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Abstract

In this paper, we analyze the long-term impact of an environmental policy on economic growth, pollution and welfare. A standard growth model with horizontal innovation is modified by including pollution which comes from the use of intermediate goods production. Taxation on pollution reduces profits of final good producer as well as intermediate good producers. In this setting, profit gains is explained by a realloaction of labor from intermediate goods sector to R&D sector which enhances innovation, growth and welfare while it reduces pollution.

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1 Introduction

The "Porter Hypothesis" claims that an environmental policy enhances innovation and growth because the long-term benefits offsets the short-term losses and simultaneously reduces pollution (Porter and van der Linde (1995)). From a theoretical point of view,[1] a growing literature has investigated the theoretical foundations of the "Porter Hypothesis".[2]

In models of endogenous growth with vertical innovation à la Aghion and Howitt (1992), Hart (2004) analyzes the impact of environmental sales taxes on growth. He shows the possibility that such sales taxes enhance growth while improving the environment, with some conditions. Nakada (2004) examines the issue through a simpler model, focusing on the "profitability effect" which is the loss in the profits of the intermediate sector, caused by taxation and a "general equilibrium effect" linked to the resource constraint.[3] He shows that an environmental policy increases growth while pollution is reduced. From a modified model à la Aghion and Griffith (2005), Bianco and Salies (2016) analyze the effects of environmental tax on growth, pollution and profits, by focusing on inefficiency within firms. They find necessary conditions under which an environmental tax increases growth, profit and reduces pollution

Based on models of endogenous growth with expanding variety (Romer (1990) and Grossman and Helpman (1991)), the literature distinguishes two approaches. The first approach assumes that research can increase the productivity in two different sectors, producing two types of goods - a clean and a dirty one. In this spirit, Hung, Chang, and Blackburn (1994) using a model of Romer (1990) analyze the effects of environmental conservation on growth. They find that in most cases pollution control is not growth depressant. In the second approach, technical change either increases productivity and/or reduces the pollution intensity of a given production input. Verdier (1995) investigates the validation of the "Porter Hypothesis" using a modified model of Grossman and Helpman (1991). He shows that environmental taxation promotes growth if the environmental tax rate is low enough. Elbasha and Roe (1996) use also a model of Romer (1990) but in a small open economy in the presence of environmental externalities. They demonstrate that a

[1] See Ricci (2007a) for an exhaustive survey of the literature focused on channels of transmission of environmental policy on growth.

[2] We focuse only on the macroeconomic literature based on endogenous growth models with innovation. This kind of model appears to us to be the best framework for understanding the "Porter Hypothesis".

[3] Unlike Hart (2004) and Nakada (2004), using similar models, Grimaud (1999) and Ricci (2007b) do not find validation of the "Porter Hypothesis".

greener preference increases the growth rate if the elasticity of intertemporal substitution is small but they don't focus on the explanation of the "Porter Hypothesis".

Following the last theoretical framework based on the second approach, we propose a model to examine the theoretical foundations of the "Porter Hypothesis"without restrictions either on the level of the environmental tax rate or on the elasticity of intertemporal substitution. In addition, we also introduce a "profitability effect"in our model. For this purpose, we develop a standard endogenous growth model with horizontal innovation à la Gancia and Zilibotti (2005) modified by including the use of intermediate goods by the final good producer as source of pollution. Moreover, we assume that the pollution affects negatively the household's utility.

Nakada (2004) studies also the effects of an environmental policy but in an endogenous growth model with vertical innovation with an exogenous pollution intensity and an intertemporal elasticity of substitution equals to one. In our paper, the pollution intensity is determined endogenously by assuming that this environmental index is linked negatively to the number of varieties of intermediate goods. Moreover, we use a more general utility function in which the elasticity is assumed to be greater than zero which allow us to take into account the effect of pollution on the marginal utility of consumption.

The rest of the paper is strutured as follows. Section 2 describes the overall framework of the model. Section 3 addresses the effects of a more strict environmental regulation on growth and pollution. Section 4 analyzes the implications of this kind of policy on welfare. Finally, section 5 concludes.

