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27 May 2017

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Abstract: We consider a decentralized supply chain (DSC) under resale price maintenance (RPM) selling a limited-lifetime product to forward-looking customers with heterogeneous valuations. When customers do not know the inventory level, double marginalization in DSC leads to a higher profit and lower aggregate welfare than in centralized supply chain (CSC). When customers know the inventory, DSC coincides with CSC. Thus, overestimation of customer awareness may lead to overcentralization of supply chains with profit loss comparable with the loss from strategic customers. The case with unknown inventory is extended to an arbitrary number of retailers with inventory-independent and inventory-dependent demand. In both cases, the manufacturer, by setting a higher wholesale price, mitigates the inventory-increasing effect of competition and reaches the same profit as with a single retailer. The high viability of RPM as a strategic-behavior-mitigating tool may serve as another explanation of why manufacturers may prefer DSC with RPM to a vertically integrated firm.

Keywords: limited-lifetime product, strategic customers, limited information, aggregate welfare, oligopoly

1 Introduction

Resale price maintenance (RPM) is any of a variety of practices through which manufacturers restrict resellers in the prices that they may charge for the manufacturer's products. RPM can be imposed explicitly as a manufacturer's suggested retail price (MSRP or list price) or as a price floor (minimum RPM), or the price can be set implicitly by, for example, fixing the retailer margin, see European Commission (2010). The legal status and use of RPM has been controversial for over a century, and evidence suggests that both the scale of RPM use and the effects on the economy are underestimated. Overstreet (1983) provides an extensive review of RPM use for the period when this practice was per se legal. In 1988, when RPM was illegal in the USA, the Supreme Court adopted a wide use of the Colgate doctrine according to which a manufacturer may refuse to deal with a retailer if it does not comply with the manufacturer's price policy. Butz (1996) quoted antitrust authorities arguing that RPM became "ubiquitous" and "endemic", "but based upon 'winks and nods' rather than written agreements that could be used in court."

RPM attracted growing attention after a 2007 Supreme Court declaration that manufacturer-imposed vertical price fixing should be evaluated using a rule of reason approach. MacKay and Smith (2014) provide explicit evidence of RPM and comment that the firms involved in this practice "include manufacturers and suppliers of childcare and maternity gear, light fixtures and home accessories, pet food and supplies, and rental cars. Sony has publicly used minimum RPM on electronics such as camcorders and video game consoles, and as of mid-2012, Sony and Samsung began enforcing minimum RPM on their televisions. Other retailers do not comment on whether or not they enter minimum RPM agreements, perhaps due to negative consumer sentiment associated with higher prices."

Manufacturers of various goods including electronic devices and seasonal goods repeatedly introduce new versions of their products. The rapid pace of fashion, innovations, and change of seasons limit the lifetime of these versions, reducing customer valuations in time. As a result, such products may regularly appear on clearance sales. According to the National Retail Federation, the practice of markdown pricing has followed a growing trend since 1960's (Deneckere et al. (1997)). When product value is relatively durable, retailers may markdown in order to price-discriminate among customers with different valuations. Clearance sales may also result from inventory-dependent demand even when the demand can be predicted with a high accuracy. This may happen when sales at the list price increase with the quantity of the product exposed to customers, see, for example, Urban (2005).

Customers accustomed to markdown pricing form expectations about future clearance sales and, using these expectations, may engage in forward-looking or patient shopping behavior by delaying the purchase until price reduction. The time of discount can be easily anticipated for seasonal goods, and the size of the discount is usually predictable because it is often product specific and expressed in round numbers (20% off, 50% off, etc.). Customers realize that delaying the purchase may reduce the sense of novelty and their enjoyment of the product, but they still make this intertemporal trade-off. Since forward-looking behavior is an instance of strategic behavior and we do not consider other forms of this behavior, we use these terms as synonyms.

Starting from Coase (1972), theoretical and empirical studies confirm that strategic customer behavior hurts sellers and may reduce profit up to 50%, see, for example, a comprehensive review in Gönsch et al. (2013). Coase conjectured that a monopolistic seller of a durable good with a secondary market is not able to extract monopolistic profit when strategic customers know the total amount of the product and total demand. Even the customers with high valuations of the product may wait until the price is reduced to the competitive level because the seller has an incentive to sell the product in small portions reducing the price with time in order to capture customer surplus. Such "price skimming" decreases the market value of the product for the first buyers, and they may end up with a negative surplus. Due to the customer delays, the profit from skimming may be significantly lower than a monopolistic profit from a one-time sale of a portion of available product. However, the seller cannot credibly commit to the one-time sale because customers know that the seller may have extra profit by selling an additional product after this sale is realized. This phenomenon is referred to as "dynamic inconsistency."

Despite broad evidence of the negative effects for sellers, Desai et al. (2004), Arya and Mittendorf (2006), and Su and Zhang (2008) prove that when customers are strategic, a decentralized supply chain (DSC) may enjoy a higher total profit than that of the centralized supply chain (CSC). These studies show that the Coase problem may be solved by adding an intermediary retailer. The benefit results from double marginalization, which, without strategic customers, leads to suboptimally low inventories procured by retailer because manufacturer sets the wholesale price higher than its own unit cost. This effect provides a credible commitment of the SC to a high retail price or low quantity, which cannot be achieved for a centralized production-selling unit under the Coase setup.

Our paper complements these studies by showing that DSC with strategic customers may have a higher profit than CSC even when the seller does not suffer from the Coase problem in its extreme case, that is, when the secondary market can be neglected, making possible intertemporal price discrimination, see, for example, Bulow (1982). Under price discrimination, the two-period profit of CSC is higher than the profit if sales occur only in the first period despite the negative effect of customer delays. This situation means that the seller does not need to commit to first-period sales. In other words, even when CSC does not suffer from dynamic inconsistency, DSC may perform better due to double marginalization, which, in this case, permits higher prices.

We derive this result in a two-period model for a limited-lifetime product, a contract with RPM for DSC, and forward-looking customers with heterogeneous valuations who do not know the retailer's inventory level. In reality, customers often do not know inventories or may ignore this information even when it is available. Comparison with the case of known inventory yields the value of information disclosure or, alternatively, the value of overestimation of customers' reaction to this information. There are a many contracts that include RPM. In this paper we restrict attention to contracts with a two-part tariff where the manufacturer sets both the retail price and the wholesale price and a fixed (franchise) fee. This follows, for example, the work of Rey and Tirole (1986), Gal-Or (1991), §5.2 of Gurnani and Xu (2006), and Rey and Vergé (2010).

Our analysis is distinct from those in the above studies because our focus is on the intertemporal effects of strategic customers and limited life of the product. In particular, the retailer may sell the product in both periods but must procure the total two-period inventory in the first period, given the inputs from the manufacturer. The manufacturer solves a one-period problem because the product is not produced in the second period. As a result, we do not use subgame perfect equilibrium as do Desai et al. (2004) and Arya and Mittendorf (2006) who consider SC under two-part tariffs and wholesale price contracts respectively. In terms of the equilibrium concept, our setup is closer to that of Su and Zhang (2008) who use rational expectations equilibrium (REE) for the wholesale price, buyback, sales rebates, and markdown money contracts.

Another flexibility for the retailer in our setup is that the second-period price is not fixed by the manufacturer but determined endogenously by the retailer inventory decision and market clearing. Besides the growing empirical evidence on markdowns mentioned above, this assumption is consistent, for example, with the sales of copyrighted materials in Japan where retailers are selling first at MSRP and eventually "reducing prices for consumers who don't mind waiting a while before they buy"; see Nippop (2005). Similarly to Desai et al. (2004) and Arya and Mittendorf (2006), we consider deterministic demand in order to have a more tractable problem. Fisher and Raman (1996) shows that demand uncertainty in the fashion apparel industry can be significantly reduced by analyzing preliminary sales of the product. The effects of uncertainty are considered, for example, in Su and Zhang (2008) and Cachon and Swinney (2009).

In this framework, DSC sets prices higher and inventory lower than CSC, which is a usual effect of double marginalization. This known effect leads to an interesting result in our setup. When customers exhibit higher levels of strategic behavior (to be defined), more customers delay their purchases under CSC, which is a usual consequence of forward-looking behavior. For DSC, the number of waiting customers decreases because more strategic customers pay more attention to the expected second-period price, which is higher under DSC. As a result, DSC enjoys more sales in the first period at a higher price than CSC. This comparative increase in the first-period profit outweighs a relative loss in the second-period sales.

A frequent motivation for studying RPM is the welfare effect of this policy. We show that RPM, compared to CSC, improves neither customer surplus nor aggregate welfare. However, it is questionable that RPM is the primary culprit in these losses for two reasons. First, some customers with low valuations suffer from the strategic behavior of other customers with higher valuations even when the first-period price is fixed (no manufacturer decisions). Second, CSC can reach the same profit and hurt welfare in the same way as RPM simply by disclosing inventory information to the customers. Therefore, under DSC with known inventory, manufacturer "turns off" unnecessary double marginalization by setting the wholesale price equal to unit cost. The result implies, first, that strategic customer behavior itself may be a fundamental reason of welfare losses; and second, that overestimation of customer knowledge of inventory or underestimation of strategic customer behavior may lead to overcentralization of SC. We show that profit loss from this overcentralization may be comparable with the loss from strategic customers.

We present a review of related literature in §2, describe a general model and provide a closed-form analysis for RPM with strategic customers and one retailer in §3. The extensions for an arbitrary number of retailers are considered in §4, and the conclusions are in §5. All proofs and supplementary materials are in the online appendix.

2 Literature review

The negative effect of double marginalization has been known since Spengler (1950) who shows in a model with linear deterministic demand that "both the consumers and the firm benefit" from vertical integration, that is, when all production-selling decisions comply with a single criterion. A broad literature on SC coordination, reviewed in Cachon (2003), examines the abilities of various contracts to reach the same profit as CSC. These results raise a question: Why do decentralized supply chains exist if their profits never exceed the one of CSC? The reasons include legal issues that motivated the work of Spengler or prohibitively high costs of CSC construction for small firms. The works of Desai et al. (2004), Arya and Mittendorf (2006), and Su and Zhang (2008) reviewed in Su and Zhang (2009), provide one more reason: double marginalization can lead to a strictly greater profit of DSC than that of CSC when it serves as a commitment device to higher prices or low inventories while customers strategically delay their purchases. The current paper extends this line of research by showing that when customers are strategic, DSC under RPM outperforms CSC even without secondary markets and with competing retailers.

The study of strategic buyers starts from the famous conjecture of Coase (1972), which is formally supported in subsequent work, for example, by Bulow (1982). These early findings have led to further research in the context of intertemporal pricing, which is systematically reviewed in Gönsch et al. (2013). In particular, Liu and van Ryzin (2008) concur that "capacity decisions can be even more important than price in terms of influencing strategic customer behavior"; they study the effects of capacity decisions when prices are fixed while customers have full information and can be risk-averse. Liu and van Ryzin find that capacity rationing can mitigate strategic customer behavior but is not profitable when customers are risk neutral. Under competition, which typically increases market supply, the effectiveness of capacity rationing is reduced, and there exists a critical number of firms beyond which rationing never occurs in equilibrium. Further development of this work by Huang and Liu (2015) shows that capacity rationing is also less effective under inaccurate customer expectations about the reduced-price product availability.

Unlike Liu and van Ryzin (2008), we consider retailers' capacity decisions in a SC framework when the manufacturer uses RPM and optimally sets the first-period price. Following Liu and van Ryzin (2008), we check the robustness of RPM as a low-inventory-commitment tool with respect to the number of retailers. First, similarly to Liu and van Ryzin (2008), we consider equal allocation of the first-period demand among the retailers. Then we raise the bar by extending the test to inventory-dependent demand when the first-period sales increase in retailers' inventories, which, in addition to competition, further boosts the supply.[1] The possibility of salvage sales, included in the case of inventory-dependent demand, further increases retailers inventories. In response to these challenges, the manufacturer raises the wholesale price, thereby achieving a desirable inventory level and profit of DSC that exceeds the one of CSC for any number of competing retailers. These extensions confirm the high viability of RPM as a strategic-behavior-mitigating tool, which may serve as another explanation of why manufacturers may prefer RPM to a vertically integrated firm.

A review of the theories explaining the existence of RPM is provided in Orbach (2008). In particular, RPM can be welfare-reducing when it leads to retailer cartels. In other cases, RPM can be welfare-improving, for example, when the manufacturer uses it to protect the appeal of branded products against using them as loss leaders or supports the retailers providing costly demand-increasing services against free riders that capture the demand by cutting the price. These theories do not consider the effects of strategic customers. Other contracts with RPM considered in the literature presume, for example, that manufacturer, besides retail price, fixes the quantity of the product, procured by the retailer, see, for example, Mukhopadhyay et al. (2009). In our setting, this assumption would lead to a passive retailer without double marginalization, which is a crucial effect to increase the profit of SC when customers are strategic. A review of other vertical restraints can be found in Lafontaine and Slade (2013).

3 RPM with one retailer

We consider a two-period market where a manufacturer sells a limited-lifetime product either directly to customers (CSC) or via an intermediary retailer (DSC). Following Desai et al. (2004) and Arya and Mittendorf (2006), we assume that the manufacturer and retailer know the demand and, in particular, the first-period demand D. First-period buyers do not participate in the second period and, therefore, there is no secondary market. The manufacturer and retailer do not offer the product for rent due to high remarketing costs or legal issues, see, for example, Bulow (1982).

3.1 Model description and general results

Under CSC, the manufacturer chooses the first-period price p_1 and inventory Y. By choosing Y, the manufacturer chooses either one-period or two-period sales depending on the profitability of price-discrimination compared to the first-period sales only. If $Y > D$, the second-period price p_2 is determined by marked clearing. Following Arya and Mittendorf (2006), who adopt a basic setup from Bulow (1982), we normalize the manufacturer's cost to zero, and then the profit of CSC is

$$\Pi^C = p_1 \min\{Y, D\} + p_2(Y)(Y - D)^+, \tag{1}$$

where superscript "C" means CSC.

Under DSC, the manufacturer maximizes its profit by offering a contract with RPM at the beginning of the first period. Since the product is not produced in the second period, the manufacturer faces a one-period problem. Following Desai et al. (2004), we disregard other retailer costs except the wholesale price w. We will call an RPM contract a tuple (p_1, w, F), where p_1 is the first-period retail price or MSRP and F is a fixed fee. Equivalently, RPM can be determined by (p_1, m_r, F), where $m_r = (p_1 - w)/p_1$ is a retailer margin. According to the studies of SC contracts with fixed fee, for example, Rey and Tirole (1986) and Gurnani and Xu (2006), the manufacturer sets F that makes the retailer indifferent between accepting and rejecting the contract. When demand is deterministic, the retailer accepts any contract with nonnegative profit; otherwise, the manufacturer may not supply the product leading to zero retailer profit. Therefore, F equals retailer profit and the manufacturer total profit equals the profit of DSC. Henceforth, we denote this profit Π^D and the manufacturer and retailer parts of this profit Π^m and Π^r respectively. In practice, the difference $F - \Pi^r$ is a positive constant, which does not affect the results below.

Retailer maximizes its profit by selecting the initial inventory level Y, which may lead either to one- or two-period sales. In the latter case, the second-period price is determined by market clearing. Then the manufacturer and retailer parts of DSC profit are

$$\Pi^m = wY, \tag{2}$$

$$\Pi^r = -wY + p_1 \min\{Y, D\} + p_2(Y)(Y - D)^+. \tag{3}$$

Customers, similarly to Desai et al. (2004), arrive at the start of the first period and their valuations v are uniformly distributed on the interval $[0,1]$. We normalize the number of customers to one, that is, the potential demand in the first-period is $1-p_1$. Normalization effectively expresses revenue and inventory as "unitless" quantities and the first-period price p_1 as a share of the maximum valuation implying $p_1 \leq 1$. The demand $1-p_1$ is "potential" because it includes all customers with valuations not less than p_1. We show below that when some of the customers strategically delay their purchases, the actual first-period demand is $D = 1 - v^{\min} < 1 - p_1$, where $v^{\min}$ is the valuation of a customer who is indifferent between buying in the first period or waiting. Then, a general expression for a seller profit with a unit cost c becomes

$$\Pi = -cY + p_1 \min\{Y, 1 - v^{\min}\} + p_2(Y)(Y - 1 + v^{\min})^+, \tag{4}$$

where $c = 0$ for CSC and $c = w = (1 - m_r)p_1$ for the retailer in DSC. Note that profit (4) increases linearly in Y when $Y \leq 1 - v^{\min} = D$. Therefore, the inventory of a profit-maximizing firm is not less than D, that is, there are no stockouts, implying that the first-period sales equal D and the second-period inventory is $Y - D = Y - (1 - v^{\min})$.

A decrease in valuations, similarly to Desai et al. (2004), is captured by factor $\beta \in [0,1]$: if the customer's first-period valuation is v, the second-period valuation is βv. Suppose the second-period inventory $Y - 1 + v^{\min} > 0$. The number of customers remaining in the market is $v^{\min}$ and the maximum second-period valuation is $\beta v^{\min}$. Therefore, the market clearing condition for the second period takes the form $v^{\min}\frac{\beta v^{\min} - p_2}{\beta v^{\min}} = Y - 1 + v^{\min}$, or, equivalently,

$$p_2 = \beta(1 - Y). \tag{5}$$

We use a logical restriction $\beta > c$, which guarantees that the highest-valuation customer is prepared to pay more than the unit cost in the second period. If this restriction does not hold, the clearance price can never be above the unit cost. This second-period setup differs from Desai et al. (2004) and Arya and Mittendorf (2006) where the product is produced in both periods, a seller chooses quantities to sell in both periods, and there is second-period used-product market. The setup differs also from Su and Zhang (2008) where the second-period price is exogenously fixed. The following assumption is common for all cases considered in this paper.

Assumption 1. *Customers know their private valuations v, list price p_1, product durability β, and* *the second-period surplus discount factor* $\rho \in [0,1)$. *Customers are non-cooperative and do not know total demand.*

Undervaluation of the surplus from delaying a purchase, similarly to Desai et al. (2004), means that even for a product that does not depreciate much by the second period (β is near one), customers with any valuation may myopically ignore the second period during the first-period deliberations, which is captured by $\rho = 0$. The value of ρ may depend on the market targeted by the product, for example, for age- or culture-oriented products, and on the customer confidence in the stability of the financial situation. Customers with a higher ρ place more emphasis on the second period in their wait-or-buy decisions. Thus, unlike β, which models an objective decrease in valuations, the customer's discount factor ρ is a subjective parameter describing the level of strategic behavior. The essence of the distinct roles of β and ρ has been succinctly captured by Pigou (1932): "Everybody prefers present [that is, $\rho < 1$] pleasures or satisfaction of given magnitude to future pleasures and satisfaction of equal magnitude [that is, $\beta = 1$], even when the latter are perfectly certain to occur." Frederick et al. (2002) provide a review of empirical estimates of customers' discount rates.

Similarly to Desai et al. (2004), customers are homogeneous in ρ and β. This assumption is applicable to any products targeting specific market segments. Some empirical studies, for example, Hausman (1979), claim a dependence of the discount rate on income (which serves sometimes as a proxy for product valuation). Other studies, however, show that the discount rate does not vary significantly with income, see, for example, Houston (1983). We do not include $\rho = 1$ because the case $\rho\beta = 1$ needs a special analysis increasing the volume of the paper. When this case is interesting, it is considered as a limiting case as $\rho\beta \to 1$. Note also, that we do not use ρ to calculate the actual (realized) total customer surplus (§3.3) since ρ models only customer first-period buying behavior. However, we do use β for this goal because deterioration of the product value indeed decreases the realized second-period surplus.

Subsections below compare the cases where customers know and do not know the inventory level. A general *sequence of events for CSC* in both cases is as follows: (a) manufacturer, anticipating strategic customer behavior, chooses p_1 and Y; (b) the first-period demand D and sales are realized; (c) the remaining inventory $Y - D$ is cleared at p_2.

For DSC with unknown inventory, the timeline is: (a) manufacturer, anticipating the retailer inventory response to manufacturer decisions and customers' behavior, offers the contract with RPM; (b) the first-period demand D is realized; (c) the retailer accepts the contract and procures inventory Y; (d) the first-period sales are realized; (e) the remaining inventory $Y - D$ is cleared; (f) the retailer pays fixed fee to the manufacturer. When customers know the inventory, the only difference in this sequence is that the first-period demand is realized after retailer's inventory decision.

3.2 Customers do not know inventory level

The availability of information about total supply of the product varies among the markets. Some markets, such as land or real estate, have nearly perfect information, the assumption used, for example, in Desai et al. (2004), Arya and Mittendorf (2006), and Liu and van Ryzin (2008). In many other markets, total system-wide inventory is unobservable, which reduces the ability of retailers to use rationing as a tool for stimulating first-period demand from strategic customers. When customers do not observe total supply, the problem can be solved by assuming that a seller (either centralized or a retailer in DSC) preannounces the second-period product availability $\alpha \in \{0, 1\}$ and the price p_2, see, for example, Yin et al. (2009). Equivalently, like in Su and Zhang (2008), it can be assumed that customer expectations of these values are rational, which means that they coincide with the actual values that will be realized. Both approaches to the information about α and p_2 imply that all customers share the same values of these parameters. We stick to the second (expectations) approach in this section and, therefore, refer to the resulting outcomes as Rational Expectation Equilibria (REE).

Assumption 2. *Customers do not know total product supply but know seller's cost c and have expectations of product availability in the second period $\bar{\alpha} \in \{0, 1\}$ and second-period price $\bar{p}_2$.*

Despite knowledge of c, customers cannot infer inventory level because they do not know total demand. Given expectations $\bar{\alpha}$ and $\bar{p}_2$, customers decide whether a first or second-period purchase maximizes their surplus, which is similar to Su and Zhang (2008)[2] or Cachon and Swinney (2009):

Assumption 3. *When the product is available, a customer with valuation v buys in the first period if and only if (iff) the first-period surplus $\sigma_1 \triangleq v - p_1$ is not less than the expected second-period surplus $\bar{\sigma}_2 \triangleq \bar{\alpha}\rho(\beta v - \bar{p}_2)^+$.*

Customers do not consider rationing risk in the first period because there are no first-period stockouts due to deterministic demand and profit-maximizing firms. Since $\bar{\sigma}_2 \geq 0$, customers with $v < p_1$ never buy in the first period because such a purchase would result in a negative surplus. For the same reason, customers with $\beta v < p_2$ do not buy in the second period when p_2 is realized. The lemma below describes the first-period demand.

Lemma 1. *Given customer expectations, surplus-maximizing behavior is to buy in the first period if $v \geq v^{\min}$, where the unique valuation threshold is given by $v^{\min} = \max\left\{p_1,\ \min\left\{\frac{p_1 - \bar{\alpha}\rho\bar{p}_2}{1-\bar{\alpha}\rho\beta},\ 1\right\}\right\}$. The resulting first-period demand is $D = 1 - v^{\min}$.*

Based on this lemma and the above assumptions, the retailer profit in DSC is $\Pi^r = \Pi^r(Y, p_1, w, \bar{p}_2, \bar{\alpha})$, where $p_1, w, \bar{p}_2$, and $\bar{\alpha}$ are the parameters, and we specify *REE in pure strategies for DSC* as follows:

(1) Given p_1 and w from the manufacturer and customer expectations, let the best response of the retailer be $BR(p_1, w, \bar{p}_2, \bar{\alpha}) = \arg\max_Y \Pi^r(Y, p_1, w, \bar{p}_2, \bar{\alpha})$.

(2) The tuple $\left[\hat{Y}(p_1, w), \hat{p}_2(p_1, w), \hat{\alpha}(p_1, w)\right]$ is a REE for given p_1, w iff $\hat{Y}(p_1, w) = BR(p_1, w, \hat{p}_2, \hat{\alpha})$, $\hat{p}_2(p_1, w) = \beta\left[1 - \hat{Y}(p_1, w)\right]$, and either $\hat{\alpha}(p_1, w) = 0$ if $\hat{Y}(p_1, w) = 1 - \hat{v}(p_1, w)$ or $\hat{\alpha}(p_1, w) = 1$ if $\hat{Y}(p_1, w) > 1 - \hat{v}(p_1, w)$ where $\hat{v}(p_1, w)$ is the equilibrium value of $v^{\min}$.

(3) The tuple $(F^*, p_1^*, w^*, Y^*, p_2^*, \alpha^*)$ is a REE for DSC-profit-maximizing p_1 and w iff $F^* = \Pi^{r*} = \Pi^r(Y^*, p_1^*, w^*, p_2^*, \alpha^*)$, where $(p_1^*, w^*) = \arg\max_{p_1, w} \Pi^D(p_1, w)$, $Y^* = \hat{Y}(p_1^*, w^*)$, $p_2^* = \hat{p}_2(p_1^*, w^*)$, and $\alpha^* = \hat{\alpha}(p_1^*, w^*)$.

Similar to Desai et al. (2004), we are interested in market situations when there are sales in both periods, but, for theoretical completeness, we consider all outcomes in order to provide the conditions when both-period sales are endogenously determined by market participants. The following lemma offers these conditions for given unit cost c and p_1. This result is used below for CSC with $c = 0$ and for the retailer in DSC with $c = w = (1 - m_r)p_1$.

Lemma 2. *A unique REE for given p_1 and c with the stated structure exists iff the respective conditions hold:*

REE1 (First-period sales): $\hat{v} = p_1, \hat{\alpha} = 0, \hat{Y} = 1 - p_1$, *and* $\hat{\Pi} = (p_1 - c)(1 - p_1)$ *under condition* $p_1 \leq c/\beta$.

REE2 (Second-period sales): $\hat{v} = 1, \hat{\alpha} = 1, \hat{p}_2 = \frac{1}{2}(\beta + c), \hat{Y} = \frac{1}{2}(1 - c/\beta)$, *and* $\hat{\Pi} = \frac{(\beta - c)^2}{4\beta}$ *under condition* $p_1 \geq 1 - \frac{1}{2}\rho(\beta - c)$.

REE3 (Two-period sales): *$\hat{v} = \frac{2p_1-\rho c}{2-\rho\beta}$ (increases in ρ), $\hat{\alpha} = 1$, $\hat{p}_2 = \frac{1}{2}(\beta\hat{v} + c), \hat{Y} = 1 - \frac{1}{2}(\hat{v} + c/\beta)$, and $\hat{\Pi} = -c\hat{Y} + p_1(1-\hat{v}) + \hat{p}_2(\hat{Y} - 1 + \hat{v})$ under condition $\frac{c}{\beta} < p_1 < 1 - \frac{\rho}{2}(\beta - c)$.*

This lemma implies that, for CSC with $c = 0$ and any $p_1 > 0$, REE1 does not exist, which means that the second-period sales are always attractive for a vertically integrated seller. The existence of equilibria in other cases depends on endogenization of p_1 and shown in the proposition below.

Proposition 1. *When customers do not know the inventory, there exists only REE3 for both CSC and DSC. The equilibrium values provided in Table 1 are such that $p_1^{D*} \geq p_1^{C*}, Y^{D*} \leq Y^{C*}, p_2^{D*} \geq p_2^{C*}, v^{D*} \leq v^{C*}$, and the performance of DSC is $\eta^{\Pi} \triangleq \Pi^{D*}/\Pi^{C*} = 1 + \frac{\rho^2\beta[\beta^2+(2-\rho\beta)(2-\rho\beta-2\beta)]}{(2-\rho\beta)^2[4-\beta(1+\rho)^2]} \geq 1$. All inequalities hold as equalities only at $\rho = 0$ and $\rho\beta \to 1$.*

Table 1: Decentralized and centralized SC under incomplete information

	DSC	CSC
w^*	$\frac{\rho\beta(2-\rho\beta-\beta)}{4-\beta(1+\rho)^2} \uparrow \rho$	0
m_r^*	$\frac{2-\rho\beta(4-\rho\beta-\beta)}{2(1-\rho\beta)} \downarrow \rho$	1
p_1^*	$\frac{2(1-\rho\beta)}{4-\beta(1+\rho)^2} \downarrow \rho$	$\frac{(2-\rho\beta)^2}{2[2(2-\rho\beta)-\beta]} \downarrow \rho$
Y^*	$\frac{3-\rho\beta-\beta-\rho}{4-\beta(1+\rho)^2} \downarrow \rho$	$\frac{3(2-\rho\beta)-2\beta}{2[2(2-\rho\beta)-\beta]} \downarrow \rho$
p_2^*	$\frac{\beta(1-\rho\beta)(1+\rho)}{4-\beta(1+\rho)^2} \uparrow \rho$	$\frac{\beta(2-\rho\beta)}{2[2(2-\rho\beta)-\beta]} \uparrow \rho$
v^*	$\frac{2-\rho\beta(1+\rho)}{4-\beta(1+\rho)^2}$ max in ρ for $\beta \in (0,1)$	$\frac{2-\rho\beta}{2(2-\rho\beta)-\beta} \uparrow \rho$
Π^{r*}	$\frac{4-\beta\{1+\rho[6(2-\rho\beta-\beta)-\rho+\beta^2(1+\rho^2)]\}}{[4-\beta(1+\rho)^2]^2}$	$\frac{(2-\rho\beta)^2}{4[2(2-\rho\beta)-\beta]} \downarrow \rho$
Π^{m*}	$\frac{\rho\beta(2-\rho\beta-\beta)(3-\rho\beta-\beta-\rho)}{[4-\beta(1+\rho)^2]^2}$	0
Π^*	$\frac{1-\rho\beta}{4-\beta(1+\rho)^2} \downarrow \rho$	$\frac{(2-\rho\beta)^2}{4[2(2-\rho\beta)-\beta]} \downarrow \rho$

It is known since Mathewson and Winter (1984) that RPM coordinates DSC (the profit of DSC equals the one of CSC) even with two decision variables. Proposition 1 confirms this result in a different setting, first, for myopic customers and, second, for $\rho\beta \to 1$. In both cases, decentralized and vertically integrated firms are identical, that is, RPM indeed removes double marginalization. The case $\rho\beta \to 1$ deserves a special attention because it also yields $\eta^{\Pi} \to 1$, which means that the superiority of RPM over CSC cannot be shown for durable goods only ($\beta = 1$) and strategic behavior of customers limited to $\rho = 0$ and $\rho = 1$. The convergence of the results for CSC and DSC when $\rho\beta \to 1$ follows from the elimination of the intertemporal effects since the product is durable ($\beta = 1$) and customers do not distinguish the surpluses from the first- and second-period sales.

