License or entry decision for innovator in international duopoly with convex cost functions

Hattori, Masahiko and Tanaka, Yasuhito

7 May 2017

License or entry decision for innovator in international duopoly with convex cost functions

Masahiko Hattori*
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.
and
Yasuhito Tanaka†
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.

May 7, 2017

Abstract

We consider a choice of options for a foreign innovating firm to license its new cost-reducing technology to a domestic incumbent firm or to enter the domestic market with or without license under convex cost functions. With convex cost functions the domestic market and the foreign market are not separated, and the results depend on the relative size of those markets. In a specific case with linear demand and quadratic cost, entry without license strategy is never the optimal strategy for the innovating firm; if the ratio of the size of the foreign market relatively to the domestic market is small, license with entry strategy is optimal; and if the ratio of the size of the foreign market relatively to the domestic market is not small, license without entry strategy is optimal.

Keywords: license with or without entry, duopoly, foreign and domestic markets, foreign innovating firm.

JEL Code: D43; L13.

*M. Hattori, mhattori@mail.doshisha.ac.jp
†Y. Tanaka, yasuhito@mail.doshisha.ac.jp

1. Introduction

We consider a choice of options for a foreign innovating firm to license its new cost-reducing technology to a domestic incumbent firm or to enter the domestic market with or without license under convex cost functions. With convex cost functions the domestic market and the foreign market are not separated, and the results depend on the relative size of those markets. We will show the following results. In a case with linear demand and quadratic cost, entry without license strategy is never the optimal strategy for the innovating firm; if the ratio of the size of the foreign market relatively to the domestic market is small, entry with license strategy is optimal; and if the ratio of the size of the foreign market relatively to the domestic market is not small, license without entry strategy is optimal.

In the next section we briefly review some related studies. In Section 3 we present the model. In Section 4 we study the general case, and in Section 5 we investigate the optimal strategies for the foreign innovating firm in the linear demand and quadratic cost functions case.

2. Brief literature review

Various studies focus on technology adoption or R&D investment in duopoly or oligopoly. Most of them analyze the relation between the technology licensor and licensee. The difference of means of contracts, which comprise royalties, upfront fixed fees, combinations of these two, and auctions, are well discussed (Katz and Shapiro (1985)). Kamien and Tauman (2002) showed that outside innovators prefer auctions, but industry incumbents prefer royalty. This topic is discussed by Kabiraj (2004) under the Stackelberg oligopoly; here, the licensor does not have production capacity. Wang and Yang (2004) considered the case when the licensor has production capacity. Sen and Tauman (2007) compared the license system in detail, namely, when the licensor is an outsider and when it is an incumbent firm, using the combination of royalties and fixed fees. However, the existence of production capacity was externally given, and they did not analyze the choice of entry. Therefore, the optimal strategies of outside innovators, who can use the entry as a threat, require more discussion. Regarding the strategies of new entrants to the market, Duchene, Sen and Serfes (2015) focused on future entrants with old technology, and argued that a low license fee can be used to deter the entry of potential entrants. However, the firm with new technology is incumbent, and its choice of entry is not analyzed. Also, Chen (2016) analyzed the model of the endogenous market structure determined by the potential entrant with old technology and showed that the licensor uses the fixed fee and zero royalty in both the incumbent and the outside innovator cases, which are exogenously given. Creane, Chiu and Konishi (2013) examined a firm that can license its production technology to a rival when firms are heterogeneous in production costs, and showed that a complete technology transfer from one firm to another always increases joint profit under weakly concave demand when at least three firms remain in the industry. Hattori and Tanaka (2014), Hattori and Tanaka (2015) studied the adoption of new technology in Cournot duopoly and Stackelberg duopoly. Hattori and Tanaka (2016) analyzed problems about product innovation, that is, introduction of higher quality good in a duopoly with vertical product differentiation.

3. The model

There are two countries and two firms, Firm A in Country A and Firm B in Country B. Call Country A the foreign country and Country B the domestic country; Firm A the foreign firm and Firm B the domestic firm. At present each firm produces the same good in each country. Firm A has a superior cost-reducing technology, and can produce the good at lower cost than Firm B.