2 The model

Our starting point is the standard version of the Romerian growth scheme as exposed in Gancia and Zilibotti (2005).[4] In this framework, the economy consists of four types of agents. Producers of a final good who use intermediate goods input to produce their output which is used only for consumption. Producers of intermediate good who use labor as single production factor. The technology to produce this output is assumed to be obtained from a research firm. This firm uses only labor in order to produce a new variety of intermediate goods. Finally, an infinitely lived representative household determines his consumption by maximizing an intertemporal utility function. We modify this standard framework by introducing pollution as a by-product

[4] We use the variation of the benchmark model called "labor for intermediates"but with a notation close to Acemoglu (2009) which is more clear.

of intermediate goods which affects negatively the utility of the representative household.

2.1 The final good sector

The unique final good is produced competitively with the production function

$$Y(t) = \frac{1}{1-\beta} \int_0^{N(t)} x(v,t)^{1-\beta} dv, \tag{1}$$

where $0 < \beta < 1$, $N(t)$ represents the different number of varieties of intermediate goods available to be used in the production process at time t and $x(v,t)$ is the total amount of intermediate goods of variety v used at time t. For notational simplicity, the term $1-\beta$ in the denominator is included and we assume that the price of final good at each date is equal to one.

In an interesting paper, Koesler (2010) mentions that the integration of pollution in an economic growth model requires the consideration of three aspects: the nature of pollution, the source of pollution and the effects of pollution on the agents in the economy. Concerning the nature of pollution, as in Elbasha and Roe (1996) and Koesler (2010) among others, we model the pollution as a flow variable. This means that pollution exclusively depends on the amount of pollution generates in the present period. Concerning the source of pollution, we support the idea that the flow of pollution is being related to an externality of the production process. More precisely, as in Nakada (2004), agregate level of pollution $P(t)$ depends on the level of intermediate goods $x(v,t)$ and an environmental technology index $z(v,t) \in [0,1]$[5]

$$P(t) = \int_0^{N(t)} z(v,t)x(v,t)dv. \tag{2}$$

Unlike Nakada (2004), we assume that the pollution intensity depends negatively on the number of variety of intermediate goods ($z(v,t) = 1/N(t)$).[6] Finally, we shall see how we model the effects of pollution when we shall present the household's preference.

[5]Hung, Chang, and Blackburn (1994) also assume that the agregate pollution is determined by the production of intermediate goods but in a more general case. Indeed, they assume that the environmental damage is a function $z(x_D(i,\tau))$ where $\partial z(.)/\partial x \geq 0$. However, unlike us, they don't take into account the intensity of pollution.

[6]In the same spirit, Bianco and Salies (2016) determine endogenously the pollution intensity by assuming that this index is linked negatively to the quality of intermediate goods instead of the number of intermediate goods because they use a schumpeterian endogenous growth model.

Unlike Koesler (2010), we assume that the government is to levy an environmental tax on polluters in order to offer an incentive to reduce pollution. We assume that this tax is levied proportionally to the level of pollution from the final production sector as in Nakada (2004) and Bianco and Salies (2016).

The profit function of the final good producer in the presence of environmental tax is given by

$$\pi_Y(t) = \frac{1}{1-\beta}\int_0^{N(t)} x(v,t)^{1-\beta}dv - \int_0^{N(t)} p(v,t)x(v,t)dv - h(t)\int_0^{N(t)} \frac{x(v,t)}{N(t)}dv, \tag{3}$$

where $p(v,t)$ is the price of a variety v of intermediate goods and $h(t) \in (0,1)$ represents the environmental tax rate.

The first order condition gives us the inverse demand of intermediate good by the final good producer

$$p(v,t) = x(v,t)^{-\beta} - \frac{h(t)}{N(t)}. \tag{4}$$

2.2 The intermediate goods sector

Like Verdier (1995) and Nakada (2004), we assume that the only input used in the intermediate goods production is labor.[7] But we depart from our analysis on two points. First, we assume that for each firm to produce one unit of intermediate goods, ψ unit of labor is required instead of only one. Second, we assume that this latter is inversely linked to the number of variety as follows $\psi = 1/N(t)$. Thus, the profit of each intermediate good producer is given by

$$\pi(v,t) = x(v,t)^{1-\beta} - \frac{h(t)}{N(t)}x(v,t) - \psi w(t)x(v,t). \tag{5}$$

At this stage, as in Verdier (1995), we assume that the government controls the environmental tax by the indicator ϕ which denotes the emission tax in labor units, i.e., $\phi = \frac{h(t)}{w(t)}$ which is constant in time.[8] So, the equation (5) can be rewritten as follows

$$\pi(v,t) = x(v,t)^{1-\beta} - \left(\frac{1+\phi}{N(t)}\right)x(v,t)w(t), \tag{6}$$

[7]Hung, Chang, and Blackburn (1994) assume that γ_s unit of the final goods is needed to produce one unit of the intermediate input.

