Multidimensional Rank Based Poverty Measures A Case Study: Tunisia

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Abstract

For a long time, poverty measurement has been based strictly on a monetary approach. Since Sen (1976), many poverty measures have been proposed based on an axiomatic foundation, like the Sen (1976) measure, the class of FGT measures (1984), the Shorrocks (1995) measure, otherwise known as the Sen-Shorrocks-Thon (SST) measure.

Due to capabilities approach (first developed by Sen (1985)) and basic needs approach, we realize that the poverty of a person is not only a lack of income but an insufficiency in various attributes of well-being. For better representing the multidimensional aspect of poverty, many approaches of multidimensional poverty measurement have been proposed like the multidimensional axiomatic approach.

We propose in this paper to contribute to the latter approach. For that, we use a two-stage aggregation procedure to develop classes of multidimensional poverty measures which are extension to the multidimensional context of classes of generalized SST measures developed by Chtioui and Ayadi (2013).

We apply our measures on Tunisia using the bootstrap method to see the evolution of multidimensional poverty between 1994 and 2006.

JEL classification: D63, I32, C02, C12, C14

Keywords: multidimensional poverty, multidimensional generalized SST, ethical, separability, bootstrap

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MULTIDIMENSIONAL RANK BASED POVERTY MEASURES

1 Introduction

For a long time, poverty measurement has been based strictly on a monetary approach[1] where poverty is considered as the consequence of unequal distribution of income. Booth (1902) and Rowntree (1901) proposed the first measure of poverty, called the headcount ratio, which has been used accurately for its simplicity. Since Sen (1976) and his criticism of the headcount index, many poverty measures have been proposed based on an axiomatic foundation[2] that can be ranged in two general classes. The first is the class of additively separable poverty measures, which contains the well-known class of FGT measures thanks to Foster, Greer and Thorbecke (1984) and the more general class of subgroup consistent measures developed by Foster and Shorrocks (1991). The second one is the class of non additively separable poverty measures which contains the rank-based measures with the Sen (1976) measure and some of its variations, like the Shorrocks (1995) measure, also known as the Sen-Shorrocks-Thon (SST) measure.

Due to capabilities approach (first developed by Sen (1985)) and basic needs approach, which offer a good framework for the multidimensional approach of poverty, we realize that the poverty of a person is not only due to a lack of income but also to insufficiency in various attributes of well-being.

Focussing on basic needs and capabilities approaches, and for better representing the multidimensional aspect of poverty scientific research has also proposed the following multidimensional poverty measures since 1990:

1. the fuzzy approach of multidimensional poverty, based on the theory of fuzzy sets, which construct multidimensional fuzzy poverty measures (example Cerioli and Zani (1990), Dagum (2002)).
2. the axiomatic approach which consists of an extension to the multidimensional context of some members of the class of additively separable measures. Bourguignon and Chakravarty (1998, 2003) and Alkire and Foster (2008) generalized the FGT class of measures while Tsui(2002) generalized the class of subgroup consistent poverty measures. This multidimensional generalization has been possible by the property of additive separability between individuals.

Based on another property of separability, between attributes, we propose in this paper to develop classes of multidimensional non-additively separable poverty measures. For that aim, we will develop an extension to the multidimensional context of classes of relative and absolute[3] generalized SST measures developed by Chtioui and Ayadi (2013).

Following Sen (1976); Chakravarty, Mukherjee and Ramade (1998), Bourguignon and

[1] The monetary approach has a foundation related to welfare. It considers that the well-being of a person is represented by his utility. The latter is indirectly measured by income or consumption expenditures.

[2] It exists actually literature reviews on one-dimensional poverty measures (see for example, Foster (1984); Seidl (1988); Chakravarty (1990); Foster and Sen (1997) and Zheng (1997)).

[3] The value of a relative poverty measure should remain unchanged after a relative change in both real incomes and the real poverty line. Examples of a relative change are doubling real incomes and the real poverty line or dividing them by the poverty line. The value of a absolute poverty measure should remain unchanged after an absolute change of both real incomes and the real poverty line. Example of an absolute change is adding a real value to all real incomes and the real poverty line. In the monetary approach, a poverty line (or threshold) is defined as the necessary of income needed to attain the minimum of well-being and a person is poor if her income is less than the poverty line.

Chakravarty (1998,2003), Tsui (2002) and Alkire and Foster (2008) who developed multidimensional poverty measures based on a multidimensional axiomatic foundation. Using the normative approach[4], we propose to develop multidimensional social evaluation functions (or social evaluation orderings) that derive both relative and absolute generalized SST poverty measures. For that, we establish the axioms that characterize the multidimensional generalized SST social evaluation orderings (rather than characterizing the measures as with the above-mentioned economists).

Chakravarty et al. (1998), Bourguignon and Chakravarty (1998,2003), Tsui (2002) and Alkire and Foster (2008) used a two-stage aggregation procedure. The first is an aggregation among attributes to construct individual multidimensional privations. And the second is aggregation among individuals to obtain multidimensional poverty measures. Following Gajdos and Weymark (2005), we will develop a reversed order of two-stage aggregation procedure to construct classes of multi-attribute poverty measures, where, in the first step, we will develop classes of generalized SST social evaluation orderings.

We will apply the multidimensional poverty measures to be developed using the bootstrap method to measure the evolution of multidimensional poverty between 1994 and 2006. For that, we construct two indicators: a composite welfare indicator, as an aggregation of different attributes of well-being, using the multivariate correspondence analysis and an indicator of human capital represented by number of years of education of the household head.

The remainder of the article is organized as follows. Notations are introduced in section 2. Section 3 presents the development of multidimensional poverty measures. Within section 4, we present the methodology used with an empirical illustration on Tunisia in section 5. Finally, the last section concludes and points out further research directions.

2 Preliminaries and Notations

We consider distributions represented by matrices $\mathrm{X}_{(n,k)} = (\underline{x}_{.j}, j = 1, \ldots, k)$ representing distributions of k attributes among population of n persons drawn from the set $\mathsf{N} = \{X \subset \bigcup_n \bigcup_k \Re_{(n,k)}\}$ where $\Re_{(n,k)} = \{X_{(n,k)} / x_{ij} \in R$ or $x_{ij} \in R^{+}$ or $x_{ij} \in R^{++}\}$.

The set of individuals is N={i; i=1,...,n} and the set of attributes is K={j; j=1,...,k}.

Also, we consider distributions represented by matrices $\mathrm{X}^{*}_{(n,k)} = (\underline{x}^{*}_{.j}, j = 1, \ldots, k)$ representing censored distributions of k attributes among population of n persons drawn from the set $\mathsf{N}^{*} = \{X^{*} \subset \bigcup_n \bigcup_k \Re_{(n,k)}\}$ where $\Re_{(n,k)} = \{X^{*}_{(n,k)} / x^{*}_{ij} \in R$ or $x^{*}_{ij} \in R^{+}$ or $x^{*}_{ij} \in R^{++}\}$.

