Option contracts in a vertical industry

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Abstract

We examine, in a vertical industry, the strategic role of horizontal subcontracting through option contracts by a downstream dominant firm competing with a competitive fringe. Downstream production requires an input from an upstream component-producing industry composed of imperfectly competitive suppliers. We characterize how the dominant firm may outsource downstream production from fringe firms in order to gain bargaining clout in the upstream input market. It is shown that option contracts are preferred to fixed-quantity forward contracts, because leverage against upstream suppliers is gained at lower contract prices. When there is no market uncertainty option contracts do not alter spot prices beyond that caused by unavoidable market power, whereas they increase price volatility whenever demand is subject to uncertainty.

1. Introduction

An option contract is an arrangement whereby one of the involved parties, in exchange of a payment, has the right but not the obligation to buy or sell a specified item at a specified price (the strike or exercise price) up to a specified date. A forward contract may be viewed as equivalent to an option contract which is always exercised because the strike price is set at a sufficiently low level that makes its execution always profitable.

The potential strategic effects of forward contracts on the performance of spot markets have been largely explored by the literature on industrial organization. From the seminal work of Allaz and Vila (1993) examining this question in a vertically integrated industry, a number of papers have addressed the issue in relation with outsourcing of production among firms competing in the same market, which may be referred as horizontal subcontracting (Kamien et al., 1989; Spiegel, 1993; Shy and Stenbacka, 2003). In this line, Antelo and Bru (2002) showed that subcontracting downstream production through forward contracts has an anticompetitive effect on spot prices in a vertical industry, in sharp contrast with Allaz and Vila's finding concerning forwards in a vertically integrated industry.

Options contracts are a more sophisticated contracting form than forward contracts and perhaps more realistic too. A stylized fact in real-world industries is that a number of business relationships and contractual arrangements among firms feature characteristics of an option contract. To illustrate, *quantity-flexibility contracts* in the electronics industry (Barnes-Schuster et al., 2002), *backup agreements* in the catalog companies and manufacturers industry (Eppen and Iyer, 1997), or *allotment contracts* and *free sale contracts* to book hotel accommodation in the travel industry (Castellani and Mussoni, 2005) can be understood as option contracts.

The operations research literature (Zhao et al, ; Hammond, 1992; Burnetas and Ritchken, 2005; Wang and Liu, 2007) does not place the rationale of these option contracts on their potential strategic role. The light is placed on the need to manage the risk of inventories associated with uncertain demand, which leads firms to use contracts that provide flexibility and "coordinate" decisions between various links in the retailer-manufacturer supply chain when there is aversion to incurring inventory costs. Indeed, a number of supply-chain models identify the optimal actions for firms and, due to the lack of an incentive to implement those actions, how firms can adjust their terms of trade via a contract that establishes a transfer payment scheme to create that incentive (Cachon, 2001).

Given that neither the supply-chain management literature nor the research on forward markets consider the potential strategic role played by option contract, our primary goal in this paper is to fill this gap by extending the analysis on the strategic role of forward markets to option contracts.[1] In particular, we want to examine the strategic effects of option contracts in a framework in which a dominant firm competing with a fringe of price-taking firms have to acquire an essential input from imperfectly competitive upstream suppliers to produce a final good. Our objective is to investigate how subcontracting production by the dominant firm to the competitive fringe through option contracts and forward contracts may facilitate the acquisition of inputs at more favorable marginal prices. Put differently, how these contracts regarding subcontracting downstream production to the fringe has a potential to gain strategic benefits in the upstream market. This is because these contracts strengthen the bargaining position of the contractor firm in dealing with the supplier, by creating a more valuable alternative in case of a breakdown in negotiations.

Our research adds to the literature four important findings. First, when subcontracting of downstream production takes the form of option contracts, the dominant firm outsources production to the fringe.[2] Hence, it can strategically use option contracts to subcontract with the fringe as a means to increase the clout in negotiating the acquisition of the input with upstream suppliers by reinforcing its market power in the downstream market. Second, although both option contracts and forward contracts facilitate better deals with input suppliers, option contracts are more profitable than forward contracts for the dominant firm. As a consequence, it

[1] Contract theory has also analyzed a different role for option contracts; namely, its usefulness to alleviate moral hazard problems in trades when contracts are incomplete and specific investments are needed, in which case efficiency may be achieved with an option contract (Noldeke and Schmidt, 1995; Segal and Whinston, 2002). But both the operations research literature and that on industrial organization show the use of these contracts with no specific investment whatsoever.

[2] Antelo and Bru (2002) found a similar result when subcontracts are binding forward contracts for fixed quantities. They also analyzed the balance between vertical and horizontal effects when the downstream segment consists of an oligarchic dominant group together with a price-taking fringe.

will prefer to use option contracts to outsource downstream production, whenever feasible, instead of forward contracts. Third, if the market demand of final good is certain, option contracts do not lead to price manipulation in the downstream market beyond that caused by market power. Thus, unlike forward contracts, option contracts are innocuous for consumers. Fourth, when demand is variable and subject to uncertainty, option contracts may increase volatility on spot prices and, as a result, the price of final good may increase.

The dominant firm may gain leverage against upstream suppliers by subcontracting downstream production to its rivals either through forward contracts or through option contracts (not necessarily exercised). Although both types of contracts do not seem very different, their strategic effects in the contractor's behaviour and the performance in the spot market are quite different, at least when the demand of final good is not subject to uncertainty. Regarding forward contracts, they are strategically equivalent to a consolidation process of the dominant firm that unfolds through purchasing productive capacity and lead it to achieve stronger market power downstream. This is because the dominant firm only gains leverage against input suppliers if it actually purchases production from fringe firms. The effect of the stronger market power by dominant firm is then an increase in the price of final good.

Contrariwise, when the dominant firm is not restricted to outsource production through forward contracts only, but may also resort to option contracts, it stops manipulating the spot price and uses option contracts with the sole purpose of increasing its bargaining power in the input market. Hence, a dominant firm with idle capacity may even subcontract production to the fringe, but this will be made through option contracts that are sometimes not exercised. Option contracts, although likewise anti-competitive, do not increase the price of final good (at least in the absence of demand uncertainty), because the dominant firm only needs to sign option contracts, i.e. to *threaten* to buy some downstream production from the fringe, to facilitate better deals in the input market. Indeed, it turns out that options are preferable to forwards not only for the dominant firm, but also for final consumers, although fringe firms would prefer forward contracts.

That option contracts do not lead to downstream market distortions beyond those caused by unavoidable market power whenever there is no uncertainty in demand of final good is due to the fact that they are signed featuring a strike price such that in equilibrium the contracts are never exercised. Thus, they only lead to a redistribution of rents from the crucial upstream supplier to the dominant downstream firm.

However, under demand uncertainty, the optimal option contracts regarding subcontracting production to the fringe feature a such an equilibrium strike price that they are exercised when the demand realization is high. Although option contracts could be designed featuring a such strike price that they are never executed, usually this is not the optimal option contract. Thus, option contracts lead to stronger market power in the downstream side, which in turn causes market distortions in the form of a more volatile price for final good and reduced expected consumer surplus.

That forward and option contracts are both anti-competitive in a vertical industry sharply contrasts with Allaz and Vila (1993)'s finding concerning forward contracting in a vertically integrated industry.[3] What we show in this paper is, firstly, that option contracts are more profitable than forward contracts and, as a consequence, they will be preferred whenever they are feasible. Secondly, option contracts have similar anti-competitive consequences to the ones proved for forward contracts in Antelo and Bru (2002). In particular, the conclusion that option contracts in a vertically industry do not lead to price distortion may not hold under market uncertainty. As a result, option contracts are better than forward contracts for both the dominant firm and consumers. To assure fringe's production, the dominant firm pays less for the outsourced production through option contracts than through forward contracts because only the latter lead to an increase in equilibrium price. Thus, the dominant firm subcontracts more production when option contracts are available, and this allows it to increase its bargaining power in the upstream market. In addition, even if the dominant firm does not execute the option contracts in equilibrium, this does not worry consumers, because is merely a struggle

[3] Note, however, that Allaz and Vila's results have been qualified by Ferreira (2003).

between the dominant firm and upstream supplier. Finally, fringe firms would prefer forward contracts to option contracts.