[8]Nakada (2004) and Bianco and Salies (2016) use the same assumption in our models.

where $w(t)$ is the wage rate at time t.

The first order condition of the firm gives the labor demand function

$$x(v,t) = \left[\frac{1-\beta}{(1+\phi)\omega}\right]^{\frac{1}{\beta}} \equiv \tilde{x}(\omega), \tag{7}$$

where $\omega = \frac{w(t)}{N(t)}$ represents the wage rate adjusted to the productivity which is constant in time and $\tilde{x}(\omega)$ denotes the productivity adjusted labor demand of each firm.

Substituting (7) into (4) gives the price of intermediate goods

$$p(v,t) = \tilde{x}(\omega)^{-\beta} - \phi\omega. \tag{8}$$

The profit of each firm is obtained by substituting (7) into (6)

$$\pi(v,t) = \left(\frac{\beta}{1-\beta}\right)(1+\phi)\,\omega\tilde{x}(\omega). \tag{9}$$

We can note that the profit of the intermediate goods producer is determined by two opposite effects as in Nakada (2004). The first one, called "profitability effect"represents the positive effect of environmental tax on profit. Indeed, an increase in environmental tax increases the mark-up rate and thus increases the profit. The second one which is negative and offsets the first one is the "traditional effect"which decreases the profit by reducing the labor demand. Finally, the composition of these two effects represents what Porter and van der Linde (1995) called the short-term loss of the environmental policy.

2.3 The R&D sector

As in Gancia and Zilibotti (2005), the innovation possibilities frontier is given by

$$\dot{N}(t) = \eta N(t) H_R(t), \tag{10}$$

where $\eta > 0$ represents the productivity of workers and $H_R(t)$ is the number of workers employed in the R&D sector at time t.

As usual, the Hamilton-Bellman-Jacobi equation is

$$r(t)V(v,t) - \dot{V}(v,t) = \pi(v,t), \tag{11}$$

where $V(v,t)$ is the net present discounted value of owning the blueprint of an intermediate good of variety v at time t and $r(t)$ represents the interest rate at time t.

When there is positive research, the free entry condition is given by

$$\eta V(v,t) = \omega. \tag{12}$$

The left-side of the equation (12) is the return from hiring one more worker for R&D and the right-side represents the flow cost of hiring one more worker for R&D, $w(t)$.

2.4 The household and the government

We model the effects of pollution by making the household's preference negatively dependent on the flow of pollution present in the economy. Unlike Elbasha and Roe (1996) who use a variable measuring the quality of the environment, we prefer to use a simpler way and follow Hung, Chang, and Blackburn (1994), Michel and Rotillon (1995), Nakada (2004) and Verdier (1995). Indeed, we incorporate pollution directly in the utility of household which causes disutility[9]

$$U(t) = \begin{cases} \frac{1}{1-\theta}\left(\left[C(t)P(t)^{-\delta}\right]^{1-\theta} - 1\right) & \text{for} \quad 0 < \theta < \infty \quad \text{and} \quad \theta \neq 1, \\ \ln C(t) - \delta \ln P(t) & \text{for} \quad \theta = 1. \end{cases} \tag{13}$$

where $\delta > 1 - \beta$ and θ represent parameters determining respectively the household's preference toward a clean environment and the multiplicative inverse of the elasticity of intertemporal substitution and $C(t)$ is the consumption of the household at time t.

