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Financial Infrastructure, Technological Shift, and Inequality in Economic Development*

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Abstract

This paper presents an overlapping generations model with technology choice and imperfect financial markets, and examines the evolution of income distribution in economic development. The model shows that improvements in financial infrastructure facilitate economic development both by raising the aggregate capital-labor ratio and by causing a technological shift to more capital-intensive technologies. While a higher capital-labor ratio under a given technology reduces inequality, a technological shift can lead to a concentration of the economic rents among a smaller number of agents. We derive the condition under which an improvement in financial infrastructure actually decreases the average utility of agents.

JEL Classification Numbers: O14, O16.

Key words: Technological Shift; Income Distribution; Rents; Enforcement; Credit Rationing.

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1 Introduction

One important aspect of economic development is that less productive technologies, which are often labor-intensive, are replaced with more productive ones. However, major technological shifts have often been accompanied by conflicts among individuals or different parties in the economy. Mokyr (1990) documents that before and during the Industrial Revolution, there were numerous examples of anti machinery agitation in Britain. In 1768, 500 sewers attacked a mechanical sawmill in London. In 1792, a Manchester-based firm that pioneered Cartwright’s power loom was burnt down. Between 1811 and 1816, the “Luddite” riots occurred in the Midlands and the industrial counties. Also in continental Europe, resistance came from guilds of skilled artisans. In 1780, anti-machinery vandalism occurred in the city of Rouen and then spread to Paris, destroying spinning machines imported from Britain and locally made devices such as pitchfork making machines. These episodes clearly show that not everyone benefits from technological shifts.

This paper focuses on the effects of technological shifts on income distribution and welfare. Any resistance to a new technology suggests that there is a group of agents who earns economic rents that are related to the existing technology. It is popularly believed that this fact actually indicates that technological shifts are desirable for the economy as a whole; that is, while some agents may lose their vested interests, improved productivity of new technologies can be enjoyed by all agents in the economy. This view suggests that the degree of inequality would fall when a new technology is adopted after overcoming the resistance to it.

However, historical evidence shows that this is not always the case. For example, between 1759 and 1801, the nominal Gini index rose from 52.2 to 59.3 in England, when the textile and many other industries shifted from cottage to manufacturing technologies (Lindert 2000). Further, Morrison (2000) argues that in continental Europe (e.g., France and Germany), the income share of the top decile increased five to ten percent in the mid-nineteenth century, making this period reach the peak of the Kuznets curve. If the rise in inequality implies a further concentration of economic rents among a smaller number of agents, the welfare effect of a technological shift is
In this paper, we theoretically examine the process of economic development and technological shift, as well as their effects on income distribution and welfare, by constructing an overlapping generations model with multiple technologies and imperfect financial markets. In particular, this study focuses on financial infrastructure, such as legal and accounting systems, because recent studies have suggested that financial infrastructure is closely related to both technological shifts and income distribution. La Porta et al. (1997, 1998) and Levine, Loayza, and Beck (2000) provided convincing evidence that the development of financial markets is strongly influenced by the financial infrastructure that determines the enforceability of financial contracts. Since “financial revolutions” have often preceded major technological shifts (e.g., Sylla 2002), this evidence justifies the consideration of financial markets as an important source of technological shift. Also, Galor and Zeira (1993) and Matsuyama (2000) theoretically demonstrated that limited enforcement of financial contracts gives rise to credit rationing, which limits the number of entrepreneurs who earn economic rents. This paper incorporates multiple technologies with different capital intensities into their settings, and examines how technological shifts affect income distribution.

Our analysis reveals that improvements in the financial infrastructure facilitate economic development in two ways, which have contrasting implications for income distribution. First, as long as the same technology is used, improved financial infrastructure makes credit accessible to an increased number of agents, which raises the aggregate capital-labor ratio and hence per capita income. In this case, the amount of

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1Sylla (2002) reports, “The Dutch financial revolution had occurred by the first decades of the seventeenth century, before the Dutch Golden Age... The British financial revolution in the late seventeenth and early eighteenth centuries, before the English industrial revolution. The U.S. financial revolution occurred ..., before the U.S. economy accelerated its growth in the ‘statistical dark age’ of the early nineteenth century.” He also notes, “In the early Meiji era of the 1870s and 1880s, Japan had a financial revolution ... Once their financial revolution was in place, the Japanese were off and running.” See also Dickson (1967) for similar arguments. Levine (1997, 2005) provides an extensive survey on the role of a country’s financial system in economic development. Christopoulos and Tsionas (2004) shows that the causality runs from finance to economic development and not vice versa.
rent received by each entrepreneur declines and the income of wage earners increases. Consequently, inequality falls and welfare improves.

The second way in which improvements in financial infrastructure facilitate economic development is through a technological shift. While the economy’s financial infrastructure is underdeveloped, agents must rely on labor-intensive technologies. However, once the financial infrastructure improves to a certain extent, some agents can obtain sufficient funds to adopt capital-intensive technology. At this point, entrepreneurs relying upon labor-intensive technologies are in effect driven out from the factor markets, while only those who can adopt the capital-intensive technology begin to attract most of the surplus from the higher productivity, without distributing much to others. We derive a condition under which the rise in inequality is so substantial, that the average utility of agents actually declines after the technological shift.

The rest of this paper is organized as follows. Section 2 briefly reviews the literature related to this topic and compares their distributional implications to those of ours. Section 3 constructs an overlapping generations model with technology choice under imperfect financial markets. In Section 4, we derive the equilibrium distribution of income and explain why significant inequality emerges among agents. Section 5 clarifies how financial infrastructure affects the choice of technology in equilibrium. Section 6 examines the effects of improvements in financial infrastructure on the income distribution and welfare in the steady state. Policies are discussed in Section 7, and Section 8 provides the conclusion. The Appendix contains the proofs for propositions.

2 Comparison with the Literature

In the literature, there are various approaches to theoretically analyze the evolution of income distribution through the process of economic development. Among these, close to our approach are the studies by Rajan and Zingales (2003) and Erosa and Hidalgo-Cabrillana (2005), which consider the effect of improved financial infrastructure on income distribution. These studies have found that economic develop-
ment that results from improvements in the financial infrastructure will reduce the amount of economic rent received by each incumbent rent-earner, thereby decreasing inequality (See Figure 1(i)). Such a redistribution of income generally improves the economy’s welfare, although it will not be supported by incumbent rent earners.\textsuperscript{2}

While the above studies suggest that the income inequality reduces under a given production technology, other studies focusing on the technological shift explain the rise in income inequality at the early stages of economic development. Specifically, with a fixed degree of credit market imperfection, Banerjee and Newman (1998) and Greenwood and Jovanovic (1990) show that when agents gradually shift to a new technology, the degree of inequality in the economy rises temporarily due to the disparity in income levels between the new and old sectors. In these studies, those who moved to the new sector are better off because they voluntarily chose to move, while those who remained in the old sector can earn an income that was as high as what they were earning before the technological shift (See Figure 1(ii)).\textsuperscript{3} Therefore,

\textsuperscript{2}See Drazen (2000) for general discussions about the conflicting interests in economic reforms.

\textsuperscript{3}They actually show that the reduced labor supply in the old sector increases the wages of those who remain in the old sector. Aghion and Bolton (1997) also show that the rise in inequality in the early phases of development is beneficial to the poor since it enhances capital accumulation. See Barro (2000, p. 9) for a survey of related studies.
while a technological shift increases inequality, it weakly increases every agent’s utility and is necessarily welfare improving.

Thus, although these two existing strands of studies found opposite implications for inequality, both concluded that development always improves welfare. This paper obtains a different welfare implication when it simultaneously considers the possibility of technological shift and improvements in financial infrastructure. Naturally, we obtain a result similar to that of Rajan and Zingales (2003) and Erosa and Hidalgo-Cabrillana (2005) in the case where an improvement in financial infrastructure does not cause a technological shift. However, in the case where it causes a technological shift, it gives rise to a new economic rent and increases inequality. As shown in Banerjee and Newman (1998) and Greenwood and Jovanovic (1990), under a certain condition, a technological shift makes every agent better off. However, under a different condition, the technological shift deprives the majority of incumbent entrepreneurs of economic rents, and then, these rents are redistributed to a smaller number of agents (See Figure 1(iii)). Incumbent entrepreneurs are strictly worse off, and only a limited number of agents benefit from a technological shift. In such cases, overcoming resistances from incumbent rent-earners does not lead to an improvement in the economy’s welfare.

A critical difference between the results of our paper and existing theories on technological shifts is that in our model, agents do not necessarily shift voluntarily from the old technology to the new one. Once the improvement in the financial infrastructure permits the adoption of a capital-intensive technology, entrepreneurs who are equipped with that technology employ workers at a marginally higher wage. The incumbent entrepreneurs cannot afford to pay their workers at this wage level since the profitability of the labor-intensive technology falls more sensitively with an increase in wage level than in the case of the capital-intensive technology. Due to this general equilibrium effect, the economy cannot continue with the old technology even when the majority of the agents are against the new technology.

While we focus on capital intensity, there are several other mechanisms through which financial markets affect technological choice. To mention a few, Saint-Paul
(1992) shows that without a well-functioning financial market, risk-averse agents may choose less specialized and less productive technologies. Castro et al. (2005) demonstrate that stronger investor protection facilitates economic development, given that the technology for producing investment goods involves a higher idiosyncratic risk than does the technology for producing consumption goods. In contrast, Ben- civenga et al. (1995) show that a technological shift resulting from a better financial infrastructure may reduce the growth rate if the new technology requires a longer duration for which investments must be committed. Each of these studies focuses on a particular aspect of technology; however, they are not concerned with income distribution and welfare. In our model, agents choose from among technologies with different capital intensities, and in this setting, we demonstrate that improvements in financial infrastructure do not necessarily improve the economy’s welfare.

3 The Model

3.1 Economic Environments

Consider an overlapping generations economy, where each generation contains a unit mass of agents who live for two periods (young and old). The life of an agent who was born in period $t$ proceeds as follows. In the first period, he supplies one unit of labor inelastically to the competitive labor market and receives the market wage $w_t$, measured in terms of consumption goods. For the purpose of simplicity, we assume that the agent’s utility depends only on the amount of consumption in the second period, $c_{t+1}$. In order to finance this consumption, the agent makes use of his first-period income $w_t$ in one of two ways. First, he may save it entirely and consume $c_{t+1} = rw_t$ in the second period. Interest rate $r \geq 1$ is constant either because the economy under consideration is a small open economy or because there is a storage technology that yields the gross rate of return $r$.

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4Recent empirical studies suggest that financial markets promote economic development not by enhancing overall capital accumulation but by efficiently allocating capital across sectors (e.g., Wurgler 2000). The open economy assumption enables us to focus on the role of financial markets in
His second option is to become an entrepreneur and start a project. At most each agent can undertake one project, and a project cannot be shared by multiple entrepreneurs due to information and enforcement problems among them. When starting a project, an agent chooses from among a discrete set of technologies, \( \mathcal{J} \). Every technology produces a homogeneous consumption good from capital and labor with constant returns to scale. Specifically, if the agent adopts technology \( j \in \mathcal{J} \), his project produces the consumption good according to

\[
y_{t+1} = \begin{cases} 
  k_{t+1}f_j(\ell_{t+1}/k_{t+1}), & \text{if } k_{t+1} \geq I_j, \\
  0, & \text{if } k_{t+1} < I_j,
\end{cases}
\]

where \( \ell_{t+1} \) and \( k_{t+1} \) are the amounts of labor and capital inputs, respectively, and \( f_j(\cdot) \) is the per unit capital (not per capita) production function of technology \( j \). Equation (1) shows that exploiting the potential of each technology requires at least a certain amount of investment. The minimal required amount of capital, denoted by \( I_j \geq 0 \), differs across technologies, and depends on technical aspects (e.g., the scope of scale economy for that technology) and various barriers to the adoption of the technologies, which may be specific to each economy. Capital depreciates completely within one period and \( f_j(\cdot) \) satisfies the standard Inada conditions for all \( j \in \mathcal{J} \).

