Generalized Constant Market Shares (G-CMS) Analysis: Composition and Partition Approach

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Abstract

Even though Constant Market Shares (CMS) analysis is already established as an analytical tool to analyze one or more country export performances, it still has several fundamental problems within the method of analysis. In this research, we try to provide a solution of those weaknesses by constructing a Generalized CMS equation. By this model we can construct a more precise and flexible CMS, which is able to capture all possible variation of observation points connecting two primary distant points of analysis.

JEL: F14, F15.

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The Constant Market Shares (CMS) is a tool to analyze factors, which determines changes in export value of a country between two distinct years of observation, initial year and observed year, denoted as $V_T - V_0$ expression. Tyszynki (1951) was the first researcher who introduced this method. Later on, the CMS has expanded as a more potent analytical tool. The researchers who contribute to the development of CMS are Baldwin (1958), Leamer and Stern (1970), Richardson (1971), Fageberg and Sollie (1987), Milana (1988), Widodo (2010), Guo *et al* (2011), Feng *et al* (2014), and Dyadkova and Momchilov (2014).

Despite the model that is already expanded, there are several problems, which exist in the model. The first problem is the previous research of constant market shares analysis does not contain theoretical and/or technical design to construct sub-period analysis within CMS. This condition could led unclear formulation of sub-period within CMS analysis. Besides, CMS has not yet a proper design to capture the behavioral data for any level period of data.

Therefore, we try to provide a generalized model of CMS analysis in order to answer those weaknesses. This general term provides a theoretical foundation of sub-period construction within primary objectives of CMS analysis. In this research we will use the "primary objectives" to represent the value of export changes between V_0 and V_T. Besides, the generalized term also provides a structure or equation, which is able to capture the data behavior in higher level of frequency. Technically, in this generalized form, we can capture CMS analysis for any level of period.

I. Literature Review

Early formulation of CMS analysis, by Tyszynki (1951), defines that changes in value of export of a country is determined by general rise of world export and a residual term effect. He defines this effect as a changes of a country competitiveness compared to the world competitiveness in the export.

The development of CMS analysis by later researchers is divided into two main groups. The first group of researchers focus on expanding the model by decomposing the residual term—stated by Tyszynki beforehand—in order to get more detailed effects that determine the changes in value of export. Researchers belong to this group are Baldwin (1958), Leamer and Stern (1970), Fageberg and Sollie (1987), and Widodo (2010).

The other group of researchers focus on expanding the analysis by using a technical development. This group developed a new methods on the analysis. Researchers among this group basically improved the methods to answer several criticisms by Richardson (1971) to CMS method based on Leamer-Stern model. He states that there are two main technical weaknesses on that model[1], which are: 1) differences in order of calculation between commodity effect and market effect would differ the result at individual commodity and market effect; and 2) Index number problem which is caused by different year used to index the model.

Several researchers among the second group are Richardson (1971), Fageberg and Sollie (1987), Milana (1988), Guo *et al* (2011), Feng *et al* (2014), and Dyadkova and Momchilov (2014). Fageberg and Sollie (1987), in this case, applied a new method to calculate the Leamer-Stern CMS model by using Laspeyers indices and the result found new effects as compositions of residual term. By using this methods, they introduced sub-period analysis, which captured the value of analysis at some points of observation within the primary observation. This methods used as a solution of the second point of Richardson's critique.

Milana (1988), on the other hand, also attempted to answer the second point of Richardson's critique. However, Milana (1988) utilized the different ways to construct the solution compared to Fageberg-Sollie method. Instead of using fixed weighted calculation such as Laspeyers, he used Dweirt's superlative class indices, Törnqvist index as a price-weighted method. His methods worked under quadratic function of CMS model[2].

At this point forward, the development of CMS analysis are using two different ways of calculation as a basis equation. They are either using Fageberg-Sollie weighted method or Milana weighted method.

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[1] There are four critical weakness stated by Richardson (1971) in total, the other two are:
(1) The various components in basic identity will vary with the level of commodity aggregation.
(2) The same problem will also happen in market consolidation.

[2] This model is also mentioned by Richardson (1971) in his paper as a basis proof of Leamer-Stern model weakness in index number problem.

II. Construction of Generalized Model

In this research we used three steps in order to construct generalized equation of CMS. The first step is called a breakdown process of fundamental concept of CMS, which is analyzing the value of $V_T - V_0$. This process aims to identify the nature of the components of CMS structure. The second step is construction of topological space structure of CMS by using those fundamental components. Thus, the last step is applying composition and partition concept on its structure to construct generalized model of CMS analysis.

A. The Nature of Constant Market Shares Analysis

CMS is used to analyze the changes of value of export between two distant points, namely V_0 and V_t. Technically, we analyze the value of V in time t_T compared to condition on t_0 as initial time. By using CMS we can determine the most determinant factors of those changes. Thus, in this research, we calls V_t as the *main objective*, $V_t - V_0$ as *primary observation*, and all possible period variation between V_t and V_0—if it exists—as *sub-period*. Besides, we also knew that

Definition 1. Value of export at a given time period is a summation of all recorded export quantities times each price within that time period.

Therefore—by **definition 1**—abstractly, we can separate them into two different sets, which are: 1) set of value of export, denoted as $\mathbf{V}$, for $v_n = \sum_{i=1}^{N} p_i \cdot q_i$; $v_n \in \mathbf{V}$; and 2) set of time, denoted as $\mathbf{T}$, given $t \in \mathbf{T}$. Thus, by crossing product of those two sets we will get

(1) $$\mathbf{V_T} \subset \mathbf{V} \times \mathbf{T}$$

where, $V_T \in \mathbf{V_T}$. However, this particular relationship, is only reflected into singleton cross product between v_n and t in order to construct V_T. That process is described on proposition hereunder:

Proposition 1.A. $v_n \in \mathbf{V}$ mapping into $V_T \in \mathbf{V_T}$ by ordered of time $O(t): v_n \to V_T$, expressed by following map,

(2) $$v_n \xrightarrow{O(t)} V_T$$

Proof.

As we mentioned before, v_n is a summand of multiplication of all export's prices and quantities that recorded during one period of time (**definition 1**).

(3) $$v_n = \sum_{i=1}^{N} p_i . q_i$$

v_n—on *Eq. 3*[3]—technically are computed externally by each recorded export's prices and quantities at the certain period. Hence we need to label the value of v_n into each recorded period, then we need to map v_n into those period by the ordered of time. Therefore, every singleton set of **V** will be paired into each period of **T**.

(4) $$\mathbf{V_T} = \{(v_a, t_0), (v_b, t_{0+k}), \dots, (v_n, t_T)\}$$

Besides, we are also able to conclude, there are bijection between $\mathbf{V_T}$ and **T**, by using pairing condition below,

(5) $$V_0 \leftrightarrow t_0, V_T \leftrightarrow t_T$$

where, t_0 is initial time, t_T is a point T in time, and $V_0, V_T \in \mathbf{V_T}; t_0, t_T \in \mathbf{T}$.

Another approach to proof **Proposition 1.A** is by using philosophical approach by following Reichenbach's definition on van Frassen (2013: 42)

Definition 2. "(The object) X has (the property) F at time t" is true if and only if "A (case of) being F of X occurred at time t" is true.

Hence v_n is a summand of $(p_i . q_i)$, then the premises become,

[3] Henceforth equation will be shorten by using an "*Eq.*" notation.

Proposition 1.B. "v_n becoming V_T is true if and only if $(p_i.q_i)$ as components of v_n is recorded at period t_T is true".

The statement above implies that V_T is a summand of,

$$V_T = \left[\sum_{i=1}^{N}(p_i.q_i)\right]_T \tag{6}$$

If and only if *Eq. 6* is true then **Proposition 1.B** is true. The "true" value of *Eq. 6* is proven by the data that are constructed by V_T. The data of $(p_i.q_i)_T$ should consists only the recorded data at t_T period. Hence **Proposition 1.B** is true then **Proposition 1.A** is also true.

After we conceived the core of $\mathbf{V^T}$, we begin to analyze more important part within CMS analysis, that is set of time, **T**. Defined by expression above, that the existence of V_T are depend on the existence of t, hence we must construct properties that bounded to **T** . This process undergone to create time "infrastructure" on behalf of further analysis.

In order to understand set of time, we need to understand the construction of time as a basis or the core of the set's build system. The concept of time that use in this research is referred to individual system of time itself, separated from the concept of space-time. This emphasis will become an important foundation for the further analysis.

Property 1. Let **T** is a set of time, which are $t \in \mathbf{T}$.

Property 1.1. $\forall t\ t_A \neq t_B,\ t_A, t_B \in \mathbf{T}$.

Time is a mass noun that is describes as an indefinite continued progress of existence and events in the past, present, and future regarded as a whole[4]. On

[4] Time definition from Oxford dictionaries, released by Oxford University Press, 2011.

the other hand, we can define the time as experience events in time as occurring in succession (Oaklander, 2004).

Philosophically, there are two fundamental properties bounded into time as a mathematical object[5], there are: 1) The product of a Continuous process; and 2) Elements are non-contemporaneous to each other. In order to represent them into time structure we implant them onto Peano's series. Thus, we map that series onto time series by unit of time mapping.

Property 2. $\forall t$ are constructed by mapping $\tau: P \rightarrow \mathbf{T}$, τ is unit of time and P is Peano Postulate's natural number construction, implies $\mathbf{T}$ bijective with $\mathbb{N}$.

Property 2 above describes how to construct the value for each element of $\mathbf{T}$. The constructive method based on "continuing process" and "non-contemporaneous" attributes—which we described before on property 1. The foundation of the construction is also related to Aristotle's question about the entity of time and Leibniz's critique on Aristotle's perception of beginning of time (van Frassen, 2013: p. 24, 32). As summarize, we construct the following Kant perspective of topological structure of time:

> *We represent the time-sequence by a line progressing to infinity, in which the manifold constitutes a series of one dimension only; and we reason from the properties of this line to all the properties of time…*
>
> (citied on van Frassen, 2013: p. 115)

One-dimensional object—line of time or *timeline*—that phrased above can be constructed by number theory construction. In this research we constructed

[5] The concept is constructed since Aristotle and Aquinas points of view of time structure, which are defined each element of time is a product of precession and succession (citied on van Frassen, 2013: p. 10-11, 20, 22-23). Barrow and Newton are also added that time structure is an independent product of internal flows of process (citied on van Frassen, 2013: p. 30-31)

them by natural number system, instead of real number system that used by Leibniz or Kant. Arguments we used are based on:

Lemma 1. **T** is a *countable set*.

Lemma 1 describes every element of time as a value, which each of them are "continuous process" product. The "continuous process" product refers to Kant's fragment definition of time above[6]. In order to construct **T** properties we need to understand how the element of time are valued and build from two basic properties: 1) continuous; and 2) non-contemporaneous. Therefore, we need Peano postulate as a basis of natural number construction as a construction mapping for the value of each element of time.

According to Peano on Webber (1966: p. 58) natural number system is consists of a set of $\mathbb{N}$ undefined elements, and an undefined mapping within elements, $p: x \to x'$, such that $x, x' \in \mathbb{N}$ and x is constructed if following postulates hold:

A1. There exists an element in $\mathbb{N}$, to be denoted by 1.

A2. $x' \neq 1$, for all $x \in \mathbb{N}$.

A3. **If** $x' = y'$, then $x = y$, for all $x, y \in \mathbb{N}$.