Summing up, our analysis of option contracts extends that of forwards in the sense that the strategic *vertical* effect implied by both contracts is clearer for options than for forwards. The dominant firm manipulates the price of final good when it is restricted to use forward contracts, but if option contracts are feasible, the strategic effect stops increasing such a price and instead increases bargaining power in the upstream market. Option contracts then do not introduce additional distortions to those caused by a market power firm since they are used to increase its leverage against suppliers, which do not lead to additional price distortions in the price of final good. Hence, if a vertical industry can use option contracts allowing the internalization of profits, the vertical structure leads to distortions in the price of final good, even when double marginalization is not an issue,[4] and such distortions ought to be compared with the risk of vertical foreclosure from a vertically integrated structure.[5]

The finding whereby manipulation in the price of final good would not be observed when the dominant firm uses option contracts may not hold under demand uncertainty. In this case, option contracts lead to greater price volatility and, as a result, higher prices for consumers. The dominant firm wishes to increase its bargaining power in the upstream market, but it does not have an incentive to increase the price of final good, since such an increase is passed on to fringe firms as more expensive option contracts. The increase in the downstream market price is a by-product of the fact that option contracts are not contingent on the state of demand.

The rest of the paper is organized as follows. Section 2 outlines the model. In Section 3 we examine the use of forward contracts and option contracts in a vertically integrated industry, where these contracts cannot be used for vertical strategic purposes. The effects of such

[4] The problem with vertical separation is an outcome of limited contractual options between the firms (both in vertical and in horizontal contractual relationship). On one hand, we may conjecture that complete contingent contracts between downstream firms would lead, in equilibrium, to contracts not exercised in equilibrium; these contracts would not be used in order to increase price manipulation, but only to obtain better deals from suppliers. On the other hand, if suppliers and retailers could set long-term contracts, then firms would internalize the vertical profits and no further distortions would appear; the threat of short-term option contracts would serve only to shift rents from suppliers to retailers.

[5] See Rey and Tirole, 2004, for a survey.

contracts on the dominant firm's behaviour, the (rational-expectations equilibrium) price of contracts and the market performance in general is investigated. Section 4 examines the strategic use of these contracts in a vertical industry showing that a dominant firm may subcontract downstream production to the fringe not for horizontal motives but for vertical reasons; namely, to gain leverage in the input market. Despite having idle capacity, the dominant firm may subcontract production through option contracts that it may not even exercise. This section also shows that option contracts are innocuous for final consumers. Section 5 is devoted to uncertainty in demand of final good and shows that in this case the use of options may increase price volatility in such market. Finally, Section 6 concludes the paper.

2. The model

We consider an industry in which a dominant firm and a fringe of small firms all produce a homogeneous final good. In Section 3 the industry is viewed as a vertically integrated industry with all firms having identical marginal (and average) costs. Thereafter, it adopts a vertical shape, being the dominant firm and the fringe the downstream side, both producing a final good for which an essential input (intermediate good) is needed. The input is purchased to an upstream supplier at marginal and average cost c, $c>0$.

In this framework, the dominant firm may subcontract downstream production to the fringe firms through option contracts (where options may not be exercised) or through binding forward contracts. In order to concentrate on the vertical strategic role of such contracts, we first assume, in Sections 3 and 4, that downstream demand is certain and unchanging. Later, in Section 5, we consider that demand is subject to uncertainty, in which case all firms are assumed to be risk-neutral.

In what follows, the cost to the dominant firm and the fringe of acquiring the intermediate good will be denoted by c_D and c_F, respectively. The strike price of option contracts and the delivery price of forward contracts is referred as p_s,[6] whereas the number of contracts signed is X, and the number of option contracts exercised is x (for forward contracts, if follows that $x = X$). The dominant firm's production when it executes x option contracts is $q(x,c)$, and the productive capacity of the competitive fringe as a whole is m. Finally, the inverse demand function for final good is given by $P(Q)$, satisfying $P'(Q) < 0$ and $P''(Q)(q + x) + 2P'(Q) \leq 0$, for all $q \in [0,Q]$.

We assume that $P(Q) > c_F$, so the fringe always produce at full capacity m, whereby $Q = q(x,c) + m$. The value of an option contract is $p_c \equiv \max\{p - p_s, 0\}$, where p is the expected spot price. Clearly, in a rational expectations equilibrium, a fringe firm expects the spot price to be $p \equiv P(Q^{eq})$, whereas the spot price for a dominant firm is endogenously determined and depends on its own level of production $q(x,c)$.

Note that an option contract to subcontract the amount of production x_i with a fringe firm i is equivalent to set a non-linear tariff $T(x_i) = F_i + w_i x_i$, where production acquired by the dominant firm must satisfy $x_i \leq X_i$, the fixed fee of the tariff must be $F_i = p_s X_i$, and the linear part must be $w_i = p_s x_i$. On the other hand, we will see below that, for sufficiently low levels of the strike price, the dominant firm exercises all the option contracts signed, in which case option contracts are equivalent to signing X forward contracts with price $p_f = p_c + p_s$ (subscript f stands for forward price) and to signing X_i forward contracts with any fringe firm i for a fixed payment $F_i = (p_c + p_s) X_i$.

3. Subcontracting production through option contracts

What, in a vertically integrated industry, are the incentives of a dominant firm to outsource production to the fringe firms either through a futures market or through an options market? Ex-

[6] Given the assumptions made on demand, in Sections 3 and 4 there is no opportunity for speculation in option or forward markets, so contracts are effectively non-transferable agreements between the dominant firm and fringe firms and there is no need to consider an option premium or a forward premium independently of the strike price and the market price at execution.

post, a dominant firm may manipulate the price of final good by signing option contracts, but ex-ante it has no incentive to sign them, at least not at a strictly positive price. It will only then sign innocuous option contracts with price $p_c = 0$ that are not subsequently executed and hence that do not affect its market behaviour and profits.

Let us first assume that the dominant firm signs, with the fringe, $X \in [0,m]$ option contracts at strike price p_s, and that it exercises x of them later on. In these circumstances, its profits amount to

$$\pi_D(q(x,c),x) = [P(q+m) - c]q + [P(q+m) - p_s]x \tag{1}$$

and are composed of two terms: the first added is the profit derived from internal production, whereas the second one is the profit due to the execution of a certain volume of option contracts. It also can be noted from (1) that the dominant firm finds it profitable to exercise x, $x > 0$, option contracts if, and only if, the resulting market price is above the strike price, $P(q+m) > p_s$.[7]

It is illustrative to solve the problem stated in (1) by proceeding in two stages. We first determine the optimal level of internal production for the dominant firm, $q(x,c)$, given the number of executed option contracts, x, and then we determine the optimal number of option contracts executed. For a given level x of executed option contracts, the internal production of the dominant firm, $q(x,c)$, is that which satisfies the first-order condition[8]

$$P'(q(x,c)+m)\,(q(x,c)+x) + P(q(x,c)+m) - c = 0\,. \tag{2}$$

[7] The difference between the strike price and the spot price of the ontract will determine its value or moneyness.
[8] For use in Section 4, where c is a variable, note that this result implies that $d\Pi_D/dc = -q^* < 0$.

Differentiating (2) with respect to x shows that $\frac{\partial q(x,c)}{\partial x} = -\frac{P'(Q)}{P''(Q)(q+x)+2P'(Q)}$ is negative by virtue of the assumptions made on demand. It then follows that: *a)* $P(q(x,c)+m)$ increases with x, and *b*) since $\frac{\partial \pi_D(q(x,c),x)}{\partial x} = P(q(x,c)+m) - p_s = 0$, it holds that

$$\frac{\partial^2 \pi_D(q(x,c),x)}{\partial x^2} = P'(q(x,c)+m)\frac{\partial q(x,c)}{\partial x} \tag{3}$$

is positive. Then $\pi_D(q(x,c),x)$, the profit given in (1), is a convex function of x and, therefore, its maximum in $[0,X]$ is attained at $x=0$ or $x=X$. If we define

$$h(X,c) \equiv \frac{1}{X}\int_0^X P(q(x,c)+m)\,dx\,, \tag{4}$$

we can obtain the following result by comparing profits in both corner solutions.

Lemma 1. *If the dominant firm signs X option contracts with the fringe, then all of them are exercised (x=X) whenever the strike price satisfies* $p_s < h(X,c)$*, whereas no one is exercised (x=*0*) if the strike price satisfies* $p_s \geq h(X,c)$.

Proof. See the Appendix.

Lemma 1 establishes that only option contracts that feature a strike price $p_s < h(X,c)$ will be executed by the dominant firm. Note that, accordingly, an option contract featuring a strike price $p_s < h(X,c)$ is strategically equivalent to a forward contract.

Since $h(X,c) < P(q(X,c)+m)$, the price for the option contracts must compensate the difference between the expected final price and the strike price, i.e. fringe firms will only sign option contracts that feature $p_c + p_s \geq P(q(X,c)+m)$. At the cheaper option contract that is accepted by fringe firms, the dominant firm's profit is

$$\pi_D(q(X,c),X) - p_c X = \left[P(q(X,c)+m) - c\right] q(X,c) \qquad (5)$$

and the following result holds.

