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Abstract

We investigate a choice of options for a foreign innovating firm to license its technology for producing the high quality good to a domestic incumbent firm or to enter the domestic market with or without license under vertical differentiation with convex cost functions. If cost functions are non-linear, the domestic market and the foreign market are not separated, and the results depend on the relative size of those markets. We consider product innovation as quality improvement of goods not process innovation such as cost-reducing. If the size of the foreign market is small, the foreign innovating firm chooses license with entry strategy, and if the foreign market is not small, it chooses license without entry strategy.

Keywords: license with or without entry; convex cost function; vertical differentiation

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1 Introduction

We investigate a choice of options for a foreign innovating firm to license its technology for producing the high quality good to a domestic incumbent firm or to enter the domestic market with or without license under vertical differentiation with convex cost functions. If cost functions are non-linear, the domestic market and the foreign market are not separated, and the results depend on the relative size of those markets. We consider product innovation as quality improvement of goods not process innovation such as cost-reducing.

In Proposition 4 of Kamien and Tauman (1986) under the assumption of linear demand and cost functions with cost-reducing innovation it was argued that in an oligopoly when the number of firms is small (or large), entry with license strategy by the innovating firm, which is a strategy to enter the market and at the same time license its cost-reducing technology to an incumbent firm, is more profitable than license without entry strategy, which is a strategy to license its technology to an incumbent firm without entering the market. However, their result depends on their definition of a license fee in the case where the innovating firm licenses its technology to an incumbent firm and does not enter the market. Interpreting their analyses in a duopoly model, they defined the license fee in the license without entry case by the difference between the profit of an incumbent firm in that case and its monopoly profit before entry and license by the innovating firm. However, we can think that if the negotiation between the innovating firm and an incumbent firm about the license fee breaks down, the innovating firm can enter the market without license to an incumbent firm. If the innovating firm does not enter the market nor license, its profit is zero. However, if it enters the market, its profit is positive. Therefore, such a threat is credible, and hence an incumbent firm must pay the difference between its profit in the license without entry case and its profit in the entry without license case as a license fee. Using an alternative definition of a license fee we can show the result which is converse to the result in Kamien and Tauman (1986) that in a duopoly license without entry strategy is optimal for the innovator. In this paper we will extend this result to a situation of product innovation in an international duopoly.

We assume that the cost functions of Firms A and B are quadratic. They are denoted by $c(x_A + y_A)^2$ and cx_B^2, where c is a positive constant. Since we focus attention to the problem of quality choice, we assume that both low-quality and high quality goods are produced at the same cost. With non-linear cost functions the domestic market and the foreign market are not separated, and the results depend on the relative size of the foreign market to the domestic market. We will show the following results. When the foreign market is small, the foreign innovating firm chooses license with entry strategy, and the foreign market is not small, it chooses license without entry strategy.

In the next section we review some related studies. In Section 3 we present the model. In Section 4 we study the general case, and in Section 5 we investigate the optimal strategies for the foreign innovating firm in the case of uniform distribution of consumers' taste parameter.

2 Literature review

Various studies focus on technology adoption or R&D investment in duopoly or oligopoly. Most of them analyze the relation between the technology licensor and licensee. Sinha (2009) compared the strategy to license with entry by FDI and the strategy to license with entry by export. He showed that each of this two strategy have different optimal combination of fixed fee and royalty. However, the strategy to license without entry is not considered. It is the main target of this paper. The difference of means of contracts, which comprise royalties, upfront fixed fees, combinations of these two, and auctions, are well discussed (Katz and Shapiro (1985)). Kamien and Tauman (2002) showed that outside innovators prefer auctions, but industry incumbents prefer royalty. This topic is discussed by Kabiraj (2004) under the Stackelberg oligopoly; here, the licensor does not have production capacity. Wang and Yang (2004) considered the case when the licensor has production capacity under the Stackelberg duopoly. Sen and Tauman (2007) compared the license system in detail, namely, when the licensor is an outsider and when it is an incumbent firm, using the combination of royalties and fixed fees. However, the existence of production capacity was externally given, and they did not analyze the choice of entry. Therefore, the optimal strategies of outside innovators, who can use the entry as a threat, require more discussion. Regarding the strategies of new entrants to the market, Duchene, Sen and Serfes (2015) focused on future entrants with old technology, and argued that while a low license fee can be used to deter the entry of potential entrants, the firm with new technology is incumbent, and its choice of entry is not analyzed. Also, Chen (2016) analyzed the model of the endogenous market structure determined by the potential entrant with old technology and showed that the licensor uses the fixed fee and zero royalty in both the incumbent and the outside innovator cases, which are exogenously given.

Below, we present a brief review of studies that analyzed related topics. A Cournot oligopoly with fixed fee under cost asymmetry was analyzed by La Manna (1993). He showed that if technologies can be replicated perfectly, a lower cost firm always has the incentive to transfer its technology; hence, while a Cournot-Nash equilibrium cannot be fully asymmetric, there exists no non-cooperative Nash equilibrium in pure strategies. On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyzed bargaining between a licensor with no production capacity and oligopolistic firms. Recent research focuses on market structure and technology improvement. Boone (2001) and Matsumura et. al. (2013) found a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) showed that technology adoption may change the market outcome. The social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may result in higher social welfare than Bertrand competition under a differentiated goods market. Hattori and Tanaka (2014), (2015) studied the adoption of new technology in Cournot duopoly and Stackelberg duopoly. Rebolledo and Sandonís (2012) presented an analysis of the effectiveness of research and development (R&D) subsidies in an oligopolistic model in the cases of international competition and cooperation in R&D. Hattori and Tanaka (2016) analyzed similar problems about product innovation, that is, introduction of higher quality good in a duopoly with vertical product differentiation.

Empirical studies focus on the relationship between multinational corporation's behavior and development country's economic growth. For example, Gould and Gruben (1996) focused on the licensing and estimated the relationship between intellectual property rights protection level and economic growth. Also, Saggi (2002) investigated the spillover caused by FDI and trade, and the relationship between FDI and economic growth of the host country.

3 The model

There are two countries and two firms Firm A in Country A and Firm B in Country B. Call Country A the foreign country and Country B the domestic country; Firm A the foreign firm and Firm B the domestic firm. Our model of vertical product differentiation is according to Mussa and Rosen (1978), Bonanno and Haworth (1998) and Tanaka (2001). At present, Firm B supplies the low-quality good, whose quality is k_L, to the domestic market. Firm A has a technology to produce the high-quality good whose quality is k_H, where $k_H > k_L > 0$, and at present it supplies the high-quality good to the foreign market. k_H and k_L are fixed. Firm A have three options. One option is to enter the domestic market without licensing its technology for producing the high-quality good to Firm B, the second option is to license its technology to Firm B without entering the domestic market, and the third option is to enter the domestic market with license to Firm B. If Firm A enters with license, both firms supply the high-quality good in the domestic market. If Firm A enters without license, it supplies the high-quality good and Firm B supplies the low-quality good in the domestic market. Since the focus of this paper is a choice of entry or license by the foreign innovating firm, we assume that Firm B does not enter the foreign market.

