License fees in oligopoly when outside innovator can enter the market: two-step auction

Hattori, Masahiko and Tanaka, Yasuhito

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Masahiko Hattori*
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.
and
Yasuhito Tanaka†
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.

Abstract

When an outside innovating firm has a cost-reducing technology, it can sell licenses of its technology to incumbent firms, or enter the market and at the same time sell licenses, or enter the market without license. We examine the definitions of license fees in such situations under oligopoly with three firms, one outside innovating firm and two incumbent firms, considering threat by entry of the innovating firm using a two-step auction.

Keywords: license; entry; oligopoly; innovating firm; two-step auction.

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1. Introduction

In Proposition 4 of Kamien and Tauman (1986) it was argued that in an oligopoly when the number of firms is small (or very large), strategy to enter the market and at the same time

*M. Hattori, mhattori@mail.doshisha.ac.jp
†Y. Tanaka, yasuhito@mail.doshisha.ac.jp

license the cost-reducing technology to the incumbent firm (license with entry strategy) is more profitable than strategy to license its technology to the incumbent firm without entering the market (license without entry strategy) for the innovating firm. However, their result depends on their definition of license fee. They defined the license fee in the case of licenses without entry by the difference between the profit of an incumbent firm in that case and its profit before it buys a license without entry of the innovating firm. However, it is inappropriate from the game theoretic view point. If an incumbent firm does not buy a license, the innovating firm may punish the incumbent firm by entering the market. The innovating firm can use such a threat if and only if it is a credible threat. In a duopoly case with one incumbent firm, when the innovating firm does not enter nor sell a license, its profit is zero; on the other hand, when it enters the market without license, its profit is positive. Therefore, threat by entry without license is credible under duopoly, and then even if the innovating firm does not enter the market, the incumbent firm must pay the difference between its profit when it uses the new technology and its profit when the innovating firm enters without license as a license fee. For example, Hattori and Tanaka (2016) presented analyses of license and entry choice by an innovating firm in a duopoly.

However, in an oligopoly with more than one incumbent firms, the credibility of threat by entry is a more subtle problem. In this paper we examine definitions of license fees under oligopoly with three firms, one outside innovating firm and two incumbent firms, considering a two-step auction in the case of licenses without entry. A two-step auction, for example, in the case of a license to one incumbent firm without entry is as follows.

(1) The first step.

The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells a license to one firm at auction *with its entry*.

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when only this firm uses the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step each incumbent firm has an incentive to make a bid when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid.

We need the minimum bidding price because if there is no minimum price, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

A two-step auction in the case of licenses to two incumbent firms without entry is similar[1], and at the first step of the auction the incumbent firm has a will to pay the following license fee;

> the difference between its profit when both firms use the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step each incumbent firm has an incentive to make a bid when the other firm makes a bid because if it does not make a bid, the auction proceed to the next step.

Threat by such a two-step auction is credible if and only if the profit of the innovating firm when it enters the market with a license to one firm is larger than its profit when it licenses to one incumbent firm without entering the market.

The analyses of optimum strategy, to enter with or without license or to license without entry for the innovating firm is the theme of the future research.

In the next section we present literature review. In Section 3 the model of this paper is described. In Section 4 we consider various equilibria of the oligopoly. In Section 5 we present the license fees under the license with entry strategy. In Section 6 we consider a two-step auction and present the definitions of license fees under the license without entry strategy. In Section 7 we present an example. Section 8 is a concluding section. In the appendix we briefly mention the two-step auctions when there are more than two incumbent firms.

