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Abstract

This study develops a Schumpeterian growth model with endogenous entry of heterogeneous firms to analyze the effects of monetary policy on economic growth via a cash-in-advance constraint on R&D investment. Our results can be summarized as follows. In the special case of a zero entry cost, an increase in the nominal interest rate decreases R&D, the arrival rate of innovations and economic growth as in previous studies. However, in the general case of a positive entry cost, an increase in the nominal interest rate affects the distribution of innovations that are implemented and would have an inverted-U effect on economic growth if the entry cost is sufficiently large. We also calibrate the model to aggregate data of the US economy and find that the growth-maximizing inflation rate is about 3%, which is consistent with recent empirical estimates. Finally, we also explore the welfare effects of inflation and consider a number of extensions to the benchmark model.

JEL classification: O30, O40, E41
Keywords: monetary policy, inflation, economic growth, heterogeneous firms

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1 Introduction

This study develops a Schumpeterian growth model with endogenous entry of heterogeneous firms to analyze the effects of monetary policy on economic growth. The canonical Schumpeterian growth model in seminal studies such as Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) features an identical step size of quality improvements across firms. In this study, we consider a Schumpeterian model with random quality improvements as in Minniti *et al.* (2013) but with the addition of a fixed entry cost to generate endogenous entry of firms with heterogeneous step sizes of quality improvements. To incorporate money demand into this growth-theoretic framework, we impose a cash-in-advance (CIA) constraint on R&D investment. Berentsen *et al.* (2012), Chu and Cozzi (2014) and Chu *et al.* (2015) provide extensive discussion on evidence for the presence of cash requirements on R&D expenditures.[1] We capture these cash requirements using a CIA constraint on R&D.

In this monetary growth-theoretic framework, we derive the following results. In the special case of a zero entry cost, an increase in the nominal interest rate decreases R&D, the arrival rate of innovations and economic growth as in previous studies, such as Chu and Cozzi (2014) who consider a monetary Schumpeterian growth model with an identical step size of quality improvements, because the distribution of innovations that are implemented is exogenous under a zero entry cost despite random quality improvements. However, in the general case of a positive entry cost, monetary policy affects the distribution of innovations that are implemented. Specifically, an increase in the nominal interest rate decreases R&D and the arrival rate of innovations, which increases the present value of future profits. The resulting higher value of inventions leads to a lower threshold of quality improvements above which an innovation is implemented generating a positive effect on economic growth due to more entries. Together with the negative effect on the arrival rate of innovations, an increase in the nominal interest rate would have an inverted-U effect on economic growth if the entry cost is sufficiently large. Because the Fisher equation gives rise to a positive *long-run* relationship between the nominal interest rate and the inflation rate that is supported by empirical studies such as Mishkin (1992) and Booth and Ciner (2001), our result also implies an inverted-U relationship between inflation and economic growth. This theoretical prediction on an inverted-U relationship between inflation and economic growth is supported by empirical studies such as Bick (2010) and López-Villavicencio and Mignon (2011). We calibrate the model to aggregate data of the US economy to provide a quantitative analysis and find that the growth-maximizing inflation rate is 2.9%, which is close to the empirical estimate in López-Villavicencio and Mignon (2011) who identify a threshold inflation rate of 2.7% for industrialized countries.

[1] For example, early empirical studies such as Hall (1992) and Opler *et al.* (1999) find a positive and significant relationship between R&D and cash flows in US firms. More recently, Bates *et al.* (2009) document that the average cash-to-assets ratio in US firms increased substantially from 1980 to 2006 and argue that this is partly driven by their rising R&D expenditures. Brown and Petersen (2011) provide evidence that firms smooth R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves. Falato and Sim (2014) use firm-level data in the US to show that firms' cash holdings increase (decrease) significantly in response to a rise (cut) in R&D tax credits. These results suggest that due to financial frictions, firms need to use cash to finance their R&D investment.

We also simulate the welfare effects of inflation and find that the relationship between inflation and social welfare is positive. Intuitively, an increase in the nominal interest rate reduces the threshold of quality improvement (and equivalently markup) for entries. The resulting decrease in the overall price level of monopolistic intermediate goods stimulates the demand for them and increases the production of final good. The increase in output leads to more consumption, which represents a positive welfare effect, and this positive effect dominates other welfare effects of inflation under our calibrated parameter values.

Furthermore, we consider two extensions to the benchmark model. Our model with a Pareto distribution of random quality improvements and a Cobb-Douglas aggregator implies that some of the monopolistic prices can be arbitrarily large, which is empirically unrealistic. Therefore, we generalize our benchmark model by imposing an upper bound on equilibrium prices. In this case, we find that if the upper bound on equilibrium prices is sufficiently large, then our model would still feature an inverted-U effect of inflation on growth. As for the simulated welfare effects of inflation, they remain positive under our calibrated parameter values. Finally, given that our benchmark model features inelastic labor supply, we consider another extension by allowing for elastic labor supply and a CIA constraint on consumption. In this case, we find that the welfare effects of inflation are sensitive to the strength of the CIA constraint on consumption.

This study relates to the literature on innovation and economic growth. The R&D-based growth model originates from Romer (1990), who develops a variety-expanding growth model in which economic growth is driven by the development of new products. Then, Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) develop the Schumpeterian quality-ladder growth model in which economic growth is driven by the quality improvement of existing products. For simplicity, these studies assume an identical step size for all quality improvements. A recent study by Minniti *et al.* (2013) generalizes the Schumpeterian model by allowing for heterogeneous step sizes of quality improvements that are randomly drawn from a Pareto distribution. Our study extends the elegant framework of Minniti *et al.* (2013) by introducing a fixed entry cost of implementing a developed invention in order to generate endogenous entries of heterogeneous firms,[2] which turn out to have important implications on the effects of monetary policy. Recently, Iwaisako and Ohki (2017) also consider a quality-ladder model with random quality improvements, and they consider a uniform distribution with an upper bound on the profits of monopolistic firms.

This study also relates to the literature on inflation and innovation. In this literature, Marquis and Reffett (1994) is the seminal study that analyzes the effects of inflation on innovation in the Romer variety-expanding growth model. In contrast, we analyze the effects of inflation in a Schumpeterian quality-ladder model as in Chu and Lai (2013), Chu and Cozzi (2014), Chu *et al.* (2015), He and Zou (2016), Huang *et al.* (2017) and Neto *et al.* (2017), whose models however feature an identical step size of quality improvements across firms. Chu and Ji (2016) and Huang *et al.* (2015) consider monetary policy in a Schumpeterian growth model with both variety expansion and (identical) quality accumu-

[2] See also Baldwin and Robert-Nicoud (2008), Haruyama and Zhao (2008) and Gustafsson and Segerstrom (2010) who adapt this fixed entry cost into the R&D-based growth model, but they do not consider random increments on the quality ladder.

lation across firms. Arawatari *et al.* (2016) and Hori (2017) consider monetary policy in the Romer variety-expanding model with heterogeneity in the productivity of R&D entrepreneurs. As in Marquis and Reffett (1994), these studies predict a monotonic relationship between inflation and economic growth.[3] The present study contributes to this literature by allowing for the endogenous entry of firms with heterogeneous step sizes of quality improvements, which gives rise to a novel channel through which monetary policy affects innovation and growth. As a result, the model generates an inverted-U relationship between inflation and economic growth, which is supported by recent empirical studies.

The rest of this study is organized as follows. Section 2 presents and solves the model. Section 3 analyzes the growth and welfare effects of monetary policy. Section 4 considers a number of extensions to the benchmark model. The final section concludes.

2 A Schumpeterian model with heterogeneous firms

The Schumpeterian quality-ladder growth model is based on Grossman and Helpman (1991). We extend their model by (a) introducing money demand via a CIA constraint on R&D to analyze monetary policy, (b) considering lab-equipment innovation and entry processes that use final good (instead of labor) as the input, (c) allowing for random quality improvements as in Minniti *et al.* (2013), and (d) incorporating a fixed entry cost to generate endogenous entry of heterogeneous firms as in Melitz (2003). In summary, when a firm invents a higher quality product, the step size of the quality increment is randomly drawn from a Pareto distribution. If and only if the quality increment is sufficiently large, then the firm would pay the fixed entry cost to implement the invention and enter the market.

2.1 Household

In the economy, there is a representative household which has the following lifetime utility function:

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt, \tag{1}$$

where the parameter $\rho > 0$ is the subjective discount rate and c_t denotes consumption of final good (numeraire) at time t. The household maximizes utility subject to an asset-accumulation equation (expressed in real terms) given by

$$\dot{a}_t + \dot{m}_t = r_t a_t - \pi_t m_t + i_t b_t + w_t + \tau_t - c_t. \tag{2}$$

a_t is the real value of financial assets (in the form of equity shares in monopolistic intermediate goods firms) owned by the household. r_t is the real interest rate. π_t is the inflation rate. m_t is the real money balance accumulated by the household. b_t is the amount of money borrowed by R&D entrepreneurs subject to the following constraint: $b_t \leq m_t$. i_t is the interest rate on money b_t borrowed by R&D entrepreneurs, and it can be shown as a no-arbitrage condition

[3] The relationship between the two variables is usually found to be monotonically negative, but some of these studies also find that the relationship can be monotonically positive under some conditions.

that i_t must be equal to the nominal interest rate such that $i_t = r_t + \pi_t$ from the Fisher equation. To earn the wage rate w_t, the household inelastically supplies one unit of labor.[4] τ_t is a lump-sum transfer from the government to the household. From standard dynamic optimization, the familiar Euler equation is

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{3}$$

2.2 Final good

Final good is produced by perfectly competitive firms that employ labor and a composite of intermediate goods as inputs. The production function of final good is $Y_t = L_t^{\theta} K_t^{1-\theta}$, where $L_t = 1$ is labor input. K_t is a composite of intermediate goods produced with the following Cobb-Douglas aggregator:

$$K_t = \exp\left\{\int_0^1 \ln\left[\sum_j q_t(\omega, j) y_t(\omega, j)\right] d\omega\right\}, \tag{4}$$

where the integer j in $q_t(\omega, j)$ denotes the quality vintage of intermediate good ω. Let j_ω denotes the highest-quality vintage in industry ω. Firms are indifferent between the highest-quality vintage and the second-highest-quality vintage if their relative price is

$$\frac{p_t(\omega, j_\omega)}{p_t(\omega, j_\omega - 1)} = \frac{q_t(\omega, j_\omega)}{q_t(\omega, j_\omega - 1)} \equiv \lambda_t(\omega), \tag{5}$$

where $\lambda_t(\omega) > 1$ is the quality increment between the two consecutive vintages of intermediate good ω at time t. As usual, whenever this equality holds, we focus on the case in which firms buy the highest-quality intermediate good only. In equilibrium, only the highest quality intermediate goods are traded. From profit maximization, the conditional demand function for intermediate good ω is given by

$$y_t(\omega, j_\omega) = \frac{(1-\theta) Y_t}{p_t(\omega, j_\omega)} = \frac{(1-\theta) K_t^{1-\theta}}{p_t(\omega, j_\omega)}. \tag{6}$$

Multiplying $q_t(\omega, j_\omega)$ to both sides of (6) and then aggregating the natural log of the resulting equation with respect to ω, we derive

$$K_t = \left[(1-\theta) Q_t / P_t\right]^{1/\theta}, \tag{7}$$

where $Q_t \equiv \exp\left[\int_0^1 \ln q_t(\omega, j_\omega) d\omega\right]$ and $P_t \equiv \exp\left[\int_0^1 \ln p_t(\omega, j_\omega) d\omega\right]$ denote respectively the aggregate quality index and the aggregate price index of intermediate goods.