Lemma 2. *It is never profitable for the dominant firm to sign option contracts that feature a strike price satisfying* $p_s < h(X,c)$*, i.e., that will be later on executed.*

Proof. See the Appendix.

In a rational expectations equilibrium, the dominant firm has no incentive to manipulate the price of final good by subcontracting production through option contracts that are exercised in equilibrium. Since fringe firms ask for a total price $p_c + p_s$ at least equal to the expected spot price $P(Q)$, then if the dominant firm subcontracts production to the fringe, its incentive to manipulate spot prices increases, but any additional rent of the industry thanks to any price increase is passed from the dominant firm to the fringe in a rational expectations equilibrium. Therefore, one cannot expect the dominant firm to sign option contracts in a vertically integrated industry.[9]

In the next section, we will examine how the strategic behaviour of the dominant firm in subcontracting (downstream) production to fringe firms is affected when they all form the downstream segment of a vertical industry and deal with imperfectly competitive upstream suppliers of an essential input.

[9] Allaz and Vila (1993) show that a dominant firm signs contracts (sells forward its production to final consumers and/or to firms on the fringe) in order to gain market share at the expense of rivals.

4. Horizontal subcontracting in a vertical industry

We now consider a vertical industry with upstream suppliers of an input (intermediate good) and downstream manufacturers-retailers of a final good. Upstream, a super-competitive supplier (efficient supplier) is capable of producing an essential input that is required by downstream firms at marginal cost $\underline{c}$, $\underline{c} \geq 0$, jointly with an alternative but less efficient source of the input at marginal cost $\overline{c}$, $\overline{c} > \underline{c}$.[10] For the sake of simplicity, the input is transformed with no additional production costs into the final good on a one-to-one basis by downstream firms. Furthermore, we assume that the lowest conceivable spot price, $P(q(0,\underline{c}) + m)$, is such that $P(q(0,\underline{c}) + m) > \overline{c}$, which guarantees positive profits to fringe firms and, therefore, implies that all their production capacity *m* will be always in use.

The efficient supplier can charge the dominant firm a two-part tariff $T(q) = F + wq$ for the input. To simplify, we assume that the supplier can make a take-it-or-leave-offer, where the alternative for the dominant firm is to produce at marginal cost $\overline{c}$, i.e. the cost of acquiring the input to the alternative source of less efficient suppliers.

To this quite standard model of vertical relations, we add the possibility that the dominant firm subcontracts downstream production to the fringe through publicly observable option contracts. The timing of the game is as follows. First, the dominant firm subcontracts production to fringe firms through option contracts. Second, the upstream efficient supplier offers an input contract to the dominant firm. Third, the dominant firm decides whether to accept this proposal. Fourth, the dominant firm decides the number of options it exercises on the production of the fringe (if any). Finally, the dominant firm sets its own production level. This timing is at reflecting the fact that subcontracting downstream production to the fringe is the longest-term action in the process.[11] As usual, we look for a subgame perfect equilibrium.

[10] Instead of a competitive supply of the input, the analysis would be unchanged if $\overline{c}$ were a price ceiling fixed by price regulation.
[11] The influence of this timing on our results is discussed below.

If there has been an agreement with the input supplier in the third stage of the game, the dominant firm produces $q(x,w)$, where x denotes its decision whether to execute its option contracts (in the equilibrium), with $0 \leq x \leq X$. In equilibrium, the dominant firm accepts the input contract and has profits

$$\pi_D(p_s, w) - F - p_c X \ , \tag{6}$$

where $\pi_D(p_s, w) = [P(q(x^{eq}, w) + m) - w]q(x^{eq}, w) + [P(q(x^{eq}, w) + m) - p_s]x^{eq}$, being x^{eq} its decision whether to execute its option contracts in equilibrium. According to Lemma 2, it will execute all the contracts if $p_S < h(X, w)$ and will execute none of them if $p_S > h(X, w)$.

On the contrary, in case of disagreement with the input supplier, the dominant firm produces $q(x^{off}, \overline{c})$, where x^{off} denotes its decision whether to execute its option contracts (off the equilibrium). Again according to Lemma 2, it will execute all the contracts if $p_s < h(X, \overline{c})$ and will execute none of them if $p_S \geq h(X, \overline{c})$. In this case, its profits amount to

$$\pi_D(p_s, \overline{c}) - p_c X \ , \tag{7}$$

where $\pi_D(p_s, \overline{c}) = [P(q(x^{off}, \overline{c}) + m) - \overline{c}]q(x^{off}, \overline{c}) + [P(q(x^{off}, \overline{c}) + m) - p_s]x^{off}$. Therefore, the dominant firm accept the input contract proposal if

$$\pi_D(p_s, w) - F \geq \pi_D(p_s, \overline{c}) \ . \tag{8}$$

Since the input supplier has all the bargaining power, it can charge the largest fixed fee that satisfies (8), $F = \pi_D(p_s, w) - \pi_D(p_s, \overline{c})$, and has profits

$$\pi_D(p_s, w) - \pi_D(p_s, \overline{c}) + (w - \underline{c})q(x^{eq}, w) \ . \tag{9}$$

It is immediate that the input supplier maximizes profits when setting an input contract with a wholesale price $w = \underline{c}$ and a fixed-fee payment $F = \pi_D(p_s, \underline{c}) - \pi_D(p_s, \overline{c})$. Hence, in equilibrium the dominant firm will obtain the profit

$$\pi_D(p_s, \underline{c}) - F - p_c X = \pi_D(p_s, \overline{c}) - p_c X \ . \tag{10}$$

We can now analyse the subcontracting decision of the dominant firm in the first stage of the game. First note that $h(X, \underline{c}) < h(X, \overline{c})$, where $h(X, \underline{c}) = \frac{1}{X}\int_0^X P(q(s, \underline{c}) + m)ds$ and $h(X, \overline{c}) = \frac{1}{X}\int_0^X P(q(s, \overline{c}) + m)ds$;[12] and that, by virtue of Lemma 1, the optimal decision of the dominant firm is always either to execute all option contracts, x=X, or not to execute any of them, x=0. We have then to distinguish three possibilities:

(i) Option contracts feature a strike price p_s that satisfies $h(X, \overline{c}) < p_s$.

(ii) Option contracts feature a strike price p_s that satisfies $p_s < h(X, \underline{c})$.

(iii) Option contracts feature a strike price p_s satisfying $h(X, \underline{c}) < p_s < h(X, \overline{c})$.

(i) If option contracts feature a strike price p_s satisfying $h(X, \overline{c}) < p_s$, we know from Lemma 1 that they will not be executed, neither if there is an agreement with the efficient input supplier and marginal costs of production are $\underline{c}$ nor if the dominant firm uses the alternative input source and thus its marginal cost of production becomes $\overline{c}$, $x^{eq} = x^{off} = 0$. Since fringe firms expect

[12] Since $\frac{\partial h(X,c)}{\partial c} = \frac{1}{X}\int_0^X P'(q(s,c)+m)\frac{\partial q(s,c)}{\partial c}ds = \frac{1}{X}\int_0^X \frac{P'}{P''(q+s)+2P'}ds > 0$, $h(X,c)$ is an increasing function in c.

that option contracts will not be exercised, they will accept to sign them at any slightly non-negative price $p_c = \varepsilon$, and the dominant firm has profits

$$[P(q(0,\overline{c})+m)-\overline{c}]q(0,\overline{c}) . \tag{11}$$

(ii) If the strike price is such that $p_S < h(X,\underline{c})$, we know that both in the equilibrium and off the equilibrium all option contracts are executed, $x^{eq} = x^{off} = X$. Fringe firms will sign the option contracts only if they compensated the expected final price,[13] $p_c + p_s = P(q(X,\underline{c})+m)$, and the dominant firm has profits

$$[P(q(X,\overline{c})+m)-\overline{c}]q(X,\overline{c})+[P(q(X,\overline{c})+m)-P(q(X,\underline{c})+m)]X . \tag{12}$$

(iii) Finally, if the strike price featured by option contracts is such that $h(X,\underline{c}) \leq p_S < h(X,\overline{c})$, we know, from Lemma 1, that the dominant firm sets option contracts that are exercised only off the equilibrium path, $x^{eq} = 0$ but $x^{off} = X$. Note that, in addition to option contracts not being exercised in equilibrium, the strike price is well above the expected spot price, $p_S \geq h(X,\underline{c}) > P(q(0,\underline{c})+m)$; hence, fringe firms are willing to sign option contracts with exercise price $p_s \geq h(X,\underline{c})$ at a call price $p_c = 0$, and the dominant firm has profits

$$[P(q(X,\overline{c})+m)-\overline{c}]q(X,\overline{c})+[P(q(X,\overline{c})+m)-p_s]X . \tag{13}$$

From the analysis of dominant firm's profits given in (11)-(13), the following result holds.