2. Literature review

Various studies focus on technology adoption or R&D investment in duopoly or oligopoly. Most of them analyze the relation between the technology licensor and licensee. The difference of means of contracts, which comprise royalties, upfront fixed fees, combinations of these two, and auctions, are well discussed (Katz and Shapiro (1985)). Kamien and Tauman (2002) show that outside innovators prefer auctions, but industry incumbents prefer royalty. This topic is discussed by Kabiraj (2004) under the Stackelberg oligopoly; here, the licensor does not have production capacity. Wang and Yang (2004) consider the case when the licensor has production capacity. Sen and Tauman (2007) compared the license system in detail, namely, when the licensor is an outsider and when it is an incumbent firm, using the combination of royalties and fixed fees. However, the existence of production capacity was externally given, and they did not analyze the choice of entry. Therefore, the optimal strategies of outside innovators, who can use the entry as a threat, require more discussion. Regarding the strategies of new entrants to the market, Duchene, Sen and Serfes (2015) focused on future entrants with old technology, and argued that while a low license fee can be used to deter the entry of potential entrants, the firm with new technology is incumbent, and its choice of entry is not analyzed. Also, Chen (2016) analyzed the model of the endogenous market structure determined by the potential entrant with old technology and showed that the licensor uses the fixed fee and zero royalty in both the incumbent and the outside innovator cases, which are

[1]Please see Section 6.2.2.

exogenously given. Below, we present a brief review of studies that analyzed related topics. A Cournot oligopoly with fixed fee under cost asymmetry was analyzed by La Manna (1993). He showed that if technologies can be replicated perfectly, a lower cost firm always has the incentive to transfer its technology; hence, while a Cournot-Nash equilibrium cannot be fully asymmetric, there exists no non-cooperative Nash equilibrium in pure strategies. On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyzed bargaining between a licensor with no production capacity and oligopolistic firms. Recent research focuses on market structure and technology improvement. Boone (2001) and Matsumura et. al. (2013) found a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) showed that technology adoption may change the market outcome. The social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may result in higher social welfare than Bertrand competition under a differentiated goods market. Hattori and Tanaka (2016), Hattori and Tanaka (2015) studied the adoption of new technology in Cournot duopoly and Stackelberg duopoly. Rebolledo and Sandonís (2012) presented an analysis of the effectiveness of research and development (R&D) subsidies in an oligopolistic model in the cases of international competition and cooperation in R&D. Hattori and Tanaka (2016) analyzed similar problems about product innovation, that is, introduction of higher quality good in a duopoly with vertical product differentiation.

3. The model

There are three firms, Firms A, B and C. At present two of them, Firms B and C, produce a homogeneous good. Firm A, which is an outside firm, has a superior cost-reducing technology and can produce the good at lower cost than Firms B and C. We call Firm A the innovating firm, and Firms B and C the incumbent firms. Firm A have the following five options.

(1) To enter the market without license to incumbent firms.

(2) To enter the market and license its technology to one incumbent firm.

(3) To enter the market and license its technology to two incumbent firms.

(4) To license its technology to one incumbent firm, but not enter the market.

(5) To license its technology to two incumbent firms, but not enter the market.

Let p be the price, x_A, x_B and x_C be the outputs of Firms A, B and C. Then, the inverse demand function of the good is written as follows.

$$p = p(x_A + x_B + x_C), \text{ when Firm A enters,}$$

$$p = p(x_B + x_C), \text{ when Firm A does not enter.}$$

The cost functions of Firms A, B and C are denoted by $c_A(x_A)$, $c_B(x_B)$ and $c_C(x_C)$. $c_B(\cdot)$ and $c_C(\cdot)$ are the same functions without license. If Firm A licenses its technology to two incumbent firms, all cost functions are the same, and if Firm A licenses its technology to one incumbent firm (for example Firm C), then the cost functions of Firms A and C are the same.

4. Equilibria of the oligopoly

4.1. Entry without license case

We suppose that Firm A enters the market without license to incumbent firms. Then, the market becomes a tripoly. The cost function of Firm C is c_B. The profits of Firms A, B and C are written as

$$\pi_A = p(x_A + x_B + x_C)x_A - c_A(x_A),$$
$$\pi_B = p(x_A + x_B + x_C)x_B - c_B(x_B),$$
$$\pi_C = p(x_A + x_B + x_C)x_C - c_B(x_C).$$

We assume Cournot type behavior of the firms. The conditions for profit maximization are

$$p + p'x_A - c'_A = 0,\ p + p'x_B - c'_B = 0,\ p + p'x_C - c'_B = 0.$$

The second order conditions are

$$2p' + p''x_A - c''_A < 0,\ 2p' + p''x_B - c''_B < 0,\ 2p' + p''x_C - c''_B < 0.$$

Hereafter we assume that the second order conditions in each case are satisfied.

