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ABSTRACT

The present study examines the long-term relationship between aggregate price and dividend data and the corresponding mechanism for short-term error correction using the rational valuation formula and time-varying cointegration and based on Muth's (1961) theory of rational expectations and price movements. The study assumes the variability of asset rates of return and tests the null hypotheses of error-correction mechanisms for time-constant cointegration vectors and inequality between fundamental value and share price. The series used were provided by Shiller (2005) and refer to aggregate price and dividend data for the U.S. stock market over the period 1871 to 2010. The data were analyzed using Johansen's cointegration models with the use of restricted variables resulting from the combination of the variables studied with the Chebyshev time polynomial, as proposed by Bierens and Martins (2010). The results indicate rejection of the null hypothesis of constancy of cointegration vectors as well as the non-rejection of the null hypothesis of inequality between fundamental value and share price. These results are consistent with those obtained by Bierens and Martins (2010) and do not corroborate Muth's (1961) theory of rational expectations. It is therefore concluded that investors have different expectations of return for different future periods. The results suggest the validation of the model used and that there is a possibility of the occurrence of speculative movements supported by rationality or rational speculative bubbles.

Keywords: Rational expectations. Cointegration. Chebyshev. RVF. TV VECM.

1 INTRODUCTION

The extent to which a security is traded in capital markets, its variation over time, the returns obtained from it, and the attempt to predict results have all been studied in finance. The ability to estimate with reasonable accuracy how much a specific share is worth or how much its price could fluctuate over a given period has significant empirical implications. In cases of developed financial markets with strong informational efficiency (Fama, 1970, 1991), the price should reflect only the future information present in predictions about the benefits of the respective asset. Moreover, given the possibility of information asymmetry, the importance of models that explain prices and returns is even greater because the need for specificity leads to gathering information on numerous accounting and market variables as well as the cash flow generated by the asset. This information is not simultaneously available to both managers and investors, thus explaining the existence of numerous models for evaluating the fundamental value of the share that are based on the dividend discounts and cash flows of the company or the shareholder, among other factors (Cuthbertson & Nitzsche, 2004). In the present study, however, only the fundamental value of the share obtained through the dividend discount, commonly known as the rational valuation formula (RVF), is considered.

The RVF is supported by the theory of rational expectations (Muth, 1961) and, in this context, would not be consistent in the presence of asymmetric information in financial markets, as amply demonstrated in the literature (Miller & Modigliani, 1961), particularly because the determination of the dividend policy does not have to be linked to profits obtained by corporations. Therefore, the application of the RVF implies the (possibly intuitive) need to adopt dividends and discount rates that change over time, which results in an operational difficulty that may be insurmountable. The present study aims to test whether dividend discount rates vary over time and whether the estimation of such rates through an original method would enable the use of dividends to explain share price.

The abovementioned tests were performed through econometric techniques of cointegration time series using the Chebyshev time polynomial (Bierens & Martins, 2010).

The discussion of the issue of share valuation has significant ramifications. Any calculation and use of time-varying discount rates that could cause the fundamental value to be close to the share price might contradict studies that admit the bounded rationality of investors, provide a new way to test informational efficiency, and indicate the importance of accounting information in differences between value and price.

This study was conducted for the aggregate price and dividend data of the U.S. stock market over the period 1871 to 2010. In summary, the results indicated that discount rates varied over time and that the fundamental value of the stock did not equate to its price in any period, particularly since 2000. Thus, the results are consistent with Bierens and Martins' (2010) study and inconsistent with Muth's (1961) theory of rational expectations, with important implications regarding periods of greater and lesser inequality.

The study is divided into the following sections: the literature on the main concepts of the study are reviewed; methodology, with specific methodological issues; and results and concluding remarks, in which the main results are presented and compared with the literature and the considerations arising from these results are discussed.