Proposition 1 is of a particular interest because previous studies, reviewed in Su and Zhang (2009), discovered that DSC may outperform CSC only when the latter cannot achieve its highest profit due to the lack of low-inventory commitment and inexistence of the first-best equilibrium. We consider a setup where CSC does not suffer from dynamic inconsistency and attains its best profit in the equilibrium REE3 with sales in both periods. However, DSC performs even better when customer discount factor is not zero. This result contributes one more explanation to the reasons why manufacturers may prefer RPM over vertical integration when customers are forward-looking.

It is easy to show that the superiority of DSC arises from double marginalization, which manufacturer "turns on" to mitigate strategic delays when customers are not myopic. Indeed, according to Proposition 1, DSC-prices p_1^* and p_2^* are higher and inventory is smaller than for CSC, which is a known effect of double marginalization. The higher ρ, the more customers delay their purchases under CSC (v^* increases in ρ), which is a known effect of strategic behavior. Meanwhile, under DSC, the number of waiting customers is always less than for CSC ($v^{D*} < v^{C*}$ for $\rho \in (0, 1)$) and may even decrease in ρ for $\beta < 1$. This occurs because customers with higher ρ pay more attention on the expected second-period price, which is higher under DSC. As a result, DSC enjoys more first-period sales at a higher price than CSC. As Proposition 1 shows, this comparative increase in the first-period profit exceeds a relative loss in the second-period sales.

A value of decentralization can be estimated by the relative difference $\left[\left(\Pi^{D*} - \Pi^{C*}\right)/\Pi^{C*}\right]|_{\rho\to 1} = \frac{\beta(1-\beta)}{(2-\beta)^2}$. A unique maximizer $\beta^{\max} = 2/3$ leads to $\max_\beta \left[\left(\Pi^{D*} - \Pi^{C*}\right)/\Pi^{C*}\right]|_{\rho\to 1} = 1/8$, that is, *when customers are strategic and do not know the inventory, RPM can improve supply chain profit up to 12.5%* (Figure 1 (a)). For comparison, the loss of CSC profit due to strategic customers at $\beta^{\max}$ is $\left(\Pi^{C*}|_{\rho=0} - \Pi^{C*}|_{\rho\to 1}\right)/\Pi^{C*}|_{\rho=0} = 7/27$ or 26%, which is in the middle of the range reported in studies reviewed in Gönsch et al. (2013). Not surprisingly, the loss from strategic customers at $\beta^{\max}$ is less for DSC: $\left(\Pi^{D*}|_{\rho=0} - \Pi^{D*}|_{\rho\to 1}\right)/\Pi^{D*}|_{\rho=0} = 1/6$ or 16.6%.

The main motivation for studying RPM is usually a welfare effect of this policy. We showed that RPM is attractive for the manufacturer compared to a vertically integrated firm when customers are strategic and do not know the inventory. The next subsection provides the comparison of aggregate welfare.

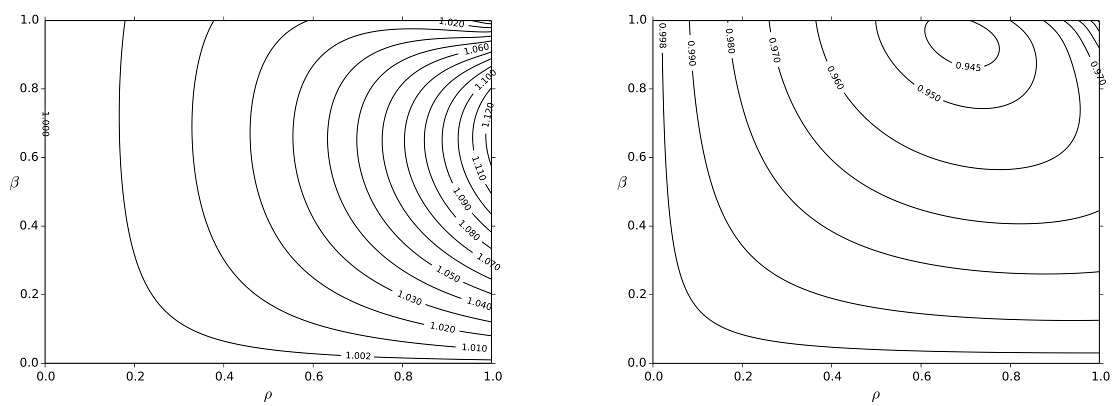
3.3 Welfare effect of RPM vs. CSC

The aggregate welfare W is a sum of a SC profit and the total customer surplus. In a two-period model, the *total (realized) customer surplus* is $\Sigma \triangleq \Sigma_1 + \Sigma_2$, where Σ_1 and Σ_2 are the total surpluses of buyers in the first and second periods. Recall that Σ_2 is not discounted by ρ because ρ is a subjective behavioral parameter and such a discount would not reflect the actual surplus. In the extreme case of $\rho = 0$, such discounting would completely disregard the second-period surplus of myopic customers. The expressions for Σ_1 and Σ_2 are given by the following:

Lemma 3. $\Sigma_1 = (1 - v^{\min})\left[\frac{1+v^{\min}}{2} - p_1\right]$ *and* $\Sigma_2 = \frac{(\beta v^{\min} - p_2)^2}{2\beta}$.

Figure 1: RPM performance with respect to vertically integrated firm

(a) Profit performance η^{Π} (b) Welfare performance η^{W}



Substitution of the corresponding equilibrium values from Table 1 leads to Σ^{D*} for DSC and Σ^{C*} for CSC. Then $W^{D*} = \Sigma^{D*} + \Pi^{D*}, W^{C*} = \Sigma^{C*} + \Pi^{C*}$, and the equilibrium welfare performance of RPM is $\eta^W \triangleq W^{D*}/W^{C*}$. The plot of η^W in Figure 1 (b) shows that this measure, unlike the profit performance η^{Π} in Figure 1 (a), does not exceed one, which means that *RPM is not welfare-improving* compared to vertically integrated firm with the maximum loss around 6%. The definition of W and Proposition 1 imply that RPM is also *not surplus-improving.* This observation is intuitive because, by Proposition 1, RPM leads to higher prices in both periods and smaller total inventory than CSC. Similar to η^{Π} and by the same reasons, $\eta^W = 1$ only when customers are myopic ($\rho = 0$) or fully strategic and the product is durable ($\rho\beta \to 1$).

It is known that in some cases RPM improves welfare, for example, when it is used to protect the retailers providing demand-enhancing services against free-riders or to support the appeal of branded products, see the review in Orbach (2008). However, when the only goal of RPM is to mitigate strategic customer behavior, the welfare may decrease in comparison with a centralized firm.

In order to understand the nature of this decrease, it is illustrative to consider the effect of an increase in strategic behavior on customer surpluses. We start from the case without RPM (p_1 and c are fixed). Consider $0 \leq \rho' < \rho'' < 1$. By part RESE3 of Lemma 2, $\hat{v}|_{\rho=\rho'} < \hat{v}|_{\rho=\rho''}$ because a higher ρ means that customers pay more attention on the second-period surplus in their buy-or-wait decision, and more customers delay the purchase, which is the essence of forward-looking behavior. To compare the outcomes at ρ' and ρ'', we split the customer population with $v \in [0, 1]$ as follows:

(a) customers with $v \in [\hat{v}|_{\rho=\rho''}, 1]$ buy in the first period at both ρ' and ρ'', and their realized surplus does not change;

(b) customers with $v \in [\hat{v}|_{\rho=\rho'}, \hat{v}|_{\rho=\rho''})$ buy at ρ' and wait at ρ'', which, by Assumption 3, means that $v - p_1 < \rho(\beta v - \bar{p}_2)$ ($\bar{\alpha} = 1$ in RESE3) implying, by rationality of $\hat{p}_2$, that $v - p_1 < \beta v - \hat{p}_2|_{\rho=\rho''}$, that is, the *realized surplus* $\beta v - \hat{p}_2|_{\rho=\rho''}$ *is greater* than the one at $\rho = \rho'$ *due to the increase in their own strategicity* and despite the increase in $\hat{p}_2$ (by Lemma 2, $\hat{p}_2$ increases in $\hat{v}$);

(c) customers with $v \in \left[\frac{\hat{p}_2|_{\rho=\rho''}}{\beta}, \hat{v}|_{\rho=\rho'}\right)$ buy in the second period at both ρ' and ρ'', and their *realized surplus* at ρ'' *is less* than at $\rho = \rho'$ *because of the retailer equilibrium reaction to the buying behavior of the customers from group (b)* ($\hat{p}_2|_{\rho=\rho''} > \hat{p}_2|_{\rho=\rho'}$);

(d) customers with $v \in \left[\frac{\hat{p}_2|_{\rho=\rho'}}{\beta}, \frac{\hat{p}_2|_{\rho=\rho''}}{\beta}\right)$ buy in the second period at ρ' and do not buy at all at ρ'' with the *surplus decreased to zero* by the same reason as in (c);

(e) customers with $v \in \left[0, \frac{\hat{p}_2|_{\rho=\rho'}}{\beta}\right)$ do not buy at both ρ' and ρ''.

One can see that even with sticky first-period price and retailer cost (no manufacturer decisions), not all customers are better off from strategic behavior. Only group (b) benefits from being more strategic. The surpluses of groups (c) and (d) are less for higher ρ because the retailer, responding to the behavior of group (b), increases the second-period price, which reduces the size of group (b).

Aflaki et al. (2016) provide a detailed analysis of changes in surpluses in groups (a)-(d) for CSC selling a durable good ($\beta = 1$) in a similar setup. The difference from the case with sticky p_1 is that group (a) benefits from strategic behavior of group (b) because the first-period price decreases in ρ, reducing the size of group (b). Proposition 1 confirms this observation for any $\beta > 0$.

Under RPM, a surplus redistribution among groups (a)-(d) leads to a smaller total customer surplus than under CSC because RPM is more efficiently mitigates this behavior: $v^{D*} \leq v^{C*}$, and the manufacturer transfers more customer surplus to DSC profit ($\Pi^{D*} \geq \Pi^{C*}$). However, the fundamental problem of losses from strategic customer behavior is still present under RPM because, by Proposition 1, DSC profit decreases in ρ.

The next subsection provides alternative evidence that the comparative welfare decrease under RPM results from customer behavior rather than from RPM itself. This subsection shows that a vertically integrated firm can reach the performance of RPM (with the same welfare-decreasing effect) by revealing the inventory information to strategic customers. Then, if strategic customer behavior is not considered as a primary cause of decrease in welfare, the legal status of opening information about inventories to customers should be questioned by the same reason as RPM because it decreases inventory and increases prices.

3.4 Customers know inventory level

In this section, Assumption 2 is replaced by

Assumption 4. *Customers know total product supply, the seller's cost c, second-period product availability $\alpha \in \{0, 1\}$ and price p_2.*

Customer awareness changes the form of the second-period surplus in Assumption 3. Now, $\sigma_2 \triangleq \alpha\rho(\beta v - p_2) = \alpha\rho\beta(v - 1 + Y)$, which, similarly to Lemma 1, implies that the customer valuation threshold is $v^{\min} = \max\left\{p_1,\ \min\left\{\frac{p_1 - \alpha\rho\beta(1-Y)}{1-\alpha\rho\beta},\ 1\right\}\right\}$. The result below, similarly to Lemma 2, provides the structure of market outcomes under complete information. These outcomes are distinct from REE and we call them Complete Information Equilibria (CIE).

Lemma 4. *A unique CIE with the stated structure exists iff the respective conditions hold:*

CIE1 (First-period sales): $\hat{v} = p_1, \hat{\alpha} = 0, \hat{Y} = 1 - p_1,$ *and* $\hat{\Pi} = (p_1 - c)(1 - p_1)$ *under condition* $p_1 \leq \frac{c(1-\rho\beta)}{\beta(1-\rho)}$.

CIE2 (Second-period sales): $\hat{v} = 1, \hat{\alpha} = 1, \hat{p}_2 = \frac{1}{2}(\beta + c), \hat{Y} = \frac{1}{2}(1 - c/\beta)$, *and* $\hat{\Pi} = \frac{(\beta-c)^2}{4\beta}$ *under condition* $p_1 \geq P_2$, *where* $P_2 = P_2(\rho, \beta, c)$ *is provided in the proof.*

CIE3 (Two-period sales): $\hat{v} = p_1\frac{2-\rho\beta-\rho^2\beta}{2(1-\rho\beta)} - c\frac{\rho}{2}, \hat{\alpha} = 1,\ \hat{p}_2 = \frac{1}{2}[\beta p_1(1+\rho) + c(1-\rho\beta)], \hat{Y} = 1 - \frac{1}{2}[p_1(1+\rho) + c(1/\beta - \rho)]$, *and* $\hat{\Pi} = -c\hat{Y} + p_1(1-\hat{v}) + \hat{p}_2(\hat{Y} - 1 + \hat{v})$ *under condition* $\frac{c(1-\rho\beta)}{\beta(1-\rho)} < p_1 < P_2$.

This lemma shows that customer knowledge of Y changes p_1-boundaries and the form of Y-response of the seller to p_1 and c under CIE3 compared to Lemma 2. In particular, the definition of the boundary P_2 between CIE2 and CIE3 is more complicated. As shown in the proof, this boundary is not less than the boundary between REE2 and REE3. However, Proposition 2 below shows that only the equilibrium with two-period sales (CIE3) exists for both CSC and DSC.

Proposition 2. *When customers know the inventory, there exists only CIE3 for both CSC and DSC. The equilibrium values $p_1^*, Y^*, p_2^*, v^*,$ and Π^* of CSC coincide with the correspondent values of DSC under incomplete info provided in Table 1. Under DSC, the manufacturer sets $m_r^* \equiv 1$ (or $w^* \equiv 0$) leading to the same result as CSC.*

Proposition 2 confirms that SC can use customer knowledge of inventory as an efficient tool for mitigating strategic behavior even when customers are risk neutral. This tool is at least as efficient as double marginalization since the manufacturer in DSC endogenously chooses the contract equivalent to a vertically integrated firm by setting the retailer margin to one and, respectively, the wholesale price to zero, which effectively "turns off" double marginalization.

At the same time, Propositions 1 and 2 imply that overestimation of customer reaction on information about inventory may lead to overcentralization of SC. A profit loss in this case can be estimated by *the value of inventory disclosure*, which, similarly to the value of decentralization, can be evaluated at $\rho \to 1$ because myopic customers do not use this information. Indeed, for $\rho = 0$, the profits of CSC under complete and incomplete information coincide: $\Pi^{CC*}|_{\rho=0} = \Pi^{CI*}|_{\rho=0} = \frac{1}{4-\beta}$. Since the profit of CSC under complete info coincides with the one of DSC under incomplete info, the value of disclosure for CSC equals the value of decentralization: $\max_\beta \left[\left(\Pi^{CC*} - \Pi^{CI*}\right)/\Pi^{CI*}\right]|_{\rho\to 1} = \max_\beta \left[\left(\Pi^{DI*} - \Pi^{CI*}\right)/\Pi^{CI*}\right]|_{\rho\to 1} = 1/8$, that is, *customer reaction on information about inventory can change the profit of CSC up to 12.5%.*

This result is consistent with Li and Yu (2016) who show in a setup similar to Su and Zhang (2008) that DSC profit is not higher than the one of CSC when customers take into account the inventory level. These findings complement Yin et al. (2009) who discovered that "display one" format (unknown total current inventory) can benefit a seller by increasing a sense of scarcity when customers know the demand and compete with each other.

The comparison of cases with incomplete and complete info sheds more light on why RPM may have higher profit than CSC. Suppose that CSC under incomplete info, for a given first period price, sets a lower inventory, for example, equal to Y^* for DSC in Table 1. This decision would be irrational and lead to a lower profit because customers do not observe changes in inventory but know that a vertically integrated firm is a low-cost seller and rationally expect low second-period price. Therefore, the first-period sales would remain the same whereas the second period sales would decrease due to a higher price. The situation changes when customers know and do not ignore the information about inventory. In this case, CSC does not need a commitment device and reaches the same profit as DSC with RPM for any level of strategic customer behavior.

It is known that the total supply to the market typically increases in the number of competing retailers. Then, intuitively, an inventory-reducing tool for mitigating strategic customer behavior may become less efficient with the growing level of competition. For example, Liu and van Ryzin (2008) showed that retailers may not use capacity rationing starting from rather small numbers of sellers. The next section studies the effects of the level of retailer competition on the performance of RPM under incomplete info.

4 RPM with Oligopolistic retailers

Similarly to §3.1 and under the assumptions of §3.2 (customers do not know inventory), manufacturer offers the same contract with RPM to an arbitrary number of identical retailers. The assumption of retailer symmetry is common for studying the effects of the *level* of competition, when retailers do not differ in their cost structure or brand value, see, for example, Liu and van Ryzin (2008).

When retailers procure more inventory than for the first-period sales only, they engage in clearance sales in the second period. As the product offerings are undifferentiated, the retailers lower their prices until all remaining inventory is cleared, that is, the second-period price p_2 (identical for all retailers) is sufficiently low for the total clearance demand to equal the total remaining inventory. Alternatively, Liu and van Ryzin (2008), §4.4, assume that the same second-period price is exogenously fixed for all retailers. Each retailer maximizes its profit by selecting the initial inventory level. The resulting game among the retailers is similar to the classical Cournot-Nash model, but with a distinct two-period structure. There are studies confirming that Cournot assumption, leading to the same price among retailers, is not implausible in cases of non-price competition, see, for example, Karnani (1984), and Perakis and Sun (2014). One of the arguments is that retailers choose their inventory-based decisions independently, whereas price cuts are easily observable and can be matched almost instantaneously. Flath (2012) shows that the markets of music records, bicycles, and thermos bottles are appropriately described by the Cournot model. For example, the Japanese market of music records, besides plausibility of the Cournot model, is characterized by legal use of RPM system (saihan seido) and strategic customer behavior (Nippop (2005)).

We now describe the market dynamics. Let retailers be indexed by set I of size $n = |I|$, and retailer $i \in I$ inventory and sales in the first period be y^i and q^i. As the second-period market is cleared, each retailer's second-period supply and sales are equal to $y^i - q^i$. Then the total product supply and first-period sales are $Y = \sum_{i \in I} y^i$ and $Q = \sum_{i \in I} q^i$ respectively. The total second-period supply is $Y - Q$, the retailer i profit is

$$\Pi^i = -wy^i + p_1 q^i + p_2(y^i - q^i), \tag{6}$$

and the profit of DSC is $\Pi^D = \Pi^m + \Pi^r$, where $\Pi^r = \sum_{i \in I} \Pi^i$ and Π^m is given by (2). First-period sales q^i are determined based on a customer decision model.

4.1 Customer decision model

The customer decision model includes two aspects: demand allocation between two periods and among the retailers. The first aspect remains the same as in §3.2, that is, customers decide to buy or wait using their expectations of the second-period product availability $\bar{\alpha}$ and price $\bar{p}_2$. In particular, by Lemma 1, the first-period demand is $D = 1 - v^{\min}$. For allocation of the demand among the retailers we use two cases of a well-known attraction model with inventory-dependent demand.

Studies such as Yin et al. (2009) reasonably assume that if all inventory is displayed to the customers, the customers know the total amount of this inventory. In some markets, however, this assumption may exaggerate customer rationality. For example, buyers of goods such as apparel or music records attracted by displayed inventory usually do not count all available units in all outlets in order to make a purchase. Therefore, an outlet with a higher inventory attracts more customers, but customers do not use the information about total inventory and rely on their expectations while deciding to buy or wait. In this sense, we consider a complimentary case to "display all" format in

Yin et al. (2009) by assuming that customers can observe the inventory but do not know its total level, and demand is allocated according to the resulting vector of inventory-driven attractions of all retailers.[3]

According to studies reviewed in Urban (2005), a typical form of attraction associated with inventory y is y^γ where $\gamma \in [0,1]$ is the inventory elasticity of attraction. Then the attraction model for the first-period demand d^i of retailer i is

$$d^i(D, y^i, \mathbf{y}^{-i}) \triangleq D \frac{(y^i)^\gamma}{\sum_{j \in I} (y^j)^\gamma}, i \in I, \tag{7}$$

where $\mathbf{y}^{-i} \triangleq (y^1, \ldots, y^{i-1}, y^{i+1}, \ldots, y^n)$ is the vector of inventories of other retailers. Function (7) is a symmetric form of the *general attraction model*. This form is widely used both in theoretical and empirical research, see, for example, Karnani (1984) and Gallego et al. (2006). An empirical study of Naert and Weverbergh (1981) concludes that the attraction model is "more than just a theoretically interesting specification." This model "may have a significantly better prediction power than the more classic market share specifications." This conclusion is supported by later research, see, for example, Klapper and Herwartz (2000).

The case $\gamma = 0$ means that a retailer's attraction does not depend on y^i, and $d^i \equiv \frac{D}{n}$ for any $y^i > 0$ and $i \in I$.[4] Liu and van Ryzin (2008), in §4.4, use this case to study the effect of rationing on strategic behavior of risk-averse customers. Cachon (2003), in §6.5, considers a newsvendor competition model where retail demand is "divided between the n firms proportional to their stocking quantity," which matches the case of $\gamma = 1$ in (7). This case can be viewed as a fluid limit of the following simple randomized allocation model. Suppose all retailers pool their (discrete) inventory into an urn (one may think of different retailers' inventory being identified by different colors). Each customer randomly picks an item from the urn (without replacement), and the retailer to whom the item belongs is credited for the sale. In such allocation model, the case of intermediate $0 < \gamma < 1$ corresponds to pooling of attractions rather than inventories.

Model (7) allows for tractable analysis when $\gamma = 0$ or $\gamma = 1$. In both cases, similarly to §3.2, deterministic demand and profit-maximizing retailers immediately imply that there are no stockouts in the first period. Therefore, the total first-period sales are $Q = D = 1 - v^{\min}$, the individual first-period sales are $q^i = d^i(D, y^i, \mathbf{y}^{-i})$, and the resulting second-period inventories are

$y^i - q^i, i \in I$. Since $D = D(\bar{p}_2, \bar{\alpha})$, implying $d^i = d^i(\bar{p}_2, \bar{\alpha}, y^i, \mathbf{y}^{-i})$, the retailer i profit is

$$\Pi^i = \Pi^i(y^i, \mathbf{y}^{-i}, p_1, w, \bar{p}_2, \bar{\alpha}) = -wy^i + p_1 d^i(\bar{p}_2, \bar{\alpha}, y^i, \mathbf{y}^{-i}) + p_2(Y)\left[y^i - d^i(\bar{p}_2, \bar{\alpha}, y^i, \mathbf{y}^{-i})\right]. \quad (8)$$

Using the same notion of rationality as in §3.2, we extend the definition of REE to n symmetric retailers and define the rational expectations symmetric Cournot-Nash equilibrium (RESE) in pure strategies for DSC as follows:

1. Given p_1 and w from the manufacturer, customer expectations $\bar{\alpha}$ and $\bar{p}_2$, and $\mathbf{y}^{-i}$, let the best response of retailer i be $BR^i(\mathbf{y}^{-i}, p_1, w, \bar{p}_2, \bar{\alpha}) = \arg\max_{y^i} \Pi^i(y^i, \mathbf{y}^{-i}, p_1, w, \bar{p}_2, \bar{\alpha})$.

2. For given $\bar{\alpha}$ and $\bar{p}_2$, let $\tilde{y} = \tilde{y}(p_1, w, \bar{p}_2, \bar{\alpha})$ denote a symmetric Cournot-Nash equilibrium inventory level in the retailer game, that is, $\tilde{y}(p_1, w, \bar{p}_2, \bar{\alpha}) = BR^i\left[(\tilde{y}, \dots, \tilde{y}), p_1, w, \bar{p}_2, \bar{\alpha}\right]$, where $(\tilde{y}, \dots, \tilde{y}) \in \mathbb{R}_+^{n-1}$, and $\tilde{Y}(p_1, w, \bar{p}_2, \bar{\alpha}) = n\tilde{y}(p_1, w, \bar{p}_2, \bar{\alpha})$ be the corresponding total inventory.

3. The tuple $\left[\hat{Y}(p_1, w), \hat{p}_2(p_1, w), \hat{\alpha}(p_1, w)\right]$ is a RESE for given (p_1, w) iff $\hat{Y}(p_1, w) = \tilde{Y}(p_1, w, \hat{p}_2, \hat{\alpha})$, $\hat{p}_2(p_1, w) = \beta\left[1 - \hat{Y}(p_1, w)\right]$, and either $\hat{\alpha}(p_1, w) = 0$ if $\hat{Y}(p_1, w) = 1 - \hat{v}(p_1, w)$ or $\hat{\alpha}(p_1, w) = 1$ if $\hat{Y}(p_1, w) > 1 - \hat{v}(p_1, w)$ where $\hat{v}(p_1, w)$ is the equilibrium value of $v^{\min}$.

4. The tuple $(F^*, p_1^*, w^*, Y^*, p_2^*, \alpha^*)$ is a RESE for Π^D-maximizing (p_1, w) iff $F^* = \sum_{i \in I} F^{i*}, F^{i*} = \Pi^{i*} = \Pi^i(y^{i*}, \mathbf{y}^{-i*}, p_1^*, w^*, p_2^*, \alpha^*)$ for all $i \in I$, where $(p_1^*, w^*) = \arg\max_{p_1, w} \Pi^D(p_1, w)$, $y^{i*} = \frac{1}{n}\hat{Y}(p_1^*, w^*), p_2^* = \hat{p}_2(p_1^*, w^*)$, and $\alpha^* = \hat{\alpha}(p_1^*, w^*)$.

The cases $\gamma = 0$ and $\gamma = 1$ of model (7) are studied below in §4.2 and §4.3 respectively.

4.2 Inventory-independent demand

Following §4.4 of Liu and van Ryzin (2008), this subsection assumes that the first-period demand is equally distributed among the retailers, which is a particular case of (7) with $\gamma = 0$. Using (8), retailer i profit with the unit cost $c = w$ and $p_2 = \beta(1 - Y)$ is

$$\Pi^i = -wy^i + p_1 \frac{1 - v^{\min}}{n} + \beta(1 - Y)\left(y^i - \frac{1 - v^{\min}}{n}\right). \quad (9)$$

The lemma below extends the result of Lemma 2 for the case of n symmetric retailers with inventory-independent demand and $c = w$.

Lemma 5. *For demand (7) with $\gamma = 0$, a unique RESE with the stated structure exists iff the respective conditions hold:*

RESE1 (First-period sales): *$\hat{v} = p_1, \hat{\alpha} = 0, \hat{Y} = 1 - p_1$, and $\hat{\Pi}^r = (p_1 - c)(1 - p_1)$ under condition $p_1 \leq c/\beta$.*

RESE2 (Second-period sales): *$\hat{v} = 1, \hat{\alpha} = 1, \hat{p}_2 = \frac{\beta+cn}{n+1}, \hat{Y} = \frac{n}{n+1}(1 - c/\beta)$, and $\hat{\Pi}^r = \frac{n(\beta-c)^2}{(n+1)^2\beta}$ under condition $p_1 \geq 1 - \frac{n}{n+1}\rho(\beta - c) \triangleq P_2$.*

RESE3 (Two-period sales): *$\hat{v} = \frac{(n+1)p_1 - n\rho c}{1+n(1-\rho\beta)}, \hat{\alpha} = 1, \hat{p}_2 = \frac{\beta p_1 + cn(1-\rho\beta)}{1+n(1-\rho\beta)}, \hat{Y} = 1 - \frac{p_1 + cn(1-\rho\beta)/\beta}{1+n(1-\rho\beta)}$, which increases in n, and $\hat{\Pi}^r = -c\hat{Y} + p_1(1 - \hat{v}) + \hat{p}_2(\hat{Y} - 1 + \hat{v})$ under condition $\frac{c}{\beta} < p_1 < P_2$.*

Lemma 5 confirms, in particular, that under RESE3, for given p_1 and $w = c$, the total supply to the market $\hat{Y}$, indeed, increases in the number of retailers n, which may challenge the efficacy of manufacturer's RPM-policy as an inventory-reducing tool in response to strategic customer delays. The following proposition provides the equilibrium reaction of the manufacturer to the level of competition n. The proposition analyses only DSC since CSC is the same as in §3.2.