In the domestic market there is a continuum of consumers with the same income, denoted by y, but different values of the taste parameter θ. Each consumer buys at most one unit of the good. If a consumer with parameter θ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \theta k$. If a consumer does not buy the good, his utility is equal to his income y. The parameter θ is distributed according to a smooth distribution function $\rho = F(\theta)$ in the interval $0 < \theta \leq 1$. ρ denotes the probability that the taste parameter is smaller than or equal to θ. The inverse function of $F(\theta)$ is denoted by $G(\rho)$. The size of consumers in the domestic market is normalized as one. The structure of the foreign market is essentially the same as that of the domestic market, and so consumers' taste parameter distributes according to the distribution function $F(\theta)$. However, only the high-quality good is supplied there, and the size of consumers in the foreign market may be different from 1. It is denoted by t, which is a positive number. If $t > 1$, the foreign market is larger than the domestic market, and if $t < 1$, the foreign market is smaller than the domestic market.

Let p_L be the price of the good of quality k_L and p_H be the price of the good of quality k_H; and let x_A and x_B be the supplies of Firms A and B in the domestic market. Let q_H be the price of the high-quality good, and y_A be the supply of Firm A in the foreign market.

Assume that the cost functions of the firms are quadratic. The cost function of Firm A is $c_A(x_A + y_A)^2$ and the cost function of Firm B is $c_B x_B^2$.

4 General analysis

4.1 Firms' behavior

(1) Assume that Firm A enters the domestic market without license to Firm B. Then, Firm A supplies the high-quality good, and Firm B supplies the low-quality good in the domestic market. Let θ_L be the value of θ for which the corresponding consumer in the domestic market is indifferent between buying nothing and buying the low-quality good. Then, $\theta_L = \frac{p_L}{k_L}$. Let θ_H be the value of θ for which the corresponding consumer in the domestic market is indifferent between buying the low-quality good and the high-quality good. Then, $\theta_H = \frac{p_H - p_L}{k_H - k_L}$. We assume $0 < \theta_L < \theta_H < 1$. The direct demand functions of the high-quality and the low-quality goods are

$$x_H = 1 - F\left(\frac{p_H - p_L}{k_H - k_L}\right)$$

and

$$x_L = F\left(\frac{p_H - p_L}{k_H - k_L}\right) - F\left(\frac{p_L}{k_L}\right).$$

x_H and x_L denote the supplies of, respectively, the good of quality k_H and the good of quality k_L in the domestic market. We have $0 < x_L < 1$, $0 < x_H < 1$, $x_A = x_H$ and $x_B = x_L$. The inverse demand functions of the high-quality and the low-quality goods in the domestic market are

$$p_H = (k_H - k_L)G(1 - x_A) + k_L G(1 - x_A - x_B)$$

and

$$p_L = k_L G(1 - x_A - x_B).$$

Similarly, let θ_H^* be the value of θ for which the corresponding consumer in the foreign market is indifferent between buying nothing and buying the high-quality good. Then $\theta_H^* = \frac{q_H}{k_H}$. The direct demand function is $y_H = t\left(1 - F\left(\frac{q_H}{k_H}\right)\right)$. y_H denotes the supply of the good of quality k_H in the foreign market. We have $0 < y_H < t$ and $y_A = y_H$. The inverse demand function of the high quality good in the foreign market is

$$q_H = k_H G\left(1 - \frac{y_A}{t}\right).$$

The profits of Firms A and B are written as

$$\pi_A = [(k_H - k_L)G(1 - x_A) + k_L G(1 - x_A - x_B)]x_A + k_H G\left(1 - \frac{y_A}{t}\right) y_A - c(x_A + y_A)^2$$

and

$$\pi_B = k_L G(1 - x_A - x_B)x_B - c x_B^2.$$

The first order conditions for profit maximization of Firms A and B are

$$(k_H - k_L)G(1 - x_A) + k_L G(1 - x_A - x_B) - (k_H - k_L)G'(1 - x_A)x_A$$
$$- k_L G'(1 - x_A - x_B)x_A - 2c(x_A + y_A) = 0,$$

$$k_H G\left(1-\frac{y_A}{t}\right) - k_H G'\left(1-\frac{y_A}{t}\right)\frac{y_A}{t} - 2c(x_A + y_A) = 0$$

and

$$k_L G(1 - x_A - x_B) - k_L G'(1 - x_A - x_B)x_B - 2cx_B = 0.$$

Denote the profits of Firms A and B in this case by π_A^e and π_B^e.

(2) Assume that Firm A licenses its technology for producing the high-quality good to Firm B without entering the domestic market. Then, only the high-quality good is supplied in both the domestic and the foreign markets. Let θ_H be the value of θ for which the corresponding consumer in the domestic market is indifferent between buying nothing and buying the high-quality good. Then $\theta_H = \frac{p_H}{k_H}$. The direct demand function is $x_H = 1 - F\left(\frac{p_H}{k_H}\right)$. x_H is the supply of the high-quality good in the domestic market. We have $0 < x_H < 1$ and $x_B = x_H$. The inverse demand function of the high quality good in the domestic market is

$$p_H = k_H G(1 - x_B).$$

The profit and the condition for profit maximization of Firm B are

$$\pi_B = k_H G(1 - x_B)x_B - cx_B^2 - L,$$

$$k_H G(1 - x_B) - k_H G'(1 - x_B)x_B - 2cx_B = 0.$$

L is the license fee. The structure of the foreign market is the same as that in the previous case. The profit and the condition for profit maximization of Firm A are

$$\pi_A = k_H G\left(1-\frac{y_A}{t}\right) y_A - cy_A^2 + L,$$

$$k_H G\left(1-\frac{y_A}{t}\right) - k_H G'\left(1-\frac{y_A}{t}\right)\frac{y_A}{t} - 2cy_A = 0.$$

If the negotiation between Firms A and Firm B about the license fee breaks down, Firm A can enter the market without license. Therefore, Firm B must pay the difference between its profit excluding the license fee in this case and its profit in the previous entry without license case as a license fee. The license fee should be equal to

$$L = \pi_B + L - \pi_B^e.$$

This equation means that the license fee is determined so that $\pi_B = \pi_B^e$ is satisfied. Denote the profits of Firms A and B in this case by π_A^l and π_B^l, and denote the license fee by L^l.