Firm A has three options. The first option is to enter the domestic market without license to Firm B, the second option is to license its technology to Firm B without entry, and the third option is to enter the domestic market with license to Firm B. If Firm A enters, the domestic market becomes a duopoly. Since the focus of this paper is a choice of entry or license by Firm A, we assume that Firm B does not enter the foreign market. Let p be the price, X be the total supply in the domestic market. The inverse demand function is written as

$$p = p(X).$$

The supplies of Firms A and B are denoted by, respectively, x_A and x_B. Thus, $X = x_A + x_B$. In the foreign market the supply of Firm A and the price of the good are denoted by y_A and q. The inverse demand function is written as

$$q = q(y_A/t).$$

t is a positive number. It represents the ratio of the size of the foreign market relatively to the domestic market. If $t < 1$, the size of the foreign market is smaller than the size of the domestic market. If $t > 1$, the size of the foreign market is larger than the size of the domestic market.

We assume that the cost functions of Firms A and B are convex. They are $c_A(x_A + y_A)$ and $c_B(x_B)$.

4. General analysis

4.1. Firms' behavior

(1) When Firm A enters the domestic market without license to Firm B, the profits of Firms A and B are

$$\pi_A = px_A + qy_A - c_A(x_A + y_A),$$
$$\pi_B = px_B - c_B(x_B).$$

The conditions for profit maximization of Firms A and B are

$$p + x_A p' - c'_A(x_A + y_A) = 0,$$
$$q + \frac{y_A}{t}q' - c'_A(x_A + y_A) = 0,$$
$$p + x_B p' - c'_B(x_B) = 0.$$

Denote the profits of Firms A and B in this case by π^e_A and π^e_B.

(2) When Firm A licenses its technology to Firm B without entering the domestic market, the profits of Firms A and B in the markets are

$$\pi_A = qy_A - c_A(y_A),$$
$$\pi_B = px_B - c_A(x_B) - L.$$

L is the license fee. The cost functions of both Firms A and B are c_A. The conditions for profit maximization of Firms A and B are

$$q + \frac{y_A}{t}q' - c'_A(y_A) = 0,$$
$$p + x_B p' - c'_A(x_B) = 0.$$

If the negotiation between Firm A and Firm B about the license fee breaks down, Firm A can enter the domestic market without license. Therefore, Firm B must pay the difference between its profit excluding the license fee in this case and its profit in the entry without license case. Denote the profits of Firms A and B by π_A^l and π_B^l, and denote the license fee by L^l. It should be equal to

$$L^l = \pi_B^l + L^l - \pi_B^e.$$

This equation means that the license fee is determined so that $\pi_B^l = \pi_B^e$ holds. The total profit of Firm A is

$$\pi_A^l + L^l.$$

(3) When Firm A enters the domestic market and at the same time licenses its technology to Firm B, the profits of Firms A and B are

$$\pi_A = px_A + qy_A - c_A(x_A + y_A),$$
$$\pi_B = px_B - c_A(x_B) - L.$$

The cost functions of both Firms A and B are c_A. L is the license fee. The conditions for profit maximization of Firms A and B are

$$p + x_A p' - c'_A(x_A + y_A) = 0,$$
$$q + \frac{y_A}{t}q' - c'_A(x_A + y_A) = 0,$$
$$p + x_B p' - c'_A(x_B) = 0.$$

Similarly to the previous case, if the negotiation between Firms A and Firm B about the license fee breaks down, Firm A can enter the domestic market without license. Therefore, Firm B must pay the difference between its profit excluding the license fee in this case and its profit in the entry without license case. Denote the profits of Firms A and B by π_A^{el} and π_B^{el}, and denote the license fee by L^{el}. It should be equal to

$$L^{el} = \pi_B^{el} + L^{el} - \pi_B^e.$$

This equation means that the license fee is determined so that $\pi_B^{el} = \pi_B^e$ holds. The total profit of Firm A is

$$\pi_A^{el} + L^{el}.$$

4.2. The optimal strategies

Comparing $\pi_A^l + L^l$, π_A^e and $\pi_A^{el} + L^{el}$, the optimal strategies for Firm A are as follows.

(1) If $\pi_A^l + L^l$ is the maximum, license without entry strategy is optimal.

(2) If π_A^e is the maximum, entry without license strategy is optimal.

(3) If $\pi_A^{el} + L^{el}$ is the maximum, entry with license strategy is optimal.

5. Linear demand and quadratic cost functions case

About details of Γ_A, Γ_B, Γ_C, Γ_D, Γ_E, Γ_F, Γ_G and Γ_H in calculations please see Appendix.