Proposition 3. *For DSC with n retailers and demand (7) with $\gamma = 0$, there exists only RESE3. The manufacturer sets $w^* = \frac{\beta\{n-1+\rho[1+n(1-\rho\beta-\beta)]\}}{n\left[4-\beta(1+\rho)^2\right]} \in \left[0, \frac{1}{2}\right)$, which increases in n and in ρ, and leads to the equilibrium values Y^*, p_1^*, p_2^*, v^*, and Π^{D*} that do not depend on n and coincide with the correspondent values for $n = 1$ provided in Table 1.*

Proposition 3 shows that RPM overpowers the force of competition when demand is inventory-independent. The manufacturer, as in Proposition 1, uses the wholesale price, which is absent in CSC, in order to adjust total supply to the market and achieve the highest possible profit. Indeed, as can be seen from Lemma 5, the total inventory $\hat{Y}$ decreases in w; therefore, the manufacturer sets a higher w^* for a higher number of retailers and enjoys the same profit as for SC with monopolistic retailer regardless of the level of competition. Comparing this result with the findings of §4.4 in Liu and van Ryzin (2008), we can conclude that RPM is a more effective inventory-reducing tool under competition than retailer capacity-rationing.

The following subsection examines RPM in another extreme case $\gamma = 1$ of demand allocation model (7). This case is of a particular interest because, in addition to inventory-increasing force of competition considered for $\gamma = 0$, retailers have one more incentive to increase inventory since their market shares directly depend on displayed inventories.

4.3 Inventory-dependent demand

Following §6.5 in Cachon (2003), this subsection assumes that the first-period demand is distributed among the retailers proportionally to their inventory levels, which is a particular case of (7) with $\gamma = 1$. Then, by (8), retailer i profit is $\Pi^i = -cy^i + p_1 y^i \frac{1-v^{\min}}{Y} + p_2(Y) y^i \left[1 - \frac{1-v^{\min}}{Y}\right]$.

An important difference of case $\gamma = 1$ from the above is that retailers' market share competition may drive the second-period price below cost (Corollary 1 below), which is usually a peculiarity of models with random demand. Since the second-period price results from market clearing, sales at loss obviously indicate an increase in total product supply compared to the cases above.

While clearance sales train strategic customers, sales at loss foster *bargain hunters* whose valuations are below the retailer unit cost. Some studies, for example Cachon and Swinney (2009) and Su and Zhang (2008), assume that, in the second period, there is a market of bargain hunters who can buy any remaining product at a unit salvage value $s < c$. Unlike these studies, we assume that the participants of the second-period market endogenously choose between clearance and "salvaging" sales. We need this endogeneity to determine manufacturer's reaction to this choice. Besides "bargain hunters" interpretation, salvage value allows for availability of alternative sales channels for retailers such as `liquidations.walmart.com`, `www.shoplc.com`, and `www.salvagesale.com`. As a result, p_2 never goes below s, and Eq. (5) becomes

$$p_2 = \max\{s,\ \beta(1-Y)\}. \tag{10}$$

An opportunity to sell large quantities at a fixed (not decreasing in Y like in clearance sales) price may serve as an additional incentive for retailers to oversupply the market and additionally challenge RPM policy as an inventory-reducing tool.[5]

Given the above, Proposition 4 below extends the result of Lemma 2 on retailer's reaction to given p_1 and c for n symmetric retailers under inventory-dependent demand[6] with $\gamma = 1$.

Proposition 4. *For demand (7) with $\gamma = 1$, a unique RESE with the stated structure exists iff the respective conditions hold:*

RESE1 (First-period sales): $\hat{v} = p_1, \hat{\alpha} = 0, \hat{Y} = 1 - p_1$, *and* $\hat{\Pi}^r = (p_1 - c)(1 - p_1)$ *under condition* $p_1 \le \frac{nc}{n-1+\beta} \triangleq P_1$.

RESE2 (Second-period sales): $\hat{v} = 1, \hat{\alpha} = 1, \hat{p}_2 = c + \frac{\beta - c}{n+1}, \hat{Y} = \frac{n}{n+1}(1 - c/\beta)$, *and* $\hat{\Pi}^r = \frac{n(\beta-c)^2}{(n+1)^2\beta}$ *under condition* $p_1 \ge 1 - \frac{n}{n+1}\rho(\beta - c) \triangleq P_2$.

RESE3 (Two-period sales, $\hat{p}_2 > s$): *$\hat{v} = \frac{p_1 - \rho\beta(1-\hat{Y})}{1-\rho\beta}$, $\hat{\alpha} = 1$, $\hat{p}_2 = \beta(1-\hat{Y})$, where $\hat{Y}$ is the larger root of equation (29) in Appendix, and $\hat{\Pi}^r = -c\hat{Y} + p_1(1-\hat{v}) + \hat{p}_2(\hat{Y} - 1 + \hat{v})$, under condition $P_1 < p_1 < P_2$ and, for $n \geq 2$, one of the following: (a) $\frac{n-1}{n}(p_1 - s)(1-\hat{v})\hat{Y} \leq (c-s)(1-s/\beta)^2$, or (b) condition (a) does not hold, $\hat{Y} < 1 - \frac{s}{\beta}$, and $\frac{1}{n}\hat{\Pi}^r \geq \check{\Pi}^i \triangleq \left(\check{Y} - \frac{n-1}{n}\hat{Y}\right)\left[-(c-s) + (p_1 - s)(1-\hat{v})/\check{Y}\right]$, where $\check{\Pi}^i$ is the maximum profit of a firm deviating from RESE3 in such a way that $p_2 = s$ (total inventory exceeds $1 - s/\beta$), $\check{Y} = \min\left\{\hat{v} - \frac{s}{\beta} + B, \sqrt{\frac{n-1}{n}\frac{\hat{Y}(p_1-s)(1-\hat{v})}{c-s}}\right\}$, and B is the number of bargain hunters.*

The equilibrium characteristics $\hat{Y}$, $\hat{v}$, and $\hat{\Pi}^r$ are continuous on the boundaries between these forms of RESE. Moreover, in RESE3, $\hat{Y} \geq \frac{n}{n+1}(1 - c/\beta)$.

In this proposition, the p_1-bounds P_1 and P_2 separate RESE3 from RESE1 and 2 respectively. Similarly to the case $\gamma = 0$, the input area of RESE1 shrinks in β, disappearing for $\beta = 1$, because of increasing profitability of the second-period market when retailers can gain from two-period price discrimination. Unlike $\gamma = 0$, this area shrinks also in n, disappearing for $n \to \infty$, due to increasing quantity competition for the market share, which may force retailers to procure more inventory than just for the first period. Another important difference from $\gamma = 0$ is the additional conditions (a) and (b) of RESE3 existence for $n \geq 2$, which result from the presence of bargain hunters. These conditions are discussed after Proposition 5 below.

The following corollary shows that competition with $\gamma = 1$ may lead to the second-period price below cost, which contrasts the case $\gamma = 0$ where the second-period sales are always profitable. We demonstrate this effect in a market for a durable good with myopic customers and some $n > 2$. The second-period price in this case remains above the unit cost in a duopoly.

Corollary 1. *For $\beta = 1$, $\rho = 0$, and $c < p_1 < 1$, RESE1 and RESE2 cannot be realized and, in RESE3, the second-period price is below cost iff $n > 2 + \frac{p_1 - c}{1 - p_1}$.*

Since a lower price corresponds to a higher inventory, the case $\gamma = 1$ provides an additional challenge to RPM as an inventory-reducing tool for mitigating strategic delays. Assuming that $c = w$, the following proposition answers the question: does there exist a feasible wholesale price w that, similarly to the case $\gamma = 0$, leads to a one-retailer profit for DSC under oligopoly with inventory-dependent demand?[7]

Proposition 5. *For DSC with n retailers and demand (7) with $\gamma = 1$, the wholesale price $w^* = \frac{1}{n\left[4-\beta(1+\rho)^2\right]}\left(\rho\beta(2-\rho\beta-\beta) + \frac{(n-1)(1-\rho\beta)(4-\beta-2\rho\beta-\rho^2\beta)}{3-\rho\beta-\beta-\rho}\right) \in \left[0, \frac{1}{2}\right]$ leads to the equilibrium values Y^*, p_1^*, p_2^*, v^*, and Π^{D*} that do not depend on n and coincide with the correspondent values for $n = 1$ provided in Table 1. This w^* increases in n, increases in ρ except for $w^*|_{n\to\infty} \equiv \frac{1}{2}$ at $\beta = 1$, and $w^*|_{\gamma=1} > w^*|_{\gamma=0}$ for any $n \geq 2$. For these equilibrium values, the condition $P_1 < p_1 < P_2$ of RESE3 existence always holds and condition (a) always holds for $s = 0$.*

This proposition confirms that even under oligopoly with an arbitrary number of retailers and inventory-dependent demand, there may exist a contract with RPM leading to the same profit of DSC as with a single retailer, which, by Proposition 1, is more profitable than CSC. It is easy to check that, for $n = 1$, the expression for w^* in Proposition 5 coincides with w^* in Table 1. Inequality $w^*|_{\gamma=1} > w^*|_{\gamma=0}$ explains that the manufacturer overcomes the challenge of inventory-dependent demand ($\gamma = 1$) by setting a higher wholesale price than in the case $\gamma = 0$ (Proposition 3).

Proposition 5 does not guarantee RESE3 existence for $n \geq 2$ in general because conditions (a) and (b) in Proposition 4 depend on salvage value s, which is not controlled by the manufacturer. These conditions hold only if s is sufficiently lower than retailer's unit cost w^*. Condition (a) means that the profit of a potential deviator from RESE3 to salvage-value sales monotonically decreases in inventory, while condition (b) means that the deviator's profit has a local maximum, which does not exceed the profit under RESE3. According to Proposition 5, RESE3 always exists when the salvage value equals manufacturer's cost ($s = 0$). As was discussed in Su and Zhang (2008), salvage value can be relatively high for mass markets with ample salvage opportunities whereas for niche markets these opportunities are rare leading to small values of s.

If there is a liquidation channel with $s > 0$, RESE3 may not exist because the retailers may prefer this channel to market clearing. This can happen when customers' strategicity is low because, by Proposition 5, w^* is minimized at $\rho = 0$, and so the salvage sales are most attractive for the retailers. A formal analysis in the appendix shows that DSC profit in this case may even exceed Π^{D*} given in Table 1 if the number of bargain hunters B and s are relatively high. However, the equilibrium with $p_2 = s$ may exist for small s and B at a high level of competition leading to DSC profit less than Π^{D*} given in Table 1.[8] The case of small DSC profit in the "salvaging" outcome marginally benefits the retailers because the manufacturer reduces fixed fee under the condition in the contract that retailers ignore salvaging opportunities and stick to RESE3. The resulting profit of DSC will still exceed the one of CSC.

5 Conclusions

A number of theories explain why manufacturers may use resale price maintenance (RPM) in a supply chain (SC) contract. It is known, in particular, that even simplest forms of RPM may coordinate SC, that is, RPM does not suffer from double marginalization and lead to the same profit as a centralized SC (CSC). However, these theories do not consider a pervasive phenomenon of forward-looking customers, who hurt sellers' profits and reallocate customer surplus. This paper contributes to the RPM literature by showing that SC profit under RPM is higher than the one of CSC when customers are strategic and do not know or ignore the inventory level.

This study extends also the line of research initiated by Spengler (1950) who argued that antitrust law should differ vertical integration from the horizontal one because double marginalization in decentralized SC (DSC) hurts both the seller and the aggregate welfare. Subsequent work provides the forms of double-marginalization-free contracts that lead to the same profits of DSC and CSC, see a review in Cachon (2003); and recent studies, reviewed in Su and Zhang (2009), find that when customers are strategic, the profit of DSC may even exceed the one of CSC when CSC cannot credibly commit to low inventory. Our paper complements these recent findings by including RPM into the list of such contracts. Another new insight is that the seller does not need to suffer from the Coase problem in its extreme form in order to benefit from double marginalization. In particular, secondary market can be neglected and intertemporal price discrimination can be more profitable than one-period sales despite customer strategic delays. In this case, double marginalization benefits the seller as a commitment device to a higher prices than under CSC in both periods, which mitigates strategic delays and reduces profit loss.

One more qualitative impact of this paper is that the efficacy of double marginalization as a low-inventory commitment device can be robust with respect to the number of competing retailers. This conclusion holds for RPM under two types of competition: with inventory-independent demand, when the first-period demand is allocated equally among the retailers, and with inventory-dependent demand when retailer's demand increases in inventory level. For the former case, we borrowed the model of demand allocation from Liu and van Ryzin (2008) who showed that retailer capacity rationing is not robust under competition. The latter case is more challenging because the retailers procure more inventories to increase their market shares in the first period. Nevertheless, in both cases, RPM enjoys the same profit as with a single retailer by setting a higher wholesale price.

If customers know the inventory level, CSC effectively uses customer awareness and has the same profit as DSC, which coincides with the DSC profit under unknown inventory. In this case, the manufacturer in DSC sets the wholesale price equal to its unit cost effectively "turning off" unnecessary double marginalization. The comparison with the case of known inventory implies that inventory disclosure can be equivalent to double marginalization as a low-inventory commitment device. On the other hand, if customer reaction on disclosed inventories is overestimated (or strategicity is underestimated), a SC can be overcentralized leading to a profit loss comparable with the loss from strategic customers.

The presence of strategic customers changes the conclusions of Spengler (1950) because DSC with RPM is preferable for manufacturer but still hurts welfare, that is, the manufacturer has an incentive for welfare-reducing decentralization. Moreover, for both types of competition, RPM performs like with a single retailer, that is, formally, the case of RPM use for mitigating strategic behavior can be qualified as a subcase of conspiracy theory leading to a retailer cartel. However, this simple argument can be misleading. First, because CSC can also achieve the same result by disclosing inventory level, that is, consistently using this argument, the legal status of inventory disclosure should be equivalent to the one of RPM. This "strange" equivalency questions the argument. Second, because even when the first-period price is sticky (no manufacturer decisions), the surplus of low-valuation customers decreases due to strategic delays of a part of high-valuation customers. The decrease results from inventory-reducing response of retailers.

The observations above imply that a primary source of welfare loss in this problem is the behavior of strategic customers. Therefore, a future research may determine the best SC-profit improving strategies that mitigate strategic delays and, at the same time, do not decrease welfare because welfare-decreasing strategies may be infeasible due to legal actions. In particular, we show that the maximum loss of welfare under RPM compared to CSC is around 6%; that is, if RPM is multipurpose, for example, besides mitigating customers' delays, it also protects the retailers providing demand-enhancing services against free-riders or supports the appeal of branded products, the combined effect may be welfare-improving. Another promising welfare-increasing strategy is studied in Aflaki et al. (2016), where retailers increase customer cost of being strategic, for example, by making markdowns less predictable. Aviv et al. (2016) show that when customers are strategic and the first-period price is sticky, a most-favored-customer clause eliminates strategic delays and increases welfare in the majority of market situations especially at high levels of competition.

Notes

[1]Urban (2005) provides a review of 60 theoretical and empirical papers studying inventory-dependent demand in various industries without strategic customers.

[2]As suggested by Su and Zhang (2008), §6.1, the form of $\bar{\sigma}_2$ below implies that ρ can be interpreted also as customer risk aversion or systematic misestimation of $\bar{\alpha}$. In these cases, ρ may exceed one.

[3]Using "cost" terminology, see, e.g., Aflaki et al. (2016), this assumption means that the customer cost of estimating total inventory is prohibitively higher than the cost of expectations $\bar{\alpha}$ and $\bar{p}_2$, which are a byproduct of a regular buying practice and do not require additional efforts.

[4]Even though attractions are not continuous at 0 in this case, we demonstrate that the analysis is still possible.

[5]A similar effect in a different setup is discussed in §6.3 of Su and Zhang (2008).

[6]This result and the case $p_2 = s$ are discussed in more detail in a working paper Bazhanov et al. (2015).

[7]The expression for w^* provided in Proposition 5 can be easily generalized for any $0 < \gamma < 1$ using the approach in the proof and the equation for Y with general γ in Bazhanov et al. (2015).

[8]Exogenously restricted B, resulting in the total inventory less than the one that maximizes *individual* retailer profits, leads to an interesting effect when the total, and therefore, individual profits of symmetric retailers are greater due to reduced sales at loss in the second period. This known effect results from market share competition of noncooperative retailers with inventory-dependent demand. A similar effect increased the profits of tobacco companies after the advertising ban on TV and radio for the cigarette industry in 1971.

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A Online Appendix

A.1 Proof of Lemma 1 (first-period demand)

By Assumption 3, $\sigma_1 \geq 0$ is equivalent to $v \geq p_1$ and $\sigma_1 \geq \bar{\sigma}_2$ is equivalent to $v - p_1 \geq \rho\bar{\alpha}(\beta v - \bar{p}_2) \Leftrightarrow v \geq \frac{p_1 - \bar{\alpha}\rho\bar{p}_2}{1-\bar{\alpha}\rho\beta}$. Combining these inequalities, we obtain the stated expression for $v^{\min}$. Because all customers with $v \geq v^{\min}$ would buy in the first period, the total demand is $D = 1 - v^{\min}$.

A.2 Proof of Lemma 2 (REE, $n = 1$)

REE1 (only first-period sales) simplifies (4) to $\Pi = (p_1 - c)Y$ yielding a unique profit-maximizing $Y = 1 - v^{\min}$ and the maximum profit $\Pi = (p_1 - c)(1 - v^{\min})$, which can be formulated as

Lemma 6. *For given model inputs and customer expectations, retailer rationality implies that the effective domain of the inventory decision is $Y \geq 1 - v^{\min}$ and $(p_1 - c)(1 - v^{\min})$ is the lower bound for the optimal profit.*

Customer rationality demands that $\bar{\alpha} = 0$ and, by Lemma 1, $v^{\min} = p_1$, implying that the candidate REE is described by $\hat{v} = p_1, \hat{Y} = 1 - \hat{v}$, and, therefore, $\hat{\alpha} = 0$ and $\hat{\Pi} = (p_1 - c)(1 - p_1)$.

Since $v^{\min}$ does not depend on Y and, by (5), $p_2 = \beta(1 - Y)$, profit (4) is concave quadratic in Y when $Y > 1 - v^{\min}$. Therefore, the candidate REE1 exists iff there is a local maximum of Π at $Y = 1 - \hat{v}$, i.e., profit (4) is not increasing in Y for any $Y > 1 - \hat{v} : \frac{\partial \Pi}{\partial Y}|_{Y=1-\hat{v}+0} \leq 0$, which, using $Y = 1 - \hat{v}$, is $-c + \beta\hat{v} \leq 0 \Leftrightarrow p_1 \leq c/\beta$.

REE2 (only second-period sales) exists iff all customers delay their purchases, i.e., $v^{\min} = 1$, which simplifies profit (4) to $\Pi = -cY + \beta(1 - Y)Y$. FOC yields the candidate REE with $\hat{Y} = \frac{1}{2}(1 - c/\beta), \hat{p}_2 = \beta(1 - \hat{Y}) = \frac{1}{2}(\beta + c)$, and $\hat{\Pi} = -c\hat{Y} + \beta(1 - \hat{Y})\hat{Y} = \frac{1}{2\beta}(\beta - c)\left[-c + \frac{1}{2}(\beta - c)\right] = \frac{(\beta-c)^2}{4\beta}$. The condition of existence $v^{\min} = 1$ holds only if $\bar{\alpha} = 1$ and, by Lemma 1, if $\frac{p_1 - \rho\bar{p}_2}{1-\rho\beta} \geq 1$. By customer rationality, $\bar{p}_2 = \hat{p}_2 = \frac{1}{2}(\beta + c)$ leading to $p_1 - \frac{\rho}{2}(\beta + c) \geq 1 - \rho\beta$ or $p_1 \geq 1 - \frac{\rho}{2}(\beta - c)$.

REE3: There are sales in both periods iff $Y > 1 - v^{\min}$ and $p_1 \leq v^{\min} < 1$ (there are sales in the first period) with $v^{\min} = p_1$ only if $\rho = 0$. Profit (4) is $\Pi = -cY + p_1(1 - v^{\min}) + \beta(1 - Y)(Y - 1 + v^{\min})$, which is concave quadratic in Y ($v^{\min}$ is constant in Y). Therefore, the candidate

REE exists iff the local maximum of Π at $Y = \hat{Y}$ is such that $\hat{Y} > 1 - \hat{v}$ and $\hat{v} < 1$. FOC is $-c - \beta(Y - 1 + v^{\min}) + \beta(1 - Y) = 0 \Leftrightarrow 2\beta Y = \beta(2 - v^{\min}) - c \Leftrightarrow \hat{Y} = 1 - \frac{1}{2}(\hat{v} + c/\beta)$, where $\hat{v} = \frac{p_1 - \rho\hat{p}_2}{1-\rho\beta}$ and, by customer rationality, $\hat{p}_2 = \beta(1 - \hat{Y})$. Substitution of $\hat{Y}$ and collection of terms with $\hat{v}$ leads to $\hat{v} = \frac{2p_1 - \rho\beta(\hat{v}+c/\beta)}{2(1-\rho\beta)} \Leftrightarrow \hat{v}(1 + \frac{\rho\beta}{2(1-\rho\beta)}) = \frac{2p_1 - \rho c}{2(1-\rho\beta)} \Leftrightarrow \hat{v} = \frac{2p_1 - \rho c}{2-\rho\beta}$, which increases in ρ since $\frac{\partial\hat{v}}{\partial\rho} = \frac{2(\beta p_1 - c)}{(2-\rho\beta)^2} > 0$. The last inequality follows from condition $\hat{Y} > 1 - \hat{v}$, which becomes $-\frac{1}{2}(\hat{v} + c/\beta) > -\hat{v} \Leftrightarrow \hat{v} > c/\beta \Leftrightarrow 2p_1 - \rho c > 2c/\beta - \rho c \Leftrightarrow p_1 > c/\beta$. Condition $\hat{v} < 1$ is $2p_1 - \rho c < 2 - \rho\beta \Leftrightarrow p_1 < 1 - \frac{\rho}{2}(\beta - c)$. Note that $\hat{v} = p_1 \Leftrightarrow \rho c = \rho\beta p_1$, which, indeed, holds only if $\rho = 0$ whenever $p_1 > c/\beta$.

A.3 Proof of Proposition 1 (RPM, incomplete info, $n = 1$)

We start from determining a profit-maximizing $p_1(c)$ for REE3. Substitution of $\hat{v}(p_1, c)$ into $\hat{Y}[\hat{v}(p_1, c), c]$, and $\hat{p}_2[\hat{v}(p_1, c), c]$ given by Lemma 2 leads to $\hat{Y}(p_1, c) = 1 - \frac{\beta p_1 + c(1-\rho\beta)}{\beta(2-\rho\beta)}$ and $\hat{p}_2(p_1, c) = c + \frac{\beta p_1 - c}{2-\rho\beta}$. Plugging in these expressions into $\hat{\Pi}$ yields $\hat{\Pi}(p_1, c) = (p_1 - c)\frac{2(1-p_1) - \rho(\beta - c)}{2-\rho\beta} + \frac{(\beta p_1 - c)^2}{\beta(2-\rho\beta)^2}$, which is concave quadratic in p_1. FOC is $\frac{2(1-p_1)-\rho(\beta-c)}{2-\rho\beta} - 2\frac{p_1 - c}{2-\rho\beta} + 2\frac{\beta p_1 - c}{(2-\rho\beta)^2} = 0$, which is equivalent to $p_1\left(\frac{2\beta}{2-\rho\beta} - 4\right) = \frac{2c}{2-\rho\beta} - 2c - 2 + \rho(\beta - c)$ or $p_1\frac{\beta - 2(2-\rho\beta)}{2-\rho\beta} = \frac{c + (2-\rho\beta)(\frac{\rho}{2}(\beta-c) - 1 - c)}{2-\rho\beta}$, yielding $p_1(c) = \frac{(2-\rho\beta)[1 - \frac{\rho}{2}(\beta - c)] + c(1-\rho\beta)}{2(2-\rho\beta) - \beta}$.

For CSC ($c = 0$), a profit-maximizing p_1 is $p_1^* = \frac{(2-\rho\beta)^2}{2[2(2-\rho\beta)-\beta]}$. Plugging in p_1^* and $c = 0$ into the formulas for $\hat{v}, \hat{Y}, \hat{p}_2$, and $\hat{\Pi}$ in Lemma 2 leads to the expressions in Table 1: $v^* = \frac{2-\rho\beta}{2(2-\rho\beta)-\beta}, Y^* = 1 - \frac{v^*}{2} = \frac{4(2-\rho\beta) - 2\beta - (2-\rho\beta)}{2[2(2-\rho\beta)-\beta]} = \frac{3(2-\rho\beta) - 2\beta}{2[2(2-\rho\beta)-\beta]}, p_2^* = \frac{\beta(2-\rho\beta)}{2[2(2-\rho\beta)-\beta]}$. In order to obtain the expression for Π^{C*}, note that $1 - v^* = \frac{2-\rho\beta-\beta}{2(2-\rho\beta)-\beta}$ and $Y^* - (1 - v^*) = \frac{v^*}{2} = \frac{2-\rho\beta}{2[2(2-\rho\beta)-\beta]}$. Then $\Pi^{C*} = p_1^*(1 - v^*) + p_2^*[Y^* - (1 - v^*)] = \frac{(2-\rho\beta)^2}{2[2(2-\rho\beta)-\beta]}\frac{2-\rho\beta-\beta}{2(2-\rho\beta)-\beta} + \frac{\beta(2-\rho\beta)}{2[2(2-\rho\beta)-\beta]}\frac{2-\rho\beta}{2[2(2-\rho\beta)-\beta]} = \frac{(2-\rho\beta)^2[2(2-\rho\beta)-2\beta+\beta]}{4[2(2-\rho\beta)-\beta]^2} = \frac{(2-\rho\beta)^2}{4[2(2-\rho\beta)-\beta]}$. The intuitive monotonicity of these values in ρ can be shown by direct differentiation. By Lemma 2, there exists only REE3 since p_1^* always satisfies the condition of REE3 existence and profit Π^{C*} under REE3 always exceeds profits Π^{C1*} and Π^{C2*} under REE1 and REE2 respectively. Indeed, $p_1^* > 0$, and condition $p_1^* < 1 - \rho\beta/2 \Leftrightarrow 2 - \rho\beta > \beta$ always holds. Profit $\Pi^{C*} \equiv \Pi^{C3*}$ has infimum at $\rho \to 1$ (since $\frac{\partial\Pi^{C*}}{\partial\rho} < 0$), which is $\inf_{\rho\to 1}\Pi^{C3*} = \frac{(2-\beta)^2}{4(4-3\beta)}$. This expression is maximal at $\beta = 2/3$ (since $\frac{\partial\Pi^{C3*}|_{\rho\to 1}}{\partial\beta} = \frac{2-3\beta}{2(4-3\beta)^2}$) and minimal at the boundaries: $\inf_{\rho\to 1}\Pi^{C3*}|_{\beta=1} = \inf_{\rho\to 1}\Pi^{C3*}|_{\beta=0} = \frac{1}{4}$, whereas, by Lemma 2, $\Pi^{C1*} = \max_{p_1} p_1(1 - p_1) = \frac{1}{4}$ and $\Pi^{C2*} = \frac{\beta}{4}$.

For DSC, By Lemma 1 and the general equations for profits (2) and (3),

$$\Pi^D(p_1, w) = p_1[1 - v(p_1, w)] + p_2(p_1, w)[Y(p_1, w) - 1 + v(p_1, w)] \tag{11}$$

when sales are in both periods. The proof is based on the following lemma.

Lemma 7. *Assume that* $Y = a_0 - a_1 p_1 - a_w w, p_2 = b_1 p_1 + b_w w, v = d_1 p_1 - d_w w$ *and the resulting quadratic profit* $\Pi^D(p_1, w)$ *is concave in* p_1 *and* w*. Then maximization of* Π^D *yields a unique solution*

$$w^* = -\frac{g_{12}g_{10} + 2g_{20}g_{11}}{4g_{11}g_{22} + g_{12}^2} \text{ and } p_1^* = -\frac{g_{10} + w^* g_{12}}{2g_{11}} \tag{12}$$

where $g_{10} = 1+b_1(a_0-1), g_{11} = -d_1+b_1(d_1-a_1), g_{12} = d_w-b_1(a_w+d_w)+b_w(d_1-a_1), g_{20} = b_w(a_0-1),$ *and* $g_{22} = b_w(a_w + d_w)$ *are the coefficients of* $\Pi^D(p_1, w) = p_1^2 g_{11} - w^2 g_{22} + p_1 w g_{12} + p_1 g_{10} - w g_{20}$.

Proof Substitution of Y, p_2, and v into (11) leads to $\Pi^D(p_1, w) = p_1^2 g_{11} - w^2 g_{22} + p_1 w g_{12} + p_1 g_{10} - w g_{20}$ and $\frac{\partial \Pi^D}{\partial p_1} = 0$ yields p_1^* given in (12). Substitution of this p_1^* into $\frac{\partial \Pi^D}{\partial w} = 0$ leads to $-2wg_{22} - \frac{g_{12}}{2g_{11}}(g_{10} + wg_{12}) - g_{20} = 0$ yielding w^* in (12)□

Part REE3 of Lemma 2 with $c = w$ provides the coefficients of Y, p_2, and v as functions of p_1 and $w : a_0 = 1, a_1 = \frac{1}{2-\rho\beta}, a_w = \frac{1-\rho\beta}{\beta(2-\rho\beta)}, b_1 = \frac{\beta}{2-\rho\beta}, b_w = \frac{1-\rho\beta}{2-\rho\beta}, d_1 = \frac{2}{2-\rho\beta}$, and $d_w = \frac{\rho}{2-\rho\beta}$. Substitution of these coefficients into the expressions for g_{ij} yields $g_{10} = 1, g_{11} = \frac{\beta}{2-\rho\beta}\left(\frac{2}{2-\rho\beta} - \frac{1}{2-\rho\beta}\right) - \frac{2}{2-\rho\beta} = \frac{\beta-2(2-\rho\beta)}{(2-\rho\beta)^2}, g_{12} = \frac{\rho}{2-\rho\beta} - \frac{\beta}{2-\rho\beta}\left(\frac{1-\rho\beta}{\beta(2-\rho\beta)} + \frac{\rho}{2-\rho\beta}\right) + \frac{1-\rho\beta}{2-\rho\beta}\frac{1}{2-\rho\beta} = \frac{\rho(2-\rho\beta-\beta)}{(2-\rho\beta)^2}, g_{20} = 0$, and $g_{22} = \frac{1-\rho\beta}{2-\rho\beta}\left(\frac{1-\rho\beta}{\beta(2-\rho\beta)} + \frac{\rho}{2-\rho\beta}\right) = \frac{1-\rho\beta}{\beta(2-\rho\beta)^2}$.