As is standard in overlapping generation models, we assume that the output (consumption goods) in period \( t \) can be used as the capital in period \( t+1 \). However, the agent’s first-period income, \( w_t \), often falls short of the minimum required amount of capital, \( I_j \). In that case, he must finance the gap by borrowing from the competitive financial intermediaries, which we call banks. Banks can borrow from the international credit market at the constant world interest rate \( r \), whereas agents cannot do so because of the issue of limited enforcement, as explained below. In order to obtain the loan, \( k_{t+1} - w_t \), which is needed to finance the investment of size \( k_{t+1} \), the agent applies to banks by announcing the plan of his project, comprising three determining the demand for capital and its composition rather than the supply of capital, which is given by the amount of savings in the closed economy setting. The assumption of storage technology is more suitable for low income countries, where inventories are the principal substitutes for investment (see discussions by Bencivenga and Smith 1993, Section 5). In the latter case, we implicitly assume that the demand for capital never exceeds the amount of aggregate savings.
elements—the choice of technology, \( j \in J \), the size of investment, \( k_{t+1} \geq I_j \), and the amount of his own fund, \( w_t \)—which are verifiable, and thus, contractable. If the agent is approached by several banks, he chooses a loan contract from the bank that offers the lowest gross interest rate, denoted by \( R_t \). If the agent is denied the loan at any interest rate—i.e., if he is credit rationed—he gives up becoming an entrepreneur and lends his entire first-period income to the credit market.

At period \( t+1 \), an entrepreneur (an agent who has successfully obtained credit or has managed his investment fully using his own funds) decides the number of young workers to hire, \( \ell_{t+1} > 0 \), at the market wage rate \( w_{t+1} \). The revenue from the project is \( y_{t+1} - w_{t+1} \ell_{t+1} \). The entrepreneur is obliged to repay the loan from this revenue; however, he has an option to default at a certain cost. We assume that the cost of default is proportional to the revenue from the project, \( \lambda(y_{t+1} - w_{t+1} \ell_{t+1}) \), where \( \lambda \in (0,1) \). If he defaults, his consumption becomes \( c_{t+1} = (1 - \lambda)(y_{t+1} - w_{t+1} \ell_{t+1}) \); otherwise, he repays the loan and consumes \( c_{t+1} = y_{t+1} - w_{t+1} \ell_{t+1} - R_{t+1}(k_{t+1} - w_t) \) units of the good.

This setting is equivalent to assuming that lenders can capture only \( 100\lambda \) percent of the cash flow from any project. Thus, parameter \( \lambda \) represents the quality of the economy’s financial infrastructure, such as legal and accounting systems, which determines the enforceability of financial contracts.

### 3.2 Behaviors of Households and Banks

This subsection examines the rational behaviors of generation-\( t \) households (who become entrepreneurs at period \( t+1 \) if they obtain credit) and banks, taking as given their first-period income \( w_t \) and the market wage rate at their second period \( w_{t+1} \). The decision processes are sequential, and therefore, can be solved backward. The final decision is to determine the number of workers to hire \( \ell_{t+1} \), given that the entrepreneur has already chosen technology \( j \) and the amount of capital \( k_{t+1} \geq I_j \).

Whether or not the entrepreneur decides to default, her objective at this stage is to maximize revenue \( y_{t+1} - w_{t+1} \ell_{t+1} \) with respect to labor input \( \ell_{t+1} \), where output \( y_{t+1} \) is given by (1). A straightforward differentiation shows that it is optimal to
choose

\[ \ell_{t+1} = f_j^{-1}(w_{t+1})k_{t+1} = \tilde{\ell}_j(w_{t+1})k_{t+1}, \]

(2)

where \( \tilde{\ell}_j(w_{t+1}) \equiv f_j^{-1}(w_t) \) represents the optimal labor input per unit capital as a decreasing function of market wage \( w_{t+1} \). The rate of return from this project (the amount of maximized revenue divided by the amount of capital) is

\[ \rho_j(w_{t+1}) = f_j(\tilde{\ell}_j(w_{t+1})) - w_{t+1}\tilde{\ell}_j(w_{t+1}), \]

(3)

which is decreasing in market wage \( w_{t+1} \). Out of revenue \( \rho_j(w_{t+1})k_{t+1} \), the entrepreneur repays the loan unless it exceeds the cost of default. That is, the loan will be repaid if and only if

\[ R_{t+1}(k_{t+1} - w_t) \leq \lambda \rho_j(w_{t+1})k_{t+1}. \]

(4)

Banks offer loans to potential entrepreneurs if and only if the entrepreneurs are willing to repay them and banks can earn an interest at least as large as the market interest rate \( r \). As long as repayment from entrepreneurs is expected, competition among banks brings the interest rate down to \( r \). Banks are assured of the repayment if a prospective entrepreneur’s planned project, summarized by \((j, k_{t+1}, w_t)\), satisfies condition (4) at interest rate \( R_{t+1} = r \). Using the size of investment for the proposed project, this condition can be written as:

\[ k_{t+1} \leq \frac{w_t}{1 - \lambda \rho_j(w_{t+1})/r} \quad \text{if} \quad \lambda \rho_j(w_{t+1}) < r. \]

(5)

If the proposed plan fails to satisfy (5), the project cannot obtain credit at any interest rate.\(^5\) It can be observed from (3) and (5) that the equilibrium wage in period \( t+1 \), \( w_{t+1} \), must satisfy \( \lambda \rho_j(w_{t+1}) < r \) for any technology \( j \in J \). If it is not satisfied (i.e., when the rate of return from the investment satisfies \( \rho_j(w_{t+1}) \geq r/\lambda > r \)), entrepreneurs can obtain an infinite payoff by investing an infinite amount of capital and hiring an unbounded number of workers, which clearly results in excess demand

\(^5\)Note that a higher interest rate makes condition (4) stricter and gives borrowers more incentive to default. Thus, banks cannot make a profit (even zero profit) by offering a loan for projects that do not satisfy (5) with an interest rate higher than \( r \).
in the labor market. Thus, the equilibrium wage $w_{t+1}$ must satisfy
\[ w_{t+1} > \max_{j \in J} \rho_j^{-1}(r/\lambda) \equiv \bar{w}(\lambda). \] (6)

Now let us return to the choice of technology and the size of investment. A prospective entrepreneur chooses $j$ and $k_{t+1}$ in order to maximize her second-period consumption,
\[ c_{t+1} = rw_t + (\rho_j(w_{t+1}) - r) k_{t+1}. \] (7)
This expression shows that she wants to become an entrepreneur (i.e., she wants to choose some $j$ and set $k_{t+1} > 0$ rather than save all her first-period income and choose $k_{t+1} = 0$) only when the rate of return from the investment $\rho_j(w_{t+1})$ is at least as high as the interest rate. Since the rate of return depends on the market wage $w_{t+1}$, this condition can be written as
\[ w_{t+1} \leq \rho_j^{-1}(r) \equiv P_j, \] (8)
which we call the *profitability constraint*. The constant $P_j$ represents the level of market wage at which a project with technology $j$ breaks even. We assume that $P_j$ is smaller than the size of minimum investment $I_j$.

When the profitability constraint is satisfied, the agent is willing (at least weakly) to start a project. In particular, when the profitability constraint holds with strict inequality, she wants to invest as much as possible. Under (6), however, condition (5) implies that there is an upper bound for the size of investment and this upper bound depends on the amount of the entrepreneur’s own fund, $w_t$. In addition, to adopt technology $j$, at least $I_j$ units of capital must be invested. This implies that the entrepreneur must provide sufficient own funds such that the upper bound is at least as large as $I_j$. Comparing the right hand side (RHS) of (5) with $I_j$, we obtain
\[ w_t \geq \left(1 - \frac{\lambda \rho_j(w_{t+1})}{r}\right) I_j \equiv \eta_j(w_{t+1}, \lambda), \] (9)

6If entrepreneurs have ample own funds, they will be able adopt the most profitable technology without relying on the financial market. However, historical instances wherein financial markets affected economic performance imply that entrepreneurs usually have insufficient funds to self-finance their projects. Accordingly, we assume $I_j > P_j$, where $P_j$ is the upper bound of the first-period income when the economy specializes in a technology $j$. 

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where function $\eta_j(\cdot)$ represents the minimum amount of own funds required to borrow from banks to start a project with technology $j$. Since this minimum requirement is increasing in the market wage $w_{t+1}$, condition (9) can be stated in terms of the market wage $w_{t+1}$, given the amount of own fund $w_t$:

$$w_{t+1} \leq \rho_j^{-1} [(r/\lambda) (1 - w_t/I_j)] \equiv B_j(w_t, \lambda).$$

(10)

We call (10), or, equivalently, (9), the *borrowing constraint* for technology $j$. Agents can adopt technology $j$ unless the market wage exceeds $B_j(w_t, \lambda)$. This borrowing constraint relaxes (i.e., $B_j(w_t, \lambda)$ increases) when the agent has more own funds $w_t$ or the economy’s financial infrastructure $\lambda$ improves.

Now we are ready to describe the occupational choice of agents in terms of the market wage $w_{t+1}$ and the amount of own funds $w_t$. Combining (8) and (10), we see that technology $j$ satisfies both the profitability and borrowing constraints if and only if

$$w_{t+1} \leq \min\{P_j, B_j(w_t, \lambda)\} \equiv \phi_j(w_t, \lambda).$$

(11)

If market wage $w_{t+1}$ is below or equal to $\phi_j(w_t, \lambda)$, an agent with own fund $w_t$ is both able and willing to become an entrepreneur with technology $j$, rather than merely save her first-period income. Among the potentially usable technologies $\mathcal{J}$, there exists at least one of such technology if

$$w_{t+1} \leq \max_{j \in \mathcal{J}} \phi_j(w_t, \lambda) \equiv \theta(w_t, \lambda).$$

(12)

In this case, the agent becomes an entrepreneur, invests as much as she can borrow (see condition 5):

$$k_{t+1} = \frac{w_t}{1 - \lambda \rho_j(w_{t+1})/r} = \frac{w_t}{\eta_j(w_{t+1}, \lambda) I_j}.$$  

(13)

When condition (12) is satisfied, (11) implies that there must be some technology $j$ such that $w_{t+1} < \theta(w_t, \lambda) = \min\{P_j, B_j(w_t, \lambda)\}$. It follows that $w_{t+1} < B_j(w_t, \lambda)$ and $w_{t+1} < P_j$; that is, technology $j$ satisfies the credit constraints, and its rate of return is strictly larger than $r$. If there are more than two such technologies, the entrepreneur chooses the most profitable technology. In equilibrium, as we will consider in the next section, there is generically only one technology that satisfies this condition.
Note that \( \frac{w_t}{\eta_j(w_{t+1}, \lambda)} \) in equation (13) represents the ratio of actual own funds to the amount required to obtain the credit and is therefore always above 1. From (2), (3), and (7), the consumption of the entrepreneur and the individual labor demand from this project are

\[
\ell_{t+1} = k_{t+1} \tilde{e}_j(w_{t+1}) = \frac{w_t}{\eta_j(w_{t+1}, \lambda)} I_j \tilde{e}_j(w_{t+1}),
\]

(14)

\[
c_{t+1} = rw_t + (\rho_j(w_{t+1}) - r) \frac{w_t}{\eta_j(w_{t+1}, \lambda)} I_j.
\]

(15)

The second term in (15) represents the surplus income obtained by virtue of becoming an entrepreneur.