5.1. Demand and cost functions

Specifically we consider a case of linear demand and quadratic cost functions. The inverse demand function in the domestic market is

$$p = a - X.$$

a is a positive constant. The inverse demand function in the foreign market is

$$q = a - \frac{y_A}{t}.$$

The cost functions of Firms A and B are $c_A(x_A + y_A)^2$ and $c_B x_B^2$, where c_A and c_B are positive constants such that $c_A < c_B$. We consider three cases about the value of t.

(1) $t > \frac{2c_B+1}{c_A}$.

Then, Firm A never enters the domestic market, and the entry without license case and the entry with license case in the next sub-section do not exist.

(2) $\frac{2c_A+1}{c_A} < t \leq \frac{2c_B+1}{c_A}$.

Then, Firm A does not enter the domestic market with license to Firm B, and the entry with license case does not exist.

(3) $t \leq \frac{2c_A+1}{c_A}$.

Then, Firm A may enter the domestic market with or without license.

5.2. Firms' behavior

About firms' behavior we obtain the following results.

(1) Entry without license case.

Suppose that Firm A enters the domestic market without license to Firm B. The profits of the firms are

$$\pi_A = px_A + qy_A - c_A(x_A + y_A)^2,$$
$$\pi_B = px_B - c_B x_B^2.$$

The equilibrium profits are obtained as follows.

$$\pi_A^e = \frac{a^2\Gamma_A}{4(4c_Ac_Bt + 3c_At + 4c_Ac_B + 4c_B + 4c_A + 3)^2},$$

$$\pi_B^e = \frac{a^2(c_B + 1)(2c_At + 2c_A + 1)^2}{(4c_Ac_Bt + 3c_At + 4c_Ac_B + 4c_B + 4c_A + 3)^2}.$$

(2) License without entry case.

Suppose that Firm A licenses its technology to Firm B without entering the domestic market. The profits of the firms are

$$\pi_A = qy_A - c_A y_A^2,$$

$$\pi_B = px_B - c_A x_B^2 - L^l.$$

The equilibrium profits are

$$\pi_A^l = \frac{a^2 t}{4(c_At + 1)},$$
$$\pi_B^l = \frac{a^2}{4(c_A + 1)}.$$

The license fee is equal to

$$L^l = \frac{a^2\Gamma_B}{4(c_A + 1)(4c_Ac_Bt + 3c_At + 4c_Ac_B + 4c_B + 4c_A + 3)^2}.$$

The total profit of Firm A including the license fee is

$$\pi_A^l + L^l = \frac{a^2\Gamma_C}{4(c_A + 1)(c_At + 1)(4c_Ac_Bt + 3c_At + 4c_Ac_B + 4c_B + 4c_A + 3)^2}.$$

(3) Entry with license case.

Suppose that Firm A enters the domestic market and at the same time licenses its technology to Firm B. The profits of the firms are

$$\pi_A = px_A + qy_A - c_A(x_A + y_A)^2,$$

$$\pi_B = px_B - c_A x_B^2 - L^{el}.$$

The equilibrium profits are

$$\pi_A^{el} = \frac{a^2\Gamma_D}{4(4c_A^2t + 3c_At + 4c_A^2 + 8c_A + 3)^2},$$

$$\pi_B^{el} = \frac{a^2(c_A+1)(2c_At + 2c_A + 1)^2}{(4c_A^2t + 3c_At + 4c_A^2 + 8c_A + 3)^2}.$$

The license fee is equal to

$$L^{el} = \frac{a^2(c_B - c_A)(2c_At + 2c_A + 1)^2\Gamma_E}{(4c_A^2t + 3c_At + 4c_A^2 + 8c_A + 3)^2(4c_Ac_Bt + 3c_At + 4c_Ac_B + 4c_B + 4c_A + 3)^2}.$$

The total profit of Firm A including the license fee is

$$\pi_A^{el} + L^{el} = \frac{a^2\Gamma_F}{4(4c_A^2t + 3c_At + 4c_A^2 + 8c_A + 3)^2(4c_Ac_Bt + 3c_At + 4c_Ac_B + 4c_B + 4c_A + 3)^2}.$$

5.3. The optimal strategies

We consider the optimal strategies for Firm A in each case.

(1) If $t > \frac{2c_B+1}{c_A}$, Firm A never enters the domestic market. Thus, its optimal strategy is license without entry.