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[13] When the strike price satisfies $p_S < h(X,\underline{c})$, option contracts are equivalent to forward contracts with a futures price $p_f = P(q(X,\underline{c})+m)$, both in their impact on final prices and in the rents accrued by the dominant and fringe firms.

Proposition 1. *The dominant firm subcontracts downstream production by signing option contracts with all the fringe,* X^{*}=*m, at a strike price* $p_s = h(X, \underline{c})$ *such that, in equilibrium, the option contracts are never exercised.*

Proof. See the Appendix.

The dominant firm could subcontract downstream production to fringe firms by signing option contracts that are exercised in equilibrium (and are thus equivalent to forward contracts); the derivative of profits given in (11) with respect to *X* is

$$P(q(X,\overline{c})+m)-P(q(X,\underline{c})+m)-\frac{\partial P(q(X,\underline{c})+m)}{\partial X}X \ , \tag{14}$$

which is strictly positive when evaluated at *X*=0, since $P(q(0,\overline{c})+m)-P(q(0,\underline{c})+m)>0$. However, the dominant firm obtain larger profits with option contracts that are not executed in equilibrium, because the "call price plus the strike price" of an option contract is below the price to be paid with a forward contract.

That price of forward contracts is larger is due to the dominant firm is committed to buy downstream production to the fringe in all circumstances, both in equilibrium (i.e. when an agreement with the upstream supplier is reached) and off-the-equilibrium path (i.e. in the case of a breakdown in negotiations with the input supplier). As a consequence, forward contracts lead to an increase in the price of final good that are reflected in the forward price, $p_f = P(q(X,\underline{c})+m)$, which is larger than prices when contracts are not exercised, $P(q(0,\underline{c})+m) < p_f$.

It is worth noting that, since in equilibrium the dominant firm does not introduce any further price manipulation in the spot market beyond that deriving from exerting its market power in the downstream market, the only impact of the option contracts is a redistribution of rents from the input supplier to the downstream dominant firm. Subcontracting downstream production through option contracts is then innocuous both for consumers and the fringe when there is no market uncertainty. Subcontracting through a forward market is, in contrast, anti-competitive, since it leads the dominant firm to reduce its production in equilibrium, which increases the price of final good.

This is a quite interesting result to the point that one could say that a social planner worried about consumer surplus should promote the use of option contracts. But, does this result survive in a market uncertainty context? We will see in the next Section that the absence of anti-competitive effects of option prices may not persist if the demand of final good is subject to uncertainty.

Note that our result does not depend on the assumption that the upstream supplier offers take-it-or-leave-it input contracts to the dominant firm. If for instance it is the dominant firm who propose contracts with probability β , $\beta \in (0,1)$, its net profits become

$$\beta\pi_D(x^{eq},\underline{c}) + (1-\beta)\pi_D(x^{off},\bar{c}) , \tag{15}$$

and it can be shown that the optimal option contract (and the quantity of contracts) remains the same; hence, in equilibrium, the dominant firm still does not acquire any downstream production from fringe firms.

We can more generally discuss what happens if the upstream supplier and the downstream dominant firm can deal for the input exchange under different scenarios; namely, (a) *both before and after* the futures market opens, (b) *only after* the futures market opens, (c) *only after* the options market opens, (d) *only after* the dominant firm acquires productive capacity from the fringe. For the dominant firm, scenario (c) is the best one, whereas the remaining scenarios lead it to have the same net profits. For consumers, however, scenarios (a) and (c) are equivalent in terms of consumer surplus since contracts do not induce further distortions in the price of final good other that the unavoidable existence of a dominant firm that exerts market power in the downstream market; moreover, either scenario (a) or scenario (c) are preferred to scenarios (b) and (d), where the dominant firm's strategic actions have anti-competitive consequences.

5. Option contracts when there is demand uncertainty

In Section 4 we obtain the striking result that, in equilibrium, option contracts are not exercised and, as a consequence, their signing do not affect final prices. In this section we deal with the impact of option contracts when there is demand uncertainty. We see that option contracts are sometimes exercised in equilibrium when demand is high, and as a consequence final prices are affected by the existence of such contracts.

Let $P(Q,\theta)$ be the inverse demand function of the final good and θ the uncertainty parameter for this demand, uniformly distributed in $[\underline{\theta},\overline{\theta}]$ with $\underline{\theta}<\overline{\theta}$. We assume that this demand satisfies $P_\theta(Q,\theta)>0$ and that final prices (taking into account production $q(s,c,\theta)$ of the dominant firm) increase in θ, $\frac{\mathrm{d}P(q(s,c,\theta)+m,\theta)}{\mathrm{d}\theta}=\frac{\partial P}{\partial Q}\frac{\partial q(s,c,\theta)}{\partial \theta}+\frac{\partial P}{\partial \theta}>0$.[14] We moreover restrict ourselves to parameter values for which (i) the price of the final good is always above $\overline{c}$, and (ii) the dominant firm always produces internally.[15]

The full game develops now as follows. First, under demand uncertainty, the dominant firm subcontracts downstream production by signing X option contracts with fringe firms, $0\le X\le m$. Second, the dominant firm deals with the efficient input supplier. Third, the dominant firm decides whether to accept the supplier's proposal. In case of disagreement, the dominant firm is restricted to use the alternative source of inputs. Fourth, demand uncertainty is

[14] This is indeed the case, for instance, for a linear demand as $P(Q,\theta)=\theta-Q$ and for a constant-elasticity demand as $P(Q,\theta)=\theta\, Q^{-\gamma}$ with $\gamma\in(0,1)$.

[15] For a linear demand, this amounts to assume parameter values that satisfy $m<\min\left\{\underline{\theta}+\underline{c}-2\overline{c},\frac{\underline{\theta}-\overline{c}}{2}\right\}$.

revealed.[16] Fifth, the dominant firm decides how many option contracts to exercise. Finally, it decides its level of internal production. As usual, we look for a subgame perfect equilibrium.

We may readily characterize the dominant firm's behaviour regarding the existence of demand uncertainty. Now (4) depends on the particular realization of demand,

$$h(X,c,\theta) \equiv \frac{1}{X}\int_0^X P(q(s,c,\theta)+m,\theta)\,\mathrm{d}s\,, \tag{16}$$

where $q(s,c,\theta)$ is the optimal production of the dominant firm, given the executed option contracts *s*, production cost *c* and demand $P(Q,\theta)$. Given production cost *c* and demand $P(Q,\theta)$, Lemma 2 still applies: the dominant firm executes all the *X* option contracts signed, $x^*=X$, whenever the strike price p_S satisfies $p_S \le h(X,c,\theta)$; otherwise, the dominant firm exercises no option contract, $x^*=0$.

It is straightforward to see that[17] $\dfrac{\partial h(X,c,\theta)}{\partial c} > 0$ and $\dfrac{\partial h(X,c,\theta)}{\partial \theta} > 0$. This means that, given option contracts *X* and cost *c,* there are three relevant intervals:

(i) If the strike price p_S satisfies $p_S \le h(X,c,\underline{\theta})$, then the dominant firm exercises all the option contracts in any demand state;

(ii) if the strike price p_S satisfies $h(X,c,\underline{\theta}) < p_S < h(X,c,\overline{\theta})$, there is a demand state, $\hat{\theta} = \theta(p_S, X, c) \in (\underline{\theta}, \overline{\theta})$, in which the dominant firm exercises all contracts whenever demand satisfies $\theta \in (\hat{\theta}, \overline{\theta})$, and exercises no contract when $\theta \in (\underline{\theta}, \hat{\theta})$;

[16] A change in the order of stages 2 and 4, i.e. that demand is known when negotiation of input contracts takes place, does not affect the choices of the dominant firm (the number of option contracts signed and exercised and the level of production). Of course it would affect the dominant firm's payment to the input supplier that would depend on the realization of demand, but the overall expected payment would remain the same.