If the market wage \( w_{t+1} \) is above the threshold \( \theta(w_t, \lambda) \), the rate of return from any technology that satisfies the credit constraint falls short of \( r \). Then, it is best for the agent to save her entire first-period income (i.e., \( k_{t+1} = \ell_{t+1} = 0 \)) and receive \( c_{t+1} = rw_t \). Finally, if \( w_{t+1} = \theta(w_t, \lambda) \), then either the profitability or profitability constraint is exactly binding. If the profitability constraint is not binding (then the borrowing constraint must be binding), the agent strictly prefers to start a project, similar to the case of \( w_{t+1} < \theta(w_t, \lambda) \). Otherwise, she is indifferent as to whether or not to start a project: investment \( k_{t+1} \) can be zero or anywhere between the minimum amount \( I_j \) and the RHS of (13); the labor demand is \( \ell_{t+1} = k_{t+1} \tilde{e}_j(w_{t+1}) \), and any choice results in \( c_{t+1} = rw_t \).

4 Inequality in Equilibrium

This section establishes the existence of an equilibrium wage rate at which the aggregate supply of and demand for labor are equalized, and then it examines the extent of inequality that arises in equilibrium. Before proceeding to the formal analysis, we first present an intuitive explanation of how and when a significant income inequality arises among old agents in equilibrium. For this purpose, it is convenient to temporarily introduce a small ex ante heterogeneity among agents. In particular, for the time being, we assume that each agent, while in his/her youth, experiences an exogenous income shock, \( \epsilon_t \), which is chosen randomly from a uniform distribution between 0 and \( \tau > 0 \).
Suppose that the agents in each generation are now indexed by \( i \in [0, 1] \) and that agent \( i \)'s realized first-period income is given by \( w_t + \epsilon_{it} \). In the previous section, we showed that an agent with own fund \( w_t \) is willing to become an entrepreneur if the market wage \( w_{t+1} \) is below the threshold level of \( \theta(w_t, \lambda) \). Since we now assume that agents have heterogeneous amounts of own funds, \( w_t + \epsilon_{it} \), the threshold \( \theta(w_t + \epsilon_{it}, \lambda) \) may also vary across agents. From its definition, function \( \theta(w_t + \epsilon_{it}, \lambda) \) is increasing in \( w_t + \epsilon_{it}, \theta(0, \lambda) = w(\lambda) > 0 \), and \( \lim_{w_t + \epsilon_{it} \to \infty} \theta(w_t + \epsilon_{it}, \lambda) = \max_{j \in J} P_j < \infty \). Therefore, the threshold level for any agent is within a (small) finite interval \([\theta_t, \overline{\theta}_t]\), where

\[
\theta_t \equiv \theta(w_t, \lambda), \quad \overline{\theta}_t \equiv \theta(w_t + \bar{\epsilon}, \lambda).
\]

From this observation, it follows that the equilibrium level of market wage, \( w_{t+1} \), must be somewhere between \( \theta_t \) and \( \overline{\theta}_t \). If \( w_{t+1} > \overline{\theta}_t \), then no agent starts a project, and therefore the aggregate labor demand would be zero. Conversely, if \( w_{t+1} < \theta_t \), then all old agents strictly prefer to start projects, which (under Assumption 1 below) necessarily results in excess demand for labor. Therefore, if there exists an equilibrium wage level \( w_{t+1} \) such that the aggregate labor demand coincides with the aggregate labor supply, it must be within interval \([\theta_t, \overline{\theta}_t]\).

Figure 2 depicts a typical shape of function \( \theta(\cdot) \) against the amount of own funds, \( w_t + \epsilon_{it} \), which we call the \( \theta \) curve. The shape of the \( \theta \) curve on a short interval \([w_t, w_t + \bar{\epsilon}]\) determines \( \theta_t \) and \( \overline{\theta}_t \). One possibility is that the curve is entirely flat in that interval. In this case, \( \theta_t \) and \( \overline{\theta}_t \) are the same, and the equilibrium wage is uniquely determined at this level. Note that a flat segment of the \( \theta \) curve corresponds to the profitability constraint for some technology \( j \). In equilibrium, \( w_{t+1} = \overline{\theta}_t = \theta_t = P_j \) holds, which means from (8) that the rate of return from investment \( \rho_j(w_{t+1}) \) is the same as the interest rate \( r \). Therefore, all agents are indifferent regarding their choice to become entrepreneurs or save their income. Irrespective of what they decide, they obtain \( c_{t+1} = r(w_t + \epsilon_{it}) \). Given that the magnitude of random income \( \epsilon_{it} \) is marginal, the inequality of consumption in the second period is also marginal.

However, we have a different distributional consequence when the \( \theta \) curve is upward sloping in interval \([w_t, w_t + \bar{\epsilon}]\). Since an upward sloping segment corresponds to
Figure 2: An Example of the $\theta$ curve. It depicts a case of two technologies, $\mathcal{J} = \{A, M\}$. The gray area represents the distribution of the agents’ own funds when they face small income shocks.

a borrowing constraint for a particular technology, the profitability constraint is not (generically) binding in this case. This implies that the rate of return from starting a project is strictly higher than $r$, and that every agent strictly prefers to start a project. However, if this were the case, the overall labor demand would exceed the aggregate supply. Thus, the equilibrium wage $w_{t+1}$ must be between $\underline{\theta}_t$ and $\overline{\theta}_t$ so that some agents (whose $\theta(w_t + \epsilon_{it}, \lambda)$ is below $w_{t+1}$) do not satisfy the borrowing constraint. In other words, some agents must be rationed from the credit market. The consumption of these credit-rationed agents is significantly lower than that of the entrepreneurs, generating a non-trivial inequality among old agents.

In the remainder of this section, we formally establish the existence of the market equilibrium and explicitly derive the equilibrium income distribution. In this economy, the aggregate supply of labor is given by the population of the young agents, which has been normalized to 1. Given this period’s market wage $w_{t+1}$ and the labor income in the previous period, $w_t$, the aggregate labor demand is obtained by
summing the decisions of all the old agents,

$$\sum_{i} \ell_{i,t+1}(w_{t+1}; w_t) \equiv \left[ \int_{\theta(w_{t+\epsilon_i})<w_{t+1}} \ell_{i,t+1} di, \int_{\theta(w_{t+\epsilon_i})>w_{t+1}} \ell_{i,t+1} di \right]$$

(16)

where $\ell_{i,t+1}$ is given by (14), with $w_t$ being replaced by $w_t + \epsilon_i$. As shown in (16), function $L^D_{t+1}(w_{t+1}; w_t)$ is a set-valued function (or correspondence) because agents may be indifferent with regard to whether or not to start a project (and hire a worker). We assume that, if all old agents start projects, the aggregate labor demand will exceed the aggregate labor supply. More specifically,

**Assumption 1** $I_j > 1/\bar{e}_j(P_j)$ for all $j \in \mathcal{J}$.

From (14), Assumption 1 means that each project requires hiring more than one worker, which we reasonably assume to be satisfied throughout the paper. Now, we can put forward the following proposition.

**Proposition 1** Suppose that Assumption 1 holds and that the number of intersections between functions $\rho_j(w_{t+1})$ and $\rho_{j'}(w_{t+1})$ for any $j \neq j'$ is not infinite. Then, given the previous period’s equilibrium wage $w_t > 0$, there is an equilibrium level of $w_{t+1} \in [\theta_t, \bar{\theta}_t]$, with which $1 \in L^D_{t+1}(w_{t+1}; w_t)$ holds.

**Proof:** In Appendix

Although the proof is technical (mainly because aggregate labor demand is given by a set-valued function), the intuition is clear. Aggregate labor demand is above one for $w_{t+1} < \theta_t$, and is zero for $w_{t+1} > \bar{\theta}_t$. Moreover, in the Appendix, we show that aggregate labor demand is continuous with respect to $w_{t+1}$. Thus, it follows that there must be a level of $w_{t+1}$ between $\theta_t$ and $\bar{\theta}_t$, at which the aggregate labor demand coincides with its supply.

It is noteworthy that the result of Proposition 1 does not depend on the size of the heterogeneity term in the income of young agents. In particular, even in the case of the limit in which the heterogeneity is almost negligible (more specifically, when

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8Since $L^D_{t+1}(\cdot)$ is set-valued, the notion of continuity is slightly different from that for a function. Precisely, in the Appendix, we show that $L^D_{t+1}(\cdot)$ is convex-valued, non-empty, and upper hemi continuous, which implies that the graph of labor demand in $(L^D_{t+1}, w_{t+1})$ space is jointed.
the distribution of $\epsilon_{it} \in [0, \tau]$ is almost degenerate; i.e., $\tau \to 0$), Proposition 1 still shows that there exists an equilibrium level of market wage, $w_{t+1}$. In that case, we can state the properties of the equilibrium more explicitly.

**Proposition 2** Suppose that the assumptions in Proposition 1 hold and the ex ante heterogeneity is negligible ($\tau \to 0$); then, the equilibrium at the limit is characterized as follows:

a. Equilibrium wage $w_{t+1}$ is determined by $\theta(w_t, \lambda)$.

b. The choice of technology in equilibrium, denoted by $j^*$, is such that $\phi_{j^*}(w_t, \lambda)$ is the highest from among all the technologies.

c. Credit rationing occurs if and only if the amount of own funds $w_t$ satisfies

$$w_t < (1 - \lambda)I_{j^*}. \quad (17)$$

d. When credit rationing occurs, the number of entrepreneurs $n_{t+1}$ and their consumption $c^e_{t+1}$ are given by

$$n_{t+1} = \left( I_{j^*} \tilde{I}_{j^*} (B_{j^*}(w_t, \lambda)) \right)^{-1} < 1, \quad (18)$$

$$c^e_{t+1} = rw_t + r((1 - \lambda)I_{j^*} - w_t)/\lambda, \quad (19)$$

while the consumption of the other agents is $rw_t < c^e_{t+1}$. When there is no credit rationing, the consumption of all the agents is $rw_t$.

Proof: In Appendix.

In the remainder of the paper, we continue to consider the limiting case of $\tau \to 0$ and omit the $\epsilon_{it}$ term in the analysis. Proposition 2 implies that only the technology that can offer the highest wage within its profitability and borrowing constraints can operate in equilibrium. Entrepreneurs with other technologies cannot operate because they cannot hire workers at the market wage level. The proposition also shows that, even at the limit where almost no ex ante heterogeneity exists, a significant inequality arises between those who obtain credit and those who do not. More specifically, in a credit constrained equilibrium, each of $n_{t+1} (< 1)$ entrepreneurs obtains economic rents of $r((1 - \lambda)I_{j^*} - w_t)$ over the income of others, $rw_t$. Using this result, the following two sections consider the effects of improvements in financial infrastructure on the equilibrium distribution of income and the welfare of agents.
5 Technological Shift

Proposition 2 implies that other things being equal, a better financial infrastructure results in a more equal income distribution through the easing of credit rationing. From (18) and (19), a stronger enforcement of financial contract (a larger $\lambda$) increases the number of entrepreneurs, $n_{t+1}$, and reduces the size of economic rent, $r((1-\lambda)I_j^* - w_t)$, received by each of them. In addition, (17) shows that credit rationing disappears when $\lambda$ is above the threshold level of $1 - w_t/I_j^*$. However, these observations do not allow us to conclude that there is a monotonic relationship between the quality of financial infrastructure and the extent of inequality, because the threshold level for credit rationing, $1 - w_t/I_j^*$, as well as $n_{t+1}$ and $c_{t+1}^e$, depends on the equilibrium choice of technology $j^*$, which in turn depends on the financial infrastructure $\lambda$. Thus, we need to clarify when an increase in $\lambda$ causes a technological shift.