2 THEORETICAL FRAMEWORK

2.1 Rational Expectations.

The Theory of Rational Expectations and Price Movement (RE) developed by John F. Muth (1961) is the basis of the present work. The rational expectations hypothesis (REH), which supports the theory, states that information is scarce, the economic system generally does not waste information, expectations are formed based on the structure of the relevant system describing the economy, and public prediction does not substantially influence the operation of the economic system (Muth, 1961). Thus, specifically for the present study, the price of a stock would represent all the information about future benefits arising from it. Price movements should therefore occur only when new information arrives on the market.

RE has had a profound effect on economic theory, but a strategic foundation is still required (Reny & Perry, 2006). Milgron (1981) analyzed RE and market trading mechanisms and concluded that there is no tension between information gathering and informational efficiency. RE has also been analyzed from the perspective of the effect that changes in economic policies have on production and inflation expectations, resulting in the observation that the theory is eventually violated for the following reasons or cases: production (Ball & Croushore, 2003); to solve the problem of infinite regress in expectations (Binder & Pesaran, 1998); to analyze the persistence of production shocks under less strict REH assumptions (Bonfim & Diebold, 1997); with the introduction of rational learning in arbitrage or rejection of price patterns (Brav & Heaton, 2002); with the indication of misspecification of agent models (Bray & Savin, 1986); with nominal anchors adopted by governments (Bruno & Fischer, 1990); with the volatility of shares being associated with failure in forming expectations (Burkley & Harris, 1997); to compare surveys conducted among housewives and economic analysts (Carroll, 2003); to set insurance premiums (Cummins & Outreville, 1987); and with the assumption that investment in shares does not constitute protection against monetary inflation (Danthine & Donaldson, 1986), among other studies.

2.2 Cointegration.

The long-term relationship between prices and dividends can be estimated using the econometric techniques of cointegration. The estimate resulting from this method corresponds to the discount rate applied to dividends to calculate the fundamental value of the share.

Cointegration provides a meaning for the regression of two or more non-stationary variables individually (Wooldridge, 2008), the combination of which may enable the elimination of non-stationarity (Asteriou & Hall, 2007), thus indicating the presence of a vector that transforms stationarity (Engle and Granger, 1987; Johansen, 1991) and eliminates the problem of spurious regressions. The most commonly used measures of cointegration (Gregory, Haug, & Lomuto, 2004) are those represented by the Augmented Dickley Fuller (ADF) (Engle and Granger, 1987), $\hat{Z}_\alpha$ (Phillips & Ouliaris, 1990), Trace (TR), maximum eigenvalue or maximum likelihood (MAX) (Johansen, 1988, 1991), and Reinsel and Ahn (Gregory et al. 2004) tests.

However, it is necessary to use cointegration vector error correction mechanisms (VECM), which allow a part of the imbalance of the relationship between variables in one period to be corrected in the next period (Engle and Granger, 1987), so that short- and long-term relationships between variables can be captured by the models (Asteriou & Hall, 2007; Cuthbertson & Nitzsche, 2004, Engle and Granger, 1987) and so that the advantages of imbalance correction, elimination of spurious regressions, better selection of specific models, and the prevention of error growth in long-term relationships are incorporated into the cointegration models (Asteriou & Hall, 2007).

Even so, the error-correction mechanisms assume as a premise cointegration time-invariant vectors (TI VECM) (Bierens & Martins, 2010), which indicate the possibility of not adequately capturing existing structural breaks in long-term time series, resulting in model misspecification (Martins, 2005). For structural breaks to be identified, the vectors must vary over time, enabling the study of time-varying cointegration vector error-correction mechanisms (TV VECM) (Bierens & Martins, 2010).

2.3 TV VECM.

As noted in the previous section, the long-term relationship between price and dividends does not remain constant over time, being subject first to error correction arising from possible residuals and then from the intrinsic variation of estimated discount rates. Error correction is dealt with by specific models, for which the rate would be constant, while the intrinsic variation requires specification that includes time-varying vectors.