(3) Assume that Firm A enters the domestic market and at the same time licenses its technology to Firm B. Then, only the high-quality good is supplied in both the domestic and the foreign markets. The direct demand function in the domestic market is $x_H =$

$1 - F\left(\frac{p_H}{k_H}\right)$. We have $0 < x_H < 1$ and $x_A + x_B = x_H$. The inverse demand function of the high quality good in the domestic market is

$$p_H = k_H G(1 - x_A - x_B).$$

The structure of the foreign market is the same as that in the previous cases. The profits of Firms A and B are

$$\pi_A = k_H G(1 - x_A - x_B)x_A + k_H G\left(1 - \frac{y_A}{t}\right) y_A - c(x_A + y_A)^2 + L$$

and

$$\pi_B = k_H G(1 - x_A - x_B)x_B - c x_B^2 - L.$$

L is the license fee. The conditions for profit maximization of Firms A and B are

$$k_H G(1 - x_A - x_B) - k_H G'(1 - x_A - x_B)x_A - 2c(x_A + y_A) = 0,$$

$$k_H G\left(1 - \frac{y_A}{t}\right) - k_H G'\left(1 - \frac{y_A}{t}\right)\frac{y_A}{t} - 2c(x_A + y_A) = 0$$

and

$$k_H G(1 - x_A - x_B) - k_H G'(1 - x_A - x_B)x_B - 2c x_B = 0.$$

Similarly to the previous case, if the negotiation between Firms A and Firm B about the license fee breaks down, Firm A can enter the market without license. Therefore, Firm B must pay the difference between its profit excluding the license fee in this case and its profit in the previous entry without license case as a license fee. The license fee should be equal to

$$L = \pi_B + L - \pi_B^e.$$

This means $\pi_B = \pi_B^e$. Denote the profits of Firms A and B in this case by π_A^{el} and π_B^{el}, and denote the license fee by L^{el}.

4.2 The optimal strategies

Comparing $\pi_A^l + L^l$, π_A^e and $\pi_A^{el} + L^{el}$, the optimal strategies for Firm A are as follows.

(1) If $\pi_A^l + L^l$ is the maximum, license without entry strategy is optimal.

(2) If π_A^e is the maximum, entry without license strategy is optimal.

(3) If $\pi_A^{el} + L^{el}$ is the maximum, entry with license strategy is optimal.

5 Uniform distribution case

5.1 Three cases

Let us suppose that $\rho = F(\theta)$ has a uniform distribution, then $\rho = \theta$. We consider three cases about the value of t as follows.

(1) $t > \frac{k_H(2k_Hk_L+2ck_H-k_L^2)}{ck_L^2}$.

Then, Firm A never enters the domestic market, and the entry without license case and the entry with license case in the following sub-section do not exist.

(2) $\frac{k_H}{c} + 2 < t \leq \frac{k_H(2k_Hk_L+2ck_H-k_L^2)}{ck_L^2}$.

Then, Firm A does not enter the domestic market when it licenses its technology to Firm B, and the entry with license case does not exist.

We can verify

$$\frac{k_H(2k_Hk_L + 2ck_H - k_L^2)}{ck_L^2} - \left(\frac{k_H}{c} + 2\right) = \frac{2(k_H - k_L)(k_Hk_L + ck_L + ck_H)}{ck_L^2} > 0.$$

(3) $t \leq \frac{k_H}{c} + 2$.

Then, Firm A may enter the domestic market with or without license.

5.2 Firms' behavior

(1) Entry without license case.

Assume that Firm A enters the domestic market without license to Firm B. Then, Firm A supplies the high-quality good, and Firm B supplies the low-quality good in the domestic market. The first order conditions for profit maximization of Firms A and B are

$$(k_H - k_L)(1 - x_A) + k_L(1 - x_A - x_B) - k_Hx_A - 2c(x_A + y_A) = 0,$$

$$k_H\left(1 - \frac{y_A}{t}\right) - k_H\frac{y_A}{t} - 2c(x_A + y_A) = 0$$

and

$$k_L(1 - x_A - x_B) - k_Lx_B - 2cx_B = 0.$$

The equilibrium values of the variables are obtained as follows.

$$p_H = \frac{k_H(2k_Hk_L + 2ck_H - k_L^2)(2ct + k_H + 2c)}{4ck_Hk_Lt + 4c^2k_Ht - k_Hk_L^2 + 4k_H^2k_L + 4ck_Hk_L + 4ck_H^2 + 4c^2k_H - ck_L^2t},$$

$$p_L = \frac{k_Hk_L(k_L + 2c)(2ct + k_H + 2c)}{4ck_Hk_Lt + 4c^2k_Ht - k_Hk_L^2 + 4k_H^2k_L + 4ck_Hk_L + 4ck_H^2 + 4c^2k_H - ck_L^2t},$$

$$x_A = \frac{2k_H^2 k_L + 2ck_H^2 - ck_L^2 t - k_H k_L^2}{4ck_H k_L t + 4c^2 k_H t - k_H k_L^2 + 4k_H^2 k_L + 4ck_H k_L + 4ck_H^2 + 4c^2 k_H - ck_L^2 t},$$

$$x_B = \frac{k_H k_L (2ct + k_H + 2c)}{4ck_H k_L t + 4c^2 k_H t - k_H k_L^2 + 4k_H^2 k_L + 4ck_H k_L + 4ck_H^2 + 4c^2 k_H - ck_L^2 t},$$

$$\pi_A = \frac{k_H A}{4(ck_L^2 t - 4ck_H k_L t - 4c^2 k_H t + k_H k_L^2 - 4k_H^2 k_L - 4ck_H k_L - 4ck_H^2 - 4c^2 k_H)^2}.$$

About the value of A please see Appendix.

$$\pi_B = \frac{k_H^2 k_L^2 (k_L + c)(2ct + k_H + 2c)^2}{(ck_L^2 t - 4ck_H k_L t - 4c^2 k_H t + k_H k_L^2 - 4k_H^2 k_L - 4ck_H k_L - 4ck_H^2 - 4c^2 k_H)^2},$$

$$q_H = \frac{k_H (4k_H k_L + 4ck_H - k_L^2)(2ct + k_H + 2c)}{2(4ck_H k_L t + 4c^2 k_H t - k_H k_L^2 + 4k_H^2 k_L + 4ck_H k_L + 4ck_H^2 + 4c^2 k_H - ck_L^2 t)}$$

and

$$y_A = \frac{(2ck_L^2 + 4k_H^2 k_L + 4ck_H^2 - k_H k_L^2)t}{2(4ck_H k_L t + 4c^2 k_H t - k_H k_L^2 + 4k_H^2 k_L + 4ck_H k_L + 4ck_H^2 + 4c^2 k_H - ck_L^2 t)}.$$

If $t > \frac{k_H(2k_H k_L + 2ck_H - k_L^2)}{ck_L^2}$, $x_A = 0$, that is, Firm A does not enter the domestic market.

(2) License without entry case.