Note that the coefficients in front of p_1^2 and w^2 are negative and $\Pi^D(p_1, w)$ is indeed concave. Then the substitution of g_{ij} into (12) yields w^* and p_1^*. Namely, the numerator of the fraction for w^* is g_{12} and the denominator is $4g_{22}g_{11} + g_{12}^2 = \frac{4(1-\rho\beta)[\beta-2(2-\rho\beta)]+\rho^2\beta(2-\rho\beta-\beta)^2}{\beta(2-\rho\beta)^4}$ leading to $w^* = \frac{\rho\beta(2-\rho\beta)^2(2-\rho\beta-\beta)}{4(1-\rho\beta)[2(2-\rho\beta)-\beta]-\rho^2\beta(2-\rho\beta-\beta)^2}$, where the denominator can be written as $8(2-\rho\beta)(1-\rho\beta)-4\beta(1-\rho\beta)-\rho^2\beta(2-\rho\beta)^2+2(2-\rho\beta)\rho^2\beta^2-\rho^2\beta^3 = 2(2-\rho\beta)(4-4\rho\beta+\rho^2\beta^2)-\beta(4-4\rho\beta+\rho^2\beta^2)-\rho^2\beta(2-\rho\beta)^2$, which equals $(2-\rho\beta)^2[4-\beta(1+\rho)^2]$. Substitution of this expression yields $w^* = \frac{\rho\beta(2-\rho\beta-\beta)}{4-\beta(1+\rho)^2}$, which increases in ρ since $\frac{\partial w^*}{\partial \rho}$ can be written as $\frac{\beta\left[(1-\rho\beta)^2+(3-\beta-2\rho\beta)(1-\beta)+4-4\rho\beta+2\rho^2\beta-2\beta\right]}{[4-\beta(1+\rho)^2]^2}$, where the denominator and the first two terms in the square bracket in the numerator are strictly positive, and $\inf_\rho\left(4-4\rho\beta+2\rho^2\beta-2\beta\right) = 4(1-\beta) \geq 0$ since this sum decreases in ρ. Monotonicity of w^* in ρ implies $w^* \in [0, \beta/2)$ for $\rho \in [0, 1)$. Plugging in w^* into the formula for p_1^* in (12) leads after simplifications to $p_1^* = \frac{1}{2}\left(-1 - w^*\frac{\rho(2-\rho\beta-\beta)}{(2-\rho\beta)^2}\right)\frac{(2-\rho\beta)^2}{\beta-2(2-\rho\beta)} = \frac{2(1-\rho\beta)}{4-\beta(1+\rho)^2}$ with $\frac{\partial p_1^*}{\partial \rho} = -\frac{2\beta(1-\rho)[2-\beta(1+\rho)]}{[4-\beta(1+\rho)^2]^2} < 0$. Then $m_r^* = 1 - \frac{w^*}{p_1^*} = \frac{2-\rho\beta(4-\rho\beta-\beta)}{2(1-\rho\beta)}$ with $\frac{\partial m_r^*}{\partial \rho} = -\frac{\beta[1-\beta+(1-\rho\beta)^2]}{2(1-\rho\beta)^2} < 0$.

Substitution of p_1^* and w^* into the expression for Y, p_2, and v yield their equilibrium values: $Y^* = 1 - a_1 p_1^* - a_w w^* = 1 - \frac{(1-\rho\beta)(2+2\rho-\rho^2\beta-\rho\beta)}{(2-\rho\beta)[4-\beta(1+\rho)^2]} = 1 - \frac{(1-\rho\beta)(1+\rho)}{4-\beta(1+\rho)^2} = \frac{3-\rho\beta-\beta-\rho}{4-\beta(1+\rho)^2}$, which is decreasing in ρ since $\frac{\partial Y^*}{\partial \rho} = \frac{3\beta-\rho^2\beta^2-2\rho\beta^2-\beta^2-4-\rho^2\beta+6\rho\beta}{[4-\beta(1+\rho)^2]^2}$, where the numerator increases in β (its derivative in

β decreases in β and is positive at $\beta = 1$) and equals $-2(1-\rho)^2 < 0$ at $\beta = 1$. Given Y^*, we have $p_2^* = \beta(1-Y^*) = \frac{\beta(1-\rho\beta)(1+\rho)}{4-\beta(1+\rho)^2}$, which increases in ρ since $\frac{\partial Y^*}{\partial \rho} < 0$. $v^* = d_1^* p_1^* - d_w w^* = \frac{2}{2-\rho\beta}\frac{2(1-\rho\beta)}{4-\beta(1+\rho)^2} - \frac{\rho}{2-\rho\beta}\frac{\rho\beta(2-\rho\beta-\beta)}{4-\beta(1+\rho)^2} = \frac{2-\rho\beta(1+\rho)}{4-\beta(1+\rho)^2}$ with $\frac{\partial v^*}{\partial \rho} = \frac{\beta[\beta(1+\rho)^2-4\rho]}{[4-\beta(1+\rho)^2]^2}$, which is positive for $\beta = 1$: $\frac{\partial v^*}{\partial \rho}\Big|_{\beta=1} = \frac{(1-\rho)^2}{[4-(1+\rho)^2]} > 0$. For $\beta \in (0,1)$, there exists a maximum of v^* in ρ since $\frac{\partial v^*}{\partial \rho} = 0 \Leftrightarrow \beta = 4\rho/(1+\rho)^2$ or $\rho^0 = 2(1-\sqrt{1-\beta})/\beta - 1$ (the larger root of the quadratic equation in ρ is greater than 1). This unique ρ^0 corresponds to a maximum of v^* since $\frac{\partial v^*}{\partial \rho}\Big|_{\rho=0} = \frac{\beta^2}{(4-\beta)^2} > 0$ and $\frac{\partial v^*}{\partial \rho}\Big|_{\rho\to 1} = \frac{4\beta(\beta-1)}{16(1-\beta)^2} < 0$.

Substitution of w^*, Y^*, p_1^*, p_2^*, and v^* into the formulas for profits leads after simplifications to Π^{m*}, Π^{r*}, and Π^{D*} : $\Pi^{m*} = w^* Y^* = \frac{\rho\beta(2-\rho\beta-\beta)(3-\rho\beta-\beta-\rho)}{[4-\beta(1+\rho)^2]^2}$; $\Pi^{D*} = p_1^*(1-v^*) + p_2^*(Y^* - 1 + v^*)$, where $1 - v^* = \frac{2-\rho\beta-\beta}{4-\beta(1+\rho)^2}$ and $Y^* - (1-v^*) = \frac{1-\rho}{4-\beta(1+\rho)^2}$. After substitution, it becomes $\Pi^{D*} = \frac{2(1-\rho\beta)(2-\rho\beta-\beta)}{[4-\beta(1+\rho)^2]^2} + \frac{\beta(1-\rho^2)(1-\rho\beta)}{[4-\beta(1+\rho)^2]^2} = \frac{1-\rho\beta}{4-\beta(1+\rho)^2}$ with $\frac{\partial \Pi^{D*}}{\partial \rho} = \frac{\beta\{\rho(2-\rho\beta)+\beta-2\}}{[4-\beta(1+\rho)^2]^2} < 0$ since the bracket $\{\cdot\}$ in the numerator increases in β and $\{\cdot\}|_{\beta=1} = \rho(2-\rho) - 1 < 0$ because the supremum of $\rho(2-\rho)$ equals 1 at $\rho \to 1$. The retailer profit is $\Pi^{r*} = \Pi^{D*} - \Pi^{m*} = \frac{1-\rho\beta}{4-\beta(1+\rho)^2} - \frac{\rho\beta(2-\rho\beta-\beta)(3-\rho\beta-\beta-\rho)}{[4-\beta(1+\rho)^2]^2} = \frac{4-\beta\{1+\rho[6(2-\rho\beta-\beta)-\rho+\beta^2(1+\rho^2)]\}}{[4-\beta(1+\rho)^2]^2}$.

We can show now that there exists only equilibrium REE3. First, we show that FOC for p_1^* and w^* always lead to an interior solution for REE3, i.e., when the manufacturer sets p_1^* and w^* under REE3, the retailer does not deviate to other equilibria. Indeed, the left inequality in the condition of REE3 existence $\frac{c}{\beta} < p_1 < \frac{1}{2}(2-\rho\beta+\rho c)$ in Lemma 2 with $c = w^*$ becomes $\rho(2-\rho\beta-\beta) < 2(1-\rho\beta) \Leftrightarrow \rho\beta(1-\rho) < 2(1-\rho)$, which always holds. The right inequality is equivalent to $4(1-\rho\beta) < (2-\rho\beta)[4-\beta(1+\rho)^2] + \rho^2\beta(2-\rho\beta-\beta) \Leftrightarrow 2-\rho\beta < 4-\beta(1+\rho)^2 + \rho^2\beta$ or $2-\rho\beta-\beta > 0,$ which also always holds.

The manufacturer has no incentives to set p_1^* and w^* leading to REE1 or REE2 because REE3-profit Π^{D3*} always exceeds the profits of other equilibria. Indeed, REE1-profit Π^{D1*} is $\Pi^{D1*} = \max_{p_1,w}[w(1-p_1) + (p_1 - w)(1-p_1)] = \max_{p_1} p_1(1-p_1) = \frac{1}{4}$ attained at $p_1^* = \frac{1}{2}$ for any $w \geq \beta/2$ since $p_1 \leq c/\beta$ must hold for REE1. REE2-profit Π^{D2*} is $\Pi^{D2*} = \max_{p_1,w}[w(\beta-w)/(2\beta) + (\beta-w)^2/(4\beta)] = \beta/4$ attained at $w^* = 0$ for any $p_1 \geq \frac{1}{2}(2-\rho\beta)$, which is the condition of REE2 existence. The infimum of REE3 profit $\Pi^{D3*} = \frac{1-\rho\beta}{4-\beta(1+\rho)^2}$ is at $\rho \to 1$ since $\frac{\partial \Pi^{D3*}}{\partial \rho} = \frac{\rho(2-\rho\beta)-(2-\beta)}{[4-\beta(1+\rho)^2]^2}$, where the numerator is maximal at $\beta = 1$ and equals $\rho(2-\rho) - 1$, which is negative for any $\rho \in [0,1)$. Therefore, the infimum of REE3 profit is $\Pi^{D3*}|_{\rho\to 1} = \frac{1-\beta}{2(2-\beta)-2\beta} = \frac{1}{4}$, i.e., $\Pi^{D3*} > \Pi^{D1*} \geq \Pi^{D2*}$ for any $\rho \in [0,1)$ and $\beta \in (0,1]$.

Finally, using the formula for profit of CSC Π^{C*} in Table 1, the profit-performance of RPM is $\eta^{\Pi} = \Pi^{D*}/\Pi^{C*} = \frac{1-\rho\beta}{4-\beta(1+\rho)^2}\frac{4[2(2-\rho\beta)-\beta]}{(2-\rho\beta)^2}$, which can be written as $\eta^{\Pi} = 1 + \frac{\rho^2\beta[\beta^2+(2-\rho\beta)(2-\rho\beta-2\beta)]}{(2-\rho\beta)^2[4-\beta(1+\rho)^2]}$.

Inequality $p_1^{D*} \geq p_1^{C*}$ is equivalent to $(2-\rho\beta)^2(4-\beta-2\rho\beta-\rho^2\beta) \leq 4(1-\rho\beta)(4-\beta-2\rho\beta)$, which holds as equality at $\rho = 0$. Considering $\rho > 0$, this inequality simplifies to $(2-\rho\beta)[\rho\beta - 2(1-\beta)] - \beta^2 \leq 0$. Denote $x \triangleq [\cdot]$, implying that $\rho\beta = x + 2(1-\beta)$. Then the last inequality is $(2\beta - x)x - \beta^2 \leq 0$, where the first term $(2\beta - x)x$ attains maximum in x, which equals β^2 at $x = \beta$, i.e., at $\beta = \rho\beta - 2(1-\beta) \Leftrightarrow \rho = (2-\beta)/\beta$, which holds at $\rho\beta \to 1$.

Inequality $Y^{D*} \leq Y^{C*}$ is equivalent to $[3(2-\rho\beta)-2\beta][4-\beta(1+\rho)^2] \geq [4-(1+\rho)(\beta+1)]2[2(2-\rho\beta)-\beta]$, which holds as equality at $\rho = 0$. Considering $\rho > 0$, it simplifies to $(2-\rho\beta)[3\beta(1+\rho) - 4] - \beta^2(1+\rho) \leq 0$. The LHS is increasing in β since $\frac{\partial LHS}{\partial\beta} = (1+\rho)[3(2-\rho\beta) - 2\beta - 3\rho\beta] + 4\rho$ decreases in β and its minimum is $\frac{\partial LHS}{\partial\beta}\big|_{\beta=1} = 2[2 + 3\rho(1-\rho)] > 0$, i.e., the LHS is maximal at $\beta = 1$. The maximum of LHS in β is $LHS|_{\beta=1} = 3[\rho(2-\rho) - 1] \leq 0$ because $\max_{\rho}\rho(2-\rho) = 1$ at $\rho = 1$, i.e., another case when $Y^{*CI} \geq Y^{*CC}$ holds as equality is $\rho\beta \to 1$. Since $p_2^* = \beta(1-Y^*)$, inequality $Y^{D*} \leq Y^{C*}$ implies $p_2^{D*} \geq p_2^{C*}$.

Inequality $v^{D*} \leq v^{C*}$ also holds as equality at $\rho = 0$. Considering $\rho > 0$, it simplifies to $\rho^2\beta(2-\rho\beta-\beta) \geq 0$, which holds as an equality only at $\rho\beta \to 1$.

Inequality $\eta^{\Pi} \geq 1$ follows from simple observations that the fraction in the formula for η^{Π} is zero only if $\rho = 0$ or the square bracket in the numerator is zero since the denominator is positive for any $\rho \in [0,1)$ and $\beta \in [0,1]$. The square bracket in the numerator has a unique minimum in ρ. This minimum equals 0 at $2-\rho\beta = \beta \Leftrightarrow \rho = (2-\beta)/\beta \geq 1$, which, for $\rho \in [0,1)$ and $\beta \in [0,1]$, can hold only as equality in the limit when $\rho\beta \to 1$. In this case, the L'Hospital's rule yields $\eta^{\Pi} \to 1$.

A.4 Proof of Lemma 3 (total surplus)

By the definition of $v^{\min}$, the total customer surplus in the first period is $\Sigma_1 = \int_{v^{\min}}^{1}(v-p_1)dv = \left(\frac{v^2}{2} - p_1 v\right)\Big|_{v^{\min}}^{1} = \frac{1}{2} - p_1 - \frac{\left(v^{\min}\right)^2}{2} + p_1 v^{\min} = \frac{1-\left(v^{\min}\right)^2}{2} - p_1(1-v^{\min}) = (1-v^{\min})\left[\frac{1+v^{\min}}{2} - p_1\right]$; and $\Sigma_2 = \int_{p_2}^{\beta v^{\min}}(\tilde{v} - p_2)\frac{d\tilde{v}}{\beta} = \frac{1}{\beta}\left(\frac{\tilde{v}^2}{2} - p_2\tilde{v}\right)\Big|_{p_2}^{\beta v^{\min}} = \frac{1}{\beta}\left(\beta v^{\min} - p_2\right)\left(\frac{\beta v^{\min}+p_2}{2} - p_2\right) = \frac{\left(\beta v^{\min}-p_2\right)^2}{2\beta}$. Hence, $\Sigma = \Sigma_1 + \Sigma_2 = (1-v^{\min})\left[\frac{1+v^{\min}}{2} - p_1\right] + \frac{\left(\beta v^{\min}-p_2\right)^2}{2\beta}$.

A.5 Proof of Lemma 4 (CIE)

CIE1: Information about Y does not change the candidate equilibrium compared to incomplete info case in Lemma 2 because $\hat{\alpha} = 0$ and $\hat{v} = p_1$, i.e., customer behavior does not depend on Y when $Y \le 1 - v^{\min}$. The existence condition, however, is different because it involves the derivative of the two-period profit (4). Under complete info, this profit is $\Pi = -cY + p_1\left(1 - \frac{p_1 - \rho\beta(1-Y)}{1-\rho\beta}\right) + \beta(1 - Y)\left(Y - 1 + \frac{p_1-\rho\beta(1-Y)}{1-\rho\beta}\right) = \frac{1}{1-\rho\beta}\left\{-cY(1-\rho\beta) + p_1(1 - p_1 - \rho\beta Y) + \beta(1-Y)(Y - 1 + p_1)\right\}$ or

$$\Pi = \frac{1}{1-\rho\beta}\left\{(1-p_1)(p_1-\beta) + Y[\beta(2-p_1) - c - \rho\beta(p_1 - c)] - \beta Y^2\right\}. \tag{13}$$

Since Π is concave in Y, the candidate CIE exists iff there is a local maximum of Π at $Y = 1 - \hat{v}$, i.e., (13) is not increasing in Y for any $Y > 1 - \hat{v}$: $\frac{\partial \Pi}{\partial Y}|_{Y=1-\hat{v}+0} \le 0$, which, using $Y = 1 - p_1$ is equivalent to $\beta(2-p_1) - c - \rho\beta(p_1 - c) \le 2\beta(1-p_1) \Leftrightarrow p_1\beta(1-\rho) \le c(1-\rho\beta) \Leftrightarrow p_1 \le \frac{c(1-\rho\beta)}{\beta(1-\rho)}$.

CIE2: This equilibrium is similar to REE2 given by Lemma 2 because it also exists only if $v^{\min} = 1$ leading to the same form of profit and, therefore, the candidate CIE with $\hat{Y} = \frac{1}{2}(1 - c/\beta)$. The condition of existence is also determined as a p_1-boundary between CIE2 ($v^{\min} = 1$) and CIE3 ($v^{\min} < 1$). We derive this condition in the proof of CIE3 below.

CIE3: The difference from REE3 is that profit (4) becomes $\Pi = -cY + p_1[1 - v^{\min}(Y)] + \beta(1 - Y)[Y - 1 + v^{\min}(Y)]$, where $v^{\min}(Y) = \frac{p_1 - \rho\beta(1-Y)}{1-\rho\beta}$ (not a constant in Y). Therefore, FOC leads to a different candidate equilibrium: $-c - p_1\frac{\partial v^{\min}}{\partial Y} - \beta(Y - 1 + v^{\min}) + \beta(1-Y)\left(1 + \frac{\partial v^{\min}}{\partial Y}\right) = 0$, where $\frac{\partial v^{\min}}{\partial Y} = \frac{\rho\beta}{1-\rho\beta}$ and $Y - 1 + v^{\min} = \frac{(Y-1)(1-\rho\beta) + p_1 - \rho\beta(1-Y)}{1-\rho\beta} = \frac{p_1 + Y - 1}{1-\rho\beta}$ result in $-c(1-\rho\beta) - p_1\rho\beta - \beta(p_1 + Y - 1) + \beta(1-Y) = 0 \Leftrightarrow 2\beta Y = -p_1\beta(1+\rho) - c(1-\rho\beta) + 2\beta$, which yields $\hat{Y} = 1 - \frac{1}{2}\left[p_1(1+\rho) + c\left(\frac{1}{\beta} - \rho\right)\right]$. Substitution of $\hat{Y}$ leads to $\hat{p}_2 = \frac{1}{2}\left[p_1\beta(1+\rho) + c\,(1-\rho\beta)\right]$ and $\hat{v} = \frac{1}{1-\rho\beta}\left\{p_1 - \frac{\rho\beta}{2}\left[p_1(1+\rho) + c\left(\frac{1}{\beta} - \rho\right)\right]\right\} = p_1\frac{2-\rho\beta-\rho^2\beta}{2(1-\rho\beta)} - c\rho/2$.

Similarly to Lemma 2, the conditions of CIE3 existence are $\hat{Y} > 1 - \hat{v}$ and $\hat{v} < 1$. The first is equivalent to $\frac{\partial \Pi}{\partial Y}|_{Y=1-\hat{v}+0} > 0$ and becomes $\frac{1}{2}\left[p_1(1+\rho) + c\left(\frac{1}{\beta} - \rho\right)\right] < p_1\frac{2-\rho\beta-\rho^2\beta}{2(1-\rho\beta)} - c\rho/2 \Leftrightarrow p_1(1+\rho)(1-\rho\beta) + \frac{c}{\beta}(1-\rho\beta) < p_1(2 - \rho\beta - \rho^2\beta) \Leftrightarrow p_1(1-\rho) > \frac{c}{\beta}(1-\rho\beta) \Leftrightarrow p_1 > \frac{c(1-\rho\beta)}{\beta(1-\rho)}$, which is the p_1-boundary between CIE3 and CIE1.

Condition $\hat{v} < 1$ leads to p_1-boundary between CIE3 ($\hat{v} < 1$) and CIE2 ($\hat{v} = 1$). By Lemma 1, $v^{\min} = 1$ iff $\frac{p_1-\rho\beta(1-Y)}{1-\rho\beta} \ge 1$ since $\bar{\alpha} = 1$ in both CIE3 and CIE2. Then the Y-boundary between CIE3 and CIE2 follows from $\frac{p_1-\rho\beta(1-Y)}{1-\rho\beta} = 1$ yielding $Y^B = \frac{1-p_1}{\rho\beta}$. Since $v^{\min}$ increases in Y, inequality $\frac{p_1-\rho\beta(1-\hat{Y}^2)}{1-\rho\beta} \ge 1$ is equivalent to $\hat{Y}^2 \ge Y^B \Leftrightarrow \frac{1}{2}(1-c/\beta) \ge \frac{1-p_1}{\rho\beta} \Leftrightarrow p_1 \ge 1 - \frac{\rho}{2}(\beta - c) \triangleq P^L$, where P^L equals the p_1-boundary between REE2 and REE3 in Lemma 2.

Unlike Lemma 2, condition $\hat{v} < 1$, or $\frac{p_1 - \rho\beta(1-\hat{Y}^3)}{1-\rho\beta} < 1$, or $\hat{Y}^3 < Y^B$ leads to a different p_1-bound: $p_1(2 - \rho\beta - \rho^2\beta) - c\rho(1-\rho\beta) < 2(1-\rho\beta) \Leftrightarrow p_1 < \frac{(1-\rho\beta)(2+c\rho)}{2-\rho\beta-\rho^2\beta} = P^U$. It can be shown that $P^U \geq P^L$. Indeed, $P^U \geq P^L \Leftrightarrow (2-2\rho\beta)(2+c\rho) \geq (2-\rho\beta)^2 + \rho c(2-\rho\beta-\rho^2\beta) - \rho^2\beta(2-\rho\beta) \Leftrightarrow \rho^2\beta(2-\rho\beta) \geq \rho^2\beta^2 + \rho^2\beta c(1-\rho)$, i.e., $P^U = P^L$ when $\rho = 0, \beta = 0$, and $\rho\beta \to 1$. Otherwise, the last inequality is equivalent to $2 - \rho\beta \geq \beta + c(1-\rho)$, which holds for any $c \leq 1$.

When $P^L \leq p_1 < P^U$, both outcomes $v^{\min} < 1$ and $v^{\min} = 1$ are possible depending on the seller's choice of inventory ($\hat{Y}^3$ or $\hat{Y}^2$). Therefore, the p_1-boundary between CIE3 and CIE2 in this p_1-range is determined by comparing the seller's profit. Since in this range $\hat{Y}^3 < Y^B \leq \hat{Y}^2$ (by construction), the maximum profit in the correspondent CIE is determined by FOC, i.e., by $\hat{Y}^2$ or $\hat{Y}^3$ (no boundary maximum). Therefore, the p_1-boundary between CIE3 and CIE2 follows from the indifference condition, i.e., $\tilde{P}_2 = \left\{\tilde{p}_1 : P^L \leq \tilde{p}_1 < P^U \text{ and } \hat{\Pi}(\hat{Y}^3)|_{p_1=\tilde{p}_1} = \frac{(\beta-c)^2}{4\beta}\right\}$. Combining $\tilde{P}_2$ with the cases when $p_1 \notin [P^L, P^U)$, we have $P_2 \triangleq \max\{P^L, \min\{P^U, \tilde{P}_2\}\}$. We do not provide a closed form for $\tilde{P}_2$ since it involves cumbersome expressions and is irrelevant for further analysis.

A.6 Proof of Proposition 2 (complete info)

For CSC ($c = 0$), a profit-maximizing p_1^* under CIE3 follows from Lemma 4. In this case, profit $\hat{\Pi}$ is $\hat{\Pi} = p_1\left(1 - p_1\frac{2-\rho\beta-\rho^2\beta}{2(1-\rho\beta)}\right) + \frac{1}{2}p_1\beta(1+\rho)\left[p_1\frac{2-\rho\beta-\rho^2\beta}{2(1-\rho\beta)} - \frac{1}{2}p_1(1+\rho)\right]$. FOC in p_1 is $\frac{\partial\hat{\Pi}}{\partial p_1} = 1 - p_1\frac{2-\rho\beta-\rho^2\beta}{1-\rho\beta} + p_1\beta(1+\rho)\left[\frac{2-\rho\beta-\rho^2\beta}{2(1-\rho\beta)} - \frac{1}{2}(1+\rho)\right] = 0$, which, after collecting the terms with p_1 is $p_1\left[2(2-\rho\beta-\rho^2\beta) - (\beta+\rho\beta)(2-\rho\beta-\rho^2\beta) + (1-\rho\beta)\beta(1+\rho)^2\right] = 2(1-\rho\beta)$, where the bracket $[\cdot]$ in LHS is $[\cdot] = 4 - 2\rho\beta - \rho^2\beta - \beta$ yielding the same $p_1^* = \frac{2(1-\rho\beta)}{4-\beta(1+\rho)^2}$ as for DSC under incomplete info given in Table 1. Substitution of this p_1^* into formulas in Lemma 4 for $\hat{Y}, \hat{p}_2, \hat{v}$, and $\hat{\Pi}$ leads to the same equilibrium expressions as Y^*, p_2^*, v^*, and Π^* for DSC in Table 1.

For DSC, similarly to the proof of Proposition 1, part CIE3 of Lemma 4 with $c = w$ provides the coefficients of $Y = a_0 - a_1p_1 - a_ww, p_2 = b_1p_1 + b_ww$, and $v = d_1p_1 - d_ww : a_0 = 1, a_1 = \frac{1+\rho}{2}, a_w = \frac{1}{2}(1/\beta - \rho), b_1 = \frac{1}{2}\beta(1+\rho), b_w = \frac{1}{2}(1-\rho\beta), d_1 = \frac{2-\rho\beta-\rho^2\beta}{2(1-\rho\beta)}$, and $d_w = \frac{\rho}{2}$. The coefficients g_{11} and $-g_{22}$ in front of p_1^2 and w^2 respectively in the expression for profit $\Pi^D(p_1, w)$, given by Lemma 7, are negative and $\Pi^D(p_1, w)$ is concave quadratic. Indeed, $g_{11} = -d_1 + b_1(d_1 - a_1) = -\frac{2-\rho\beta-\rho^2\beta}{2(1-\rho\beta)} + \frac{1}{2}\beta(1+\rho)\left(\frac{2-\rho\beta-\rho^2\beta}{2(1-\rho\beta)} - \frac{1+\rho}{2}\right) = \frac{\beta(1-\rho^2)-2(2-\rho\beta-\rho^2\beta)}{4(1-\rho\beta)}$ leading to $g_{11} = -\frac{4-\beta(1+\rho)^2}{4(1-\rho\beta)} < 0$ and $-g_{22} = -b_w(a_w + d_w) = -\frac{1}{2}(1-\rho\beta)\left[\frac{1}{2}(1/\beta - \rho) + \frac{\rho}{2}\right] = -\frac{1-\rho\beta}{4\beta} < 0$. Then the unique profit-maximizing

w^* is $w^* = -\frac{g_{12}g_{10}+2g_{20}g_{11}}{4g_{11}g_{22}+g_{12}^2}$, where $g_{10} = 1+b_1(a_0-1) = 1$; $g_{12} = d_w - b_1(a_w+d_w)+b_w(d_1-a_1) = \frac{\rho}{2} - \frac{1}{2}\beta(1+\rho)\left[\frac{1}{2}(1/\beta-\rho)+\frac{\rho}{2}\right]+\frac{1}{4}\left[2-\rho\beta-\rho^2\beta-(1+\rho)(1-\rho\beta)\right] = \frac{1}{2}\left\{\rho-\frac{1}{2}(1+\rho)+\frac{1}{2}(1-\rho)\right\} = 0$, and $g_{20} = b_w(a_0-1) = 0$ yielding $w^* = 0$ and $p_1^* = -\frac{g_{10}+w^*g_{12}}{2g_{11}} = \frac{2(1-\rho\beta)}{4-\beta(1+\rho)^2} > 0$. Then $m_r^* = 1 - w^*/p_1^* = 1$ implying the main result of the Proposition.