[4] Given that our model is already quite complex, we normalize the aggregate supply of labor to unity in order to sidestep the issue of scale effects; see for example, Peretto (1998, 2007) and Segerstrom (1998) for important ways of removing the strong scale effect in the Schumpeterian growth model. In the conclusion, we discuss implications of scale effects in our model.

2.3 Intermediate goods

There is a unit continuum of industries $\omega \in [0,1]$ producing differentiated intermediate goods. Each industry is temporarily dominated by a quality leader until the arrival and implementation of the next higher-quality product. The owner of the new innovation becomes the next quality leader.[5] The current quality leader in industry ω uses one unit of final good to produce one unit of intermediate good $y_t(\omega, j_\omega)$, so that the marginal cost of production is one. From Bertrand competition,[6] limit pricing yields the equilibrium price given by

$$p_t(\omega, j_\omega) = \lambda_t(\omega). \tag{8}$$

Therefore, the amount of monopolistic profit in industry ω is

$$\Pi_t(\omega, j_\omega) = [\lambda_t(\omega) - 1]\, y_t(\omega, j_\omega) = \left[\frac{\lambda_t(\omega) - 1}{\lambda_t(\omega)}\right](1-\theta)Y_t, \tag{9}$$

where the second equality uses (6) and (8).

2.4 R&D

R&D is performed by a unit continuum of competitive entrepreneurs. If an R&D entrepreneur employs $R_t(\omega)$ units of final good to engage in innovation in industry ω, then she is successful in inventing the next higher-quality product in the industry with an instantaneous probability given by

$$\phi_t(\omega) = R_t(\omega)/\alpha_t, \tag{10}$$

where $\alpha_t \equiv \alpha Q_t^{(1-\theta)/\theta}$ inversely measures R&D productivity and is proportional to $Q_t^{(1-\theta)/\theta}$ to ensure balanced growth. To facilitate the payment of $R_t(\omega)$, the entrepreneur needs to borrow cash from the household, and the cost of borrowing is determined by the nominal interest rate i_t. Therefore, the cost of R&D is $(1+i_t)\, R_t(\omega)$. Let $v_t^e(\omega, j_\omega + 1)$ denotes the expected value of an innovation before the realization of its quality increment. Then, the R&D free-entry condition is given by

$$v_t^e(\omega, j_\omega + 1)\phi_t(\omega) = (1+i_t)\, R_t(\omega) \Leftrightarrow v_t^e(\omega, j_\omega + 1)/\alpha_t = 1 + i_t. \tag{11}$$

2.5 Random quality improvements

As in Minniti *et al.* (2013), when an R&D entrepreneur invents a higher-quality product in industry ω, the quality increment $\lambda_t(\omega) > 1$ is drawn from a stationary Pareto distribution with the following probability density function:

$$f(\lambda) = \frac{1}{\kappa}\lambda^{-\frac{1+\kappa}{\kappa}}, \tag{12}$$

[5] This is known as the Arrow replacement effect; see Cozzi (2007) for a discussion of the Arrow effect.

[6] See Denicolò and Zanchettin (2010) for an analysis of Cournot competition in the Schumpeterian model.

where the parameter $\kappa \in (0,1)$ determines the shape of the Pareto distribution. Given that the expected value of $\lambda_t(\omega)$ is equal across industries, (9) implies that the expected value of $\Pi_t(\omega, j_\omega)$ is also the same across industries. Therefore, we will follow the standard treatment in the literature to focus on the symmetric equilibrium in which the arrival rate of innovations is equal across industries,[7] such that $\phi_t(\omega) = \phi_t$ for $\omega \in [0,1]$.

2.6 Endogenous firm entry

To generate an endogenous distribution of heterogeneous firms, we follow Melitz (2003) and others to consider a fixed entry cost. The entry cost is given by $\beta_t \equiv \beta Q_t^{(1-\theta)/\theta}$,[8] which is proportional to $Q_t^{(1-\theta)/\theta}$ to ensure balanced growth. Given the entry cost, a firm enters the market if and only if $v_t(\lambda) \geq \beta_t$, where $v_t(\lambda)$ denotes the *ex post* value of an innovation (i.e., after the realization of the quality increment λ).[9] $v_t(\lambda)$ is monotonically increasing in λ because $\Pi_t(\lambda) = (1-\theta)Y_t(\lambda - 1)/\lambda$ is increasing in λ. Given that $v_t(1) = 0$ and $v_t(\lambda)/Q_t^{(1-\theta)/\theta}$ is stationary in equilibrium, it can be shown that there exists a stationary threshold value of λ,[10] denoted as $\tilde{\lambda}$, above which firms implement their innovations and enter the market generating endogenous entry of firms with heterogeneous quality improvements.

2.7 Asset prices

The ex-ante value of an innovation (i.e., before the realization of λ) is formally defined as

$$v_t^e(\omega, j_\omega + 1) = \int_1^{\tilde{\lambda}} 0 \cdot f(\lambda) d\lambda + \int_{\tilde{\lambda}}^{\infty} [v_t(\lambda) - \beta_t] f(\lambda) d\lambda = \int_{\tilde{\lambda}}^{\infty} v_t(\lambda) f(\lambda) d\lambda - \Pr(\lambda \geq \tilde{\lambda})\beta_t,$$

where $\Pr(\lambda \geq \tilde{\lambda})$ denotes the probability of the innovation being implementable. In the symmetric equilibrium with $v_t^e(\omega, j_\omega + 1) \equiv v_t^e$, the no-arbitrage condition for the ex-ante value of innovation can be derived as[11]

$$r_t = \frac{\Pi_t^e + \dot{v}_t^e + \Pr(\lambda \geq \tilde{\lambda})\dot{\beta}_t - \Pr(\lambda \geq \tilde{\lambda})\phi_t \left[v_t^e + \Pr(\lambda \geq \tilde{\lambda})\beta_t \right]}{v_t^e + \Pr(\lambda \geq \tilde{\lambda})\beta_t}, \tag{13}$$

[7] Cozzi *et al.* (2007) provide a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the Schumpeterian model.

[8] We do not impose a CIA constraint on entry for the following reasons. Unlike R&D investment that is subject to uncertainty in innovation success, the entry cost is incurred after an innovation is already developed and patented. Therefore, banks should be available to extend credits to the firm, which can use the patent as a collateral.

[9] In a symmetric equilibrium with $\phi_t(\omega) = \phi_t$, the value of innovations does not depend on ω.

[10] See Appendix A for the proof.

[11] See Appendix A for the proof. To be more precise, we should refer to (13) as the no-arbitrage condition for the expected value of an implemented innovation; i.e., $\int_{\tilde{\lambda}}^{\infty} v_t(\lambda) f(\lambda) d\lambda$.

where ϕ_t is the arrival rate of innovation. $\Pr(\lambda \geq \tilde{\lambda})\phi_t$ is the instantaneous probability that an innovation is created and implemented in an industry. The Pareto probability density function implies that

$$\Pr(\lambda \geq \tilde{\lambda}) = \int_{\tilde{\lambda}}^{\infty} f(\lambda) d\lambda = \tilde{\lambda}^{-1/\kappa}. \tag{14}$$

Substituting (14) into (13) and rearranging terms yield

$$\frac{\Pi_t^e}{v_t^e + \tilde{\lambda}^{-1/\kappa}\beta_t} = r_t + \tilde{\lambda}^{-1/\kappa}\phi_t - \frac{\dot{v}_t^e + \tilde{\lambda}^{-1/\kappa}\dot{\beta}_t}{v_t^e + \tilde{\lambda}^{-1/\kappa}\beta_t}, \tag{15}$$

where the ex-ante value of monopolistic profits can be shown to be

$$\Pi_t^e = \left[\int_{\tilde{\lambda}}^{\infty} \left(\frac{\lambda - 1}{\lambda}\right) f(\lambda) d\lambda\right](1-\theta)Y_t = \left[\frac{\tilde{\lambda} - 1/(1+\kappa)}{\tilde{\lambda}^{\frac{1+\kappa}{\kappa}}}\right](1-\theta)Y_t. \tag{16}$$

Similarly, the no-arbitrage condition for the *ex-post* value of an innovation with $\lambda \geq \tilde{\lambda}$ is

$$\frac{\Pi_t(\lambda)}{v_t(\lambda)} = r_t + \tilde{\lambda}^{-1/\kappa}\phi_t - \frac{\dot{v}_t(\lambda)}{v_t(\lambda)}, \tag{17}$$

where the ex-post value of monopolistic profits with $\lambda \geq \tilde{\lambda}$ is given by

$$\Pi_t(\lambda) = \left(\frac{\lambda - 1}{\lambda}\right)(1-\theta)Y_t. \tag{18}$$

2.8 Monetary authority

The monetary policy instrument that we consider is the nominal interest rate i_t, which is exogenously set by the monetary authority. Given i_t, the inflation rate π_t is endogenously determined according to the Fisher equation such that $\pi_t = i_t - r_t$, where r_t is the real interest rate and determined from the Euler equation in (3). Then, the growth rate of the nominal money supply is given by $\mu_t = \pi_t + \dot{m}_t/m_t$, which becomes $\mu = i - \rho$ on the balanced growth path.[12] Finally, the monetary authority returns the seigniorage revenue as a lump-sum transfer $\tau_t = \dot{m}_t + \pi_t m_t$ to the household.

