovating firm and its profit when only the rival firm buys the license without entry of the innovating firm.

In this case there is a minimum bidding price which is equal to the willingness to pay for the incumbents. The license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{l1}.$$

This means $\pi_C^{l2} = \pi_B^{l1}$. Denote L^{l2} in this case by $\tilde{L}^{l2}$. In the case of uniform distribution and zero cost we have

$$\tilde{L}^{l2} = \frac{k_L t}{9} - \frac{k_L t^2}{(4t-1)^2} = \frac{k_L(t-1)t(16t-1)}{9(4t-1)^2}.$$

6.2. Two-step auction

We consider a two-step auction for each case.

6.2.1. License to one firm

In this case the two-step auction is practiced as follows.

(1) The first step.

The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells a license to one firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when only this firm uses the technology for producing the high-quality good *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

Then, the license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{l1} = \pi_B^{e1}$. Denote L^{l1} in this case by $\hat{L}^{l1}$. In the case of uniform distribution and zero cost we have

$$\hat{L}^{l1} = \frac{k_L t(2t-1)^2}{(4t-1)^2} - \frac{k_L t^2}{4(3t-1)^2} = \frac{k_L t(144t^4 - 256t^3 + 156t^2 - 41t + 4)}{4(3t-1)^2(4t-1)^2}.$$

In the first step each incumbent firm has an incentive to make a bid with the license fee $\hat{L}^{l1}$ when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid.

We need the effective minimum bidding price $\hat{L}^{l1}$ because the profit of a non-licensee is π_B^{l1} which is larger than π_B^{e1}. If the minimum price does not function effectively, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

6.2.2. Licenses to two firms

We consider the following two-step auction

(1) The first step.

The innovating firm sells licenses to two firms at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below, and both firms make bids. If both firms make bids, they get licenses. If at least one of the firms does not make a bid, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells a license to *one* firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when two firms use the technology for producing the high-quality good *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

The minimum bidding price should be equal to this willingness to pay. Then, the license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{e1}.$$

This means $\pi_C^{l2} = \pi_B^{e1}$. Denote L^{l2} in this case by $\hat{L}^{l2}$. In the case of uniform distribution and zero cost we have

$$\hat{L}^{l2} = \frac{k_L t}{9} - \frac{k_L t^2}{4(3t-1)^2} = \frac{k_L t(36t^2 - 33t + 4)}{36(3t-1)^2}.$$

In the first step each incumbent firm has an incentive to make a bid when the other firm makes a bid because if it does not make a bid, the auction proceeds to the next step.

6.3. Credibility of two-step auction

The innovating firm uses a two-step auction if and only if the threat by the existence of the second step of the auction is credible, and it is credible if and only if the total profit of the innovating firm when it enters the market with a license to one firm is larger than its profit when it does not enter and sells a license to one firm. Therefore, if

$$\pi_A^{e1} + L^{e1} \geq \tilde{L}^{l1},$$

the two-step auction is credible. On the other hand, if

$$\tilde{L}^{l1} > \pi_A^{e1} + L^{e1},$$

the two-step auction is not credible.

Uniform distribution and zero cost case

Comparing π_B^{e1} and π_B^{l1} in the case of uniform distribution and zero cost,

$$\pi_B^{e1} - \pi_B^{l1} = -\frac{k_L t^2(2t-1)(10t-3)}{4(3t-1)^2(4t-1)^2} < 0.$$

Thus, threat by entry with a license to the rival firm is more severe than non-entry with license to the rival firm for the incumbent firms. The total profit of the innovating firm when it enters the market with a license to one firm is

$$\pi_A^{e1} + L^{e1} = \frac{k_L t(8t^2 - 9t + 2)}{4(3t-1)^2}.$$

On the other hand, the profit of the innovating firm when it sells a license to one firm conditional on that it does not enter the market is $\tilde{L}^{l1}$.

Comparing them,

$$\pi_A^{e1} + L^{e1} - \tilde{L}^{l1} = -\frac{k_L t(2t-1)(2t^2 - 7t + 2)}{4(3t-1)^2(4t-1)}.$$

This is positive if $q < \frac{\sqrt{33}+7}{4}$, and is negative if $q > \frac{\sqrt{33}+7}{4}$. Therefore, we obtain the following result.