(2) If $\frac{2c_A+1}{c_A} < t \leq \frac{2c_B+1}{c_A}$, Firm A does not enter the domestic market when it licenses its technology to Firm B. Comparing the profit of Firm A in that case and its profit when it enters the market without license,

$$\pi_A^l + L^l - \pi_A^e = \frac{\Gamma_G}{4(c_A+1)(c_At+1)(4c_Ac_Bt + 3c_At + 4c_Ac_B + 4c_B + 4c_A + 3)^2}.$$

This is positive for reasonable values of variables if $t > \frac{2c_A+1}{2}$. In Figure 1 we depict an example of this case assuming $c_A = 1$ and $c_B = 5$. Then, $\frac{2c_B+1}{c_A} = 11$ and $\frac{2c_A+1}{c_A} = 3$.

Some discussion about Case 1 and Case 2 When Firm A licenses its technology to Firm B without its entry, the domestic market becomes a monopoly in which Firm B produces the good at lower cost. Then, π_B^l is larger than π_B^e plus the profit of Firm A in the domestic market when it enters without license. The license fee in the case of license without entry is $\pi_B^l - \pi_B^e$ which is larger than the profit of Firm A in the domestic market. Then, the total profit of Firm A when it licenses its technology to Firm B without entry should be larger than the total profit when it enters the domestic market without license, and license without entry strategy is optimal for Firm A.

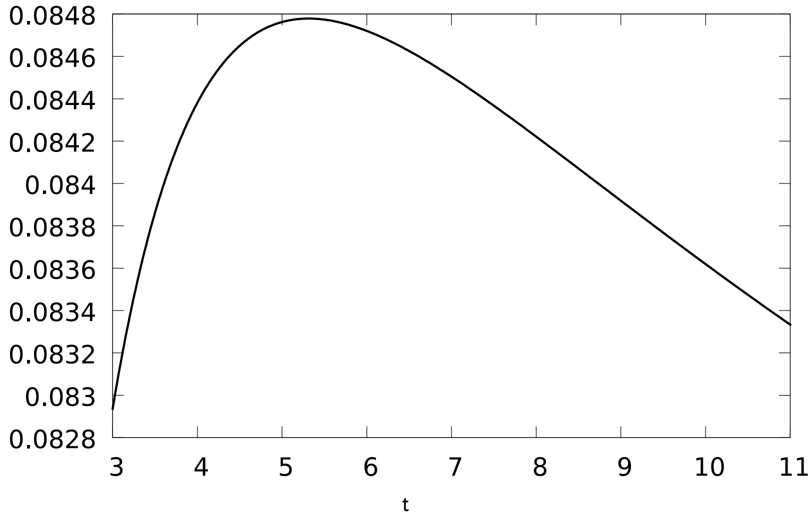

Figure 1: Illustration of $\pi_A^l + L^l - \pi_A^e$

(3) Now consider the case where $t \leq \frac{2c_A+1}{c_A}$. Let us compare the profit of Firm A when it licenses its technology to Firm B with entry and its profit when it enters the market without license. Then,

$$\pi_A^{el} + L^{el} - \pi_A^e$$
$$= \frac{(c_B - c_A)(2c_A t + 2c_A + 1)\Gamma_H}{(4c_A^2 t + 3c_A t + 4c_A^2 + 8c_A + 3)^2(4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)^2}.$$

This is positive for reasonable values of variables. Thus, $\pi_A^{el} + L^{el} > \pi_A^e$, and entry only (entry without license) strategy is never the optimal strategy for Firm A. In Figure 2 we depict an example of $\pi_A^{el} + L^{el} - \pi_A^e$ assuming $c_A = 1$ and $c_B = 5$.

Comparing the profit of Firm A when it licenses its technology to Firm B with entry and its profit when it licenses its technology to Firm B without entry yields

$$\pi_A^{el} + L^{el} - (\pi_A^l + L^l) = \frac{a^2(c_A t - 2c_A - 1)\Gamma_I}{4(c_A + 1)(c_A t + 1)(4c_A^2 t + 3c_A t + 4c_A^2 + 8c_A + 3)^2}$$

where

$$\Gamma_I = 12c_A^3 t^2 + 11c_A^2 t^2 + 4c_A^3 t + 18c_A^2 t + 12c_A t - 8c_A^3 - 12c_A^2 - 2c_A + 1.$$

This depends on the values of t and c_A, but does not depend on the value of c_B. Solving $\pi_A^{el} + L^{el} - (\pi_A^l + L^l) = 0$, we obtain the following solution.