TV VECM have been the subject of several studies. TV VECM models can include the following features: periodic cointegration, which enables the seasonal variation of the cointegration vector, called periodic cointegration (Boswijk & Franses, 1995); fractional cointegration, in which vectors are fractioned in orthogonal cointegrating subspaces (Chen & Hurvich, 2006); intercept subspaces for ascertaining unobservable variables (Deschamps, 2003); average, space, and quantile for the design of cointegrating subspaces (Granger, 2010); incorporation of the Markov chain (Hall, Psaradakis, & Sola, 1997); incorporation of canonical cointegration regression (Kim & Lee, 2001); incorporation of deviations from the unit root in the test of interest rate spreads (Lanne, 2000); no prior knowledge of the memories of time series in fractionally integrated components (Marmol & Velasco, 2004); the combination of Markov chain with Monte Carlo simulation (Koop, Leon-Gonzalez, & Strachan, 2008); pre-filtering and pre-estimation of models with time-varying coefficients (Park & Hahn, 1999); maximum likelihood to estimate the Capital Asset Pricing Model - CAPM (Engel & Rodrigues, 1989); transience testing with permanent structural breaks (Engle & Smith, 1999); discrete time systems on the errors generated (Phillips, 1991); and a time-variant discount rate for studying cointegration failure (Timmermann, 1995). Surveying cointegration models and ECM (Error Correction Model) demonstrates the explanatory superiority of TV VECM (Chan, Koop, Leon-Gonzalez, & Strachan, 2010). Some studies have used the Chebyshev time polynomial (Boyd, 2000) for the test of unit roots in a time series (Bierens, 1982, 1997), while others have considered the mentioned polynomial in MAX, TR, and LR (Likelihood Ratio) tests to study TV VECM (Martins, 2005; Bierens & Martins, 2010). Finally, TV VECM has been the object of study using the Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) model (Williams & Ioannidis, 2010).

2.4 Chebyshev.

As noted, one way to estimate time-varying discount rates is by applying cointegration techniques in combination with the Chebyshev time polynomial. The polynomial allows estimation of the long-term relationship, error correction, and the temporal variability of vectors and is also easy to apply. Thus, this technique, first used by Bierens and Martins (2010), will be used in the present study.

The Chebyshev time polynomial (CTP) shown in (1) and in Figure 1 is considered appropriate for solving time-dependent or finite-interval problems that can be scaled or translated, where $x \in [-1,1]$, where the expansion of the CTP is considered identical to the Fourier cosine coefficients (Boyd, 2000).

Bierens (1982) initially used the CTP in developing misspecification tests and considered it not very refined but having the advantage of computational simplicity. In another study, Bierens (1997) noted that due to its orthogonality property, the CTP is recommended for solving nonlinear functions and that any time function can be arbitrary and powerfully approximated by a CTP linear function. In this study, ADF applications with time variations were developed, considering the alternative hypothesis that time series are stationary in any time-determining arbitrary function. Martins (2005) used the CTP explicitly in an attempt to capture time in the VECM parameters, making it possible to use MAX, TR, and LR directly with critical values of LR developed by simulation for different sample sizes and without resorting to other effects not directly related to time, such as the use of level variables transformed by the respective natural logarithms. Bierens and Martins (2010) perfected some aspects of Martins' (2005) work and validated the use of the CTP in capturing the time variability of cointegrated time-series regression parameters and their respective correction mechanisms, that is, cointegration and VECM variables at different time periods.

$$P_{0,T}(t) = 1,\ P_{i,T}(t) = \sqrt{2}\cos\left(i\pi\,\frac{(t-0.5)}{T}\right), \qquad (1)$$