[12] It is useful to note that in this model, it is the *growth rate* of the money supply that affects the real economy in the long run, and a one-time change in the *level* of money supply has no long-run effect on the real economy. This is the well-known distinction between the neutrality and superneutrality of money. Empirical evidence generally favors neutrality and rejects superneutrality, consistent with our model; see Fisher and Seater (1993) for a discussion on the neutrality and superneutrality of money.

2.9 Dynamics

This section characterizes the dynamics of the model. Lemma 1 shows that given a constant nominal interest rate i, the economy immediately jumps to a balanced growth path. On this balanced growth path, each variable grows at a constant (possibly zero) growth rate.

Lemma 1 *The economy jumps to a unique and saddle-point stable balanced growth path.*

Proof. See Appendix B. ∎

2.10 Economic growth

Recall that the (log of) aggregate quality index is $\ln Q_t \equiv \int_0^1 \ln q_t(\omega, j_\omega) d\omega$. In industry ω, the quality $q_t(\omega, j_\omega)$ jumps to $q_t(\omega, j_\omega+1) = \lambda(\omega) q_t(\omega, j_\omega)$ with probability $\Pr(\lambda \geq \tilde{\lambda})\phi = \tilde{\lambda}^{-1/\kappa}\phi$. The continuum of industries shares this random process of quality improvements. Therefore, the time derivative of $\ln Q_t$ is given by

$$\frac{\dot{Q}_t}{Q_t} = \left\{ \int_0^1 \left[\ln q_t(\omega, j_\omega + 1) - \ln q_t(\omega, j_\omega)\right] d\omega \right\} \tilde{\lambda}^{-1/\kappa} \phi = \left[\int_0^1 \ln \lambda(\omega) d\omega \right] \tilde{\lambda}^{-1/\kappa} \phi. \tag{19}$$

Using the law of large numbers, we obtain[13]

$$\frac{\dot{Q}_t}{Q_t} = \left[\int_{\tilde{\lambda}}^{\infty} (\ln \lambda) \tilde{f}(\lambda) d\lambda \right] \tilde{\lambda}^{-1/\kappa} \phi = (\ln \tilde{\lambda} + \kappa) \tilde{\lambda}^{-1/\kappa} \phi, \tag{20}$$

where $\ln \tilde{\lambda} + \kappa$ captures the average step size of implemented quality improvements and $\tilde{f}(\lambda)$ is defined as

$$\tilde{f}(\lambda) \equiv \frac{f(\lambda)}{\int_{\tilde{\lambda}}^{\infty} f(\lambda) d\lambda} = \tilde{\lambda}^{\frac{1}{\kappa}} f(\lambda).$$

Finally, the growth rate of output Y_t and consumption c_t is equal to

$$g = \frac{1-\theta}{\theta} \frac{\dot{Q}_t}{Q_t} = \frac{1-\theta}{\theta} (\ln \tilde{\lambda} + \kappa) \tilde{\lambda}^{-1/\kappa} \phi. \tag{21}$$

Equation (21) shows that the equilibrium growth rate depends on two endogenous variables, the arrival rate ϕ of innovations and the threshold step size $\tilde{\lambda}$. We can determine ϕ using the R&D condition $v_t^e = (1+i)\alpha Q_t^{(1-\theta)/\theta}$, where the balanced-growth value of v_t^e is given by $v_t^e = \Pi_t^e / (\rho + \tilde{\lambda}^{-1/\kappa} \phi) - \tilde{\lambda}^{-1/\kappa} \beta Q_t^{(1-\theta)/\theta}$ using (15) and the Euler equation. Then, substituting (16) into the R&D condition, we obtain

$$(1-\theta) \left[\frac{\tilde{\lambda} - 1/(1+\kappa)}{\tilde{\lambda}^{\frac{1+\kappa}{\kappa}}} \right] \frac{Y_t}{Q_t^{(1-\theta)/\theta}} = \left[(1+i)\alpha + \tilde{\lambda}^{-1/\kappa} \beta \right] (\rho + \tilde{\lambda}^{-1/\kappa} \phi). \tag{22}$$

[13] Derivations are available in an unpublished appendix; see Appendix C.

In Appendix B, we show that the production function of final good can be expressed as

$$Y_t = \left(\frac{1-\theta}{\tilde{\lambda}e^{\kappa}}\right)^{(1-\theta)/\theta} Q_t^{(1-\theta)/\theta}. \tag{23}$$

Similarly, we can determine $\tilde{\lambda}$ using the entry condition $v_t(\tilde{\lambda}) = \beta Q_t^{(1-\theta)/\theta}$, where the balanced-growth value of $v_t(\tilde{\lambda})$ is given by $v_t(\tilde{\lambda}) = \Pi_t(\tilde{\lambda})/(\rho + \tilde{\lambda}^{-1/\kappa}\phi)$ using (17) and the Euler equation. Then, substituting (18) into the entry condition, we obtain

$$(1-\theta)\left(\frac{\tilde{\lambda}-1}{\tilde{\lambda}}\right)\frac{Y_t}{Q_t^{(1-\theta)/\theta}} = \beta(\rho + \tilde{\lambda}^{-1/\kappa}\phi). \tag{24}$$

Combining (22) and (24), we have the $\tilde{\lambda}$ condition given by

$$(\tilde{\lambda}-1)\tilde{\lambda}^{1/\kappa} = \frac{1}{1+i}\frac{\beta}{\alpha}\frac{\kappa}{1+\kappa}, \tag{25}$$

where the left-hand side is increasing in $\tilde{\lambda}$. Therefore, (25) implicitly determines the unique equilibrium value of $\tilde{\lambda}$. Using (23)-(25), we obtain the ϕ condition given by

$$\phi = \frac{\tilde{\lambda}^{-1/\theta}}{1+i}\frac{\kappa}{1+\kappa}\frac{(1-\theta)^{1/\theta}}{\alpha e^{\kappa(1-\theta)/\theta}} - \rho\tilde{\lambda}^{1/\kappa}. \tag{26}$$

Given $\tilde{\lambda}$ from (25), equation (26) determines the unique equilibrium value of ϕ.

3 Growth and welfare effects of monetary policy

In this section, we explore the effects of monetary policy on economic growth and social welfare. In Section 3.1, we analytically derive the effects of the nominal interest rate on economic growth. In Section 3.2, we calibrate the model to quantify the relationship between inflation and growth and the relationship between inflation and welfare.

3.1 Qualitative analysis

Here we first derive the effects of increasing the nominal interest rate i on the innovation-arrival rate ϕ and the threshold step size $\tilde{\lambda}$. Lemma 2 shows that ϕ is decreasing in i for a given $\tilde{\lambda}$. Lemma 3 shows that $\tilde{\lambda}$ is decreasing in i. The intuition can be explained as follows. An increase in the nominal interest rate i increases the cost of R&D and reduces the incentives for innovation; as a result, the innovation rate ϕ decreases for a given $\tilde{\lambda}$. From the balanced-growth version of (15), we have $v_t^e = \Pi_t^e/(\rho+\tilde{\lambda}^{-1/\kappa}\phi) - \tilde{\lambda}^{-1/\kappa}\beta Q_t^{(1-\theta)/\theta}$, which shows that the decrease in ϕ, by reducing creative destruction, increases the present value of the profit stream generated by implementing an innovation. This induces the implementation of innovations associated with smaller profit margins, thereby reducing the threshold markup $\tilde{\lambda}$ for entry.

Lemma 2 *For a given $\tilde{\lambda}$, the innovation rate ϕ is decreasing in the nominal interest rate i.*

Proof. Use (26). ■

Lemma 3 *The threshold step size $\tilde{\lambda}$ is decreasing in the nominal interest rate i.*

Proof. Use (25). ■

When the entry cost β_t is zero, the nominal interest rate has no effect on the distribution of innovations that are implemented because all firms enter the market regardless of the size of quality increments. In this case, $\tilde{\lambda} = 1$, and $g = \frac{1-\theta}{\theta}\kappa\phi$ is monotonically decreasing in i via ϕ. This result is the same as in Chu and Cozzi (2014), who consider a Schumpeterian growth model with an identical step size of quality improvements across firms. However, when the entry cost β_t is positive, the nominal interest rate i affects both $\tilde{\lambda}$ and ϕ. In this case, $\Pr(\lambda \geq \tilde{\lambda}) = \tilde{\lambda}^{-1/\kappa}$ is increasing in i. In other words, an increase in the nominal interest rate reduces the threshold value $\tilde{\lambda}$ for entry and leads to more innovations being implemented. When the entry cost β_t is sufficiently large, the overall effects of i on the composite innovation rate $\tilde{\lambda}^{-1/\kappa}\phi$ and the equilibrium growth rate $g = \frac{1-\theta}{\theta}(\ln\tilde{\lambda}+\kappa)\tilde{\lambda}^{-1/\kappa}\phi$ become non-monotonic. Specifically, we find that when the nominal interest rate i increases, $\tilde{\lambda}^{-1/\kappa}\phi$ and g first increase and eventually decrease. We summarize these results in Proposition 1.

Proposition 1 *If the entry cost is sufficiently large (small), an increase in the nominal interest rate has an inverted-U (negative) effect on the composite innovation rate $\tilde{\lambda}^{-1/\kappa}\phi$ and the equilibrium growth rate g*

Proof. See the Appendix B. ■

Before we conclude this section, we explore the relationship between inflation and economic growth. The Fisher equation gives rise to a positive *long-run* relationship between the inflation rate and the nominal interest rate that is supported by empirical studies such as Mishkin (1992) and Booth and Ciner (2001). In our model, the inflation rate is given by the Fisher equation $\pi = i - r = i - g(i) - \rho$, where the second equality follows from the Euler equation. Therefore, so long as $\partial g(i)/\partial i < 1$, we have $\partial\pi/\partial i = 1 - \partial g(i)/\partial i > 0$.[14] Given this positive relationship, inflation and economic growth would also exhibit an inverted-U relationship. Recent empirical studies such as Bick (2010) and López-Villavicencio and Mignon (2011) provide evidence that supports an inverted-U relationship between inflation and economic growth.

[14] Under our calibrated parameter values, steady-state inflation is increasing in the nominal interest rate.