[17] That $\dfrac{\partial h(X,c,\theta)}{\partial c} > 0$ is a consequence of $\dfrac{\partial P(q(s,c,\theta)+m,\theta)}{\partial c} = P'\dfrac{\partial q(s,c,\theta)}{\partial c} = \dfrac{P'}{P''(q+s)+P'-c} > 0$. In turn, that $\dfrac{\partial h(X,c,\theta)}{\partial \theta} > 0$ is due to $\dfrac{\mathrm{d}P(q(s,c,\theta)+m,\theta)}{\mathrm{d}\theta} = \dfrac{\partial P}{\partial Q}\dfrac{\partial q(s,c,\theta)}{\partial \theta} + \dfrac{\partial P}{\partial \theta} > 0$.

(iii) if the strike price p_s satisfies $p_s > h(X, c, \overline{\theta})$, there is no demand state for which the dominant firm exercises the option contracts.

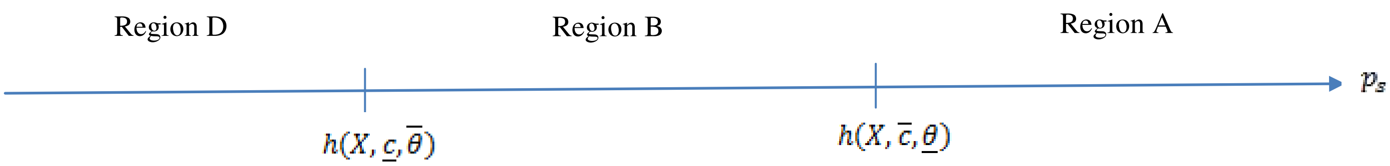
We can easily discard that the optimal strike price lies in (i) or (iii). For $h(X, \underline{c}, \underline{\theta}) < p_s < h(X, \overline{c}, \overline{\theta})$, if the dominant firm produces at marginal cost $c = \underline{c}$ we define θ^{eq} as the minimum value of θ for which options are exercised. This is either $\theta^{eq} = \overline{\theta}$ (and contracts are never exercised) or the value of the demand parameter that satisfies $h(X, \underline{c}, \theta^{eq}) = p_s$. If marginal costs are $c = \overline{c}$ we define similarly θ^{off} as the minimum value of θ for which options are exercised. We have either $\theta^{off} = \underline{\theta}$ or the value of the demand parameter that satisfies $p_s = h(X, \overline{c}, \theta^{off})$. It is immediate to see that $\theta^{off} < \theta^{eq}$. For instance, with a linear demand as $P(Q, \theta) = \theta - Q$, we have $h(X, c, \theta) \equiv \dfrac{\theta + c - m}{2} + \dfrac{X}{4}$ and therefore

$$\theta^{off} = m - \overline{c} + 2p_s - \frac{X}{2} < \theta^{eq} = m - \underline{c} + 2p_s - \frac{X}{2}.$$

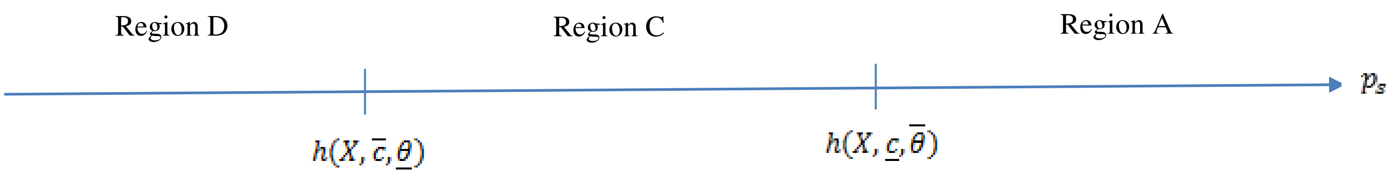
In this interval of values of the strike price p_s, Figure 1 shows the distinct possibilities that we must consider.

Figure 1. Dominant firm behavior according to the strike price values (linear demand).

Case (i): $\overline{\theta} - \underline{\theta} < \overline{c} - \underline{c}$.



Case (ii): $\overline{c} - \underline{c} < \overline{\theta} - \underline{\theta}$.



Note: We are in Region A when $\underline{\theta} < \theta_{off} < \overline{\theta}$ and $\theta_{eq} = \overline{\theta}$; in Region B when $\theta_{off} = \underline{\theta}$ and $\theta_{eq} = \overline{\theta}$; in Region C when $\underline{\theta} < \theta_{off} < \overline{\theta}$ and $\underline{\theta} < \theta_{eq} < \overline{\theta}$, with $\theta_{off} < \theta_{eq}$; and in Region D when $\theta_{off} = \underline{\theta}$ and $\underline{\theta} < \theta_{eq} < \overline{\theta}$.

First at all, we may have either $h(X,\underline{c},\overline{\theta}) < h(X,\overline{c},\underline{\theta})$ or $h(X,\overline{c},\underline{\theta}) < h(X,\underline{c},\overline{\theta})$.[18] Then we can have two different configurations in the behaviour of the dominant firm:

(i) In Regions A and B, where the strike price satisfies $p_s > h(X,\underline{c},\overline{\theta})$, option contracts are never exercised in equilibrium (when the dominant firm produces at marginal cost $c = \underline{c}$). The price of the option contracts is therefore $p_C(X, p_S) = 0$ for these values of the strike price.

(ii) For a strike price in Regions C and D, option contracts are exercised in equilibrium whenever the demand state satisfies $\theta \in [\theta^{eq}, \overline{\theta}]$. In these regions, the price of the option contract must compensate fringe firms for the expected revenues in the market: Expected revenues for those fringe firms that sign an option contract are

$$p_C(X, p_S) + \int_{\underline{\theta}}^{\theta^{eq}} P(q(\theta, 0, \underline{c}) + m, \theta) f(\theta) \mathrm{d}\theta + (1 - F(\theta^{eq})) p_S, \tag{17}$$

[18] With linear demand, we have $h(X,\overline{c},\underline{\theta}) < h(X,\underline{c},\overline{\theta})$ if and only if $\overline{c} - \underline{c} < \overline{\theta} - \underline{\theta}$.

whereas those for fringe firms that sell directly in the market are

$$\int_{\underline{\theta}}^{\theta^{eq}} P(q(\theta,0,\underline{c})+m,\theta) f(\theta)\,\mathrm{d}\theta + \int_{\theta^{eq}}^{\overline{\theta}} P(q(\theta,X,\underline{c})+m,\theta)\; f(\theta)\,\mathrm{d}\theta\,. \tag{18}$$

Thus the option contract must compensate for the market revenues that are resigned in high demand states,

$$p_C(X,p_S) = \int_{\theta^{eq}}^{\overline{\theta}} [P(q(\theta,X,\underline{c})+m,\theta) - p_S]\; f(\theta)\,\mathrm{d}\theta > 0\,. \tag{19}$$

If the dominant firm were to choose option contracts featuring a strike price in Regions C or D, then the result in Proposition 1 would no longer extend to arguably more realistic situations in which there is demand uncertainty. Before considering this possibility, we can state the following result.

Lemma 3. *The optimal contract in Regions A and B has a strike price* $p_S = h(X,\underline{c},\overline{\theta})$.

Proof. See Appendix.

According to this lemma, the optimal option contract along those that are never exercised in equilibrium is the one with the lowest strike price. The dominant firm has a positive margin on these contracts off the equilibrium path, and therefore it makes sense to increase the number of demand states in which the option contracts are exercised.

However, we are mostly interested in the possibility that the dominant firm chooses strike prices in regions C and D. In both regions, option contracts are exercised more frequently off the equilibrium path, $\theta^{off} < \theta^{eq}$ and moreover final prices are always higher off the equilibrium price, $P(q(\theta, x, \underline{c}) + m, \theta) < P(q(\theta, x, \overline{c}) + m, \theta)$. Therefore, there is a potential to increase expected profits if strike prices are set below $p_s = h(X, \underline{c}, \overline{\theta})$. However, the dominant firm must also consider that fringe firm will ask for a strictly positive option contract according to (19) that compensates for losses in market revenues. In other words, the dominant firm must trade off savings on payments to fringe firms (an increase of p_s reduces the demand states where option contracts are exercised in equilibrium) against lower bargaining power in the input market (an increase of p_s reduces the demand states where option contracts are exercised off the equilibrium path).