The representative household sets a consumption plan to maximize the present value stream of utility subject to an intertemporal budget constraint. The consumption plan satifies a standard Euler equation[10]

$$g_C \equiv \frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\theta}, \tag{14}$$

where $\rho > 0$ is the time preference rate and the following transversality condition

$$\lim_{t\to\infty} \left[\mu(t)A(t)e^{-\rho t}\right] = 0. \tag{15}$$

[9]This utility function is a generalization of the utility function used by Hung, Chang, and Blackburn (1994) and Nakada (2004). Indeed, if we assume $\theta = 1$ and $\delta = \beta$, we obtain the utility function assumed by Hung, Chang, and Blackburn (1994) and if we assume $\theta = 1$ and $\delta = 1$, we obtain the utility function assumed by Nakada (2004). Althrough Verdier (1995) uses a utility function quite different, if we assume $\theta = 1$, we obtain a utility function close to Verdier (1995).

[10]Inserting equation (7) into equation (2) yields $P(t) = \tilde{x}(\omega)$ which is constant over time because ω and ϕ are also constant.

The governement regulates the economy by using the environmental tax $(h(t))$ and tranfers this fiscal revenue to the household $(T(t))$.

$$h(t)P(t) = T(t). \tag{16}$$

2.5 The balanced growth path

The resource constraint of the economy at time t is

$$C(t) \leq Y(t). \tag{17}$$

In the labor market, workers are assumed to be employed either in production or in research

$$H = H_x(t) + H_R(t), \tag{18}$$

where H is the total number of workers,[11] $H_x(t)$ is the number of workers in production and $H_R(t)$ is the number of workers in research.

Since the interest rate (14) and the profit of intermediate good producer (9) are constant, thus $\dot{V}(t) = 0$. This implies that the net present discounted value of the firm is given by

$$V^* = \frac{\pi(v,t)}{r^*} = \frac{\left(\frac{\beta}{1-\beta}\right)(1+\phi)\omega\tilde{x}(\omega)}{r^*}. \tag{19}$$

Hence the balanced growth path equilibrium interest rate must be

$$r^* = \eta\left(\frac{\beta}{1-\beta}\right)(1+\phi)H_x^*. \tag{20}$$

The fact that the number of workers in production must be constant in the balanced growth path follows from (18).

Now using the Euler equation of the representative household, (14), the growth rate of consumption is

$$g_C^* = \frac{\eta\left(\frac{\beta}{1-\beta}\right)(1+\phi)H_x^* - \rho}{\theta} \equiv g^*. \tag{21}$$

To complete the characterization of the balanced growth path equilibrium, the balanced growth path level of employment, H_x^* needs to be determined. From the innovation possibilities frontier, (10), we obtain $\dot{N}(t)/N(t) =$

[11] As in the standard endogeneous growth model with horizontal innovation, we assume that the labor force is constant.

$\eta H_R^* = \eta(H - H_x^*)$. Moreover, by definition, the balanced growth path growth rate of consumption must be equal to the rate of technological progress. Thus $g^* = \dot{N}(t)/N(t)$. This implies that the balanced growth path level of employement is uniquely determined as follows

$$H_x^* = \frac{\eta\theta H + \rho}{\eta\left(\frac{\beta}{1-\beta}\right)(1+\phi) + \eta\theta}. \tag{22}$$

Finally, the balanced growth path equilibrium interest rate is given by

$$r^* = \frac{\left(\frac{\beta}{1-\beta}\right)(1+\phi)(\eta\theta H + \rho)}{\left(\frac{\beta}{1-\beta}\right)(1+\phi) + \theta}, \tag{23}$$

and the long-run growth rate of the economy is then obtained as

$$g^* = \frac{\eta\left(\frac{\beta}{1-\beta}\right)(1+\phi)H - \rho}{\left(\frac{\beta}{1-\beta}\right)(1+\phi) + \theta}. \tag{24}$$

Let us assume that $\eta\left(\frac{\beta}{1-\beta}\right)(1+\phi)H > \rho$ and $\rho > \frac{\beta(1-\theta)(1+\phi)\eta H}{1+\beta\phi}$.

The first inequality ensures that the long-run growth rate is positive while the second one ensures that the representative household's utility is finite and the transversality condition is satisfied.

3 Effects of a stricter environmental policy

In this section, we analyse the effects of a stricter environmental taxation on long-run growth rate and pollution.