3.2 Quantitative analysis

In this section, we calibrate the model to aggregate data of the US economy to provide a quantitative illustration on the growth and welfare effects of monetary policy. The model features the following structural parameters $\{\rho, \theta, \alpha, \beta, \kappa\}$ and policy variable i. For the discount rate, we set ρ to a standard value of 0.05. For the labor share, we set θ to a value of 0.59; see Elsby *et al.* (2013) who document that the labor share in the US has fallen to less than 0.60 recently. According to the Conference Board Total Economy Database, the average growth rate of total factor productivity (TFP) in the US is about 0.6% from 1990 to 2014. We calibrate the R&D cost parameter α by targeting the scenario in which domestic innovation drives half of the TFP growth in the US (i.e., $g = 0.3\%$).[15] For the cost of entry, we calibrate β by setting the time between arrivals of innovation $1/\phi$ to about 3 years as in Acemoglu and Akcigit (2012). For the Pareto distribution parameter, we follow Minniti *et al.* (2013) to consider $\kappa = 0.21$ as our benchmark, but we also explore another value $\kappa = 0.16$ that has interesting implications. Finally, we calibrate the value of i by targeting the average inflation rate π in the US, which is about 2.5% in the past two decades. The parameter and variable values are summarized in Table 1.

Table 1: Calibration

Targets		r	wL/Y	g	ϕ	π
		0.053	0.590	0.003	0.338	0.025
Parameters	κ	ρ	θ	α	β	i
	0.210	0.050	0.590	0.0013	1.1249	0.078
	0.160	0.050	0.590	0.0023	1.0951	0.078

To explore the welfare effects of monetary policy, we need to derive an expression for social welfare. Given that the economy is always on a balanced growth path, we impose balanced growth on (1) to derive the steady-state welfare function as

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g}{\rho}\right), \tag{27}$$

where c_0 is the balanced-growth level of consumption at time 0.[16] We know that final good Y_t is used for consumption c_t, production of intermediate goods X_t^m, R&D X_t^r and entry X_t^e.[17] Using the market-clearing condition $Y_t = c_t + X_t^m + X_t^r + X_t^e$ and normalizing the initial quality index Q_0 to unity, we derive c_0 as

$$c_0 = \left[1 - \frac{1-\theta}{(1+\kappa)\,\tilde{\lambda}}\right]\left(\frac{1-\theta}{\tilde{\lambda}e^{\kappa}}\right)^{(1-\theta)/\theta} - \alpha\phi - \beta\phi\tilde{\lambda}^{-1/\kappa}. \tag{28}$$

Under our benchmark parameter values, we find that economic growth is an inverted-U function of the nominal interest rate. In Figures 1a and 2a, we plot the equilibrium

[15] See Chu (2010) who finds that domestic R&D drives less than half of the TFP growth in the US.

[16] Here we define time 0 as the instant when the economy jumps to the new balanced growth path as a result of any policy change.

[17] See equations (B3)-(B5) for the definitions.

growth rate g against the inflation rate π, which is monotonically increasing in the nominal interest rate i. Figure 1a presents our benchmark result and shows that the relationship between economic growth and inflation follows an inverted-U shape. Furthermore, the growth-maximizing inflation rate is about 2.9%, which is close to the empirical estimate in López-Villavicencio and Mignon (2011) who find a threshold inflation rate of 2.7% for industrialized countries. As for the welfare effect of inflation, we find that social welfare is increasing in the inflation rate. We report this result in Figure 1b, in which the welfare effects are expressed in the usual equivalent variations in consumption. The intuition can be explained as follows. An increase in the nominal interest rate decreases the entry threshold $\tilde{\lambda}$, which in turn reduces the average markup and the overall price level $P = \tilde{\lambda} e^{\kappa}$.[18] This lower price level increases the demand for intermediate goods and the production of final good $Y_0 = \left[(1-\theta)/(\tilde{\lambda} e^{\kappa})\right]^{(1-\theta)/\theta}$, which in turn increases the initial level of consumption c_0. Although the growth effect is non-monotonic, the positive consumption-level effect dominates in this case. Therefore, our model with endogenous entry generates a positive relationship between inflation and welfare over a wide range of parameter values.

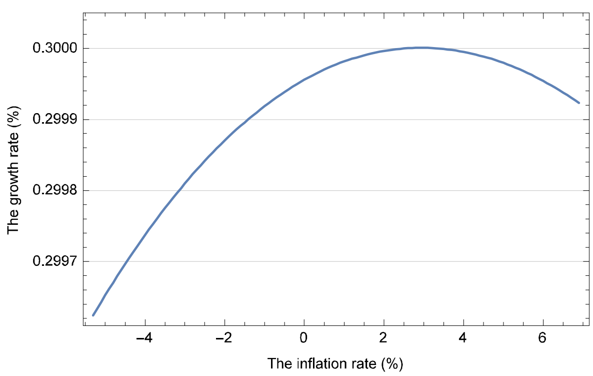

Figure 1a: Inflation and economic growth ($\kappa = 0.21$)

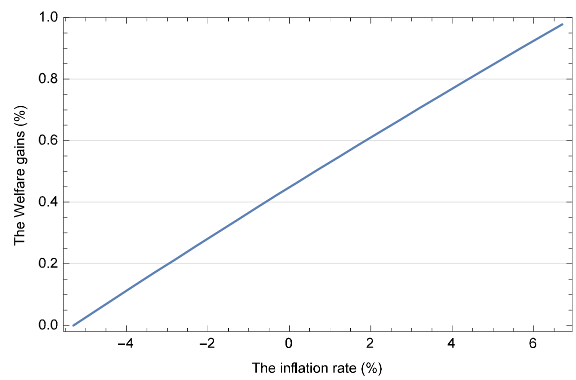

Figure 1b: Inflation and social welfare ($\kappa = 0.21$)

In the empirical literature, studies sometime find a monotonically negative effect of inflation on economic growth; see for example, Guerrero (2006) and Vaona (2012). Indeed, we find that our model is flexible enough to deliver a negative relationship between inflation and economic growth under reasonable parameter values. When we decrease the value of κ to 0.16 and recalibrate the rest of the parameters, we find that the relationship between economic growth and inflation becomes monotonically negative. In this case, the smaller value of κ implies a smaller ratio of β/α, such that the negative growth effect of inflation dominates the positive growth effect. Although the growth effect of inflation becomes negative, the welfare effect of inflation remains positive over a wide range of parameter values due to the increase in the initial level of output and consumption.

[18] See equation (B1).

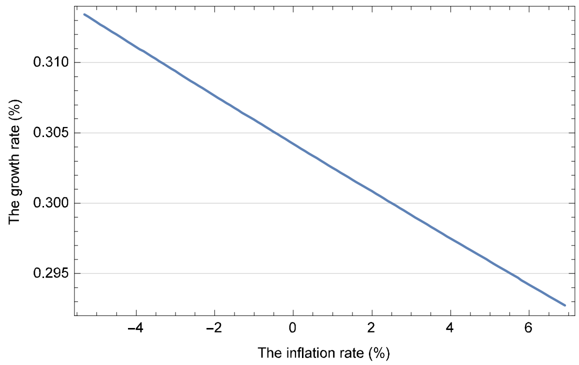

Figure 2a: Inflation and economic growth ($\kappa = 0.16$)

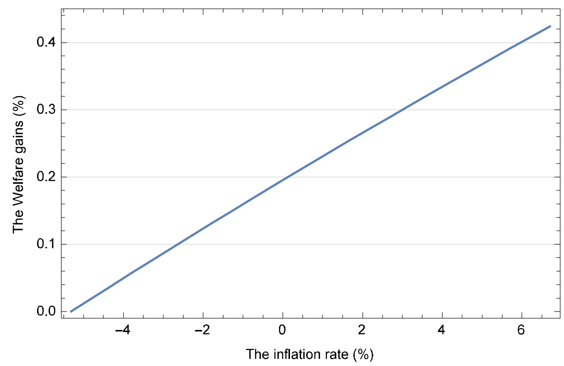

Figure 2b: Inflation and social welfare ($\kappa = 0.16$)

4 Extensions

In this section, we explore two extensions of our benchmark model.[19] First, our model implies that some prices of intermediate goods can be arbitrarily high due to the Pareto distribution. Therefore, we modify the model by imposing an upper bound on equilibrium prices. Second, our model features inelastic labor supply, under which inflation does not cause a consumption-leisure tradeoff that is commonly discussed in monetary growth models. Therefore, we modify the benchmark model by allowing for elastic labor supply and imposing also a CIA constraint on consumption.

4.1 Upper bound on monopolistic prices

In this section, we follow Evans *et al.* (2003) to impose an upper bound μ on the monopolistic prices of intermediate goods.[20] In this case, the prices of intermediate goods are given by

$$p_t(\omega, j_\omega) = \min\{\lambda_t(\omega), \mu\}. \tag{29}$$

In the following derivations, we present the changes caused by the introduction of upper bound μ and show that our inverted-U relationship between inflation and growth can still hold. Given μ, monopolistic profits become

$$\Pi_t(\omega, j_\omega) = \min\left\{\frac{\lambda_t(\omega) - 1}{\lambda_t(\omega)}, \frac{\mu - 1}{\mu}\right\}(1 - \theta)Y_t. \tag{30}$$

[19] We would like to thank the referees for these suggestions.

[20] Alternatively, one can impose an upper bound on equilibrium prices by following (a) Minniti *et al.* (2013) to replace the Cobb-Douglas aggregator in (4) by a CES aggregator or (b) Iwaisako and Ohki (2017) to impose an upper bound on the support of the distribution. For simplicity, we use the approach in Evans *et al.* (2003) to cosider price regulation that imposes an upper bound directly on monopolistic prices.

Accordingly, the ex-ante and ex-post equilibrium profits in (16) and (18) become[21]

$$\Pi_t^e = \left[\left(\frac{\tilde{\lambda} - 1/(1+\kappa)}{\tilde{\lambda}^{\frac{1+\kappa}{\kappa}}} \right) - \left(\frac{\kappa}{1+\kappa} \frac{1}{\mu^{\frac{1+\kappa}{\kappa}}} \right) \right] (1-\theta) Y_t \tag{31}$$

and

$$\Pi_t(\lambda) = \min \left\{ \frac{\lambda - 1}{\lambda}, \frac{\mu - 1}{\mu} \right\} (1-\theta) Y_t. \tag{32}$$

We follow the same procedures as in Appendix A to derive the revised $\tilde{\lambda}$ condition in (25) as follows:

$$\tilde{\lambda}^{1/\kappa} (\tilde{\lambda} - 1) = \frac{1}{1+i} \frac{\kappa}{1+\kappa} \frac{\beta}{\alpha} \left[1 - \left(\frac{\tilde{\lambda}}{\mu} \right)^{\frac{1+\kappa}{\kappa}} \right]. \tag{33}$$

Equation (33) uniquely determines the equilibrium value of $\tilde{\lambda}$ as a function of i. From (7), (8), (17), (32) and (33), we can derive the revised ϕ condition in (26) as follows:

$$\phi = \frac{\tilde{\lambda}^{-1/\theta}}{1+i} \frac{\kappa}{1+\kappa} \frac{(1-\theta)^{1/\theta}}{\alpha \left[e^{\kappa(1-\theta)/\theta} \right]^{1 - \left(\tilde{\lambda}/\mu \right)^{1/\kappa}}} \left[1 - \left(\frac{\tilde{\lambda}}{\mu} \right)^{\frac{1+\kappa}{\kappa}} \right] - \rho \tilde{\lambda}^{1/\kappa}. \tag{34}$$

Given the equilibrium value of $\tilde{\lambda}$ from (33), equation (34) determines the unique equilibrium value of ϕ, analogous to (25) and (26).