Proposition 1. *In the case of uniform distribution of consumers' taste parameter and zero cost, if $t \leq \frac{\sqrt{33}+7}{4}$, the two-step auction is credible, and if $q > \frac{\sqrt{33}+7}{4}$, the two-step auction is not credible.*

This means that when the quality improvement (the difference between the quality of the high-quality good and the quality of the low-quality good) is small (or large), the two-step auction is (or is not) credible.

We illustrate the relations among q, $\tilde{L}^{l1}$ and $\pi_A^{e1} + L^{e1}$ in Figure 1.

Comparing $\hat{L}^{l1}$ and $\tilde{L}^{l1}$ yields

$$\hat{L}^{l1} - \tilde{L}^{l1} = \frac{k_L t^2(2t-1)(10t-3)}{4(3t-1)^2(4t-1)^2} > 0.$$

We illustrate the license fee in the case of license to one firm without entry in Figure 2. It is discontinuous at $q = \frac{\sqrt{33}+7}{4}$. Since $\hat{L}^{l1} > \tilde{L}^{l1}$, we can define that the license fee when $q = \frac{\sqrt{33}+7}{4}$ is

$$\frac{k_L t(144t^4 - 256t^3 + 156t^2 - 41t + 4)}{4(3t-1)^2(4t-1)^2} = \hat{L}^{l1}.$$

Comparing $\hat{L}^{l2}$ and $\tilde{L}^{l2}$ yields

$$\hat{L}^{l2} - \tilde{L}^{l2} = \frac{k_L t^2(2t-1)(10t-3)}{4(3t-1)^2(4t-1)^2} > 0.$$

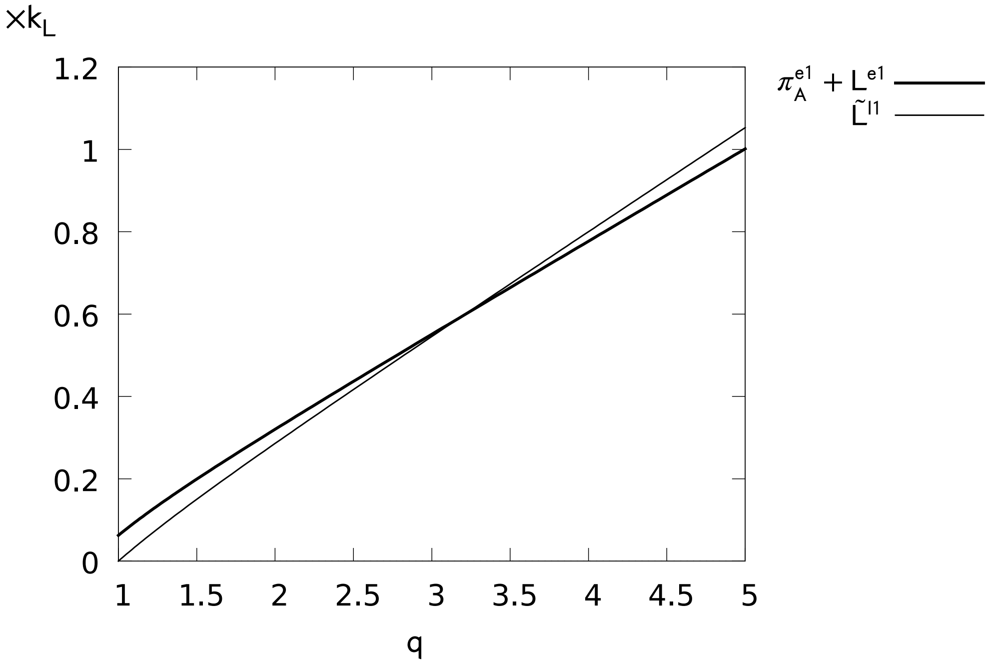

Figure 1: Relations among q, $\pi_A^{e1} + L^{e1}$ and $\tilde{L}^{l1}$

We illustrate the license fee in the case of licenses to two firms without entry in Figure 3. It is also discontinuous at $q = \frac{\sqrt{33}+7}{4}$. Since $\hat{L}^{l2} > \tilde{L}^{l2}$, we can define that the license fee when $q = \frac{\sqrt{33}+7}{4}$ is

$$\frac{k_L t(36t^2 - 33t + 4)}{36(3t-1)^2} = \hat{L}^{l2}.$$

Note that we do not assume any specific value of each variable. Therefore, the results of this section are general for situations of uniform distribution and zero cost.

