Negative royalty in duopoly and definition of license fee: general demand and cost functions

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Abstract

We examine the relationship between the definition of license fee and a possibility of negative royalty in a duopoly with an outside innovator which has an option to enter the market and imposes a combination of a royalty per output and a fixed fee under general demand and cost functions. We consider two scenarios about determination of license fee. One is a scenario which does not assume entry of the innovator, and the other is a scenario which takes a possibility of entry of the innovator into the market. We will show that the optimal royalty rate for the innovator in the former case is smaller than that in the latter case, and the sign of the optimal royalty rate depends on whether the goods of firms are strategic substitutes or strategic complements.

1 Introduction

Liao and Sen (2005) analyzed a problem of licensing by a combination of a royalty per output and a fixed fee under oligopoly with an outside or an incumbent innovator. Assuming linear demand and cost functions, they showed that when there are one licensee and one non-licensee, the innovator imposes a negative royalty with a positive fixed fee on the licensee. Similarly to the definition of license fee in Kamien and Tauman (1986), they defined the total license fee in the outside innovator case by the difference between the profit of the licensee when it buys the license and its profit when it does not buy the license and the other incumbent firm (non-licensee) buys the license. However, if the outside innovator has an option to sell a license to the other incumbent firm and, at the same time, enter the market when the potential licensee refuses to buy the license, we should use a different definition of license fee. In this paper we consider two scenarios about the license fee.

Scenario 1: If the potential licensee refuses the payment of license fee, the other

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incumbent firm buys the license and the innovator does not enter the market. Then, the willingness to pay of the licensee is the difference between its profit as a licensee and the profit of a non-licensee. It is the definition used in Liao and Sen (2005).

Scenario 2: If the potential licensee refuses the payment of license fee, the other incumbent firm buys the license and, at the same time, the innovator enters the market. It may be more severe punishment than selling the license to the other firm without entering the market. The willingness to pay of the licensee is the difference between its profit as a licensee and its profit when the innovator enters the market with a license to the other incumbent firm.

We consider a model of an oligopoly in which firms produce substitutable goods under general demand and cost functions[3], and the innovator has a new cost reducing technology which can commonly used by all firms. We will show the following results.

1. The optimal royalty rate in Scenario 1 is smaller than that in Scenario 2.
2. If the goods of the firms are strategic substitutes, the optimal royalty rates in Scenario 1 and Scenario 2 are negative.
3. If the goods of the firms are strategic complements, the optimal royalty rate in Scenario 1 may be positive or negative, but that in Scenario 2 is positive.
4. When the non-licensee drops out of the market at the optimal royalty rate, Scenario 1 and Scenario 2 are equivalent.

We consider only the outside innovator case. In the incumbent innovator case there is no problem of definition of license fee because in that case the innovator does not have an option whether its enters market or not.

Also in this paper we analyse only a problem of a possibility of negative royalty with one licensee and one non-license. For an outside innovator with two potential licensees whether it sells a license to one firm, or sells licenses to two firms is an important problem. We will study such a problem in the future research,

2 The model

There are three firms. One outside innovator and two incumbent firms. The innovator has a superior cost reducing technology which can commonly used by all firms, and licenses its technology to the licensee. Another incumbent firm is the non-licensee. The licensee is called Firm A, the non-licensee is called Firm B and the innovator is called Firm I. Firm I is an outside innovator at present. But it may enter the market if Firm A refuses to buy the license. Therefore, we consider a possibility of entry of Firm I so as to determine the license fee imposed on Firm A. Firm I does not really enter the market.

Denote the outputs of Firms A and B by x_A and x_B. The output of Firm I when it enters is x_I. The firms produce substitutable goods. The prices of the goods of Firms I, A and B

[3] Sen and Stamatopoulos (2016) presented an analysis of royalty and fixed fee in a duopoly under general demand and cost functions.

are p_I, p_A, p_B, and the inverse demand functions are written as $p_A(x_A, x_B)$ and $p_B(x_A, x_B)$, or $p_I(x_I, x_A, x_B)$, $p_A(x_I, x_A, x_B)$ and $p_B(x_I, x_A, x_B)$. We assume that all partial derivatives of the inverse demand functions are negative. They are twice differentiable. The cost function of Firm A with a license of the new technology is $c_A(x_A)$. The cost function of Firm B is $c_B(x_B)$, and the cost function of Firm I when it enters is written as $c_A(x_I)$. They are increasing and twice differentiable. We assume $c_A(x_A) < c_B(x_B)$ and $c'_A(x_A) < c'_B(x_B)$ for $x_A = x_B$. Firm I imposes a royalty per output and a fixed fee on Firm A. Denote the royalty rate by r.