$$t^* = \frac{(c_A + 1)\sqrt{100c_A^2 + 68c_A + 25} - 2c_A^2 - 9c_A - 6}{12c_A^2 + 11c_A}.$$

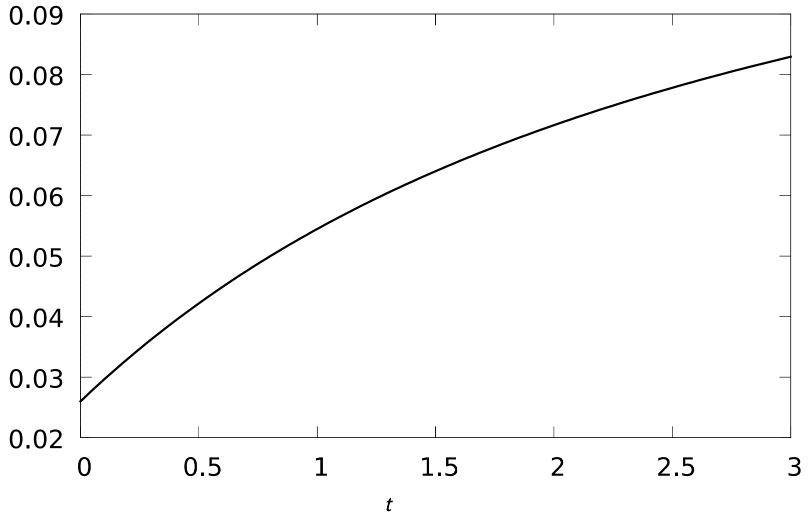

Figure 2: Illustration of $\pi_A^{el} + L^{el} - \pi_A^e$

This is the threshold value of the relative size of the foreign market to the domestic market. It depends on only c_A. If $t < t^*$, $\pi_A^l + L^l < \pi_A^{el} + L^{el}$, and if $t > t^*$, $\pi_A^l + L^l > \pi_A^{el} + L^{el}$. Thus, if the foreign market is small relatively to the domestic market, license with entry strategy is optimal for Firm A, and if the foreign market is not small relatively to the domestic market, license without entry strategy is optimal. In Figure 3 we depict an example of $\pi_A^{el} + L^{el} - (\pi_A^l + L^l)$ assuming $c_A = 1$.

When $c_A \leq \frac{\sqrt{2}-1}{2}$, $t^* \leq 0$. Therefore, if $c_A \leq \frac{\sqrt{2}-1}{2}$, there is no positive t^*, and license without entry strategy is optimal. In Figure 4 we depict the relation between c_A and t^*.

Some discussion about Case 3 Let us provide some intuition behind this result. If the foreign innovating firm chooses license without entry strategy, the domestic market becomes a monopoly, and the incumbent firm gets the large profit. Then, the license fee is large, and as larger the magnitude of the innovation (the smaller the value of c_A is), the larger the license fee is. If the foreign market is not small relatively to the domestic market, the profit of the foreign firm when it enters the domestic market as a duopolist is small relatively to the profit it earns in the foreign market as a monopolist. Therefore, if the size of the foreign market is not small, the foreign innovating firm does not have an incentive to enter the domestic market. On the other hand, if the size of the foreign market is small and the magnitude of the innovation is not so large, its total profit when it enters the domestic market with license is larger than the total profit when it chooses license without entry strategy.

We have shown the following results.