In the rest of this section, we recalibrate the model to aggregate data of the US economy to provide a numerical analysis on the growth and welfare effects of inflation. Here we present results for different values of the upper bound $\mu = \{4, 8, 12\}$. The calibrated parameter and variable values are summarized in Table 2. As for social welfare, the steady-state welfare function is the same as (27). The initial level of consumption c_0 is revised as follows in the presence of the upper bound μ:

$$c_0 = \left\{ 1 - \frac{1-\theta}{(1+\kappa)\tilde{\lambda}} \left[1 + \kappa \left(\frac{\tilde{\lambda}}{\mu} \right)^{(1+\kappa)/\kappa} \right] \right\} \left[\frac{1-\theta}{\tilde{\lambda} \left(e^{\kappa} \right)^{1 - \left(\tilde{\lambda}/\mu \right)^{1/\kappa}}} \right]^{(1-\theta)/\theta} - \alpha\phi - \beta\phi\tilde{\lambda}^{-1/\kappa}. \tag{35}$$

Table 2: Calibration ($\kappa = 0.21$)

Targets		r	wL/Y	g	ϕ	π
		0.053	0.590	0.003	0.338	0.025
Parameters	μ	ρ	θ	α	β	i
	4	0.050	0.590	0.0012	1.145	0.078
	8	0.050	0.590	0.0013	1.126	0.078
	12	0.050	0.590	0.0013	1.125	0.078

Under an upper bound of $\mu = 12$, we find that economic growth is an inverted-U function of inflation and the growth-maximizing inflation rate is about 2.7%, which corresponds to the

[21] We focus on the case with $\tilde{\lambda}_t < \mu$ because it can be shown that $\tilde{\lambda}_t \geq \mu$ does not hold in equilibrium.

empirical estimate in López-Villavicencio and Mignon (2011). This result is shown in Figure 3a. We also consider an upper bound of $\mu = 8$, under which the growth effect of inflation continues to be an inverted-U function. In Figure 4a, we see that the growth-maximizing inflation rate is lower at 1.2%. As for the case of $\mu = 4$, the result is shown in Figure 5a. Figure 5a indicates that the relationship between economic growth and inflation becomes monotonically negative in this case. Therefore, we need a sufficiently large upper bound μ to generate an inverted-U relationship between inflation and growth. According to the data in Barsky *et al.* (2003), an upper bound of 8 to 12 on the markup is not unreasonable given that some products do charge quite a sizable markup.[22] Finally, we also present the welfare effects of inflation in Figures 3b, 4b and 5b and find that they are all positive over a wide range of parameter values. These results show that despite the presence of an upper bound on prices, social welfare is still increasing in inflation.

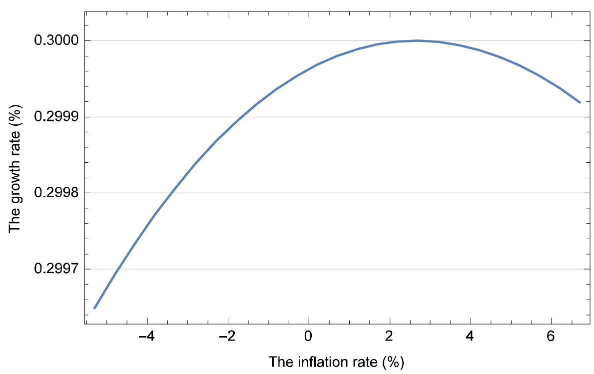

Figure 3a: Inflation and economic growth ($\mu = 12$)

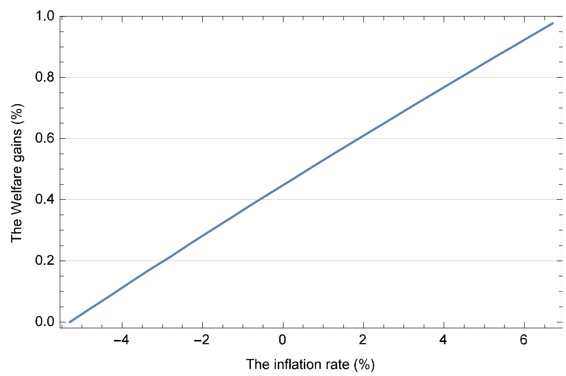

Figure 3b: Inflation and social welfare ($\mu = 12$)

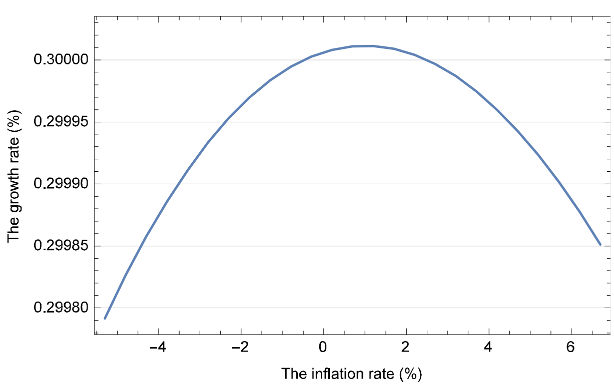

Figure 4a: Inflation and economic growth ($\mu = 8$)

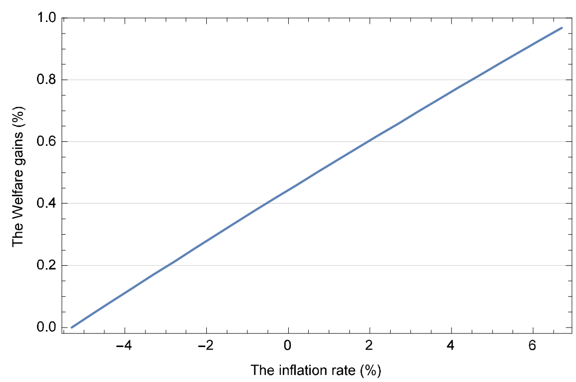

Figure 4b: Inflation and social welfare ($\mu = 8$)

[22] Furthermore, it is well known that markups in the pharmaceutical industry, which is an important innovative sector, are very high.

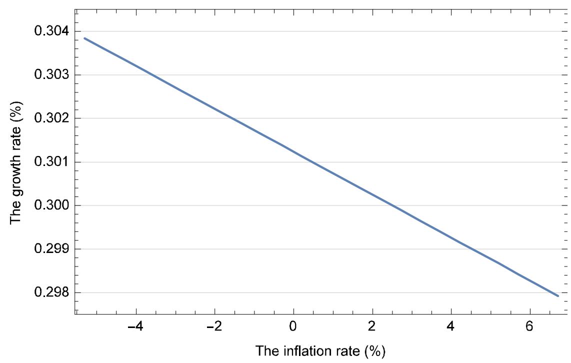

Figure 5a: Inflation and economic growth ($\mu = 4$)

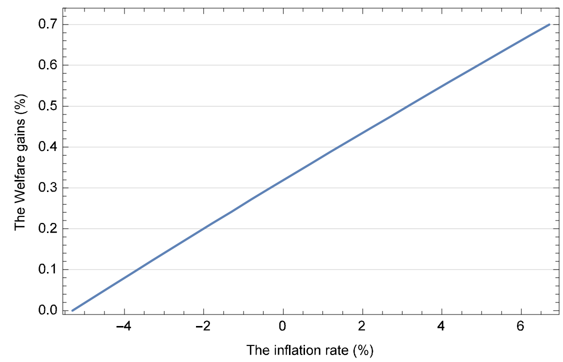

Figure 5b: Inflation and social welfare ($\mu = 4$)

4.2 Elastic labor supply and CIA constraint on consumption

For simplicity, we now relax the upper bound on prices to $\mu \to \infty$ as in the benchmark model. In this section, we explore the general case with elastic labor supply and impose a CIA constraint on consumption in addition to the CIA constraint on R&D. To consider this case, we generalize the utility function to

$$U = \int_0^\infty e^{-\rho t}[\ln c_t + \eta \ln(1 - l_t)]dt,$$

where l_t is the supply of labor and η determines the disutility of labor supply. Furthermore, we generalize the CIA constraint to $b_t + \psi c_t \leq m_t$, where $\psi \in [0, 1]$ measures the strength of the CIA constraint on consumption.

From standard dynamic optimization, the optimality condition for labor supply is

$$w_t(1 - l_t) = \eta c_t(1 + \psi i_t), \tag{36}$$

where $i_t = r_t + \pi_t$. From the profit maximization of final-good firms, the conditional demand functions for labor and intermediate goods are respectively[23]

$$w_t = \theta Y_t / l_t, \tag{37}$$

$$K_t = l_t \left[(1 - \theta) Q_t / P_t\right]^{1/\theta}. \tag{38}$$

Using (38), we express the aggregate production function of final good as

$$Y_t = l_t \left(\frac{1 - \theta}{\tilde{\lambda} e^{\kappa}}\right)^{(1-\theta)/\theta} Q_t^{(1-\theta)/\theta}. \tag{39}$$

[23]It is helpful to note that we set $L_t = l_t$ so that the production function of final good becomes $Y_t = l_t^\theta K_t^{1-\theta}$.

Combining (24), (25) and (39), the ϕ condition in (26) can be revised as follows:

$$\phi = \frac{\tilde{\lambda}^{-1/\theta}}{1+i} \frac{\kappa}{1+\kappa} \frac{(1-\theta)^{1/\theta}}{\alpha e^{\kappa(1-\theta)/\theta}} l - \rho \tilde{\lambda}^{1/\kappa}, \tag{40}$$

whereas the $\tilde{\lambda}$ condition in (25) remains unchanged.