The profit of Firm A net of the royalty and the profit of Firm B are written as

$$\pi_A = p_A x_A - c_A(x_A) - r x_A,$$

and

$$\pi_B = p_B x_B - c_B(x_B).$$

To determine the total license fee we consider two scenarios.

Scenario 1: The innovator does not enter the market. According to auction policy by the innovator in Liao and Sen (2005), if Firm A refuses the payment of license fee, Firm B buys the license, and the willingness to pay of Firm A is the difference between its profit as a licensee and the profit of a non-licensee, that is, $\pi_A - \pi_B$. Let L be the fixed license fee. Then,

$$L = \pi_A - \pi_B.$$

The payoff of the innovator is the sum of the royalty and the fixed license fee. We denote it by

$$\varphi_1 = L + r x_A = p_A x_A - c_A(x_A) - (p_B x_B - c_B(x_B)).$$

Scenario 2: The innovator has an option to enter the market when Firm A refuses to buy a license. Then, Firm B buys the license and at the same time Firm I enters the market. Firm A must pay the difference between its profit as a licensee when Firm I does not enter and its profit when Firm I enters the market with a license to Firm B. The fixed license fee, L, is given by

$$L = \pi_A - \pi_A^e.$$

π_A^e denotes the profit of Firm A when Firm I enters the market and Firm B buys the license. Then, Firm A is a non-licensee. Concise description of the equilibrium in that case is included in Appendix.

We denote the payoff of the innovator in this case by

$$\varphi_2 = L + r x_A = p_A x_A - c_A(x_A) - \pi_A^e.$$

Note that π_A^e is constant, that is, it does not depend on the royalty rate when Firm I does not enter because the optimal royalty rate when Firm I enters the market is different from (and independent of) that when it does enter. On the other hand π_B in φ_1 is not constant. It depends on the optimal royalty rate without entry of Firm I.

Severity and credibility of punishment

If $\pi_A^e < \pi_B$, entry of Firm I with a license to Firm B is more severe punishment for Firm A than a license to Firm B without entry of Firm I when it does not buy the license. π_B is the profit of a non-licensee in a duopoly. On the other hand, π_A^e is the profit of a non-licensee in an oligopoly with three firms in which two firms, Firm I and B, use the new technology. Thus, we can think that π_A^e is usually smaller than π_B

When $\pi_A^e < \pi_B$, we have $\varphi_2 > \varphi_1$, and Scenario 2 is more profitable than Scenario 1 for Firm I. However, its entry is not necessarily a credible threat. The payoff of Firm I when it enters the market with a license to Firm B is the sum of its profit as a firm in an oligopoly and the licensee fee imposed on Firm B. If it is larger than φ_1, entry into the market is a credible threat.

We assume that the output of Firm B, x_B, is positive when the royalty rate is zero. However, if the innovator imposes a negative royalty on Firm A, x_B may be zero. We consider two cases about demand functions. A case where the goods are strategic substitutes and a case where the goods are strategic complements. Also we consider two cases about the innovation. The first is a case where the non-licensee continues to operate when a negative royalty is imposed on the licensee, and the second is a case where the non-licensee drops out of the market with the optimal royalty rate. In the former case the innovation is non-drastic, and in the latter case it is drastic.

We will show that in the case where the non-licensee drops out of the market, Scenario 1 and Scenario 2 are equivalent.

3 The results

3.1 Firm behavior

The first order conditions for profit maximization of Firm A and Firm B are

$$p_A + \frac{\partial p_A}{\partial x_A} x_A - c'_A(x_A) - r = 0, \tag{1}$$

and

$$p_B + \frac{\partial p_B}{\partial x_B} x_B - c'_B(x_B) = 0. \tag{2}$$

The second order conditions are

$$2\frac{\partial p_A}{\partial x_A} + \frac{\partial^2 p_A}{\partial x_A^2} x_A - c''_A(x_A) < 0,$$

and

$$2\frac{\partial p_B}{\partial x_B} + \frac{\partial^2 p_B}{\partial x_B^2} x_B - c''_B(x_B) < 0.$$