A4. **If** $G \subseteq \mathbb{N}$ such that

a. $1 \in G$, and

b. $n \in G \Rightarrow n' \in G$, for all $n \in G$,

[6] In this section we will not discuss about Kant's perception of time as an unreal object and merely as subjective constitution of mind. We are also not debate either time and space are *real* or *ideal*. We limited our research with a concept that time is individual body of structure with value and direction (Reichenbach, 1971). Moreover, in our research context, construction of time is related how to construct generalized analysis of CMS. Within CMS construction value of export merely a discrete product for each period of time (**definition 10** and following existence identity of **T** and **V**). Therefore, in our construction limited into how we valued and understand the properties of them).

Both hold, **then** $G = \mathbb{N}$

The first postulate on A.1 axiom determines that the least possible element of natural number is denoted by "1", which is supported by axiom A.2. Axiom A.2 make a clear statement that "1" as an element of $\mathbb{N}$ is not a successor for any other element of $\mathbb{N}$.

The third postulate declares that, if two elements of $\mathbb{N}$ have equivalence relation for their successor, then both of them are equal. Lighstone (1965: p. 35-36) described third postulated by the inversed logic version. The third postulate stated by following sentence,

(A.3)' **If** $x' \neq y'$, then $x \neq y$, for all $x, y \in \mathbb{N}$.

Axiom (A.3)' basically has the same meaning with axiom A3, but Axiom (A.3)' are using the inverse premise of axiom A3.

Peano postulates become the basis concept of mathematical induction. Term of successor on natural number denoted by binary symbol "$+_N$", which is implies that the system is built on $\mathbb{N}$. This property implies,

Theorem 1. $m \neq m +_N 1$, for $m \in \mathbb{N}$.

Proof. $m +_N 1$ and m are two distinct elements of $\mathbb{N}$, then simply said $m \neq m +_N 1$ by axiom (A.3)'.

By theorem above, then we can construct $\mathbb{N}$ by using Peano postulates and mathematical induction, $+_N$. We will also have a constant value "0" which satisfies converse **theorem 1** conditions.

Theorem 2. $m = m +_N k$ iff k does not have properties, which is represented by a constant "0".

Theorem 2 justifies the existences of an element of $\mathbb{N}$, that does not change other element's value. Let, P is natural number series constructed by Peano postulates, then we get,

(7) $$P = 0, 1, (1 +_N 1), [1 +_N (1 +_N 1)], 1 +_N [1 +_N (1 +_N 1)], \ldots$$

By property 2 we know that every t elements on **T** are constructed by mapping Peano series into elements of time. Let, there is mapping $\tau: P \rightarrow \mathbf{T}$ with, $\tau(p) = p.(T)$ such that $p \in P$ and (T) is unit of time, then we will get,

(8) $$\mathbf{T} = \{0, 1.T, (1 +_N 1).T, [1 +_N (1 +_N 1)].T, (1 +_N [1 +_N (1 +_N 1)]).T, \ldots\}$$

On the other hand, if we construct **T** into each element value, we will get,

(9) $$\mathbf{T} = \{0, 1(T), 2(T), 3(T), 4(T), \ldots\}$$

Property 3. **T** is a finite set.

Proposition 2. **T** is a well-ordered set, such that exists $\min \mathbf{T}$ and $\max \mathbf{T}$.

However, in this research we assumed **T** is a finite set. By this property, we can declare that there is exists one greatest element on **T**, say $\varphi(T)$ that is greater value than any t. By those properties we are able to say that **T** is a well-ordered set too. Besides, we already knew that structure of **T** represented by summation of "1" property for each t element.

Proposition 3. t_m have k-more "T" properties than t_n iff there is exist $m = n + k$ (**Theorem 2**) for k is not a constant "0", implies $t_m > t_n$ and $t_n < t_m$.

By proposition 3 we are able to declare that **T** is strictly totally ordered set, such that exist chain $\mathcal{C}$ connecting all of element t.

Proof.

$$t_0 = 0$$
$$t_1 = 1(T),$$
$$t_2 = (1 + 1).T = 2(T),$$
$$t_n = \underbrace{(1 + 1 + \cdots + 1)}_{n\text{-times}}.T = n.(T),$$

The t-series above provide us that t_2, which is technically t_{0+2}, has 2 more (T) properties than t_0. Therefore we can rewrite those condition into $t_2 >_T t_0$ or more general as $t_2 > t_0$. Otherwise, $t_0 <_T t_2$ or more general as $t_0 < t_2$. Hence **T** is totally ordered set and a finite set, then **T** is also well-ordered if and only if they have minimum and maximum element. Proposition 2 is proven by following sentences:

Proof.

Hence t is mapping product from $\mathbb{N}$, then 0-point become least element of **T** and by Axiom A3 on Peano postulates, we can state there is no distinct element of **T** that value as "0". Under those circumstances we also able to state that 0-point is minimum of **T**.

On the other hand, under property 3, we have an upper bound to restrict **T** from infinite value. Let U is value for t, where time is finished. Then, to reach U value we need summation of "1" to a certain amount, let say it is φ.

$$U = \underbrace{(1 + 1 + \cdots + 1)}_{\varphi\text{-times}}$$

Therefore, U-value will reach at $\varphi(T)$ point on **T**, so we says $\varphi(T)$ is greatest element of **T**. Assume, there is $\rho(T)$, if $\rho = \varphi$, technically $\rho(T)$ is $\varphi(T)$, otherwise $\rho(T) \neq \varphi(T)$, which is implies there will not be any element $t(T)$ such that $t = \varphi$. Hence **T** is a finite set, then $\varphi(T)$ is maximum element of **T**.

Property 4. $\boldsymbol{O_n}$ is an order of element t, such that O_T is a representative value for T-order t within $\boldsymbol{T}$, $O_T \in \boldsymbol{O_n}$, implies $O_T \in \mathbb{N}$.

Property 4.1. 0_T point for "0" valued on t represented by 0_T which is called point of origin.

By **T** construction under proposition 2 we know that **T** is a well-ordered set and $t(T)$ point are exclusive to each other, $r(T) \neq s(T)$ such that $r(T) < s(T)$

for $s = r + k$ and $k \neq 0$; $r(T), s(T) \in \mathbf{T}$. Relation within **T** for all t are reflected by "<" and ">", then there will exist $\mathcal{O}$ as ordered of elements within **T**.

$\boldsymbol{O_n}$, which is constructed by property 4, is an ordered elements for all t with 0_T represent min **T**, which is the first element of **T**. In this research we calls that point as "Beginning of Time" and denoted by B_T. In term of order theory there are successor and predecessor within an order $\mathcal{O}$ (Jech, 2002: p. 18, 20). Thus,

Proposition 4. s_T is successor of r_T iff $s > r$ such that $s, r \in \mathcal{O}$ and r_T is predecessor of s_T.

Proposition 4.1. s_T is immediate successor[7] of r_T iff $s > r$ such that $s, r \in \mathcal{O}$ and $s = r + 1$.

By those two propositions above we understand that given any point r_T, every point which is ordered beyond r_T are successor of r_T and a point ordered after r_T is immediate successor of r_T. Hence **T** already have B_T and E_T as a representation of minimum and maximum element respectively, then we will have,

$$(B+1)_T \text{ as } \mathit{immediate\ successor} \text{ of } B_T$$
$$[(B+1)+1]_T \text{ as } \mathit{immediate\ successor} \text{ of } (B+1)_T,$$
$$\{[(B+1)+1]+1\}_T \text{ as } \mathit{immediate\ successor} \text{ of } [(B+1)+1]_T,$$
$$\vdots$$
$$\vdots$$
$$[(E-1)-1]_T \text{ as } \mathit{immediate\ successor} \text{ of } \{[(E-1)-1]-1\}_T,$$
$$(E-1)_T \text{ as } \mathit{immediate\ successor} \text{ of } [(E-1)-1]_T,$$
$$E_T \text{ as } \mathit{immediate\ successor} \text{ of } (E-1)_T$$

[7] *Immediate successor* term is also called *successor ordinal* (Jech, 2002: p. 20). In our case if s_T is not a successor ordinal of r_T, then s_T is called *limit ordinal*, which is satisfying $s_T = \sup\{r_T : r_T < s_T\}$ condition.

Hence B_T is equal to 0_T, then $\boldsymbol{O_n}$ is constructed by following series,

(10) $\boldsymbol{O_n} = \{0_T, (0+1)_T, [(0+1)+1]_T, \dots, [(E-1)-1]_T, (E-1)_T, E_T\}$

Therefore, we also could rearrange them into,

(11) $$\boldsymbol{O_n} = \{0_T, 1_T, 2_T, \dots, (E-2)_T, (E-1)_T, E_T\}$$

where, O_T series from equation above are following Peano constructive in property 1, implies $O \in \mathbb{N}$.

Property 5. T as unit of time for $\forall t$ are consistent.

T as unit of time for any element reflected time unit for each value of time. By general agreement there are several unit of time that exist,

Property 5.1. There is exist T_k that is a collection of k-number of T_l such that $k < l$ and $k \neq l$, given $k, l = \{0,1,2, \dots, \Omega\}$.

Property 5.1 describes the existences of "level" for unit of time, given 0th-level is a base unit of time. T_k is denoted for k^{th}-level of magnitude for the unit of time and T_l is denoted for l^{th}-level of magnitude. Unit with a lower magnitude on those level is a collection or union of several values of unit of time with higher magnitude. Suppose $k^{th} < l^{th}$ in level of magnitude, then exists satisfied condition:

Corollary 1. $\mathrm{k}.T_k = \mathrm{k}.f^{k \to l}\, T_l$

Corollary 2. $l.T_l = a.T_k \; and \; b.T_l$; $l \equiv b \bmod f^{l \to k}$, such that $l = b + a.f^{l \to k}$

where, $f^{k \to l}$ is constant value to convert T_k unit into T_l unit and $f^{l \to k}$ is constant value to convert T_l unit into T_k unit. Level system above is reflected the measurement system that called as time-measurement system. Moreover, that system are divided into two classes: calendar system and clock system.

$$T^{calendar} = \begin{cases} Y, \text{for "year" time unit} \\ M, \text{for "month" time unit} \\ W, \text{for "week" time unit} \\ D, \text{for "day" time unit} \end{cases}$$

and T^{clock} as a set of unit of time on "clock" system,

$$T^{clock} = \begin{cases} D, \text{for "day" time unit} \\ H, \text{for "hour" time unit} \\ Ms, \text{for "minutes" time unit} \\ S, \text{for "second" time unit} \\ and\ soon \end{cases}$$

If we defines unit of time as international standard, then it is referred as "second", S. However, in this research we use term of "base unit of time" to create order $\boldsymbol{O_n}$ instead of using second, S. We define "base unit of time" as a unit of time that is used as basis measurement of time within **T**. Therefore, we use year basis (Y) because we defines year as one earth revolution to the sun period. Besides, "year" is the least subset that is not repeated in counting series. We write relationship of all those unit of time into,

Definition 3. Calendar system is measured into several level of magnitude, such that:

T is **a union** of "year" magnitude, such that year is a base unit of time.

$$\mathbf{T} = \bigcup_{\boldsymbol{i=0}} \mathbf{Y}_{\boldsymbol{i}}$$

and a "year" unit is also a collection of "day",

$$\mathbf{T} = \bigcup_{\boldsymbol{i=0}} \bigcup_{\boldsymbol{j=0}} (\mathbf{D}_{\boldsymbol{j}})_{\boldsymbol{i}}$$

Hence $\bigcup_{j=0}^{m}(\mathrm{D}_j) = \mathbf{W}eek$ and $\bigcup_{j=0}^{m}(\mathrm{D}_j) = \mathbf{M}onth$, then

(12) $$\mathbf{T} = \bigcup_{i=0} \bigcup_{j=0} (\mathbf{W}_j)_i \text{ and } \mathbf{T} = \bigcup_{i=0} \bigcup_{j=0} (\mathbf{M}_j)_i$$

On the other hand, clock system is measured "day" as a basis unit of time that measures a moon's rotation to the earth. On this system, there are a lot of level of magnitude, such as

$$\mathbf{D} = \bigcup_{i=0} \mathbf{H}_i$$

where, **H** is "hour" magnitude unit. Hence "hour" is also collection of "minute" and "minute" is a collection of "second" unit, then

(13) $$\mathbf{D} = \bigcup_{i=0} \bigcup_{j=0} (\mathbf{Ms}_j)_i \text{ and } \mathbf{Ms} = \bigcup_{k=0} \mathbf{SE}_k \ then\ \mathbf{D} = \bigcup_{i=0} \bigcup_{k=0} (\mathbf{SE}_k)_i$$

where, $\mathbf{Ms}_j$ is "minute" level unit and $\mathbf{SE}_k$ is "second" level unit. Hence there are also a lot of lower level of magnitudes, then we can generalize them into following expression,

(14) $$\mathbf{D} = \bigcup_{i=0} \bigcup_{j=0} \cdots \bigcup_{\omega=0} ([\{\boldsymbol{Z}_\omega\}_{\dots}]_j)_i$$

where, $\boldsymbol{Z}$ is the smallest possible subse t of time and Z_ω is smallest possible unit of time.