N^{*} can take three forms of sets $\{\mathsf{A}^{*}_{1}, \mathsf{A}^{*}_{2}, \mathsf{A}^{*}_{3}\}$ with A^{*}_{1} : the set of all possible distribution matrices, $\mathsf{A}^{*}_{2} = \{X^{*} / X^{*} \subset \mathsf{A}^{*}_{1}$ with non-negative $x^{*}_{ik}\}$ and $\mathsf{A}^{*}_{3} = \{X^{*} / X^{*} \subset \mathsf{A}^{*}_{2}$ with at least one

[4] The normative approach was proposed by Dalton (1920) for the measurement of income inequality. He recommended that an inequality index should have a normative foundation in terms of social evaluation and should incorporate society's judgments regarding inequality. Chakravarty (1983) among others introduced this approach to the literature on poverty measurement.

positive element in each column}.

We denote $\underline{x}^*_{.k}$ by $\underline{x}^*$ in the one-dimensional case.

Let $\underline{x}^*$ be a permutation of $\underline{x}^*$ such that $x_1^* \geq x_2^* \geq \cdots \geq x_n^*$ and $\hat{\underline{x}}^*$ be a permutation of $\underline{x}^*$ such that $\hat{x}_1^* \leq \hat{x}_2^* \leq \cdots \leq \hat{x}_n^*$.

$\underline{z}(z_1, \ldots, z_k)$ represents a vector of k poverty lines, $\underline{z} \in Z, Z \subset \bigcup_k R^k_{++}$.

Let W($\underline{x}$) be a social evaluation function, the equally distributed equivalent (EDE) income $\Xi(\underline{x})$ is given by[5]

$$W(\Xi(\underline{x})) = W(x_1, \ldots, x_n). \quad (1)$$

3 Development of multidimensional poverty measures

In developing a multidimensional poverty measure, Chakravarty et al. (1998), Bourguignon and Chakravarty (1998,2003), Tsui (2002) and Alkire and Foster (2008) all followed the same schema as in the one-dimensional context, which is identification of the poor, and then in the construction of a multidimensional poverty measure.

For identification of the poor, Bourguignon et Chakravarty (1998,2003) and Tsui (2002) adopted the notion of the poverty line and generalized it. Their generalization takes into account the multidimensional nature of poverty. Inspite of having one poverty line, there will be a vector of poverty lines which corresponds to the vector of the attributes of well-being. Tsui (2002) developed a union definition of multidimensional poverty : a person is poor if he is deprived in at least one attribute (a union definition of poverty). For example, if it is rich and illiterate. Others consider an intersection definition of poverty where a person is considered a poor if its dotation in each attribute is below its corresponding poverty line. Bourguignon and Chakravarty (1998) gave another identification of the poor in relation to the nature of attributes and the relation between them in the case of substitute-attributes and in the case of complementary attributes. We adopt in this article the union definition of poverty which requires independence between attributes regarding our use of attribute separability axiom.

To avoid the choice's arbitrariness of one multidimensional poverty measure, the adopted solution is to develop a reasonable axiomatic structure that leads to a class of multidimensional poverty measures . For Bourguignon and Chakravarty (2003) and Alkire and Foster (2008) they developed axioms that characterize classes of multidimensional FGT poverty measures while Tsui (2002) developed axioms that characterize class of multidimensional relative and absolute subgroup consistent poverty measures[6].

Now that the axiomatic foundation is established (in section 3), we construct the

[5] If each person receives the equally distributed equivalent income then we obtain a distribution which is socially equivalent to the initial distribution but is more equal.
[6] For a recent survey on the axiomatic approach of multidimensional poverty measurement see Bibi(2005).

corresponding multidimensional poverty measures using a two-stage aggregation procedure where we extend the generalized SST poverty measures to the multidimensional context.

3.1 Multidimensional Axiomatic Foundation

The SST poverty measure uses the Gini social evaluation function. It is iportant to mention that the generalized SST poverty measures developed in Chtioui and Ayadi (2013) use a generalized Gini social evaluation function. That's why, the axioms developed by Gajdos and Weymark (2005) to characterize a multidimensional generalized Gini social evaluation ordering (and its corresponding social evaluation function) can be taken to the multidimensional poverty context to characterize multidimensional generalized SST social evaluation ordering that we will develop in this article.

A social evaluation ordering is a binary relation $\succeq$ on the set of distribution matrices A^*. The relation $\succeq$ is interpreted as "weakly socially preferred to". The symmetric and asymmetric factors of $\succeq$ are $\sim$ and $\succ$, respectively. This order will be represented by a function $W^* : A^* \to R$.

The multidimensional generalized SST social evaluation ordering satisfies the following axioms.

(A1) **Ordering**(**ORD**) : The binary relation is reflexive, complete and transitive on A^*.

(A2) **Continuity**(**CON**) : $\forall X^* \in A^*$, the sets $\{ Y^* | Y^* \succ X^* \}$ and $\{ Y^* | X^* \succ Y^* \}$ are open sets.

It ensures that W^* is a continuous function.

(A3) **Monotonicity**(**MON**) : $X^* \succ Y^*$ with $x^*_{ij} \geq y^*_{ij}$ for all i $\in$ N and all j $\in$ k whenever X^* is obtained from Y^* by increasing the quantity of some attributes for one (or many) persons without decreasing the quantities of the others.

(A4) **Anonymity**(**ANON**) : $X^* : \Pi X^*$ for all n*n permutation matrices Π.

It states that W^* is indifferent to the characteristics of the persons like their identity : name,age etc and it is only concerned by the dotations.

According to Sen (1976), a poverty measure should be sensitive to inequality among the poor. If inequality between poor diminishes incomes, then the poverty measure should decrease and if inequality increases, then the poverty measure should increase. One way of representing the increase or the decrease of income inequality is the transfer of incomes between two individuals. That was first proposed in the literature on income inequality by Dalton (1920) and Pigou (1914) involving the redistribution on the income in the whole population. Sen (1976) and other scholars proposed transfer axioms that are concerned with transfers among the poor class. Multidimensional generalizations of the Pigou-Dalton principle were first developed by Kolm (1977). We retain here the two well-known which are uniform to Pigou-Dalton majorization transfer and uniform majorization transfer.

Definition 1 *The distribution X^* uniformly Pigou-Dalton majorizes the distribution Y^* if X^* is obtained from Y^* by a finite sequence of Pigou-Dalton transfers*[7] *and denoted by $X^* \succ_{PD} Y^*$.*

Then, the distribution X^* is socially weakly preferred to the distribution Y^* if X^* uniformly Pigou-Dalton majorizes Y^*.

(A5) **Weak Uniform Pigou-Dalton Majorization**(**WUPM**)
$X^* \succ_{PD} Y^*$ then $X^* \succeq Y^*, \forall X^*, Y^* \in \mathsf{A}^*$.

The second multi-attribute generalization Pigou-Dalton transfer principle can be stated as
(A5') **Weak Uniform Majorization**(**WUM**) $X^* \succ_U Y^*$ then $X^* \succeq Y^*, \forall X^*, Y^* \in \mathsf{A}^*$.