Differentiating (1) and (2) with respect to r yields

$$\left(2\frac{\partial p_A}{\partial x_A} + \frac{\partial^2 p_A}{\partial x_A^2} x_A - c''_A(x_A)\right)\frac{dx_A}{dr} + \left(\frac{\partial p_A}{\partial x_B} + \frac{\partial^2 p_A}{\partial x_A x_B} x_A\right)\frac{dx_B}{dr} = 1,$$

and

$$\left(\frac{\partial p_B}{\partial x_A} + \frac{\partial^2 p_B}{\partial x_A x_B} x_B\right)\frac{dx_A}{dr} + \left(2\frac{\partial p_B}{\partial x_B} + \frac{\partial^2 p_B}{\partial x_B^2} x_B - c''_B(x_B)\right)\frac{dx_B}{dr} = 0.$$

From them we obtain

$$\frac{dx_A}{dr} = \frac{2\frac{\partial^2 p_B}{\partial x_B^2} + \frac{\partial^2 p_B}{\partial x_B^2} x_B - c''_B(x_B)}{\Delta},$$

and

$$\frac{dx_B}{dr} = -\frac{\frac{\partial p_B}{\partial x_A} + \frac{\partial^2 p_B}{\partial x_A x_B} x_B}{\Delta},$$

where

$$\Delta = \left(2\frac{\partial^2 p_B}{\partial x_B^2} + \frac{\partial^2 p_B}{\partial x_B^2} x_B - c''_B(x_B)\right)\left(2\frac{\partial p_A}{\partial x_A} + \frac{\partial^2 p_A}{\partial x_A^2} x_A - c''_A(x_A)\right) - \left(\frac{\partial p_B}{\partial x_A} + \frac{\partial^2 p_B}{\partial x_A x_B} x_B\right)\left(\frac{\partial p_A}{\partial x_B} + \frac{\partial^2 p_A}{\partial x_A x_B} x_A\right).$$

We assume

$$\Delta > 0.$$

Also we assume

$$\left|2\frac{\partial p_A}{\partial x_A} + \frac{\partial^2 p_A}{\partial x_A^2} x_A - c''_A(x_A)\right| > \left|\frac{\partial p_A}{\partial x_B} + \frac{\partial^2 p_A}{\partial x_A x_B} x_A\right|,$$

and

$$\left|2\frac{\partial p_B}{\partial x_B} + \frac{\partial^2 p_B}{\partial x_B^2} x_B - c''_B(x_B)\right| > \left|\frac{\partial p_B}{\partial x_A} + \frac{\partial^2 p_B}{\partial x_A x_B} x_B\right|.$$

These assumptions are derived from the stability conditions for duopoly (see Seade (1980) and Dixit (1986)). We get

$$\frac{dx_A}{dr} < 0,$$

and

$$\left|\frac{dx_A}{dr}\right| > \left|\frac{dx_B}{dr}\right|.$$

The goods of the firms are strategic substitutes when $\frac{\partial p_B}{\partial x_A} + \frac{\partial^2 p_B}{\partial x_A x_B} x_B < 0$ and strategic complements when $\frac{\partial p_B}{\partial x_A} + \frac{\partial^2 p_B}{\partial x_A x_B} x_B > 0$. Then,

1. When the goods of the firms are strategic substitutes, $\frac{dx_B}{dr} > 0$.
2. When the goods of the firms are strategic complements, $\frac{dx_B}{dr} < 0$.

3.2 Comparison of two scenarios

Suppose that Firm B does not drop out of the market.

Scenario 1: The innovator does not enter the market.

The condition for maximization of φ_1 with respect to r is

$$\frac{d\varphi_1}{dr} = \left(p_A + \frac{\partial p_A}{\partial x_A} x_A - c'_A(x_A) - \frac{\partial p_B}{\partial x_A} x_B\right)\frac{dx_A}{dr} - \left(p_B + \frac{\partial p_B}{\partial x_B} x_B - c'_B(x_B) - \frac{\partial p_A}{\partial x_B} x_A\right)\frac{dx_B}{dr}$$
$$= \left(r - \frac{\partial p_B}{\partial x_A} x_B\right)\frac{dx_A}{dr} + \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr} = 0. \quad (3)$$

We get the optimal royalty rate for the innovator as follows.

$$\tilde{r}_1 = \frac{1}{\frac{dx_A}{dr}}\left(\frac{\partial p_B}{\partial x_A} x_B \frac{dx_A}{dr} - \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr}\right). \quad (4)$$

Scenario 2: The innovator enters the market when Firm A refuses to buy a license.