Property 5.1. Given s_T represent s point on "year" unit of time, then s_T can be write as "s_Y" or "s".

Property 5.2. Given r_T represent r point on subset unit of time of "year", then r_T can be write as "$s: \ldots : r_T$" or can be simplified as "$s: r_T$" for $s \in \mathbf{Y}$.

Property 5.1 and 5.2 provide guidelines for writing notation for every point in **T**. By property 5.1, if there is s_T is denoted for s-th order in "year" unit of time, says $s(Y)$, then we can write s_T order as "s_Y" point or just "s" as simplified form. The simplified form in "s" point can be used because "year" unit of time is the "base unit of time" and to represent base unit we can use the magnitude of point to denote every point in **T**.

Furthermore, if there is r_T point denoted r-th order in "Z-subset" unit of time, then we can write r order as "$s: \dots : r_T$" point or "$s: r_T$" point. We use this method because we know that every subset unit of time are repeated after one "year" period. Let, Z-subset is "month" unit of time, then we can write r_T as "$s: r_T$", which is represented point of r-month within year-s.

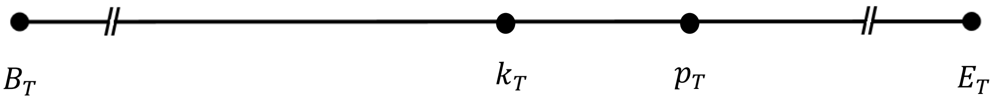
Property 6. (*Optional Property*: *Horizontal Chain Connection*)

Let $C^{\top}$ is vertical chain that is connecting $\boldsymbol{T}$ by Hasse diagram, then there is exist $C^{\dashv}$ which is constructed by rotate $C^{\top}$ to 90^0 clock-wisely.

Property 6.1. Let, k_T and p_T is a distinct point within $\boldsymbol{T}$, then p_T is located on the right side of k_T iff $p > k$.

Figure 1. Original *Timeline* connected by $C^{\dashv}$.

$$B_T \ll k_T < p_T \ll E_T$$



Property 7. (*Gregorian Calendars synchronization process*)

There are several model that construct calendar system. However, Calendar system that is used in this research refers to Gregorian system. The system is chosen because Gregorian Calendars is used as International Calendar system and used globally as standard time-measurement system. In order to apply

Gregorian system into our timeline we construct them, which can be viewed on **Appendix 1**.

Afterwards we can apply or synchronize *primary observation* on CMS into timeline above. Suppose, $\mathbf{T}^{\mathbf{PO}}$ is a set, which consists of $\{t_0, t_T\}$ and t_0, t_T are represented two distinct points in time T, then we can determine that $\mathbf{T}^{\mathbf{PO}} \subset \mathbf{T}$. Moreover, imagine given a point n_T in timeline are t_0, and $(n+\tau)_T$ as a t_T then we also can define that,

Proposition 5. Length of time $|t_0, t_T|$ is τ unit of time, with $\tau > 0$.

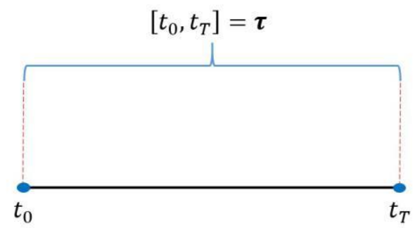
Proof.

Length between two points of time is measured by differences between those two point's magnitudes in *timeline*, reflected by expression

(15) $$[(n+\tau) - n]_T = \tau$$

Magnitude of each point at *timeline* reflected by Gregorian-order of element (by Property 7.1 to 7.3.2). By those construction, then we get a part of *timeline* that consist of $\mathbf{T}^{\mathbf{PO}}$,

Figure 2. $\mathbf{T}^{\mathbf{PO}}$**-construction** Connecting t_0 and t_T Points.



Proposition 6. There is exist at least one element t with value $t \geq t_0$ and $t \leq t_T$.

Proof.

Given t_0 and t_T are totally ordered within $\mathbf{T}^{\mathbf{PO}}$ (hence $\mathbf{T}^{\mathbf{PO}} \subset \mathbf{T}$ and $\mathbf{T}$ is well-ordered set, then $\mathbf{T}^{\mathbf{PO}}$ is also well-ordered too), we can construct inverse transitivity properties of totally ordered set by following logical expression,

(16) $$t_T \geq t_0 \rightarrow \forall t\ \exists w\ [w: w \in t, w \geq t_0 \wedge w \leq t_T]$$

B. Construction of Topological Spaces Structure of CMS

By using all of characteristic of both components of CMS analysis, then we are able to process the generalization form for them. The very first step in the process is to construct the *topological structure* of CMS itself. In this research we are using two different approaches in order to construct the topological structure.

The first approach construct CMS as one-dimensional structure and the second one construct them into two-dimensional topological space. By using those two structures, we can approach generalized constant market share with two method, they are: 1) Path-connectedness on $\mathbf{T^{PO}}$; or, 2) cartesian product of partition of $\mathbf{T^{PO}}$.

Composition of Path-connectedness on $\mathbf{T^{PO}}$—the first approach we view CMS equation as one-dimensional space, which is implied that all of events that are *coincide* together (contemporaneous event) at period t are lies in the same point at space. Therefore, the one-dimensional space structure are constructed coincide with time construction. Implies, in order to construct topological structure of CMS we need to define our time construction as a topological object and then *translate* those definitions into CMS construction.

Definition 4. *Reichenbach's Causal Theory of Time Order*

> Given two events, E_1 and E_2, both of them are *coincide* if and only if $t(E_1) = t(E_2)$.

The first step is to define our time construction as a topological spaces construction. In order to do that, we need to specify topological structure of **T**. This procedure made us recall property 1.1 on time construction and create several arrangement. Recall,

Property 1.1. $\forall t\ t_A \neq t_B, t_A, t_B \in \mathbf{T}$

where, each value of $t \in \mathbf{T}$ constructed by mapping $\tau: P \rightarrow \mathbf{T}$ and give us,

$$\mathbf{T} = \{0, 1(T), 2(T), 3(T), 4(T), \dots\}$$

Thus, imagine that following proposition,

Proposition 7. **T** is a topological space and it is **closed**.

Proof.

$\mathbf{T} = [O_T, E_T]$ is closed hence complement of $\mathbf{T}^\infty$,

(17) $$\mathbf{T}^\infty - \mathbf{T} = [E_T, \infty[$$

is open. This condition is satisfying De Morgan's Law (Munkres, 2000: 94), given X topological sets, then subset A_1 of X is closed if and only if,

(18) $$X - \bigcup_{i=1}^{n} A_i = \bigcap_{i=1}^{n} (X - A_i)$$

Proposition 7.1. Any element of **T** is separate to each other.

Proof.

Proposition 7.1 already proven by property 1.1 and explained further by property 5.1 on construction of time.

Proposition 7.2. **T** is a **connected** space.

Proof.

T is a totally ordered set, by property 2 and constructed by property 2 and the form designed by proposition 4 and 4.1. By those properties, then **T** is only have subsets of **T** and $\emptyset$ element.

Given by proposition 7.1 and 7.2, we can imagine topological structure of **T** as a **closed-connected** space. Moreover, by *chain* properties, we are also able to define them as one-dimensional space, restricted by chain "line" named as *timeline*.

Proposition 7.3. $\mathbf{T}^{\mathbf{PO}}$ is also a **closed-connected** space.

Proof.

In order to proof proposition 7.2 we must consider $\mathbf{T}^{\mathbf{PO}}$ as one of connected component of $\mathbf{T}$. Connected component of a space is a maximal subset of that space (Viro *et al*, 2008: 69). Hence $\mathbf{T}^{\mathbf{PO}} \subset \mathbf{T}$ and supported by following definition,

Definition 5. Connected components are closed. (Viro *et al*, 2008: 69, *Definition 12.K*)

then, $\mathbf{T}^{\mathbf{PO}}$ is closed.

Under all available proposition, we are cleared that $\mathbf{T}^{\mathbf{PO}}$ is a closed-connected space within timeline and it complies with what illustrated by Figure 2 above. Hence this properties, we can address CMS analysis into,

Proposition 8. $|t_T - t_0|$ is a length of path-connectedness of $\mathbf{T}^{\mathbf{PO}}$ space by segment $I = [t_T, t_0]$.

Recall, $\mathbf{T}^{\mathbf{PO}} = \{t_T, t_0\}$ and t_T and t_0 is lie in the same component due to both of them are element of $\mathbf{T}^{\mathbf{PO}}$, which is connected. This statement supported by following definition,

Definition 6. Two points lie in the same components iff they are belong to the same connected space. (Viro *et al*, 2008: 69, *Definition 12.I*)

At this point, we also can connect both of them by a continuous connected-line, given one of them as a starting point and the other is a end point.

Definition 7. Given points x and y of the space X, a **path** in X from x to y is a continuous map $f: [a, b] \to X$ of some closed interval in the real line, such that $f(a) = x$ and $f(b) = y$. A space of X is said to be **path-connected** if every pair of points of X can be joined by a path in X. (Munkres, 2000: 155)

By this definition we conclude that $\mathbf{T^{PO}}$ is connected by a path with segment $I = [t_T, t_0]$, as t_0 is starting point and t_T is end point. Henceforth we denote this path of $\mathbf{T^{PO}}$ as a "path$^H(t_0, t_T)$". Moreover, we also can draw a conclusion that **T** is also **path-connected**. This statement is satisfied by **property 6** and **property 7** on construction time above. Furthermore, the length of the **path** is the interval between t_T and t_0, given by definition,

Definition 8. (*Intermediate Value Theorem*)

A continuous function

$$f: [a, b] \to \mathbb{R}$$

Takes every value between $f(a)$ and $f(b)$. (Viro *et al*, 2008: 73, *Definition 13`1*)

and

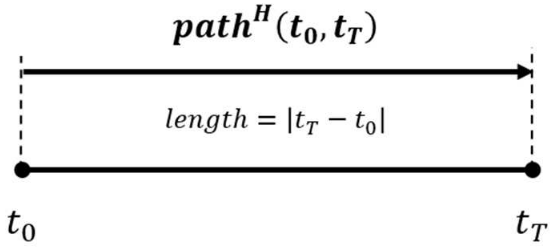
Definition 9. Let X is be a connected space, $f: X \to \mathbb{R}$ a continuous funtion. Then $f(X)$ is an interval of $\mathbb{R}$. (Viro *et al*, 2008: 73, *Generalization of definition 13.A*)

Both of those properties above defined that we can draw that a path is connecting all intervals between two points. However, instead of $\mathbb{R}$ we use $\mathbb{N}$ system. This system still hold since, both of them are also well-ordered sets. In order to measure the length of the path we follow a definition hereunder,

Definition 10. The interval on $\mathbb{R}$ reflected by $|b - a|$. (Simmons, 1963: p. 143-144)

The differences between **definition 10** and our research is lies on number system that we used. Interval on $\mathbb{N}$ system is only captured counting numbers.