Assume that Firm A licenses its technology for producing the high-quality good to Firm B without entering the domestic market. The first order conditions for profit maximization of Firms A and B are

$$k_H(1 - x_B) - k_H x_B - 2cx_B = 0,$$

$$k_H\left(1 - \frac{y_A}{t}\right) - k_H \frac{y_A}{t} - 2cy_A = 0.$$

The equilibrium values of the variables are obtained as follows.

$$p_H = \frac{k_H(k_H + 2c)}{2(k_H + c)}, \; x_B = \frac{k_H}{2(k_H + c)},$$

$$\pi_B^l = \frac{k_H^2}{4(k_H + c)},$$

$$y_A = \frac{k_H t}{2(ct + k_H)},$$

$$\pi_A^l = \frac{k_H^2 t}{4(ct + k_H)}$$

and

$$q_H = \frac{k_H(2ct + k_H)}{2(ct + k_H)}.$$

The license fee is equal to

$$L^l = \frac{k_H^2 B}{\Delta_B}$$

where

$$\begin{aligned}\Delta_B =& 4(k_H + c)(ck_L^2 t - 4ck_H k_L t - 4c^2 k_H t + k_H k_L^2 - 4k_H^2 k_L - 4ck_H k_L \\ &- 4ck_H^2 - 4c^2 k_H)^2.\end{aligned}$$

About the value of B please see Appendix.

The total profit of Firm A including the license fee is

$$\pi_A^l + L^l = \frac{k_H^2 C}{\Delta_C}$$

where

$$\begin{aligned}\Delta_C =& 4(k_H + c)(ct + k_H)(ck_L^2 t - 4ck_H k_L t - 4c^2 k_H t + k_H k_L^2 - 4k_H^2 k_L \\ &- 4ck_H k_L - 4ck_H^2 - 4c^2 k_H)^2.\end{aligned}$$

About the value of C please see Appendix.

(3) Entry with license case.

Assume that Firm A enters the domestic market and at the same time licenses its technology to Firm B. The first order conditions for profit maximization of Firms A and B are

$$k_H(1 - x_A - x_B) - k_H x_A - 2c(x_A + y_A) = 0,$$
$$k_H \left(1 - \frac{y_A}{t}\right) - k_H \frac{y_A}{t} - 2c(x_A + y_A) = 0$$

and

$$k_H(1 - x_A - x_B) - k_H x_B - 2cx_B = 0.$$

The equilibrium values of the variables are

$$p_H = \frac{k_H(k_H + 2c)(2ct + k_H + 2c)}{3ck_H t + 4c^2 t + 3k_H^2 + 8ck_H + 4c^2},$$

$$x_A = \frac{k_H(k_H + 2c - ct)}{3ck_H t + 4c^2 t + 3k_H^2 + 8ck_H + 4c^2},$$

$$x_B = \frac{k_H(2ct + k_H + 2c)}{3ck_H t + 4c^2 t + 3k_H^2 + 8ck_H + 4c^2},$$

$$y_A = \frac{3k_H(k_H + 2c)t}{2(3ck_H t + 4c^2 t + 3k_H^2 + 8ck_H + 4c^2)},$$

$$\pi_A = \frac{k_H^2 D}{4(3ck_H t + 4c^2 t + 3k_H^2 + 8ck_H + 4c^2)^2}$$

where

$$\begin{aligned} D =& 9ck_H^2 t^2 + 28c^2 k_H t^2 + 16c^3 t^2 + 9k_H^3 t + 40ck_H^2 t + 60c^2 k_H t \\ &+ 32c^3 t + 4k_H^3 + 20ck_H^2 + 32c^2 k_H + 16c^3, \end{aligned}$$

$$\pi_B = \frac{k_H^2 (k_H + c)(2ct + k_H + 2c)^2}{(3ck_H t + 4c^2 t + 3k_H^2 + 8ck_H + 4c^2)^2}$$

and

$$q_H = \frac{k_H (3k_H + 4c)(2ct + k_H + 2c)}{2(3ck_H t + 4c^2 t + 3k_H^2 + 8ck_H + 4c^2)}.$$

The license fee is equal to

$$L^{el} = \frac{k_H^2 (k_L - k_H)(k_H k_L + ck_L + ck_H)(2ct + k_H + 2c)^2 E}{\Delta_E}$$

where

$$\begin{aligned} \Delta_E =& (3ck_H t + 4c^2 t + 3k_H^2 + 8ck_H + 4c^2)^2 (ck_L^2 t - 4ck_H k_L t - 4c^2 k_H t + k_H k_L^2 \\ &- 4k_H^2 k_L - 4ck_H k_L - 4ck_H^2 - 4c^2 k_H)^2 \end{aligned}$$

and

$$\begin{aligned} E =& c^2 k_L^2 t^2 - 16c^2 k_H k_L t^2 - 16c^3 k_L t^2 - 16c^3 k_H t^2 - 16c^4 t^2 + 2ck_H k_L^2 t \\ &- 32ck_H^2 k_L t - 64c^2 k_H k_L t - 32c^3 k_L t - 32c^2 k_H^2 t - 64c^3 k_H t - 32c^4 t \\ &+ k_H^2 k_L^2 - 16k_H^3 k_L - 48ck_H^2 k_L - 48c^2 k_H k_L - 16c^3 k_L - 16ck_H^3 \\ &- 48c^2 k_H^2 - 48c^3 k_H - 16c^4. \end{aligned}$$

The total profit of Firm A including the license fee is

$$\pi_A^{el} + L^{el} = \frac{k_H^2 F}{\Delta_F}$$

where

$$\begin{aligned} \Delta_F =& 4(3ck_H t + 4c^2 t + 3k_H^2 + 8ck_H + 4c^2)^2 (ck_L^2 t - 4ck_H k_L t - 4c^2 k_H t \\ &+ k_H k_L^2 - 4k_H^2 k_L - 4ck_H k_L - 4ck_H^2 - 4c^2 k_H)^2. \end{aligned}$$

About the value of F please see Appendix.

If $t > \frac{k_H + 2}{c}$, $x_A = 0$, that is, Firm A does not enter the market when it licenses its technology to Firm B.

5.3 The optimal strategies

We explore the optimal strategies of Firm A in each case of the value of t.

(1) Assume $t > \frac{k_H(2k_Hk_L+2ck_H-k_L^2)}{ck_L^2}$. Then, Firm A never enters the domestic market. Thus, its optimal strategy is to license without entry.