The condition for maximization of φ_2 with respect to r is

$$\frac{d\varphi_2}{dr} = \left(p_A + \frac{\partial p_A}{\partial x_A} x_A - c'_A(x_A)\right)\frac{dx_A}{dr} + \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr} = r\frac{dx_A}{dr} + \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr} = 0. \quad (5)$$

Then, we get the optimal royalty rate for the innovator as follows.

$$\tilde{r}_2 = -\frac{1}{\frac{dx_A}{dr}} \frac{dx_B}{dr} \frac{\partial p_A}{\partial x_B} x_A. \quad (6)$$

Suppose that $\tilde{r}_1 = \tilde{r}_2$ and (5) is satisfied. $\frac{dx_A}{dr}$ and $\frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr}$ in (3) and those in (5) are equal. Thus, we have

$$\frac{d\varphi_1}{dr} = -\frac{\partial p_B}{\partial x_A} x_B \frac{dx_A}{dr} < 0.$$

Therefore, $\tilde{r}_1 < \tilde{r}_2$. We have shown

Proposition 1 *The optimal royalty rate in Scenario 1 is smaller than that in Scenario 2.*

From (4) and (6) we can show the following result.

Proposition 2 *Suppose that Firm B does not drop out of the market.*

1. *If the goods of the firms are strategic substitutes, the optimal royalty rates in Scenario 1 and Scenario 2 are negative.*
2. *If the goods of the firms are strategic complements, the optimal royalty rate in Scenario 1 may be positive or negative, but that in Scenario 2 is positive.*

Proof.

1. If the goods of the firms are strategic substitutes, we have $\frac{dx_B}{dr} > 0$. Then, $\tilde{r}_1 < 0$ because $\frac{dx_A}{dr} < 0$. Also we have $\tilde{r}_2 < 0$.

2. If the goods of the firms are strategic complements, $\frac{dx_B}{dr} < 0$. Then, we have $\tilde{r}_1 > 0$ or $\tilde{r}_1 < 0$ depending on $\frac{\partial p_B}{\partial x_A} x_B \frac{dx_A}{dr} - \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr} < 0$ or $\frac{\partial p_B}{\partial x_A} x_B \frac{dx_A}{dr} - \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr} > 0$. On the other hand, $\frac{dx_B}{dr} < 0$ means $\tilde{r}_2 > 0$.

If x_{B} is sufficiently smaller than x_{A} although Firm B does not drop out, it is likely that $\frac{\partial p_B}{\partial x_A} x_B \frac{dx_A}{dr} - \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr} < 0$ and $\tilde{r}_1 > 0$.

A case where Firm B drops out

Suppose that at some royalty rate Firm B drops out of the market. Then, in Scenario 1 we have

$$\left.\frac{d\varphi_1}{dr}\right|_{x_B=0} = r\frac{dx_A}{dr} + \frac{\partial p_A}{\partial x_B}x_A\frac{dx_B}{dr}.$$

From (5) also in Scenario 2 we get the same relation. Thus, when Firm B drops out of the market, Scenario 1 and Scenario 2 are equivalent, and we obtain the following result.

Proposition 3 *In the case where Firm B drops out of the market we obtain the following results.*

1. *If the goods of the firms are strategic substitutes, the optimal royalty rate is negative.*
2. *If the goods of the firms are strategic complements, the optimal royalty rate is positive.*

Proof.

1. If

$$\left.\frac{d\varphi}{dr}\right|_{x_B=0} = r\frac{dx_A}{dr} + \frac{\partial p_A}{\partial x_B}x_A\frac{dx_B}{dr} > 0.$$

Then, $x_B > 0$ at the optimal state for the innovator and we have the case in the previous proposition.

On the other hand, if $\frac{d\varphi}{dr} \leq 0$ when $x_B = 0$, then the licensee is a monopolist and the optimal royalty rate for the innovator is one such that $x_B = 0$. It is negative because $x_B > 0$ with zero royalty and $\frac{dx_B}{dr} > 0$.

2. If $\frac{d\varphi}{dr} < 0$ at $x_B = 0$, then $x_B > 0$ at the optimal state for the innovator and we have the case in the previous proposition.

On the other hand, if $\frac{d\varphi}{dr} \geq 0$ at $x_B = 0$, then the licensee is a monopolist and the optimal royalty rate for the innovator is one such that $x_B = 0$. It is positive because $x_B > 0$ with zero royalty and $\frac{dx_B}{dr} < 0$.