$X^* \succ_U Y^*$ means that the distribution X^* uniformly majorizes the distribution Y^* and this if $X^* = BY^*$, B is a bistochastic matrix[8].

These two principles are equivalent in the univariate case(k=1) but not in the multi-attribute case (when $k > 1$ and $n \geq 3$). By theorem 1 in Gajdos and Weymark (2005), if the social evaluation function satisfies anonymity (A4) and shows strong attribute separability (A10) then the two multi-attribute transfer axioms are equivalents. It is important to note that we only consider, in this case, transfers done within the poor population and involving one attribute where the redistribution in one attribute is independent from the redistribution in another attribute.

In order to characterize the class of absolute generalized Gini inequality indexes (and in consequence their underlying social evaluation function), Weymark (1981) added to the preceding axioms a comonotonic additivity axiom. Gajdos and Weymark (2005) generalized it to the multidimensional context to axiomatize multidimensional generalized Gini indexes (and their underlying social evaluation functions). Based on the relation between Gini inequality index and the Sen-Shorrocks-Thon poverty measure, we introduce here similar comonotonic additivity axioms to characterize our multidimensional generalized Sen-Shorrocks-Thon social evaluation orderings.

Definition 2 *A distribution matrix X^* is non-increasing comonotonic if*

$$x^*_{1j} \geq x^*_{2j} \geq \cdots \geq x^*_{nj} \text{ for all } j \in K.$$

This means that person 1 has at least more that person 2 in all attributes until the person n has less than the others in all attributes.

Let A^*_{WC} be the set of non-increasing comonotonic matrices in A^*.

[7] A Pigou-Dalton transfer between two persons leads to convex post-transfer dotation. Let x_1 and x_2 be the dotation respectively of person 1 and person 2 before the transfer and x'_1 and x'_2 be their dotation after the transfer. Then, we have $x'_1 = \alpha x_1 + (1-\alpha)x_2$ and $x'_2 = (1-\alpha)x_1 + \alpha x_2$ with $\alpha \in (0,1)$.

[8] An n*n bi-stochastic matrix is a matrix with the sum of each of its columns and of each of its lines is equal to 1.

(A6) **Welfare Weak Comonotonic Additivity** (WWCA):

$\forall X^* \in A^*_{WC}, Y^* \in A^*_{WC}$ and $W \in A^*_{WC} \cup \Im_{WC}$

$X^* \succeq Y^*$ if and only $X^* + W \pm Y^* + W$,

where X^* and Y^* differ in only one column and W differs from the null matrix in only the same column, $\Im_{WC}$ is the set of non-negative and non-increasing comonotonic matrices.

The following axiom threatens the separability between attributes where attributes in one set (K_1) are separable from attributes belonging to another set (K_2).

(A7) **Strong Attribute Separability** (SAS): $\forall (K_1, K_2) \in \overline{K}, X^*_{K_1}, Y^*_{K_1}, X^*_{K_2}$ and $W^*_{K_2}$

$(X^*_{K_1}, X^*_{K_2}) \succeq (Y^*_{K_1}, X^*_{K_2})$ if and only if $(X^*_{K_1}, W^*_{K_2}) \succeq (Y^*_{K_1}, W^*_{K_2})$

$\overline{K} = \{(K_1, K_2) \subset K \times K, K_1 \cap K_2 = \emptyset, K_1 \cup K_2 = K, K_1 \neq \emptyset, K_2 \neq \emptyset\}$.

In order to directly compare the poverty level between two distributions with different sizes, we need first to replicate them to the same size.

(A8) **Population Principle, Replication Invariance**(**PP,RI**) : $X^* : X^{*q}$ where X^{*q} is the q-fold replication of X^* and q is a positive integer.

The axioms (A1),(A2),(A3),(A4),(A5),(A6),(A7)[9] characterize classes of multidimensional generalized SST poverty measures that are continuous, monotonic, symmetric, verify one of the transfer axioms that involve one attribute, satisfy the comonotonic additivity axiom and the attribute separability axiom.

Adding the axiom (A8) leads to obtain a particular family of multidimensional generalized SST poverty measures that are replicant invariant.

3.2 The two-stage aggregation procedure

The construction of a multidimensional generalized SST poverty measure is obtained via a two-stage aggregation procedure. The first stage consists to aggregate the distributions of each attribute in order to obtain a univariate generalized SST social evaluation function or its particular representation, the EDE attribute function defined as $\Xi^n_{GSST}(\underline{x}^*) : A^* \to R$ and has the form

$$\begin{aligned} \Xi^n_{GSST}(\underline{x}^*) &= \Sigma^n_{i=1} a_i x^*_i ; a_1 \le a_2 \le \cdots \le a_n \\ &= \frac{\Sigma^n_{i=1} w^n_i x^*_i}{\Sigma^n_{i=1} w^n_i} \end{aligned} \tag{2}$$

in the case of welfare-ranked attribute ($x^*_1 \ge x^*_2 \ge \cdots \ge x^*_n$)[10].

9 The one-dimensional counterparts of these axioms characterize classes of generalized SST poverty measures. (see Chtioui and Ayadi 2013)

[10] Note that each attribute has its own weighting system $\{a_1, \ldots, a_n\}$.

In the second-stage, the different univariate generalized SST social evaluation functions are aggregated among attributes in order to construct multidimensional EDE attributes function which represent the underlying multidimensional generalized SST social evaluation ordering.

To construct a multidimensional relative generalized SST poverty measure, the multidimensional EDE attributes function $\Xi_R^M(X^*): \mathrm{A}^* \to R$ is function of $\Theta_R^M(X^*)$ which is equal to the scalar that solves

$$\Xi_R^M(X^*)\mathbf{1} = \Theta_R^M(X^*)Z : X^*, \forall X^* \quad \in \mathrm{A}^*, \tag{3}$$

where **1** is a matrix whose elements are all equal to 1 and Z denotes the matrix for which every entry in the j^{th} column is equal to z_j.

Then, the multidimensional relative generalized SST poverty measure associated with $\succeq$ is the function $MP_{GSST}^R: \mathrm{A}^* \to R$ defined by

$$MP_{GSST}^R(X^*, \underline{z}) = 1 - \Theta_R^M(X^*), \forall X^* \quad \in \mathrm{A}^*. \tag{4}$$

To construct a multidimensional absolute generalized SST poverty measure,the multidimensional EDE attributes function $\Xi_A^M(X^*): \mathrm{A}^* \to R$ is function of $\Theta_A^M(X^*)$ which is equal to the scalar that solves

$$\Xi_A^M(X^*)\mathbf{1} = Z - \Theta_A^M(X^*)\mathbf{1} : X^*, \forall X^* \quad \in \mathrm{A}^*. \tag{5}$$

Then, the multidimensional absolute generalized SST poverty measure associated with $\succeq$ is the function $MP_{GSST}^A: \mathrm{A}^* \to R$ defined by

$$MP_{GSST}^A(X^*, \underline{z}) = \Theta_A^M(X^*), \forall X^* \quad \in \mathrm{A}^*. \tag{6}$$

The number of attributes and the version of comonotonic additivity axiom retained imply different functional forms of the second-stage aggregation function leading to different characterizations of the multidimensional generalized SST social evaluation ordering and also to different characterizations of multidimensional generalized SST measures.