Figure 3. path$^H(t_0, t_T)$ connecting t_0 and t_T points.



Suppose we have $|t_T - t_0| = \tau$, then $\tau \neq 0$ iff $T = 0 + 1$ (**proposition 5**). This condition is reflected on proposition 3, where $\forall t \in \mathbf{T}$ are constructed by Peano continuous mapping $t \to t'$ by "1" property from his predecessor

Proposition 9. Sub-path within $\text{path}^H(t_0, t_T)$ denoted by $\text{path}^{H_\text{J}}(t_\alpha, t_\beta)$, with $\text{J} = 1, \ldots, \omega$ and $\alpha, \beta, \ldots$ are value of point between t_0 and t_T. ω indicate as a last sub-path that connecting point φ_T to t_T, given $0 \leq \varphi \leq T$ and $\varphi \in \mathbb{N}$.

Sub-path by definition are one or more paths which are connecting the same points as $\text{path}^H(t_0, t_T)$. Viro *et al* (2008: p. 76) defined this relation as "Path is a product of sub-paths" and demonstrated them into following example. Let $u : I \mapsto X$ and $v : I \mapsto X$ are two paths given $u(1) = v(0)$, then

(19)
$$uv: I \mapsto X: t \mapsto \begin{cases} u(2t) & if\ t \in [0,1/2] \\ v(2t-1) & if\ t \in [1/2,1] \end{cases}$$

Hence u end point is equal to v initial point, then uv is also close-connected path. Symmetrically, this logical expression concludes that close-connected path are able to construct by one or more paths connected in-between. The length of path uv is calculated by,

(20)
$$uv[1] - uv[0] = [u(1) - u(0)] + [v(1) - v(0)]$$

If we apply expression 19 and 20 into proposition 9, then we also can calculated the length for each sub-path between $\text{path}^H(t_0, t_T)$ as follow,

Proposition 10. Suppose $lt\ \mathbf{T^{PO}}$ is length of $|t_T - t_0|$, then length for each sub-period $\text{path}^{H_\text{J}}(t_\alpha, t_\beta)$, constructed by composition of $lt\ \mathbf{T^{PO}}$.

Proposition 10 is hold, because length of $|t_T - t_0|$ is positive integers in order to defined the path is a *forward path*[8]. That condition is given by definition $t_0 \neq t_T$. Beside, there are two main objectives of existences of proposition 10: 1) Aim to identify point of observation between t_0 and t_T; and 2) calculate each length of sub-paths between them. Suppose $lt\ \mathbf{T^{PO}}$ valued as τ^{PO} (or can be abbreviated into τ) by proposition 5, then we have

$$\tau^{PO} = \tau_a + \tau_b + \cdots + \tau_k \ \text{ for } \tau_a, \tau_b, \dots, \tau_\omega \in \mathbb{N}$$

(21)
$$\tau^{PO} = \sum_{\mathrm{J}=1}^{\omega} \tau_{\mathrm{J}} \ \text{ for } \tau_a, \tau_b, \dots, \tau_\omega \in \mathbb{N}$$

where, J is the component index of composing configuration and ω is total component to compose τ^{PO}. Moreover, τ_{J} is also represents the length of sub-paths from each component index, J. Hence every τ is denoted the length for each path, then we also can arrange them into,

(22)
$$\text{path}^{H}(t_0, t_T) = \text{path}^{H_1}(t_0, t_\alpha) + \text{path}^{H_2}(t_\alpha, t_\beta) + \cdots + \text{path}^{H_\omega}(t_\varphi, t_T)$$
$$|t_T - t_0| = |t_\alpha - t_0| + |t_\beta - t_\alpha| + \cdots + |t_T - t_\varphi|$$

We also can generalize them by plug *Eq. 22* into *Eq. 21* as follow,

(23)
$$\text{path}^{H}(t_0, t_T) = \sum_{J=1}^{\omega} \text{path}^{H_J}\left[t_{a_{\mathrm{J}}},\ t_{(a_{\mathrm{J}}+\tau_{\mathrm{J}})}\right]$$

where, each $t_{a_{\mathrm{J}}}$ is represents initial point for each sub-paths. Hence all sub-paths $\text{path}^{H_J}[\cdot,\cdot]$ construct a continuous $\text{path}^{H}(t_0, t_T)$, then every initial point $t_{a_{\mathrm{J}}}$ will have the same value for each end-point $t_{(a_{\mathrm{J}}+\tau_{\mathrm{J}})}$ from the previous component index. Specific to the t_{a_1} and t_{a_ω} points, both of them will have the same t value with t_0 and t_T respectively. Furthermore, *Eq. 21* can be satisfied

[8] The positive integer values for the length is reflected a *forward continuous function* that is constructed every (next) elements of *time construction*. Moreover, this condition is also created by forward continuous function, which is used to construct the Peano series.

by more than one composing formation if and only if the value of τ_{J} as a component the same.

Proposition 11. Suppose $\boldsymbol{\Xi}_{\mathbf{S}}$ denote ***s***-th sub-period's variance of configuration, then there will be $\mathrm{S} = 2^{[lt\mathbf{T}^{\mathbf{PO}}-\mathbf{1}]}$ variances of sub-period.

<u>Proof</u>.

Given a condition of *Eq. 21*, we have,

$$\tau^{PO} = \tau_a + \tau_b + \cdots + \tau_\omega \text{ for } \tau_a, \tau_b, \ldots, \tau_\omega \in \mathbb{N}$$

The solution for τ^{PO} created by the addendum of components τ_ω, such that ω components composed those structure is valued between $1 \leq \omega \leq \tau^{PO}$. Therefore for each ω composition to construct τ^{PO} the composing configuration is construct as follows,

Table 1. The Composing Configuration of τ^{PO}.

	ω=1	ω=2	ω=3	…	ω=$\tau^{PO}-1$	ω=τ^{PO}
$\mathbf{S}=1$	τ^{PO}	0	0	…	0	0
$\mathbf{S}=2$	$\tau^{PO}-1$	1	0	…	0	0
$\mathbf{S}=3$	$\tau^{PO}-2$	1	1	…	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\mathbf{S}=c_s-1$	1	1	1	…	2	0
$\mathbf{S}=c_s$	1	1	1	…	1	1

where, c_s is the total variance of composing configuration. Furthermore, hence each τ_ω value represents sub-path's length, which is connecting two sub-period observation, then their value for each τ are independent for each others. This condition implies such following condition is hold,

$$\boldsymbol{\Xi}_{\mathbf{1}}: \tau^{PO} = \tau_{1,1} + \tau_{1,2} + \tau_{1,3}$$

$$\Xi_2: \tau^{PO} = \tau_{2,1} + \tau_{2,2} + \tau_{2,3}$$

(24) $$\Xi_3: \tau^{PO} = \tau_{3,1} + \tau_{3,2} + \tau_{3,3}$$

where, τ_1-components of Ξ_1 has similar component's value to the τ_2 and τ_3 - components of Ξ_2. For example, if we have condition τ_1, τ_2, τ_3 satisfying,

$$\Xi_1: \tau^{PO} = a + b + c$$

$$\Xi_2: \tau^{PO} = b + a + c$$

$$\Xi_3: \tau^{PO} = c + a + b$$

then, all those 3 composing configuration will be treated as separated configuration. By this condition, we are able to calculate total variance of configuration, c_s, by using following equation,

(25) $$c_s = \sum_{n=1}^{\tau^{PO}} \binom{\tau^{PO} - 1}{n - 1} = \sum_{n=0}^{\tau^{PO}} \binom{\tau^{PO} - 1}{n} = 2^{\tau^{PO} - 1}$$

The last step in this process is to apply this construction into CMS analysis. In order to complete this step, we combine definition from **Definition 1** onto **Definition 4** to specify that event that occurs on each timeline—in our cases—is export quantities recording event. Under that condition, then

Proposition 12.A. There is exists *Recorded* events of export quantities at any point of $s_T \in \mathcal{O}_T$ denoted as V_T and has a v_T value (*Eq. 3*).

By condition on **Proposition 12.A** then we will have the sub-period analysis between primary observations can be constructed by using Composing Configuration as follows,

$$V_T - V_0 = \sum_{J=1}^{\omega} (v_n, t)_{a_J + \tau_J} - (v_n, t)_{a_J}$$

$$V_T - V_0 = \sum_{J=1}^{\omega} V_{a_J + \tau_J} - V_{a_J}$$

(26)

where, t_{n_J} is time configuration, which is constructed by $\text{path}^{H_J}\left[t_{a_J},\ t_{(a_J+\tau_J)}\right]$ under J composing configuration on **Table 1**. v_n is the *recorded* value of export quantities at 1 T_k-unit level of time. Each *recorded* value is calculated by a *recording* process at 1 T_k-unit level of time during some T_l-unit

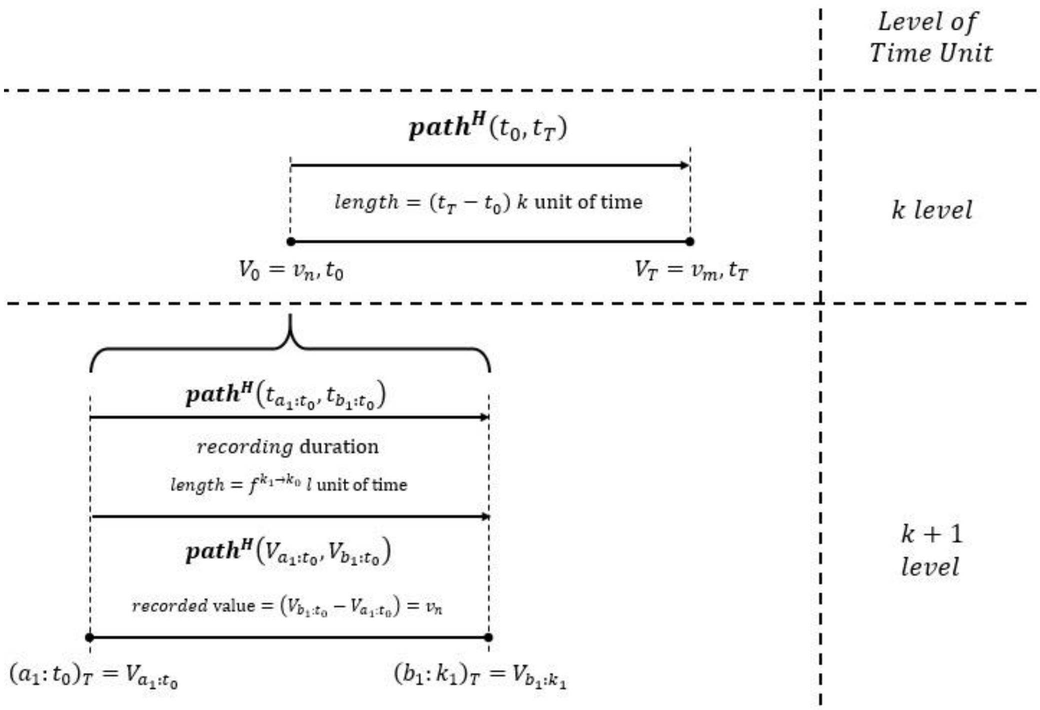
Proposition 12.B. Assume we have s_T lies on T_k-unit level of time then v_T value is calculated by a *recording* events, constructed by a $\text{path}^H(V_a, V_b)$ connecting $a_{T_l:T_k}$ to $b_{T_l:T_k}$ point given by conditions:

1) $a_{T_l:T_k} = 0 \cdot f^{k \to l}$ point of s_T as initial point; and
2) $b_{T_l:T_k} = 1 \cdot f^{k \to l}$ point of s_T as end-point.