7. The optimal strategy for the innovator

In this section we examine the optimal strategy for the innovator using the case of uniform distribution and zero cost. It is determined by comparing its payoff in various situations. We consider two cases. One is a case where the two-step auction is credible, and the other is a case where the two-step auction is not credible.

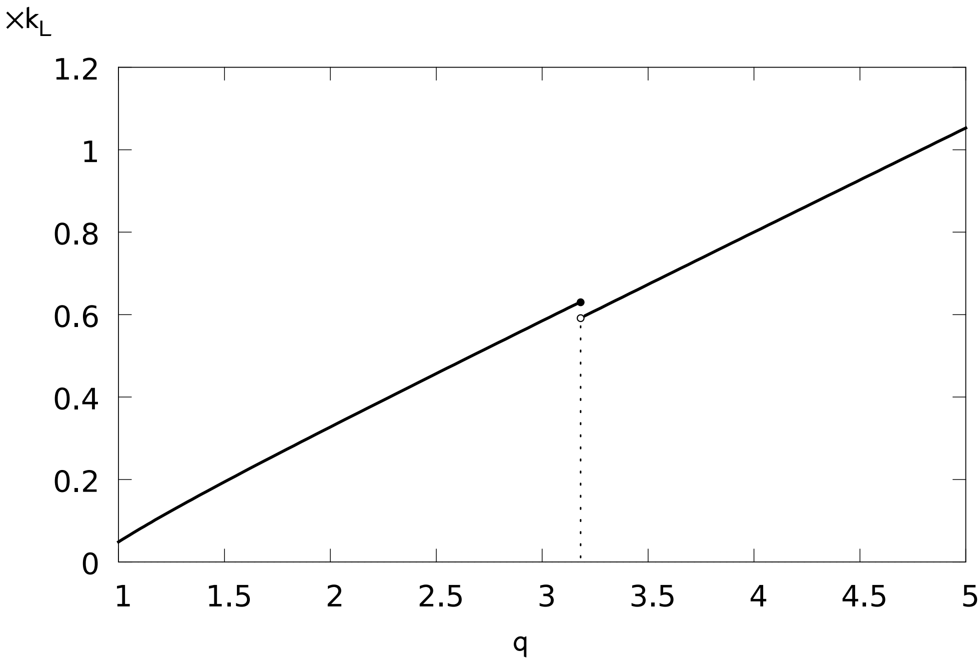

Figure 2: License fee in the case of license to one firm without entry

7.1. Case 1: Two-step auction is credible

When $1 < q \leq \frac{\sqrt{33}+7}{4}$, the two-step auction is credible. Then, we have to compare the following payoffs of the innovator.

$$\pi_A^{e0} : \text{ Entry without license strategy,}$$
$$\hat{L}^{l1} : \text{ License to one firm without entry strategy,}$$
$$2\hat{L}^{l2} : \text{ Licenses to two firms without entry strategy,}$$
$$\pi_A^{e1} + L^{e1} : \text{ Entry with license to one firm strategy,}$$
$$\pi_A^{e2} + 2L^{e2} : \text{ Entry with licenses to two firms strategy.}$$

The values of them other than $\pi_A^{e2} + 2L^{e2}$ are obtained in the previous sections. The total profit of the innovating firm when it enters the market with licenses to two firms is

$$\pi_A^{e2} + 2L^{e2} = \frac{k_L t(27t^2 - 26t + 3)}{16(3t-1)^2}.$$

Please see Figure 4. In this case $2\hat{L}^{l2}$ is the maximum. Thus, license to two firms without entry strategy is optimal for the innovator.