This section examines the role of financial infrastructure in determining the equilibrium choice of technology. For concreteness, suppose that the set of usable technologies are composed of Cobb-Douglas technologies, and their per unit capital production functions are given by

$$f_j(\ell/k) = A_j(\ell/k)^{1-\alpha_j}, \quad k \geq I_j,$$  \hspace{1cm} (20)

where productivity $A_j$, capital intensity $\alpha_j$, and the minimum size of investment $I_j$ are different among technologies. Substituting (20) into (3) and then into (8) and (10), the profitability and borrowing constraints now become the following:

$$w_{t+1} \leq (1 - \alpha_j) (\alpha_j/r)^{\tilde{\alpha}_j} A_j^{\tilde{\alpha}_j+1} \equiv P_j,$$  \hspace{1cm} (21)

$$w_{t+1} \leq P_j (\lambda I_j/(I_j - w_t))^{\tilde{\alpha}_j} \equiv B_j(w_t, \lambda),$$  \hspace{1cm} (22)

where $\tilde{\alpha}_j \equiv \alpha_j/(1 - \alpha_j) > 0$. The value of $\phi_j(w_t, \lambda)$ is given by the smaller of $P_j$ and $B_j(w_t, \lambda)$. As stated in Proposition 2, the equilibrium choice of technology is such that $\phi_j(w_t, \lambda)$ is the highest from among all the technologies.

While our concern is when a marginal increase in $\lambda$ causes a technological shift, it is insightful to see how the pattern of technological specialization is affected by large changes in $\lambda$. In particular, when the economy’s financial infrastructure is quite
primitive ($\lambda \to 0$), then the borrowing constraint becomes very tight ($B_j(w_t, \lambda) \to 0$ for every $j$), which means that $\phi_j(w_t, \lambda)$ is determined by $B_j(w_t, \lambda)$. In addition, (22) indicates that the higher the capital intensity, the more rapidly $B_j(w_t, \lambda)$ converges to 0 as $\lambda \to 0$. This implies that, with a sufficiently low $\lambda$, the economy specializes in a labor-intensive technology. Intuitively, if the enforcement of financial contracts is weak, only a small number of agents obtain funds due to tight credit rationing. In such a situation, entrepreneurs who have successfully obtained funds can hire a large number of workers at a low wage level, in which case, the labor-intensive technology is more suitable. Conversely, when the enforcement of financial contracts is nearly perfect ($\lambda \to 1$), the borrowing constraint becomes weaker than the profitability constraint ($B_j(w_t, \lambda) > P_j$ for every $j$), which implies that $\phi_j(w_t, \lambda)$ is determined by $P_j$. Hence, the economy specializes in the technology with the highest profitability $P_j$, which is largely determined by the technology’s productivity $A_j$. Thus, when development in financial infrastructure triggers a technological shift, the new technology tends to have higher capital intensity and higher profitability. If the profitability of a capital-intensive technology is low, it will never be adopted at any stage of economic development. If the capital intensity of a highly profitable technology is low, it would be adopted from the beginning and therefore there would be no technological shift.

Let us derive the precise condition under which a technological shift occurs. By denoting the technology before the shift $A$ and that after the shift $M$, the above observation implies that the new technology has a higher capital intensity $\alpha_M > \alpha_A$ and a higher profitability $P_M > P_A$. In this setting, the technological shift occurs when $\phi_A(w_t, \lambda) \equiv \min\{P_A, B_A(w_t, \lambda)\}$ is overtaken by $\phi_M(w_t, \lambda) \equiv \min\{P_M, B_M(w_t, \lambda)\}$. Since $P_M > P_A \geq \phi_A(w_t, \lambda)$, the value of $\phi_M(w_t, \lambda)$ is larger than that of $\phi_A(w_t, \lambda)$ when either $B_M(w_t, \lambda) \geq P_A$ or $B_M(w_t, \lambda) \geq B_A(w_t, \lambda)$ holds.\(^9\) Intuitively, as long as technology $A$ is used, the equilibrium market wage $w_{t+1}$ is bounded above by both the profitability constraint $P_A$ and the borrowing constraint $B_A(w_t, \lambda)$ for technology

\[^9\]To verify this, observe that $\phi_M \geq \phi_A \iff \min\{P_M, B_M\} \geq \phi_A \iff P_M \geq \phi_A$ and $B_M \geq \phi_A$. In the last condition, $P_M \geq \phi_A$ always holds because $P_M > P_A \geq \min\{P_A, B_A\} \equiv \phi_A$. Therefore $\phi_M \geq \phi_A \iff B_M \geq \phi_A \iff B_M \geq \min\{P_A, B_A\} \iff B_M \geq P_A$ or $B_M \geq B_A$. 

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A. If the borrowing constraint for technology $M$ is weaker than (i.e., $B_M(w_t, \lambda)$ is higher than) either of those two constraints, it means that agents can borrow enough fund to adopt technology $M$. In fact, agents always shift to technology $M$ if this condition holds, since technology $M$ is more profitable than technology $A$.

From (22), solving $B_M(w_t, \lambda) \geq PA$ gives

$$\lambda \geq \left( \frac{PA}{PM} \right)^{1/\alpha_M} \left( 1 - \frac{w_t}{I_M} \right) \equiv \Lambda_1(w_t).$$

Similarly $B_M(w_t, \lambda) \geq B_A(w_t, \lambda)$ holds if and only if $w < I_A$ and

$$\lambda \geq \left[ \frac{PA}{PM} \left( 1 - \frac{w_t}{I_M} \right) \frac{\alpha_M}{\alpha_A} \left( 1 - \frac{w_t}{I_A} \right)^{-\alpha_A} \right]^{1/(\alpha_M - \alpha_A)} \equiv \Lambda_2(w_t).$$

For convenience, let us define $\Lambda_2(w_t) = \infty$ when $w_t \geq I_A$. Combining these two conditions shows that the economy shifts to technology $M$ whenever

$$\phi_M(w_t, \lambda) \geq \phi_A(w_t, \lambda) \iff \lambda \geq \min\{\Lambda_1(w_t), \Lambda_2(w_t)\} \equiv \Lambda(w_t).$$

This clearly shows that the economy shifts from a labor-intensive technology to a more capital-intensive technology when the financial infrastructure improves to a certain extent. A typical shape of function $\Lambda(w_t)$ has been calculated numerically in Figure 3.

Figure 3 also depicts the regions in which credit rationing occurs. (From condition 17, credit rationing occurs whenever $\lambda < 1 - w_t/I_j$.). We observe that a simple relationship does not exist between the degree of contract enforcement and the existence of credit rationing. Specifically, economies in region $B_M$ experience credit rationing even though they have a better financial infrastructure than economies in region $P_A$, where no such rationing occurs. Similarly, credit rationing in region $B_M$ can be more fierce than in $B_A$, particularly when technology $M$ requires a larger scale of production. In other words, better financial infrastructure enables the economy to adopt more productive technologies, but at the same time, may cause greater inequality. While it may appear strange, this is not particularly at odds with reality. Credit rationing is not necessarily most prevalent at the initial stage of economic development, when the financial infrastructure is weak. Our model shows that such
Figure 3: Technology choice and credit regime. Numerically calculated using parameter values of $\alpha_A = .20$, $\alpha_M = .45$, $r = 2.0$, $P_A = 1.20$, $P_M = 2.25$, $I_A = 1.8$, and $I_M = 3.5$. Region $P_A$ disappears when $I_A > I_M$.

non-monotonic behavior arises because the degree of enforcement $\lambda$ not only affects the difficulty of obtaining credit for a given technology but is also a determinant of the economy’s technology specialization.

6 Dynamic Effects of Improved Financial Infrastructure

Now, we will investigate the dynamic effects of financial infrastructure. Li, Squire, and Zou (1998) showed that the degree of credit market imperfections can differ markedly across countries but change only slowly within countries. La Porta et al. (1998) found the dependence of the current performance of the financial market on an economy’s colonial and legal origins. Both these observations suggest that improvements in financial infrastructure $\lambda$, if any, must be gradual. This section examines how such gradual improvements affect the economy’s income distribution and welfare.
6.1 Wage Dynamics over Generations

Until this section, we have assumed as given the value of \( w_t \), the amount of own funds held by each entrepreneur. However, since an entrepreneur’s own funds are her first-period income, it is in fact endogenously determined by the equilibrium level of wage in one period before. In other words, the equilibrium wage in this period determines the amount of own funds for the next generation, which in turn affects the equilibrium wage in the next period. In this way, the equilibrium wage evolves dynamically over generations.

Recall from Proposition 2 that \( w_t \) evolves over generations according to \( w_{t+1} = \theta(w_t, \lambda) \). When credit rationing is absent, i.e., when the \((\lambda, w_t)\) pair is in region \( P_A \) or \( P_M \) of Figure 3, the equilibrium wage is determined by the profitability constraint: \( w_{t+1} = P_A \) or \( P_M \). When credit rationing occurs, i.e., when the \((\lambda, w_t)\) pair is in region \( B_A \) or \( B_M \), the equilibrium wage is determined by the borrowing constraint: \( w_{t+1} = B_A(w_t, \lambda) \) or \( B_M(w_t, \lambda) \). In the latter case, equation (22) implies that equilibrium wage \( w_{t+1} \) is higher (or lower) than the previous period’s wage \( w_t \), if financial infrastructure \( \lambda \) is better (or worse) than \( B^*_j(w_t) \equiv (1 - w_t/I_j)(w_t/P_j)^{1/\alpha_j} \).

Function \( B^*_j(w_t) \) gives the quality of financial infrastructure such that the market wage becomes stationary at \( w_t \) under technology \( j \).

Figure 4 depicts the steady state level of \( w_t \) against \( \lambda \) and also indicates the direction of its movement when \( w_t \) is off the steady state. A number of properties can be observed from this. First, for a given level of \( \lambda \), there is at least one steady state. There can be multiple steady states, but the lowest steady state (i.e., the steady state with the lowest \( w_t \)) is always stable. This means that, as long as the amount of own funds held by the initial old agents, \( w_0 \), is sufficiently small, the economy converges to the lowest steady state, which we denote by \( w^*(\lambda) \). We assume that this is the case and suppose that the economy always stays near the lowest steady state \( w^*(\lambda) \) in the long run.

Second, the steady state income of young agents \( w^*(\lambda) \) grows with the financial infrastructure. This implies that the income of credit-rationed agents, \( rw^*(\lambda) \), in-
creases when the financial infrastructure improves. Third, there is a threshold level of financial infrastructure, denoted by $\lambda^{st}$, such that a technology shift occurs. More specifically, in the steady state, the economy specializes in technology $j^*(\lambda) = A$ if $\lambda < \lambda^{st}$ and $j^*(\lambda) = M$ if $\lambda \geq \lambda^{st}$. In particular, observe that the steady state wage $w^*(\lambda)$ is continuous with respect to $\lambda$ even at $\lambda^{st}$. This means that a technological shift only marginally affects the income of credit-rationed agents, $rw^*(\lambda)$. In other words, if a technological shift changes the income distribution drastically, it occurs only through the changes in the way in which economic rents are distributed.

Finally, the precise pattern of the evolution depends on the minimum size of investment. The locus of the steady state transits region $P_A$, as shown by panel (i), only if $I_A$ is smaller than a threshold of

$$\zeta(I_M) \equiv P_A \left(1 - (P_A/P_M)^{1/\delta M} (1 - P_A/I_M)\right)^{-1}. \quad (27)$$

In this case, there are no rent earners immediately before the technological shift. However, if the minimum size of the old technology is larger than $\zeta(I_M)$, credit rationing exists immediately before the technological shift, as shown by panel (ii). This means that there exist a group of agents who lose economic rents when the technological shift occurs.
In Appendix, Lemma 1 formally establishes the above four properties under reasonably weak conditions.

### 6.2 Income Distribution in the Steady State

With the economy’s technological specialization $j^*(\lambda)$ and the amount of own funds $w^*(\lambda)$ in hand, we can characterize the income distribution of agents in the steady state as a function of the economy’s financial infrastructure, $\lambda$. More specifically, we consider the distribution of consumption among old agents (which coincides with their gross income) since in this economy, only old agents are assumed to obtain utility from consumption.