Example

Now let us consider an example. The inverse demand functions are

$$p_A = a - x_A - kx_B,$$

and

$$p_B = a - x_B - kx_A,$$

where $0 < k < 1$. The cost functions of Firm A and Firm B after adoption of the new technology by Firm A are, respectively, $(c-\varepsilon)x_A$ and cx_B. c and ε are positive constants such that $c > \varepsilon$. Suppose that both firms produce. Then,

$$x_A = \frac{(2-k)(a-c) - 2r + 2\varepsilon}{(2-k)(2+k)},$$

and

$$x_B = \frac{(2-k)(a-c)+kr-\varepsilon k}{(2-k)(2+k)}.$$

From them we have

$$\frac{dx_A}{dr} = -\frac{2}{(2-k)(2+k)},$$

and

$$\frac{dx_B}{dr} = \frac{k}{(2-k)(2+k)}.$$

From (4) and (6) the optimal royalty rate in Scenario 1 and that in Scenario 2 are

$$\tilde{r}_1 = -\frac{k(a-c)}{2} < 0,$$

and

$$\tilde{r}_2 = -\frac{(a-c)k^3+2(a+\varepsilon-c)k^2}{4(2-k^2)} < 0.$$

4 Concluding Remark

We have examined the relationship between the definition of license fee and a possibility of negative royalty in an oligopoly with an outside innovator which has an option to enter the market and imposes a combination of a royalty per output and a fixed fee under general demand and cost functions.

In the future research we want to extend the analysis in this paper to more general oligopolistic situation with more than three firms.

Appendix: Main points of a case where the innovator enters the market with a license to Firm B.

Suppose that Firm A refuses to buy the license, and Firm I enters the market with a license to Firm B. Then, Firm B uses the new technology, and Firm A uses the old technology. Denote the output and the profit of the innovator by x_I and π_I. The profits of the firms are

$$\pi_I = p_I x_I - c_A(x_I),$$

$$\pi_B = p_B x_B - c_A(x_B) - r x_B,$$

and

$$\pi_A = p_A x_A - c_B(x_A).$$

In this case Firm B pays the license fee. The first order conditions for profit maximization of Firms I, A and B are

$$p_I + \frac{\partial p_I}{\partial x_I} x_I - c'_A(x_I) = 0, \tag{7}$$

$$p_B + \frac{\partial p_B}{\partial x_B} x_B - c'_A(x_B) - r = 0, \tag{8}$$

$$p_A + \frac{\partial p_A}{\partial x_A} x_A - c'_B(x_A) = 0. \tag{9}$$

The second order conditions are

$$2\frac{\partial p_I}{\partial x_I} + \frac{\partial^2 p_I}{\partial x_I^2} x_I - c''_A(x_I) < 0,$$

$$2\frac{\partial p_B}{\partial x_B} + \frac{\partial^2 p_B}{\partial x_B^2} x_B - c''_A(x_B) < 0,$$

and

$$2\frac{\partial p_A}{\partial x_A} + \frac{\partial^2 p_A}{\partial x_A^2} x_A - c''_B(x_A) < 0.$$

Differentiating (7), (8) and (9) with respect to r yields

$$\lambda_I \frac{dx_I}{dr} + \sigma_{IB} \frac{dx_B}{dr} + \sigma_{IA} \frac{dx_A}{dr} = 0,$$

$$\sigma_{BI} \frac{dx_I}{dr} + \lambda_B \frac{dx_B}{dr} + \sigma_{BA} \frac{dx_A}{dr} = 1,$$

and

$$\sigma_{AI} \frac{dx_I}{dr} + \sigma_{AB} \frac{dx_B}{dr} + \lambda_A \frac{dx_A}{dr} = 0.$$