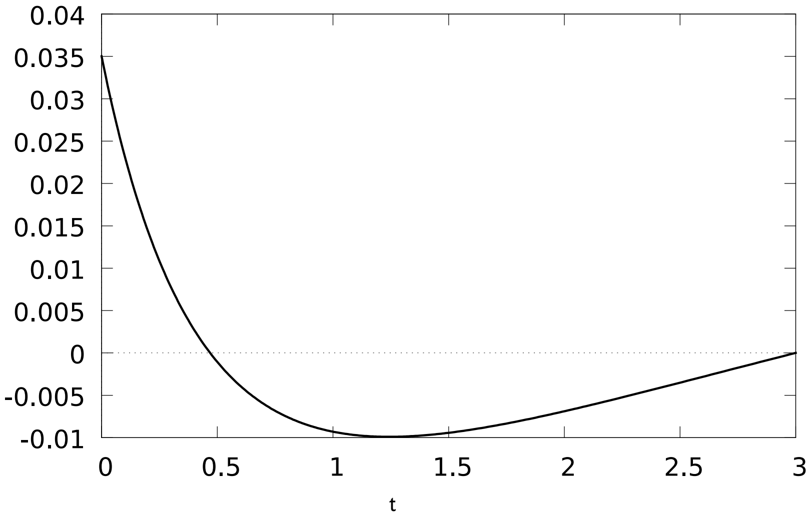

Figure 3: Illustration of $\pi_A^{el} + L^{el} - (\pi_A^{l} + L^{l})$

Proposition 1. *(1) When $t > \frac{2c_B+1}{c_A}$ or $\frac{2c_A+1}{c_A} < t \leq \frac{2c_B+1}{c_A}$, Firm A never enters the domestic market, and its optimal strategy is to license without entry.*

(2) When $t \leq \frac{2c_A+1}{c_A}$;

- *i) Entry without license strategy is never the optimal strategy for Firm A.*
- *ii) If the ratio of the size of the foreign market relatively to the domestic market is small, license with entry strategy is optimal for Firm A.*
- *iii) If the ratio of the size of the foreign market relatively to the domestic market is not small, license without entry strategy is optimal for Firm A.*
- *iv) If c_A is small ($c_A \leq \frac{\sqrt{2}-1}{2}$), license without entry strategy is always optimal.*

6. Concluding Remark

We have examined the optimal strategies for the foreign innovator in international duopoly when it can enter the domestic market, and have shown that its optimal strategy depends on the relative size of the foreign and the domestic markets. In the future research we want to extend the analysis to an oligopolistic situation.

References

Chen C.-S. (2016), "Endogenous market structure and technology licensing," *Japanese Economic Review*, forthcoming.

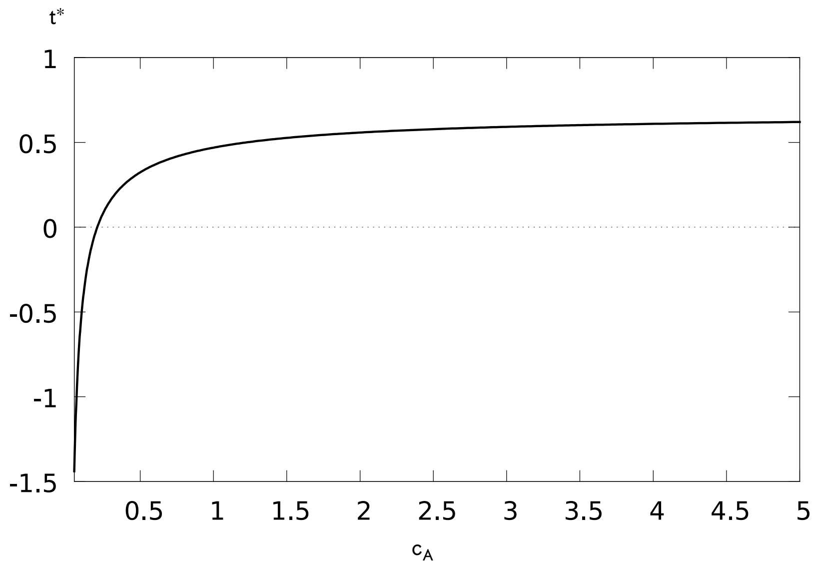

Figure 4: Relation between c_A and t^*

Creane, A., Chiu, Y. K. and Konishi, H. (2013), "Choosing a licensee from heterogeneous rivals," *Games and Economic Behavior*, **82**, pp. 254-268.

Duchene, A., Sen, D. and Serfes, K. (2015), "Patent licensing and entry deterrence: The role of low royalties," *Economica*, **82**, pp. 1324-1348.

Hattori, M. and Tanaka, Y. (2014), "Incentive for adoption of new technology in duopoly under absolute and relative profit maximization," *Economics Bulletin*, **34**. pp. 2051-2059.

Hattori, M. and Tanaka, Y. (2015), "Subsidy or tax policy for new technology adoption in duopoly with quadratic and linear cost functions," *Economics Bulletin*, **35**, pp. 1423-1433.

Hattori, M. and Tanaka, Y. (2016), "License or entry with vertical differentiation in duopoly," *Economics and Business Letters*, **5**, pp. 17-29.

Kabiraj, T. (2004), "Patent licensing in a leadership structure," *The Manchester School*, **72**, pp.188-205.