When $w^*(\lambda) \geq (1 - \lambda)I_j(\lambda)$, there is no credit rationing. In this case, the consumption of all the old agents is $rw^*(\lambda)$. When $w^*(\lambda) < (1 - \lambda)I_j(\lambda)$, credit rationing occurs, and only a limited number of agents can start projects. By substituting (2), (20), and (22) into (18), we obtain the number of entrepreneurs in a credit rationing steady state as

$$n^*(\lambda) \equiv \frac{\alpha_j P_j}{r I_j} \left( \frac{\lambda I_j}{I_j - w^*(\lambda)} \right)^{1+\alpha_j}, \quad \text{where } j = j^*(\lambda).$$

Among the old agents, $n^*(\lambda)$ of them start projects, and from (19), their earnings are

$$e^*(\lambda) \equiv \frac{1 - \lambda}{\lambda} r \left( I_j - w^*(\lambda) \right).$$

The remaining $1 - n^*(\lambda)$ agents are rationed from the credit market and end up consuming $rw^*(\lambda)$ in the steady state.

Before formally characterizing the effect of financial infrastructure on the income distribution and welfare, it is illustrative to consider its effects on the economy’s aggregates, such as aggregate consumption (which coincides with the gross national production in our model) and the Gini coefficient. Recall that only the old agents consume and their population is 1. When there is no credit rationing, the aggregate consumption $C^*(\lambda)$ is the same as every agent’s consumption $rw^*(\lambda)$. Since there is no inequality, the Gini coefficient $G^*(\lambda)$ is obviously zero. With credit rationing,
aggregate consumption and the Gini coefficient are, from (28) and (29),

\[ C^*(\lambda) = rw^*(\lambda) + n^*(\lambda)\left(\frac{r}{\lambda}\right)\left((1 - \lambda)I_j^*(\lambda) - w^*(\lambda)\right) , \]  
(30)

\[ G^*(\lambda) = (1 - n^*(\lambda))\left(1 - rw^*(\lambda)/C^*(\lambda)\right). \]  
(31)

When the financial infrastructure improves (\( \lambda \) increases), the aggregate consumption and the Gini coefficient respond in the following way.

**Proposition 3**  
(a) \( C^*(\lambda) \) is weakly increasing in \( \lambda \) for all \( \lambda \);  
(b) \( G^*(\lambda) \) is weakly decreasing in \( \lambda \) for all \( \lambda \) except at \( \lambda = \lambda_{st} \).

**Proof:** In Appendix.

Property (a) shows that improvements in financial infrastructure facilitate economic development in the sense that it increases the aggregate consumption. In particular, function \( C^*(\lambda) \) is strictly upward sloping when the economy faces credit rationing, and it rises discretely when a technological shift occurs. Flat segments of function \( C^*(\lambda) \) correspond to the region of \( \lambda \) under which no credit rationing occurs. In that case, a marginal change in \( \lambda \) has no effect.

In addition, property (b) shows that a better financial infrastructure reduces inequality as long as the same technology is used. However, we cannot determine its effect on inequality when a technological shift occurs. In fact, as depicted in Figure 5, the Gini coefficient tends to increase when a technological shift occurs. (Observe
that $G^*(\lambda)$ rises discontinuously at $\lambda = \lambda^{st}$.) This confirms our earlier observation that the degree of inequality changes non-monotonically when an economy develops though improvements in the financial infrastructure.

6.3 Distribution of Economic Rents and Welfare Effects

In the remainder of this section, we consider the welfare effects of the changes in income distribution. Specifically, we examine the effect of improved financial infrastructure on the average (or, equivalently, sum) of utility among all the old agents in the steady state:

$$U^*(\lambda) = \begin{cases} 
  n^*(\lambda)u(c^*(\lambda)) + (1 - n^*(\lambda))u(rw^*(\lambda)) & \text{with credit rationing} \\
  u(rw^*(\lambda)) & \text{without credit rationing},
\end{cases}$$

where individual utility function satisfies $u' > 0$ and $u'' < 0$. As we have observed above, a marginal improvement in financial infrastructure has different effects on the economy depending on whether it causes a technological shift or not. Let us consider possible cases in turn.

**Case 1: When an increase in $\lambda$ does not cause a technological shift.**

As long as the same technology is used, an increase in $\lambda$ has effects on income distribution only when credit rationing occurs, i.e., when the economy is either in region $B_A$ or in $P_A$. In that case, the equilibrium wage $w^*(\lambda)$ rises (see Figure 4), which means that the consumption of credit-rationed agents, $rw^*(\lambda)$, also rises. In addition, from equations (28) and (29), we observe that the number of rent earners, $n^*(\lambda)$, increases and the income of each of them, $c^*(\lambda)$, falls. Intuitively, an improved

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10 We use this specification of $U^*(\lambda)$ for two other reasons besides simplicity. First, among various welfare criteria, the Benthamian welfare function is considered as paying relatively little attention to inequality. We will show that the rise in inequality at the point of technological shift can be welfare reducing even under such a welfare function. Second, $U^*(\lambda)$ can be interpreted as the expected utility of young agents when they are uncertain about whether they can obtain funds or be credit rationed in the future. Given that the reforms in the financial infrastructure take considerable time, it is reasonable to evaluate the desirability of a change in $\lambda$ based on its effect on the current young generation’s expected utility.
financial infrastructure enables more agents to obtain funds and thereby increases the aggregate supply of capital. Increased supply of capital, in turn, raises the equilibrium wage and therefore the income of credit-rationed agents. However, a rise in the equilibrium wage erodes the rate of return from the project, and as a result, the income of each entrepreneur falls.

As illustrated in Figure 1(i), these changes imply that some of the economic rents received by initial entrepreneurs are redistributed to credit-rationed agents, some of whom have now become entrepreneurs and receive rents, while the others also benefit from the increased labor income. Moreover, the aggregate consumption, or the total pie, also increases. As a result, $U^*(\lambda)$ unambiguously improves.

**Case 2: When a technological shift occurs in an economy without credit rationing.**

Consider the case in which there is an absence of credit rationing in the economy when $\lambda$ is slightly below $\lambda^{st}$. Such a case occurs when the size of minimum investment for technology $A$ satisfies $I_A \leq \zeta(I_M)$ (see Figure 4(i)). Before the technological shift, the economy is in region $P_A$ and all the old agents earn $rw^*(\lambda) = rP_A$. When $\lambda$ is raised slightly above $\lambda$, the economy moves to region $B_M$, where credit rationing occurs. This creates substantial income inequality, but, nonetheless, all the agents are better off. To verify this, recall that the equilibrium wage $w^*(\lambda)$ does not fall by a technological switch. In fact, we have seen that $w^*(\lambda)$ is weakly increasing and continuous at $\lambda = \lambda^{st}$. Thus, the income of credit-rationed agents, $rw^*(\lambda)$ is at least as high as the income before the shift (see Figure 1(ii)). In addition, in region $B_M$, entrepreneurs earn surplus rents over $rw^*(\lambda)$, and therefore, their income is now discretely higher than before. Intuitively, the high productivity of the new technology allows them to earn economic rents without exploiting workers. As a result, welfare $U^*(\lambda)$ unambiguously improves.

**Case 3: When a technological shift occurs in a credit-rationed economy.**

Finally, let us consider the case in which $I_A > \zeta(I_M)$, for which a comparison must be made between income distribution in region $B_A$ and that in region $B_M$ (see Figure 4(ii)). In this case, technological shift may not make every agent better off, since some entrepreneurs adopting technology $A$ may lose their economic rents when
Figure 6: Welfare consequence of the technological shift. The number of rent earners discretely declines at the point of the technological shift if the \((I_A, I_M)\) pair is within the dark-gray area between the two thick curves. The curves are calculated numerically under parameter values of \(\alpha_A = .20\), \(\alpha_M = .45\), \(r = 2.0\), \(P_A = 1.20\), and \(P_M = 2.25\).

the economy shifts to technology \(M\). Nonetheless, the technological shift would be welfare improving if it distributes economic rents more widely among agents. However, the following proposition shows that this is not necessarily the case.

**Proposition 4** When a slight increase in \(\lambda\) causes a technological shift, the number of old agents who earn more than \(rw_t\) in the lowest steady state decreases if and only if \(I_A \in (\zeta(I_M), \chi(I_M))\) and \(I_M \geq \tilde{T}_M\), where \(\chi(I_M)\) is a continuous function satisfying \(\zeta(I_M) < \chi(I_M) < I_M\) and \(\tilde{T}_M \equiv P_A (\tilde{\alpha}_M/\tilde{\alpha}_A - 1) / ((P_M/P_A)^{1/\tilde{\alpha}_M} - 1)\).

Proof: In Appendix.

Figure 6 shows the representative shapes of functions \(\zeta(I_M)\) and \(\chi(I_M)\). When the size of investment is within an intermediate range, \(I_A \in (\zeta(I_M), \chi(I_M))\), the technological shift discretely reduces the number of rent earners even though it increases the amount of rent received by each entrepreneur.\(^{11}\) Such a concentration

\(^{11}\)Strictly speaking, Proposition 4 requires an additional condition \(I_M > \tilde{T}_M\). However, under reasonable parameter values for \(\alpha_A, \alpha_M,\) and \(P_M/P_A\), we found \(\tilde{T}\) is often close to \(P_M\) or even below \(P_M\). Since \(I_M > P_M\), condition \(I_M > \tilde{T}_M\) is usually satisfied.
of economic rents deteriorates welfare $U^*(\lambda)$ if agents have a sufficiently high risk aversion or, equivalently, their utility function $u(\cdot)$ is sufficiently concave. To observe this point, note that the technological shift caused by a marginal increase in $\lambda$ affects the steady state income distribution in three respects (see Figure 1(iii)). First, the income of credit-rationed agents, $ru^*(\lambda)$ increases only marginally. Second, the number of entrepreneurs $n^*(\lambda)$ falls discretely. Third, the amount of income received by each entrepreneur $c^*(\lambda)$ rises discretely. The first effect is marginal and is therefore dominated by the second and the third effects. The change in the number of entrepreneurs linearly affects $U^*(\lambda)$, while the increase in the entrepreneurs’ income is subject to decreasing marginal utility. Therefore, when the degree of risk aversion is sufficiently high, the second effect dominates. In this case, welfare $U^*(\lambda)$ falls discretely at $\lambda = \lambda^{\text{eff}}$.

This result can be interpreted as a certain type of crowding-out effect. Note that the rate of return from the capital-intensive technology responds less sensitively to changes in the market wage, since it relies less on labor than labor-intensive technology. As a result, the borrowing constraint for technology $M$ is less sensitive to an increase in $w_{t+1}$ than the borrowing constraint for technology $A$ (see equation 22). Therefore, once financial infrastructure is improved to slightly above $\lambda^{\text{eff}}$, entrepreneurs adopting the capital-intensive technology can attract workers by paying wage rates that are marginally higher than wages that could be offered by entrepreneurs using the labor-intensive technology, within their respective borrowing constraints. In effect, entrepreneurs with labor-intensive technologies are crowded out from the factor markets. Thus, even if the society as a whole does not want a technological shift, people cannot stay with the old technology once the improved financial infrastructure enables the adoption of the new technology.

7 Discussion: Policy Implications

Economic historians have documented many instances of social conflicts through the process of economic development. Our model shows that such conflicts occur both when inequality rises and falls. When a reform in the financial infrastructure enables
more agents to start businesses by adopting the existing technology, as in Case 1 in the previous section, the degree of inequality falls and welfare improves. However, since the income of the incumbent entrepreneurs falls due to increased competition, they may act against the reform. In this case, the desirability of the reform is not a question, but whether it can actually be carried out depends on the political process (e.g., how disproportionately the incumbent entrepreneurs have political powers).