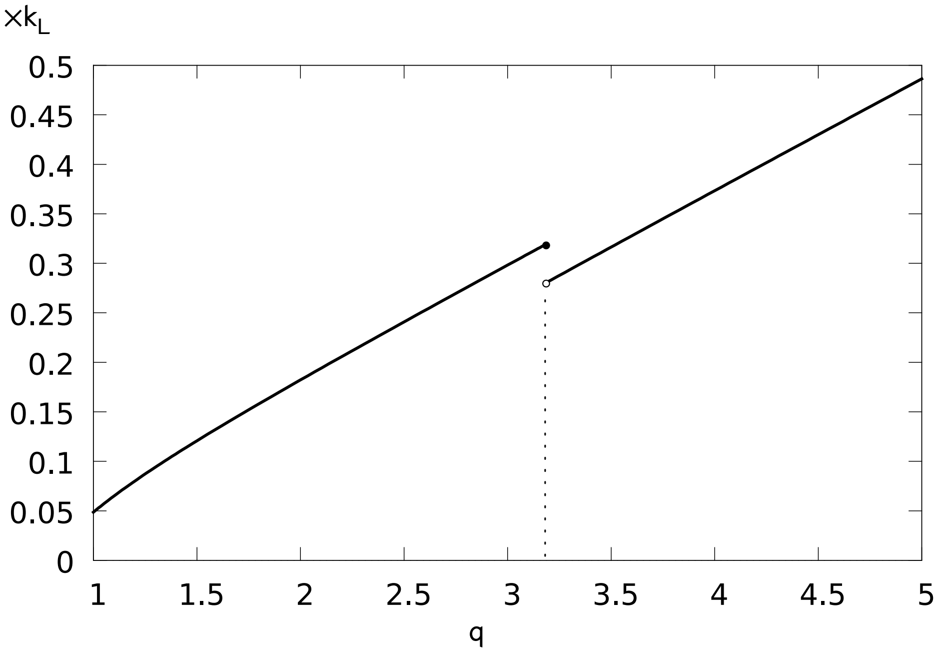

Figure 3: License fee in the case of licenses to two firms without entry

7.2. Case 2: Two-step auction is not credible

When $q > \frac{\sqrt{33}+7}{4}$, the two-step auction is not credible. Then, we have to compare the following payoffs of the innovator.

$$\pi_A^{e0}:\ \text{Entry without license strategy,}$$
$$\tilde{L}^{l1}:\ \text{License to one firm without entry strategy,}$$
$$2\tilde{L}^{l2}:\ \text{Licenses to two firms without entry strategy,}$$
$$\pi_A^{e1}+L^{e1}:\ \text{Entry with license to one firm strategy,}$$
$$\pi_A^{e2}+2L^{e2}:\ \text{Entry with licenses to two firms strategy.}$$

Please see Figure 5. In this case π_A^{e0} is the maximum. Thus, entry without license strategy is optimal for the innovator.

We have shown the following results.

Proposition 2. *In the case of uniform distribution of consumers' taste parameter and zero cost, if the two-step auction is credible, license to two firms without entry strategy is optimal for the innovator, and if the two-step auction is not credible, entry without license strategy is optimal.*