(2) Assume $\frac{k_H}{c}+2 < t \leq \frac{k_H(2k_Hk_L+2ck_H-k_L^2)}{ck_L^2}$. Firm A does not enter the domestic market when it licenses its technology to Firm B. Comparing the profit of Firm A when it licenses its technology to Firm B without entry and its profit when it enters the market without license,

$$\pi_A^l + L^l - \pi_A^e = \frac{k_H G}{\Delta_G}$$

where

$$\begin{aligned}\Delta_G =&4(k_H+c)(ct+k_H)(ck_L^2t-4ck_Hk_Lt-4c^2k_Ht+k_Hk_L^2-4k_H^2k_L\\&-4ck_Hk_L-4ck_H^2-4c^2k_H)^2\end{aligned}$$

and

$$\begin{aligned}G =&3c^3k_Hk_L^4t^3+4c^4k_L^4t^3+24c^3k_H^2k_L^3t^3+16c^4k_Hk_L^3t^3-16c^3k_H^3k_L^2t^3\\&+24c^4k_H^2k_L^2t^3+16c^5k_Hk_L^2t^3-32c^4k_H^3k_Lt^3-16c^5k_H^3t^3+9c^2k_H^2k_L^4t^2\\&+16c^3k_Hk_L^4t^2+4c^4k_L^4t^2+40c^2k_H^3k_L^3t^2+56c^3k_H^2k_L^3t^2+32c^4k_Hk_L^3t^2\\&-48c^2k_H^4k_L^2t^2+8c^3k_H^3k_L^2t^2+56c^4k_H^2k_L^2t^2+32c^5k_Hk_L^2t^2-96c^3k_H^4k_Lt^2\\&-64c^4k_H^3k_Lt^2-48c^4k_H^4t^2-32c^5k_H^3t^2+9ck_H^3k_L^4t+20c^2k_H^2k_L^4t+8c^3k_Hk_L^4t\\&+12ck_H^4k_L^3t+36c^2k_H^3k_L^3t+48c^3k_H^2k_L^3t+16c^4k_Hk_L^3t-32ck_H^5k_L^2t-36c^2k_H^4k_L^2t\\&+20c^3k_H^3k_L^2t+48c^4k_H^2k_L^2t+16c^5k_Hk_L^2t-64c^2k_H^5k_Lt-96c^3k_H^4k_Lt-32c^4k_H^3k_Lt\\&-32c^3k_H^5t-48c^4k_H^4t-16c^5k_H^3t+3k_H^4k_L^4+8ck_H^3k_L^4+4c^2k_H^2k_L^4-4k_H^5k_L^3\\&-4ck_H^4k_L^3+16c^2k_H^3k_L^3+16c^3k_H^2k_L^3-4ck_H^5k_L^2-4c^2k_H^4k_L^2+16c^3k_H^3k_L^2+16c^4k_H^2k_L^2.\end{aligned}$$

This is positive for reasonable values of variables if $t > \frac{k_H}{c}+2$. In Figure 1 we depict an example of this case assuming $k_H = 10$ and $k_L = c = 5$. Then, $\frac{k_H}{c}+2 = 4$ and $\frac{k_H(2k_Hk_L+2ck_H-k_L^2)}{ck_L^2} = 14$.

Some discussion When Firm A licenses its technology to Firm B, the domestic market becomes a monopoly in which Firm B produces the good at lower cost. Then, π_B^l is larger than π_B^e plus the profit of Firm A in the domestic market when it enters. The license fee in the case of license without entry is $\pi_B^l - \pi_B^e$ which is larger than the profit

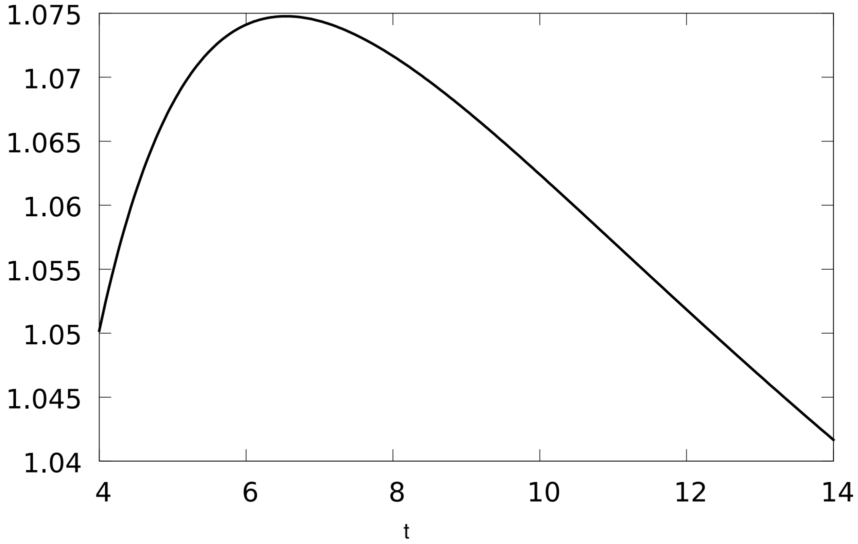

Figure 1: Illustration of $\pi_A^l + L^l - \pi_A^e$

of Firm A in the domestic market. Then, the total profit of Firm A when it licenses its technology to Firm B without entry should be larger than the total profit when it enters the domestic market without license, and license without entry strategy is optimal for Firm A.

(3) Assume $t \leq \frac{k_H}{c} + 2$. Let us compare the profit of Firm A when it licenses its technology to Firm B with entry and its profit when it enters the market without license. Then,

$$\pi_A^{el} + L^{el} - \pi_A^e = \frac{k_H(k_H - k_L)(k_H k_L + ck_L + ck_H)(2ct + k_H + 2c)H}{\Delta_H}$$

where

$$\begin{aligned}\Delta_H =&(3ck_Ht + 4c^2t + 3k_H^2 + 8ck_H + 4c^2)^2(ck_L^2t - 4ck_Hk_Lt - 4c^2k_Ht + k_Hk_L^2 - 4k_H^2k_L \\ &- 4ck_Hk_L - 4ck_H^2 - 4c^2k_H)^2\end{aligned}$$

and

$$\begin{aligned}H =&2c^3k_Hk_L^2t^3 + 8c^4k_L^2t^3 + 40c^3k_H^2k_Lt^3 + 32c^4k_Hk_Lt^3 + 40c^4k_H^2t^3 + 32c^5k_Ht^3 \\ &+ 11c^2k_H^2k_L^2t^2 + 38c^3k_Hk_L^2t^2 + 16c^4k_L^2t^2 + 76c^2k_H^3k_Lt^2 + 160c^3k_H^2k_Lt^2 \\ &+ 96c^4k_Hk_Lt^2 + 76c^3k_H^3t^2 + 160c^4k_H^2t^2 + 96c^5k_Ht^2 + 16ck_H^3k_L^2t + 56c^2k_H^2k_L^2t \\ &+ 44c^3k_Hk_L^2t + 8c^4k_L^2t + 32ck_H^4k_Lt + 132c^2k_H^3k_Lt + 200c^3k_H^2k_Lt + 96c^4k_Hk_Lt \\ &+ 32c^2k_H^4t + 132c^3k_H^3t + 200c^4k_H^2t + 96c^5k_Ht + 7k_H^4k_L^2 + 26ck_H^3k_L^2 + 28c^2k_H^2k_L^2 \\ &+ 8c^3k_Hk_L^2 - 4k_H^5k_L + 4ck_H^4k_L + 56c^2k_H^3k_L + 80c^3k_H^2k_L + 32c^4k_Hk_L - 4ck_H^5 \\ &+ 4c^2k_H^4 + 56c^3k_H^3 + 80c^4k_H^2 + 32c^5k_H.\end{aligned}$$