Denote the equilibrium profits in this case by π_A^{e0}, π_B^{e0} and π_C^{e0}.

4.2. License to one firm without entry case

Suppose that Firm A licenses its technology to one firm, Firm C, but it does not enter the market. Then, the market is a duopoly. The cost function of Firm C is c_A. The profits of the firms are written as

$$\pi_B = p(x_B + x_C)x_B - c_B(x_B),$$
$$\pi_C = p(x_B + x_C)x_C - c_A(x_C) - L.$$

L denotes the license fee. The conditions for profit maximization are

$$p + p'x_B - c'_B = 0,\ p + p'x_C - c'_A = 0.$$

Denote the equilibrium profits and the license fee in this case by π_B^{l1}, π_C^{l1} and L^{l1}.

4.3. Licenses to two firms without entry case

Suppose that Firm A licenses its technology to two firms, Firms B and C, but it does not enter the market. The cost functions of Firms B and C are c_A. The profits of the firms are written as

$$\pi_B = p(x_B + x_C)x_B - c_A(x_B) - L,$$
$$\pi_C = p(x_B + x_C)x_C - c_A(x_C) - L.$$

L denotes the license fee. The conditions for profit maximization are

$$p + p'x_B - c'_A = 0,\ p + p'x_C - c'_A = 0.$$

Denote the equilibrium profits and the license fee in this case by π_B^{l2}, π_C^{l2} and L^{l2}.

4.4. Entry with license to one firm case

Next suppose that Firm A enters the market and sells a license to one firm, Firm C. The cost function of Firm C is c_A. The profits of Firms A, B and C are written as

$$\pi_A = p(x_A + x_B + x_C)x_A - c_A(x_A),$$
$$\pi_B = p(x_A + x_B + x_C)x_B - c_B(x_B),$$
$$\pi_C = p(x_A + x_B + x_C)x_C - c_A(x_C) - L.$$

L is the license fee. The conditions for profit maximization are

$$p + p'x_A - c'_A = 0,\ p + p'x_B - c'_B = 0,\ p + p'x_C - c'_A = 0.$$

Denote the equilibrium profits and the license fee in this case by π_A^{e1}, π_B^{e1}, π_C^{e1} and L^{e1}.

4.5. Entry with licenses to two firms case

Next suppose that Firm A enters the market and sells licenses to Firms B and C. The cost functions of Firms B and C are c_A. The profits of Firms A, B and C are written as

$$\pi_A = p(x_A + x_B + x_C)x_A - c_A(x_A),$$
$$\pi_B = p(x_A + x_B + x_C)x_B - c_A(x_B) - L,$$
$$\pi_C = p(x_A + x_B + x_C)x_C - c_A(x_C) - L.$$

L is the license fee. The conditions for profit maximization are

$$p + p'x_A - c'_A = 0,\ p + p'x_B - c'_A = 0,\ p + p'x_C - c'_A = 0.$$

Denote the equilibrium profits and the license fee in this case by π_A^{e2}, π_B^{e2}, π_C^{e2} and L^{e2}.

5. License fees in the case of licenses with entry

In the case of licenses with entry the license fees are equal to the usual willingness to pay for the incumbent firms. We follow the arguments by Kamien and Tauman (1986) and Sen and Tauman (2007) about license fees by auction.

5.1. License to one firm

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when only this firm uses the new technology with entry of the innovating firm and its profit when only the rival firm buys the license with entry of the innovating firm.

This is because the incumbent firms know that there will be one licensee regardless of whether or not it buys a license. Then, the license fee is

$$L^{e1} = (\pi_C^{e1} + L^{e1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{e1} = \pi_B^{e1}$.

5.2. Licenses to two firms

The willingness to pay for each incumbent firm in this case is equal to

> the difference between its profit when two firms use the new technology with entry of the innovating firm and its profit when only the rival firm buys the license with entry of the innovating firm.