We substitute (37) into (36) to derive $\theta Y(1-l)/l = \eta(1+\psi i)c$. Combining this condition with the resource constraint $Y = c + X^m + X^r + X^e$ and (39), we obtain

$$\theta \left(\frac{1-\theta}{\tilde{\lambda} e^{\kappa}}\right)^{(1-\theta)/\theta} (1-l) = \eta (1+\psi i) \left\{ \left[1 - \frac{1-\theta}{(1+\kappa)\tilde{\lambda}}\right] \left(\frac{1-\theta}{\tilde{\lambda} e^{\kappa}}\right)^{(1-\theta)/\theta} l - \alpha\phi - \beta\phi\tilde{\lambda}^{-1/\kappa} \right\}. \tag{41}$$

Therefore, we can solve the three endogenous variables $\{\tilde{\lambda}, \phi, l\}$ using (25), (40) and (41).

In the rest of this section, we calibrate the model to aggregate data of the US economy in order to provide a numerical analysis on the growth and welfare effects of inflation. Here we explore the implications of different degrees ψ of the CIA constraint on consumption. We calibrate the parameter η by setting the supply of labor l to a standard value of 0.33. The parameter and variable values are summarized in Table 3. As for social welfare, we make use of an analogous derivation as before to obtain

$$U = \frac{1}{\rho} \left[\ln c_0 + \frac{g}{\rho} + \eta \ln(1-l) \right], \tag{42}$$

where

$$c_0 = \left[1 - \frac{1-\theta}{(1+\kappa)\tilde{\lambda}}\right] \left(\frac{1-\theta}{\tilde{\lambda} e^{\kappa}}\right)^{(1-\theta)/\theta} l - \alpha\phi - \beta\phi\tilde{\lambda}^{-1/\kappa}. \tag{43}$$

Table 3: Calibration ($\kappa = 0.21$)

Targets		r	wL/Y	g	ϕ	l	π
		0.053	0.590	0.003	0.338	0.330	0.025
Parameters	ψ	ρ	θ	α	β	η	i
	0.00	0.050	0.590	0.0004	0.371	1.410	0.078
	0.05	0.050	0.590	0.0004	0.371	1.405	0.078
	0.13	0.050	0.590	0.0004	0.371	1.396	0.078
	1.00	0.050	0.590	0.0004	0.371	1.308	0.078

First, we consider the implications of elastic labor supply without the CIA constraint on consumption (i.e., $\psi = 0$) in Figure 6. In this case, the growth effect becomes positive because equilibrium labor l is increasing in the nominal interest rate i. The intuition can be explained as follows. Although the initial level of consumption is increasing in i, the consumption-output ratio c/Y is decreasing in i, which in turn implies that labor is increasing in i because equilibrium labor is given by $\theta(1-l)/l = \eta c/Y$. The increase in l represents an additional positive effect on the innovation-arrival rate ϕ as (40) shows; therefore, the overall growth effect of inflation becomes positive. Furthermore, the increase in l also represents an additional positive effect on the level of output Y_0 as (39) shows; therefore, the overall welfare effect of inflation remains positive and becomes quantitatively stronger than before.

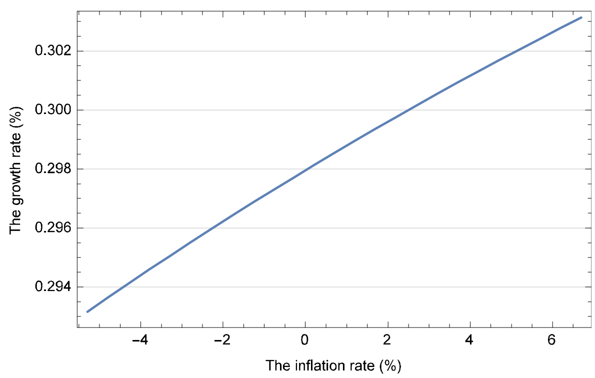

Figure 6a: Inflation and economic growth (ψ= 0)

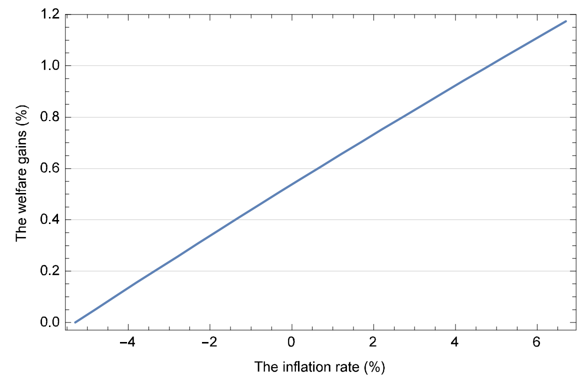

Figure 6b: Inflation and social welfare (ψ= 0)

We now explore the implications of the CIA constraint on consumption. We begin by considering a small value of $\psi = 0.05$ in Figure 7. In this case, the growth effect becomes negative because equilibrium labor l is now decreasing in the nominal interest rate i. To see this, when $\psi > 0$, equilibrium labor is given by $\theta(1-l)/l = \eta(1+\psi i)c/Y$, where ψi exerts a negative effect on l through the consumption-leisure tradeoff. The decrease in l now represents a negative effect on the innovation-arrival rate ϕ as (40) shows; therefore, the overall growth effect of inflation becomes negative. Furthermore, the decrease in l also represents a negative effect on the level of output Y_0 as (39) shows; however, the overall welfare effect of inflation remains positive but becomes quantitatively weaker than before.

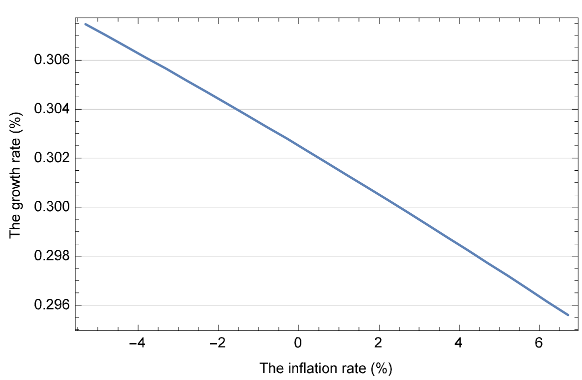

Figure 7a: Inflation and economic growth (ψ= 0.05)

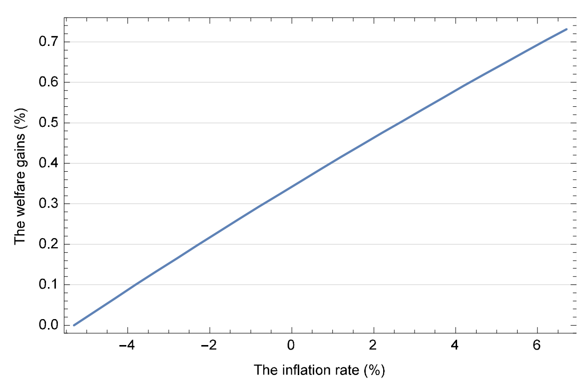

Figure 7b: Inflation and social welfare (ψ= 0.05)

We now increase the strength of the CIA constraint on consumption by raising ψ to 0.13 in Figure 8. In this case, the growth effect of inflation continues to be negative but the welfare effect of inflation becomes an inverted-U function. The reason is that the negative effect of i on labor l, which in turn exerts a negative effect on output Y_0, becomes stronger. In this case, the welfare effect of inflation eventually becomes negative when the inflation rate is sufficiently high.

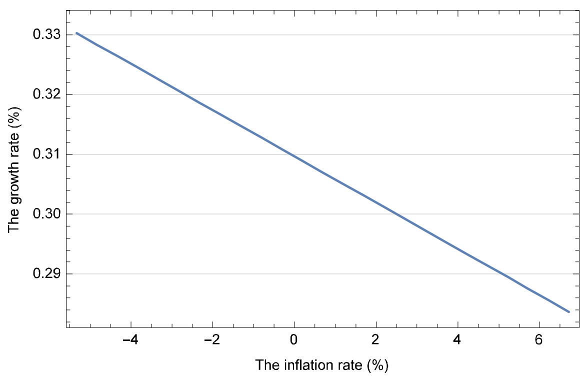

Figure 8a: Inflation and economic growth (ψ= 0.13)

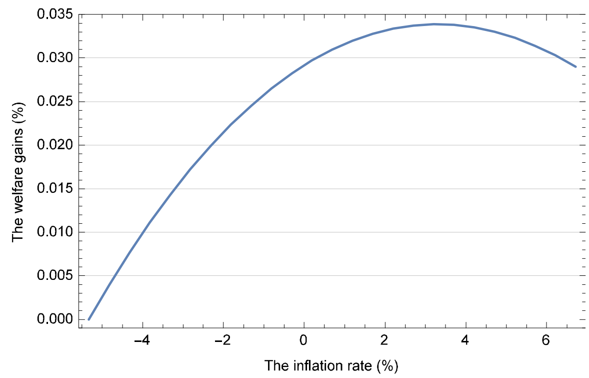

Figure 8b: Inflation and social welfare (ψ= 0.13)

Finally, we consider the maximum strength of the CIA constraint on consumption by raising ψ to 1 in Figure 9. In this case, the negative effect of i on labor becomes stronger and causes the welfare effect of inflation to become monotonically negative. Furthermore, the negative welfare effect of inflation becomes very significant.

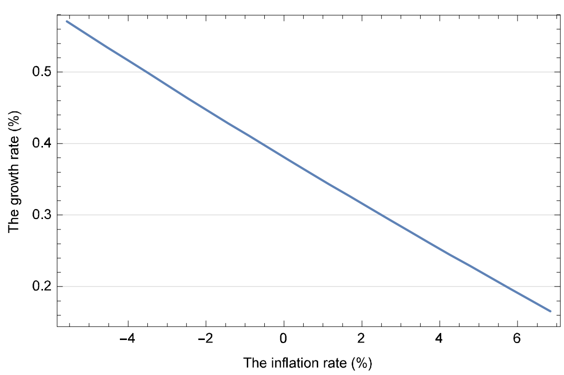

Figure 9a: Inflation and economic growth (ψ= 1)

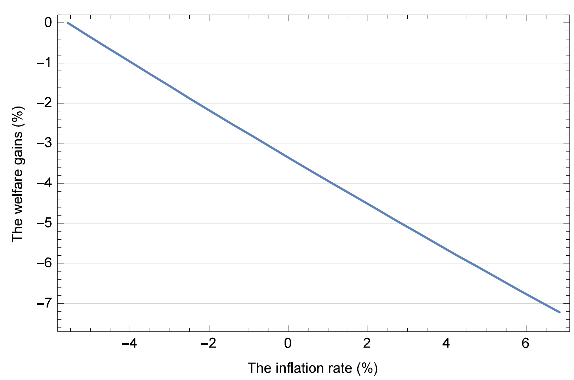

Figure 9b: Inflation and social welfare (ψ= 1)

5 Conclusion

In this study, we have developed a monetary Schumpeterian growth model with endogenous entry of firms and random quality improvements. Given this monetary growth-theoretic framework, we explore the effects of monetary policy on economic growth and find that inflation could have an inverted-U effect on economic growth. Furthermore, we calibrate the model to aggregate data of the US economy to provide a quantitative investigation. Under our benchmark parameter values, we find that the growth-maximizing inflation rate is about 2.9%, which is consistent with recent empirical estimates. However, given that we have a stylized model, the quantitative analysis should be viewed as an illustrative exercise. We have also explored the welfare effects of inflation and considered a number of extensions to the benchmark model.