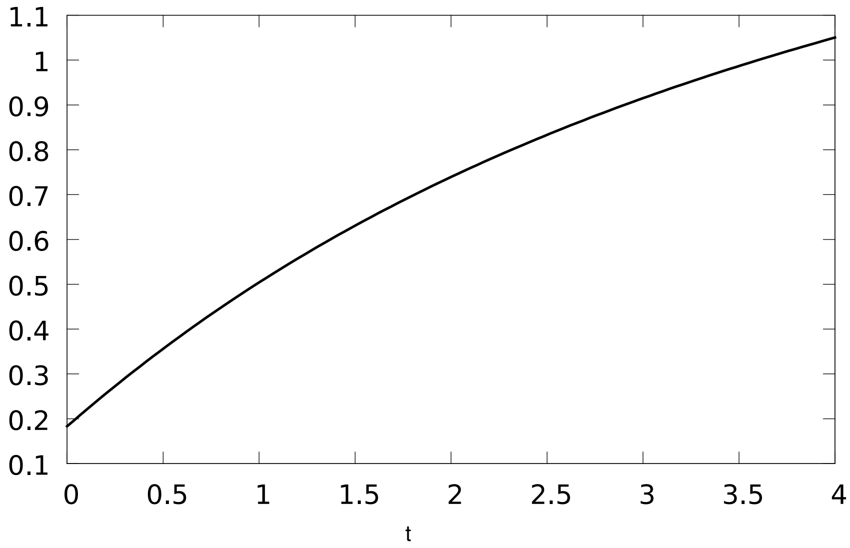

Figure 2: Illustration of $\pi_A^{el} + L^{el} - \pi_A^e$

This is positive for reasonable values of the variables. Thus, $\pi_A^{el} + L^{el} > \pi_A^e$, and entry only (entry without license) strategy is never the optimal strategy for Firm A.

In Figure 2 we depict an example of $\pi_A^{el} + L^{el} - \pi_A^e$ assuming $k_H = 10, k_L = 5$ and $c = 5$.

Let us compare the profit of Firm A when it licenses its technology to Firm B without entry and its profit when it enters the market with license to Firm B. Then,

$$\pi_A^{el} + L^{el} - (\pi_A^l + L^l) = \frac{k_H^3(ct - k_H - 2c)I}{4(k_H + c)(ct + k_H)(3ck_Ht + 4c^2t + 3k_H^2 + 8ck_H + 4c^2)^2}$$

where

$$I = 11c^2k_Ht^2 + 12c^3t^2 + 12ck_H^2t + 18c^2k_Ht + 4c^3t + k_H^3 - 2ck_H^2 - 12c^2k_H - 8c^3.$$

Solving $\pi_A^l = \pi_A^{el}$, we get the following solution

$$t^* = \frac{(k_H + c)\sqrt{25k_H^2 + 68ck_H + 100c^2} - 6k_H^2 - 9ck_H - 2c^2}{11ck_H + 12c^2}.$$

This is the threshold value of the relative size of the foreign market to the domestic market. If $t < t^*$, $\pi_A^l + L^l < \pi_A^{el} + L^{el}$, and if $t > t^*$, $\pi_A^l + L^l > \pi_A^{el} + L^{el}$. Thus, if the foreign market is small relatively to the domestic market, license with entry strategy is optimal for Firm A, and if the foreign market is not small relatively to the domestic market, license without entry (license only) strategy is optimal. In Figure 3 we depict an example of $\pi_A^{el} + L^{el} - (\pi_A^l + L^l)$ assuming $k_H = 10, k_L = 5$ and $c = 5$.

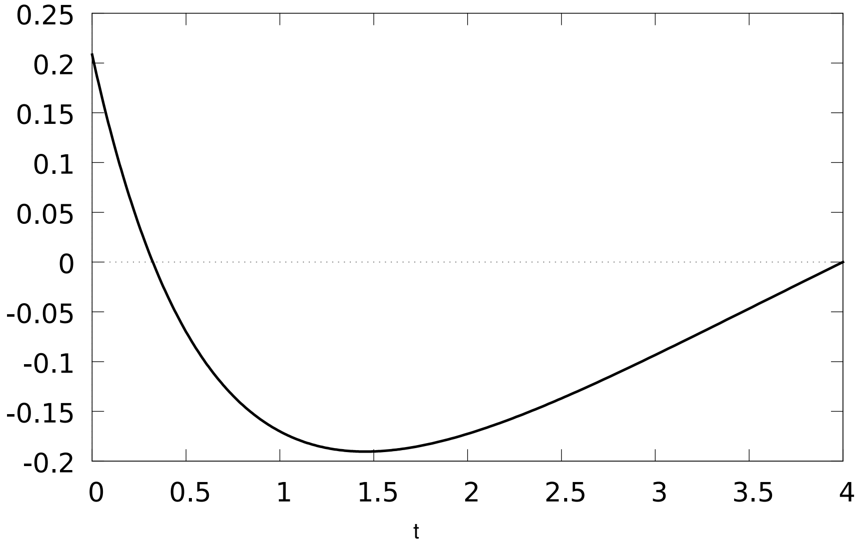

Figure 3: Illustration of $\pi_A^{el} + L^{el} - (\pi_A^l + L^l)$

When $k_H \geq 10(\sqrt{2}+1)$, $t^* \leq 0$. Therefore, if $k_H \geq 10(\sqrt{2}+1)$, there is no positive t^*, and license without entry strategy is optimal. In Figure 4 we depict the relation between k_H and t^* assuming $c = 5$.

Some discussion Let us provide some intuition behind this result. If the foreign innovating firm chooses license without entry strategy, the domestic market becomes a monopoly, and the incumbent firm gets the large profit. Then, the license fee is large, and the larger the magnitude of the innovation (k_H is large) is, the larger the license fee is. If the foreign market is not small relatively to the domestic market, the profit of the foreign firm when it enters the domestic market as a duopolist is small relatively to the profit it earns in the foreign market as a monopolist. Therefore, if the size of the foreign market is not small, the foreign innovating firm does not have an incentive to enter the domestic market. On the other hand, if the size of the foreign market is small and the magnitude of the innovation is not so large, its total profit when it enters the domestic market with license is larger than the total profit when it chooses license without entry strategy.

We have shown the following results.