In contrast, when a reform in the financial infrastructure triggers a technological shift, as in Case 3 in the previous section, the degree of inequality generally rises. Nonetheless, the incumbent entrepreneurs again act against the reform since they know that only a few of them will be able to adopt the new technology, while the remaining will be driven out from the business. In this case, the desirability of the reform is questionable. The non-implementation of reforms that are essential for further development does not necessarily indicate problems in the political process but can be a righteous decision by the society. If so, then, is there any remedy for this “development trap?”

The following presents two examples of such a remedy. Recall that a technological shift can be welfare reducing only when the minimum amount of investment for the old technology is of a substantial size (i.e., $I_A \in (\zeta(I_M), \chi(I_M))$). In reality, the minimum size of the investment is determined not only by the technical aspects of the technology but also by political and social constraints. For example, before the French Revolution, the old urban guilds in continental Europe restricted new entrants to their industry, effectively maintaining at a $I_A$ high level. Mokyr (1990) documents that in those days, the society chose to slow down the rate of technological progress by a vast body of regulations and restrictions on inputs and outputs.

Based on our theory, one way to escape from this development trap is to facilitate the adoption of the currently used technology by small businesses so that the minimum size of investment is reduced. Such a policy is beneficial even when one does not consider a technological shift, since it weakens credit rationing and thereby distributes the rent obtained by the incumbent entrepreneurs to workers (in fact, it can be confirmed that the income of credit-rationed agents, $rw^*(\lambda)$, increases as $I_A$
falls). Moreover, if the minimum size can be reduced to a level below the threshold (specifically, when $I_A < \zeta(I_M)$), credit rationing disappears, and the technological shift is now Pareto improving.\textsuperscript{12} Therefore, even when the long-term object is to promote the adoption of the new technology, the immediate policy is to remove barriers to the adoption of the currently used technology rather than the new technology. In the above example of continental Europe, the French Revolution abolished guilds in 1791 and subsequently in areas that fell under French domination. After that, Europe followed Britain in revolutionizing its production system. Even today, barriers to entry into the existing industries are substantial. Djankov \textit{et al.} (2002) report that the official cost of following the procedures required to start up a simple firm averages 46 percent of annual per capita GDP in the world, with this number being systematically high in low-income countries. Our theory implies that simplifications of procedures in low-income countries would be essential to break free from the development trap.

The reduction of $I_A$ is thus quite effective; however, there would be many instances in which it cannot be reduced further. In that case, a second best policy suggested by our analysis is to protect the currently used technology, or even to deter the adoption of new technology, so that the technological shift does not occur until $\lambda_t$ becomes large enough to resolve credit rationing. Specifically, equation (17) shows that credit rationing is resolved if the adoption of technology $M$ is somehow deterred until $\lambda_t$ reaches $1 - w_t/I_A$. Once this is accomplished, no party earns rents from the current technology, and the adoption of the new technology is beneficial for every agent. In the process of economic development after World War II, Okazaki (1996) reported that the Japanese government subsidized small-scale firms with primitive technologies while continually strengthening Japan’s financial infrastructure. Our model confirms that this combination of policies was effective in fostering economic development while controlling social conflicts.

\textsuperscript{12}Note that the threshold level of $\lambda$, given by $\Lambda(w_t)$ in (25), gets larger when $I_A$ is smaller. Thus, when $I_A$ is small, the credit rationing immediately after shifting to technology $M$ is not as strong as when $I_A$ is large. This is another reason why a smaller $I_A$ makes the technological shift welfare improving.
8 Concluding Remarks

We have presented a model in which economic development is facilitated by improvements in the financial infrastructure, such as the legal and accounting systems, and their enforcements. As improved financial infrastructure strengthens enforcement of financial contracts, the economy shifts from having a labor intensive technology to a technology with a higher capital intensity. The technological shift discretely increases the aggregate income, and at the same time, creates social conflicts since the incumbent entrepreneurs will lose the economic rent they are currently receiving. However, this does not necessarily mean that the technological shift brings about more equality in income distribution. In fact, the technological shift generally raises the degree of inequality, and, under a certain condition, it concentrates the economic rents among a smaller number of agents and reduces the average utility among all the agents. This creates the possibility of a development trap, in which the society as a whole is reluctant to improve its financial infrastructure for fear of causing a welfare-reducing technological shift.

We found that the precise condition under which the concentration of rents occurs is when the adoption of the currently used technology requires a considerable amount of fixed costs, which can be associated with entry barriers (e.g., trade guilds or complex procedures to set up a firm). This observation implies that a way to avoid the welfare loss is to facilitate the adoption of the currently used, not new, technology by small businesses. It even legitimizes the protection of current technology against new technologies for some time while promoting the development of financial markets. Such policies will gradually redistribute the rent received by existing entrepreneurs to the broader population and thus mitigate the welfare loss and opposition associated with the technological shift.

Does this mean that we should protect particular firms with vested interests in the status quo, despite the conventional wisdom that they are obstacles to economic development? No, it does not. Facilitating the adoption of the currently used technology by reducing the fixed cost actually decreases the income of the incumbent firms using that technology since it increases competition. The same holds when the
authority promotes the development of financial markets while deterring the adoption of newer technologies. Our analysis suggests the importance of making a clear distinction between protecting a particular technology and protecting particular firms that are connected to that technology.
Appendix

Proof of Proposition 1

Here, we establish the existence of the equilibrium market wage at period \( t + 1 \), denoted simply by \( w \), assuming as given the predetermined market wage of the previous period \( w_t \). As explained in the text, \( \min L_{t+1}^D(w; w_t) > 1 \) for all \( w \in (w(\lambda), \theta_t) \) and \( L_{t+1}^D(w; w_t) = \{0\} \) for all \( w > \bar{\theta}_t \). Thus, the intermediate value theorem implies the existence of \( w \in [\theta_t, \bar{\theta}_t] \) such that \( L_{t+1}^D(w; w_t) \ni 1 \) if the graph of \( L_{t+1}^D(w; w_t) \) is jointed for all \([\Theta, \bar{\Theta}]\), where \( \Theta \) and \( \bar{\Theta} \) are arbitrary constants with \( \Theta \in (\bar{w}(\lambda), \theta_t) \) and \( \bar{\Theta} > \bar{\theta} \) (note that we can always choose such constants since \( \tilde{\Theta} \) is finite and \( \theta_t > w(\lambda) \) from \( w_t > 0 \)). For this, it is sufficient to show that \( L_{t+1}^D(w; w_t) \) is convex-valued, non-empty, and upper hemi continuous (hereafter u.h.c.) for all \( w \in [\Theta, \bar{\Theta}] \).

From the definition that \( L_{t+1}^D(w; w_t) \) is an aggregation of individual labor demand, its non-emptiness and convexity are obvious; there is a continuum of agents, and the set of most preferred actions of each agent is convex-valued, non-empty and does not depend on the action of others given \( w \). It is also easy to see that set \( L_{t+1}^D(w; w_t) \) is compact. For any \( w \in [\Theta, \bar{\Theta}] \), condition (6) implies that the size of investment of each project is finite. Then, there exists a finite upper bound in labor demand \( \bar{L} \) in \( L_{t+1}^D(w; w_t) \), from (2). Since \( L_{t+1}^D(w; w_t) \) is closed by construction, it is compact.

According to the definition of Stokey and Lucas (1989, p. 56), a compact-valued correspondence \( L_{t+1}^D : [\Theta, \bar{\Theta}] \to [0, \bar{L}] \) is u.h.c. at \( w^* \), if, for every sequence \( \{w_n\} \) such that \( w_n \in [\Theta, \bar{\Theta}] \) and \( w_n \to w^* \), and for every sequence \( \{L_n\} \) such that \( L_n \in L_{t+1}^D(w_n; w_t) \), there exists a convergent subsequence of \( \{L_n\} \) whose limit point \( L^* \) is in \( L_{t+1}^D(w^*; w_t) \). To show that \( L_{t+1}^D(w; w_t) \) is u.h.c., fix \( w^* \in [\Theta, \bar{\Theta}] \) and pick any arbitrary sequences \( \{w_n\} \) and \( \{L_n\} \) such that \( w_n \in [\Theta, \bar{\Theta}] \), \( w_n \to w^* \), and \( L_n \in L_{t+1}^D(w_n; w_t) \).

If \( \bigcup_{n=1}^{\infty} w_n \) is finite, there must be some \( N > 0 \) such that \( w_n = w^* \) for all \( n \geq N \). Then, since \( L_n \in L_{t+1}^D(w^*; w_t) \) for all \( n \geq N \) and \( L_{t+1}^D(w^*; w_t) \) is compact, there is a convergent subsequence of \( \{L_n\} \) whose limit point \( L^* \) is in \( L_{t+1}^D(w^*; w_t) \). Therefore, the conditions for u.h.c. are satisfied.

The remainder of the case is that \( \bigcup_{n=1}^{\infty} w_n \) is infinite. Note that, since \( \rho_j(w) \)'s
intersect with each other only a finite number of times, we can choose a subsequence of \( \{w_n\} \) so that each element in set \( \{\rho_j(w_n)\}_{j \in J} \cup \mathcal{r} \) has a distinct value, i.e., so that there is always a strict ordering of profitability among technologies as well as between investment and saving. Since the set of technologies \( \mathcal{J} \) is finite, the number of patterns in the ordering of profitability that possibly appear in that subsequence is also finite. Therefore, there is at least one pattern of the strict ordering of profitability that appears infinite times in sequence \( \{w_n\} \), from which we can construct a subsequence \( w_{n_k} \to w^* \) such that

\[
\rho_1(w_{n_k}) < \cdots < \rho_{j-1}(w_{n_k}) < r < \rho_j(w_{n_k}) < \cdots < \rho_J(w_{n_k}) \text{ for all } k, \tag{32}
\]

where each of the available technologies is numbered in ascending order of profitability. In this particular ordering, \( J \) is the number of technologies, and \( \hat{j} \) is the index of the least profitable technology whose rate of return is higher than that on saving.

Let us derive \( L_{t+1}^D(w_{n_k}; w_t) \) using (32). Note that, from (5) and (7), it is optimal for each agent to invest in the most profitable technology (that with the largest index) under his specific borrowing constraint \( w_t + \epsilon_{it} \geq \eta_j(w_{n_k}, \lambda) \) as long as there is such a technology above \( \hat{j} \). More specifically, he adopts technology \( j \) if and only if the amount of his own funds is within the range of \( w_t + \epsilon_{it} \in W_j(w_{n_k}) \), where \( W_j(w_{n_k}) \) is defined recursively for \( j = J, J-1, \ldots, \hat{j} \) by

\[
W_j(w_{n_k}) \equiv \begin{cases} 
[\eta_j(w_{n_k}, \lambda), \infty) & \text{for } j = J; \\
[\eta_j(w_{n_k}, \lambda), \infty) \setminus W_{j+1}(w_{n_k}) & \text{for } j = J-1, \ldots, \hat{j}.
\end{cases}
\]

From (10) and (14), an entrepreneur with own funds \( w_t + \epsilon_{it} \) demands \( \bar{\ell}_j(w_{n_k})(w_t + \epsilon_{it})/(1 - \lambda \rho_j(w_{n_k})/r) \) units of labor whenever \( w_t + \epsilon_{it} \in W_j(w_{n_k}) \) for some \( j \geq \hat{j} \). Recall that \( \bar{\ell}_j(w_{n_k}) \equiv f_j^{-1}(w_{n_k}) \), which means that the individual labor demand is a value rather than a set. Since \( \epsilon_{it} \) is distributed uniformly between 0 and \( \overline{\epsilon} \), the aggregate labor demand is given by

\[
\sum_{j=\hat{j}}^J \int_{w_t+\epsilon \in E_j(w_{n_k}; w_t)} \frac{(w_t + \epsilon)\bar{\ell}_j(w_{n_k}) \, d\epsilon}{1 - \lambda \rho_j(w_{n_k})/r} \equiv \bar{L}_{t+1}(w_{n_k}; w_t), \tag{33}
\]

where set \( E_j(w_{n_k}; w_t) \equiv \{\epsilon \in [0, \overline{\epsilon}] | w_t + \epsilon \in W_j(w_{n_k})\} \) represents the range of random incomes with which technology \( j \) is chosen. Note that \( \bar{L}_{t+1}(w_{n_k}; w_t) \) is a function
and, therefore, set \( L_{t+1}(w_{n_k}; w_t) \) has only one element. The only choice of sequence \( \{L_{n_k}\} \) is such that \( L_{n_k} = \tilde{L}_{t+1}(w_{n_k}; w_t) \) for all \( k \). When viewed as a correspondence, \( E_j(w; w_t) \) is well defined and continuous for all \( w \in [\Theta, \overline{\Theta}] \). From (33), function \( \tilde{L}_{t+1}(w; w_t) \) is also well defined and continuous for all \( w \in [\Theta, \overline{\Theta}] \). Thus, given that \( w_{n_k} \in [\Theta, \overline{\Theta}] \) converges to \( w^* \), \( L_{n_k} = \tilde{L}_{t+1}(w_{n_k}; w_t) \) converges to \( L^* \equiv \tilde{L}_{t+1}(w^*; w_t) \).