For $l = k + 1$ level of unit of time.

Assume we have value v_n for each T_k-unit calculated during T_{k+1} unit of time, then we can illustrate that process as follows,

Figure 4. *Recording* Value (v_n) Process for Any k from $k + 1$ Level of Time.



Example for proposition above. Given a v_n value at t_{2016} (year 2016) period called V_{2016} calculates during a *recording* event between initial *recording* point at $t_{M_1:2016}$ (January, 2016) for any v goods until the end-point at $t_{M_{12}:2016}$ (December 2016). Thus, those recorded v goods multiplied the export price, p_t to create the export value V_{2016}. If we apply this construction into any t_Y points, then we will get a series of discrete value of V_Y.

Therefore, under those construction building of V_T, V_0 (*primary observations*) sub-period observations, we are able to generalized CMS construction for them into more commonly used *Eq. 26* by using equation from *Eq. 22* as structure. Those will has construction as follows,

$$(27) \qquad \Xi_{\mathbf{S}}^{\mathbf{V}}: V_T - V_0 = (V_\alpha - V_0) + \left(V_\beta - V_\alpha\right) + \cdots + \left(V_T - V_\varphi\right)$$

for $t_0, t_\alpha, t_\beta, \dots, t_\varphi, t_T \in \boldsymbol{O_T}$ and order-condition $t_0 \leq t_\alpha \leq t_\beta \leq \cdots \leq t_\varphi \leq t_T$ are satisfied. $\Xi_{\mathbf{S}}^{\mathbf{V}}$ is variances of constant market shares analysis, which are constructed between V_0 and V_T.

Partition of $\boldsymbol{T^{PO}}$ *(Hausdorff Construction of* $\mathbf{V_T}$*-space Approach)*—the second approach in this research are based on two-dimensional space analysis of $\mathbf{V_T}$-space. $\mathbf{V_T}$ -space is used in order to capture one-to-one correspondence relationship between **V** and **T**. The relationship which is then reflected in the pairing condition on *Eq. 5*. In order to accomplish this process, we need to construct **V** and **T** into $\mathbf{V_T}$-space. Particularly this process will provide more detailed proof for **Proposition 1.A** and *Eq. 4*.

The first step are focused on **T** construction. Hence, **T** is a totally ordered set and $\mathbf{T^{PO}}$ is a proper subset of **T,** then $\mathbf{T^{PO}}$ is totally ordered too. Elements of $\mathbf{T^{PO}}$ are two distinct points, implies $t_0 \neq t_T$. Unless we have closed interval, $]t_0, t_T[= \delta$, with $\delta = 0$, then we can define[9],

[9] If we have closed interval $]t_0, t_T[= 0$, then we will have $\mathbf{S^{PO}} \equiv \mathbf{T^{PO}}$.
Proof**:**

$$]t_0, t_T[= 0 \rightarrow [t_0, t_T] = 1, \qquad \text{for } t_0, t_T \in \mathbb{N}$$

That condition give us properties for n_T-point that valued more or equal than t_0 is equal to t_T, in the same way, we will also have n_T-point that valued less or equal than t_T is identical to t_0. Therefore, we can conclude $\mathbf{S^{PO}} = \{t_0, t_T\}$ by using axiom of extensionality (**Axiom 2**).

Proposition 13. There exists subset $\mathbf{S^{PO}}$, within $\mathbf{T^{PO}}$ that also totally ordered, and consists of all t between t_0 and t_T.

By property 6, we can declare that $\inf \mathbf{S^{PO}} = t_{0+1}$ and $\sup \mathbf{S^{PO}} = t_{T-1}$, with length $|t_r, t_s|$ equal to 1 unit of time. Therefore, hence $\mathbf{S^{PO}} \subset \mathbf{T^{PO}}$ then we will get,

$$\mathbf{S^{PO}} \cup \mathbf{T^{PO}} = \{t_0, \mathbf{S^{PO}}, t_T\}$$

$$\mathbf{S^{PO}} \cup \mathbf{T^{PO}} = \{t_0, t_{0+1}, t_{0+2}, \dots, t_{T-1}, t_T\} \tag{28}$$

Now, set $\mathbf{T}^{A} = \mathbf{S^{PO}} \cup \mathbf{T^{PO}}$, become a union of whole integer number between t_0 and t_T with interval sequential order $[t_i, t_{i+1}] = 1$ unit of time, $t_i, t_{i+1} \in \mathbf{T}^{A}$. Furthermore, we can partition them into κ-blocks subset. We arrange blocks with only one elements each, to make them into singleton sets, then we get single time-frame[10],

$$\kappa_1 = \{t_0\};\ \kappa_2 = \{t_1\};\ \dots;\ \kappa_{T-1} = \{t_{T-1}\};\ \kappa_T = \{t_T\} \tag{29}$$

The next step of that procedure into the value of export set. Let, **V** is a set of each value of export that recorded on period series on *Eq. 29*. Suppose we have $\mathbf{V} = \{V_a, V_b, \dots, V_m, V_n\}$ which is related to each period on *Eq. 29*, then we are able to partition them into ν-blocks,

$$\nu_1 = \{V_a\};\ \nu_2 = \{V_b\};\ \dots;\nu_m = \{V_m\};\nu_n = \{V_n\} \tag{30}$$

Henceforth, we are able to construct relationship between value of export and time set into cartesian product. The procedures are referred to relationship in *Eq. 4*,

[10] A little proof for partitioning procedure.

1. $\kappa_1 = \{t_0\};\ \kappa_2 = \{t_1\};\ \dots;\ \kappa_{T-1} = \{t_{T-1}\};\ \kappa_T = \{t_T\}$ are non-empty sets, with each of κ have $|\kappa| = 1$.
2. Union for all of κ, $\bigcup_{n=0}^{T} \kappa_n = \{t_0, t_{0+1}, t_{0+2}, \dots, t_{T-1}, t_T\}$, which is equal to all elements of $\mathbf{T}^{A}$.
3. Intersection for $\kappa_1 \cap \kappa_2 = \{t_0\} \cap \{t_1\} = \emptyset$.

$$\begin{aligned} \kappa_1 \times \nu_1 &= \{(t_0, V_a)\} \\ \kappa_2 \times \nu_2 &= \{(t_1, V_b)\} \\ &\vdots \\ \kappa_{T-1} \times \nu_m &= \{(t_{T-1}, V_m)\} \\ \kappa_T \times \nu_n &= \{(t_T, V_n)\} \end{aligned} \tag{31}$$

Condition 31 implies,

Proposition 14. Each pairs of $\kappa \times \nu$ is an element for $\mathbf{V_T}$ for each period t

<u>Proof</u>

Let us plug every cross product in *Eq. 31* into *Eq 4a*. This process are applied of *Eq. 1*, which is implied that $\mathbf{V_T} = \bigcup(\kappa \times \nu)$, then we will get,

$$\mathbf{V_T} = \{\kappa_1 \times \nu_1, \kappa_2 \times \nu_2, \dots, \kappa_{T-1} \times \nu_m, \kappa_T \times \nu_n\}$$

$$\mathbf{V_T} = \{(t_0, V_a), (t_1, V_b), \dots, (t_{T-1}, V_m), (t_T, V_n)\}$$

$$\mathbf{V_T} = \{V_0, V_1, \dots, V_{T-1}, V_T\} \tag{32}$$

where, each 2-tuplets element of $\mathbf{V_T}$ is a Cartesius coordinate point of $\mathbf{V_T}$-space. Those pairing conditions declare that each $V_n \in \mathbf{V}$ is solely owned by each $t_n \in$ **T** (**Proposition 1.A**).

The next process is constructing every element of $\mathbf{V^T}$ into constant market shares equation. In order to construct them we use combination approach as an analytical tool. Suppose we have $\mathbf{V^T}$ consists of all elements on *Eq. 32*, then we will have,

Proposition 15. Let $\mathcal{N}$ is cardinality of $\mathbf{V^T}$ and $\mathscr{p}$ is point observation in group of observation G, such that G is consists of $\mathscr{k}$-tuplets of $\mathscr{p}$, then

$$\mathrm{G}_{\mathscr{k}} = (\mathscr{p}_1, \dots, \mathscr{p}_{\mathscr{k}}) \text{ for } 2 \leq \mathscr{k} \leq \mathcal{N}$$

$\mathrm{G}_{\mathscr{k}}$ groups of observations are constructed by following binomial expression,

(33) $$\mathrm{G}_{k} = \binom{\mathcal{N}-2}{k}$$

where, $\mathcal{N}-2$ condition at *Eq. 33* are presented hence the combinations are only applied into combinations of $\mathbf{S^{PO}}$. V_0 and V_T are already "chosen" in order to create $\mathbf{T^{PO}}$. Therefore, we have $\mathrm{G_{PO}} = (V_0, V_T)$ as primary group of observations in constant market shares analysis.

G_{k} at *Eq. 33* provide us a group of observation within $\mathbf{T^{PO}}$. However, G_{k} for $\mathcal{N} \geq k \geq 2$, can provide more than one G_{k}.

Proposition 16. Let $\Xi_{\mathbf{G}}$ is all possible G_{k} which can be constructed within $\mathbf{T^{A}}$, then

(34) $$\Xi_{\mathbf{G}} = \sum_{k=0}^{\mathcal{N}-2} \binom{\mathcal{N}-2}{k} = 2^{\mathcal{N}-2}$$

Implies,

Proposition 17. The value of $\Xi_{\mathbf{G}}$ is equal to $\Xi_{\mathbf{S}}$.

<u>Proof</u>

Assume $\Xi_{\mathbf{G}} = \Xi_{\mathbf{S}}$ is true, then implies

$$2^{\mathcal{N}-2} = 2^{(\tau^{PO}-\mathbf{1})}$$

(35a) $$\mathcal{N}-2 = \tau^{PO} - 1$$

Recall, $\mathcal{N} = |\mathbf{T^{A}}|$ and $\mathbf{T^{A}} = \mathbf{S^{PO}} \cup \mathbf{T^{PO}}$, then we will have

$$|\mathbf{T^{A}}| = |\mathbf{S^{PO}} \cup \mathbf{T^{PO}}|$$

By using De Morgan's Law at *Eq. 18*, then

$$|\mathbf{T^{A}}| = |\mathbf{S^{PO}}| + |\mathbf{T^{PO}}| - |\mathbf{S^{PO}} \cap \mathbf{T^{PO}}|$$
$$\mathcal{N} = (\tau^{PO} - 1) + 2 - 0$$
$$\mathcal{N} = (\tau^{PO} + 1)$$

(35b)

Plug *Eq. 35b* into *Eq. 35a*,

$$(\tau^{PO} + 1) - 2 = \tau^{PO} - 1$$

(35c) $$\tau^{PO} - 1 = \tau^{PO} - 1$$

Equal sign on *Eq. 35c* proves that statement on **Proposition 17** is true.