Kamien, T. and Tauman, Y. (2002), "Patent licensing: the inside story," *The Manchester School*, **70**, pp.7-15.

Katz, M. and Shapiro, C. (1985), "On the licensing of innovations," *Rand Journal of Economics*, **16**, pp. 504-520.

Sen, D. and Tauman, Y. (2007), "General licensing schemes for a cost-reducing Innovation," *Games and Economic Behavior*, **59**, pp.163-186.

Wang, X. H. and Yang, B. Z. (2004), "On technology licensing in a Stackelberg duopoly," *Australian Economic Papers*, **43**, pp. 448-458.

A. Appendix: Details of calculation

$$\begin{aligned}\Gamma_A =&16c_Ac_B^2t^2 + 24c_Ac_Bt^2 + 4c_A^2t^2 + 9c_At^2 + 32c_Ac_B^2t + 16c_B^2t + 40c_Ac_Bt + 24c_Bt\\ &+ 4c_A^2t + 16c_At + 9t + 16c_Ac_B^2 + 16c_B^2 + 16c_Ac_B + 16c_B + 4c_A + 4,\end{aligned}$$

$$\begin{aligned}\Gamma_B =&16c_A^2c_B^2t^2 - 16c_A^3c_Bt^2 + 8c_A^2c_Bt^2 - 16c_A^3t^2 - 7c_A^2t^2 + 32c_A^2c_B^2t + 32c_Ac_B^2t\\ &- 32c_A^3c_Bt + 8c_A^2c_Bt + 32c_Ac_Bt - 32c_A^3t - 24c_A^2t + 2c_At + 16c_A^2c_B^2 + 32c_Ac_B^2 + 16c_B^2\\ &- 16c_A^3c_B + 36c_Ac_B + 20c_B - 16c_A^3 - 16c_A^2 + 4c_A + 5,\end{aligned}$$

$$\begin{aligned}\Gamma_C =&32c_A^3c_B^2t^3 + 16c_A^2c_B^2t^3 - 16c_A^4c_Bt^3 + 32c_A^3c_Bt^3 + 24c_A^2c_Bt^3 - 16c_A^4t^3 + 2c_A^3t^3\\ &+ 9c_A^2t^3 + 64c_A^3c_B^2t^2 + 112c_A^2c_B^2t^2 + 32c_Ac_B^2t^2 - 32c_A^4c_Bt^2 + 48c_A^3c_Bt^2 + 144c_A^2c_Bt^2\\ &+ 48c_Ac_Bt^2 - 32c_A^4t^2 - 16c_A^3t^2 + 37c_A^2t^2 + 18c_At^2 + 32c_A^3c_B^2t + 112c_A^2c_B^2t + 96c_Ac_B^2t\\ &+ 16c_B^2t - 16c_A^4c_Bt + 132c_A^2c_Bt + 132c_Ac_Bt + 24c_Bt - 16c_A^4t - 32c_A^3t + 20c_A^2t + 40c_At + 9t\\ &+ 16c_A^2c_B^2 + 32c_Ac_B^2 + 16c_B^2 - 16c_A^3c_B + 36c_Ac_B + 20c_B - 16c_A^3 - 16c_A^2 + 4c_A + 5,\end{aligned}$$

$$\Gamma_D =16c_A^3t^2 + 28c_A^2t^2 + 9c_At^2 + 32c_A^3t + 60c_A^2t + 40c_At + 9t + 16c_A^3 + 32c_A^2 + 20c_A + 4,$$

$$\begin{aligned}\Gamma_E =&16c_A^3c_Bt^2 + 16c_A^2c_Bt^2 + 16c_A^3t^2 + 15c_A^2t^2 + 32c_A^3c_Bt + 64c_A^2c_Bt + 32c_Ac_Bt\\ &+ 32c_A^3t + 64c_A^2t + 30c_At + 16c_A^3c_B + 48c_A^2c_B + 48c_Ac_B + 16c_B + 16c_A^3 + 48c_A^2 + 48c_A + 15,\end{aligned}$$