This is because the incumbent firms know that there will be one licensee when it does not buy a license. In this case there is a minimum bidding price which is equal to the willingness to pay for the incumbents because without the minimum bidding price no firm makes a positive bid. The license fee is

$$L^{e2} = (\pi_C^{e2} + L^{e2}) - \pi_B^{e1}.$$

This means $\pi_C^{e2} = \pi_B^{e1}$.

6. License fees in the case of licenses without entry: two-step auction

6.1. One-step auction

If the licenses are auctioned off to the incumbent firms by one-step auction, the license fee is determined by the usual willingness to pay for the incumbent firms described in Kamien and Tauman (1986) and Sen and Tauman (2007).

6.1.1. License to one firm

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when only this firm uses the new technology without entry of the innovating firm and its profit when only the rival firm buys the license without entry of the innovating firm.

Then, the license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{l1}.$$

This equation means $\pi_C^{l1} = \pi_B^{l1}$. Denote L^{l1} in this case by $\tilde{L}^{l1}$.

6.1.2. Licenses to two firms

The willingness to pay for each incumbent firm in this case is equal to

> the difference between its profit when two firms use the new technology without entry of the innovating firm and its profit when only the rival firm buys the license without entry of the innovating firm.

In this case there is a minimum bidding price which is equal to the willingness to pay for the incumbents. The license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{l1}.$$

This means $\pi_C^{l2} = \pi_B^{l1}$. Denote L^{l2} in this case by $\tilde{L}^{l2}$.

6.2. Two-step auction

We consider a two-step auction for each case.

6.2.1. License to one firm

In this case the two-step auction is practiced as follows.

(1) The first step.

The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells a license to one firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when only this firm uses the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

Then, the license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{l1} = \pi_B^{e1}$. Denote L^{l1} in this case by $\hat{L}^{l1}$.

In the first step each incumbent firm has an incentive to make a bid with the license fee L^{l1} when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid.

We need the minimum bidding price L^{l1} because the profit of a non-licensee is π_B^{l1} which is larger than π_B^{e1}. If there is no minimum price, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

6.2.2. Licenses to two firms

We consider the following two-step auction

(1) The first step.

The innovating firm sells licenses to two firms at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below, and both firms make bids. If both firms make bids, they get licenses. If at least one of the firms does not make a bid, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells a license to one firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when two firms use the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

The minimum bidding price should be equal to this willingness to pay. Then, the license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{e1}.$$

This means $\pi_C^{l2} = \pi_B^{e1}$. Denote L^{l2} in this case by $\hat{L}^{l2}$.

In the first step each incumbent firm has an incentive to make a bid when the other firm makes a bid because if it does not make a bid, the auction proceeds to the next step.

6.3. Credibility of two-step auction

The innovating firm uses a two-step auction if and only if threat by the existence of the second step auction is credible, and it is credible if and only if the total profit of the innovating firm when it enters the market with a license to one firm is larger than its profit when it does not enter and sells a license to one firm. Therefore, if

$$\pi_A^{e1} + L^{e1} > \tilde{L}^{l1},$$

the two-step auction is credible. On the other hand, if

$$\tilde{L}^{l1} > \pi_A^{e1} + L^{e1},$$

the two-step auction is not credible.

7. An example: Linear demand and cost functions

As an example we assume that demand and cost functions are linear. The inverse demand function is written as follows.

$$p = a - (x_A + x_B + x_C)$$

where a is a positive constant. The cost function of Firm A is $c_A x_A$, and the cost functions of Firms B and C are $c_B x_B$ and $c_B x_C$, where $0 < c_A < c_B$. However, if Firm B (or C) buys a license for using the cost-reducing technology of Firm A, its cost function is $c_A x_B$ (or $c_A x_C$). There is no fixed cost. Let

$$a = 2c_B - c_A + t(c_B - c_A),\ t > 0.$$

Then, when $t \geq 1$ the equilibrium values of the profits of the firms in various cases are obtained as follows.