$$t = 1, 2, \ldots, T, \qquad i = 1, 2, 3, \ldots$$

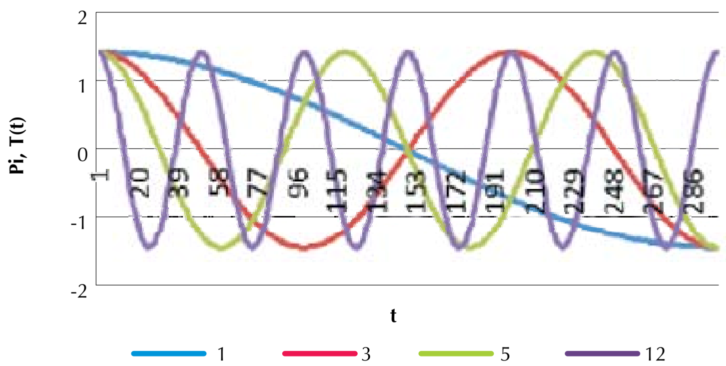

Figure 1 *The Chebyshev time polynomial for T = 298 and i = 1, 3, 5, and 12*

2.5 RVF.

The CTP has been used in several manners: to develop algorithms to solve overdetermined systems of linear equations (Abdelmalek, 1976); to solve nonlinear equations in one dimension (Berzins & Dew, 1981); to develop an algorithm for linear constraints with rank deficient or ill-conditioned matrices (Brannigan, 1981); with logarithmic singularity in regression coefficient estimates (Chawla, 1967); in the method for the numerical solution of the heat transfer equation (Dew & Scraton, 1972); and in parabolic partial differential equations in a region that permits transformation in quadratic equations or a circular cylinder (Dew & Scraton, 1975).

Furthermore, the CTP has been used in the best approximation for all sets of polynomials and the alternation of error curves and uniqueness (Dunham, 1972); in polynomial interpolation with the conversion of expressions in an algebraic nonlinear programming problem (Elanagar & Khamayseh, 1997); and in the development of monomial basis modification algorithms to find CTP zeros and extension to other bases (Grant & Ghiatis, 1983).

Finally, the CTP has been used in tests of misspecification based on the concurrent validity of the CTP (Hong & White, 1995; Bierens, 1982); to solve time-delay dynamic equations (Hsu & Chou, 2007); to solve infinite triangular arrays of points that constitute the roots of the unit (Ivanov, Rivlin, & Saff, 1990); to develop a model of smooth partitioning of blocks from a set (Mansour, 2009); using the barycentric form of the Lagrange interpolation formula and in confluent divided differences (Salzer, 1971); to solve numerically linear and nonlinear differential and integral equations (Sweilam & Khader, 2010); to develop an algorithm to solve the overdetermined system of complex linear equations (Watson, 1988); and to accelerate the convergence of iterative solutions of simultaneous equations originated while solving partial differential equations (Wrigley, 1963).

Present value and asset pricing models have significant relevance as study objects in finance. In their seminal studies, Campbell and Shiller (1987a, 1987b) assessed cointegration for stock prices, government bonds, dividends, and earnings per share, where the rate or discount factor $\delta/1\text{-}\delta$ was obtained through the cointegration vector between prices and dividends, where another factor may have been the average. They found that short- and long-term interest rates were cointegrated, with persistent disturbances between prices and stocks and dependence on the discount factor used, and that moving average profits over long periods have explanatory power in the regression against stock prices. Scott (1985) compared estimates using two methods to test propositions that he considered easily adaptable to long-term dividends. In addition, West (1987) developed a test to identify speculative bubbles and ended up rejecting the null hypothesis of the inexistence of such bubbles.

West (1988) also supported the quantitative and statistically significant rejection of RVF by analyzing the relationship between price and dividend variances. Chow (1989) rejected REH, concluding that RVF data are not explained by RE. DeJong and Whiteman (1991) claimed the impossibility of existence of cointegration between stock prices and dividends. Froot and Obstfeld (1991) sought to detect the induction of persistent deviations in RVF due to intrinsic bubbles in the U.S. market by linking the bubbles to the aggregate dividend and exogenous macroeconomic variables. Lee (1995) attributed the volatility of stock prices to the sum of permanent and transitory shocks to dividends. Donaldson and Kamstra (1996) demonstrated the inexistence of speculation in the American crisis of 1929 using an autoregressive conditional heteroskedasticity (ARCH) model, an autoregressive moving average (ARMA) model, and artificial neural networks. Timmermann (1996) attributed the excess in volatility to the phenomenon known as present value with learning or something like RVF with learning. Chow and Liu (1999) argued that the memory of dividend changes might explain the volatility of stock prices.