Proposition 1. *(1) When $t > \frac{k_H(2k_Hk_L+2ck_H-k_L^2)}{ck_L^2}$ or $\frac{k_H}{c}+2 < t \leq \frac{k_H(2k_Hk_L+2ck_H-k_L^2)}{ck_L^2}$, Firm A does not enter the domestic market, and its optimal strategy is to license without entry.*

(2) When $t \leq \frac{k_H}{c}+2$;

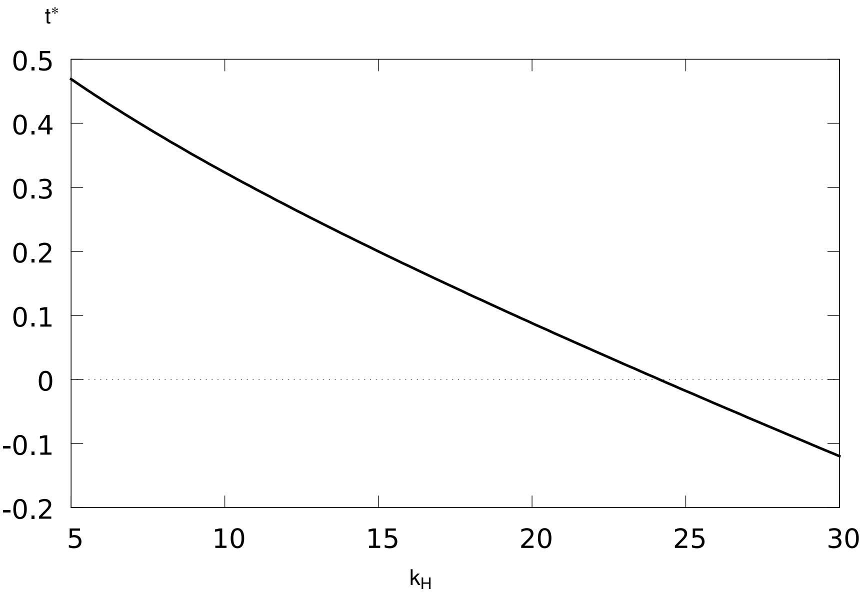

Figure 4: Relations among k_H and t^*

i) Entry without license strategy is never the optimal strategy for Firm A.

ii) If the ratio of the size of the foreign market relatively to the domestic market is small, license with entry strategy is optimal for Firm A.

iii) If the ratio of the size of the foreign market relatively to the domestic market is not small, license without entry strategy is optimal for Firm A.

iv) If k_H is large ($k_H \geq 10(\sqrt{2}+1)$), license without entry strategy is always optimal.

6 Concluding Remark

We have examined the optimal strategies for the foreign innovator in international duopoly when it can enter the domestic market under vertical differentiation, and have shown that its optimal strategy depends on the relative size of the foreign and the domestic markets.

In the future research we will extend the analysis to a case of more general demand and cost functions, and oligopoly with more than two firms.

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Appendix: Details of calculation

$$\begin{aligned}
A =& ck_Hk_L^4t^2 + 4c^2k_L^4t^2 - 8ck_H^2k_L^3t^2 + 16ck_H^3k_L^2t^2 - 8c^2k_H^2k_L^2t^2 + 32c^2k_H^3k_Lt^2 + 16c^3k_H^3t^2 \\
&+ k_H^2k_L^4t + 8ck_Hk_L^4t + 4c^2k_L^4t - 8k_H^3k_L^3t - 24ck_H^2k_L^3t + 16k_H^4k_L^2t + 24ck_H^3k_L^2t - 24c^2k_H^2k_L^2t \\
&+ 32ck_H^4k_Lt + 64c^2k_H^3k_Lt + 16c^2k_H^4t + 32c^3k_H^3t + 4k_H^2k_L^4 + 4ck_Hk_L^4 - 16k_H^3k_L^3 \\
&- 16ck_H^2k_L^3 + 16k_H^4k_L^2 - 16c^2k_H^2k_L^2 + 32ck_H^4k_L + 32c^2k_H^3k_L + 16c^2k_H^4 + 16c^3k_H^3.
\end{aligned}$$

$$\begin{aligned}
B =& c^2k_L^4t^2 - 24c^2k_Hk_L^3t^2 - 16c^3k_L^3t^2 + 16c^2k_H^2k_L^2t^2 - 24c^3k_Hk_L^2t^2 - 16c^4k_L^2t^2 + 32c^3k_H^2k_Lt^2 \\
&+ 16c^4k_H^2t^2 + 2ck_Hk_L^4t - 32ck_H^2k_L^3t - 56c^2k_Hk_L^3t - 32c^3k_L^3t + 32ck_H^3k_L^2t - 56c^3k_Hk_L^2t - 32c^4k_L^2t \\
&+ 64c^2k_H^3k_Lt + 64c^3k_H^2k_Lt + 32c^3k_H^3t + 32c^4k_H^2t + k_H^2k_L^4 - 12k_H^3k_L^3 - 28ck_H^2k_L^3 - 32c^2k_Hk_L^3 \\
&- 16c^3k_L^3 + 16k_H^4k_L^2 + 20ck_H^3k_L^2 - 12c^2k_H^2k_L^2 - 32c^3k_Hk_L^2 - 16c^4k_L^2 + 32ck_H^4k_L + 64c^2k_H^3k_L \\
&+ 32c^3k_H^2k_L + 16c^2k_H^4 + 32c^3k_H^3 + 16c^4k_H^2.
\end{aligned}$$

$$\begin{aligned}
C =& c^2k_Hk_L^4t^3 + 2c^3k_L^4t^3 - 8c^2k_H^2k_L^3t^3 - 32c^3k_Hk_L^3t^3 - 16c^4k_L^3t^3 + 16c^2k_H^3k_L^2t^3 + 24c^3k_H^2k_L^2t^3 \\
&- 32c^4k_Hk_L^2t^3 - 16c^5k_L^2t^3 + 32c^3k_H^3k_Lt^3 + 64c^4k_H^2k_Lt^3 + 16c^4k_H^3t^3 + 32c^5k_H^2t^3 + 2ck_H^2k_L^4t^2 \\
&+ 5c^2k_Hk_L^4t^2 - 16ck_H^3k_L^3t^2 - 80c^2k_H^2k_L^3t^2 - 80c^3k_Hk_L^3t^2 - 32c^4k_L^3t^2 + 32ck_H^4k_L^2t^2 \\
&+ 96c^2k_H^3k_L^2t^2 - 16c^3k_H^2k_L^2t^2 - 80c^4k_Hk_L^2t^2 - 32c^5k_L^2t^2 + 64c^2k_H^4k_Lt^2 + 224c^3k_H^3k_Lt^2 \\
&+ 128c^4k_H^2k_Lt^2 + 32c^3k_H^4t^2 + 112c^4k_H^3t^2 + 64c^5k_H^2t^2 + k_H^3k_L^4t + 4ck_H^2k_L^4t - 8k_H^4k_L^3t \\
&- 60ck_H^3k_L^3t - 92c^2k_H^2k_L^3t - 64c^3k_Hk_L^3t - 16c^4k_L^3t + 16k_H^5k_L^2t \\
&+ 88ck_H^4k_L^2t + 52c^2k_H^3k_L^2t - 60c^3k_H^2k_L^2t - 64c^4k_Hk_L^2t - 16c^5k_L^2t \\
&+ 32ck_H^5k_Lt + 192c^2k_H^4k_Lt + 224c^3k_H^3k_Lt + 64c^4k_H^2k_Lt + 16c^2k_H^5t \\
&+ 96c^3k_H^4t + 112c^4k_H^3t + 32c^5k_H^2t + k_H^3k_L^4 - 12k_H^4k_L^3 - 28ck_H^3k_L^3 \\
&- 32c^2k_H^2k_L^3 - 16c^3k_Hk_L^3 + 16k_H^5k_L^2 + 20ck_H^4k_L^2 - 12c^2k_H^3k_L^2 - 32c^3k_H^2k_L^2 \\
&- 16c^4k_Hk_L^2 + 32ck_H^5k_L + 64c^2k_H^4k_L + 32c^3k_H^3k_L + 16c^2k_H^5 + 32c^3k_H^4 + 16c^4k_H^3.
\end{aligned}$$