The final task is to show that \( L^* \in L_{t+1}(w^*; w_t) \). Consider the relative profitability of each technology when the market wage is given by \( w^* \). Since \( \rho_j(w) \) is continuous, taking limit \( w_{n_k} \to w^* \) in (32) implies

\[
\rho_1(w^*) \leq \cdots \leq \rho_{j-1}(w^*) \leq r \leq \rho_j(w^*) \leq \cdots \leq \rho_J(w^*). \tag{34}
\]

In the limit, we only have a weak ordering of profitability. Nonetheless, agents with own funds \( \epsilon_{it} \in W_j(w^*) \) for some \( j \geq \hat{J} \) find it at least weakly optimal to choose technology \( j \), while other agents find it at least weakly optimal to save. Thus, \( L_{t+1}(w^*; w_t) \equiv \tilde{L}_{t+1}(w^*; w_t) = L^* \). This establishes that \( L_{t+1}(w; w_t) \) is u.h.c. at \( w = w^* \). Since \( w^* \in [\Theta, \overline{\Theta}] \) is arbitrary, the correspondence \( L_{t+1}(w; w_t) \) is u.h.c. for all \( w \in [\Theta, \overline{\Theta}] \). This completes the proof.

**Proof of Proposition 2**

*Proof of parts a and b.* Those properties are directly obtained by taking limit \( \overline{\tau} \to 0 \) in Proposition 1.

*Proof of part c.* Given technology \( j^* \), credit rationing occurs if borrowing constraint (10) is stronger than profitability constraint (8). A comparison of these conditions gives \( B_{j^*}(w_t, \lambda) < P_{j^*} \Leftrightarrow w_t < (1 - \lambda)I_{j^*} \).

*Proof of part d.* The following derives the income distribution when credit rationing occurs. Note that, from the continuity of function \( B_j(\cdot) \) for all \( j \), we can choose sufficiently small \( \overline{\tau} > 0 \) such that \( \theta(w_t + \epsilon_{it}, \lambda) = B_{j^*}(w_t + \epsilon_{it}, \lambda) \) for all \( \epsilon_{it} \in [0, \overline{\tau}] \).

Then, Proposition 1 states that \( w_{t+1} \in [B_{j^*}(w_t, \lambda), B_{j^*}(w_t + \overline{\tau}, \lambda)] \) or, equivalently, \( \eta_{j^*}(w_{t+1}, \lambda) \in [w_t, w_t + \overline{\tau}] \) from (9). Now, consider the limiting case in which the degree of heterogeneity \( \overline{\tau} \) is infinitesimally small \( (\overline{\tau} \to 0) \). In the limit, the previous relationships indicate \( w_{t+1} \to B_{j^*}(w_t, \lambda) \) and \( \eta_{j^*}(w_{t+1}, \lambda) \to w_t \). Applying
these for (14) shows that the limiting value of labor demand by any entrepreneur is $I_j \tilde{\ell}_j, (B_j, (w_t, \lambda))$. Since the total labor demand should be 1 in equilibrium, it suggests that the number of entrepreneurs in the limit is 

$$\left(I_j \tilde{\ell}_j, (B_j, (w_t, \lambda))\right)^{-1},$$

which is smaller than the number of agents, 1, under Assumption 1. Similarly, taking the limit in (15) and eliminating $B_j, \cdot$ by (10) show that the limiting value of consumption of entrepreneurs is $rw_t + r((1 - \lambda)I_j - w_t)/\lambda$, where the second term is positive since $w_t < (1 - \lambda)I_j$ from part c.

Lemma 1 and Proof

In this appendix, we formally establish the four properties discussed in subsection 6.1. Recall that we have assumed $P_M > P_A$. In the following, we assume a slightly stronger version, $P_M/P_A \geq 1/(1 - \alpha_M)$, in order to reduce the number of cases to be analyzed without affecting the main findings.

**Lemma 1** Let $j^*(\lambda)$ and $w^*(\lambda)$ denote the choice of technology and wage rate at the lowest steady state. Suppose that $P_M/P_A \geq 1/(1 - \alpha_M)$ and that increases in $\lambda$ do not cause $j^*(\lambda) \in \{A, M\}$ to change more than twice.\(^{13}\) Then, the following properties hold:

a. For any $\lambda \in (0, 1)$, there exists $w^*(\lambda) \in (0, P_M]$. In addition, if $w_t \leq w^*(\lambda)$ for some $t$, then $w_t$ converges to $w^*(\lambda)$.

b. There exists $\lambda^{st} \in (0, 1)$ such that $j^*(\lambda) = A$ if $\lambda < \lambda^{st}$ and $j^*(\lambda) = M$ if $\lambda > \lambda^{st}$.

c. $w^*(\lambda)$ is weakly increasing in $\lambda$; in addition, it is continuous at $\lambda = \lambda^{st}$.

d. $w^*(\lambda^{st}) < P_A$ whenever $I_A > \zeta(I_M)$.

*Proof of part a.* As shown by Figure 2, $0 < w(\lambda) \leq \theta(w, \lambda) \leq P_M$ for all $w \geq 0$ from the definition of function $\theta(\cdot, \lambda)$ in (12) and the assumption that $P_M > P_A$. Therefore,

\(^{13}\)By this technical assumption, we ignore an unlikely possibility of temporarily shifting back to the old, labor-intensive technology as a result of improvements in the financial infrastructure. Specifically, for this assumption to be violated, functions $B^*_A(w)$ and $B^*_M(w)$ have multiple points of intersection in the range of $w \in (0, P_A)$, which we numerically found to occur only for a very narrow range of parameters.
$\theta(0, \lambda) - 0 > 0 \geq \theta(P_M, \lambda) - P_M$. Since $\theta(w, \lambda) - w$ is continuous with respect to $w$, the intermediate value theorem shows that there is at least one $w \in (0, P_M]$ such that $\theta(w, \lambda) - w = 0$, the smallest of which is denoted by $w^*(\lambda)$.

Note that $\theta(w, \lambda) - w > 0$ for all $w \in [0, w^*(\lambda))$. Suppose that $w_t \in [0, w^*(\lambda))$. Then $w_t$ follows $w_{t+1} = \theta(w_t, \lambda) > w_t$ and thus gets higher overtime. In addition, since $\theta(w, \lambda)$ is weakly increasing in $w$, $w_{t+1} = \theta(w_t, \lambda) \leq \theta(w^*(\lambda), \lambda) = w^*(\lambda)$, which means that $w_t$ never exceeds $w^*(\lambda)$. Therefore, $w_t$ converges to $w^*(\lambda)$ whenever $w_t \leq w^*(\lambda)$ for some $t$.

Proof of part b. Define $h(\lambda) \equiv \Lambda(w^*(\lambda)) - \lambda$. Then, (25) implies that $j^*(\lambda) = A$ if $h(\lambda) > 0$ and $j^*(\lambda) = M$ if $h(\lambda) < 0$. From (23), (24), (25), and $P_M > P_A$, it follows that $\Lambda(0) = \Lambda_2(0) = (P_A/P_M)^{1/(\hat{\alpha}_M - \hat{\alpha}_A)} > 0$ and that $\Lambda(w) \leq \Lambda_1(w) \leq (P_A/P_M)^{1/(\hat{\alpha}_M - \hat{\alpha}_A)} < 1$ for any $w > 0$. Using these, we first show that $j^*(\lambda) = A$ if $\lambda$ is sufficiently small. From (22), as $\lambda \to 0$, $B_j(w, \lambda) \to 0$ for every $j$ and $w$ and, therefore, $\theta(w, \lambda) \to 0$ for all $w$ from (12). Then, $w^*(\lambda) = \theta(w^*(\lambda), \lambda) \to 0$. Therefore, $\lim_{\lambda \to 0} h(\lambda) = \Lambda(0) - 0 = (P_A/P_M)^{1/(\hat{\alpha}_M - \hat{\alpha}_A)} > 0$, which means that $\lim_{\lambda \to 0} j^*(\lambda) = A$. Conversely, $\lim_{\lambda \to 1} h(\lambda) = \Lambda(1) - 1 \leq (P_A/P_M)^{1/(\hat{\alpha}_M - \hat{\alpha}_A)} - 1 < 0$, which implies that $\lim_{\lambda \to 1} j^*(\lambda) = M$. Since we are considering the situation in which $j^*(\lambda)$ does not change more than twice, it immediately follows that there is a unique threshold $\lambda^{\text{sh}} \in (0, 1)$ such that $j^*(\lambda) = A$ if $\lambda < \lambda^{\text{sh}}$ and $j^*(\lambda) = M$ if $\lambda > \lambda^{\text{sh}}$.

Proof of part c. Choose arbitrary $\lambda_1, \lambda_2 \in (0, 1)$ such that $\lambda_1 < \lambda_2$ and suppose that $w^*(\lambda_1) > w^*(\lambda_2)$. Since $w < \theta(w, \lambda_1)$ for all $w < w^*(\lambda_1)$, as shown in part a, we have $w^*(\lambda_2) < \theta(w^*(\lambda_2), \lambda_1)$. Recall that $\theta(w, \lambda)$ is weakly increasing with respect to $\lambda$, as can be confirmed from its definition (12). Then, $\theta(w^*(\lambda_2), \lambda_1) \leq \theta(w^*(\lambda_2), \lambda_2) = w^*(\lambda_2)$. Combining the above two results yields $w^*(\lambda_2) < w^*(\lambda_2)$, which is a contradiction. Therefore, $w^*(\lambda)$ is weakly increasing.

For later use, we prove that if the $B_M$ curve crosses the 45 degree line at $w \leq P_A$, then its slope at the intersecting point is less than one. Differentiating $B_M(w, \lambda)$ with respect to $w$ and equating $B_M(w_t, \lambda)$ with $w$ show that the gradient is less than one whenever $w < (1 - \alpha_M)I_M$. Since the parameters satisfy $I_M > P_M$ and
\( P_M \geq \frac{P_A}{(1 - \alpha_M)} \) (as assumed at the beginning of this appendix), this property holds for all \( w \leq P_A \leq (1 - \alpha_M)P_M < (1 - \alpha_M)I_M \).