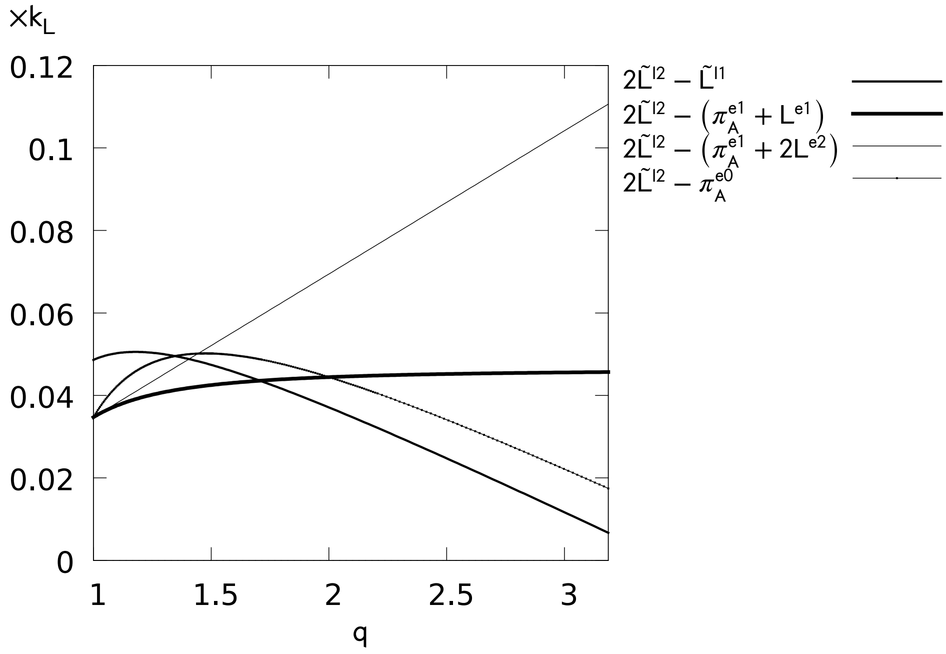

Figure 4: Comparison of payoffs of the innovator: Case 1

8. Concluding remarks and the future research

We have examined the definitions of license fees for the technology to produce a higher quality good than the good produced at present developed by an outside innovator in an oligopoly under vertical product differentiation when the innovator may enter the market with or without licensing. In the future research we will investigate the optimal strategy, to sell licenses to one or two incumbent firms without entry, or to enter the market with or without license, for the innovating firm based on the definitions of license fees presented in this paper under general distribution and cost functions, and we want to extend the analysis to more general oligopolistic setting with $n \geq 3$ incumbent firms.

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Appendix

A. Detailed analysis of demand functions

If a consumer with taste parameter θ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \theta k$. Let θ_0 be the value of θ for which the corresponding consumer is

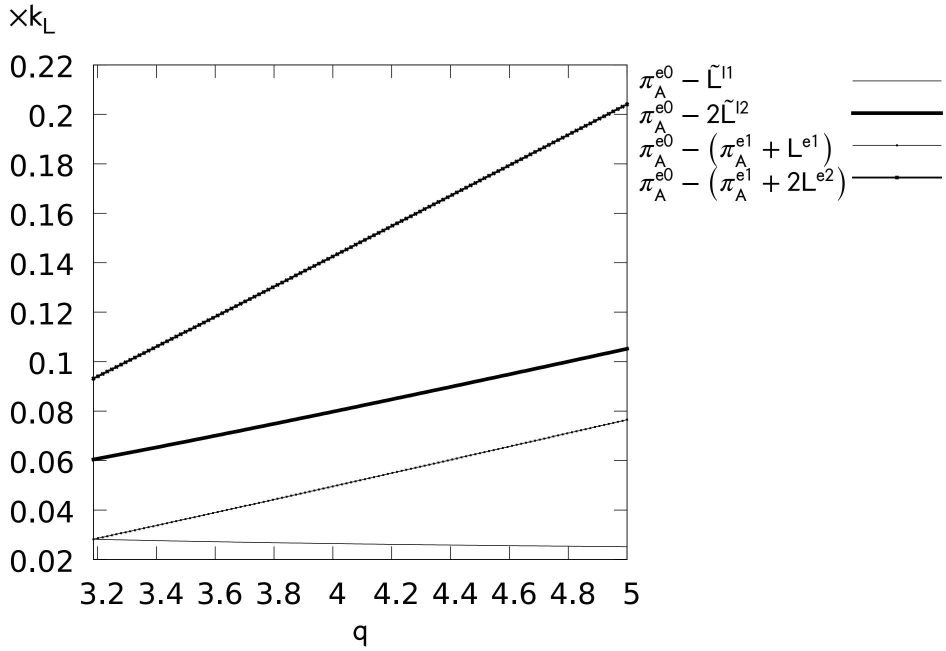

Figure 5: Comparison of payoffs of the innovator: Case 2

indifferent between buying nothing and buying the high-quality good. Then,

$$\theta_0 = \frac{p_H}{k_H}.$$

Let θ_L be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality good. Then,

$$\theta_L = \frac{p_L}{k_L}.$$

Let θ_H be the value of θ for which the corresponding consumer is indifferent between buying the low-quality good and buying the high-quality good. Then