$$\pi_A^{e0} = \frac{(c_B - c_A)^2(t+4)^2}{16},\ \pi_B^{e0} = \pi_C^{e0} = \frac{(c_B - c_A)^2 t^2}{16},$$

$$\pi_A^{e1} = \frac{(c_B - c_A)^2(t+3)^2}{16},\ \pi_B^{e1} = \frac{(c_B - c_A)^2(t-1)^2}{16},\ \pi_C^{e1} = \frac{(c_B - c_A)^2(t+3)^2}{16} - L^{e1},$$

$$\pi_A^{e2} = \frac{(c_B - c_A)^2(t+2)^2}{16},\ \pi_B^{e2} = \pi_C^{e2} = \frac{(c_B - c_A)^2(t+2)^2}{16} - L^{e2},$$

$$\pi_B^{l1} = \frac{(c_B - c_A)^2 t^2}{9},\ \pi_C^{l1} = \frac{(c_B - c_A)^2(t+3)^2}{9} - L^{l1},$$

$$\pi_B^{l2} = \pi_C^{l2} = \frac{(c_B - c_A)^2(t+2)^2}{9} - L^{l2},$$

On the other hand, when $t < 1$, $x_B^{e1} = 0$, and the equilibrium values of the profits of the firms in the case of entry with a license to one firm are

$$\pi_A^{e1} = \frac{(c_B - c_A)^2(t+2)^2}{9},\ \pi_B^{e1} = 0,\ \pi_C^{e1} = \frac{(c_B - c_A)^2(t+2)^2}{9}.$$

The equilibrium profits in other cases are the same as those when $t \geq 1$.

Comparing π_B^{e1} and π_B^{l1}, when $t > 1$

$$\pi_B^{e1} - \pi_B^{l1} = -\frac{(c_B - c_A)^2(t+3)(7t-3)}{144} < 0$$

and when $t < 1$,

$$\pi_B^{e1} - \pi_B^{l1} = -\frac{(c_B - c_A)^2 t^2}{9} < 0.$$

Thus, threat by entry with a license to the rival firm is more severe than non-entry with license to the rival firm for the incumbent firms. The total profit of the innovating firm when it enters the market with a license to one firm is, when $t > 1$

$$\pi_A^{e1} + L^{e1} = \frac{(c_B - c_A)^2(t^2 + 14t + 17)}{16}$$

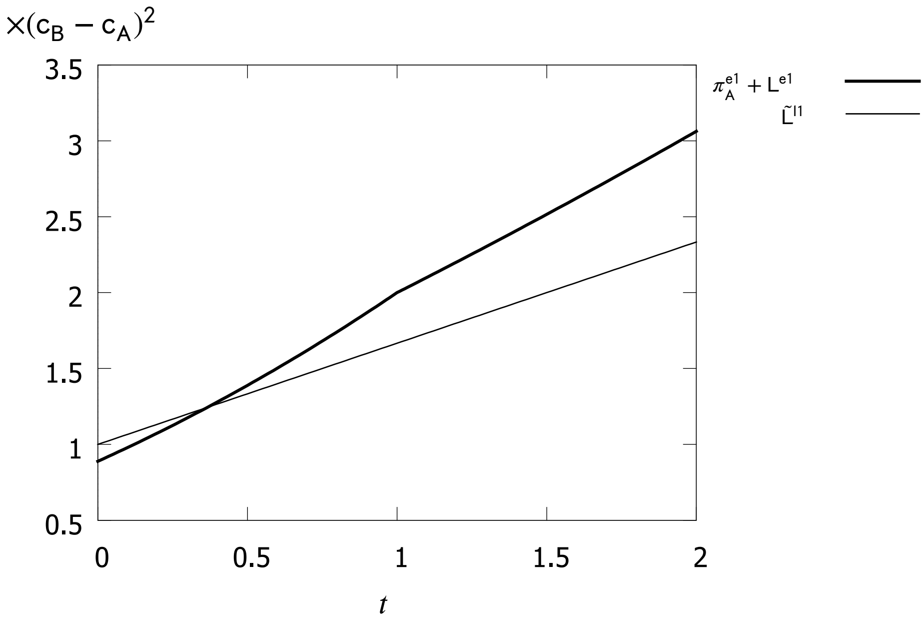

Figure 1: Relations among t, $\pi_A^{e1} + L^{e1}$ and $\tilde{L}^{l1}$

when $t < 1$,

$$\pi_A^{e1} + L^{e1} = \frac{2(c_B - c_A)^2(t+2)^2}{9}.$$

On the other hand, the profit of the innovating firm when it sells a license to one firm conditional on that it does not enter the market is