LHS of the condition of CIE3 existence $\frac{c(1-\rho\beta)}{\beta(1-\rho)} < p_1 < P_2$ holds for $p_1 = p_1^*$ since $c = w^* = 0$ for both CSC and DSC, and RHS follows from the result of Proposition 1 for DSC ($p_1^* < 1 - \frac{\rho}{2}(\beta - c)$).

A.7 Proof of Lemma 5 (RESE, inventory-independent demand)

The proof uses the same arguments as the proof of Lemma 2, which are applied to symmetric retailers with profit (9).

RESE1 (only first-period sales) yields the same candidate RESE as for $n = 1$ (Lemma 2) in terms of $\hat{v} = p_1$, the total inventory $\hat{Y} = 1 - \hat{v}$, implying $\hat{y}^i = \frac{1}{n}(1-\hat{v})$, and total retailer profit $\hat{\Pi}^r = (p_1 - c)(1 - p_1)$, which follows from retailers' symmetry and Lemma 6. The candidate RESE1 exists iff two-period profit (9) is not increasing in y^i for any $y^i > \frac{1}{n}(1-\hat{v}) : \frac{\partial \Pi^i}{\partial y^i}\big|_{y^i=\frac{1-\hat{v}}{n}+0} \le 0$, which, using $\hat{v} = p_1$, becomes $-c - \beta\left[y^i - \frac{1}{n}(1-p_1)\right] + \beta(1-Y)\big|_{y^i=\frac{1-p_1}{n}+0} \le 0 \Leftrightarrow \beta p_1 \le c \Leftrightarrow p_1 \le c/\beta$.

RESE2 (only second-period sales) exists iff $\bar{\alpha} = 1$ and $v^{\min} = 1$, which simplifies profit (9) to $\Pi^i = -cy^i + \beta(1-Y)y^i$. FOC is $\frac{\partial \Pi^i}{\partial y^i} = -c + \beta(1-Y) - \beta y^i = 0$. By symmetry, $y^i = \frac{Y}{n}$, which leads to $\beta - c = \beta(1+\frac{1}{n})Y$ or $\hat{Y} = \frac{n}{n+1}(1 - c/\beta)$. Substitution of this $\hat{Y}$ and $\hat{y}^i = \frac{\hat{Y}}{n}$ into the expressions for p_2 and $\Pi^r = n\Pi^i$ yields $\hat{p}_2 = \beta\left[1 - \frac{n}{n+1}(1-c/\beta)\right] = \frac{1}{n+1}(\beta + cn)$ and $\hat{\Pi}^r = (\hat{p}_2 - c)\hat{Y} = \frac{\beta-c}{n+1}\frac{n(1-c/\beta)}{n+1} = \frac{n(\beta-c)^2}{\beta(n+1)^2}$. The condition of existence $v^{\min} = 1$ holds only if $\bar{\alpha} = 1$ and, by Lemma 1, if $\frac{p_1-\rho\bar{p}_2}{1-\rho\beta} \ge 1$. By customer rationality, $\bar{p}_2 = \hat{p}_2 = \frac{1}{n+1}(\beta + cn)$ leading to $p_1 - \frac{\rho}{n+1}(\beta+cn) \ge 1 - \rho\beta \Leftrightarrow p_1(n+1) \ge n+1-n\rho(\beta-c) \Leftrightarrow p_1 \ge 1 - \frac{n}{n+1}\rho(\beta-c) = P_2$.

RESE3 exists iff $\bar{\alpha} = 1$, $v^{\min} < 1$ (there are sales in both periods) and any retailer i has no incentive to deviate to sales only in the first period, i.e., $\frac{\partial \Pi^i}{\partial y^i}\Big|_{y^i=\frac{1-v^{\min}}{n}+0} > 0$. Then profit (9) is $\Pi^i = -cy^i + \frac{p_1}{n}(1-v^{\min}) + \beta(1-Y)\left[y^i - \frac{1}{n}(1-v^{\min})\right]$ with FOC $\frac{\partial \Pi^i}{\partial y^i} = 0 = -c + \beta(1-Y) - \beta\left[y^i - \frac{1}{n}(1-v^{\min})\right] \Leftrightarrow \beta - c + \frac{\beta}{n}(1-v^{\min}) = \beta(y^i + Y)$, which, divided by β, using symmetry ($y^i = \frac{Y}{n}$) and customer rationality (substitute $\bar{p}_2 = \beta(1-Y)$ into $v^{\min}$), is $Y\frac{n+1}{n} = 1 - c/\beta + \frac{1}{n}\frac{1-\rho\beta-p_1+\rho\beta(1-Y)}{1-\rho\beta} \Leftrightarrow Y\left(\frac{n+1}{n} + \frac{\rho\beta}{n(1-\rho\beta)}\right) = 1 - c/\beta + \frac{1}{n}\frac{1-p_1}{1-\rho\beta}$. Multiplication by $n(1-\rho\beta)$ leads to $Y\left[(n+1)(1-\rho\beta)+\rho\beta\right] = (1-c/\beta)n(1-\rho\beta)+1-p_1 \Leftrightarrow \hat{Y} = \frac{1+n(1-\rho\beta)-p_1-cn(1-\rho\beta)/\beta}{1+n(1-\rho\beta)}$. Substitution of $1-\hat{Y} = \frac{p_1+cn(1-\rho\beta)/\beta}{1+n(1-\rho\beta)}$ into $\hat{p}_2 = \beta(1-\hat{Y})$ and $\hat{v}^* = \frac{p_1-\rho\hat{p}_2}{1-\rho\beta}$ results in the corresponding expressions.

Condition $\hat{v} < 1$ is $p_1 + n(p_1 - \rho c) < 1 + n(1 - \rho\beta)$ or $p_1(n+1) < 1 + n[1 - \rho(\beta - c)]$ yielding $p_1 < P_2$ — the boundary with RESE2. Condition $\hat{v} \geq p_1$ is $p_1 + n(p_1 - \rho c) \geq p_1 + np_1(1 - \rho\beta)$ or $\rho c \leq p_1\rho\beta$, which holds for $\rho = 0$. For $\rho > 0$, it becomes $p_1 \geq \frac{c}{\beta}$.

Condition $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = \frac{1-\hat{v}}{n}+0} > 0$ is $-2\beta\frac{1-\hat{v}}{n} + \beta\frac{1-\hat{v}}{n} + \beta\left(1 - \frac{n-1}{n}\hat{Y}\right) > c$. Multiplication by $\frac{n}{\beta}$ leads to $n\left(1 - c/\beta\right) - (n-1)\hat{Y} > 1 - \hat{v}$, and, after the substitutions of $\hat{Y}$ and $1 - \hat{v} = \frac{1-p_1+n[1-p_1-\rho(\beta-c)]}{1+n(1-\rho\beta)}$, the last inequality, multiplied by $1+n(1-\rho\beta) > 0$, becomes $n(1-c/\beta)+n^2(1-c/\beta)(1-\rho\beta)-(n-1)(1-p_1)-n(n-1)(1-c/\beta)(1-\rho\beta) > 1-p_1+n[1-p_1-\rho(\beta-c)]$ or $n(1-c/\beta)+n(1-c/\beta)(1-\rho\beta) > 2n(1-p_1) - n\rho\beta(1-c/\beta)$, which, after simplifications, yields $p_1 > \frac{c}{\beta}$. This inequality implies that $\hat{v} = p_1$ only if $\rho = 0$. Treating n as a continuous variable, $\frac{\partial \hat{Y}}{\partial n} = \frac{(1-\rho\beta)(p_1\beta - c)}{\beta[1+n(1-\rho\beta)]^2} > 0$ since $p_1 > \frac{c}{\beta}$.

A.8 Proof of Proposition 3 (RPM with inventory-independent demand)

Similarly to the proof of Proposition 1, part RESE3 of Lemma 5 provides the coefficients of $Y = a_0 - a_1 p_1 - a_w w, p_2 = b_1 p_1 + b_w w$, and $v = d_1 p_1 - d_w w$, which, for $\gamma = 0$, are still linear functions of p_1 and w : $a_0 = 1, a_1 = \frac{1}{1+n(1-\rho\beta)}, a_w = \frac{n(1-\rho\beta)}{\beta[1+n(1-\rho\beta)]}, b_1 = \beta a_1, b_w = \beta a_w, d_1 = \frac{n+1}{1+n(1-\rho\beta)}$, and $d_w = \frac{n\rho}{1+n(1-\rho\beta)}$. The coefficients g_{11} and $-g_{22}$ in front of p_1^2 and w^2 respectively in the expression for profit $\Pi^D(p_1, w)$, given by Lemma 7, are negative and $\Pi^D(p_1, w)$ is concave quadratic. Indeed, $g_{11} = -d_1 + b_1(d_1 - a_1) = -\frac{n+1}{1+n(1-\rho\beta)} + \frac{\beta}{1+n(1-\rho\beta)}\frac{n}{1+n(1-\rho\beta)} = \frac{\beta n - (n+1)[1+n(1-\rho\beta)]}{[1+n(1-\rho\beta)]^2} < 0$ and $-g_{22} = -b_w(a_w + d_w) = -\frac{n(1-\rho\beta)}{1+n(1-\rho\beta)}\left[\frac{n(1-\rho\beta)}{\beta[1+n(1-\rho\beta)]} + \frac{n\rho}{1+n(1-\rho\beta)}\right] = -\frac{n^2(1-\rho\beta)}{\beta[1+n(1-\rho\beta)]^2} < 0$. Then the unique profit-maximizing w^* is $w^* = -\frac{g_{12}g_{10}+2g_{20}g_{11}}{4g_{11}g_{22}+g_{12}^2}$, where $g_{10} = 1 + b_1(a_0 - 1) = 1$; $g_{12} = d_w - b_1(a_w + d_w) + b_w(d_1 - a_1) = \frac{n\rho}{1+n(1-\rho\beta)} - \frac{\beta}{1+n(1-\rho\beta)}\left[\frac{n(1-\rho\beta)}{\beta[1+n(1-\rho\beta)]} + \frac{n\rho}{1+n(1-\rho\beta)}\right] + \frac{n^2(1-\rho\beta)}{[1+n(1-\rho\beta)]^2} = \frac{n[n(1-\rho\beta)(1+\rho)+\rho-1]}{[1+n(1-\rho\beta)]^2}$ and $g_{20} = b_w(a_0 - 1) = 0$ yielding $4g_{11}g_{22}+g_{12}^2 = \frac{4n^2(1-\rho\beta)\{\beta n-(n+1)[1+n(1-\rho\beta)]\}+n^2\beta[n(1-\rho\beta)(1+\rho)+\rho-1]^2}{\beta[1+n(1-\rho\beta)]^4}$, which numerator is $4n^3\beta(1-\rho\beta) - 4n^2(1-\rho\beta)(n+1)[1+n(1-\rho\beta)] + n^2\beta\left\{[1+n(1-\rho\beta)](1+\rho) - 2\right\}^2$. The second term can be written as $n^2\beta\left\{[1+n(1-\rho\beta)]^2(1+\rho)^2 - 4n(1-\rho\beta)(1+\rho) - 4\rho\right\}$ or $n^2\beta\left\{[1+n(1-\rho\beta)]^2(1+\rho)^2 - 4n(1-\rho\beta)\rho - 4\rho\right\} - 4n^3\beta(1-\rho\beta)$. Then the numerator becomes $-4n^2(1-\rho\beta)(n+1)[1+n(1-\rho\beta)] + n^2\beta\left\{[1+n(1-\rho\beta)]^2(1+\rho)^2 - 4\rho[1+n(1-\rho\beta)]\right\}$. This expression can be rearranged as $[1+n(1-\rho\beta)]\left[-4n^3(1-\rho\beta) - 4n^2\right] + n^2\beta[1+n(1-\rho\beta)]^2(1+\rho)^2$ or as $[1+n(1-\rho\beta)]^2 n^2\left[\beta(1+\rho)^2 - 4\right]$. Then $w^* = -\frac{n[n(1-\rho\beta)(1+\rho)+\rho-1]}{[1+n(1-\rho\beta)]^2}\frac{\beta[1+n(1-\rho\beta)]^2}{n^2[\beta(1+\rho)^2-4]}$ or $w^* = \frac{\beta\{n-1+\rho[1+n(1-\rho\beta-\beta)]\}}{n[4-\beta(1+\rho)^2]}$. Another form is $w^* = \frac{\beta[1+\rho(1-\rho\beta-\beta)]}{4-\beta(1+\rho)^2} - \frac{\beta(1-\rho)}{n[4-\beta(1+\rho)^2]}$, i.e., w^* increases in n with the maximum $w^*|_{n\to\infty} = \frac{\beta\{1+\rho(1-\rho\beta-\beta)\}}{4-\beta(1+\rho)^2} \in \left[\frac{\beta}{4-\beta}, \frac{\beta}{2}\right)$ for $\rho \in [0, 1)$. The last formula for w^* implies that $\frac{\partial w^*}{\partial \rho}$ is monotonic in n. Therefore, $\frac{\partial w^*}{\partial \rho} > 0$ for any n since, by Proposition 1, $\frac{\partial w^*}{\partial \rho}|_{n=1} > 0$

and $\frac{\partial w^*}{\partial \rho}|_{n\to\infty} = \frac{\beta\left[(1-\rho\beta)^2+3(1-\beta)+\beta(\rho^2+2\rho\beta+\beta-4\rho)\right]}{[4-\beta(1+\rho)^2]^2} > 0$ because $\rho^2+2\rho\beta+\beta-4\rho$ decreases in ρ and $\inf_\rho\left(\rho^2+2\rho\beta+\beta-4\rho\right) = 0$ at $\rho \to 1$.

By Lemma 7, $p_1^* = \frac{g_{10}+w^* g_{12}}{-2g_{11}} = \left\{1+\frac{\beta\{n-1+\rho[1+n(1-\rho\beta-\beta)]\}}{n[4-\beta(1+\rho)^2]}\frac{n[n(1-\rho\beta)(1+\rho)+\rho-1]}{[1+n(1-\rho\beta)]^2}\right\}\frac{-[1+n(1-\rho\beta)]^2}{2\{\beta n-(n+1)[1+n(1-\rho\beta)]\}}$, which can be written as $-\frac{\left[4-\beta(1+\rho)^2\right][1+n(1-\rho\beta)]^2+\beta\{n-1+\rho[1+n(1-\rho\beta-\beta)]\}[n(1-\rho\beta)(1+\rho)+\rho-1]}{2[4-\beta(1+\rho)^2]\{\beta n-(n+1)[1+n(1-\rho\beta)]\}}$, where the second term in the numerator equals $\beta\{n(1-\rho\beta)+1+n\rho(1-\rho\beta)+\rho-2\}[n(1-\rho\beta)+1+n\rho(1-\rho\beta)+\rho-2] = \beta\{[n(1-\rho\beta)+1](1+\rho)-2\}^2 = [n(1-\rho\beta)+1]^2\beta(1+\rho)^2-4\beta[n(1-\rho\beta)+1](1+\rho)+4\beta$. Then $p_1^* = 2\frac{-[n(1-\rho\beta)+1]^2+\beta[n(1-\rho\beta)+1](1+\rho)-\beta}{[4-\beta(1+\rho)^2]\{\beta n-(n+1)[1+n(1-\rho\beta)]\}}$, where the second term in the numerator is $n\beta(1-\rho\beta)(1+\rho)+\rho\beta = n\beta(1-\rho\beta)+\rho\beta[n(1-\rho\beta)+1]$ yielding $p_1^* = 2\frac{n\beta(1-\rho\beta)-[n(1-\rho\beta)+1][n(1-\rho\beta)+1-\rho\beta]}{[4-\beta(1+\rho)^2]\{\beta n-(n+1)[1+n(1-\rho\beta)]\}} = \frac{2(1-\rho\beta)\{n\beta-(n+1)[n(1-\rho\beta)+1]\}}{[4-\beta(1+\rho)^2]\{\beta n-(n+1)[1+n(1-\rho\beta)]\}} = \frac{2(1-\rho\beta)}{4-\beta(1+\rho)^2}$, which coincides with p_1^* for $n=1$ given in Table 1.

Given p_1^* and w^*, $Y^* = 1-a_1p_1^*-a_ww^* = 1-\frac{2(1-\rho\beta)+(1-\rho\beta)\{n-1+\rho[1+n(1-\rho\beta-\beta)]\}}{[1+n(1-\rho\beta)][4-\beta(1+\rho)^2]}$, where the numerator of the fraction is $(1-\rho\beta)\{1+n(1-\rho\beta)+\rho[1+n(1-\rho\beta)]\} = (1-\rho\beta)(1+\rho)[1+n(1-\rho\beta)]$ yielding $Y^* = 1-\frac{(1-\rho\beta)(1+\rho)}{4-\beta(1+\rho)^2} = \frac{3-\rho\beta-\beta-\rho}{4-\beta(1+\rho)^2}$, which coincides with Y^* for $n=1$ given in Table 1. Given these p_1^*, w^*, and Y^*, the equilibrium values of $p_2^* = \beta(1-Y^*)$ (by 5), $v^* = \frac{p_1^*-\rho p_2^*}{1-\rho\beta}$ (by Lemma 1 with $\bar{\alpha}=1$), and $\Pi^{D*} = \Pi^{m*}+\Pi^{r*} = p_1^*(1-v^*)+p_2^*(Y^*-1+v^*)$ (by Eqs. (2) and (3) with $D = 1-v^* < Y^*$) also do not depend on n and equal the correspondent values in Table 1.

However, since w^* depends on n, Π^{m*} and Π^{r*} also depend on n. After substitutions of equilibrium values into (2) and (3) and simplifications, $\Pi^{m*} = \frac{\beta\{n-1+\rho[1+n(1-\rho\beta-\beta)]\}(3-\rho\beta-\beta-\rho)}{n[4-\beta(1+\rho)^2]^2}$ and $\Pi^{r*} = \frac{4n-\beta\left\{4n-3-\beta(n-1)+\rho\left[4(2n+1)-\rho\beta(5n+1)-6n\beta-\rho+n\beta^2(1+\rho)^2\right]\right\}}{n[4-\beta(1+\rho)^2]^2}$, which, for $n=1$, coincide with the corresponding values in Table 1.

As to the type of RESE, recall that the actual manufacturer profit, after applying the fixed fee, is Π^{D*}, which equals the one in the case of $n=1$ (Proposition 1). Therefore, as is shown in the proof of Proposition 1, the manufacturer has no incentives to deviate from RESE3 because RESE3-profit Π^{D3*} always exceeds the profits of other equilibria.

The retailers also have no incentives to deviate from RESE3. Indeed, LHS of the condition of RESE3 existence $\frac{w^*}{\beta} < p_1^* < 1-\frac{n}{n+1}\rho(\beta-w^*) = P_2$ is $\frac{\beta\{n-1+\rho[1+n(1-\rho\beta-\beta)]\}}{n[4-\beta(1+\rho)^2]} < \frac{2\beta(1-\rho\beta)}{4-\beta(1+\rho)^2}$ or $n-1+\rho[1+n(1-\rho\beta-\beta)] < 2n(1-\rho\beta) \Leftrightarrow n[1+\rho(1-\rho\beta)-\rho\beta-2(1-\rho\beta)] < 1-\rho$ $\Leftrightarrow n(1-\rho\beta)(\rho-1) < 1-\rho$, which always holds. The right inequality is $\frac{2(1-\rho\beta)}{4-\beta(1+\rho)^2} < 1-\frac{n\rho\beta}{n+1}+\frac{n\rho}{n+1}\frac{\beta\{n-1+\rho[1+n(1-\rho\beta-\beta)]\}}{n[4-\beta(1+\rho)^2]} \Leftrightarrow \left[4-\beta(1+\rho)^2\right](n+1-n\rho\beta)+\rho\beta\{n-1+\rho[1+n(1-\rho\beta-\beta)]\} > 2(n+1)(1-\rho\beta)$, where LHS is $\left[4-\beta(1+\rho)^2\right][1+n(1-\rho\beta)]+\rho\beta\{n-1+\rho[1+n(1-\rho\beta)]-n\rho\beta\} = [1+n(1-\rho\beta)](4-\beta-2\rho\beta)+\rho\beta[n(1-\rho\beta)-1]$ and RHS is $2n(1-\rho\beta)+2-2\rho\beta = 2[1+n(1-\rho\beta)]-$

$2\rho\beta$. Then condition $p_1^* < P_2$ is $[1+n(1-\rho\beta)]\,(2-\beta-\rho\beta) > 0$, which always holds.

A.9 Profit function for inventory-dependent demand ($\gamma = 1$)

Recall that the first-period total sales are $Q = 1 - v^{\min}$ and retailer i sales are $q^i = d^i(y^i, \mathbf{y}^{-i})$, which, for $\gamma = 1$, is $y^i \frac{Q}{Y}$. The second-period sales of retailer i equal its second-period inventory $y^i\left(1-\frac{Q}{Y}\right)$. Then the general expression for retailer i profit, using (6) and (10), takes the form

$$\Pi^i = -cy^i + p_1 \frac{y^i}{Y}(1-v^{\min}) + \max\{s, \beta(1-Y)\}\left\{y^i - \frac{y^i}{Y}(1-v^{\min})\right\}. \tag{14}$$

Although this expression is continuous in all parameters and inventory y^i, it is generally not globally differentiable. Next, we consider all possible subintervals for y^i. Each subinterval results in a differentiable expression for the profit function and a qualitatively distinct market outcome.

A.9.1 No sales in the second period

Formula (6) for profit becomes $\Pi^i = (p_1 - c)y^i$, which yields a unique profit-maximizing inventory $y^i = \left(1 - v^{\min} - Y^{-i}\right)^+$, where $Y^{-i} = \sum_{j\neq i} y^j$, and the maximum $\Pi^i = (p_1 - c)\left(1 - v^{\min} - Y^{-i}\right)^+$, leading to the result similar to Lemma (6) for $n = 1$:

Lemma 8. *For given model inputs and customer expectations, retailer rationality implies that the effective domain of the inventory decision is $y^i \geq (1 - v^{\min} - Y^{-i})^+$ and $(p_1 - c)(1 - v^{\min} - Y^{-i})^+$ is the lower bound for the optimal profit.*

This lemma and the rationality of customer expectations immediately imply the following result.

Lemma 9. *In any rational expectations equilibrium, (1) $p_2 < \beta p_1$ if there are sales in the second period; (2) $Y \geq 1 - p_1$, which holds as an equality only if there are no sales in the second period; (3) $\rho\beta Y < 1 - p_1$ if there are sales in both periods and $p_2 > s$; $\rho\beta Y \geq 1 - p_1$ and $p_2 \geq c$ if there are sales only in the second period; and (4) $v^{\min} = p_1$ iff $\bar{\alpha} = 0$ or $\rho = 0$.*

Proof From Lemma 1, we have $v^{\min} = p_1$ iff $\frac{p_1 - \bar{\alpha}\rho\bar{p}_2}{1-\bar{\alpha}\rho\beta} \leq p_1$, which is equivalent to $\bar{\alpha}\rho\beta p_1 \leq \bar{\alpha}\rho\bar{p}_2$. Within feasible parameter values, the later holds iff either $\bar{\alpha} = 0$, $\rho = 0$, or $\beta p_1 \leq \bar{p}_2$. By Lemma (8), $Y \geq 1 - v^{\min}$. Thus, either of $\rho = 0$, $\bar{\alpha} = 0$ or $\beta p_1 \leq \bar{p}_2$ implies that $Y \geq 1 - p_1$. Moreover, $Y = 1 - p_1$ means there are no sales in the second period, whereas $Y > 1 - p_1$ means that these sales occur at price $p_2 < \beta p_1$ according to the market clearing condition (10).

Part 1: We conclude that $\bar{p}_2 \geq \beta p_1$ would never be rational and, in any rational expectations equilibrium, we must have $p_2 < \beta p_1$.

Part 2: By the above reasoning, $\bar{\alpha} = 0$ implies $v^{\min} = p_1$ and $Y \geq 1 - p_1$. However, $Y > 1 - p_1$ in combination with $v^{\min} = p_1$ means that there *are* second-period sales and $\bar{\alpha} = 0$ is not rational.

If $\bar{\alpha} = 1$, by part 1 and condition (10), we have $\beta(1-Y) \leq \max\{s, \beta(1-Y)\} = p_2 < \beta p_1$. Thus, $Y > 1 - p_1$ in any rational expectations equilibrium with $\bar{\alpha} = 1$.

Part 3: Because in any rational expectations equilibrium, $\bar{p}_2 = p_2$ and $\bar{\alpha} = 1$ if there are sales in the second period, Lemma 1 implies that, if there are sales in both periods, $v^{\min} < 1$, which, using (10), is equivalent to $p_1 - \rho\beta(1-Y) < 1 - \rho\beta$ or $\rho\beta Y < 1 - p_1$. If there are sales only in the second period, $p_1 - \rho\beta(1-Y) \geq 1 - \rho\beta$ or $\rho\beta Y \geq 1 - p_1$; $p_2 \geq c$ because, in this case, $\Pi^i = (p_2 - c)y^i$, and retailers are profit-maximizing.

Part 4: As $\bar{p}_2 \geq \beta p_1$ would never be rational, $v^{\min} = p_1$ can occur in a rational expectations equilibrium iff $\bar{\alpha} = 0$ or $\rho = 0$$\square$

A.9.2 Second-period sales with $p_2 > s$

If $v^{\min} > 1 - Y$ (or $y^i > 1 - v^{\min} - Y^{-i}$), there are sales in the second period. If $0 < y^i < 1 - s/\beta - Y^{-i}$, then $p_2 > s$ and the profit is $\Pi^i = -cy^i + p_1 \frac{y^i}{Y}\left(1 - v^{\min}\right) + \beta\left(1-Y\right)y^i\left(1 - \frac{1-v^{\min}}{Y}\right)$

$$= \quad y^i\left[\beta\left(1-Y\right) - c + \left(p_1 - \beta\left(1-Y\right)\right)\frac{1-v^{\min}}{Y}\right] \tag{15}$$

$$= \quad y^i\left[\beta\left(1-Y\right) - c + \beta\left(1 - v^{\min}\right) + \frac{(p_1 - \beta)(1 - v^{\min})}{Y}\right] \tag{16}$$

with $\frac{\partial \Pi^i}{\partial y^i} = y^i\left[\beta\left(-1 + \frac{1-v^{\min}}{Y}\right) - \frac{(p_1 - \beta(1-Y))(1-v^{\min})}{Y^2}\right] + \beta\left(1-Y\right) - c + \frac{[p_1 - \beta(1-Y)](1-v^{\min})}{Y}$

$$= \beta\left(1-Y\right) - c + \beta(1 - v^{\min}) - \beta y^i + (p_1 - \beta)(1 - v^{\min})(Y - y^i)/Y^2, \tag{17}$$

which, using equations $Y = y^i + Y^{-i}$ and (16), can be rewritten as

$$\frac{\partial \Pi^i}{\partial y^i} = \beta\left(1 - Y^{-i}\right) - c + \beta\left(1 - v^{\min}\right) - 2\beta y^i + (p_1 - \beta)(1 - v^{\min})\frac{Y^{-i}}{Y^2}. \tag{18}$$

The second derivative is

$$\frac{\partial^2 \Pi^i}{\partial \left(y^i\right)^2} = -2\left[\beta + (p_1 - \beta)(1 - v^{\min})\frac{Y^{-i}}{Y^3}\right]. \tag{19}$$

A.9.3 Second-period sales with $p_2 = s$

This case is possible only under oligopoly, i.e., $Y^{-i} > 0$ (for a monopolistic retailer, any $p_2 \leq c$ is not rational) and only for $v^{\min} < 1$ (there are first-period sales, otherwise profit is negative). If there are sales in the second period and $y^i \geq \left(1 - s/\beta - Y^{-i}\right)^+$ (or $Y \geq 1 - s/\beta$), then $p_2 = s$ and (14) becomes $\Pi^i = -cy^i + p_1 y^i \left(1 - v^{\min}\right)/Y + sy^i \left[1 - (1 - v^{\min})/Y\right]$

$$= -(c-s)y^i + y^i\,(p_1 - s)\left(1 - v^{\min}\right)/Y \tag{20}$$

with the derivative

$$\frac{\partial \Pi^i}{\partial y^i} = -(c-s) + \frac{Y - y^i}{Y^2}\,(p_1 - s)\left(1 - v^{\min}\right) = -(c-s) + Y^{-i}\,(p_1 - s)\left(1 - v^{\min}\right)/Y^2, \tag{21}$$

which is monotonically strictly decreasing in y^i when $v^{\min} < 1$.

A.9.4 Properties of the profit function

The following lemma provides the properties of retailer i profit Π^i, using the continuity of Π^i in y^i. The best response in the retailer game depends on $Y^{-i} = Y - y^i$ – total inventory less the inventory of retailer i. If $Y^{-i} < 1 - s/\beta$, retailer i can influence p_2. Namely, $p_2 > s$ if $y^i < 1 - s/\beta - Y^{-i}$ (no salvaging) or $p_2 = s$ if $y^i \geq 1 - s/\beta - Y^{-i}$ (salvaging). If $Y^{-i} \geq 1 - s/\beta$, salvaging is forced on retailer i, i.e., $p_2 = s$ regardless of y^i.

Lemma 10. *The profit function Π^i is such that*

(1) If $1 - s/\beta - Y^{-i} > 0$, then

(1.1) $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} - 0} < \left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} + 0}$;

(1.2) $\Pi^i(1 - s/\beta - Y^{-i}) \leq 0$ *iff*

$$\frac{(p_1 - s)\left(1 - v^{\min}\right)}{(1 - s/\beta)(c - s)} \leq 1; \tag{22}$$

(1.3) Π^i is pseudoconcave in y^i and strictly concave if $p_1 \geq \beta v^{\min}$ on the interval $(1 - v^{\min} - Y^{-i})^+ \leq y^i \leq 1 - s/\beta - Y^{-i}$;

(1.4) Π^i is strictly concave on the interval leading to $p_2 = s$, i.e. $y^i \geq 1 - s/\beta - Y^{-i}$; and

(1.5) Π^i is pseudoconcave on the interval $y^i \geq (1 - v^{\min} - Y^{-i})^+$ if either

$$\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} + 0} \leq 0 \text{ or } \left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} - 0} \geq 0.$$

(2) If $1 - s/\beta - Y^{-i} \leq 0$, Π^i is strictly concave on its entire domain $y^i \geq 0$.