$$\theta_H = \frac{p_H - p_L}{k_H - k_L}.$$

We find

$$\theta_0 = \frac{(k_H - k_L)\theta_H + k_L\theta_L}{k_H}.$$

Therefore, $\theta_L \geq \theta_0 \geq \theta_H$ or $\theta_H > \theta_0 > \theta_L$.

For $\theta > (<)\theta_L$,

$$y - p_L + \theta k_L > (<) y.$$

For $\theta > (<)\theta_0$,

$$y - p_H + \theta k_H > (<) y.$$

For $\theta > (<)\theta_H$,

$$y - p_H + \theta k_H > (<) y - p_L + \theta k_L.$$

A.1. Licenses to two firms without entry

In this case Firms B and C produce the high-quality good. Let q_H be the demand for the high-quality good. Then, we get

(1) When $p_H \geq k_H$ ($\theta_0 \geq 1$), we have $q_H = 0$.

(2) When $p_H < k_H$ ($\theta_0 < 1$), we have $q_H = 1 - F(\theta_0)$.

The inverse demand function is described as follows.

(1) When $q_H > 0$, we have $p_H = k_H G(1 - q_H)$.

(2) When $q_H = 0$, we have $p_H = k_H$.

This is a continuously differentiable function with the domain $0 \leq q_H \leq 1$. We have $q_H = q_B + q_C$.

A.2. Licenses to two firms with entry

In this case all firms produce the high-quality good. Let $q_H = q_A + q_B + q_C$. The inverse demand function is the same as that in Case A.1.

A.3. License to one firm without entry

In this case Firm C produces the high-quality good, and Firm B produces the low-quality good. Let q_H be the demand for the high-quality good and q_L be the demand for the low-quality good. Then, we get[3]

(1) When $p_H \geq k_H$ ($\theta_0 \geq 1$) and $p_L \geq k_L$ ($\theta_L \geq 1$), we have $q_H = 0$ and $q_L = 0$.

(2) When $p_H < k_H$ ($\theta_0 < 1$) and $p_L \geq \frac{p_H}{k_H} k_L$ ($\theta_L \geq \theta_0 \geq \theta_H$), we have $q_H = 1 - F(\theta_0)$ and $q_L = 0$.

(3) When $p_L < k_L$ ($\theta_L < 1$), $p_H > \frac{p_L}{k_L} k_H$ ($\theta_H > \theta_0 > \theta_L$) and $p_H - p_L \geq k_H - k_L$ ($\theta_H \geq 1$), we have $q_H = 0$ and $q_L = 1 - F(\theta_L)$.

(4) When $p_L < k_L$ ($\theta_L < 1$), $p_H > \frac{k_H}{k_L} p_L$ ($\theta_H > \theta_0 > \theta_L$) and $p_H - p_L < k_H - k_L$ ($\theta_H < 1$), we have $q_L = F(\theta_H) - F(\theta_L)$ and $q_H = 1 - F(\theta_H)$.

From this demand function we obtain the inverse demand function as follows.

(1) When $q_H > 0$ and $q_L > 0$, we have $p_H = (k_H - k_L)G(1 - q_H) + k_L G(1 - q_H - q_L)$ and $p_L = k_L G(1 - q_H - q_L)$.

(2) When $q_H > 0$ and $q_L = 0$, we have $p_H = k_H G(1 - q_H)$ and $p_L = k_L G(1 - q_H)$.

[3] We owe this formulation to an anonymous referee.

(3) When $q_H = 0$ and $q_L > 0$, we have $p_H = k_H - k_L + k_L G(1 - q_L)$ and $p_L = k_L G(1 - q_L)$.

(4) When $q_H = 0$ and $q_L = 0$, we have $p_H = k_H$ and $p_L = k_L$.