Proposition 1 *A higher environmental tax increases the long-run growth rate of the economy.*

Proof. *The derivative of g^* with respect to ϕ is positive.*

$$\frac{\partial g^*}{\partial \phi} = \frac{\left(\frac{\beta}{1-\beta}\right)(\eta H\theta + \rho)}{\left[\left(\frac{\beta}{1-\beta}\right)(1+\phi) + \theta\right]^2} > 0.$$

■

Proposition 1 shows the positive impact of environmental policy on growth. On the one hand, a tax on pollution reduces the intermediate profits because it decreases not only final output but also intermediate demand. This negative effect represents the short-term loss caused by the environmental taxation and called “profitability effect”by Nakada (2004). As in Nakada (2004), this effect is alleviated due to an increase in the price elasticity of demand but its overall impact on the profit of intermediate good producers remains negative. On the other hand, environmental tax has an indirect effect on the labor market called “labor reallocation effect”. Indeed, this tax reduces final good production which decreases the supply of each intermediate good producer and thus implies a reallocation of labor from intermediate goods sector to R&D sector. Overall, the “labor reallocation effect”offsets the “profitability effect”. Finally, the tax on pollution enhances innovation and growth.

Proposition 2 *A higher environmental tax decreases pollution.*

Proof. *The derivative of $P(t)$ with respect to ϕ is negative.*

$$\frac{\partial P(t)}{\partial \phi} = \left(\frac{-1}{\beta}\right)\left(\frac{1-\beta}{\omega}\right)^{\frac{1}{\beta}}(1+\phi)^{\frac{-(1+\beta)}{\beta}} < 0.$$

■

Proposition 2 shows the negative effect of environmental policy on pollution. As we already shows a tax on pollution reduces the level of intermediate goods used by the final good producer and thus the level of pollution because it is proportional to intermediate goods production.

4 Welfare analysis

In this subsection, we examine the impact of environmental policy on welfare. From the equation (13), the welfare along the balanced growth path is given by the following equation[12] :

$$W(t) = \int_0^\infty \frac{1}{1-\theta}\left([C(t)P(t)^{-\delta}]^{1-\theta} - 1\right)e^{-\rho t}dt. \tag{25}$$

Using (17), the welfare can be rewritten as

$$W = \left(\frac{1}{1-\theta}\right)\{\left[Y(0)P(0)^{-\delta}\right]^{1-\theta}\left(\frac{1}{\rho - g^*(1-\theta)}\right) - \frac{1}{\rho}\}, \tag{26}$$

[12]We focus only on the more general case where the elasticity of substitution is different from one ($\theta \neq 1$).

where $Y(0) > 0$ (respectively $P(0)$) is the level of the production (respectively the pollution) in the economy at time 0.

In order to highlight the different impacts of the environmental tax upon the consumer's welfare, we analyze the first derivative of the expression (26) with respect to the environmental tax ϕ

$$\begin{aligned}\frac{dW}{d\phi} &= \left[\frac{\partial Y(0)}{\partial \phi} P(0)^{-\delta}\right] \left[Y(0)P(0)^{-\delta}\right]^{-\theta} \left[\frac{1}{\rho - g^*(1-\theta)}\right] \\ &- \left[\delta P(0)^{-\delta-1} Y(0) \frac{\partial P(0)}{\partial \phi}\right] \left[Y(0)P(0)^{-\delta}\right]^{-\theta} \left[\frac{1}{\rho - g^*(1-\theta)}\right] \\ &+ \left[\frac{\frac{\partial g^*}{\partial \phi}}{\left[\rho - g^*(1-\theta)\right]^2}\right] \left[Y(0)P(0)^{-\delta}\right]^{1-\theta}.\end{aligned} \tag{27}$$

The first term of the equation (27) or the production effect refers to the impact of the environmental policy upon the consumer's welfare through its effect on the production of the final good. Indeed, an increase of the environmental tax reduces the demand of the final good producer in intermediate goods and leads to a decrease in the production of final good. This explains why the production effect is negative.

The second term of the equation (27) or the pollution effect refers to the impact of the envrionmental policy upon the consumer's welfare through its effect on pollution. Indeed, as the polution is determined by the quantity of intermediate goods used in the production, an increase in the environmental tax reduces the latter and therefore leads to a decrease in pollution. But a decrease in pollution affects positively the consumer's welfare, that why the pollution effect is positive.