Possibility of asymmetric equilibria When there are no sales in the second period, profit-maximizing inventory $y^i = \left(1 - v^{\min} - Y^{-i}\right)^+$ is determined up to a redistribution of inventory among the retailers. In this case, the model allows for a continuum of combinations of profit-maximizing y^i, satisfying $\sum_{i=1}^n y^i = Y = 1 - v^{\min}$.

When there are second period sales ($y^i > \left(1 - v^{\min} - Y^{-i}\right)^+$), parts 1.3 and 1.4 of Lemma 10 imply that in both cases $p_2 > s$ and $p_2 = s$, profit-maximizing y^i results from $\frac{\partial \Pi^i}{\partial y^i} = 0$.

When $p_2 > s$, using (17) for $\frac{\partial \Pi^i}{\partial y^i}$, for any y^i and y^j ($j \neq i$) satisfying $\frac{\partial \Pi^i}{\partial y^i} = \frac{\partial \Pi^j}{\partial y^j} = 0$ we have $\frac{\partial \Pi^i}{\partial y^i} - \frac{\partial \Pi^j}{\partial y^j} = 0 = (y^j - y^i)\left[\beta + (p_1 - \beta)(1 - v^{\min})/Y^2\right]$, yielding $y^j = y^i$ because the bracket $[\cdot]$ is always positive. Indeed, $[\cdot] > 0 \Leftrightarrow p_1(1 - v^{\min}) + \beta[Y^2 - (1 - v^{\min})] > 0$. As $v^{\min} \geq p_1$, by part 2 of Lemma 9, $Y^2 > (1-p_1)(1-v^{\min})$. Then $p_1(1-v^{\min}) + \beta[Y^2 - (1-v^{\min})] > (1-v^{\min})[p_1 - \beta p_1] \geq 0$.

When there are sales in both periods ($v^{\min} < 1$) and $p_2 = s$, the first equation in (21) implies that any y^i and y^j ($j \neq i$), satisfying $\frac{\partial \Pi^i}{\partial y^i} = \frac{\partial \Pi^j}{\partial y^j} = 0$, are such that $\frac{\partial \Pi^i}{\partial y^i} - \frac{\partial \Pi^j}{\partial y^j} = 0 = (y^j - y^i)(p_1 - s)(1 - v^{\min})/Y^2$, i.e., $y^j = y^i$ because $(p_1 - s)(1 - v^{\min})/Y^2 > 0$.

A.10 Proof of Proposition 4 (RESE1-3, inventory-dependent demand, $\gamma = 1$)

The proposition exhaustively covers all market outcomes without salvaging: only first-period sales (RESE1), only second-period sales (RESE2), and two-period sales (RESE3). Logically, these outcomes are mutually exclusive but it is not obvious *a priori* that they cannot exist under the same model inputs. In the course of the proof we establish that these outcomes do not overlap in the sense of their necessary and sufficient conditions. RESE definition (§4.1) rely on the notion of a symmetric equilibrium for given customer expectations. The structure of such an equilibrium is one of the major sources of necessary and sufficient conditions. Another source is the rationality of customer expectations. We first classify the outcomes by the presence of second-period sales.

First-period sales: RESE1 By Lemma 8, the absence of second-period sales and retailer rationality imply that the best response in a symmetric equilibrium occurs with $Y = 1 - v^{\min}$. Customer rationality demands that $\bar{\alpha} = 0$ and $v^{\min} = p_1$ implying that the candidate RESE is described by $\hat{v} = p_1, \hat{Y} = 1 - \hat{v}$, and, therefore, $\hat{\alpha} = 0$ and total retailer profit is $\hat{\Pi}^r = (p_1 - c)(1 - p_1)$. This implies that $\frac{n-1}{n}\hat{Y} = \frac{n-1}{n}(1 - p_1) < 1 - p_1 < 1 - \frac{s}{\beta}$ and condition of part 1 of Lemma 10 is satisfied.

Since, by part 1.3 of Lemma 10, Π^i is pseudoconcave on the interval $(1 - v^{\min} - Y^{-i})^+ \leq y^i <$

$1 - s/\beta - Y^{-i}$, the candidate RESE exists iff

(i) there is a local maximum of Π^i at $y^i = 1 - \hat{v} - \frac{n-1}{n}\hat{Y} = \hat{Y}/n$ and

(ii) the profit Π^i at this maximum is greater than at a potential local maximum on the interval $y^i > 1 - \frac{s}{\beta} - \frac{n-1}{n}\hat{Y}$.

Condition (i) is equivalent to $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1-\hat{v}-\frac{n-1}{n}\hat{Y}+0} \leq 0$. As $y^i = \frac{1}{n}(1-p_1)$, the last inequality, using (17), becomes $\beta\hat{v} - c + p_1 - \beta\hat{v} + \frac{1}{n}(1-p_1)\left[-(p_1 - \beta\hat{v})\frac{1}{1-\hat{v}}\right] \leq 0$, which, after the substitution for $\hat{v} = p_1$ and multiplication by n, takes the form $np_1 - p_1(1-\beta) \leq nc$ or $p_1 \leq \frac{nc}{\beta+n-1} = P_1$. We showed that this condition is necessary.

Condition (ii) is satisfied if Π^i is nonincreasing for $y^i > 1 - s/\beta - \frac{n-1}{n}\hat{Y}$. Because Π^i is concave on this interval by part 1.4 of Lemma 10, it is nonincreasing if $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1-s/\beta-\frac{n-1}{n}\hat{Y}+0} \leq 0$. The latter, using (21), can be written as

$$-c + s + \frac{n-1}{n}\frac{1-p_1}{(1-s/\beta)^2}(p_1 - s)(1-p_1) \leq 0 \quad \text{or} \quad \frac{n-1}{n}\frac{(p_1-s)(1-p_1)^2}{(c-s)(1-s/\beta)^2} \leq 1. \tag{23}$$

As $p_1 > s/\beta$, we have $\frac{(1-p_1)^2}{(1-s/\beta)^2} < 1$, and (23) is implied by $(n-1)(p_1 - s) \leq n(c-s)$. The latter holds because, by (already proved as necessary) condition $p_1 \leq P_1$, $n(c-s) \geq (n-1+\beta)p_1 - ns = (n-1)(p_1-s) + \beta p_1 - s > (n-1)(p_1-s)$. Therefore, condition $p_1 \leq P_1$ is necessary and sufficient for the existence of RESE1.

There are second-period sales: RESE2 or 3 When there are second-period sales, a symmetric equilibrium $Y = \hat{Y} > 1 - v^{\min}$, by Lemma 10, is an internal maximum of the profit function for each retailer. Using (18) for $\frac{\partial \Pi^i}{\partial y^i}$ with $y^i = Y/n$ and $Y^{-i} = \frac{n-1}{n}Y$, FOC $\frac{\partial \Pi^i}{\partial y^i} = 0$ is $0 = \beta\left(1 - \frac{n-1}{n}Y\right) - c + \beta(1-v^{\min}) - 2\beta\frac{Y}{n} + (p_1 - \beta)(1-v^{\min})\frac{n-1}{n}\frac{Y}{Y^2} = -\beta\frac{n+1}{n}Y - c + \beta(2 - v^{\min}) + (p_1-\beta)(1-v^{\min})\frac{n-1}{n}\frac{1}{Y}$. Multiplication of the last expression by $-\frac{n}{\beta(n+1)}Y$ yields

$$Y^2 - Y\frac{n}{n+1}\left(2 - v^{\min} - \frac{c}{\beta}\right) - \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)(1 - v^{\min}) = 0. \tag{24}$$

Equation (24) along with the relation between $v^{\min}$ and Y from Lemma 1 and inequality $Y > 1 - p_1$ (from part 2 of Lemma 9) provide the necessary conditions for any equilibria with sales in the second period and $p_2 = \beta(1-Y) > s$.

Consider $v^{\min}$ as a function of Y. For rational expectations $\bar{\alpha} = 1$ and $\bar{p}_2 = p_2 = \beta(1-Y)$, denote the mapping from Y to $v^{\min}$ resulting from Lemma 1 as function

$$v_1^{\min}(Y) \triangleq \max\left\{p_1,\ \min\left\{\frac{p_1 - \rho\beta(1-Y)}{1-\rho\beta},\ 1\right\}\right\}. \tag{25}$$

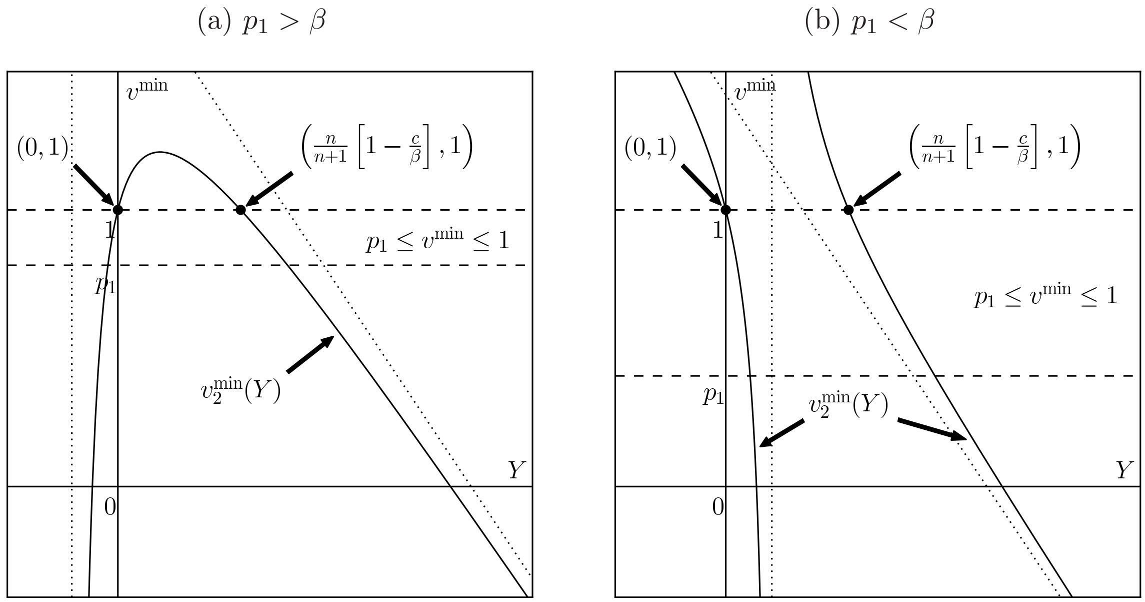

Figure 2: Possible appearance of $v_2^{\min}(Y)$ and the relevant range of $v^{\min}$

When $\rho > 0$, this function is increasing and piecewise linear with two breakpoints. It is straightforward to check that the first break-point occurs exactly at $Y = 1 - p_1$ whereas the second at $Y = \frac{1-p_1}{\rho\beta}$. When $\rho = 0$, $v_1^{\min} \equiv p_1$.

Equation (24) yields another mapping from Y to $v^{\min}$:

$$v_2^{\min}(Y) \triangleq 1 - \frac{Y^2 - Y\frac{n}{n+1}(1 - c/\beta)}{Y\frac{n}{n+1} + \frac{n-1}{n+1}(p_1/\beta - 1)}. \tag{26}$$

When $p_1 \neq \beta$ and $n > 1$, this function is a hyperbola with a vertical asymptote $Y = \frac{n-1}{n}(1 - p_1/\beta)$ and an asymptote with a negative slope $-\frac{n+1}{n}$. When $Y = 0$ or $Y = \frac{n}{n+1}(1 - c/\beta)$, $v_2^{\min}(Y) = 1$. Implicit differentiation of (24) yields $2Y - \frac{n}{n+1}\left(2 - v_2^{\min} - \frac{c}{\beta}\right) + Y\frac{n}{n+1}\frac{\partial v_2^{\min}}{\partial Y} + \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)\frac{\partial v_2^{\min}}{\partial Y} = 0$ resulting in $(n-1)(p_1 - \beta)\left.\frac{\partial v_2^{\min}}{\partial Y}\right|_{Y=0} = n(\beta - c)$.

When $p_1 > \beta$ and $n > 1$, the vertical asymptote is to the left of $Y = 0$ implying that points $(0, 1)$ and $\left(\frac{n}{n+1}[1 - c/\beta], 1\right)$ in the $(Y, v^{\min})$-plane belong to the same branch of the hyperbola, see Figure 2 (a), where a solid curve is $v_2^{\min}(Y)$ and dotted lines represent its asymptotes. In this case, $\left.\frac{\partial v_2^{\min}}{\partial Y}\right|_{Y=0} > 0$ and it must be true that $\frac{\partial v_2^{\min}}{\partial Y} < 0$ for all $Y \geq \frac{n}{n+1}(1 - c/\beta)$. Relevant equilibrium candidates can only be on the downward-sloping segment of $v_2^{\min}(Y)$ to the right of $Y = \frac{n}{n+1}(1 - c/\beta)$ and in the range $p_1 \leq v^{\min} \leq 1$, depicted in dashed lines.

When $p_1 < \beta$ and $n > 1$, the vertical asymptote is to the right of $Y = 0$ implying that points

$(0,1)$ and $\left(\frac{n}{n+1}\left[1-\frac{c}{\beta}\right],1\right)$ belong to different branches of the hyperbola, see Figure 2 (b). We have $\frac{\partial v_2^{\min}}{\partial Y}<0$ for all Y, and the entire left branch is irrelevant because the vertical asymptote is to the left of $Y=1-p_1$. Indeed, $\frac{n-1}{n}\left(1-\frac{p_1}{\beta}\right)<1-p_1$ is equivalent to $np_1-(n-1)\frac{p_1}{\beta}<1$ which always holds for $p_1<\beta$. All possible equilibrium candidates are again on the downward-sloping segment of $v_2^{\min}(Y)$ to the right of $Y=\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)$ and in the range $p_1\leq v^{\min}\leq 1$.

When $p_1=\beta$ or $n=1$, the relevant part of $v_2^{\min}(Y)$ is decreasing linear: $v_2^{\min}(Y)=2-\frac{c}{\beta}-\frac{n+1}{n}Y$, which also satisfies $v_2^{\min}\left(\frac{n}{n+1}\left[1-\frac{c}{\beta}\right]\right)=1$. Thus, regardless of n and the relation between p_1 and β, the geometric structure of potential equilibrium candidates is essentially the same.

RESE2: There are no sales in the first period at a RESE iff $\hat{v}=1$. The geometric structure described above implies that such an equilibrium can be realized only if $v_1^{\min}(Y)$ intersects with $v_2^{\min}(Y)$ at a point corresponding to $\hat{Y}=\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)$, i.e., $v_1^{\min}(\hat{Y})=1$ or $p_1-\rho\beta\left[1-\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)\right]\geq 1-\rho\beta$, which is equivalent to $p_1\geq P_2=1-\frac{n}{n+1}\rho(\beta-c)$. This necessary condition is also sufficient for RESE2. Indeed, given that $v_1^{\min}(\hat{Y})=1$, the equilibrium values are in the form of RESE2, $\hat{p}_2=\beta\left[1-\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)\right]=\frac{nc+\beta}{n+1}>c>s$ and $y^i=\hat{Y}/n$ indeed delivers the best response of retailer i because $\hat{Y}=\frac{n}{n+1}(1-c/\beta)<1-c/\beta<1-s/\beta$ and $\left.\frac{\partial\Pi^i}{\partial y^i}\right|_{y^i=1-s/\beta-\frac{n-1}{n}\hat{Y}+0}=-c+s<0$ implying, by part 1.5 of Lemma 10, that Π^i is pseudoconcave.

The description of RESE2 is completed by substituting $\hat{p}_2$, $\hat{Y}$ and $\hat{v}$ into (16): $\hat{\Pi}^i=\frac{\hat{Y}}{n}\left[\frac{\beta+nc}{n+1}-c\right]=\frac{1}{n+1}\left(1-\frac{c}{\beta}\right)\left[\frac{\beta+nc}{n+1}-c\right]=\frac{(\beta-c)}{(n+1)\beta}\frac{\beta+nc-nc-c}{n+1}=\frac{(\beta-c)^2}{(n+1)^2\beta}$.

The p_1-ranges in RESE1 and 2 do not overlap because the minimal lower bound for p_1 in RESE2, which corresponds to $n\to\infty$, exceeds the maximal upper bound in RESE1 (at $n=1$): $1-\rho(\beta-c)>c/\beta\ \Leftrightarrow\ \beta(1-\rho\beta)>c(1-\rho\beta)$.

RESE3: In this case, $\hat{Y}>1-\hat{v}$ (there are second-period sales) and $p_1\leq\hat{v}<1$ (there are first-period sales) with $\hat{v}=p_1$ only if $\rho=0$. Translating this into the geometric structure described above, necessary conditions for RESE3 are $v_1^{\min}\left(\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)\right)<1$ and $v_2^{\min}(1-p_1)>p_1$. The first condition is equivalent to the negation of $p_1\geq P_2$, i.e., the strict upper limit of p_1-range for RESE3. The second condition ensures that $v_2^{\min}(Y)$ intersects $v_1^{\min}(Y)$ for $Y>1-p_1$ and is equivalent to $1-\frac{(1-p_1)^2-(1-p_1)\frac{n}{n+1}(1-c/\beta)}{(1-p_1)\frac{n}{n+1}+\frac{n-1}{n+1}(p_1/\beta-1)}>p_1$, and, since $(1-p_1)\frac{n}{n+1}+\frac{n-1}{n+1}(p_1/\beta-1)=\frac{1-p_1}{n+1}+\frac{(n-1)p_1(1-\beta)}{(n+1)\beta}>0$, to $(1-p_1)\left[(1-p_1)\frac{n}{n+1}+\frac{n-1}{n+1}(p_1/\beta-1)-(1-p_1)+\frac{n}{n+1}(1-c/\beta)\right]>0$. Collecting like terms inside $[\cdot]$ yields $(n-1+\beta)p_1>nc$ which is the negation of the necessary and sufficient condition $p_1\leq P_1$ of RESE1, i.e., the strict lower limit of p_1-range for RESE3.

Given that necessary condition $P_1 < p_1 < P_2$ holds and there are sales in both periods, the candidate point for the equilibrium, by Lemma 1, satisfies

$$\hat{v} = \frac{p_1 - \rho\beta(1-\hat{Y})}{1-\rho\beta} \tag{27}$$

and $\hat{v} \in [p_1, 1)$. Substitution for $v^{\min} = \hat{v}$ into (24) results in the following equation for $\hat{Y}$:

$$Y^2 - Y\frac{n}{n+1}\left(2 - \frac{p_1 - \rho\beta(1-Y)}{1-\rho\beta} - \frac{c}{\beta}\right) - \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)\left(1 - \frac{p_1 - \rho\beta(1-Y)}{1-\rho\beta}\right) = 0,$$

which, after collecting the terms with Y, becomes

$$\begin{aligned} &Y^2\left(1 + \frac{n}{n+1}\frac{\rho\beta}{1-\rho\beta}\right) - \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)\left(1 - \frac{p_1 - \rho\beta}{1-\rho\beta}\right) \\ &-Y\left[\frac{n}{n+1}\left(2 - \frac{p_1 - \rho\beta}{1-\rho\beta} - \frac{c}{\beta}\right) - \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)\frac{\rho\beta}{1-\rho\beta}\right] = 0. \end{aligned} \tag{28}$$

The coefficient in front of Y^2 is $1 + \frac{n}{n+1}\frac{\rho\beta}{1-\rho\beta} = \frac{n+1-\rho\beta}{(n+1)(1-\rho\beta)}$, and the coefficient in front of Y is $-\frac{1}{(n+1)(1-\rho\beta)}\left\{n\left[2 - 2\rho\beta - p_1 + \rho\beta - (1-\rho\beta)c/\beta\right] - (n-1)\left(p_1/\beta - 1\right)\rho\beta\right\}$, where the first term in the bracket $\{\cdot\}$ is $n[2-\rho\beta-p_1-(1-\rho\beta)c/\beta] = n(1-\rho\beta)(1-c/\beta)+n(1-p_1)$. Then multiplication of (28) by $\frac{\beta(n+1)(1-\rho\beta)}{\beta(n+1-\rho\beta)}$ results in

$$Y^2 - \frac{(\beta-c)n(1-\rho\beta) + \beta(1-p_1)n - (p_1-\beta)\rho\beta(n-1)}{\beta(n+1-\rho\beta)}Y - \frac{(p_1-\beta)(1-p_1)(n-1)}{\beta(n+1-\rho\beta)} = 0. \tag{29}$$

By geometric structure under condition $P_2 < p_1 < P_1$, the larger root of this equation does belong to the region $Y > 1 - p_1$ and the smaller root is irrelevant.

The conditions for RESE3 will become necessary and sufficient if (29), (27), and $P_1 < p_1 < P_2$ are complemented with the conditions guaranteeing that the larger root $\hat{Y}$ of (29) is such that $\hat{Y} < 1 - \frac{s}{\beta}$ (implying $\hat{p}_2 > s$ and included as a condition of the proposition) and either

(a) the profit Π^i of retailer i deviating from this RESE so that $p_2 = s$ (the total inventory is greater than $1 - \frac{s}{\beta}$) has no maximum for $Y > 1 - \frac{s}{\beta}$, or

(b) if $\check{\Pi}^i = \max_{y^i} \Pi^i$ exists for $Y > 1 - \frac{s}{\beta}$, then inequality $\check{\Pi}^i \le \hat{\Pi}^i$ holds.

Since, by part 1.4 of Lemma 10, Π^i is concave for $y^i \ge 1 - s/\beta - \frac{n-1}{n}\hat{Y}$ (or, equivalently, $Y \ge 1 - s/\beta$), Π^i is nonincreasing for $y^i \ge 1 - s/\beta - \frac{n-1}{n}\hat{Y}$ iff $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - \frac{n-1}{n}\hat{Y} + 0} \le 0$. Thus, the latter condition is equivalent to (a). Using (21) with $v^{\min} = \hat{v}$, $Y^{-i} = \frac{n-1}{n}\hat{Y}$, and $Y = 1 - s/\beta$, this condition is $-c + s + \frac{n-1}{n}\frac{\hat{Y}}{(1-s/\beta)^2}\left(p_1 - s\right)\left(1 - \hat{v}\right) \le 0$, yielding condition (a) of the proposition.

If $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} + 0} > 0$, then, since $\frac{\partial \Pi^i}{\partial y^i}$ becomes negative for sufficiently large Y by (21), $\check{\Pi}^i = \max_{y^i} \Pi^i$ exists for $Y > 1 - s/\beta$. Then RESE exists if $\hat{\Pi}^i \ge \check{\Pi}^i$ (condition (b)). To provide the

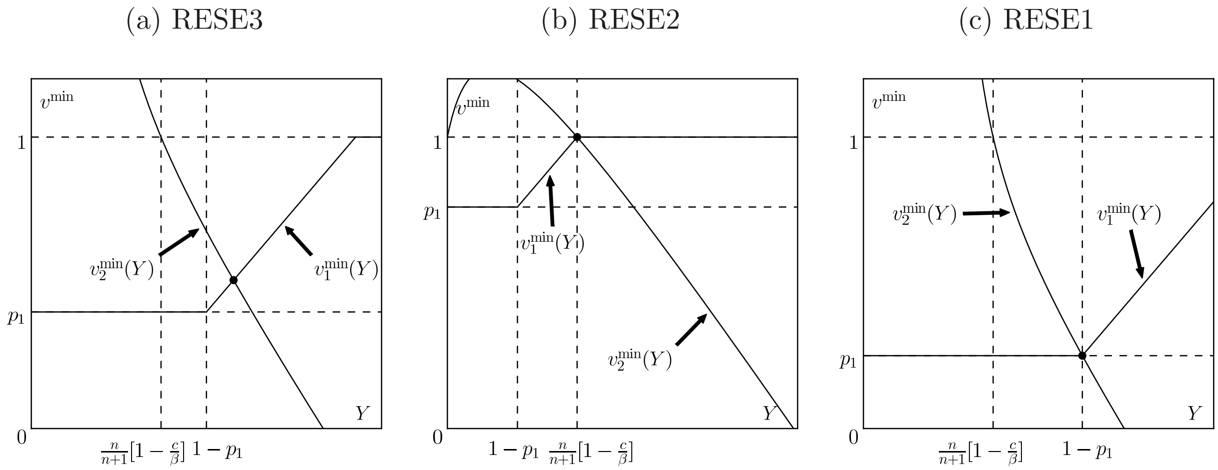

Figure 3: Changes in equilibrium structure from RESE3 to RESE2 and RESE1

expression for $\check{\Pi}^i$, denote the maximized deviator's inventory decision by $\check{y}^i \triangleq \arg\max \Pi^i > \frac{1}{n}\hat{Y}$. As a result of this deviation, the total inventory is $\check{Y} = \check{y}^i + \frac{n-1}{n}\hat{Y}$. When the number of bargain hunters B is large, then, using (21) with $v^{\min} = \hat{v}$, we obtain FOC in $\check{Y}$: $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i=\check{y}^i} = 0 = -(c-s) + \frac{n-1}{n}\frac{\hat{Y}}{\check{Y}^2}(p_1 - s)(1-\hat{v})$, which yields $\check{Y} = \sqrt{\frac{n-1}{n}\frac{\hat{Y}(p_1-s)(1-\hat{v})}{c-s}}$. In general, $\check{Y} = \min\left\{\hat{v} - \frac{s}{\beta} + B, \sqrt{\frac{n-1}{n}\frac{\hat{Y}(p_1-s)(1-\hat{v})}{c-s}}\right\}$, where $\hat{v} - \frac{s}{\beta} + B$ is the total number of the second-period customers. Substitution of $Y = \check{Y}$ and $y^i = \check{Y} - \frac{n-1}{n}\hat{Y}$ into equation for profit (20) yields $\check{\Pi}^i = \left(\check{Y} - \frac{n-1}{n}\hat{Y}\right)\left[-(c-s) + (p_1 - s)(1-\hat{v})/\check{Y}\right]$. When $\check{Y}$ is determined by FOC, $\check{\Pi}^i$ is $\check{\Pi}^i = \left\{\sqrt{\frac{n-1}{n}\frac{\hat{Y}(p_1-s)(1-\hat{v})}{c-s}} - \frac{n-1}{n}\hat{Y}\right\} \times \left\{-(c-s) + \frac{(p_1-s)(1-\hat{v})}{\sqrt{\frac{n-1}{n}\frac{\hat{Y}(p_1-s)(1-\hat{v})}{c-s}}}\right\}$, which, after factoring out $\frac{n-1}{n}\hat{Y}$ from the first curly bracket and $c-s$ from the second one, becomes $\check{\Pi}^i = \frac{n-1}{n}\hat{Y}(c-s)\left\{\sqrt{\frac{n}{(n-1)}\frac{(p_1-s)(1-\hat{v})}{(c-s)\hat{Y}}} - 1\right\}^2$ or $\check{\Pi}^i = \left\{\sqrt{(p_1-s)(1-\hat{v})} - \sqrt{\frac{n-1}{n}\hat{Y}(c-s)}\right\}^2$. Expression for $\hat{\Pi}^r$ follows from (6) and Lemma 1.

We complete the proof by a simple observation that equilibrium characteristics are continuous on the boundaries between RESE1 and 3 as well as RESE2 and 3. Figure 3 (a) depicts a typical configuration of $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ when RESE3 exists, whereas subplots (b) and (c) depict this configuration at the points of change to RESE2 and 1, respectively.

RESE3 continuously changes into RESE2 as the intersection point of $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ representing RESE3 moves toward the point $\left(\frac{n}{n+1}\left[1-\frac{c}{\beta}\right], 1\right)$ on $v_2^{\min}(Y)$ representing RESE2. The latter point is to the left of all possible candidates for RESE3 located on $v_2^{\min}(Y)$ implying that, in RESE3, $\hat{Y} \geq \frac{n}{n+1}\left[1-\frac{c}{\beta}\right]$. Similarly, RESE3 continuously changes into RESE1 as the intersection point of $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ moves toward $v_1^{\min}(Y)$'s break-point $(1-p_1, p_1)$ (representing RESE1). The continuity of $\hat{\Pi}^r$ follows from the continuity of the expression for Π^i, given by (14), in all the parameters and continuity of $\hat{v}$ and $\hat{Y}$ (using $y^i = \frac{1}{n}\hat{Y}$).

A.11 Proof of Corollary 1 (RESE3, $\rho = 0$, second-period sales at loss)

For $\beta = 1$ and $\rho = 0$, p_1-range in RESE3 is $c < p_1 < 1$. Thus, RESE1 and 2 cannot be realized. By the proof of Proposition 4, $\hat{Y}^3 \leq 1 - c/\beta$ is equivalent to $\hat{v} \geq v_2^{\min}(1 - c/\beta)$ because $v_2^{\min}(Y)$, given by (26), is decreasing in the relevant range of Y. Using $\beta = 1$ and $Y = 1 - c$ in (26), we get $v_2^{\min}(1-c) = 1 - \frac{(1-c)^2 - (1-c)^2 \frac{n}{n+1}}{(1-c)\frac{n}{n+1} + \frac{n-1}{n+1}(p_1 - 1)} = 1 - \frac{(1-c)^2}{n(p_1-c)+1-p_1}$. Thus, under conditions of the corollary, $\hat{p}_2 \geq c$ iff $\hat{v} = p_1 \geq 1 - \frac{(1-c)^2}{n(p_1-c)+1-p_1}$. Rearranging this inequality we obtain $\frac{(1-c)^2}{n(p_1-c)+1-p_1} \geq 1 - p_1$, and solving for n we get $n \leq \frac{1}{p_1-c}\left(\frac{(1-c)^2}{1-p_1} - (1-p_1)\right) = \frac{2-c-p_1}{1-p_1} = 2 + \frac{p_1-c}{1-p_1}$.