In Region C, the dominant firm expected profits are

$$E\pi_D^C(X, p_s) = \int_{\theta^{off}}^{\overline{\theta}} \{[P(q(\theta, X, \overline{c}) + m, \theta) - \overline{c}]\, q(\theta, X, \overline{c}) + [P(q(\theta, X, \overline{c}) + m, \theta) - p_s]\, X\}\, f(\theta)\,\mathrm{d}\theta +$$
$$+ \int_{\underline{\theta}}^{\theta^{off}} [P(q(\theta, 0, \overline{c}) + m, \theta) - \overline{c}]\, q(\theta, 0, \overline{c}) f(\theta)\,\mathrm{d}\theta - p_c(X, p_s)\, X\,, \tag{20}$$

where θ^{off} is the demand state satisfying $P(q(\theta^{off}, X, \overline{c}) + m, \theta^{off}) = p^s$: in the off-the-equilibrium path the dominant firm exercises the options only in high demand states, $\theta \in [\theta^{off}, \overline{\theta}]$. In region D, in off-the-equilibrium path the dominant firm always exercises the options and its expected profits are

$$E\pi_D^D(X, p_s) = \int_{\underline{\theta}}^{\overline{\theta}} \{[P(q(\theta, X, \overline{c}) + m, \theta) - \overline{c}]\, q(\theta, X, \overline{c}) + [P(q(\theta, X, \overline{c}) + m, \theta) - p_s]\, X\} f(\theta)\,\mathrm{d}\theta$$
$$- p_c(X, p_s) X\,. \tag{21}$$

In both regions, C and D, the dominant firm exercises the option contract in equilibrium in high demand states and therefore must pay the option contract in (19), $p_c(X, p_s) = \int_{\theta^{eq}}^{\bar{\theta}} [P(q(\theta, X, \underline{c}) + m, \theta) - p_s] f(\theta) \mathrm{d}\theta > 0$. The price p_c of the option contracts must compensate the fringe firms for the profits they would expect in high demand states.

In Region C, the expected profits of the dominant firm evolve with the strike price as follows

$$\frac{\partial E\pi_D^C(X, p_s)}{\partial p_s} = \left\{ [P(q(\theta^{eq}, X, \underline{c}) + m, \theta^{eq}) - p_s] f(\theta^{eq}) \frac{\partial \theta^{eq}}{\partial p_s} - [F(\theta^{eq}) - F(\theta^{off})] \right\} X \qquad (22)$$

and in Region D as

$$\frac{\partial E\pi_D^D(X, p_s)}{\partial p_s} = \left\{ [P(q(\theta^{eq}, X, \underline{c}) + m, \theta^{eq}) - p_s] f(\theta^{eq}) \frac{\partial \theta^{eq}}{\partial p_s} - F(\theta^{eq}) \right\} X \ . \qquad (23)$$

Both derivatives (22) and (23) have an ambiguous sign: In both of them, there is a negative term, $-X[F(\theta^{eq}) - F(\theta^{off})]$ in (22) and $-XF(\theta^{eq})$ in (23) (when $F(\theta^{off}) = 0$ in Region D), that reflects the fact that option contracts are exercised more frequently off the equilibrium path than in equilibrium; hence, an increase in the strike price p_s increases the cost of the contract off the equilibrium. At the same time, there is the positive term $X[P(q(\theta^{eq}, X, \underline{c}) + m, \theta^{eq}) - p_s] f(\theta^{eq}) \frac{\partial \theta^{eq}}{\partial p_s}$ either in (22) or (23), which indicates that an increase in p_s diminishes the number of demand states where the option is exercised, thereby decreasing the upfront payment p_c.

5.1. A linear demand example

In what follows, we explicitly calculate the optimal number of option contracts to be signed by considering a linear demand schedule $P(Q,\theta)=\theta-Q$, with the uncertainty parameter θ uniformly distributed in $[\underline{\theta},\overline{\theta}]$. The dominant firm's problem of finding the optimal contract $\{p_s,p_c\}$ and the optimal number of option contracts are obtained in two steps. First, we determine the optimal contract as a function of the level of contracts, and then we find the optimal level of contracts. When the optimal contract is chosen, it is done in terms of the strike price; the premium p_c — stated in terms of the strike price and the number of contracts — is the lowest premium the dominant firm must pay to fringe firms for these to accept the required number of contracts (and assuming fringe firms correctly evaluate their expected profits, i.e., assuming a rational expectations equilibrium in the contracting stage).

The dominant firm produces $q(x,c,\theta)=\frac{1}{2}(\theta-m-c-x)$ and the price of the final good is $P(q(x,c,\theta)+m,\theta)=\frac{1}{2}(\theta-m+c+x)$. The above-mentioned Regions A, B, C and D are now easily defined from $h(X,c,\theta)=\frac{1}{2}(\theta-m+c)+\frac{1}{4}X$. Hence, we are in Region C when the strike price p_s satisfies $\frac{1}{2}(\underline{\theta}-m+\overline{c})+\frac{1}{4}X<p_s<\frac{1}{2}(\overline{\theta}-m+\underline{c})+\frac{1}{4}X$ and in Region D when $p_s\le\frac{1}{2}(\underline{\theta}-m+\overline{c})+\frac{1}{4}X$.

Derivatives (22) and (23) become

$$\frac{\partial E\pi_D^C(X,p_s)}{\partial p_s}=\left\{\frac{X}{2}-(\overline{c}-\underline{c})\right\}\frac{X}{\overline{\theta}-\underline{\theta}} \tag{24}$$

and

$$\frac{\partial E\pi_D^D(X,p_s)}{\partial p_s}=\left\{\frac{X}{2}-(\theta^{eq}-\underline{\theta})\right\}\frac{X}{\overline{\theta}-\underline{\theta}} \tag{25}$$

respectively, where $\theta^{eq}=m-\underline{c}+2p_s-\dfrac{X}{2}$.

Lemma 4. *The optimal strike price is* $p_s^*=\frac{1}{2}(\underline{\theta}-m+\underline{c}+X)$ *in Region D when* $X<\min\{2(\overline{c}-\underline{c}),2(\overline{\theta}-\underline{\theta})\}$, *and* $p_s^*=h(X,\underline{c},\overline{\theta})=\dfrac{\overline{\theta}-m+\underline{c}}{2}+\dfrac{X}{4}$ *otherwise.*

Proof. See Appendix.

According to Lemma 4, we can be sure that for some parameter values, the dominant firm will choose a number of option contracts X such that they will be exercised in equilibrium when demand is high. In addition, we can evaluate in the case where $X<\min\{2(\overline{c}-\underline{c}),2(\overline{\theta}-\underline{\theta})\}$ the amount of option contracts that maximizes the expected profits of the dominant firm.

Proposition 2. *When* $X<\min\{2(\overline{c}-\underline{c}),2(\overline{\theta}-\underline{\theta})\}$, *the dominant firm chooses to sign* $X^*=m$ *option contracts with the fringe, the optimal strike price is* $p_s^*=\frac{1}{2}(\underline{\theta}+\underline{c})$ *and option contracts are executed when* $\theta\in[\underline{\theta}+\frac{m}{2},\overline{\theta}]$.

Proof. See Appendix.

According to Lemma 4 and Proposition 2, for parameter constellations such that $X<\min\{2(\overline{c}-\underline{c}),2(\overline{\theta}-\underline{\theta})\}$ the dominant firm subcontracts with all the fringe firms and exercises contracts in high demand states. For other values of the parameters, the dominant firm subcontracts with all the fringe firms, but never exercises the contracts in equilibrium.

The main consequence of Lemma 4 and Proposition 2 is that the signing of option contracts harms consumers in high demand states.

Corollary 1. *When* $X < \min\{2(\bar{c}-\underline{c}), 2(\bar{\theta}-\underline{\theta})\}$, *consumers are worse off with option contracts if* $\theta \in \left\lfloor \underline{\theta}+\frac{m}{2}, \bar{\theta} \right\rfloor$, *since final prices jump from* $\frac{1}{2}(\theta+\underline{c}-m)$ *to* $\frac{1}{2}(\theta+\underline{c})$ *in this interval of the demand parameter.*

It is instructive to compare market performance under forward and option contracts. If forward contracts are used, the dominant firm profitably acquires production from fringe firms.[19] For a number of forward contracts X_f, the forward price is $p_f = EP(q(\theta, X_f, \underline{c})+m, \theta)$, where EP denotes expected price, and expected profits are

$$E\pi^f(X_f) = \int_{\underline{\theta}}^{\bar{\theta}} \{[P(q(\theta, X_f, \bar{c})+m, \theta)-\bar{c}]\, q(\theta, X_f, \bar{c}) + [P(q(\theta, X_f, \bar{c})+m, \theta) - p_f]X_f\} f(\theta)\,\mathrm{d}\theta \tag{26}$$

With a linear demand and uniform distribution on the parameter value, expected profits stated in (26) become

$$E\pi^f(X_f) = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \frac{1}{4}(\theta - m - \bar{c})^2 - \frac{1}{4}X_f^{\,2} + \frac{1}{2}(\bar{c}-\underline{c})X_f \right\} \frac{d\theta}{\bar{\theta}-\underline{\theta}} \tag{27}$$