3.3 Multidimensional relative generalized SST poverty measures

In the literature on income poverty, Blackorby and Donaldson (1980) introduced an invariance axiom which is scale-invariance axiom that stipulates that the value of a poverty measure remains unchanged after a relative change[11] of the distribution[12] . For an ethical poverty measure this stipulates that the value of the underlying homothetic social evaluation function remains the same after such change. Poverty measures that satisfy this axiom are called relative poverty measures.

A multidimensional generalization of this axiom is as follows

(A10) **Weak Homotheticity(WHOM)**

[11] An income distribution $\underline{x}'$ is obtained from an income distribution $\underline{x}$ by a relative change if $\underline{x}' = \alpha\underline{x}, \alpha > 0$.

[12] This axiom is developed in order to let the poverty measure stay invariant to a change in the unit measurement.

$X^* \succeq Y^*$ if and only if $\alpha X^* \succeq \alpha Y^*$, $\forall X^*, Y^* \in A^*, \alpha > 0$.

It requires that doubling (for example) all dotations leaves the value of W^* unchanged and also requires the multidimensional social evaluation ordering to be homothetic.

This axiom is valid, only, in the presence of attributes with the same nature example when the attributes represent incomes in different time periods. For that, Tsui (1995) introduced in the literature of multidimensional inequality, an homothetic axiom that is appropriate to the case of attributes of different nature like income, education. Then, Tsui (2002) introduced it to the literature on multidimensional poverty.

(A11) **Strong Homotheticity(SHOM)**[13]

$X^* \succeq Y^*$ if and only if $\Lambda X^* \succeq \Lambda Y^*$, $\forall X^*, Y^* \in A^*, \Lambda = \mathrm{diag}(\alpha_1, \ldots, \alpha_k), \alpha_j > 0, j = 1, \ldots, k$.

It requires the social evaluation ordering to be invariant, and the value of the corresponding social evaluation function and the value of the derived multidimensional poverty measure remain unchanged to independent relative changes.

We assume that the dotations in the different attributes are positive ($X^* \in A_3^*$).

Following Gajdos and Weymark (2005), we develop a characterization of multidimensional generalized SST social evaluation ordering as

Proposition 1 If $k \geq 2$, then the binary relation $\succeq$ on A_3^* satisfies ORD, CON, MON, ANON, WUPM, SAS, WWCA and SHOM if and only if there exists an n*k matrix A of positive coefficients with $a_{.j}$ non-decrasing in i and $\Sigma_{i=1}^n a_{ij} = 1$ for all $j \in K$ and a positive vector $\gamma \in R_{++}^k$ with $\Sigma_{j=1}^k \gamma_j = 1$ such that $\succeq$ can be represented by a multidimensional EDE attributes function defined on A_3^* given by

$$\Xi_R^{MS}(X^*) = \Pi_{j=1}^k (\Sigma_{i=1}^n a_{ij} x_{ij}^*)^{\gamma_j}, \forall X^* \in A_3^*, \tag{7}$$

The matrix of coefficients A and the vector γ are unique.

We have $X^* \succeq Y^*$ if and only if $\Xi_R^{MS}(X^*) \geq \Xi_R^{MS}(Y^*), \forall X^*, Y^* \in A_3^*$.

Proof. Similar to the proof of theorem 6 in Gajdos and Weymark (2005).

For the functional form of the multidimensional EDE attributes function in proposition 4 (equation 7), equation (3) is satisfied if

$$\Pi_{j=1}^k [\Sigma_{i=1}^n a_{ij} \Theta_R^M z_j]^{\gamma_j} = \Pi_{j=1}^k [\Sigma_{i=1}^n a_{ij} x_{ij}^*]^{\gamma_j}, \forall X^* \in A_3^*.$$

Hence, we obtain the following multidimensional relative generalized SST poverty measure

[13] We use the terminology of Gajdos and Weymark (2005).

$$\mathrm{MP}^{\mathrm{R}}_{\mathrm{GSST}}(\mathrm{X}^*, \underline{\mathrm{z}}) = 1 - \Pi^{\mathrm{k}}_{\mathrm{j=1}}[\Sigma^{\mathrm{n}}_{\mathrm{i=1}} \mathrm{a}_{\mathrm{ij}} (\frac{\mathrm{x}^*_{\mathrm{ij}}}{\mathrm{z}_{\mathrm{j}}})]^{\gamma_{\mathrm{j}}}. \quad (8)$$

3.4 Multidimensional absolute generalized SST poverty measures

In the literature on income poverty, Blackorby and Donaldson (1980) introduced an invariance axiom which is translation invariance axiom. It stipulates that the value of a poverty measure remains unchanged after an absolute change[14] of the distribution. For an ethical poverty measure, this stipulates that the value of the underlying translatable social evaluation function remains the same after such change. Poverty measures satisfying this axiom are called absolute poverty measures.

A multidimensional generalization of this axiom is as follows
(A12) **Weak Translatability**(**WTRA**)
$X^* \succeq Y^*$ if and only if $X^* + \alpha 1 \succeq Y^* + \alpha 1, \forall X^*, Y^* \in \mathsf{A}^*$ for which $X^* + \alpha 1 \in \mathsf{A}^*, Y^* + \alpha 1 \in \mathsf{A}^*$ and $\alpha \in R$,
where **1** is a distribution matrix whose elements are all equal to 1.

It requires that adding a real value to all dotations leaves the value of W^* unchanged. It also requires that the multidimensional social evaluation ordering can be translatable.

This axiom is valid only in the presence of attributes with the same nature. For that, Tsui (1995) introduced in the literature on multidimensional inequality a translatable axiom appropriate to the case of attributes of different nature. Tsui (2002) introduced it to the literature on multidimensional poverty
(A13) **Strong Translatability**(**STRA**)[15]
$X^* \succeq Y^*$ if and only if $X^* + 1\Lambda \succeq Y^* + 1\Lambda, \forall X^*, Y^* \in \mathsf{A}^*, \Lambda = diag(\alpha_1, \ldots, \alpha_k), \alpha_j > 0, j = 1, \ldots, k$.

It requires the social evaluation ordering to be invariant and the value of the corresponding social evaluation function and the value of the derived multidimensional poverty measure to remain unchanged to independent absolute changes.