$$
\begin{aligned}
F =& 9c^3k_H^2k_L^4t^4 + 44c^4k_Hk_L^4t^4 + 32c^5k_L^4t^4 - 72c^3k_H^3k_L^3t^4 \\
&- 496c^4k_H^2k_L^3t^4 - 640c^5k_Hk_L^3t^4 - 256c^6k_L^3t^4 + 144c^3k_H^4k_L^2t^4 \\
&+ 632c^4k_H^3k_L^2t^4 + 16c^5k_H^2k_L^2t^4 - 640c^6k_Hk_L^2t^4 - 256c^7k_L^2t^4 \\
&+ 288c^4k_H^4k_Lt^4 + 1408c^5k_H^3k_Lt^4 + 1024c^6k_H^2k_Lt^4 + 144c^5k_H^4t^4 + 704c^6k_H^3t^4 \\
&+ 512c^7k_H^2t^4 + 27c^2k_H^3k_L^4t^3 + 144c^3k_H^2k_L^4t^3 + 172c^4k_Hk_L^4t^3 + 64c^5k_L^4t^3 \\
&- 216c^2k_H^4k_L^3t^3 - 1656c^3k_H^3k_L^3t^3 - 3552c^4k_H^2k_L^3t^3 - 3200c^5k_Hk_L^3t^3 \\
&- 1024c^6k_L^3t^3 + 432c^2k_H^5k_L^2t^3 + 2376c^3k_H^4k_L^2t^3 + 2504c^4k_H^3k_L^2t^3 \\
&- 1504c^5k_H^2k_L^2t^3 - 3200c^6k_Hk_L^2t^3 - 1024c^7k_L^2t^3 + 864c^3k_H^5k_Lt^3 \\
&+ 5184c^4k_H^4k_Lt^3 + 8320c^5k_H^3k_Lt^3 + 4096c^6k_H^2k_Lt^3 + 432c^4k_H^5t^3 \\
&+ 2592c^5k_H^4t^3 + 4160c^6k_H^3t^3 + 2048c^7k_H^2t^3 + 27ck_H^4k_L^4t^2 + 164c^2k_H^3k_L^4t^2 \\
&+ 288c^3k_H^2k_L^4t^2 + 192c^4k_Hk_L^4t^2 + 32c^5k_L^4t^2 - 216ck_H^5k_L^3t^2 \\
&- 1924c^2k_H^4k_L^3t^2 - 5840c^3k_H^3k_L^3t^2 - 8368c^4k_H^2k_L^3t^2 - 5760c^5k_Hk_L^3t^2 \\
&- 1536c^6k_L^3t^2 + 432ck_H^6k_L^2t^2 + 2984c^2k_H^5k_L^2t^2 + 6028c^3k_H^4k_L^2t^2 \\
&+ 2416c^4k_H^3k_L^2t^2 - 5296c^5k_H^2k_L^2t^2 - 5760c^6k_Hk_L^2t^2 - 1536c^7k_L^2t^2 \\
&+ 864c^2k_H^6k_Lt^2 + 6400c^3k_H^5k_Lt^2 + 15904c^4k_H^4k_Lt^2 + 16512c^5k_H^3k_Lt^2 \\
&+ 6144c^6k_H^2k_Lt^2 + 432c^3k_H^6t^2 + 3200c^4k_H^5t^2 + 7952c^5k_H^4t^2 + 8256c^6k_H^3t^2 \\
&+ 3072c^7k_H^2t^2 + 9k_H^5k_L^4t + 72ck_H^4k_L^4t + 188c^2k_H^3k_L^4t + 192c^3k_H^2k_L^4t \\
&+ 64c^4k_Hk_L^4t - 72k_H^6k_L^3t - 864ck_H^5k_L^3t - 3648c^2k_H^4k_L^3t - 7456c^3k_H^3k_L^3t \\
&- 8064c^4k_H^2k_L^3t - 4480c^5k_Hk_L^3t - 1024c^6k_L^3t + 144k_H^7k_L^2t + 1368ck_H^6k_L^2t \\
&+ 4336c^2k_H^5k_L^2t + 5056c^3k_H^4k_L^2t - 608c^4k_H^3k_L^2t - 6016c^5k_H^2k_L^2t - 4480c^6k_Hk_L^2t \\
&- 1024c^7k_L^2t + 288ck_H^7k_Lt + 2880c^2k_H^6k_Lt + 10400c^3k_H^5k_Lt + 17408c^4k_H^4k_Lt \\
&+ 13696c^5k_H^3k_Lt + 4096c^6k_H^2k_Lt + 144c^2k_H^7t + 1440c^3k_H^6t + 5200c^4k_H^5t \\
&+ 8704c^5k_H^4t + 6848c^6k_H^3t + 2048c^7k_H^2t + 8k_H^5k_L^4 + 40ck_H^4k_L^4 + 64c^2k_H^3k_L^4 \\
&+ 32c^3k_H^2k_L^4 - 100k_H^6k_L^3 - 720ck_H^5k_L^3 - 2096c^2k_H^4k_L^3 - 3200c^3k_H^3k_L^3 \\
&- 2752c^4k_H^2k_L^3 - 1280c^5k_Hk_L^3 - 256c^6k_L^3 + 128k_H^7k_L^2 + 796ck_H^6k_L^2 + 1712c^2k_H^5k_L^2 \\
&+ 1104c^3k_H^4k_L^2 - 1152c^4k_H^3k_L^2 - 2240c^5k_H^2k_L^2 - 1280c^6k_Hk_L^2 - 256c^7k_L^2 \\
&+ 256ck_H^7k_L + 1792c^2k_H^6k_L + 4864c^3k_H^5k_L + 6400c^4k_H^4k_L + 4096c^5k_H^3k_L \\
&+ 1024c^6k_H^2k_L + 128c^2k_H^7 + 896c^3k_H^6 + 2432c^4k_H^5 + 3200c^5k_H^4 + 2048c^6k_H^3 + 512c^7k_H^2.
\end{aligned}
$$