As observed in the reviewed literature regarding the main concepts of the present study, this is the first application of the model on data from the U.S. stock market. Although Bierens and Martins (2010) have also applied the model used in this study, the empirical evidence was based on the purchasing power parity of European countries. Campbell and Shiller (1987a, 1987b) highlighted the dependence of results on the discount factor used and did not use a methodology in which vectors varied over time.

3 METHODOLOGY

3.1 Modeling.

Assuming constant dividends and discount rates (Cuthbertson & Nitzsche, 2004), the price of a stock is equal to the present value of its dividend, which implies that the RVF demonstrated in (2), where P_t represents the fundamental value of the stock; E_t is the mathematical expectation operator based on the information set Ω_t; δ^i is the constant discount rate; and D_{t+i} is the dividend per share, which is applicable to various types of investors (Cuthbertson & Nitzsche, 2004).

$$P_t = E_t \left[\sum_{i=1}^{\infty} \delta^i D_{t+i} \right] \qquad (2)$$

Substituting (2) and writing it in a more compact notation, we obtain (3), which may be considered non-operational (Cuthbertson & Nitzsche, 2004), where the fundamental value of the share P_t is the sum of the results of the application of exponentially accumulated discount rates $\Pi_{t=1}^{j} \delta_{t+i}$ on all future dividends D_{t+j}.

$$P_t = E_t \left[\sum_{j=1}^{\infty} \left[\prod_{i=1}^{j} \delta_{t+i} \right] D_{t+j} \right] \equiv E_t \sum_{j=1}^{\infty} \delta_{t,t+j} D_{t+j} \qquad (3)$$

To circumvent the possible lack of operationalization, hypotheses about investors' dividend and discount rate forecasts and/or econometric models that achieve structural changes in time series (Bierens & Martins, 2010) should be tested, as proposed in (4), in which TV VECM is represented without intercepts and trends, with errors following a Gaussian distribution, with T being equal to the number of observations, with $\Pi'_t Y_{t-1}$ being the product of imbalances occurring at the level of the lagged variable in the previous period, and with $\Sigma_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j}$ being the sum of the products of imbalances occurring in the differences in the variable in all periods (Martins, 2005), but that would not allow the direct application of tests.

$$\Delta Y_t = \Pi'_t Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t, \qquad t=1, \ldots, T \qquad (4)$$

The direct application of the usual cointegration statistics and tests can be resolved in (4) with the CTP (Boyd, 2000) and subsequent transformation into (5), helping to operationalize (3).

$$\Delta Y_t = \alpha \left(\sum_{l=0}^{m} \xi_i P_{i,T}(t) \right)' Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t \qquad (5)$$

In the present study, the hypothesis testing arising from the application of (5) and the operationalization of (3) are the main objectives for validating the model's ability to capture the time variability of the cointegration vectors and use the corresponding estimators for estimating the discount rate, which would enable the determination of the fundamental value of the share.

3.2 Basic Aspects.

The main objective of the present study is to test the rationality of economic agents in the formation of U.S. stock prices by testing the null hypothesis of TI VECM (6) and inequality between price and fundamental value of the share (8). That is, the objective is to investigate whether the price of a share in the U.S. market can be predicted by its dividend, which corresponds to the RVF. However, the RVF will be applied with rates of return that are supposedly unstable or variable at different times and that are estimated using econometric techniques applicable to time series, with the inclusion of the CTP as a variable restricted to the model.

The variables are the aggregate price and dividend data of the corporations used by Shiller (2005) and made available electronically[1] for the years 1871 to 2010.

[1] Available at: www.econ.yale.edu/~shiller/data.htm, accessed on March 7th 2011.