$$\begin{aligned}\Gamma_F =&512c_A^5c_B^2t^4 + 704c_A^4c_B^2t^4 + 144c_A^3c_B^2t^4 - 256c_A^6c_Bt^4 + 384c_A^5c_Bt^4 + 912c_A^4c_Bt^4\\ &+ 216c_A^3c_Bt^4 - 256c_A^6t^4 - 96c_A^5t^4 + 252c_A^4t^4 + 81c_A^3t^4 + 2048c_A^5c_B^2t^3 + 4160c_A^4c_B^2t^3\\ &+ 2592c_A^3c_B^2t^3 + 432c_A^2c_B^2t^3 - 1024c_A^6c_Bt^3 + 896c_A^5c_Bt^3 + 4768c_A^4c_Bt^3 + 3528c_A^3c_Bt^3\\ &+ 648c_A^2c_Bt^3 - 1024c_A^6t^3 - 1088c_A^5t^3 + 780c_A^4t^3 + 1080c_A^3t^3 + 243c_A^2t^3 + 3072c_A^5c_B^2t^2\\ &+ 8256c_A^4c_B^2t^2 + 7952c_A^3c_B^2t^2 + 3200c_A^2c_B^2t^2 + 432c_Ac_B^2t^2 - 1536c_A^6c_Bt^2 + 384c_A^5c_Bt^2\\ &+ 8144c_A^4c_Bt^2 + 10064c_A^3c_Bt^2 + 4476c_A^2c_Bt^2 + 648c_Ac_Bt^2 - 1536c_A^6t^2 - 2656c_A^5t^2 + 80c_A^4t^2\\ &+ 2400c_A^3t^2 + 1440c_A^2t^2 + 243c_At^2 + 2048c_A^5c_B^2t + 6848c_A^4c_B^2t + 8704c_A^3c_B^2t + 5200c_A^2c_B^2t\\ &+ 1440c_Ac_B^2t + 144c_B^2t - 1024c_A^6c_Bt - 384c_A^5c_Bt + 5632c_A^4c_Bt + 9952c_A^3c_Bt + 6752c_A^2c_Bt\\ &+ 2016c_Ac_Bt + 216c_Bt - 1024c_A^6t - 2432c_A^5t - 1152c_A^4t + 1440c_A^3t + 1740c_A^2t + 648c_At + 81t\\ &+ 512c_A^5c_B^2 + 2048c_A^4c_B^2 + 3200c_A^3c_B^2 + 2432c_A^2c_B^2 + 896c_Ac_B^2 + 128c_B^2 - 256c_A^6c_B\\ &- 256c_A^5c_B + 1344c_A^4c_B + 3200c_A^3c_B + 2768c_A^2c_B + 1072c_Ac_B + 156c_B - 256c_A^6 - 768c_A^5\\ &- 704c_A^4 + 32c_A^3 + 400c_A^2 + 216c_A + 36,\end{aligned}$$

$$
\begin{aligned}
\Gamma_G =&16c_A^3c_B^2t^3 - 16c_A^4c_Bt^3 + 8c_A^3c_Bt^3 - 20c_A^4t^3 - 11c_A^3t^3 + 32c_A^3c_B^2t^2 + 48c_A^2c_B^2t^2 \\
&- 32c_A^4c_Bt^2 + 8c_A^3c_Bt^2 + 56c_A^2c_Bt^2 - 36c_A^4t^2 - 40c_A^3t^2 - c_A^2t^2 + 16c_A^3c_B^2t + 48c_A^2c_B^2t \\
&+ 32c_Ac_B^2t - 16c_A^4c_Bt - 16c_A^3c_Bt + 60c_A^2c_Bt + 52c_Ac_Bt - 16c_A^4t - 40c_A^3t - 8c_A^2t + 11c_At \\
&- 16c_A^3c_B - 16c_A^2c_B + 4c_Ac_B + 4c_B - 16c_A^3 - 20c_A^2 - 4c_A + 1,
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_H =&32c_A^4c_Bt^3 + 40c_A^3c_Bt^3 + 40c_A^4t^3 + 42c_A^3t^3 + 96c_A^4c_Bt^2 + 160c_A^3c_Bt^2 + 76c_A^2c_Bt^2 + 112c_A^4t^2 \\
&+ 198c_A^3t^2 + 87c_A^2t^2 + 96c_A^4c_Bt + 200c_A^3c_Bt + 132c_A^2c_Bt + 32c_Ac_Bt + 104c_A^4t + 244c_A^3t + 188c_A^2t \\
&+ 48c_At + 32c_A^4c_B + 80c_A^3c_B + 56c_A^2c_B + 4c_Ac_B - 4c_B + 32c_A^4 + 88c_A^3 + 84c_A^2 + 30c_A + 3.
\end{aligned}
$$