This is a continuously differentiable function with the domain $0 \le q_H \le 1$ and $0 \le q_L \le 1$. We have $q_H = q_C$ and $q_L = q_B$.

A.4. Entry with license to one firm

In this case Firms A and C produce the high-quality good, and Firm B produces the low-quality good. The inverse demand function is the same as that in Case A.3 with $q_H = q_A + q_C$ and $q_L = q_B$.

A.5. Entry without license

In this case Firm A produces the high-quality good, and Firms B and C produce the low-quality good. The inverse demand function is the same as that in Case A.3 with $q_H = q_A$ and $q_L = q_B + q_C$.

References

Bonanno, G. and Haworth, B., Intensity of competition and the choice between product and process innovations, *International Journal of Industrial Organization*, **16**, pp. 495-510, 1998.

Boone, J. (2001), "Intensity of competition and the incentive to innovate," *International Journal of Industrial Organization*, **19**, pp.705-726.

Chen C.-S. (2016), "Endogenous market structure and technology licensing," *Japanese Economic Review*, forthcoming.

Creane, A., Chiu, Y. K. and Konishi, H. (2013), "Choosing a licensee from heterogeneous rivals," *Games and Economic Behavior*, **82**, pp. 254-268.

Duchene, A., Sen, D. and Serfes, K. (2015), "Patent licensing and entry deterrence: The role of low royalties," *Economica*, **82**, pp. 1324-1348.

Hattori, M. and Tanaka, Y. (2015), "Subsidy or tax policy for new technology adoption in duopoly with quadratic and linear cost functions," *Economics Bulletin*, **35**, pp. 1423-1433.

Hattori, M. and Tanaka, Y. (2016a), "Subsidizing new technology adoption in a Stackelberg Duopoly: cases of substitutes and complements," *Italian Economic Journal*, **2**, pp. 197-215.

Hattori, M. and Tanaka, Y. (2016b), "License or entry with vertical differentiation in duopoly," *Economics and Business Letters*, **5**, pp. 17-29.

Kabiraj, T. (2004), “Patent licensing in a leadership structure,” *The Manchester School*, **72**, pp.188-205.

Kamien, T. and Tauman, Y. (1986), “Fees versus royalties and the private value of a patent,” *Quarterly Journal of Economics*, **101**, pp.471-492.

Kamien, T. and Tauman, Y. (2002), “Patent licensing: the inside story,” *The Manchester School*, **70**, pp.7-15.

Katz, M. and Shapiro, C. (1985), “On the licensing of innovations,” *Rand Journal of Economics*, **16**, pp. 504-520.

La Manna M. (1993), “Asymmetric Oligopoly and Technology Transfers,” *Economic Journal*, **103**, pp.436-443.

Matsumura, T., Matsushima, N. and Cato, S. (2013), “Competitiveness and R&D competition revisited,” *Economic modelling*, **31**, pp.541-547.

Mussa, M. and Rosen, S., “Monopoly and product quality,” *Journal of Economic Theory*, **18**, pp. 130-156, 1978.

Pal, R. (2010), “Technology adoption in a differentiated duopoly: Cournot versus Bertrand,” *Research in Economics*, **64**, pp. 128-136.

Rebolledo, M. and J. Sandonís (2012), “The effectiveness of R&D subsidies,” *Economics of Innovation and New Technology*, **21**, pp. 815-825.

Sen, D. and Tauman, Y. (2007), “General licensing schemes for a cost-reducing Innovation,” *Games and Economic Behavior*, **59**, pp.163-186.

Tanaka, Y. (2001), “Profitability of price and quantity strategies in a duopoly with vertical product differentiation,” *Economic Theory*, **17**, pp.693-700.

Wang, X. H. and Yang, B. Z. (2004), “On technology licensing in a Stackelberg duopoly,” *Australian Economic Papers*, **43**, pp. 448-458.

Watanabe N. and Muto, S. (2008), “Stable profit sharing in a patent licensing game: general bargaining outcomes,” *International Journal of Game Theory*, **37**, pp.505-523.