Table 1 *Critical values of LRtvc*

m	10%	5%	1%
1	4.790	6.275	9.530
2	8.149	10.015	14.173
3	11.181	13.197	18.042
4	14.059	16.400	21.193
5	17.059	19.452	24.749
10	31.247	34.608	40.850
15	45.621	49.515	56.899
25	76.331	81.177	91.638

Source: Bierens and Martins (2010).

$$H_0\text{: TI VECM} \quad (6)$$

$$H_1\text{: TV VECM} \quad (7)$$

After testing the null hypothesis of TI VECM and the matrix solution with the estimators obtained in this process, the present value of the dividend moving average was compared with the share price in each month. In this case, the null hypothesis H_0 is the inequality between the price and the fundamental value of the share, as expressed in (8) with the alternative hypothesis H_1 being of equality between them, as in (9).

$$H_0: P_t \neq E_t \sum_{j=1}^{\infty} \delta_{t,t+j} D_{t+j} \quad (8)$$

$$H_1: P_t \equiv E_t \sum_{j=1}^{\infty} \delta_{t,t+j} D_{t+j} \quad (9)$$

The important contribution of the studied subject is the attempt to validate the model, which will enable research on TV VECM to be conducted, such as that proposed by Williams and Ioannidis (2010).

3.3 Application of Models.

The ADF, MAX, TR, and LR models and tests were applied using two econometrics software packages. Cointegration vector coefficients and error-correction mechanisms were obtained using OxMetrics 5:10. The ADF, HQC (Hanna Quin Information Criteria), BIC (Schwarz Bayesian Information Criteria), MAX, TR, LR, and LR_{tvc} tests were all performed with the software EasyReg International, which was developed by Herman J. Bierens and is freely distributed by The Pennsylvania State University[2].

The determining factor for the selection of the OxMetrics software was to obtain the cointegration vector coefficients and the error-correction mechanisms. This software uses the multiple time-series equation models of the PC GIVE module, with the selection of price and dividend variables, with seasonal constants, and with combinations of variables with the CTP as new variables restricted to the model.

The EasyReg International software was selected due the perfect parameterization and adoption of the CTP and of the tests of null hypothesis of TI VECM with critical LR_{tvc} values and because the software was developed by Bierens, one of the authors of studies on the subject (Bierens & Martins, 2010). The statistics were obtained using Johansen's cointegration analysis, which is listed on the menu; multiple equations models, with the selection of price and dividend variables; and indication of the model with intercepts and seasonal dummies with cointegration restrictions imposed on the intercept parameter, calculation and choice of the VAR lag, the number of cointegration vectors, and the maximum number of CTP polynomials.

Other models could have been used in testing the hypothesis of non-stationarity of the data, with the possibility of generating better results for large samples (Pantula & Fuller, 1993) or even specifically for the CTP (Bierens, 1982, 1997). However, for operational reasons, the models described above were selected.

4 RESULTS

4.1 Descriptive Statistics.

There were no apparent problems in the data. Only the price variable was non-stationary, which does not affect the assumption of cointegration (Granger, 1981). The Schwartz Bayesian information criterion indicated two periods for the best choice of VAR lag, with a cointegration vector between price and dividends, as shown in Tables 2-5.

Table 2 *Mean and standard deviation*

P		D	
Mean	SD	Mean	SD
385.79208	384.68848	12.35205	5.698750

Table 3 *ADF Test*

P		D	
t adf	p-value	t adf	p-value
-0.5218	0.8900	-2.8883	0.0500 *

* - significant at 10%

Table 4 *Choice of VAR lags by HQC and BIC information criteria*

VAR	
HQC	BIC
2	2

Table 5 *MAX and TR tests for cointegration rank*

rank	MAX	TR
1	2.50	2.50

4.2 TV VECM.

The average discount rate for the Shiller frequency yielded a value of 0.046159 over the month, as shown in Table 6, was positive throughout, and did not exhibit counterintuitive values. Figure 2 shows the time of maximum rate, which occurred at approximately 1930.