Furthermore, we are able to construct group of observation, which are used into constant market shares analysis. Hence *Eq. 35c* is proven $\boldsymbol{\Xi}_{\mathbf{G}} = \boldsymbol{\Xi}_{\mathbf{S}}$, then we are able to conclude that,

(36) $$\boldsymbol{\Xi}_{\mathbf{G}}^{V}: V_T - V_0 = (V_\alpha - V_0) + (V_\beta - V_\alpha) + \cdots + (V_T - V_\varphi)$$

where $V_0, V_\alpha, V_\beta, \dots, V_\varphi, V_T$ are specifically member of $\mathrm{G}_{\Bbbk}$.

We can illustrate those processes of generalization into $\mathbf{V_T}$-space on **Figure 5**. Assume, we have 4 blocks, $\kappa = \{t_0\}, \{t_1\}, \{t_{T-1}\}, \{t_T\}$ and $\nu = \{V_a\}, \{V_b\}, \{V_m\}, \{V_n\}$, then we will have,

$$\mathbf{V_T} = \{(t_0, V_a), (t_1, V_b), (t_{T-1}, V_m), (t_T, V_T)\}$$

which are represented as coordinates points on cartesian diagram at **Figure 5**. Furthermore, we assume $V_a < V_b < V_m < V_n$.

Figure 5. Cartesian Products of $\kappa \times \nu$.

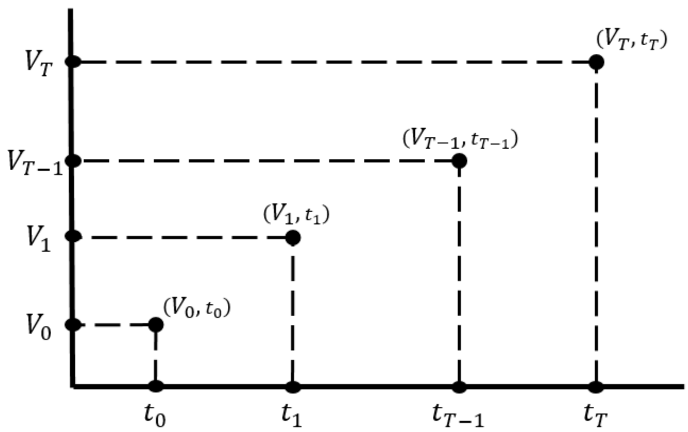


Figure 5, we generate four coordinates in the diagram, namely by order V_0, V_1, V_{T-1}, and V_T points. Graphical approach of selecting $\mathrm{G}_{\Bbbk}$ observation points by this point forward are required graphical representation of mapping $\mathbf{V} \times \mathbf{T}$ as

pictured at **Figure 6**. As we explain before that **Figure 6** are pictured all of component $\mathbf{V_T}$. Hence $\mathbf{V_T} \subset \mathbf{V} \times \mathbf{T}$, then Figure 6 will give us a complete cartesian product between **V** and **T**.

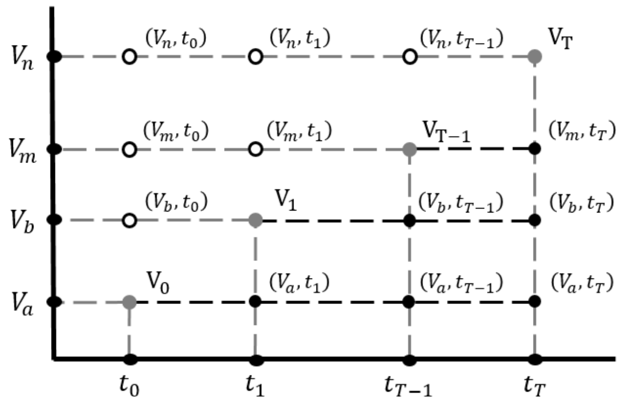
$$\begin{aligned} \mathbf{V} \times \mathbf{T} = &(\boldsymbol{t_0}, \boldsymbol{V_a}), (t_0, V_b), (t_0, V_m), (t_0, V_n), (t_1, V_a), (\boldsymbol{t_1}, \boldsymbol{V_b}), (t_1, V_m), \\ &(t_1, V_n), (t_{T-1}, V_a), (t_{T-1}, V_b), (\boldsymbol{t_{T-1}}, \boldsymbol{V_m}), (t_{T-1}, V_n), \\ &(t_T, V_a), (t_T, V_b), (t_T, V_m), (\boldsymbol{t_T}, \boldsymbol{V_n}) \end{aligned} \tag{37}$$

A complete $\mathbf{V} \times \mathbf{T}$, which is formed at *Eq. 37*, technically consists of three main parts or subsets,

$$\mathbf{V} \times \mathbf{T} \equiv \mathbf{V_T^{null}} \cup \mathbf{V_T} \cup \mathbf{V_T^{record}}$$

$\mathbf{V_T^{null}}$ is a subset of $\mathbf{V} \times \mathbf{T}$ which is consist of component $V_z = \emptyset$, for (V_z, t_t).

Figure 6. Complete $\mathbf{V} \times \mathbf{T}$ Discrete Space.



Those elements are denoted by white-rounded-by-black-ring dots. $\mathbf{V_T}$ is a subset that consists of constant market shares point of observations, denoted by red dots. $\mathbf{V_T^{record}}$ is a subset that consists of points of observations, which are valued by *record* the data V_T from t_T to any points t_{T+k}. The last subset denoted by black dots in Figure 6.

Proposition 18. Suppose $V_{n',T+k} = (V_{n'}, t_{T+k})$ is V_n that is valued at period t_{T+k}, given $V_T = (V_n, t_T)$, then $V_{n',T+k}$ is recorded V_T if and only if $V_n = V_{n'}$. Therefore, $V_{n',T+k} \in \mathbf{V_T^{record}}$.

Proof

Hence V_n is computed exogenously by *Eq. 3* and labeled by *Eq. 6*, then V_T will have the same value V_n for $V_{n',T+k}$ given value $V_{n'}$ is *recorded* at any period t_{T+k}. In our selecting process, basically, we apply combinations on $\forall V_T \in \mathbf{V_T}$, in order to construct selected point of observations by *Eq. 33*,

$$\mathrm{G}_{\mathscr{k}} = \binom{\mathcal{N}-2}{\mathscr{k}}$$

However, in order to construct CMS analysis, this process are incomplete. By definition 1, CMS is focused on analysis between two point of observation that chosen under *Eq. 33*. The CMS analysis technically measures the value differences between V_T. Therefore, we cannot directly measure[11] the difference term between V_T hence the properties bounded to **$\mathbf{V_T}$-space**.

In order to measure those differences then we need $V_{n',T+k} \in \mathbf{V_T^{record}}$ for $\forall V_T \in \mathbf{V_T}$ on the process. In our cases, the condition can be shown by **Figure 7**, which are points of observation used are concentrated on period t_T. Those points lie on the shaded area at Figure 7, $\mathrm{V_T} = (V_n, t_T)$; $\mathrm{V}_{\mathrm{T}-1,t_T} = (V_m, t_T)$; $\mathrm{V}_{1,t_T} = (V_b, t_T)$; and $\mathrm{V}_{0,t_T} = (V_a, t_T)$.

Hence those three elements $V_{m',T}$, $V_{b',T}$, and $V_{a',T}$ are *record* the value of V_{T-1}, V_1, and V_a respectively, then we can write them into their *recorded* points. Therefore, by those four elements of $\mathbf{V_T}$ we can construct $\mathrm{G}_{\mathscr{k}}$ with $\mathscr{k} = 2, 3, 4$.

[11] Direct measure of the differences between element of $\mathbf{V_T}$ is able to be done by treat **$\mathbf{V_T}$-space** as a metric discrete space $(\mathbf{V_T}, d)$, implies the distance, d between those points are measured by,

$$d_H(V_T, V_{T+k}) = \max\{d(V_T, V_{T+k}) : V_T, V_{T+k} \in \mathbf{V_T}\}$$

However, those value are not represented constant market share equation, which we are looking for. Therefore we need to measure them by indirect measurement between element of $\mathbf{V_T}$.

III. Empirical Implementation

In this research we will apply our general construction of CMS analysis into empirical calculation of Indonesia export performance to Japan's market for 2009-2014 period analysis. We used Fagerberg-Sollie "one market and several commodities" CMS model to capture effects that are determined those export performances. We also used one digit SITC Rev. 3 Indonesia to Japan's export reported by UN COMTRADE database[12]. Graphic information is shown by **Figure 7** below,

[Insert **Figure 7** Here]

By those information, we are able to define $\mathbf{T}^{\boldsymbol{PO}} = \{V_{2009}, V_{2014}\}$ as primary observation and $\mathbf{S}^{\boldsymbol{PO}} = \{V_{2010}, V_{2011}, V_{2012}, V_{2013}\}$ as subset observation. Under this circumstances[13], then we construct $\mathrm{path}^{H}(2014_{Y}, 2009_{Y})$ and their sub-paths $\mathrm{path}^{H_J}(T_Y, T_Y)$, which is connecting subset observations to "travel" between 2014_{Y} and 2009_{Y} points.

$$
\begin{aligned}
length\,\mathrm{path}^{H}(2014_{Y}, 2009_{Y}) &= (2014 - 2009)\ year \\
&= 5\ year
\end{aligned}
$$

Hence length of path^{H} is 5, then there will be $2^{[lt\ \mathrm{path}^{H} - 1]} = 2^4$ which is equal to 16 variance of path^{H_J} between 2014_Y and 2009_Y points and also 5 groups $\mathrm{G}_{\&}$. Let Ξ_{S} is variance of path^{H_J}, then we will have,

$$
\Xi_{\mathrm{S}}\colon \mathrm{path}^{H}(2014_{Y}, 2009_{Y}) = \sum_{J=1;}^{\omega} \mathrm{path}^{H_J}\left[t_{a_J},\ t_{(a_J+\tau_J)}\right]
$$

where, each Ξ_{S} is constructed in **Table 3** hereunder,

---

[12] List of product classification for 1-digit SITC Rev. 3 provided at **Table 2**

[13] By proposition 8, if there are points, which are related by

$$
n_T \neq m_T, \qquad n_T, m_T \in \hat{\mathbf{O}}_{\mathbf{n}}
$$

then exist one Hasse-path, $\mathrm{path}^{H}(m_T, n_T)$, connecting those two points on *timeline*.

(38)

[Insert **Table 3** Here]

Afterwards, we apply all of variance path to sub-period analysis of constant market shares. Recall,

(39) $$\Xi_{\mathbf{S}}^{\mathbf{V}}: V_{2014} - V_{2009} = \sum_{J=1}^{\omega} V_{a_{J}+\tau_{J}} - V_{a_{J}}$$

where Ξ_{S}^{V} is applied path system into element V within $\mathbf{T}^{\boldsymbol{A}}$.

In order to analyze this case, we use Fageberg and Sollie (1987) "several commodities, one market" model of CMS as follows,

(40) $$\Delta V^{kl} = \Delta M_{a}^{kl} + \Delta M_{b}^{kl} + \Delta M_{ab}^{kl}$$

where, ΔV^{kl} is changes of k-country value of export to l-country, ΔM_{a}^{kl} is denoted changes of *market shares* effect, ΔM_{b}^{kl} is *commodity composition* effect, and ΔM_{ab}^{kl} is *residual term*, which is constructed by inner product of a vector of ΔM_{a}^{kl} and ΔM_{b}^{kl}, called as a commodity adaptation effect. Afterwards, by plug *Eq. 40* into *Eq. 39* we will get,

(41) $$\Xi_{\mathbf{S}}^{\mathbf{V}}: V_{2014} - V_{2009} = \Delta V_{PO}^{AK} = \sum_{J=1}^{\omega} \left(\Delta M_{\alpha}^{AK} + \Delta M_{\beta}^{AK} + \Delta M_{\alpha\beta}^{AK}\right)_{J}$$

The result for every variance of *Eq. 41* shown in **Table 4**. The sixteen path connecting V_{2009} and V_{2014} give us all possible variance sub-period analysis within the primary objective. Each variance valued differently hence the point of observation within the sub-period analysis consists of different composition. However, as we found that every variance will give us the same value, -0.005176 (or simply write as -0.52%) as the differences value between V_{2014} and V_{2009}.