$$\tilde{L}^{l1} = \frac{(c_B - c_A)^2(2t+3)}{3}.$$

Comparing them, when $t > 1$

$$\pi_A^{e1} + L^{e1} - \tilde{L}^{l1} = \frac{(c_B - c_A)^2(t+3)(3t+1)}{48} > 0$$

and when $t < 1$

$$\pi_A^{e1} + L^{e1} - \tilde{L}^{l1} = \frac{(c_B - c_A)^2(2t^2 + 2t - 1)}{9}.$$

This is positive if $t > \frac{\sqrt{3}-1}{2}$, and is negative if $t < \frac{\sqrt{3}-1}{2}$. Therefore, we obtain the following results about this example.

> If $t > \frac{\sqrt{3}-1}{2}$, the two-step auction is credible, and if $t < \frac{\sqrt{3}-1}{2}$, the two-step auction is not credible.

We illustrate the relations among t, $\tilde{L}^{l1}$ and $\pi_A^{e1} + L^{e1}$ in Fig. 1.

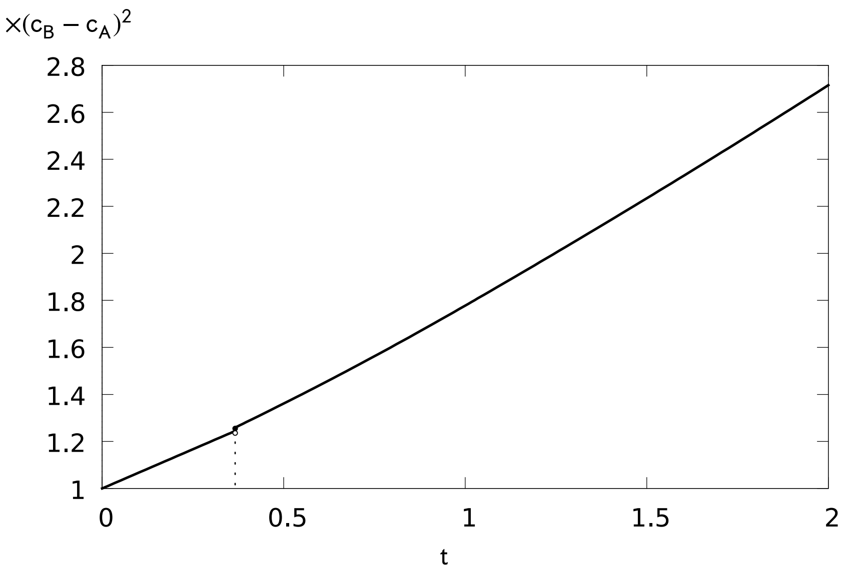

Figure 2: License fee in the case of license to one firm without entry

The license fee in the case of license to one firm without entry when the two-step auction is credible is

$$\hat{L}^{l1} = \frac{(c_B - c_A)^2(t+3)^2}{9}.$$

Comparing $\hat{L}^{l1}$ and $\tilde{L}^{l1}$ yields

$$\hat{L}^{l1} - \tilde{L}^{l1} = \frac{(c_B - c_A)^2 t^2}{9} > 0.$$

We illustrate the license fee in the case of license to one firm without entry in Fig. 2. It is discontinuous at $t = \frac{\sqrt{3}-1}{2}$. Since $\hat{L}^{l1} > \tilde{L}^{l1}$, we can define that the license fee when $t = \frac{\sqrt{3}-1}{2}$ is

$$\frac{(c_B - c_A)^2(t+3)^2}{9} = \hat{L}^{l1}.$$

The license fee in the case of licenses to two firms without entry when the two-step auction is not credible is