[19] A forward contract that pays p^f for one unit of product is equivalent to an option contract with a strike price p_s below $h(X, \underline{c}, \underline{\theta})$ (and therefore is always exercised) acquired at price $p_c = p_f - p_s$.

and

$$\frac{\partial E\pi^{f}(X)}{\partial X}=\frac{1}{2}(\overline{c}-\underline{c})-\frac{1}{2}X_{f}. \tag{28}$$

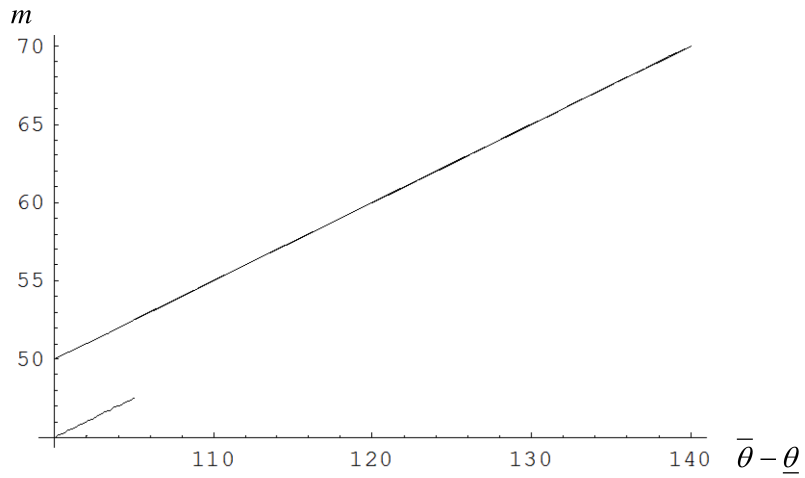
Therefore, we obtain that the dominant firm signs $X_f^*=\min\{\overline{c}-\underline{c},m\}$ forward contracts.

Figures 2 and 3 show some examples of the above analysis. For both figures we consider the linear demand case and the parameter values $(\overline{\theta},\underline{\theta},\overline{c},\underline{c})=(140,100,20,0)$.[20] For fringe sizes as $m<\overline{c}-\underline{c}=20$, the equilibrium number of option and forward contracts is the same, $X^*=X_f^*=m$. Forward contracts always have an anticompetitive effect, by increasing final prices from $\frac{1}{2}(\theta+\underline{c}-m)$ to $\frac{1}{2}(\theta+\underline{c})$. In turn, option contracts are innocuous when demand is low, $\theta\in\left[\underline{\theta},\underline{\theta}+\frac{m}{2}\right]$, but have the same anticompetitive effect than forward contracts when demand is high, $\theta\in\left[\underline{\theta}+\frac{m}{2},\overline{\theta}\right]$.

An example where final prices are more volatile with option contracts than with forwards and are above prices with forward contracts in high demand states is depicted in Figure 2 for the particular case when *m*=10. For this fringe's size, $X^*=X_f^*=m=10$ for both option and forward contracts. In equilibrium, final prices are $P(X_f^*,\theta)=\frac{1}{2}\theta$ with forward contracts and $P(X^*,\theta)=\begin{cases}\frac{1}{2}(\theta-10),\text{if } \theta<105\\ \frac{1}{2}\theta,\text{if } \theta>105\end{cases}$ with option contracts.

[20] In order to have always strictly positive production and final prices above $\overline{c}=20$, we must have *m*<40.

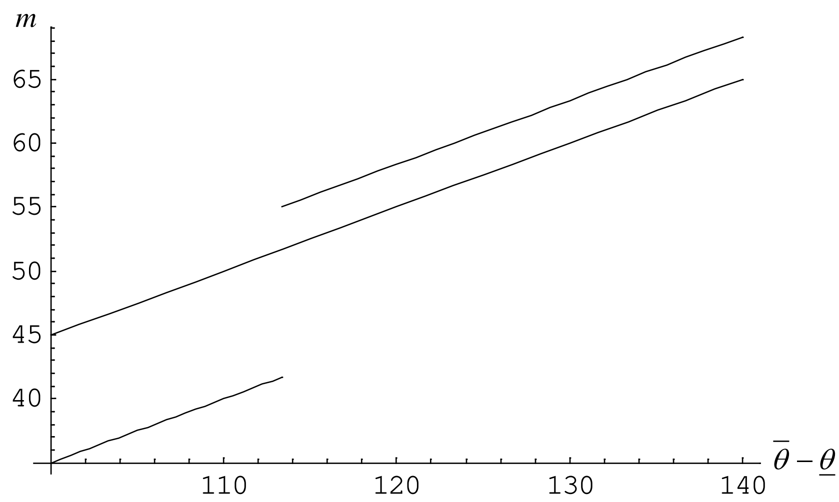
Figure 2. Final prices with forward and option contracts when the fringe's size is low (*m*=10).



Interestingly, the welfare ranking between option and forward contracts is ambiguous when $\overline{c}-\underline{c}=20<m<2(\overline{c}-\underline{c})=40$. In equilibrium, the dominant firm does not sign forward contracts with all the fringe, $X_f^*=\overline{c}-\underline{c}=20$. Under forward contract final prices always jump from $\frac{1}{2}(\theta+\underline{c}-m)$ to $\frac{1}{2}(\theta+2\underline{c}-\overline{c})$, whereas under option contracts consumers pay the same prices $\frac{1}{2}(\theta+\underline{c}-m)$ when demand is low (and are therefore better off than under forward contracts) but pay higher prices $\frac{1}{2}(\theta+\underline{c})$ when demand is high.

Figure 3 depicts the case when *m*=30 showing as final prices under option contracts exhibit more volatility than under forward contracts, and are lower than with forward contracts in high demand states.

Figure 3. Final prices with forward and option contracts when the fringe is medium-sized (*m*=30).



In fact, for this size of fringe, $X_f^* = 20$ with forward contracts, whereas $X^* = 26.6$ with option contracts, so $X_f^* < X^* < m$. In equilibrium, final prices are $P(X_f^*,\theta) = \frac{1}{2}(\theta - 10)$ with forward contracts and $P(X^*,\theta) = \begin{cases} \frac{1}{2}(\theta - 30) \text{ if } \theta < 100 + \frac{X^*}{2} \\ \frac{1}{2}(\theta - 30 + X^*) \text{ if } \theta > 100 + \frac{X^*}{2} \end{cases}$ with option contracts.

6. Concluding remarks

In this paper we have investigated the strategic role played by option contracts and forward contracts regarding outsourcing of production in a vertical industry, where the downstream industry consists of a dominant firm together with a competitive fringe of small firms. Downstream production requires an input from a component-producing industry with a super-competitive supplier as well as second supply source. In this context, it is shown that, without uncertainty in the demand of the final good, the dominant firm subcontracts downstream production to the fringe firms through forward contracts in order to increase final prices to consumers (strategic horizontal effect). However, when it can use option contracts regarding subcontracting to the fringe firms these contracts are strategically used to increase the firm's leverage in the input market (strategic vertical effect) as preferable to increasing the price of the final good. Thus, the dominant firm stops manipulating prices in the downstream market when it can sign option contracts and increases its bargaining power in the upstream market. In a sense, option contracts allow us to observe, in greater detail, a strategic effect that is absent under forward contracts.

When there is uncertainty in the market demand of the final good, option contracts regarding subcontracting production may have the unintended consequence of increasing price manipulation in the downstream market. However, this effect could be avoided if buyers and sellers were able to sign long-term contracts or even if they integrated vertically. Hence, our findings extend the well-known result that vertical integration may serve to increase both the firms' profits and the consumer surplus, through the disappearance of double marginalization, which is replaced by a more general contracting set-up. This should be contrasted with the possibility that vertical integration leads to vertical foreclosure. Future research should try to disentangle the circumstances in which each of these countervailing effects is likely to dominate.