[Insert **Table 4** Here]

The same value on the rightmost column provides us a definitive proof that our generalized model is true. This condition is due to each difference value for each sub-path is the composition of the length of the path itself. Implies, the value of -0.52% is a summand of difference value for each sub-path on each variance.

The value also provides us an information that Indonesian's share in Japan is reduced 0.52 percent in 2014 compared to 2009 export value. Its declining condition caused by the declining of micro shares effect and commodity adaptation. On the other hand, commodity composition effect is improved by almost 0.2 percent.

However, those conditions are calculated by directly measuring the changes of Indonesian's share in Japan's market. However, if we use different *path* in the analysis—as shown in **Table 4**—will provide more detailed information. As shown in $\mathbf{\Xi_{16}}$, it gives us annually more detailed information about the changes in Indonesia market shares condition.

Besides, we also found that path of primary observation, which is composed by sub-period observations within CMS, is also constructed by a summand for each effect during the period of analysis. We calls them as *the sum of partial effect* (SPE). Calculate as follows,

$$(42) \quad SPE_\alpha = \sum_{J=1}^{\omega} (\Delta S_\alpha^{AK})_J \,;\; SPE_\beta = \sum_{J=1}^{\omega} \left(\Delta S_\beta^{AK}\right)_J \,;\; SPE_{\alpha\beta} = \sum_{J=1}^{\omega} \left(\Delta S_{\alpha\beta}^{AK}\right)_J$$

where, $\Delta S_{PO}^{AK} = SPE_\alpha + SPE_\beta + SPE_{\alpha\beta}$.

Each SPE is composed by a summation of each partial effect of CMS. The values of its composition are captured in the fluctuation or movement for each effect during the primary observations. This term provides us a summation of each effect, which is composed the total value of each effect on the primary objective, $\mathbf{\Xi_1}$. If we combine each value of *the sum of partial effect* then we will also get the value of -0.52%.

The compositions of SPE depend on the CMS model used for analysis. For example, if we use Widodo (2010) model, we will get six instead of three compositions of SPE. Each part of SPE describes the value for each effect during the sub-period analysis. Therefore, the fluctuation of value shows us the dynamics behavior for each effect in the CMS analysis.

Table 4 provides us all possible variation of CMS analysis for V_{2014} and V_{2009}. If we compare or synchronize them with several existing literature of Constant Market Shares analysis, we can summarize them in **Table 5**.

Table 5. The Comparison between Several Existing Literature of CMS with All Variances on Generalized Model

CMS Model	Type of Variation
Tyszynki (1951)	Ξ_1
Baldwin[a] (1958)	Ξ_3
Leamer and Stern (1970)	Ξ_1
Richardson (1971)	Ξ_1
Fagerberg and Sollie (1987)	Not-clear[b]
Widodo[c] (2010)	Ξ_3
Dyadkova and Momchilov (2014)	Ξ_{16}

Note: a) We synchronized Baldwin (1958) method to determine the point of observations into our data shown by **Figure 7**.

b) On their papers, they did not explain how to determine points of observations for theirs sub-period analysis.

c) We synchronized Widodo (2010) method to determine the point of observations into our data shown by **Figure 7**.

Based on the note under the **Table 5**, we synchronize Baldwin (1958) and Widodo (2010) methods in order to construct the sub-period analysis. Both of them using the fluctuations of the data as a criterion. If the point of the period reaches one of the peak or bottoms on fluctuations of export's value, then those points become the one of observation point on sub-period analysis. In our cases, based on **Figure 7**, the period is V_{2011}. Therefore, we will get Ξ_3 as our synchronized variations.

As the conclusion, all of the information on **Table 5** give us a conclusion of our generalized CMS position compared to the existing model and technical analysis of CMS. Each model technically is one of CMS variation in **Table 4**.

IV. Concluding Remarks

The generalization model is constructed by the use of three steps process. The first step is decomposed or breakdown the concept of CMS analysis itself. This process is aims to understand the essences of the CMS and it will give us set of time ($\mathbf{T}$) and set of value of export ($\mathbf{V}$) as their components. Each parts of components become the fundamental elements, which are composed the model.

The second step has constructed the model of CMS into an abstract topological structure by using those two fundamental components. The topological structure is easier to us in the generalization process. However, in this research, we are using two different approaches to constructing the structure. The first one we construct them into one-dimensional space structure and the second one we used two-dimensional space as a board of construction.

The last step is applying composition and partition concept into the topological structure in order to gain the generalized structure of CMS. The applying process uses different method hence those two structures has different properties.

By the fundamental properties of time, which are constructed during the breakdown process of CMS, give us insight that the one-dimensional spaces of CMS is a closed-connected space. Under this circumstances, we can connect those two distinct points, V_T and V_0, by a path. We later measured the length of the path as a duration between t_T and t_0 and the changes of value of export between that duration of time. Hence we know the value of duration, we find that the generalized sub-period analysis within V_T and V_0 can be constructed by the composition of that duration of time between t_T and t_0.

On the other hand, the second approach is using two-dimensional spaces structure in order to give us a one-to-one correspondence relationship between $\mathbf{V}$ and $\mathbf{T}$. This relation create a discrete metric space (Hausdorff Space), $\mathbf{V_T}$, which isolated points are represented each element of constant market share given $V_T \in \mathbf{V_T}$.

The last process, we find that we can directly measure the value between those singleton sets in order to get the value or the measurement of CMS. We need every element of those metric spaces in form of complete $\mathbf{V} \times \mathbf{T}$ Cartesian product. Thus, by using elements, which are defined as *recorded elements* at a given period t, we apply combination method to create a *group*, which is consists of $\mathscr{k}$-tuplets points of observation. By those points within each group we are able to construct the generalized model of Constant Market Shares analysis.

Therefore, both approaches lead us into generalized equation of CMS that is cover any points between the primary observations. Those condition enrich the analysis of export performances with more precise and flexible construction.

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APPENDIX 1. Gregorian Calendars Synchronized System Construction

In order to apply Gregorian system into our time construction, there are several properties that are essentially satisfied.

Property 7.1. $\hat{\mathbf{O}}_{\mathbf{n}}$ is synchronized order system to Gregorian Time System, O_T-order will start from the beginning of 1 A.D.

Year order on Gregorian system starts from a "1 A.D" year, where "A.D" stand for "Anno Domini". $\hat{\mathbf{O}}_{\mathbf{n}}$ is notation for year-order with positive value order, which is grouped all possible *after* "1 A.D" year ahead. Hence "A.D" is referred to positive ordinal number, then we write notation point for "A.D" group as "O_T year", such that $O_T \in \hat{\mathbf{O}}_{\mathbf{n}}$.

Technically, hence all possible period *after* "1 A.D"—including the "1 A.D"—is ordered by positive value for O_T position, then all period *before* those point are ordered by mapping properties. Those conditions are caused by "non-negative" calendar system. Therefore, those group are mapping into "B.C" year, where there is no 0 A.D in Gregorian Calendars System, then O_T start from 1_T-order.

Property 7.2 $\hat{\mathbf{O}}_{\mathbf{n}}$ consists of $\hat{\boldsymbol{O}}_{\boldsymbol{n}}^{+}$ and $\hat{\boldsymbol{O}}_{\boldsymbol{n}}^{-}$.

Property 7.2.1 $\hat{\boldsymbol{O}}_{\boldsymbol{n}}^{+}$ subset consists of $\forall t$ such that t with $O_T \stackrel{\text{def}}{=} O_T^{+} \Leftrightarrow O > 0$.

Property 7.2.2 $\hat{\boldsymbol{O}}_{\boldsymbol{n}}^{-}$ subset consists of $\forall t$ such that t with $O_T \stackrel{\text{def}}{=} O_T^{-} \Leftrightarrow O < 0$.

Let, $И \subset \mathbb{N}^{>0}$, then there is exist $^{-}И$, such that satisfying following condition:

$$И \cap {}^{-}И = \emptyset,$$

Therefore, exists mapping $\sigma\colon И \to {}^{-}И$, such that $\sigma(n) = n.BC$.

Property 7.3 $O \in {}^{-}И$ for $O_T^{-} \in \hat{\boldsymbol{O}}_{\boldsymbol{n}}^{-}$ and $O \in И$ for $O_T^{+} \in \hat{\boldsymbol{O}}_{\boldsymbol{n}}^{+}$.

Property 7.3 give us several properties to construct Gregorian time-system by following expression:

Property 7.4. If there are k_T and p_T points at $\mathbf{T}$, then,

$$(p -_G k)_T = (p -_N k)_T, \text{for } p > k$$

$$(p -_G k)_T = \sigma[(k -_N p) - 1]_T, \text{for } p < k$$

$$(p -_G k)_T = \sigma[1]_T, \text{for } p = k$$

$$\sigma(p -_G k)_T = \sigma(p +_N k)_T$$

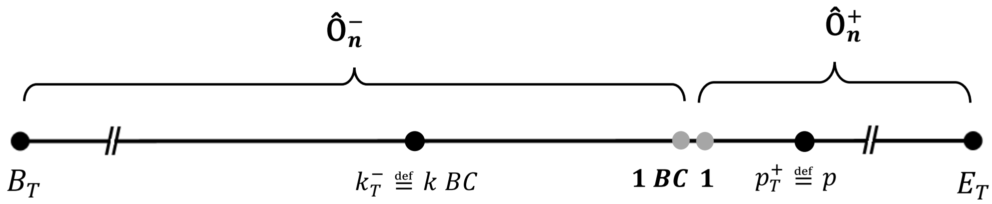
$$\sigma[p -_G \sigma(k)]_T = \sigma(p -_N k)_T, \text{for } p > k$$

$$\sigma[p -_G \sigma(k)]_T = [(k -_N p) - 1]_T, \text{for } p < k$$

Based on property 7.4, we can construct time-measurement system based on Gregorian Calendar system. Hence this construction based on calendar system, then condition on property 7.4 will be held at "$year$"-basis level unit of time. For example, suppose we have two points on this system—suppose k_T and p_T from property 6.1—then we will have conditions such as provided in Figure 1.

Assume, k_T lies on $\hat{\mathbf{O}}_{\boldsymbol{n}}^{-}$ section and p_T is pointed at $\hat{\mathbf{O}}_{\boldsymbol{n}}^{+}$ section, k_T point will defined as "$k\ BC$ year" and p_T is defined as "p year".

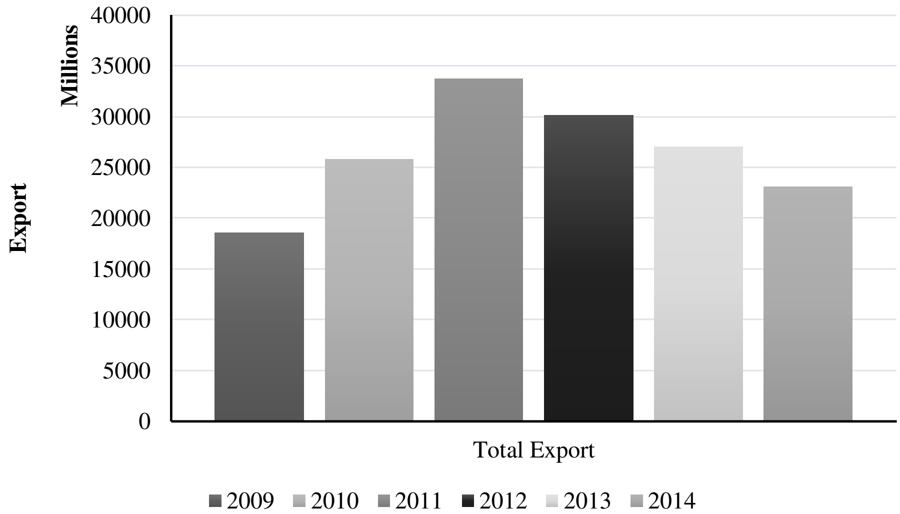
Figure 8. Gregorian *Timeline* connected by $\mathcal{C}^{\dashv}$.