A.12 Proof of Proposition 5 (RPM with inventory-dependent demand)

Since the values in Table 1 correspond to REE3 with two-period sales, formula for w^* may follow from equation (29) for $\hat{Y}$ in RESE3 by plugging in $\hat{Y} = Y^*$ and $p_1 = p_1^*$ from Table 1, i.e.,

$$\left(\frac{3-\rho\beta-\beta-\rho}{4-\beta(1+\rho)^2}\right)^2 - a_1\frac{3-\rho\beta-\beta-\rho}{4-\beta(1+\rho)^2} - a_0 = 0 \Leftrightarrow a_1 = \frac{3-\rho\beta-\beta-\rho}{4-\beta(1+\rho)^2} - a_0\frac{4-\beta(1+\rho)^2}{3-\rho\beta-\beta-\rho}, \quad (30)$$

where $a_1 = \left.\frac{(\beta-w)n(1-\rho\beta)+\beta(1-p_1^*)n-(p_1^*-\beta)\rho\beta(n-1)}{\beta(n+1-\rho\beta)}\right|_{p_1^*=\frac{2(1-\rho\beta)}{4-\beta(1+\rho)^2}}$ and $a_0 = \left.\frac{(p_1^*-\beta)(1-p_1^*)(n-1)}{\beta(n+1-\rho\beta)}\right|_{p_1^*=\frac{2(1-\rho\beta)}{4-\beta(1+\rho)^2}}$. The expressions for $1-p_1^*$ and $p_1^*-\beta$ are $1-p_1^* = \frac{2-\beta-\rho^2\beta}{4-\beta(1+\rho)^2}$ and $p_1^*-\beta = \frac{2(1-\rho\beta)-\beta[4-\beta(1+\rho)^2]}{4-\beta(1+\rho)^2}$. Then the numerators of a_1 and a_0 are $\beta n(1-\rho\beta) - wn(1-\rho\beta) + \frac{(2-\beta-\rho^2\beta)\beta n - \rho\beta(n-1)\{2(1-\rho\beta)-\beta[4-\beta(1+\rho)^2]\}}{4-\beta(1+\rho)^2}$ $= \beta(n-\rho\beta) - wn(1-\rho\beta) + \frac{(2-\beta-\rho^2\beta)\beta n - \rho\beta(n-1)2(1-\rho\beta)}{4-\beta(1+\rho)^2}$, and $\frac{(n-1)\{2(1-\rho\beta)-\beta[4-\beta(1+\rho)^2]\}(2-\beta-\rho^2\beta)}{[4-\beta(1+\rho)^2]^2}$. Substitution into (30) multiplied by $\frac{\beta(n+1-\rho\beta)}{n(1-\rho\beta)}$ leads to $w^* = \frac{\beta(n-\rho\beta)}{n(1-\rho\beta)} + \frac{(2-\beta-\rho^2\beta)\beta n - \rho\beta(n-1)2(1-\rho\beta)}{n(1-\rho\beta)[4-\beta(1+\rho)^2]} - \frac{(3-\rho\beta-\beta-\rho)\beta(n+1-\rho\beta)}{n(1-\rho\beta)[4-\beta(1+\rho)^2]} + \frac{(n-1)\{2(1-\rho\beta)-\beta[4-\beta(1+\rho)^2]\}(2-\beta-\rho^2\beta)}{n(1-\rho\beta)(3-\rho\beta-\beta-\rho)[4-\beta(1+\rho)^2]} = \frac{b_1+b_2-b_3+b_4}{n(1-\rho\beta)(3-\rho\beta-\beta-\rho)[4-\beta(1+\rho)^2]}$, where $b_1 = \beta(n-\rho\beta)(3-\rho\beta-\beta-\rho)\left[4-\beta(1+\rho)^2\right]$, $b_3 = (3-\rho\beta-\beta-\rho)^2\beta(n+1-\rho\beta)$, $b_2 = (3-\rho\beta-\beta-\rho)\left[\left(2-\beta-\rho^2\beta\right)\beta n - 2\rho\beta(n-1)(1-\rho\beta)\right]$, and b_4 is the numerator of the last fraction in w^*. Rewrite b_i as $b_1 = \beta(n-\rho\beta)[(1-\rho\beta)+(2-\beta-\rho)]\left[2(1-\rho\beta)+2-\beta-\beta\rho^2\right] = \beta(1-\rho\beta)(n-\rho\beta)[2(1-\rho\beta)+2(2-\beta-\rho)+(2-\beta-\beta\rho^2)] + (2-\beta-\rho)(2-\beta-\beta\rho^2)\beta(n-\rho\beta)$, $b_2 = -2\rho\beta(n-1)(1-\rho\beta)(3-\rho\beta-\beta-\rho) + [(1-\rho\beta)+2-\beta-\rho]\left(2-\beta-\beta\rho^2\right)\beta n$, $b_3 = (1-\rho\beta)\beta(3-\rho\beta-\beta-\rho)^2 + n\beta(1-\rho\beta+2-\beta-\rho)^2$, and $b_4 = \left(2-\beta-\rho^2\beta\right)(n-1)\left\{2(1-\rho\beta)(1-\beta) - \beta(2-\beta-\beta\rho^2)\right\}$. The sum of the terms in all b_i without multiplier $1-\rho\beta$ is $\left(2-\beta-\beta\rho^2\right)(2-\beta-\rho)\beta(2n-\rho\beta) -$

$n\beta\,(2-\beta-\rho)^2 - \beta(n-1)(2-\beta-\beta\rho^2)^2 = -\beta n\,[\rho(1-\rho\beta)]^2 - \rho\beta^2\left(2-\beta-\beta\rho^2\right)(2-\beta-\rho) + \beta(2-\beta-\beta\rho^2)^2 = -\beta n\,[\rho(1-\rho\beta)]^2 + \beta\left(2-\beta-\beta\rho^2\right)(2-\beta)(1-\rho\beta)$. Then w^* becomes $w^* = \frac{\{\cdot\}}{n[4-\beta(1+\rho)^2](3-\rho\beta-\beta-\rho)}$, where $\{\cdot\} = \beta(n-\rho\beta)[2(1-\rho\beta)+2(2-\beta-\rho)+(2-\beta-\beta\rho^2)]$ $-2\rho\beta(n-1)\,(3-\rho\beta-\beta-\rho)$ $+\left(2-\beta-\beta\rho^2\right)\beta n - \beta\,(3-\rho\beta-\beta-\rho)^2$ $-n\beta\,[1-\rho\beta+2(2-\beta-\rho)]$ $+2(1-\beta)(n-1)\left(2-\beta-\rho^2\beta\right)$ $+\beta\left(2-\beta-\beta\rho^2\right)(2-\beta) - \beta n\rho^2(1-\rho\beta)$. The sum of the terms with the multiplier βn is $2(1-\rho\beta)+2(2-\beta-\rho)+(2-\beta-\beta\rho^2) - 2\rho\,(3-\rho\beta-\beta-\rho) + \left(2-\beta-\beta\rho^2\right) - 1$ $+\rho\beta - 2(2-\beta-\rho)$ $-2(2-\beta-\rho^2\beta) - \rho^2(1-\rho\beta) = (1+\rho\beta)(1+\rho^2) - 2\rho(3-\rho\beta)$. The sum of the terms without the multiplier βn is $4(n-1) - \beta(1+\rho^2)(2n-\rho\beta^2) + 2\rho^2\beta^3 - 6\rho\beta^2 - \rho^2\beta + 6\rho\beta(2-\rho\beta) + \beta$. Then $w^* = \frac{4(n-1)-\beta(1+\rho^2)(2n-\rho\beta^2)+2\rho^2\beta^3-6\rho\beta^2-\rho^2\beta+6\rho\beta(2-\rho\beta)+\beta+\beta n\left[(1+\rho\beta)(1+\rho^2)-2\rho(3-\rho\beta)\right]}{n[4-\beta(1+\rho)^2](3-\rho\beta-\beta-\rho)} = \frac{4(n-1)+\beta\left\{1+\rho\beta^2(1+\rho)^2-6\rho\beta-\rho^2+6\rho(2-\rho\beta)-n\left[(1-\rho\beta)(1+\rho^2)+2\rho(3-\rho\beta)\right]\right\}}{n[4-\beta(1+\rho)^2](3-\rho\beta-\beta-\rho)}$ leading to the most compact formula: $w^* = \frac{4(n-1)+\beta\left\{1+\rho\left[\beta^2(1+\rho)^2-\rho+6(2-\rho\beta-\beta)\right]-n\left[(1-\rho\beta)(1+\rho)^2+4\rho\right]\right\}}{n[4-\beta(1+\rho)^2](3-\rho\beta-\beta-\rho)}$. Using intermediate expressions, w^* can be written as $w^* = \frac{1}{n\left[4-\beta(1+\rho)^2\right]}\left(\rho\beta(2-\rho\beta-\beta) + \frac{(n-1)(1-\rho\beta)(4-\beta-2\rho\beta-\rho^2\beta)}{3-\rho\beta-\beta-\rho}\right)$, which is the expression in the proposition statement. It is easy to observe from this expression that, for $n=1$, the bracket $(3-\rho\beta-\beta-\rho)$ cancels out and w^* coincides with the corresponding formula in Table 1, i.e., $w^*|_{n=1} = \frac{\rho\beta(2-\rho\beta-\beta)}{4-\beta(1+\rho)^2}$.

Another expression for w^* follows from collecting terms with n in the numerator, which leads to the two-term formula where, in the first term, the bracket $n\left[4-\beta\,(1+\rho)^2\right]$ cancels out, i.e.,

$$w^* = \frac{1-\rho\beta}{3-\rho\beta-\beta-\rho} - \frac{g(\rho,\beta)}{n}, \tag{31}$$

and $g(\rho,\beta) = \frac{(2-\rho\beta)^2(1-\rho\beta)-\beta\left\{1+\rho\left[4-\rho(1+\beta-2\beta^2)-6\beta+\beta^2\right]\right\}}{\left[4-\beta(1+\rho)^2\right](3-\rho\beta-\beta-\rho)}$. Function $g(\rho,\beta)>0$ for any $\beta\in[0,1]$ and $\rho\in[0,1)$ since the denominator is positive and it can be shown that the numerator is also positive. Indeed, the derivative of the numerator w.r.t. ρ is $\beta\left[-3\rho^2\beta^2+(2-4\beta^2+12\beta)\rho-12+6\beta-\beta^2\right]$, where $[\cdot]$ is concave quadratic in ρ with the discriminant that simplifies to $4(1+12\beta-4\beta^2-6\beta^3+\beta^4)>0$, i.e., this derivative is positive only between the roots. It can be shown that the smaller root, $\left(1-2\beta^2+6\beta-\sqrt{1+12\beta-4\beta^2-6\beta^3+\beta^4}\right)/(3\beta^2)$, takes its minimum equal to one at $\beta=1$, i.e., the range between the roots is irrelevant, and the derivative of the numerator w.r.t. ρ is negative for any $\rho\in[0,1)$. Therefore, the numerator of $g(\rho,\beta)$ is always positive if it is nonnegative at $\rho=1$, which is $4(1-\beta^3+3\beta^2-3\beta)$. The sum $\beta^3-3\beta^2+3\beta$ is maximal at the double root $\beta_{1,2}=1$ of the FOC $3\beta^2-6\beta+3=0$ because $\beta=1$ is, obviously, an inflection point, and this term increases from 0 to 1 when $\beta\in[0,1]$. Hence, $g(\rho,\beta)>0$ for any $\rho\in[0,1)$ and $\beta\in[0,1]$. Then, by (31), w^* increases in n, and $w^*|_{n\to\infty} = \frac{1-\rho\beta}{3-\rho\beta-\beta-\rho}$, which does not decrease in ρ since $\frac{\partial w^*|_{n\to\infty}}{\partial\rho} = \frac{(1-\beta)^2}{(3-\rho\beta-\beta-\rho)^2}$ and, therefore, for $\beta < 1$, $w^*|_{n\to\infty} \in \left[\frac{1}{3-\beta}, \frac{1}{2}\right)$ when $\rho \in [0,1)$, and $w^*|_{n\to\infty} \equiv \frac{1}{2}$ for $\beta = 1$. Combination of this result with monotonicity of w^* in n and $w^*|_{n=1} = \frac{\rho\beta(2-\rho\beta-\beta)}{4-\beta(1+\rho)^2} \in [0, \frac{1}{2})$ (by the proof of Proposition 1) implies that $w^* \in [0, \frac{1}{2}]$ for any $n \geq 1$. w^* increases in ρ except for $w^*|_{n\to\infty}$ at $\beta = 1$ when it is constant because, by (31), $\frac{\partial w^*}{\partial\rho}$ is monotonic in n, $\frac{\partial w^*}{\partial\rho}|_{n=1} > 0$ (by Proposition 1) and, by above, $\frac{\partial w^*|_{n\to\infty}}{\partial\rho} = \frac{(1-\beta)^2}{(3-\rho\beta-\beta-\rho)^2} \geq 0$.

Inequality $w^*|_{\gamma=1} > w^*|_{\gamma=0}$, by (30) and Proposition 3, simplifies to $\frac{f(\rho,\beta)(n-1)}{n\left[4-\beta(1+\rho)^2\right](3-\rho\beta-\beta-\rho)} > 0$, which, for $n \geq 2$ is equivalent to $f(\rho,\beta) > 0$ (denominator is positive), where $f(\rho,\beta) = 6\rho\beta^2 - 2\rho^2\beta^3 - \rho\beta^3 - 4\beta - \rho^3\beta^3 - 8\rho\beta + 5\rho^2\beta^2 + \beta^2 + 4$, which decreases in ρ since $\frac{\partial f}{\partial\rho} = -\beta\left[\beta^2(1-\rho)^2 + 2(1-\rho\beta)^2 + 6(1-\rho\beta)(1-\beta)\right] < 0$. Therefore, $f(\rho,\beta) > 0$ for any $\rho \in [0,1)$ if $f(1,\beta) \geq 0$, where $f(1,\beta) = 4+12\beta^2-4\beta^3-12\beta$ decreases in β, i.e., $\min_{\beta\in[0,1]} f(1,\beta) = f(1,1) = 0$.

The condition of RESE3 existence $P_1 < p_1^*$ holds for any $n \geq 1$ since P_1 decreases in n and $P_1|_{n=1} < p_1^*$ by Proposition 1. Condition $p_1^* < P_2$ holds because it holds for $n = 1$ (by Proposition 1) and, as we show below, because P_2 is monotonic in n and $p_1^* < P_2$ holds for $n \to \infty$. First, we show that $p_1^* < P_2|_{n\to\infty}$, which, using (31) for $c = w^*|_{n\to\infty}$ is $\frac{2(1-\rho\beta)}{4-\beta(1+\rho)^2} < (1-\rho\beta)\left(1 + \frac{\rho}{3-\rho\beta-\beta-\rho}\right)$ $\Leftrightarrow 2(3-\rho\beta-\beta) - 2\rho < (3-\rho\beta-\beta)\left[4-\beta(1+\rho)^2\right] \Leftrightarrow [3-\beta(1+\rho)]\left[\beta(1+\rho)^2 - 2\right] < 2\rho$. The LHS is concave quadratic in β and its maximum follows from FOC: $\frac{\partial \text{LHS}}{\partial\beta} = 0 \Leftrightarrow 2\beta(1+\rho)^3 = 3(1+\rho)^2 + 2(1+\rho)$ leading to $\beta^{\max} = \frac{5+3\rho}{2(1+\rho)^2}$. Since $\beta^{\max} \in (1, \frac{5}{2}]$ when $\rho \in [0,1)$, condition $p_1^* < P_2|_{n\to\infty}$ holds for any $\beta \in [0,1]$ if it holds for $\beta = 1$, which is $(2-\rho)\left[\rho(2+\rho) - 1\right] < 2\rho \Leftrightarrow \rho(3-\rho^2) < 2$. Since $\sup_\rho \text{LHS} = 2$ when $\rho \to 1$, condition $p_1^* < P_2|_{n\to\infty}$ holds for any $\rho \in [0,1)$ and $\beta \in [0,1]$.

In order to show the monotonicity of P_2 in n, we treat n as continuous and consider $\frac{\partial P_2}{\partial n} = -\frac{\rho\beta}{(n+1)^2} + \frac{\rho w^*}{(n+1)^2} + \frac{n\rho}{n+1}\frac{\partial w^*}{\partial n} = -\frac{\rho}{(n+1)^2}\left[\beta - w^* - n(n+1)\frac{\partial w^*}{\partial n}\right]$, where, by (31), $\frac{\partial w^*}{\partial n} = \frac{g(\rho,\beta)}{n^2}$. Then $\frac{\partial P_2}{\partial n} = -\frac{\rho}{(n+1)^2}\left[\beta - \frac{1-\rho\beta}{3-\rho\beta-\beta-\rho} + \frac{g(\rho,\beta)}{n} - (n+1)\frac{g(\rho,\beta)}{n}\right] = -\frac{\rho}{(n+1)^2}\left[\beta - \frac{1-\rho\beta}{3-\rho\beta-\beta-\rho} - g(\rho,\beta)\right]$, i.e., the sign of $\frac{\partial P_2}{\partial n}$ does not depend on n. Therefore, $p_1^* < P_2$ holds for any $n \geq 1$.

Note that under condition $P_1 < p_1 < P_2$, RESE3 exists for any $\rho \in [0,1)$ if condition (a) holds at $\rho = 0$ since w^* is minimal at $\rho = 0$, and LHS of condition (a) is maximal. The latter is true because, by Proposition 1, p_1^* and Y^* are decreasing in ρ, whereas $1 - v^*$ attains minimum for $\rho \in (0,1)$ and $v^*|_{\rho=0} > v^*|_{\rho=1}$. By (30), $w^*|_{\rho=0} = \frac{n-1}{n(3-\beta)}$. Then condition (a) is $\frac{n-1}{n}\left(\frac{2}{4-\beta} - s\right)\frac{2-\beta}{4-\beta}\frac{3-\beta}{4-\beta} \leq \left(\frac{n-1}{n(3-\beta)} - s\right)(1-s/\beta)^2$. For $s = 0$ this condition does not depend on n : $(4-\beta)^3 - 2(2-\beta)(3-\beta)^2 \geq 0$ and holds for any $\beta \in [0,1]$ because it holds at the boundaries of the range $[0,1]$ and the roots of the derivative of LHS $\frac{4}{3} \pm \frac{1}{3}\sqrt{34}$ are not in this range.

A.13 RPM for salvage value $s > 0$

The result below shows that when $s > 0$ and the number of bargain hunters is $B > 0$, in the p_1-range of RESE3, there may exist one more form of RESE with sales in both periods and $\hat{p}_2 = s$.

Proposition 6 **(Two-period sales,** $\hat{p}_2 = s$**).** *RESE with* $\hat{\alpha} = 1, \hat{p}_2 = s, \hat{v} = \frac{p_1 - \rho s}{1 - \rho\beta}, \hat{Y} = \min\left\{1 - \frac{s}{\beta} + B, \frac{n-1}{n}\frac{(p_1-s)(1-\hat{v})}{c-s}\right\}$, *and* $\hat{\Pi}^r = (p_1 - s)(1 - \hat{v}) - (c - s)\hat{Y}$ *exists iff one of the following holds:*

(a) *salvaging is forced on retailers, i.e.,* $\frac{n-1}{n}\hat{Y} \geq 1 - \frac{s}{\beta}$*;*

(b) *condition (a) does not hold, and* $\left(\beta\left(1 - \frac{s}{\beta}\right)^2 + (p_1 - \beta)(1 - \hat{v})\right)\frac{n-1}{n}\frac{\hat{Y}}{c + \beta\hat{v} - 2s} \geq \left(1 - \frac{s}{\beta}\right)^2$*;*

(c) *conditions (a) and (b) do not hold, and*

(c.1) *if* $1 - \frac{s}{\beta} + B < \frac{n-1}{n}\frac{(p_1-s)(1-\hat{v})}{c-s}$*, then* $n - 1 \geq \frac{c-s}{p_1-c}$*;*

(c.2) $\hat{Y} > 1 - \frac{s}{\beta}$*, and there are no real roots of the equation*

$$2Y^3 - \left(2 - \hat{v} - c/\beta + \frac{n-1}{n}\hat{Y}\right)Y^2 + (1 - p_1/\beta)(1 - \hat{v})\frac{n-1}{n}\hat{Y} = 0 \tag{32}$$

in the interval $(1-\hat{v}, 1-\frac{s}{\beta})$*, or there is only one real root of (32)* $\tilde{Y} \in \left(1-\hat{v}, 1-\frac{s}{\beta}\right)$ *and* $\frac{1}{n}\hat{\Pi}^r \geq \tilde{\Pi}^i(\tilde{Y})$*, where* $\tilde{\Pi}^i(\tilde{Y})$ *is the maximum profit of a firm deviating from this RESE in such a way that* $p_2 > s$*.*

Unlike RESE1-3, RESE4 cannot exist for $n = 1$ because a monopolist would not have an incentive to overinvest in this setting. This can be seen, e.g., from the expression for $\hat{Y}$. The larger n, the easier retailers find themselves in RESE with $\hat{p}_2 = s$. Condition (a) means that $p_2 = s$ regardless of supply y^i because the total inventory of other retailers ($\frac{n-1}{n}\hat{Y}$) is enough for the salvaging outcome. Similar to RESE3, conditions (b) and (c) correspond to different attractiveness of deviation from RESE4 by decreasing inventory. Condition (b) means that the deviator profit monotonically increases in inventory, i.e., for the inputs that satisfy (b), RESE4 is stable with respect to small parameter changes when p_1 is sufficiently far from the boundary. Condition (c.1) results from a possibility for a single retailer to deviate to the "no second-period sales" outcome when the total inventory is restricted by a small number of bargain hunters. When this condition holds, the deviator's profit is not greater than the equilibrium one. The first part of condition (c.2) – no real roots of (32) in the interval $\left(1 - \hat{v}, 1 - \frac{s}{\beta}\right)$ – means that the deviator's profit has no local maxima with $p_2 > s$, whereas inequality $\frac{1}{n}\hat{\Pi}^r \geq \tilde{\Pi}^i(\tilde{Y})$ requires that when such a maximum exists at $y^i = \tilde{Y} - \frac{n-1}{n}\hat{Y}$, it does not exceed the profit under RESE4. The inputs where RESE4 exists only by the second part of (c.2) are close to the boundary of RESE4 existence where this equilibrium may be unstable with respect to parameter misestimation. Conditions (a)-(c) hold if $c - s$ is sufficiently small, i.e., the cost is largely compensated by salvaging any excess units, which makes this outcome attractive for the retailers.

Proposition 6 implies a necessary condition $\hat{v} < 1$ (there are first-period sales). This condition is equivalent to the upper bound $p_1 < 1 - \rho(\beta - s) \triangleq P_4$ signifying that a high MSRP precludes salvaging outcome. Alternatively, this condition yields an upper bound on the customer's discount factor: $\rho < (1 - p_1)/(\beta - s)$. As long as the product is durable enough for $1 - p_1 < \beta - s$ to hold, highly strategic (with ρ near one) customers guarantee that the salvaging outcome is impossible. Since $P_4 < P_2$ (the bound that separates RESE2 and 3), P_4 separates RESE4 and 3.

The possibility of salvaging outcome raises the questions: What is the maximum DSC profit Π^{D4*} under RESE4 when $s > 0$, and what is the manufacturer strategy if $\Pi^{D4*} < \Pi^{D3*}$? The proposition and discussion below answer these questions.

Proposition 7. *When the second-period price is $s = const$ such that $0 < s < \frac{\beta(1-\rho\beta)}{2-\rho\beta-\beta} \triangleq \bar{s}$ and the number of customers with valuation s is $B > 0$, the wholesale price $w^* = s + \frac{(n-1)[1-\rho\beta-s(1-\rho)]^2}{4n(1-s/\beta+B)(1-\rho\beta)}$, which increases in n and decreases in ρ, and $p_1^* = \frac{1}{2}\left[1 - \rho\beta + s(1+\rho)\right]$ lead to $v^* = \frac{1-\rho\beta+s(1-\rho)}{2(1-\rho\beta)}$, $Y^* = 1 - s/\beta + B$, and the profit of DSC $\Pi^{D4*} = \frac{[1-\rho\beta-s(1-\rho)]^2}{4(1-\rho\beta)} + s(1 - s/\beta + B)$, which is less than Π^{D*} for $n = 1$ provided in Table 1 iff $sB < \frac{\beta(1-\rho\beta)(1+\rho)^2}{4[4-\beta(1+\rho)^2]} - \frac{s(1+\rho)}{2} + \frac{s^2(4-\beta(1+\rho)^2)}{4\beta(1-\rho\beta)}$. For these equilibrium values, condition (a) of RESE4 existence holds iff $B \geq \frac{1-s/\beta}{n-1}$.*

The salvage value in this proposition is bounded from above by $\bar{s}$ to focus on interesting cases. As shown in the proof, $\bar{s} > \beta/2$, i.e., for $\beta = 1$, $\bar{s}$ exceeds the optimal manufacturer's one-period price, which is implausible for salvage value. This bound is equivalent to the requirement $v^* > p_1^*$ (or $s < p_1^*\beta$) for any $0 < \rho < 1$ assuring a non-trivial role of strategic customers in RESE4.

Unlike the studies with exogenously fixed retailer cost, the proposition considers a limited number of bargain hunters because the manufacturer's optimal profit is unbounded when $B \to \infty$ and customer valuations exceed the manufacturer's unit cost. This effect can be seen from the expressions in Proposition 7 for Π^{D4*} and w^*, where $w^* \to s$ when $B \to \infty$, implying $Y \to \infty$.

Unlike previous cases (Propositions 1, 3, and 5), w^* decreases in ρ because, for a fixed unit cost, retailers reduce the inventory in ρ while the manufacturer keeps it at the maximum level.

The bound on the profit from sales to the bargain hunters sB shows that when this profit is relatively high, *the total DSC profit under RPM and salvage sales may exceed the two-period profit of DSC with one retailer*, which, by Proposition 1, is higher than the profit of CSC.

On the other hand, the lower bound on B, which is equivalent to condition (a) of RESE4 existence, shows that RESE4 may exist for a large number of retailers and small s and B . Then the condition for sB, which does not depend on n, may hold implying that profit Π^{D4*} is less than Π^{D*} given in Table 1. In this case, the manufacturer has at least two options to avoid RESE4. First, the wholesale price can be set sufficiently higher than s, e.g., to satisfy condition (b) of RESE3 existence in Proposition 4. However, since such a wholesale price is suboptimal, DSC profit may be less than the one of CSC. Another option follows from retailers' indifference among equilibria due to the fixed fee. Therefore, the manufacturer can "bribe" the retailers by a marginal decrease of the fixed fee for ignoring the opportunity of salvage sales. Since this decrease can be arbitrary small, the profit of DSC will still exceed the one of CSC.

A.14 Proof of Proposition 6 (RESE4)

We start by identifying candidate solutions for a symmetric equilibrium with given expectations. When $p_2 = s$, the equilibrium is possible only with sales in both periods, and rationality requires that $v^{\min} < 1$ and $\bar{\alpha} = 1$. We rule out uninteresting cases by considering $s < \beta p_1$. If this condition does not hold, it can be shown that, in a two-period RESE, the second-period price cannot exceed s, $v_0^{\min} = p_1$, and strategic customer behavior has no effect on any market outcome.

By parts 1.4 and 2 of Lemma 10, the profit function is strictly concave when $y^i \geq \left(1 - s/\beta - Y^{-i}\right)^+$ and, by part 1.1, the optimum cannot occur at $y^i = 1 - s/\beta - Y^{-i}$. Then the candidate is found either from FOC or it equals the total number of buyers, $1 - s/\beta + B$ ($1 - v^{\min}$ in the first period, $v^{\min} - s/\beta + B$ in the second one), if this number is less than the local maximum. Using (21) for $\frac{\partial \Pi^i}{\partial y^i}$ and, by symmetry, $Y^{-i} = \frac{n-1}{n} Y$, $0 = \frac{\partial \Pi^i}{\partial y^i} = -(c - s) + \frac{Y^{-i}}{Y^2} (p_1 - s) \left(1 - v^{\min}\right) = -(c - s) + \frac{n-1}{nY} (p_1 - s) \left(1 - v^{\min}\right)$. The unique solution is $\hat{Y} = \frac{n-1}{n} \frac{(p_1 - s)\left(1 - v^{\min}\right)}{c-s}$, where, by rationality of expectations and Lemma 1, $v^{\min} = \hat{v} = \frac{p_1 - \rho s}{1 - \rho\beta}$. In general, $\hat{Y} = \min\left\{1 - s/\beta + B, \frac{n-1}{n} \frac{(p_1-s)(1-\hat{v})}{c-s}\right\}$. The total equilibrium profit is $\hat{\Pi}^r = -c\hat{Y} + p_1(1 - \hat{v}) + s\left[\hat{Y} - (1 - \hat{v})\right] = -(c - s)\hat{Y} + (p_1 - s)(1 - \hat{v})$, which is the expression for $\hat{\Pi}^r$ in the proposition.