[2] Available at: http://econ.la.psu.edu/ hbierens ~ / OLDERVER.HTM, accessed April 24th of 2011.

Table 6 *Average discount or return rate*

Mean	SD	Maximum	Minimum
0.046159	0.0182803	0.165598	8.3E-05

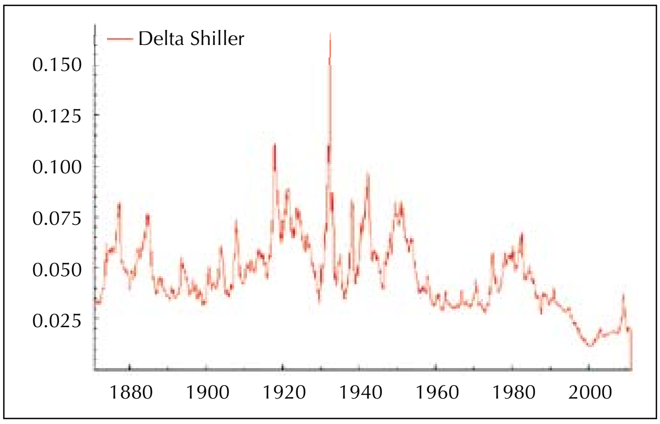

Figure 2 *Discount or return rates*

The null hypothesis of constancy of the cointegration vectors was significantly rejected in all polynomials used, from m = 1 to m = 15, as shown in Table 7, with the results being analyzed in relation to Bierens and Martins' (2010) critical LR_{tvc} values. This result is extremely relevant for studies of the U.S. stock market, suggesting a robust validation of Bierens and Martins' (2010) model for the monthly aggregated data used herein. The first null hypothesis of the study is therefore rejected.

Table 7 *LR statistics for Shiller frequency*

m=1			m=2			m=3			m=4			m=5		
LR	p-value		LR	p-value		LR	p-value		LR	p-value		LR	p-value	
7.7500	0.0208	**	10.4600	0.0334	**	11.2200	0.0817	*	15.9500	0.0431	**	23.3800	0.0094	***
m=6			**m=7**			**m=8**			**m=9**			**m=10**		
LR	p-value		LR	p-value		LR	p-value		LR	p-value		LR	p-value	
25.5800	0.0123	**	31.9600	0.0041	***	37.8800	0.0016	***	46.1300	0.0003	***	47.8400	0.0005	***
m=11			**m=12**			**m=13**			**m=14**			**m=15**		
LR	p-value		LR	p-value		LR	p-value		LR	p-value		LR	p-value	
48.8600	0.0008	***	51.9400	0.0008	***	60.4700	0.0001	***	70.3900	0.0000	***	77.9900	0.0000	***

*** - Significant at 1%. ** - Significant at 5%. * - Significant at 10%.

4.3 RVF.

As there were no apparent problems in discount or return rates, the second null hypothesis of the study can now be considered by comparing the fundamental value and price of the share. As shown in Figure 3, the null hypothesis of inequality between fundamental value and price of the share cannot be rejected for the whole study period. There was a significant difference between value and price from the mid-1990s that increased until 2000, and this difference cannot be attributed to the extension of the dividend series used in the calculation of the fundamental value.

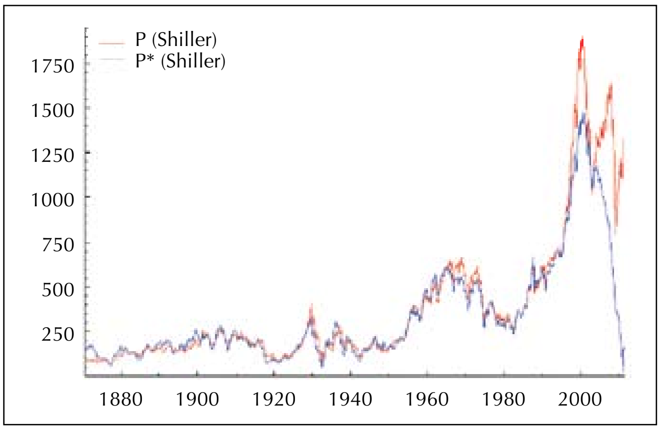

Figure 3 *Fundamental Value of share (P*) and price (P)*

The null hypothesis of inequality between fundamental value and price of the share, which is the second hypothesis of the present study, therefore also cannot be rejected in relation to the Shiller frequency. This is inconsistent with Muth's (1961) RE, which was used as a theoretical reference in this study.