APPENDIX II. Tables and Figures

1. List of Figure

Figure 7. Graph for Indonesia's export to Japan, 2009-2014.



2. List of Tables

Table 2. List of Product Classification of 1-digit SITC Rev.3

Digit Code	Name of Classification
0	Food and live animals
1	Baverages and tobacco
2	Crude materials, inedible, except fuels
3	Mineral fuels, lubricants and related materials
4	Animal and vegetable oils, fats and waxes
5	Chemicals and related products, n.e.s.
6	Manufactured goods classified chiefly by material
7	Machinery and transport equipment
8	Miscellaneous manufactured articles
9	Commodities and transactions not classified elsewhere in the SITC

Table 3. List of group G_{k} and variance Ξ_{S}, connecting V_{2014} and V_{2009}.

Group	Variance	path^{H_J}	length of path^{H_J}
G_2	Ξ_1	$(2014, 2009)$	5
G_3	Ξ_2	$(2014, 2010); (2014, 2009)$	$4+1$
	Ξ_3	$(2014, 2011); (2011, 2009)$	$3+2$
	Ξ_4	$(2014, 2012); (2012, 2009)$	$2+3$
	Ξ_5	$(2014, 2013); (2013, 2009)$	$1+4$
G_4	Ξ_6	$(2014, 2011); (2011, 2010); (2010, 2009)$	$3+1+1$
	Ξ_7	$(2014, 2012); (2012, 2010); (2010, 2009)$	$2+2+1$
	Ξ_8	$(2014, 2012); (2012, 2011); (2011, 2009)$	$2+1+2$
	Ξ_9	$(2014, 2013); (2013, 2011); (2011, 2009)$	$1+2+2$
	Ξ_{10}	$(2014, 2013); (2013, 2012); (2012, 2009)$	$1+1+3$
	Ξ_{11}	$(2014, 2013); (2013, 2010); (2010, 2009)$	$1+3+1$
G_5	Ξ_{12}	$(2014, 2012); (2012, 2011); (2011, 2010)$ $;(2010, 2009)$	$2+1+1+1$
	Ξ_{13}	$(2014, 2013); (2013, 2011); (2011, 2010)$ $;(2010, 2009)$	$1+2+1+1$
	Ξ_{14}	$(2014, 2013); (2013, 2012); (2012, 2010)$ $;(2010, 2009)$	$1+1+2+1$
	Ξ_{15}	$(2014, 2013); (2013, 2012); (2012, 2011)$ $;(2011, 2009)$	$1+1+1+2$
G_6	Ξ_{16}	$(2014, 2013); (2013, 2012); (2012, 2011)$ $;(2011, 2010); (2010, 2009)$	$1+1+1+1+1$

Table 4. Sub-Period Analysis of Constant Market Shares

Variances of Sub-Period Analysis	ΔS_{α}^{AK} (Micro shares effect)	ΔS_{β}^{AK} (Commodity Composition)	ΔS_{α}^{AK} (Commodity Adaptation)	$\mathbf{path}^{H_J}$ $[V_Y, V_{Y+k}]$
1. Ξ_1: $\text{path}^H = [V_{2014} - V_{2009}]$	-0.00641	0.001997	-0.00076	**-0.005176**
2. Ξ_2: $\text{path}^{S_1} = [V_{2014} - V_{2010}]$ $\text{path}^{S_2} = [V_{2010} - V_{2009}]$	-0.00889 0.00165	0.00026 0.00168	-0.00004 0.00017	-0.008671 0.003496
The Sum of Partial Effect	-0.007245	0.001939	0.000131	**-0.005176**
3. Ξ_3: $\text{path}^{S_1} = [V_{2014} - V_{2011}]$ $\text{path}^{S_2} = [V_{2011} - V_{2009}]$	-0.010204 0.002344	-0.00086 0.00314	0.000128 0.000284	-0.01094 0.005764
The Sum of Partial Effect	-0.007860	0.002272	0.000412	**-0.005176**
4. Ξ_4: $\text{path}^{S_1} = [V_{2014} - V_{2012}]$ $\text{path}^{S_2} = [V_{2012} - V_{2009}]$	-0.004997 -0.002197	-0.00073 0.00282	0.00018 -0.00025	-0.00554 0.00037
The Sum of Partial Effect	-0.007194	0.002090	-0.000071	**-0.005176**
5. Ξ_5: $\text{path}^{S_1} = [V_{2014} - V_{2013}]$ $\text{path}^{S_2} = [V_{2013} - V_{2009}]$	-0.00365 -0.00322	-0.00048 0.00257	0.000097 -0.000498	-0.00403 -0.00114
The Sum of Partial Effect	-0.006865	0.002090	-0.000401	**-0.005176**
6. Ξ_6: $\text{path}^{S_1} = [V_{2014} - V_{2011}]$	-0.01020	-0.00086	0.00013	-0.01094

$\text{path}^{S_2} = [V_{2011} - V_{2010}]$ $\text{path}^{S_3} = [V_{2010} - V_{2009}]$	0.00036 0.00165	0.00154 0.00168	0.00036 0.00017	0.00227 0.003496
The Sum of Partial Effect	-0.008197	0.002360	0.000661	**-0.005176**
7. Ξ_7: $\text{path}^{S_1} = [V_{2014} - V_{2012}]$ $\text{path}^{S_2} = [V_{2012} - V_{2010}]$ $\text{path}^{S_3} = [V_{2010} - V_{2009}]$	-0.004997 -0.00438 0.00165	-0.00073 0.00117 0.00168	0.00018 0.00008 0.00017	-0.00554 -0.00313 0.003496
The Sum of Partial Effect	-0.007734	0.002127	0.000431	**-0.005176**
8. Ξ_8: $\text{path}^{S_1} = [V_{2014} - V_{2012}]$ $\text{path}^{S_2} = [V_{2012} - V_{2011}]$ $\text{path}^{S_3} = [V_{2011} - V_{2009}]$	-0.004997 -0.00529 0.00234	-0.00073 0.00011 0.00314	0.00018 -0.00022 0.00028	-0.00554 -0.005396 0.00576
The Sum of Partial Effect	-0.007942	0.002521	0.000245	**-0.005176**
9. Ξ_9: $\text{path}^{S_1} = [V_{2014} - V_{2013}]$ $\text{path}^{S_2} = [V_{2013} - V_{2011}]$ $\text{path}^{S_3} = [V_{2011} - V_{2009}]$	-0.00365 -0.00656 0.00234	-0.00048 -0.00016 0.00314	0.000097 -0.00019 0.00028	-0.00403 -0.006905 0.00576
The Sum of Partial Effect	-0.007862	0.002496	0.000190	**-0.005176**
10. Ξ_{10}: $\text{path}^{S_1} = [V_{2014} - V_{2013}]$ $\text{path}^{S_2} = [V_{2013} - V_{2012}]$ $\text{path}^{S_3} = [V_{2012} - V_{2009}]$	-0.00365 -0.00131 -0.002197	-0.00048 -0.000204 0.00282	0.000097 0.00001 -0.00025	-0.00403 -0.00151 0.00037

The Sum of Partial Effect	-0.007158	0.002128	-0.000146	**-0.005176**
11. $\boldsymbol{\Xi_{11}}$: $\text{path}^{S_1} = [V_{2014} - V_{2013}]$ $\text{path}^{S_2} = [V_{2013} - V_{2010}]$ $\text{path}^{S_3} = [V_{2010} - V_{2009}]$	-0.00365 -0.00551 0.00165	-0.00048 0.00091 0.00168	0.000097 -0.00004 0.00017	-0.00403 -0.00464 0.003496
The Sum of Partial Effect	-0.007508	0.002102	0.000230	**-0.005176**
12. $\boldsymbol{\Xi_{12}}$: $\text{path}^{S_1} = [V_{2014} - V_{2012}]$ $\text{path}^{S_2} = [V_{2012} - V_{2011}]$ $\text{path}^{S_3} = [V_{2011} - V_{2010}]$ $\text{path}^{S_4} = [V_{2010} - V_{2009}]$	-0.004997 -0.00529 0.00036 0.00165	-0.00073 0.00011 0.00154 0.00168	0.00018 -0.00022 0.00036 0.00017	-0.00554 -0.005396 0.00227 0.003496
The Sum of Partial Effect	-0.008278	0.002608	0.000494	**-0.005176**
13. $\boldsymbol{\Xi_{13}}$: $\text{path}^{S_1} = [V_{2014} - V_{2013}]$ $\text{path}^{S_2} = [V_{2013} - V_{2011}]$ $\text{path}^{S_3} = [V_{2011} - V_{2010}]$ $\text{path}^{S_4} = [V_{2010} - V_{2009}]$	-0.00365 -0.00656 0.00036 0.00165	-0.00048 -0.00016 0.00154 0.00168	0.000097 -0.00019 0.00036 0.00017	-0.00403 -0.006905 0.00227 0.003496
The Sum of Partial Effect	-0.008198	0.002584	0.000439	**-0.005176**
14. $\boldsymbol{\Xi_{14}}$: $\text{path}^{S_1} = [V_{2014} - V_{2013}]$ $\text{path}^{S_2} = [V_{2013} - V_{2012}]$ $\text{path}^{S_3} = [V_{2012} - V_{2010}]$ $\text{path}^{S_4} = [V_{2010} - V_{2009}]$	-0.00365 -0.00131 -0.00438 0.00165	-0.00048 -0.000204 0.00117 0.00168	0.000097 0.00001 0.00008 0.00017	-0.00403 -0.00151 -0.00313 0.003496

The Sum of Partial Effect	-0.007697	0.002166	0.000356	**-0.005176**
15. $\mathbf{\Xi_{15}}$: $\text{path}^{S_1} = [V_{2014} - V_{2013}]$ $\text{path}^{S_2} = [V_{2013} - V_{2012}]$ $\text{path}^{S_3} = [V_{2012} - V_{2011}]$ $\text{path}^{S_4} = [V_{2011} - V_{2009}]$	-0.00365 -0.00131 -0.00529 0.00234	-0.00048 -0.000204 0.00011 0.00314	0.000097 0.00001 -0.00022 0.00028	-0.00403 -0.00151 -0.005396 0.00576
The Sum of Partial Effect	-0.007905	0.002559	0.000171	**-0.005176**
16. $\mathbf{\Xi_{16}}$: $\text{path}^{S_1} = [V_{2014} - V_{2013}]$ $\text{path}^{S_2} = [V_{2013} - V_{2012}]$ $\text{path}^{S_3} = [V_{2012} - V_{2011}]$ $\text{path}^{S_4} = [V_{2011} - V_{2010}]$ $\text{path}^{S_5} = [V_{2010} - V_{2009}]$	-0.00365 -0.00131 -0.00529 0.00036 0.00165	-0.00048 -0.000204 0.00011 0.00154 0.00168	0.000097 0.00001 -0.00022 0.00036 0.00017	-0.00403 -0.00151 -0.005396 0.00227 0.003496
The Sum of Partial Effect	-0.008241	0.002646	0.000419	**-0.005176**