$$\tilde{L}^{l2} = \frac{4(c_B - c_A)^2(t+1)}{9}.$$

When the two-step auction is credible, it is

$$\hat{L}^{l2} = \frac{(c_B - c_A)^2(t+2)^2}{9}.$$

Comparing them yields

$$\hat{L}^{l2} - \tilde{L}^{l2} = \frac{(c_B - c_A)^2 t^2}{9} > 0.$$

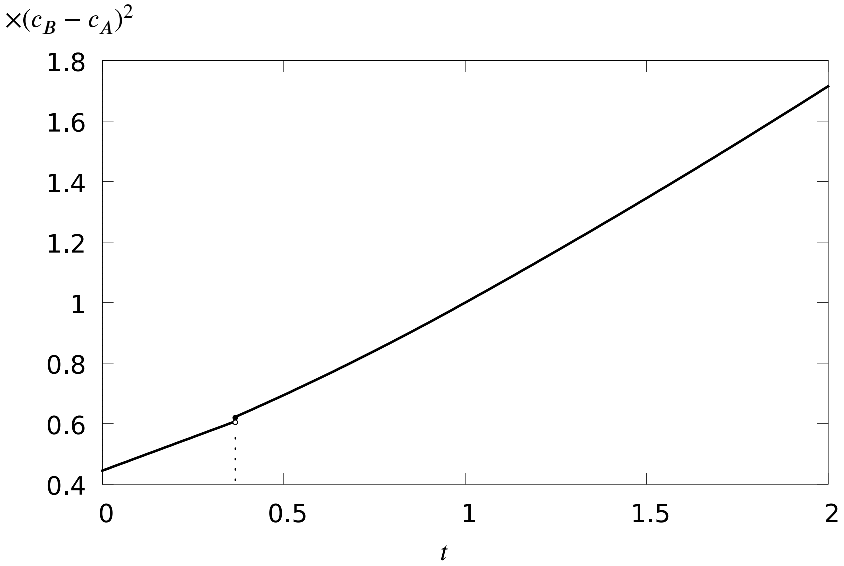

Figure 3: License fee in the case of licenses to two firms without entry

We illustrate the license fee in the case of licenses to two firms without entry in Fig. 3. It is also discontinuous at $t = \frac{\sqrt{3}-1}{2}$. Since $\hat{L}^{l2} > \tilde{L}^{l2}$, we can define that the license fee when $t = \frac{\sqrt{3}-1}{2}$ is

$$\frac{(c_B - c_A)^2(t+2)^2}{9} = \hat{L}^{l2}.$$

8. Concluding remarks and the future research

We have examined the definitions of license fees for new superior technology developed by an outside innovator in an oligopoly when the innovator may enter the market with or without licensing. In the future research we will investigate the optimum strategy, to sell licenses to incumbent firms without entry, or to enter the market with or without license, for the innovating firm based on the definitions of license fees in the various cases presented in this paper, and we want to extend the analysis to more general oligopolistic setting with $n \geq 3$ incumbent firms.

A. A note on the two-step auction with more than two incumbent firms

We briefly mention the two-step auctions when there are $n \geq 3$ incumbent firms without entry of the innovating firm. The equilibrium values of the profits of firms are denoted as follows.

π_i^{lm}	the profit of a licensee when m ($m < n$) firms buy licenses without entry of the innovating firm
π_j^{lm}	the profit of a non-licensee when m ($m < n$) firms buy licenses without entry of the innovating firm
π_i^{ln}	the profit of a licensee when n firms buy licenses without entry of the innovating firm
π_i^{em}	the profit of a licensee when m ($m < n$) firms buy licenses with entry of the innovating firm
π_j^{em}	the profit of a non-licensee when m ($m < n$) firms buy licenses with entry of the innovating firm
π_i^{en}	the profit of a licensee when n firms buy licenses with entry of the innovating firm

L^{lm}	license fee when m ($m < n$) firms buy licenses without entry of the innovating firm
L^{ln}	license fee when n firms buy licenses without entry of the innovating firm
L^{em}	license fee when m ($m < n$) firms buy licenses with entry of the innovating firm
L^{en}	license fee when n firms buy licenses with entry of the innovating firm

First we consider auctions in the case of licenses with entry of the innovating firm.

A.1. Licenses with entry case

A.1.1. Licenses to m ($m < n$) firms

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when m firms including this firm use the new technology with entry of the innovating firm and its profit when m firms other than this firm buy licenses with entry of the innovating firm.