In this study, we have sidestepped the issue of scale effects by normalizing the supply of labor to unity in the case of inelastic labor supply. As for the case of elastic labor supply, the scale of the economy becomes endogenous and exerts an influence on the relationship between inflation and growth. One can remove scale effects by endogenizing the market structure of the economy as in Chu and Ji (2016) and Huang *et al.* (2015), whose models are in turn based on the second-generation Schumpeterian model in Peretto (1998, 2007). Chu and Ji (2016) show that the growth effect of the nominal interest rate via the CIA constraint on consumption disappears under endogenous market structure because the market structure endogenously responds to the scale of the economy, measured by equilibrium labor, through which the nominal interest rate affects economic growth. Huang *et al.* (2015) show that the growth effect of the nominal interest rate via the CIA constraint on R&D continues to be present under endogenous market structure because the nominal interest rate directly affects the incentives for R&D (rather than through the scale of the economy) as in our benchmark model with inelastic labor supply. Finally, due to its complexity, we leave the development of a second-generation Schumpeterian model with random quality improvements to future research.

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Appendix A: The stationary quality threshold

In the symmetric equilibrium $v_t^e(\omega, j_\omega + 1) \equiv v_t^e$, the *ex-ante* value of an innovation is given by

$$v_t^e = \int_1^{\tilde{\lambda}_t} 0 \cdot f(\lambda)d\lambda + \int_{\tilde{\lambda}_t}^{\infty} [v_t(\lambda) - \beta_t]f(\lambda)d\lambda = \int_{\tilde{\lambda}_t}^{\infty} v_t(\lambda)f(\lambda)d\lambda - \Pr(\lambda \geq \tilde{\lambda}_t)\beta_t. \quad \text{(A1)}$$

Substituting the no-arbitrage condition for the *ex-post* innovation value $r_t v_t(\lambda) = \Pi_t(\lambda) + \dot{v}_t(\lambda) - \Pr(\lambda \geq \tilde{\lambda}_t)\phi_t v_t(\lambda)$ into (A1) yields

$$r_t v_t^e = \Pi_t^e + \int_{\tilde{\lambda}_t}^{\infty} \dot{v}_t(\lambda)f(\lambda)d\lambda - \Pr(\lambda \geq \tilde{\lambda}_t)\phi_t \int_{\tilde{\lambda}_t}^{\infty} [v_t(\lambda) - \beta_t] f(\lambda)d\lambda - \left[\Pr(\lambda \geq \tilde{\lambda}_t)\phi_t + r_t\right] \Pr(\lambda \geq \tilde{\lambda}_t)\beta_t. \quad \text{(A2)}$$

Combining (A1) and the R&D condition (11) and also using (14), we obtain

$$\int_{\tilde{\lambda}_t}^{\infty} v_t(\lambda)f(\lambda)d\lambda = (1 + i_t)\,\alpha_t + \tilde{\lambda}_t^{-1/\kappa}\beta_t, \quad \text{(A3)}$$

where i_t is chosen exogenously by the monetary authority. Differentiating (A3) with respect to t, we use the Leibniz integral rule to derive

$$\int_{\tilde{\lambda}_t}^{\infty} \dot{v}_t(\lambda)f(\lambda)d\lambda - v_t(\tilde{\lambda}_t)f(\tilde{\lambda}_t)\dot{\tilde{\lambda}}_t = (1 + i)\,\dot{\alpha}_t + \tilde{\lambda}_t^{-1/\kappa}\dot{\beta}_t - \frac{1}{\kappa}\tilde{\lambda}_t^{-\frac{1+\kappa}{\kappa}}\dot{\tilde{\lambda}}_t\beta_t. \quad \text{(A4)}$$

We substitute (12) and the entry condition $v_t(\tilde{\lambda}_t) = \beta_t$ into (A4) to obtain

$$\int_{\tilde{\lambda}_t}^{\infty} \dot{v}_t(\lambda)f(\lambda)d\lambda = (1 + i)\,\dot{\alpha}_t + \tilde{\lambda}_t^{-1/\kappa}\dot{\beta}_t. \quad \text{(A5)}$$

Substituting (A5) into (A2), the *ex-ante* no-arbitrage condition for an innovation can be expressed as

$$r_t = \frac{\Pi_t^e + \left[\dot{v}_t^e + \tilde{\lambda}_t^{-1/\kappa}\dot{\beta}_t\right] - \tilde{\lambda}_t^{-1/\kappa}\phi_t\left[v_t^e + \tilde{\lambda}_t^{-1/\kappa}\beta_t\right]}{v_t^e + \tilde{\lambda}_t^{-1/\kappa}\beta_t}, \quad \text{(A6)}$$

which uses (14) and the R&D condition (11) again. Moreover, we make use of the R&D condition (11), $\alpha_t = \alpha Q_t^{(1-\theta)/\theta}$ and $\beta_t = \beta Q_t^{(1-\theta)/\theta}$ to derive

$$\frac{\dot{v}_t^e + \tilde{\lambda}_t^{-1/\kappa}\dot{\beta}_t}{v_t^e + \tilde{\lambda}_t^{-1/\kappa}\beta_t} = \left(\frac{1-\theta}{\theta}\right)\frac{\dot{Q}_t}{Q_t}.$$

With this expression, (A6) becomes

$$r_t = \frac{\Pi_t^e}{v_t^e + \tilde{\lambda}_t^{-1/\kappa}\beta_t} + \left(\frac{1-\theta}{\theta}\right)\frac{\dot{Q}_t}{Q_t} - \tilde{\lambda}_t^{-1/\kappa}\phi_t. \quad \text{(A7)}$$

Meanwhile, the no-arbitrage condition for the *ex-post* value of the innovation with threshold quality (i.e., $\lambda = \tilde{\lambda}_t$) can be written as

$$r_t = \frac{\Pi_t(\tilde{\lambda}_t)}{v_t(\tilde{\lambda}_t)} + \left(\frac{1-\theta}{\theta}\right)\frac{\dot{Q}_t}{Q_t} - \tilde{\lambda}_t^{-1/\kappa}\phi_t. \tag{A8}$$

By the R&D condition (11), the entry condition $v_t(\tilde{\lambda}_t) = \beta_t$, $\alpha_t = \alpha Q_t^{(1-\theta)/\theta}$ and $\beta_t = \beta Q_t^{(1-\theta)/\theta}$, (A7) and (A8) imply

$$\frac{\Pi_t^e}{(1+i)\,\alpha + \tilde{\lambda}_t^{-1/\kappa}\beta} = \frac{\Pi_t(\tilde{\lambda}_t)}{\beta}. \tag{A9}$$

Given (16) and (18), (A9) can be rearranged as

$$(\tilde{\lambda}_t - 1)\tilde{\lambda}_t^{1/\kappa} = \frac{1}{1+i}\frac{\beta}{\alpha}\frac{\kappa}{1+\kappa}. \tag{A10}$$

Equation (A10) shows that $\tilde{\lambda}_t$ is always stationary.

Appendix B: Proofs

Proof of Lemma 1. Using (8), we can express the aggregate price index of intermediate goods as[24]

$$P_t = \exp\left[\int_0^1 \ln \lambda_t(\omega) d\omega\right] = \exp\left[\int_{\tilde{\lambda}}^{\infty} (\ln \lambda)\tilde{f}(\lambda)d\lambda\right] = \tilde{\lambda}e^{\kappa}, \tag{B1}$$

where $\tilde{f}(\lambda)$ is defined as

$$\tilde{f}(\lambda) \equiv \frac{f(\lambda)}{\int_{\tilde{\lambda}}^{\infty} f(\lambda)d\lambda} = \tilde{\lambda}^{\frac{1}{\kappa}} f(\lambda). \tag{B2}$$

Here we introduce a modified density function $\tilde{f}(\lambda)$ in summing λ on $[\tilde{\lambda}, \infty]$ because the distribution of λ in equilibrium is not on the original domain $[1, \infty)$, but instead on $[\tilde{\lambda}, \infty)$, due to endogenous entry. Note that $\int_{\tilde{\lambda}}^{\infty} \tilde{f}(\lambda)d\lambda = 1$. By (7) and (B1), we obtain $K_t = \left[(1-\theta)Q_t/(\tilde{\lambda}e^{\kappa})\right]^{1/\theta}$. Incorporating this condition into the production function $Y_t = L_t^{\theta} K_t^{1-\theta}$, we obtain

$$Y_t = \left[\frac{(1-\theta)Q_t}{\tilde{\lambda}e^{\kappa}}\right]^{(1-\theta)/\theta}, \tag{B3}$$

noting $L_t = 1$. Recall that final goods are used for consumption, production of intermediate goods, R&D and entry. Consumption is given by c_t. By (6) and (8), the amount of final goods used for the production of intermediate goods is

$$X_t^m = \int_0^1 y_t(\omega, j_\omega)d\omega = \int_0^1 \frac{(1-\theta)Y_t}{\lambda_t(\omega)}d\omega = (1-\theta)Y_t \int_{\tilde{\lambda}}^{\infty} \frac{1}{\lambda}\tilde{f}(\lambda)d\lambda = \frac{(1-\theta)Y_t}{(1+\kappa)\,\tilde{\lambda}}. \tag{B4}$$