Using this property, now we prove the continuity of \( w^*(\lambda) \) at \( \lambda_{sft} \). Suppose that contrary to our claim, \( w^*(\lambda) \) increases discretely at \( \lambda_{sft} \). Since the \( \theta \) curve shifts upward continuously with \( \lambda \), such a jump means that the \( \theta \) curve is tangent to the 45 degree curve at \( w^*(\lambda_{sft}) \) when \( \lambda = \lambda_{sft} \). Note that \( w^*(\lambda_{sft}) \in (0, P_A] \) is implied by the continuity of \( \theta \) curve with respect to \( w \) and \( \lambda \). In addition, \( w^*(\lambda_{sft}) \) cannot be \( P_A \) since it is impossible for the \( \theta \) curve to be tangent to the 45 degree line at \( P_A \) given that the slope of \( B_M \) curve is less than one on the 45 degree line (See Figure 7(i)). Therefore, \( w^*(\lambda_{sft}) \in (0, P_A) \). This implies that \( B_M(P_A, \lambda_{sft}) \) must be smaller than \( P_A \) since, otherwise, the \( B_M \) curve is above the 45 degree line for all \((0, P_A)\) and so is the \( \theta \) curve, contradicting the premise that \( w^*(\lambda) \in (0, P_A) \) is a steady state (See Figure 7(ii)). However, if \( B_M(P_A, \lambda_{sft}) < P_A \), then \( \theta(P_A, \lambda_{sft}) = P_A \), which means that \( P_A \) is the second smallest steady state (See Figure 7(iii)). In that case, a slight increase in \( \lambda \) lets \( w^*(\lambda) \) jump to \( P_A \), where \( j^*(\lambda) = A \) still holds. This contradicts the definition of \( \lambda_{sft} \), proving that the jump assumed at the beginning cannot occur.

\[ \Box \]

**Proof of part d.** Let us first consider the timing at which the borrowing constraint get resolved under technology \( A \), by tentatively assuming that only technology \( A \) is available. Similarly to the first half of the proof of part c, it can be shown that \( w^*(\lambda) \)
is weakly increasing in $\lambda$. Then, there is a unique $\hat{\lambda} \in (0, 1)$ such that

$$w^*(\lambda) < P_A \text{ for all } \lambda < \hat{\lambda}; \quad w^*(\hat{\lambda}) = P_A. \quad (35)$$

This means that the borrowing constraint for technology $A$ resolves in steady state when $\lambda = \hat{\lambda}$. Note that, from property $c$ of Proposition 2, $\hat{\lambda} \geq 1 - P_A / I_A$ holds.

Now we show that the economy shifts to technology $M$ before $\lambda$ reaches $\hat{\lambda}$, under the assumption that $I_A > \zeta(I_M)$. Manipulations using (24) show that the condition $I_A > \zeta(I_M)$ is equivalent to $\Lambda_2(P_A) < 1 - P_A / I_A$. From the definition of $\Lambda(\cdot)$ in (25) and the property of $\hat{\lambda}$ obtained above, it follows that $\Lambda(P_A) \leq \Lambda_2(P_A) < 1 - P_A / I_A < \hat{\lambda}$. We have seen above that if the economy has not shifted to technology $M$, the steady state when $\lambda = \hat{\lambda}$ is $w_i = w_{i+1} = P_A$. However, the derived inequality $\Lambda(P_A) \leq \Lambda(\cdot)$ means that the economy has already shifted to technology $M$ when $\lambda = \hat{\lambda}$ and $w_t = P_A$. This means that the economy shifts to technology $M$ before the credit rationing resolves. Therefore, $w^*(\lambda_{\text{sft}}) < P_A$.

**Proof of Proposition 3**

*Proof of part a.* Recall that $\lambda = B_j^*(w)$ holds in any steady state with credit rationing, where $B_j^*(w)$ is defined in (26). Using it to eliminate $\lambda$ from (28) and (30) shows that the number of entrepreneurs and the aggregate consumption can be written as a function of the steady state wage $w$ and technology $j$:

$$N^{ss}(j, w) \equiv \tilde{\alpha}_j w^{1/\bar{\gamma}_j} \left( r I_j P_j^{1/\bar{\gamma}_j} \right)^{-1}, \quad (36)$$

$$C^{ss}(j, w) \equiv r w + \tilde{\alpha}_j w \left( 1 - \left( w / P_j \right)^{1/\bar{\gamma}_j} \right). \quad (37)$$

Although the above is derived by assuming that the economy is with credit rationing, (37) still gives the right amount of aggregate consumption even when there is no credit rationing: i.e., $C^{ss}(j, P_j) = r P_j$. Therefore, $C^*(\lambda) = C^{ss}(j^*(\lambda), w^*(\lambda))$ holds for all $\lambda$.

Suppose that $\lambda$ is below $\lambda_{\text{sft}}$, in which case, $j^*(\lambda) = A$ from Lemma 1. The slope of $C^*(\lambda)$ is then given by

$$\frac{dC^*(\lambda)}{d\lambda} = \frac{dC^{ss}(A, w^*(\lambda))}{dw} \cdot \frac{dw^*(\lambda)}{d\lambda}. \quad (38)$$
Differentiating (37) shows that the first term in the RHS is $r + \hat{\alpha}_A - (1 + \hat{\alpha}_A)(w^*(\lambda)/P_A)^{1/\hat{\alpha}_A} \geq 0$, where the inequality follows from $r \geq 1$ and $w^*(\lambda) \leq P_A$. Lemma 1 shows that the second term in (38) is also nonnegative. Therefore, $C^*(\lambda)$ is weakly upward-sloping for all $\lambda \in (0, \lambda_{\text{sft}})$. The same argument applies when $\lambda \in [\lambda_{\text{sft}}, 1)$, except that $A$ should be replaced by $M$, which proves that $C^*(\lambda)$ is weakly upward-sloping in that range as well. The remaining task is to prove that $C^*(\lambda)$ does not decrease at the threshold $\lambda_{\text{sft}}$. Applying $\hat{\alpha}_A < \hat{\alpha}_M$ and $P_A < P_M$ to the definition of $C_{\text{ss}}(j, w)$ in (37) shows that $C_{\text{ss}}(A, w) \leq C_{\text{ss}}(M, w)$ for any $w$. In addition, $w^*(\lambda)$ is continuous at the threshold. Therefore, the left-hand limit $\lim_{\lambda \to \lambda_{\text{sft}} - 0} C^*(\lambda) = C_{\text{ss}}(A, w^*(\lambda_{\text{sft}}))$ is lower than $C^*(\lambda_{\text{sft}}) = C_{\text{ss}}(M, w^*(\lambda_{\text{sft}}))$. ■

Proof of part b. Substituting (36) and (37) for (31) gives the Gini coefficient in the steady state as $G(j, w, B^*_j(w)) = (1 - N_{\text{ss}}(j, w))(1 - rw/C_{\text{ss}}(j, w)) \equiv G_{\text{ss}}(j, w)$. Note that $N_{\text{ss}}(j, w)$ is increasing in $w$ and that $rw/C_{\text{ss}}(j, w) = [1 + (\hat{\alpha}_j/r)(1 - (w/P_j)^{1/\hat{\alpha}_j})]^{-1}$ is decreasing in $w$, which, together, imply that $G_{\text{ss}}(j, w)$ is decreasing in $w$. When $\lambda \in (0, \lambda_{\text{sft}})$, $G^*(\lambda) = G_{\text{ss}}(A, w^*(\lambda))$, from Lemma 1. It is weakly decreasing in $\lambda$, since a rise in $\lambda$ weakly increases $w^*(\lambda)$, from Lemma 1, which weakly decreases $G_{\text{ss}}(j, w^*(\lambda))$, as shown above. The same argument applies for the case of $\lambda \in [\lambda_{\text{sft}}, 1)$, except that $A$ should be replaced by $M$. ■

Proof of Proposition 4

If $I_A \leq \zeta(I_M)$, there is no credit rationing and the number of rent earners before the technological shift is zero. Therefore it is sufficient to consider only the case of $I_A > \zeta(I_M)$, for which case the economy is credit constrained both before and after the technological shift. Then, we can use (36) and write the steady state number of entrepreneurs at levels of enforcement slightly below and above the threshold as $N_{\text{ss}}(A, w_{\text{sft}})$ and $N_{\text{ss}}(M, w_{\text{sft}})$, respectively, where $w_{\text{sft}} \equiv w^*(\lambda_{\text{sft}})$ denote the steady state wage at the threshold.

Let $Q$ denote the ratio of these two numbers. Using (36),

$$Q \equiv \frac{N_{\text{ss}}(M, w_{\text{sft}})}{N_{\text{ss}}(A, w_{\text{sft}})} = \frac{\hat{\alpha}_M}{\hat{\alpha}_A} \frac{I_A(w_{\text{sft}}/P_A)^{1/\hat{\alpha}_A}}{I_M(w_{\text{sft}}/P_M)^{1/\hat{\alpha}_M}}. \quad (39)$$
Our concern is whether \( Q \geq 1 \) or \( Q < 1 \). Note that the continuity of the steady-state wage at the threshold means that \( B_A^s(w^{\text{stf}}) = \lambda_{\text{stf}} = B_M^s(w^{\text{stf}}) \), where \( B_j^s(\cdot) \) is given by (26). Using this relationship, (39) can be simplified as

\[
Q = \frac{\alpha_M}{\alpha_A} I_A - w^{\text{stf}}. 
\]

From assumptions \( I_j > P_j \) and \( P_M > P_A \), it follows that both \( I_A - w^{\text{stf}} \) and \( I_M - w^{\text{stf}} \) are positive, guaranteeing \( Q > 0 \). Moreover, it is implied that \( Q > 1 \) whenever \( I_A \geq I_M \) (recall that \( \alpha_A < \alpha_M \)).

Let us examine how \( Q \) responds to changes in \( I_A \) when \( I_A < I_M \). Differentiating (40) with respect to \( I_A \) gives

\[
\frac{dQ}{dI_A} = \frac{\alpha_M}{\alpha_A(I_M - w^{\text{stf}})^2} \left[ (I_M - w^{\text{stf}}) - (I_M - I_A) \frac{d w^{\text{stf}}}{dI_A} \right],
\]

the sign of which depends on that of \( d w^{\text{stf}}/dI_A \). Note that function \( B_A^s(w) \) and function \( \Lambda(w) \) intersect at the point \( (\lambda_{\text{stf}}, w^{\text{stf}}) \), as shown by Figure 4(ii). Equations (26) and (25) show that with an increase in \( I_A \), \( B_A^s(w) \) shifts to the right, whereas \( \Lambda(w) \) shifts to the left, pushing the intersecting point downward. This means that \( d w^{\text{stf}}/dI_A < 0 \) and, therefore, \( dQ/dI_A > 0 \) from (41).

We confirmed that \( Q > 1 \) when \( I_A = I_M \) and that it gradually decreases as \( I_A \) falls for all \( I_A > \zeta(I_M) \). If \( \lim_{I_A \to \zeta(I_M)} Q < 1 \), the intermediate value theorem shows that there exists a value of \( I_A \) below which \( Q < 1 \) holds. We now calculate the limiting value. From the definition \( \zeta(I_M) \) and the continuity of \( w^{\text{stf}} \) with respect to \( I_A \), observe that \( w^{\text{stf}} \to P_A \) when \( I_A \to \zeta(I_M) \) (i.e., the point of technological shift approaches region \( P_A \), where \( w = P_A \)). Substituting it into \( Q = N^{ss}(M, w^{\text{stf}})/N^{ss}(A, w^{\text{stf}}) \) and using the definition of \( \zeta(I_M) \) in (27) show

\[
\lim_{I_A \to \zeta(I_M)} Q = \frac{\alpha_M}{\alpha_A} \left( \frac{P_A}{P_M} \right)^{1/\alpha_M} \frac{\zeta(I_M)}{I_M} \leq 1 \Leftrightarrow I_M \geq \overline{I}.
\]

Therefore, given that \( I_M > \bar{I}_M \), there exists \( \chi(I_M) \in (\zeta(I_M), I_M) \) such that \( Q < 1 \) for \( I_A \in (\zeta(I_M), \chi(I_M)) \). Finally, since \( Q \) is continuous with respect to \( I_M \) from (40), the implicit function theorem guarantees that the value of \( I_A \) at which \( Q = 1 \) changes continuously with respect to \( I_M > \bar{I}_M \) and approaches \( \zeta(I_M) \) as \( I_M \to \bar{I}_M \). This indicates the continuity of function \( \chi(I_M) \).
References