This is because the incumbent firms know that there will be m licensees regardless of whether or not it buys a license. The license fee for each licensee in this case is

$$L^{em} = \pi_i^{em} + L^{em} - \pi_j^{em}.$$

This means $\pi_i^{em} = \pi_j^{em}$.

A.1.2. Licenses to n firms

The willingness to pay for each incumbent firm in this case is equal to the difference between its profit when n firms use the new technology with entry of the innovating firm and its profit when $n-1$ firms other than this firm buy licenses with entry of the innovating firm.

This is because the incumbent firms know that there will be $n-1$ licensees when it does not buy a license. In this case there is a minimum bidding price which is equal to the willingness to pay for the incumbents because without the minimum bidding price no firm makes a positive bid. The license fee for each licensee in this case is

$$L^{en} = \pi_i^{en} + L^{en} - \pi_j^{e(n-1)}.$$

This means $\pi_i^{en} = \pi_j^{e(n-1)}$.

A.2. One-step auction in the licenses without entry case

A.2.1. Licenses to m ($m < n$) firms

Similarly to the above case, the license fee for each licensee in this case is

$$L^{lm} = \pi_i^{lm} + L^{lm} - \pi_j^{lm}.$$

This means $\pi_i^{lm} = \pi_j^{lm}$. Denote this license fee by $\tilde{L}^{lm}$.

A.2.2. Licenses to n firms

The license fee for each licensee in this case is

$$L^{ln} = \pi_i^{ln} + L^{ln} - \pi_j^{l(n-1)}.$$

This means $\pi_i^{ln} = \pi_j^{l(n-1)}$.

A.3. Two-step auction in the licenses without entry case

A.3.1. Licenses to m ($m < n$) firms

(1) The first step.

The innovating firm sells licenses to m firms at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below, and at least m firms make bids. m highest bidders win the licenses, and ties are resolved at random.

If at most $m-1$ firms make bids, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells licenses to m firms at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is the difference between its profit when m firms including this firm get licenses with entry of the innovating firm and its profit when m firms other than this firm get licenses with entry of the innovating firm.

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when m firms including this firm get licenses *without entry* of the innovating firm and its profit when m firms other than this firm get licenses *with entry* of the innovating firm.

There is a minimum bidding price which is equal to the willingness to pay for the incumbent firms. The license fee for each licensee in this case is

$$L^{lm} = \pi_i^{lm} + L^{lm} - \pi_j^{em}.$$

This means $\pi_i^{lm} = \pi_j^{em}$. Denote this license fee by $\hat{L}^{lm}$.

A.3.2. Licenses to n firms

(1) The first step.

The innovating firm sells licenses to n firms at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, and n firms, that is, all firms make bids. If n firms make bids, they get licenses. If at most $n-1$ firms make bids, then the auction proceeds to the next step.

(2) The second step.

The innovating firm sells licenses to n firms at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is the difference between its profit when n firms get licenses with entry of the innovating firm and its profit when $n-1$ firm other than this firm get licenses with entry of the innovating firm.

At the first step of the auction, each incumbent firm has a will to pay the following license fee;

> the difference between its profit when n firms get licenses *without entry* of the innovating firm and its profit when $n-1$ firm other than this firm get licenses *with entry* of the innovating firm.

There is a minimum bidding price which is equal to the willingness to pay for the incumbent firms. The license fee for each licensee in this case is

$$L^{ln} = \pi_i^{ln} + L^{ln} - \pi_j^{e(n-1)}.$$

This means $\pi_i^{ln} = \pi_j^{e(n-1)}$.

Credibility of the two-step auction when there are more than two incumbent firms depends on the comparison of the total profit of the innovating firm when it sells licenses to m ($m \leq n-1$) firms with its entry and its profit when it sells licenses to m ($m \leq n-1$) firms without its entry. Formally, if

$$\pi_i^{em} + L^{em} > \tilde{L}^{lm},$$

the two-step auction is credible, and if

$$\pi_i^{em} + L^{em} < \tilde{L}^{lm},$$

the two-step auction is not credible. Whether the two-step auction is credible or not depends on the number of licensees (m). Thus, it may be complicated.

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