Following Gajdos and Weymark (2005), we develop a characterization of multidimensional generalized SST social evaluation ordering as

Proposition 2 *If* $k \geq 2$*, then the binary relation* $\succeq$ *on* A^*_3 *satisfies ORD, CON, MON, ANON, WUPM, SAS, WWCA and STRA if and only if there exists an n* matrix A of positive coefficients with* $a_{.j}$ *nondecrasing in i and* $\Sigma^n_{i=1} a_{ij} = 1$ *for all* $j \in K$ *and a positive vector* $\gamma \in R^k_{++}$ *with* $\Sigma^k_{j=1} \gamma_j = 1$ *such that* $\succeq$ *can be represented by a multidimensional EDE attributes function*

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[14] An income distribution $\underline{x}'$ is obtained from an income distribution $\underline{x}$ by an absolute change if $\underline{x}' = \underline{x} + \alpha \underline{1}, \alpha > 0$ and $\underline{1}$ is an n vector with elements equal 1.
[15] We use the terminology of Gajdos and Weymark (2005).

defined on A_1^* *given by*

$$\Xi_A^{MS}(X^*) = \Sigma_{j=1}^{k}\gamma_j(\Sigma_{i=1}^{n}a_{ij}x_{ij}^*), \forall X^* \in \mathsf{A}_1^*, \tag{9}$$

The matrix of coefficients A and the vector γ are unique.

We have $X^* \succeq Y^*$ if and only if $\Xi_A^{MS}(X^*) \geq \Xi_A^{MS}(Y^*), \forall X^*, Y^* \in \mathsf{A}_1^*$.

Proof. Similar to the proof of theorem 9 in Gajdos and Weymark (2005).

For the functional form of the multidimensional EDE attributes function in proposition 1 (equation 9), equation (5) is satisfied if

$$\Sigma_{j=1}^{k}\gamma_j[\Sigma_{i=1}^{n}a_{ij}(\Theta_A^M - z_j)] = \Sigma_{j=1}^{k}\gamma_j[\Sigma_{i=1}^{n}a_{ij}x_{ij}^*], \forall X^* \in \mathsf{A}_1^*.$$

Hence, we obtain the following multidimensional absolute generalized SST poverty measure

$$MP_{GSST}^{A}(X^*, \underline{z}) = \Sigma_{j=1}^{k}\gamma_j[\Sigma_{i=1}^{n}a_{ij}(z_j - x_{ij}^*)]. \tag{10}$$

3.5 A different system of weighting $\{a_{ij}\}$

The weighting function considered in the previous section is a weighting function for each attribute separately. The procedure has been to order the allocations of the attribute k (k=1,. . . ,K) in a decreasing way and then obtain a non-increasing distribution of an attribute k. So, we have $a_{1k} \leq a_{2k} \leq \cdots \leq a_{nk}$, k=1,. . . ,K.

We propose here a different weighting function $\{a_{ij}\}$ that involves all the attributes together and this for dotations censored at the poverty line $\{(\mathrm{x}_\mathrm{k}^{'})^*\}$. For that, we order the attributes according to their importance in representing the poverty.

The attributes are ordered in a decreasing way

1) we order decreasingly the first attribute (k=1), we obtain :

$x_{11} \geq \ldots \geq x_{n1}$: non-increasing distribution of the attribute k=1

2) for each value of $\underline{x}_{.1}$, we order decreasingly the second attribute (k=2), we obtain :

$x_{11}; x_{12} \geq \ldots \geq x_{n2}$: non-increasing distribution of the attribute k=2

$\vdots$

$x_{n1}; x_{12} \geq \ldots \geq x_{n2}$: non-increasing distribution of the attribute k=2

3) for each value of $\underline{x}_{.2}$, we order decreasingly the third attribute (k=3), we obtain :

$$x_{11};\begin{cases} x_{12};x_{13} \geq \cdots \geq x_{n3}: \text{non-increasing distribution of the attribute } k=3 \\ \vdots \\ x_{n2};x_{13} \geq \cdots \geq x_{n3}: \text{non-increasing distribution of the attribute } k=3 \end{cases}$$

$$\vdots$$

$$x_{n1};\begin{cases} x_{12};x_{13} \geq \cdots \geq x_{n3}: \text{non-increasing distribution of the attribute } k=3 \\ \vdots \\ x_{n2};x_{13} \geq \cdots \geq x_{n3}: \text{non-increasing distribution of the attribute } k=3 \end{cases}$$

4) and so on until the last attribute.

Example:
We have 8 persons with the following dotations in three attributes person A(1,1,5);person B(1,1,1);person C(2,2,2);person D(2,2,5); person E(2,1,5);person F(2,5,2);person G(1,5,1) and person H(2,5,1). By applying the procedure we obtain the following order

	First step	Second step	Third step		Weight attributed
Person F	2	5	2	$\rightarrow$	a_1
Person H			1	$\rightarrow$	a_2
Person D		2	5	$\rightarrow$	a_3
Person C			2	$\rightarrow$	a_4
Person E		1	5	$\rightarrow$	a_5
Person G	1	5	1	$\rightarrow$	a_6
Person A		1	5	$\rightarrow$	a_7
Person B			1	$\rightarrow$	a_8

We conclude that the person B is the poorest candidate and the person F is the richest.

Let's also apply the procedure of the first function of weighting to see the difference between the two systems of weighting.

Attribute 1				Attribute 2				Attribute 3			
C	2	$\rightarrow$	a_{11}	F	5	$\rightarrow$	a_{12}	A	5	$\rightarrow$	a_{13}
D	2	$\rightarrow$	a_{21}	G	5	$\rightarrow$	a_{22}	D	5	$\rightarrow$	a_{23}
E	2	$\rightarrow$	a_{31}	H	5	$\rightarrow$	a_{32}	E	5	$\rightarrow$	a_{33}
F	2	$\rightarrow$	a_{41}	C	2	$\rightarrow$	a_{42}	C	2	$\rightarrow$	a_{43}
H	2	$\rightarrow$	a_{51}	D	2	$\rightarrow$	a_{52}	F	2	$\rightarrow$	a_{53}
A	1	$\rightarrow$	a_{61}	A	1	$\rightarrow$	a_{62}	B	1	$\rightarrow$	a_{63}
B	1	$\rightarrow$	a_{71}	B	1	$\rightarrow$	a_{72}	G	1	$\rightarrow$	a_{73}
G	1	$\rightarrow$	a_{81}	E	1	$\rightarrow$	a_{82}	H	1	$\rightarrow$	a_{83}

With the first weighting function, the person H is ranked fifth with the first attribute and receives a weight a_{51}, third with the second attribute and receives a weight a_{32} and eighth with the third attribute and receives a weight a_{83}, so, he has three ranks and three weights while with the second weighting function he is ranked second and receives only one weight a_2.

In our context of multidimensional poverty, the second system of weighting is more accurate than the first one because it reflects the notion of multidimensional poverty. To relate this in a meaningful example, let the vector of threshold be $\underline{z} = \{1.5,3,2\}$. With the second weighting function, the person H(2,5,1) is poor because $x_{H,3} = 1 < 2$(union poverty), while with the first one, H is not poor in the first attribute, he is poor in the second attribute and is not poor with the last one : The person is attributed by characteristics as in one-dimensional way with the second weighting, the person is considered poor in all attributes together : a multidimensional way.

Then, the EDE attribute function corresponding to the second weighting function is:

$$\Xi^{n}_{GSST}(\underline{x}'*) = \sum_{k=1}^{K} \sum_{i=1}^{n} a_i (x')^{*}_{ik} ; a_1 \leq \cdots \leq a_n \tag{13}$$

4 Methodology

We present in this section the statistical inference procedure and the data treatment. We construct from two indicators (or variables) a composite welfare indicator and number of years of education of the household head.