The third term of the equation (27) or the growth effect refers to the impact of the environmental policy upon the consumer's welfare through its effect on the growth rate. A higher environmental tax stimulates the growth rate. This growth effect is thus positive.

To summarize, the environmental policy affects the consumer's welfare through three channels. The production effect decreases welfare because it reduces the demand of intermediate goods. In contrast, the pollution effect increases welfare because it decreases also the demand of intermediate goods. Finally, the growth effect is positive for all the consumers.

Unlike Nakada (2004),[13] we determine the overall effect of the environmental policy on welfare. Indeed, we show that the two positive effects (growth and pollution effects) offset the negative effect (production effect) without constraints on the value of parameters.

[13]Indeed, in his paper, Nakada (2004) analyzes only the different effects of the environmental tax on welfare but not the overall effect.

Proposition 3 *A higher environmental tax increases welfare if the household's preference toward a clean environment is large enough.*

Proof. *See Appendix B.* ■

5 Conclusion

In this paper, we have shown that the long-run impact of an environmental policy on economic growth and pollution. For this purpose, we modify the Gancia and Zilibotti (2005) endogenous growth model by including pollution which comes from the use of intermediate goods production and by assuming an negative externality of pollution on consumer's utility. This allows us to describe in a very simple model the "win-win" situation called in the literature the "Porter Hypothesis".

The analysis shows that environmental taxation decreases the level of final good production and the demand for intermediate goods, thus, it decreases the profit of intermediate good producers. However, in our setting, as in Nakada (2004) environmental taxation also alleviates this impact because it decreases the price elasticity of intermediate demand and raises its rate of mark-up. Moreover, the reallocation of labor from intermediate goods sector to R&D sector enhances innovation and thus growth. Finally, we find that environmental taxation increases growth as well as reduces the level of pollution which provides theoretical foundations of the "Porter Hypothesis".

In addition, the environmental policy affects the consumer's welfare through three channels. The production effect decreases welfare because it reduces the demand of intermediate goods. In contrast, the pollution effect increases welfare because it decreases also the demand of intermediate goods. Finally, the growth effect is positive for all the consumers. Unlike Nakada (2004), we show that the overall effect of the environmental policy on welfare is positive.

Appendix A : the household problem

The household maximizes its overall utility in consideration of its initial asset and income while taking the flow of pollution in the economy as exogenously given. Thus the household's optimization problem is

$$\max_{C(t)} \quad U(t) = \int_0^\infty e^{-\rho t} \frac{1}{1-\theta} \left(\left[C(t) P(t)^{-\delta} \right]^{1-\theta} - 1 \right) dt,$$

$$\text{s.t.} \quad \dot{A}(t) = r(t)A(t) + w(t) + T(t) - C(t),$$

$$A(0) = A_0,$$

where $0 < \rho < \infty$ represents the rate of time preference, $A(t)$ the assets owned by the household, $r(t)$ the interest rate, $T(t)$ the transfert of the fiscal revenue to the household and $w(t)$ the wage. The current value Hamiltonian is

$$H(t, A(t), C(t), \mu(t)) = \frac{1}{1-\theta}\left(\left[C(t)P(t)^{-\delta}\right]^{1-\theta} - 1\right)$$
$$+\mu(t)\left[r(t)A(t) + w(t) + T(t) - C(t)\right],$$

where $\mu(t)$ is the Hamiltonian multiplier.

The first order conditions are

$$\frac{\partial H(.)}{\partial C(t)} = C(t)^{-\theta}P(t)^{-\delta(1-\theta)} - \mu(t), \tag{28}$$

$$\dot{\mu(t)} = \rho\mu(t) - \frac{\partial H(.)}{\partial A(t)}, \tag{29}$$

and the transversality condition given by

$$\lim_{t\to\infty}\left[\mu(t)A(t)e^{-\rho t}\right] = 0.$$

Finally, deriving equation (28) with respect to time and combining the derivative with equation (29) yields to the following equation

$$\frac{\dot{C}(t)}{C(t)} = \frac{\eta(1-\theta)\frac{\dot{P}(t)}{P(t)} + r(t) - \rho}{\theta}. \tag{30}$$

As the pollution is constant over time, the previous equation yields to equation (21).