We now analyze when the candidate point is indeed a RESE with $\hat{p}_2 = s$, and start by checking that it is within the ranges $p_1 \le \hat{v} < 1$ and $\hat{Y} \ge 1 - s/\beta$, which provide necessary conditions for RESE existence. The second condition is the domain restriction of §A.9.3. It is equivalent to $\hat{p}_2 = s$ and follows from either of the conditions (a), (b), and (c) in the proposition statement. Since the equilibrium cannot result in $\hat{Y} = 1 - s/\beta$, by part 1.1 of Lemma 10, the second condition is strengthened to $\hat{Y} > 1 - s/\beta$ under which cases (a), (b), and (c) become exhaustive. Since $1 - s/\beta > 0$, the resulting strict positivity of $\hat{Y}$ implies that $\hat{v} < 1$. Similarly to RESE3, $\hat{v} = p_1$ if $\rho = 0$, and it can be shown that $\hat{v} > p_1$ if $\rho > 0$. Indeed, $\hat{v} > p_1$ is equivalent to $\frac{p_1 - \rho s}{1 - \rho\beta} > p_1 \Leftrightarrow p_1 - \rho s > p_1 - p_1\rho\beta \Leftrightarrow -\rho s > -p_1\rho\beta \Leftrightarrow p_1 > s/\beta$, which always holds in this problem.

It remains to establish that the exact conditions ensuring that $\hat{Y}/n$ is the global optimum of the profit function are indeed provided by the exhaustive (under condition $\hat{Y} > 1 - s/\beta$) cases (a)-(c).

Condition (a), i.e., $\frac{n-1}{n}\hat{Y} \ge 1 - \frac{s}{\beta}$, means, by (5), that $p_2 = s$ independently of the inventory decisions of individual retailers. By part 2 of Lemma 10, the profit function is globally strictly concave in this case and $\hat{Y}/n$ is indeed its unique global maximum.

In case (b) of the proposition, condition (a) does not hold, i.e., $p_2 = s$ may or may not hold depending on the decisions of individual retailers. Nevertheless, the maximum of the profit is unique and occurs when $p_2 = s$ as long as the profit function is strictly increasing in the interval corresponding to $p_2 > s$. This is ensured by $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} - 0} \ge 0$, which, by part 1.5 of Lemma 10, implies pseudoconcavity of the profit function. Using (18), the last condition is

$$\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} - 0} = \beta\left(1 - Y^{-i}\right) - c + \beta\left(1 - v^{\min}\right) - 2\beta\left(1 - \frac{s}{\beta} - Y^{-i}\right) + \frac{(p_1 - \beta)(1 - v^{\min})Y^{-i}}{(1 - s/\beta)^2} \ge 0,$$

which, after collecting the terms and substituting $Y^{-i} = \frac{n-1}{n}\hat{Y}$ and $v^{\min} = \hat{v}$, can be rewritten as $\left(\beta + \frac{(p_1 - \beta)(1 - \hat{v})}{(1 - s/\beta)^2}\right)\frac{n-1}{n}\hat{Y} \ge c + \beta\hat{v} - 2s$, yielding condition (b).

In case (c) of the proposition, condition (b) does not hold, i.e. $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} - 0} < 0$. Then, there exists a local maximum of Π^i without or with the sales in the second period and $p_2 > s$. In other words, there exists such an inventory decision $\tilde{y}^i$ of a deviating retailer that

$$\tilde{y}^i \triangleq \arg\max\left\{\Pi^i(y^i) \mid y^i \in \left[\max\left\{0, 1 - \hat{v} - \frac{n-1}{n}\hat{Y}\right\}, 1 - s/\beta - \frac{n-1}{n}\hat{Y}\right)\right\}$$

or, denoting $\tilde{Y} \triangleq \tilde{y}^i + \frac{n-1}{n}\hat{Y}$, $\tilde{Y} \in \left[\max\left\{1 - \hat{v}, \frac{n-1}{n}\hat{Y}\right\}, 1 - s/\beta\right)$. Then the equilibrium with

$\hat{p}_2 = s$ exists only if

$$\tilde{\Pi}^i \triangleq \Pi^i(\tilde{y}^i) \leq \hat{\Pi}^i = \hat{\Pi}^r/n. \tag{33}$$

Consider this condition at the left boundary of the range for y^i. If $\tilde{y}^i = 0$, then $\tilde{\Pi}^i = 0$ and (33) holds trivially. If $\tilde{y}^i = 1 - \hat{v} - \frac{n-1}{n}\hat{Y}$, then $\tilde{\Pi}^i = \left(1 - \hat{v} - \frac{n-1}{n}\hat{Y}\right)(p_1 - c)$. Note that both $\tilde{\Pi}^i$ and $\hat{\Pi}^i$ decrease in $\hat{Y}$, which is intuitive for $\tilde{\Pi}^i$ because, for a smaller $\hat{Y}$, it takes less "effort" (in terms of reduced individual inventory) for a deviator to achieve the outcome with the first-period sales only. As a result, both $\tilde{y}^i$ and $\tilde{\Pi}^i$ are greater for smaller $\hat{Y}$. For $\hat{\Pi}^i$, a smaller $\hat{Y}$, ceteris paribus, leads to a smaller second-period loss, i.e., when $\hat{Y}$ is less than the total inventory obtained from FOC for *individual* retailer profit, the total, and therefore, individual profits of symmetric retailers are greater. This effect results from retailers market share competition and inventory-dependent demand in the first period.

The case $\hat{Y} = \frac{n-1}{n}\frac{(p_1-s)(1-\hat{v})}{c-s}$ leads to $\tilde{y}^i = (1-\hat{v})\left[1 - \left(\frac{n-1}{n}\right)^2 \frac{p_1-s}{c-s}\right]$. Then, by §A.9.1, there are no sales in the second period and $\tilde{\Pi}^i = (1-\hat{v})\left[1 - \left(\frac{n-1}{n}\right)^2 \frac{p_1-s}{c-s}\right](p_1 - c)$. After substitutions for $\tilde{\Pi}^i$ and $\hat{\Pi}^i$, and multiplication of both sides by $\frac{n^2}{(1-\hat{v})(p_1-c)}$, condition (33) becomes $n^2 - (n-1)^2 \frac{p_1-s}{c-s} \leq \frac{p_1-s}{p_1-c}$, which always holds. Indeed, let $g(n) \triangleq n^2 - (n-1)^2\frac{p_1-s}{c-s}$. Then $g'(n) = 2n - 2(n-1)\frac{p_1-s}{c-s} = 2\left[-n\frac{p_1-c}{c-s} + \frac{p_1-s}{c-s}\right]$ and $g''(n) = -2\frac{p_1-c}{c-s} < 0$. Therefore, the unique maximum of g, defined by the condition $g'(n) = 0$, is $n_{\max} = \frac{p_1-s}{p_1-c}$ and

$$g_{\max} = g(n_{\max}) = \left(\frac{p_1 - s}{p_1 - c}\right)^2 - \left(\frac{p_1 - s}{p_1 - c} - 1\right)^2 \frac{p_1 - s}{c - s} = \frac{p_1 - s}{p_1 - c}\left[\frac{p_1 - s}{p_1 - c} - \frac{(c-s)^2}{p_1 - c}\frac{1}{c - s}\right] = \frac{p_1 - s}{p_1 - c}.$$

If $\hat{Y} = 1 - \frac{s}{\beta} + B$ (B is small), $\tilde{y}^i = 1 - \hat{v} - \frac{n-1}{n}\left(1 - \frac{s}{\beta} + B\right)$, which is greater than for $\hat{Y} = \frac{n-1}{n}\frac{(p_1-s)(1-\hat{v})}{c-s}$, leading to a greater $\tilde{\Pi}^i = \left[1 - \hat{v} - \frac{n-1}{n}\left(1 - \frac{s}{\beta} + B\right)\right](p_1 - c)$. RESE4 profit is $\hat{\Pi}^i = \frac{1}{n}\left[(p_1 - s)(1 - \hat{v}) - (c - s)\left(1 - \frac{s}{\beta} + B\right)\right] = \frac{1}{n}\left[(p_1 - c)(1 - \hat{v}) - (c - s)\left(\hat{v} - \frac{s}{\beta} + B\right)\right]$. Then, multiplied by $\frac{n}{p_1-c}$, condition (33) is $(1-\hat{v})n-(n-1)\left(1 - s/\beta + B\right) \leq (1-\hat{v}) - \frac{c-s}{p_1-c}\left(\hat{v} - s/\beta + B\right)$ $\Leftrightarrow -(n-1)(\hat{v} - s/\beta + B) \leq -\frac{c-s}{p_1-c}\left(\hat{v} - s/\beta + B\right) \Leftrightarrow n - 1 \geq \frac{c-s}{p_1-c}$, which is condition (c.1).

Finally, the RESE with $\hat{p}_2 = s$ may also exist if there exists an internal local maximum $\Pi^i(\tilde{y}^i) \leq \hat{\Pi}^i$ with $\tilde{y}^i = \tilde{Y} - \frac{n-1}{n}\hat{Y}$ such that $\max\left\{1 - \hat{v}, \frac{n-1}{n}\hat{Y}\right\} < \tilde{Y} < 1 - \frac{s}{\beta}$ and $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = \tilde{y}^i} = 0$. In this case, formula (16) from §A.9.2 yields the expression for $\tilde{\Pi}^i$ in condition (c.2):

$$\tilde{\Pi}^i = \left(\tilde{Y} - \frac{n-1}{n}\hat{Y}\right)\left[\beta\left(1 - \tilde{Y}\right) - c + \beta\left(1 - \hat{v}\right) + \frac{(p_1 - \beta)(1 - \hat{v})}{\tilde{Y}}\right],$$

where $\tilde{Y}$ is a zero of the profit function derivative (18), which, in this case, is

$$0 = \frac{\partial \Pi^i}{\partial y^i} = \beta\left(1 - \frac{n-1}{n}\hat{Y}\right) - c + \beta(1-\hat{v}) - 2\beta\left(Y - \frac{n-1}{n}\hat{Y}\right) + (p_1 - \beta)(1-\hat{v})\frac{n-1}{n}\frac{\hat{Y}}{Y^2}.$$

After multiplication by $-Y^2/\beta$ this equation becomes

$$2Y^3 + a_2 Y^2 + a_0 = 0, \tag{34}$$

which is equation (32) if one substitutes the coefficients $a_2 \triangleq c/\beta - (1-\hat{v}) - \left(1 + \frac{n-1}{n}\hat{Y}\right) = -(1-\hat{v}) - (1 - c/\beta) - \frac{n-1}{n}\hat{Y} < 0$, and $a_0 \triangleq (1 - p_1/\beta)(1-\hat{v})\frac{n-1}{n}\hat{Y}$. Since, by part 1.3 of Lemma 10, the profit function of the deviating retailer is pseudoconcave on $(1-\hat{v}-\frac{n-1}{n}\hat{Y})^+ \le y^i \le 1 - s/\beta - \frac{n-1}{n}\hat{Y}$, Eq. (34) may have at most one root on this interval.

A cubic equation with real coefficients has at least one and up to three real roots. If the roots are irrelevant, there is no internal maximum and a boundary maximum cannot exceed $\hat{\Pi}^i$ as shown above. If there is a relevant root, a comparison of $\hat{\Pi}^i$ and $\tilde{\Pi}^i$ determines the existence of RESE.

A.15 Proof of Proposition 7 (RPM with RESE4)

When the second-period price is $s = const > 0$, the total number of regular buyers is $1 - s/\beta$ ($1 - v^{\min}$ in the first period and $v^{\min} - s/\beta$ in the second one). DSC profit is $\Pi^{D4} = p_1\left(1 - v^{\min}\right) + s\left[Y - \left(1 - v^{\min}\right)\right]$, where, by Lemma 1, $v^{\min} = \frac{p_1 - \rho s}{1 - \rho\beta}$ if, for $\rho > 0, s < p_1\beta$ (i.e., $\frac{p_1-\rho s}{1-\rho\beta} > p_1$) and $v^{\min} = p_1$ otherwise (the values of p_1 leading to $v^{\min} = 1$ are obviously suboptimal here). Since Π^{D4} is unrestricted in Y, the manufacturer sets w^* leading to a maximal $Y = 1 - s/\beta + B$, which, using the expression for $\hat{Y}$ in Proposition 6, is $1 - s/\beta + B = \frac{n-1}{n}\frac{p_1 - s}{w - s}(1-\hat{v})$. This equation yields $w(p_1, \hat{v}) = s + \frac{(n-1)(p_1-s)(1-\hat{v})}{n(1-s/\beta+B)}$. Then $\Pi^{D4} = (p_1 - s)\left(1 - v^{\min}\right) + s\left(1 - s/\beta + B\right)$, and assuming $s < p_1\beta$, FOC $\frac{\partial \Pi^{D4}}{\partial p_1} = 0 = 1 - \frac{2p_1 - s(1+\rho)}{1-\rho\beta}$ yields $p_1^* = \frac{1}{2}\left[1 - \rho\beta + s(1+\rho)\right]$. Then condition $s < p_1\beta$ becomes $2s < \beta\left[1 - \rho\beta + s(1+\rho)\right] \Leftrightarrow s < \frac{\beta(1-\rho\beta)}{2-\rho\beta-\beta} = \bar{s}$ ($\inf_\rho \bar{s} = \frac{\beta}{2}$ since $\frac{\partial \bar{s}}{\partial \rho} = -\frac{\beta^2(1-\beta)}{(2-\rho\beta-\beta)^2} \le 0$), which holds by the assumption of the proposition. This p_1^* leads to $v^* = \frac{1-\rho\beta+s(1-\rho)}{2(1-\rho\beta)}$. Since $p_1^* - s = \frac{1}{2}\left[1 - \rho\beta - s(1-\rho)\right] = (1 - v^*)(1 - \rho\beta)$, we have $\Pi^{D4*} = \frac{\left[1-\rho\beta-s(1-\rho)\right]^2}{4(1-\rho\beta)} + s(1 - s/\beta + B)$ and the expression for w^* becomes $w^* = s + \frac{(n-1)\left[1-\rho\beta-s(1-\rho)\right]^2}{4n(1-s/\beta+B)(1-\rho\beta)}$, which increases in n and decreases in ρ because $\frac{\partial w^*}{\partial \rho} = -\frac{(n-1)\left[1-\rho\beta-s(1-\rho)\right]\left[\beta(1-\rho\beta)-s(2-\rho\beta-\beta)\right]}{4n(1-s/\beta+B)(1-\rho\beta)^2} < 0$ (since $s < \bar{s}$). Inequality $v^* < 1 \Leftrightarrow 1 - \rho\beta + s(1-\rho) < 2(1-\rho\beta) \Leftrightarrow s(1-\rho) < 1 - \rho\beta$ always holds.

Inequality $\Pi^{D4*} < \Pi^{D*}|_{n=1}$, by Proposition 1, is $\frac{\left[1-\rho\beta-s(1-\rho)\right]^2}{4(1-\rho\beta)} + s(1 - s/\beta + B) < \frac{1-\rho\beta}{4-\beta(1+\rho)^2} \Leftrightarrow$

$sB < \frac{\beta(1-\rho\beta)(1+\rho)^2}{4[4-\beta(1+\rho)^2]} - \frac{s(1+\rho)}{2} + \frac{s^2(4-\beta(1+\rho)^2)}{4\beta(1-\rho\beta)}$. Condition (a) in Proposition 6 becomes $\frac{n-1}{n}(1 - s/\beta + B) \geq 1 - s/\beta \Leftrightarrow B \geq \frac{1}{n-1}(1 - s/\beta)$.

A.16 Proofs of auxiliary statements

Proof of Lemma 10 (properties of the profit) Part 1.1 can be shown by direct substitution of $y^i = 1 - s/\beta - Y^{-i}$ (which is strictly positive by the condition of part 1) into the expressions for $\frac{\partial \Pi^i}{\partial y^i}$ defined by (18) and (21): $\frac{\partial \Pi^i}{\partial y^i}\Big|_{y^i = 1-s/\beta-Y^{-i}-0} = \beta\left(1 - Y^{-i}\right) - c + \beta\left(1 - v^{\min}\right) - 2\beta\Big(1 - \frac{s}{\beta} - Y^{-i}\Big) + \frac{Y^{-i}(p_1-\beta)(1-v^{\min})}{(1-s/\beta)^2} = -c - \beta v^{\min} + 2s + Y^{-i}\left(\beta + \frac{(p_1-\beta)(1-v^{\min})}{(1-s/\beta)^2}\right)$, and $\frac{\partial \Pi^i}{\partial y^i}\Big|_{y^i = 1-s/\beta-Y^{-i}+0} = -c + s + Y^{-i}\frac{(p_1-s)\big(1-v^{\min}\big)}{(1-s/\beta)^2}$. These expressions imply that part 1.1 holds iff $s - \beta v^{\min} + Y^{-i}\left(\beta + \frac{(p_1-\beta)(1-v^{\min})}{(1-s/\beta)^2}\right) < Y^{-i}\frac{(p_1-s)\big(1-v^{\min}\big)}{(1-s/\beta)^2}$, which is equivalent to $s - \beta v^{\min} < Y^{-i}\left[\frac{(\beta-s)\big(1-v^{\min}\big)}{(1-s/\beta)^2} - \beta\right] = Y^{-i}\left[\frac{\beta(1-v^{\min})}{1-s/\beta} - \beta\right] = \frac{Y^{-i}(s-\beta v^{\min})}{1-s/\beta}$, which holds because $s < \beta v^{\min}$ and, by condition of part 1, $Y^{-i} < 1 - s/\beta$.

As Π^i is continuous, i.e. $\Pi^i(1 - s/\beta - Y^{-i} - 0) = \Pi^i(1 - s/\beta - Y^{-i} + 0)$, we can show part 1.2 using either (16) or (20). From (20), $\Pi^i(1 - s/\beta - Y^{-i}) = \Big(1 - \frac{s}{\beta} - Y^{-i}\Big)\left[s - c + \frac{(p_1-s)\big(1-v^{\min}\big)}{1-s/\beta}\right] = \Big(1 - \frac{s}{\beta} - Y^{-i}\Big)(c - s)\left[\frac{(p_1-s)\big(1-v^{\min}\big)}{(1-s/\beta)(c-s)} - 1\right]$, which yields the result of part 1.2.

For part 1.3, rewrite (19) as $\frac{\partial^2 \Pi^i}{\partial (y^i)^2} = -\frac{2}{Y^3}\left[\beta Y^3 + (p_1 - \beta)(1 - v^{\min})Y^{-i}\right]$. As $Y \geq 0$, RHS of this equation is negative (Π^i is strictly concave) iff $\beta Y^3 + (p_1 - \beta)(1 - v^{\min})Y > 0$. Equality $Y = 1 - v^{\min}$ holds only at the left boundary of the domain of the profit function. For all other points in the domain $Y > 1 - v^{\min} \geq 0$ and we have $\beta Y^3 + (p_1 - \beta)(1 - v^{\min})Y > \beta(1 - v^{\min})^2 Y + (p_1 - \beta)(1 - v^{\min})Y = [\beta(1 - v^{\min}) + p_1 - \beta](1 - v^{\min})Y = [p_1 - \beta v^{\min}](1 - v^{\min})Y \geq 0$ if $p_1 \geq \beta v^{\min}$ (a sufficient condition for strict concavity of Π^i).

Suppose $p_1 < \beta v^{\min}$. Although Π^i may be non-concave in this case, $\frac{\partial^2 \Pi^i}{\partial (y^i)^2} = -2\beta\Big[1 + (\frac{p_1}{\beta} - 1)(1 - v^{\min})\frac{Y^{-i}}{Y^3}\Big]$ is monotonically decreasing in y^i. Therefore, if Π^i has an inflection point, this point is unique and corresponds to the total supply level $\tilde{Y}$ such that $\tilde{Y}^3 = (1 - \frac{p_1}{\beta})(1 - v^{\min})Y^{-i}$.

Consider an extension $\tilde{\Pi}^i$ of Π^i in the form (16) to the domain $y^i \geq (1 - v^{\min} - Y^{-i})^+$. In terms of the total supply, this domain is equivalent to $Y \geq \max\{(1 - v^{\min}), Y^{-i}\}$. We will prove that $\tilde{\Pi}^i$ is pseudoconcave implying the claim of part 1.3 for the case of $p_1 < \beta v^{\min}$.

Equation (16), divided through by y^i, implies that $\tilde{\Pi}^i = 0$ iff $y^i = 0$ or $\beta(1 - Y) - c + \beta\left(1 - v^{\min}\right) + \frac{(p_1-\beta)(1-v^{\min})}{Y} = 0$. After multiplying by $-Y/\beta$, this equation becomes

$$Y^2 - (2 - c/\beta - v^{\min})Y + (1 - p_1/\beta)(1 - v^{\min}) = 0. \tag{35}$$

Its properties are explored in the following lemma.

Lemma 11. *For any feasible $c, s, v^{\min}$, and $p_1 < \beta$, the real roots $Y_{1,2}$ of Eq. (35) exist and satisfy the conditions:* $0 \le Y_1 \le 1 - v^{\min} < Y_2 \le 2 - \left(c/\beta + v^{\min}\right)$ *with* $Y_1 = 1 - v^{\min}$ *only if* $v^{\min} = 1$.

By Lemma 11, the roots $Y_{1,2}$ of (35) always exist and $0 \le Y_1 \le 1 - v^{\min} < Y_2$, where $Y_1 < 1 - v^{\min}$ unless $v^{\min} = 1$. Using these roots, we can express $\tilde{\Pi}^i$ as the following function of Y: $\tilde{\Pi}^i = -\frac{\beta}{Y}(Y - Y^{-i})(Y - Y_1)(Y - Y_2)$. Moreover, by (35), $Y_1 Y_2 = (1 - p_1/\beta)(1 - v^{\min})$, and the inflection point has the form $\tilde{Y}^3 = Y_1 Y_2 Y^{-i}$, i.e., $\tilde{Y}$ is the geometric mean of Y_1, Y_2, and Y^{-i}. Because the second derivative is decreasing, $\tilde{\Pi}^i$ is strictly concave to the right of $\tilde{Y} - Y^{-i}$.

There are three possible locations of Y^{-i} relative to $Y_1 < Y_2$. First, if $Y^{-i} \ge Y_2$, then $1 - v^{\min} < Y^{-i}$, $\tilde{Y} < Y^{-i}$, and $\tilde{\Pi}^i$ is nonpositive and strictly concave for all $y^i \ge (1 - v^{\min} - Y^{-i})^+$. In this case, the claim of part 1.3 holds.

Second, if $Y^{-i} \le Y_1$, then $Y^{-i} \le 1 - v^{\min}$, $\tilde{Y} < Y_2$, $\tilde{\Pi}^i$ is nonnegative for $(1 - v^{\min} - Y^{-i})^+ \le y^i \le Y_2 - Y^{-i}$ and nonpositive for $y^i \ge Y_2 - Y^{-i}$. Because $\tilde{\Pi}^i$ is concave for $y^i \ge \tilde{Y} - Y^{-i}$ and changes its sign from positive to negative at $Y_2 - Y^{-i} \ge \tilde{Y} - Y^{-i}$, it is also decreasing for all $y^i \ge Y_2 - Y^{-i}$. However, when $1 - v^{\min} < \tilde{Y}$, $\tilde{\Pi}^i$ is convex in the interval $[1 - v^{\min} - Y^{-i}, \tilde{Y} - Y^{-i}]$.

Third, if $Y_1 < Y^{-i} < Y_2$, it is still true that $\tilde{Y} < Y_2$, $\tilde{\Pi}^i$ is nonnegative for $(1 - v^{\min} - Y^{-i})^+ \le y^i \le Y_2 - Y^{-i}$, and nonpositive, decreasing and strictly concave for $y^i \ge Y_2 - Y^{-i}$. It is also true that, when $\max\{(1 - v^{\min}), Y^{-i}\} < \tilde{Y}$, $\tilde{\Pi}^i$ is convex in the interval $[(1 - v^{\min} - Y^{-i})^+, \tilde{Y} - Y^{-i}]$.

We combine the cases two and three by observing that in both of them $\tilde{\Pi}^i$ is nonnegative for $[(1 - v^{\min} - Y^{-i})^+, Y_2 - Y^{-i}]$ and decreasing as well as concave for $y^i \ge Y_2 - Y^{-i}$. Thus, there is no local minimum for $y^i \ge Y_2 - Y^{-i}$. We complete the proof of part 1.3 using the following lemma.

Lemma 12. *If $\tilde{\Pi}^i$ has an internal (local) minimum $(y^i)_{\min}$, then* $\tilde{\Pi}^i((y^i)_{\min}) < 0$.

Lemma 12 implies that $\tilde{\Pi}^i$ has no local minimum in the interval $((1 - v^{\min} - Y^{-i})^+, Y_2 - Y^{-i})$. Thus, $\tilde{\Pi}^i$ has no internal minima in its entire domain, is strictly increasing when it is convex and, therefore, is pseudoconcave.

Parts 1.4 and 2 follow directly from (21). Part 1.5 immediately follows from parts 1.3 and 1.4. Indeed, condition $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} + 0} \le 0$ implies that Π^i is decreasing for $y^i \ge 1 - \frac{s}{\beta} - Y^{-i}$ (by concavity on this interval). Combining this observation with pseudoconcavity for $y^i \le 1 - \frac{s}{\beta} - Y^{-i}$, we obtain pseudoconcavity for the entire domain. Similarly, $\left.\frac{\partial \Pi^i}{\partial y^i}\right|_{y^i=1-\frac{s}{\beta}-Y^{-i}-0} \geq 0$ implies that Π^i is strictly increasing for $y^i \leq 1-\frac{s}{\beta}-Y^{-i}$, again, leading to pseudoconcavity for the entire domain.

Proof of Lemma 11 (the roots of $\Pi^i(Y)=0$) The discriminant of (35) is $D=(2-c/\beta-v^{\min})^2-4(1-p_1/\beta)(1-v^{\min}) \geq (2-c/\beta-v^{\min})^2-4(1-c/\beta)(1-v^{\min})=(v^{\min}-c/\beta)^2 \geq 0$, where the first inequality is strict unless $v^{\min}=1$ because $p_1>c$, whereas the second inequality is strict unless $v^{\min}=c/\beta$. Therefore, $D>0$, the real roots given by $Y_{1,2}=\frac{1}{2}(2-c/\beta-v^{\min} \pm \sqrt{D})$ always exist, and $Y_1<Y_2$. As $p_1<\beta$, we have $4\,(1-p_1/\beta)\,(1-v^{\min}) \geq 0$ and $Y_{1,2} \in \left(0, 2-c/\beta-v^{\min}\right)$.

If $v^{\min}=1$, the roots are $Y_1=0$, $Y_2=1-c/\beta$, and the claim of the lemma holds.

If $v^{\min}<1$, then $D>\left(v^{\min}-c/\beta\right)^2$, and an upper bound on Y_1 is $Y_1<1-\frac{1}{2}(c/\beta+v^{\min})-\frac{1}{2}\left|v^{\min}-c/\beta\right|=1-\max\left\{c/\beta, v^{\min}\right\} \leq 1-v^{\min}$, which, in turn, is a lower bound on $Y_2: Y_2>1-\frac{1}{2}(c/\beta+v^{\min})+\frac{1}{2}\left|v^{\min}-c/\beta\right|=1-\min\left\{c/\beta, v^{\min}\right\} \geq 1-v^{\min}$.

Proof of Lemma 12 Function $\tilde{\Pi}^i$, its first and second derivatives are given, respectively, by (15), (17), and (19). If an internal local minimum $y^i_{\min}$ of $\tilde{\Pi}^i$ exists, it must satisfy the conditions

$$\left.\frac{\partial \tilde{\Pi}^i}{\partial y^i}\right|_{y^i=y^i_{\min}} = 0, \text{ and} \tag{36}$$

$$\left.\frac{\partial^2 \tilde{\Pi}^i}{\partial \left(y^i\right)^2}\right|_{y^i=y^i_{\min}} \geq 0. \tag{37}$$

Using (36) and the expression for $\frac{\partial \tilde{\Pi}^i}{\partial y^i}$, we obtain $\beta\,(1-Y)-c+\left[p_1-\beta\,(1-Y)\right]\frac{1-v^{\min}}{Y}=$

$$=-y^i_{\min}\beta\left[-1+\frac{1-v^{\min}}{Y}-\left(\frac{p_1}{\beta}-(1-Y)\right)\frac{1-v^{\min}}{Y^2}\right]=y^i_{\min}\beta\left[1+\left(\frac{p_1}{\beta}-1\right)\frac{1-v^{\min}}{Y^2}\right]. \tag{38}$$

Since LHS of (38) multiplied by y^i matches the expression for $\tilde{\Pi}^i$, it follows that

$$\left.\tilde{\Pi}^i\right|_{y^i=y^i_{\min}}=\left(y^i_{\min}\right)^2\beta\left[1+\left(\frac{p_1}{\beta}-1\right)\frac{1-v^{\min}}{Y^2}\right]. \tag{39}$$

Condition (37) and the expression for the second derivative of $\tilde{\Pi}^i$ imply that, at $y^i=y^i_{\min}$, $\left(\frac{p_1}{\beta}-1\right)(1-v^{\min}) \leq -\frac{Y^3}{Y^{-i}}$. Combining this inequality with (39), we obtain $\left.\tilde{\Pi}^i\right|_{y^i=y^i_{\min}} \leq \left(y^i_{\min}\right)^2\beta\left[1-\frac{Y}{Y^{-i}}\right]<0$, which is strict because, here, we consider only $y^i>0$.