4.1 Statistical Inference Procedure

Estimation of the composite welfare indicator

It consists of constructing an index of the assets owned by the household based on two categories of assets : household durables (having TV,bicycle,etc) and household conditions (source of drinking water, floor material,etc).

The index of assets has the following expression

$$A_i = \Sigma_{j=1}^{J} \gamma_j a_{ij}, \tag{14}$$

where A_i is the index of assets owned by the household i (i=1,...,n), a_{ik} the different assets considered (j=1,...,J) and γ s are the coefficients to estimate.

We use the multivariate correspondence analysis (MCA) which is an appropriate method of estimation of the coefficients due to the presence in our surveys of categorical variables. In order to analyze categorical variables, we consider that each primary indicator (j) has K_j response categories so the index of assets becomes a function of the categories as follows

$$A_i = \frac{\Sigma_{j=1}^{J}\Sigma_{k=1}^{K_j} W_k^j I_{ijk}}{J}, \quad (15)$$

where I_{ijk} are binary variables (take value 1 if the household i owns the category k and 0 if not) and W_k^j are the coefficients to estimate.

Statistical Inference

For poverty measurement, we, first, need to choose a poverty threshold. For education, we choose a poverty threshold corresponding to a primary diploma achieved. For the composite welfare indicator, we adopt a poverty threshold as a proportion k of the median.

In order to see the evolution of multidimensional poverty in Tunisia, we computed a multidimensional absolute generalized SST poverty measure:

$$MP_{GSST}^{A}(X^*, \underline{z}) = \Sigma_{j=1}^{k}\gamma_j [\Sigma_{i=1}^{n} \frac{(i^{\nu} - (i-1)^{\nu})}{n^{\nu}}(z_j - x_{ij}^*)]. \quad (16)$$

With $\gamma_1 = \gamma_2 = 0.5$, $\nu = 2,3,4$; where ν is a parameter of relative deprivation aversion) for 1994, 2001 and 2006 (we only consider the values relatives to k equal to 50%).

To eliminate sampling variability, we used the bootstrap techniques in order to compute the standard errors, to construct confidence intervals for the estimates and to conduct hypothesis tests[16].

- First, we computed the estimated values of the poverty measures and their standard errors.
- Second, we used the percentile method, an alternative method developed by Hall (1992), for developing the confidence intervals of the estimates.
- Finally, we carried out hypothesis tests. We tested the null hypothesis of poverty equality between A and B against the alternative hypothesis that A has less poverty than B. In order to do this, we constructed the following test.

$$H_0 : P(x_A, z_A) = P(x_B, z_B) \text{ versus } H_1 : P(x_A, z_A) < P(x_B, z_B) \quad (17)$$

where P(x_A, z_A): poverty measure of A and P(x_B, z_B): poverty measure of B and significance level equal 5%.

We computed the confidence interval of the bootstrap estimation for the difference $P_A - P_B$. If zero is not found in this interval, we rejected H_0 and concluded that A has less or more poverty than B.

4.2 Description of Data and Selection of the attributes
Data

[16] For more details of these techniques, see Chtioui and Ayadi (2013).

Our empirical analysis will be carried out using primary micro data of three demographic and health surveys conducted by the National Office for Family and Population (ONFP) as a part of the Pan Arab Project for the development of children (PAPCHILD project) in collaboration with the Pan Arab Project for Family Health developed by the League of Arab Countries. The surveys contain demographic information, housing conditions, health status, social-cultural characteristics, education status of each member of the family. The 1994's survey covered 6085 households (with 33750 individuals) reduced to 6037 households, the 2001's survey covered 6083 households (with 31505 individuals) reduced to 6051 households and the 2006's survey covered 8682 (with 42677 individuals) reduced to 8186 households.

The surveys contain, with information on women fertility (like women fertility preferences), demographic information, housing conditions, health status, socio-cultural characteristics, education status of the family but no information on expenditures. The non-monetary data in these surveys will be used to construct a composite welfare indicator. This reduces the multidimensional non-monetary information into one-dimensional indicator (welfare indicator) but has the advantage to avoid problems encountered with income like price deflation.

Choice of the attributes for the Composite Welfare Indicator

We select fourteen variables available in the three data that can represent some primary indicators of well-being or some functionnings achieved. We classify them into four categories as shown in table 1. Then, in order to harmonize the data in the three surveys, we recode and rename the modalities of the selected variables. Next, we merge the three data and conduct the MCA by applying the program developed by Nenadic and Greenacre(2006) with the programation language R. The attributes' weights estimated (W_k^j in equation 19) are used to compute the values of the composite welfare for the three data (equation 19).

Table 1: List of 14 variables

Dimensions (Primary Indicators)	Attributes (Variables)
1. Housing Conditions	Floor materiel
	Type of house
	Number of persons per bedroom
	Source of energy
2. Water and Toilet availability	Water inside the house
	Toilet inside the house
3. Ownership of household durables	Refrigerator, Gas cooker
	Water heating, Washing machine
	Conditioner
4. Access to the technologies of information and telecommunication (TIC)	Television, Radio Telephone

Construction of the variable education

The ONFP's data contain information on the diploma obtained but not on scolarity years. That's why, we assign to each diploma achieved the corresponding number of years according to the tunisian academic system. In Tunisia,with the old academic system, we spend 6 years to have a primary diploma, 7 years to have the baccalaureate degree (within the secondary school), 5 years to have the professional diploma (within the secondary school) and 4 years to have a 4 years diploma in the university. With the new system, the secondary diploma has been divided into two diplomas : the preparatory with 3 years and the baccalaureate diploma (4years) or the professional diploma (2 years).

Table 2: Number of years of Education the household head

Diploma	Years
Never been scholarized	3
Primary diploma	3 + 6
Preparatory diploma	3 + 6 + 3
Professional diploma	3 + 6 + 3 + 2
Baccalaureate diploma	3 + 6 + 3 + 4
University diploma	3 + 6 + 7 + 4

5 Results and discussion

5.1 Results of the Multivariate Correspondence Analysis

Analysis of the MCA

We conduct the MCA on the attributes selected. The histogram of the eigenvalues (see appendix D) shows a separation between the first factorial axis and the other axes. In fact, it explains 21.30% of total inertia and each one of the rest of the axes explains less than 10%. Hence, the first factorial axis can be considered as the axis of welfare and of the analysis of the MCA carried out with regard to the relations between the variables of this axis. Also, it will be used to compute the values of the composite welfare indicator of the households in the three data and then to compute poverty measures of the years 1994, 2001 and 2006. While, the wealth in welfare is represented by variables positively correlated to the first factorial axis, the poverty in welfare is represented by variables negatively correlated to that axis.

The representation of the first factorial plan in table 3 shows the importance of the first factorial axis in representing household's welfare. The variables representing wealth in welfare are in the left side (positively correlated to the axis) and those representing poverty in welfare are in the right side (negatively correlated to the axis).