Appendix B : the welfare analysis

As the growth effect (the third term of the equation (27)) is always positive, we just have to demonstrate that the production effect (the first term of the equation (27)) which is negative is more than compensated by the pollution effect (the second term of the equation (27)) which is positive. As $\left[Y(0)P(0)^{-\delta}\right]^{-\theta}$ and $\left[\frac{1}{\rho-g^*(1-\theta)} - \frac{1}{\rho}\right]$ are positive, this condition can be rewritten as

$$-\left[\delta P(0)^{-\delta-1}Y(0)\frac{\partial P(0)}{\partial\phi}\right] > \frac{\partial Y(0)}{\partial\phi}P(0)^{-\delta}.$$

Rearranging this inequality, we get

$$\delta > \frac{\frac{\partial Y(0)}{\partial \phi}}{\frac{\partial P(0)}{\partial \phi}} \frac{P(0)}{Y(0)}$$

From equations (1), (7) and (17), we obtain the level of production at the initial date $t = 0$, $Y(0) = \left(\frac{1}{1-\beta}\right) \tilde{x}(\omega)^{1-\beta} N(0)$. From equations (2) and (7), we obtain the level of pollution at the initial date $t = 0$, $P(0) = \tilde{x}(\omega)$. Rearranging this inequality, we get the following condition $\delta > 1 - \beta$ which is always true by definition.

References

Acemoglu, D. (2009): *Introduction to Modern Economic Growth*. Princeton University Press.

Aghion, P., and R. Griffith (2005): *Competition and Growth Reconciling Theory and Evidence*. The MIT Press.

Aghion, P., and P. Howitt (1992): "A model of growth through creative destruction," *Econometrica*, 60(2), 323–51.

Bianco, D., and E. Salies (2016): "The strong Porter hypothesis in an endogenous growth model model with satisficing managers," Working Papers 2016-01, OFCE.

Elbasha, E. H., and T. L. Roe (1996): "On endogenous growth: the implications of environmental externalities," *Journal of Environmental Economics and Management*, 31(2), 240–268.

Gancia, G., and F. Zilibotti (2005): "Horizontal innovation in the theory of growth and development," in *Handbook of Economic Growth*, ed. by P. Aghion, and S. Durlauf, vol. 1. Elsevier.

Grimaud, A. (1999): "Pollution permits and sustainable growth in a schumpeterian model," *Journal of Environmental Economics and Management*, 38, 249–266.

Grossman, G. M., and E. Helpman (1991): *Innovation and Growth in the Global Economy*. The MIT Press.

Hart, R. (2004): "Growth, environment and innovation - a model with vintages and environmentally oriented research," *Journal of Environmental Economics and Management*, 48, 1078–1098.

Hung, V., P. Chang, and K. Blackburn (1994): "Endogenous growth, environment and R&D," in *Trade, Innovation and Environment*, ed. by C. Carraro. Kluwer Academic Publishers.

Koesler, S. (2010): "Pollution externalities in a schumpeterian growth model," ZEW Discussion Papers 10-055, ZEW - Zentrum für Europäische Wirtschaftsforschung / Center for European Economic Research.

Michel, P., and G. Rotillon (1995): "Disutility of pollution and endogenous growth," *Environmental & Resource Economics*, (6), 279–300.

Nakada, M. (2004): "Does environmental policy necessarily discourage growth?," *Journal of Economics*, 81(3), 249–275.

Porter, M., and C. van der Linde (1995): "Toward a new conception of the environment-competitiveness relationship," *Journal of Economic Perspectives*, 9(4), 97–118.

Ricci, F. (2007a): "Channels of transmission of environmental policy to economic growth: a survey of the theory," *Ecological Economics*, 60(4), 688–699.

——— (2007b): "Environmental policy and growth when inputs are differentiated in pollution intensity," *Environmental & Resource Economics*, 38, 285–310.

Romer, P. (1990): "Endogeneous technological change," *Journal of Political Economy*, 98(5), s71–s102.

Verdier, T. (1995): "Environmental pollution and endogenous growth: a comparaison between emission taxes and technological standards," in *Control and Game-Theoretic Models of the Environment*, ed. by C. Carraro, and J. A. Filar, pp. 175–200. Birkhäuser.