4.4 Comparison.

The result shown in Figure 3 is similar to that obtained by Campbell and Shiller (1987a) up to 1980. At approximately 1930, the price exceeds the fundamental value. However, in that study, from 1960 to 1980, the fundamental value did not exceed the share price again, unlike the results observed herein. From 1980 to 1995, the fundamental value and price of the share have some moments of equality, as shown in Figure 4. Therefore, while the second null hypothesis of the present study is not rejected, the results suggest the validation and explanatory power of the model used.

The historical background provided by Granger (2010) demonstrates the trend of TV VECM studies. In Bierens and Martins (2010), the model was used for purchasing power parity. In the present study, the use of MAX, TR, and LR with CTP in level variables allowed the capture of variability of vectors for the monthly price and dividend series, which suggests validation of the model and of LR_{tvc} (Bierens & Martins, 2010) and strengthens the abovementioned trend.

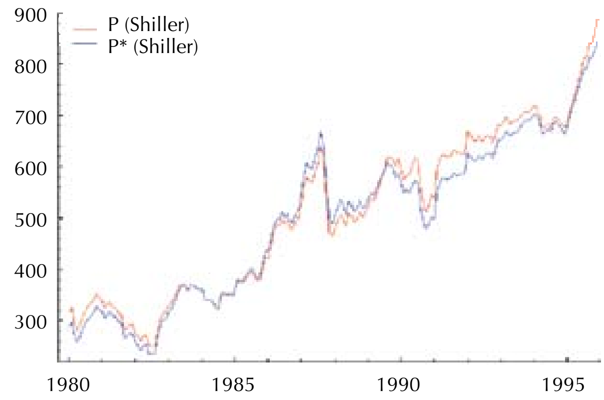

Figure 4 *Fundamental Value of share (P*) and price (P) from 1980 to 1995*

5 FINAL CONSIDERATIONS

In summary, the results demonstrate the validation of an innovative and unique model for analysis of time series and the inequality between the fundamental value and the price of a share in the U.S. stock market. This demonstration raises important considerations.

In terms of the model, unexplored research frontiers can be overcome, such as the analysis of investment decisions, financing, and distribution of corporate results, and the consideration of variables for shares, debt, capital structure, and information asymmetry at the company level. This is possible because the appropriateness of using level variables, rather than differences or changes, and, in particular, the ability to capture smooth and continuous structural changes in time series has been demonstrated, opening new perspectives for corporate finance research. Specifically, the use of level variables allows models to capture characteristics intrinsic to the variable without being affected by potential residuals arising from their variation from one period to another or from their transformation into a natural logarithm, which mistakenly leads non-stationary variables into a stationary condition and implies a significant distortion of the application of cointegration tests.

The rationality testing of speculative bubbles can also be extended, bearing in mind that the inequality of fundamental value and share price may be due to rational decisions that can now be better incorporated into the model used herein. It is possible that the limit of vector oscillation has been reached, leaving the hypothesis of rationality of speculative movements to explain such an inequality.

The role of the quality and availability of accounting information is emphasized in this sense because the period of inequality between price and fundamental value coincides with that of increased international accounting harmonization efforts, with the objective of determining their impact on information asymmetry.

Moreover, regarding the period of apparent equality between fundamental value and share price, it is important to assess whether this result was exclusively due to the model used or a potential misspecification of the models used in previous studies and whether other models using time-varying vectors would achieve a similar result.

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