Table 3: The first factorial Plan

Factor 2 (Vertical axis): 9.14% of inertia	
POVERTY	**WEALTH**
Between 2 and 4 persons per bedroom Having water but outside the house Having toilet but outside the house No Access to TIC (Telephone) No Durables (Water heating, Washing machine, Conditioner)	Floor material of high quality Popular house or Villa Source of energy : electricity Access to TIC (Television, Radio) Durables (Refrigerator, Gas cooker)
Floor material of low quality House of other types More than 4 persons per bedroom Source of energy : Gas and others No water, No toilet No Durables (Refrigerator, Gas cooker) No Access to TIC (Television, Radio)	Apartment Less than 2 persons per bedroom Having water inside the house Having toilet inside the house Durables (Water heating, Washing machine, Conditioner) Access to TIC (Telephone)
	Factor 1 (Vertical axis): 21.30% of inertia
Source : MCA on ONFP's data of 1994, 2001 and 2006	

The first factorial axis distinguishes between rich population and poor population. The second factorial axis distinguishes between two categories inside each population; rich(respectively poor) and very rich (respectively very poor) households.

Consistency of the MCA

Table 8 (see appendix) presents the relations of the variables with the two first factors and contains their scores and their contributions. All the variables verify the propriety of first axis ordering consistency (FAOC) which requires that the ordinal structure of each variable is respected by the ordinal structure of the scores of its modalities. For example, considering the variable gaz cooker, its modality "to have it" is in the left (positively correlated to the first factorial axis) and its other modality is in the right (negatively correlated to the first factorial axis). This propriety ensures that the composite welfare indicator reflects a welfare situation.

The weights analysis of the fourteen attributes shows that the ownership of durables, having floor material of high quality, having a popular house, a villa or an apartment, having less than 2 persons per bedroom, having electricity, having water and toilet inside the house and access to TIC contributes positively to the composite welfare indicator of the household. In the other side, having floor material of bad quality, not having a good house, having more than two persons per bedroom, no access to electricity, not having water and toilet inside the house, not having durables, no access to TIC contribute negatively to the composite welfare indicator of the household and represents its poverty.

We note that the attributes contributing strongly to the first factorial axis are durables (refrigerator, washing machine, water heating, gas cooker), type of house, number of persons per bedroom, source of energy, water and toilet availability and three means of TIC : television, telephone and radio. The ones that contribute less to the first factor are conditioner and floor material.

The analysis of MCA results allows to draw a multidimensional welfare profile and a multidimensional poverty profile of tunisian households.

Evolution of the composite welfare indicator between 1994, 2001 and 2006

The table below reports the average values of the composite welfare indicators for the three years.

Table 4: Descriptive Statistics of the Composite Welfare Indicator (CWI)

Variable	Mean	Median	Standard deviation	Min.	Max.	Range	N
CWI 1994	-0.153	-0.099	0.334	-1.208	0.552	1.760	6037
CWI 2001	0,000	0,061	0,276	-1,137	0,552	1,689	6051
CWI 2006	0,165	0,193	0,228	-0,873	0,552	1,424	8186
Source : Our calculations							

There is an improvement in the indicator value between 1994 and 2006, meaning an improvement of the households well-being for this period. This is confirmed by the figure below which represents the cumulative distributions of the composite welfare indicator for 1994, 2001 and 2006.

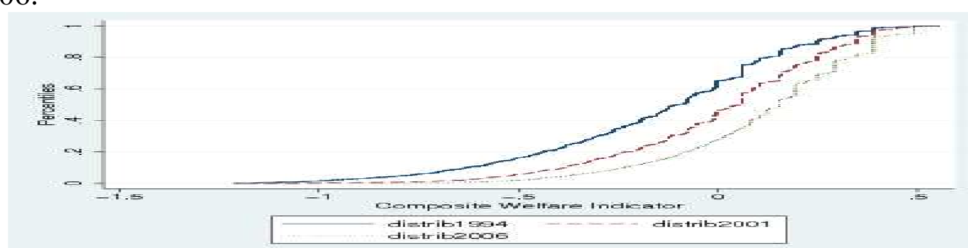

Figure 1: Cumulative Distributions of the Composite Welfare Indicator for 1994, 2001 and 2006

5.2 Analysis of Multidimensional Poverty in Tunisia

The distribution of the computed composite welfare indicator has some negative values. Hence, we transform it by adding the absolute value of the lowest score to each household's score in order to obtain a new distribution for the indicator defined on a positive support[17].

We compare poverty between 1994 and 2006 to see if poverty has increased or decreased in Tunisia during this period (table 5). The values of the three bi-dimensional poverty measures decreased between 1994 and 2001 and also decreased between 2001 and 2006. In general, we note that bi-dimensional poverty has decreased between 1994 and 2006.

Table 5: Evolution of Poverty in Tunisia between 1994. 2001 and 2006

Year	1994		2001		2006	
Bi-dimensional Poverty Measure	**Value**	**bias**	**Value**	**bias**	**Value**	**bias**
MP.2	2.8064	-0.0002	2.6778	-0.0002	2.3755	0.0008
	(0.0092)		(0.0121)		(0.0151)	

[17] As noted before, this absolute change has no effect in terms of poverty on the values of absolute poverty measures that we apply in this section.

MP.3	2.9445 (0.0071)	0.0001	2.8866 (0.0076)	0.0005	2.7081 (0.0108)	0.0006
MP.4	2.9779 (0.0084)	0.0003	2.9520 (0.0070)	0.0009	2.8566 (0.0080)	0.0005

Note: bootstrap standard errors in parentheses. * bias: difference between the mean of B bootstrap poverty measure estimates and sample poverty measure value

According to the values of the measures, we cannot conclude that multidimensional poverty has decreased between 1994 and 2006 in Tunisia . For that, we must construct confidence intervals and conduct hypothesis tests for all the measures (Table 6). The confidence intervals are an estimation of the real limits of the poverty measures of the population and the hypothesis test is a tough way to confirm or infirm our conclusion concerning the increase or decrease of poverty rate.

Table 6 : Construction of Confidence Intervals

Year	1994		2001		2006	
Method	Simple Percentile		Simple Percentile		Simple Percentile	
	Standard	Hall's	Standard	Hall's	Standard	Hall's
Bi Poverty	2.5% 97.5%		2.5% 97.5%		2.5% 97.5%	
MP,2	2.789 2.824	2.788 2.824	2.654 2.701	2.655 2.702	2.347 2.406	2.345 2.403
MP,3	2.930 2.958	2.932 2.960	2.873 2.902	2.872 2.901	2.687 2.730	2.686 2.729
MP,4	2.960 2.993	2.963 2.996	2.938 2.966	2.938 2.966	2.841 2.872	2.841 2.872

We also test the statistical significance of poverty changes in Tunisia (table 7), first between 1991 and 2001 (Case 1 with 2001 being distribution A) and then between 2001 and 2006 (Case 2 with 2006 being distribution A).