From them we obtain

$$\frac{dx_I}{dr} = -\frac{\lambda_A \sigma_{IB} - \sigma_{AB}\sigma_{IA}}{\Gamma},$$

$$\frac{dx_B}{dr} = \frac{\lambda_A \lambda_I - \sigma_{AI}\sigma_{IA}}{\Gamma},$$

$$\frac{dx_A}{dr} = -\frac{\sigma_{AB}\lambda_I - \sigma_{AI}\sigma_{IB}}{\Gamma},$$

where

$$\Gamma = \lambda_I \lambda_B \lambda_A - \lambda_I \sigma_{BA}\sigma_{AB} - \lambda_B \sigma_{IA}\sigma_{AI} - \lambda_A \sigma_{IB}\sigma_{BI} + \sigma_{IB}\sigma_{BA}\sigma_{AI} + \sigma_{IA}\sigma_{BI}\sigma_{AB},$$

$$\lambda_I = 2\frac{\partial p_I}{\partial x_I} + \frac{\partial^2 p_I}{\partial x_I^2} x_I - c''_A(x_I),$$

$$\lambda_B = 2\frac{\partial p_B}{\partial x_B} + \frac{\partial^2 p_B}{\partial x_B^2} x_B - c''_A(x_B),$$

$$\lambda_A = 2\frac{\partial p_A}{\partial x_A} + \frac{\partial^2 p_A}{\partial x_A^2} x_A - c''_B(x_A),$$

$$\sigma_{IB} = \frac{\partial p_I}{\partial x_B} + \frac{\partial^2 p_I}{\partial x_I x_B} x_I,$$

$$\sigma_{IA} = \frac{\partial p_I}{\partial x_A} + \frac{\partial^2 p_I}{\partial x_I x_A} x_I,$$

$$\sigma_{BI} = \frac{\partial p_B}{\partial x_I} + \frac{\partial^2 p_B}{\partial x_I x_B} x_B,$$

$$\sigma_{BA} = \frac{\partial p_B}{\partial x_A} + \frac{\partial^2 p_B}{\partial x_B x_A} x_B,$$

$$\sigma_{AI} = \frac{\partial p_A}{\partial x_I} + \frac{\partial^2 p_A}{\partial x_I x_A} x_A,$$

and

$$\sigma_{AB} = \frac{\partial p_A}{\partial x_B} + \frac{\partial^2 p_A}{\partial x_B x_A} x_A.$$

Firm B must pay the following total license fee.

$$r x_B + \pi_B - \pi_A.$$

If Firm B refuses to buy the license, in turn Firm A buys the license. The fixed fee is $\pi_B - \pi_A$. The payoff of Firm I is

$$\varphi_e = \pi_I + r x_B + \pi_B - \pi_A = p_I x_I - c_A(x_I) + p_B x_B - c_A(x_B) - (p_A x_A - c_B(x_A)).$$

The condition for maximization of φ_e with respect to r is

$$\frac{d\varphi_e}{dr} = \left(p_I + \frac{\partial p_I}{\partial x_I} x_I - c'_A(x_I) + \frac{\partial p_B}{\partial x_I} x_B - \frac{\partial p_A}{\partial x_I} x_A\right) \frac{dx_I}{dr}$$
$$+ \left(p_B + \frac{\partial p_B}{\partial x_B} x_B - c'_A(x_B) + \frac{\partial p_I}{\partial x_B} x_I - \frac{\partial p_A}{\partial x_B} x_A\right) \frac{dx_B}{dr}$$
$$- \left(p_A + \frac{\partial p_A}{\partial x_A} x_A - c'_B(x_A) - \frac{\partial p_I}{\partial x_A} x_I - \frac{\partial p_B}{\partial x_A} x_B\right) \frac{dx_A}{dr}$$
$$= \left(\frac{\partial p_B}{\partial x_I} x_B - \frac{\partial p_A}{\partial x_I} x_A\right) \frac{dx_I}{dr} + \left(r + \frac{\partial p_I}{\partial x_B} x_I - \frac{\partial p_A}{\partial x_B} x_A\right) \frac{dx_B}{dr}$$
$$+ \left(\frac{\partial p_I}{\partial x_A} x_I + \frac{\partial p_B}{\partial x_A} x_B\right) \frac{dx_A}{dr} = 0.$$

The optimal royalty rate for the innovator is expressed as

$$\tilde{r} = -\frac{1}{\frac{dx_B}{dr}} \left[\left(\frac{\partial p_B}{\partial x_I} x_B - \frac{\partial p_A}{\partial x_I} x_A\right) \frac{dx_I}{dr} + \left(\frac{\partial p_I}{\partial x_B} x_I - \frac{\partial p_A}{\partial x_B} x_A\right) \frac{dx_B}{dr} \right.$$
$$\left. + \left(\frac{\partial p_I}{\partial x_A} x_I + \frac{\partial p_B}{\partial x_A} x_B\right) \frac{dx_A}{dr} \right].$$

Denote the profit of Firm A in this case by π_A^e.

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