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Appendix

Proof of Lemma 1. If x option contracts, $x \in [0, X]$, are exercised in equilibrium, and production is chosen optimally at $q(x,c)$, the dominant firm's profit amounts to

$$\pi_D(q(x,c),x) = \left[P(q(x,c)+m) - c\right]q(x,c) - \left[P(q(x,c)+m) - p_s\right]x\,. \qquad \text{(A1)}$$

Profits given in (A1) are a convex function of *x*, since $\dfrac{\partial \pi_D(q(x,c),x)}{\partial x} = P(q(x,c)+m) - p_s$ and

$$\frac{\partial^2 \pi_D(q(x,c),x)}{\partial x^2} = P'(q(x,c)+m)\frac{\partial q(x,c)}{\partial x} = -\frac{[P'(q(x,c)+m)]^2}{P''(q(x,c)+m)(q+x) + 2P'(q(x,c)+m)} > 0\,.$$

(A2)

Therefore the optimal number of option contract executed is either *x*=0 or *x*=*X*. If we write the difference $\pi_D(q(X,c),X) - \pi_D(q(0,c),0)$ as

$$\int_0^X \left(P(q(x,c)+m) - p_s\right)dx = \left(h(X,c) - p_s\right)X\ , \qquad \text{(A3)}$$

it can be shown that $\pi_D(q(X,c),X) > \pi_D(q(0,c),0)$ whenever $p_s < h(X,c)$. ■

Proof of Lemma 2. The derivative of the profit stated in (5) with respect to *X* is

$$\left[P'(q(X,c)+m)q(X,c) + P(q(X,c)+m) - c\right]\frac{\partial q(X,c)}{\partial X}\,, \qquad \text{(A4)}$$

which, according to (2), can be written as

$$-P'(q(X,c)+m)X\,\frac{\partial q(X,c)}{\partial X}\,, \qquad \text{(A5)}$$

and since the derivative $\frac{\partial q(X,c)}{\partial X}$ is strictly negative, it immediately follows that $X^* = 0$. ■

Proof of Proposition 1. Since the dominant firm's profit given in (13) is a decreasing function in p_S, the optimal strike price in the interval $h(X,\underline{c}) \leq p_S < h(X,\overline{c})$ is $p_s = h(X,\underline{c})$. In the interval $p_s < h(X,\underline{c})$, an option contract has total cost $p_c + p_s = P(q(X,\underline{c}) + m)$. Since the strike price $p_s = h(X,\underline{c})$ is strictly lower than $P(q(X,\underline{c}) + m)$, profits given in (12) and evaluated at $p_s = h(X,\underline{c})$ are strictly higher than profits given in (11). The dominant firm has then profits

$$[P(q(X,\overline{c}) + m) - \overline{c}]q(X,\overline{c}) + [P(q(X,\overline{c}) + m) - h(X,\underline{c})]X \,. \tag{A6}$$

The derivative of these profits with respect of *X*, $\frac{\partial\{[P(q(X,\overline{c}) + m) - h(X,\underline{c})]X\}}{\partial X}$, is

$$P(q(X,\overline{c}) + m) - h(X,\underline{c}) - \frac{\partial h(X,\underline{c})}{\partial X}X = P(q(X,\overline{c}) + m) - P(q(X,\underline{c}) + m)\,, \tag{A7}$$

which is strictly positive. Thus, profits with $X > 0$ are larger than profits given in (11) when no option contracts are signed. The optimal number of option contracts is then $X^*=m$. ■

Proof of Lemma 3. In Region A, the option contract is exercised off the equilibrium path (if there is no agreement between the dominant firm and the input supplier and marginal costs of production are $c = \overline{c}$) only if the demand state satisfies $\theta \in [\theta^{off}, \overline{\theta}]$, with $\theta^{off} > \underline{\theta}$, whereas in Region B the option contract is always exercised off the equilibrium path. In region A, the dominant firm's expected profits are

$$E\pi^{A}(X,p^{s}) = \int_{\theta^{off}}^{\bar{\theta}} \{[P(q(\theta,X,\bar{c})+m,\theta)-\bar{c}]q(\theta,X,\bar{c})+[P(q(\theta,X,\bar{c})+m,\theta)-p_{s}]X\}f(\theta)\,\mathrm{d}\theta + \\ + \int_{\underline{\theta}}^{\theta^{off}} \{[P(q(\theta,0,\bar{c})+m,\theta)-\bar{c}]q(\theta,0,\bar{c})\}f(\theta)\,\mathrm{d}\theta, \quad \text{(A8)}$$

whereas in region B, $h(X,\underline{c},\bar{\theta}) < p^{s}$, and the dominant firm's expected profits amount to

$$E\pi^{B}(X,p^{s}) = \int_{\underline{\theta}}^{\bar{\theta}} \{[P(q(\theta,X,\bar{c})+m,\theta)-\bar{c}]q(\theta,X,\bar{c})+[P(q(\theta,X,\bar{c})+m,\theta)-p_{s}]X\}f(\theta)\mathrm{d}\theta. \quad \text{(A9)}$$

From (A8) and (A9) it can be checked that $\frac{\partial E\pi_{D}^{A}(X,p_{s})}{\partial p_{s}} = -[1-F(\theta^{off})]X < 0$ and $\frac{\partial E\pi_{D}^{B}(X,p_{s})}{\partial p_{s}} = -X < 0$, respectively. Thus, in these regions expected profits are a decreasing function of the strike price, which imply that the optimal contract features a strike price $p_{s} = h(X,\underline{c},\bar{\theta})$. ■

Proof of Lemma 4. When parameter values satisfy $\bar{\theta}-\underline{\theta} < \bar{c}-\underline{c}$, we have an interior solution in region D; namely, $\frac{\partial E\pi_{D}^{D}(X,p_{s})}{\partial p_{s}} = 0$ at $p_{s}^{*} < h(X,\underline{c},\bar{\theta}) = \frac{\bar{\theta}-m+\underline{c}}{2}+\frac{X}{4}$ only when $X < 2(\bar{\theta}-\underline{\theta})$. Thus when $X < 2(\bar{\theta}-\underline{\theta})$ the optimal strike price is $p_{s}^{*} = \frac{1}{2}\left(\underline{\theta}-m+\underline{c}+X\right)$ and when $X > 2(\bar{\theta}-\underline{\theta})$ the optimal strike price is $p_{s}^{*} = h(X,\underline{c},\bar{\theta}) = \frac{\bar{\theta}-m+\underline{c}}{2}+\frac{X}{4}$.

When parameter values satisfy $\bar{c}-\underline{c} < \bar{\theta}-\underline{\theta}$, we have a corner solution in region C: $\frac{\partial E\pi_{D}^{C}(X,p_{s})}{\partial p_{s}} < 0$ if $X < 2(\bar{\theta}-\underline{\theta})$ and $\frac{\partial E\pi_{D}^{C}(X,p_{s})}{\partial p_{s}} > 0$ otherwise; and an interior solution in region D; namely, $\frac{\partial E\pi_{D}^{D}(X,p_{s})}{\partial p_{s}} = 0$ at $p_{s}^{*} < h(X,\bar{c},\underline{\theta}) = \frac{\underline{\theta}-m+\bar{c}}{2}+\frac{X}{4}$ only when

$X < 2(\bar{c} - \underline{c})$. Therefore when $X < 2(\bar{c} - \underline{c})$ the optimal strike price is $p_s^* = \frac{1}{2}\left(\underline{\theta} - m + \underline{c} + X\right)$ and when $X > 2(\bar{c} - \underline{c})$ the optimal strike price is $p_s^* = h(X, \underline{c}, \bar{\theta}) = \frac{\bar{\theta} - m + \underline{c}}{2} + \frac{X}{4}$. ■

Proof of Proposition 2. The derivative $\frac{\partial E\pi_D^D(X, p_s^*)}{\partial X}$ is

$$\frac{\partial E\pi_D^D(X, p_s^*)}{\partial X} = \int_{\underline{\theta}}^{\bar{\theta}} [P(q(\theta, X, \bar{c}) + m, \theta) - p_s^*] f(\theta) \,\mathrm{d}\theta - p_c(X, p_s^*) - \frac{\partial p_c(X, p_s^*)}{\partial X} X \qquad \text{(A10)}$$

and taking into account that $\frac{\partial \theta^{eq}}{\partial X} = -\frac{1}{2}$, and $\theta^{eq} = \underline{\theta} + \frac{X}{2}$ at $p_s^* = \frac{1}{2}\left(\underline{\theta} - m + \underline{c} + X\right)$, we can write (A10) as

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{2} \frac{\theta - \underline{\theta}}{\bar{\theta} - \underline{\theta}} \,\mathrm{d}\theta + \frac{1}{2}\left(\bar{c} - \underline{c}\right) - \int_{\underline{\theta} + \frac{X}{2}}^{\bar{\theta}} \frac{1}{2} \frac{\theta - \underline{\theta}}{\bar{\theta} - \underline{\theta}} \mathrm{d}\theta + \frac{1}{8} \frac{X^2}{\bar{\theta} - \underline{\theta}} = \frac{1}{2}\left(\bar{c} - \underline{c}\right) + \frac{3}{16} \frac{X^2}{\bar{\theta} - \underline{\theta}} \qquad \text{(A11)}$$

which is strictly positive. Therefore, $X^* = m$. Equilibrium values of p_s^* and θ^{eq} come immediately. ■