Table 7: Rule of Decision for the Hypothesis Tests

	Confidence Interval			
	case1		case2	
Bi-dimensional Poverty Measure				
MP,2	-0.158	-0.101	-0.340	-0.264
MP,3	-0.077	-0.037	-0.205	-0.152
MP,4	-0.045	-0.003	-0.117	-0.074

We note that zero does not belong to any confidence interval in both case 1 and case 2, so we reject the null hypothesis and conclude that bi-dimensional poverty has significantly decreased between the years 1994 and 2001 and that it has significantly decreased between 2001 and 2006 in Tunisia.

5.3 Discussion

To our knowledge, there exist only one study (Bibi(2004), Ayadi et al.(2007)) which made the application of multidimensional poverty measures on Tunisia. We will discuss here our results and compare them to theirs.

We have first measured poverty in the composite welfare indicator and we found that poverty has decreased between 1994 and 2001. A result conform to the result found by Bibi (2004) who used a 1990 survey conducted by the National Institute of Statistics (INS), applying the bi-dimensional FGT poverty measures developed by Bourguignon and Chakravarty (1998, 2003) and used two variables: the total expenditures per capita and the number of rooms per capita. Also conform to the result found by Ayadi et al. (2007) which used a 1988 DHS survey and the ONFP's 1994 and 2001 surveys to apply a composite welfare indicator that they have developed. We also find a decrease of poverty between 2001 and 2006. This decrease can be explained by the economic growth in Tunisia which results, as consequence, the improvement of life quality of the tunisian household, the improvement of the household conditions, the increase of the access of the household to public services (water, electricity, health), etc.

When measuring bi-dimensional poverty, we have assigned the same weight to education and to the composite welfare indicator and the result has been that bi-dimensional poverty has decreased between 1994 and 2006. This shows that poverty is bi-dimensional and even multidimensional and that there is an inequality in the poor population and a feeling of relative deprivation that must be revealed, studied, understood and treated.

6 Conclusion

The important contributions to multidimensional poverty measurement have been the extension to the multidimensional context of some members of the general class of additively separable poverty measures. In fact, using a two-stage aggregation procedure, Chakravarty, Mukherjee and Ramade (1998), Bourguignon and Chakravarty (1998, 2003) extended the FGT (1984) class of poverty measures and Tsui (2002) extended the the more general class of sub-group consistent poverty measures. Following Gajdos and Weymark (2005), we have used a reversed-order two-stage aggregation procedure to construct classes of multidimensional non-additively separable poverty measures. For that, we have developed an extension to the multidimensional context of classes of both relative and absolute generalized Sen-Shorrocks-Thon (SST) measures developed by Chtioui and Ayadi (2013). The purpose of constructing a poverty measure is to be used by the decision-maker to help them know the extension of the poverty. It is more interesting to have a measure with a parameter where each value corresponds to a particular social judgement. This offers to the decision-maker a large choice of values, and he can choose the value that represents more his value judgements. The multidimensional poverty measures we have developed offer this property where there are function of parameters (as the parameter of aversion to relative deprivation, the parameter representing the weight to assign to the attribute in the multidimensional poverty measure, the system of weighting that is concerned with the well-being of the poor in each attribute distribution where greater weight is given to those at the bottom of the distribution).

We have developed a structure of multidimensional poverty measures based on separability between attributes. But this does not capture the important and realistic nature of the multidimensionality, that is, the possible interdependence between dimensions. Performing our structure, in order to incorporate it, can be explored in further research.

Finally, the application of the multidimensional generalized SST poverty measures that we have developed in Tunisia using the bootstrap method shows that they can be easily implemented in practice and this for any other country. It also gives an idea on the evolution of multidimensional poverty in Tunisia which has decreased between 1994 and 2006 and this based on two indicators: a composite welfare indicator and number of years of education. Performing the statistical inference, like using the U-statistics, can improve both estimations and results.

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Appendix : Multivariate Correspondence Analysis

Histogram of the first five eigenvalues of the Multivariate Correspondence Analysis

Number	Eigenvalue	Percentage	Cumulative Percentage	
1	0.3043	21.30	21.30	
2	0.1305	9.14	30,44	
3	0.0806	5.64	36,08	
4	0.0799	5.59	41,67	
5	0.0722	5.05	46,73	

Table 8: **Scores and Contributions of the modalities from the Multivariate Correspondence Analysis**

	Scores		Contributions (%)	
	Factor 1	**Factor 2**	**Factor 1**	**Factor 2**
1. Floor Material			**1.859**	**8.046**

Cemented/”Marble”/”Carrelage”	0.054	0.074	1.084	5.020
Earth or « clay » floor	-1.262	-1.778	0.065	0.287
Others	-1.995	-2.567	0.71	2.740
2. Type of house			**3.713**	**11.423**
Villa/Popular house	0.023	0.094	0.011	0.460
Apartment	0.899	-1.020	0.608	1.825
Others	-2.621	-2.950	3.094	9.138
3. Number of persons per bedroom			**6.381**	**6.786**
Less than 2 persons per bedroom	0.554	-0.451	2.477	3.822
Between 2 and 4 persons per bedroom	-0.034	0.339	0.013	2.958
More than 4 persons per bedroom	-0.949	-0.025	3.891	0.006
4. Source of Energy			**6.674**	**9.670**
Electricity	0.145	0.112	0.46	0.637
Gas	-1.593	-1.080	2.259	2.421
Other sources of Energy	-2.263	-1.916	3.955	6.612
5. Water			**10.296**	**4.688**
Having water inside the house	0.462	-0.062	3.303	0.139
Having water but outside the house	-0.564	0.707	1.018	3.726
No water	-1.117	-0.272	5.975	0.823
6. Toilet			**12.746**	**12.027**
Having toilet inside the house	0.464	-0.102	3.397	0.381
Having toilet but outside the house	-0.437	0.934	0.787	8.373
No toilet	-1.550	-0.627	8.562	3.272
7. Refrigerator			**12.721**	**0.647**
Yes	0.440	0.065	3.352	0.171
No	-1.231	-0.182	9.369	0.477
8. Gas Cooker			**3.861**	**6.108**
Yes	0.104	0.086	0.238	0.377
No	-1.581	-1.303	3.623	5.731
9. Water heating			**7.769**	**11.070**
Yes	1.029	-0.804	5.918	8.432
No	-0.322	0.252	1.851	2.638
10. Washing machine			**8.714**	**10.693**
Yes	1.001	-0.726	6.358	7.802
No	-0.371	0.269	2.356	2.891
11. Conditioner			**2.112**	**9.197**
Yes	1.357	-1.855	2.014	8.769
No	-0.066	0.091	0.098	0.428

12. Television			**4.595**	**0.021**
Yes	0.277	0.012	1.291	0.006
No	-0.708	-0.031	3.304	0.015
13. Radio			**9.246**	**5.573**
Yes	0.253	0.129	1.294	0.780
No	-1.556	-0.791	7.952	4.792
14. Telephone			**9.314**	**4.051**
Yes	0.685	-0.296	5.047	2.195
No	-0.579	0.250	4.267	1.856