Final goods for innovation and entry are given by

$$X_t^r = \int_0^1 R_t(\omega)d\omega = \alpha_t \phi_t \text{ and } X_t^e = \int_{\omega \in \Omega_t} \beta_t d\omega = \beta_t \tilde{\lambda}^{-1/\kappa}\phi_t, \tag{B5}$$

where Ω_t is the set of industries in which innovations take place and are implemented at date t. Finally, we substitute (B3), (B4) and (B5) into the market-clearing condition $Y_t = c_t + X_t^m + X_t^r + X_t^e$ to derive

$$\phi_t = \frac{1}{\alpha + \beta\tilde{\lambda}^{-1/\kappa}} \left[\left(\frac{1-\theta}{\tilde{\lambda}e^{\kappa}}\right)^{(1-\theta)/\theta} \left(1 - \frac{1-\theta}{(1+\kappa)\,\tilde{\lambda}}\right) - C_t\right], \tag{B6}$$

where $C_t \equiv c_t/Q_t^{(1-\theta)/\theta}$ is a transformed variable that is stationary. We substitute (16) and the R&D condition (11) into (A7) to derive

$$r_t = \frac{(1-\theta)Y_t}{(1+i)\alpha_t + \tilde{\lambda}^{-1/\kappa}\beta_t} \left[\frac{\tilde{\lambda} - 1/(1+\kappa)}{\tilde{\lambda}^{\frac{1+\kappa}{\kappa}}}\right] - \tilde{\lambda}^{-1/\kappa}\phi_t + \frac{1-\theta}{\theta}\frac{\dot{Q}_t}{Q_t}. \tag{B7}$$

[24] We achieve this by applying integration by parts to

$$\int_{\tilde{\lambda}}^{\infty} (\ln \lambda)\tilde{f}(\lambda)d\lambda = \frac{\tilde{\lambda}^{1/\kappa}}{\kappa} \int_{\tilde{\lambda}}^{\infty} (\ln \lambda)\left(\frac{d}{d\lambda}\frac{\lambda^{1-\frac{1+\kappa}{\kappa}}}{1-\frac{1+\kappa}{\kappa}}\right)d\lambda.$$

Finally, substituting (B3) and (B7) into (3) yields

$$\frac{\dot{C}_t}{C_t} = r_t - \rho - \frac{1-\theta}{\theta}\frac{\dot{Q}_t}{Q_t} = \frac{(1-\theta)^{1/\theta}}{[(1+i)\alpha + \tilde{\lambda}^{-1/\kappa}\beta]e^{\kappa(1-\theta)/\theta}}\left[\frac{\tilde{\lambda} - 1/(1+\kappa)}{\tilde{\lambda}^{(1/\theta)+(1/\kappa)}}\right] - \tilde{\lambda}^{-1/\kappa}\phi_t - \rho, \quad \text{(B8)}$$

noting the definitions $C_t \equiv c_t/Q_t^{(1-\theta)/\theta}$ and $\alpha_t \equiv \alpha Q_t^{(1-\theta)/\theta}$. Substituting (B6) into (B8), we have an one-dimensional differential equation in C_t.[25] Given that ϕ_t decreases with C_t in (B6), the right-hand side of (B8) is increasing in C_t, so the dynamics of C_t is characterized by saddle-point stability, such that C_t must jump to its interior steady-state value. Given a stationary value of C_t, (B6) implies that ϕ_t is also stationary. ■

Proof of Proposition 1. In this proof, we first show that the relationship between i and $\tilde{\lambda}^{-1/\kappa}\phi$ is either inverted U-shaped or negative. Combining (25) and (26), we have

$$\tilde{\lambda}^{-1/\kappa}\phi = \frac{(1-\theta)^{1/\theta}}{\beta e^{\kappa(1-\theta)/\theta}}\left(\frac{\tilde{\lambda}-1}{\tilde{\lambda}^{1/\theta}}\right) - \rho. \quad \text{(B9)}$$

By differentiating the right-hand side of (B9) with respect to $\tilde{\lambda}$, we can easily show that $d(\tilde{\lambda}^{-1/\kappa}\phi)/d\tilde{\lambda} > (<)\ 0$ if $\tilde{\lambda} < (>)\ 1/(1-\theta)$, implying an inverted-U relationship between $\tilde{\lambda}$ and $\tilde{\lambda}^{-1/\kappa}\phi$. In identifying the relationship with respect to i, we naturally focus on a non-trivial range of $\tilde{\lambda}$, i.e., $(\underline{\lambda}, \overline{\lambda})$, where $\tilde{\lambda}^{-1/\kappa}\phi > 0$ holds.[26] Given that $\tilde{\lambda}$ monotonically decreases with i (Lemma 3), $i \geq 0$ provides another natural upper bound of $\tilde{\lambda}$, say λ_i, which is defined by

$$(\lambda_i - 1)\,\lambda_i^{1/\kappa} = \frac{\beta}{\alpha}\frac{\kappa}{1+\kappa}. \quad \text{(B10)}$$

When λ_i is large enough (exceeding $1/(1-\theta)$), the relationship between i and $\tilde{\lambda}^{-1/\kappa}\phi$ is inverted U-shaped on the non-trivial range $(\underline{\lambda}, \lambda_i)$; see Figure 10a. When λ_i is small enough (falling below $1/(1-\theta)$), $\tilde{\lambda}^{-1/\kappa}\phi$ is monotonically decreasing in i on $(\underline{\lambda}, \lambda_i)$; see Figure 10b. Note that, by (B10), λ_i increases with β and, by (B9), $\overline{\lambda}$ decreases with β. This implies that for a larger (smaller) entry cost β, accompanied by a larger (smaller) λ_i, the relationship between i and $\tilde{\lambda}^{-1/\kappa}\phi$ becomes inverted-U (negative).

[25] Although $\tilde{\lambda}$ is an endogenous variable, it is stationary and a function of parameters as shown in (A10).

[26] The formal definition of $(\underline{\lambda}, \overline{\lambda})$ is given by incorporating $\tilde{\lambda}^{-1/\kappa}\phi = 0$ into (B9): $\underline{\lambda}$ and $\overline{\lambda}$ are equal to x such that $(x-1)/x^{1/\theta} = \rho\beta e^{\kappa(1-\theta)/\theta}/(1-\theta)^{1/\theta}$. This has the two solutions such as $x = \underline{\lambda}$ and $\overline{\lambda}$ if and only if $\rho\beta < \theta(1-\theta)^{\frac{2-\theta}{\theta}}/e^{\kappa(1-\theta)/\theta}$. Otherwise, $\tilde{\lambda}^{-1/\kappa}\phi$ cannot be positive.

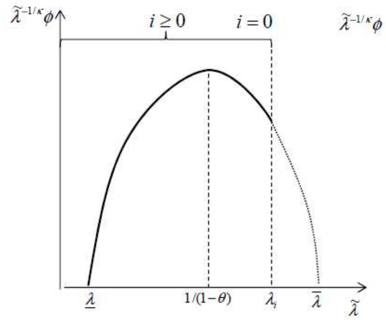

Figure 10a

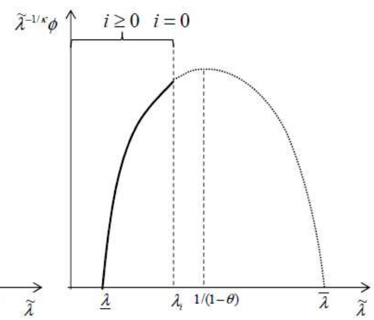

Figure 10b

In the rest of this proof, we characterize the relationship between i and g. For $\tilde{\lambda} < 1/(1-\theta)$, it holds $d(\tilde{\lambda}^{-1/\kappa}\phi)/d\tilde{\lambda} > 0$ as shown above. Given that $(\ln\tilde{\lambda} + \kappa)$ is also increasing in $\tilde{\lambda}$, this implies $dg/d\tilde{\lambda} > 0$ for $\tilde{\lambda} < 1/(1-\theta)$, by noting (21). To see the case where $1/(1-\theta) < \tilde{\lambda}$, using (21) and (B9), we can obtain

$$\frac{dg}{d\tilde{\lambda}} = \frac{1-\theta}{\theta\tilde{\lambda}^{1+1/\theta}}\left\{\underbrace{\left[\frac{(1-\theta)^{1/\theta}}{\beta e^{\kappa(1-\theta)/\theta}}\left(\tilde{\lambda}-1\right) - \rho\tilde{\lambda}^{1/\theta}\right]}_{\zeta(\tilde{\lambda})\text{: uni-modal and concave in } \tilde{\lambda}} - \underbrace{(\ln\tilde{\lambda}+\kappa)\frac{(1-\theta)^{1/\theta}}{\beta e^{\kappa(1-\theta)/\theta}}\frac{1-\theta}{\theta}\left(\tilde{\lambda}-\frac{1}{1-\theta}\right)}_{\xi(\tilde{\lambda})\text{: increasing and convex in } \tilde{\lambda}}\right\}.$$

Note the following properties: (a) $\zeta(1/(1-\theta)) > 0$ and $\xi(1/(1-\theta)) = 0$; (b) $\zeta(\tilde{\lambda})$ is an uni-modal function[27] and $\xi(\tilde{\lambda})$ is a strictly increasing function; (c) $\zeta(\underline{\lambda}) = \zeta(\overline{\lambda}) = 0$; and (d) $\zeta(\tilde{\lambda})$ is strictly concave and $\xi(\tilde{\lambda})$ is strictly convex.

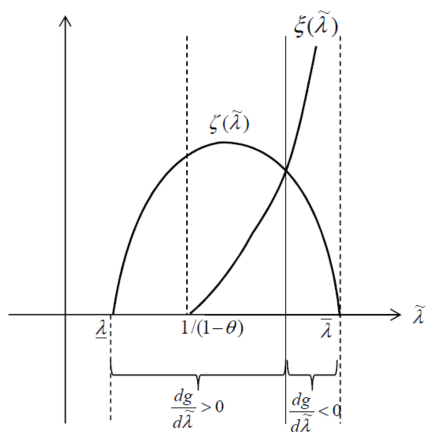

Figure 11

Using these properties, we can graphically show that $\xi(\tilde{\lambda})$ intersects $\zeta(\tilde{\lambda})$ from below only once at some point in $\tilde{\lambda} \in (1/(1-\theta), \overline{\lambda})$, below (above) which $dg/d\tilde{\lambda} > (<)\ 0$. This implies an inverted-U relation between $\tilde{\lambda}$ and g on $(\underline{\lambda}, \overline{\lambda})$. The rest of Proposition 1 straightforwardly follows, noting that λ_i is increasing in β. ■

[27] It is useful to note that $\zeta(\tilde{\lambda})$ is upward sloping at $\tilde{\lambda} = 1/(1-\theta)$.

Appendix C: Not for publication

Equation (20): Recall that the equilibrium distribution of λ is given by $\tilde{f}(\lambda)$, which is defined by (B2) in Appendix B. Then we calculate

$$\int_0^1 \ln \lambda(\omega) d\omega = \int_{\tilde{\lambda}}^{\infty} (\ln \lambda)\, \tilde{f}(\lambda) d\lambda = \frac{\tilde{\lambda}^{1/\kappa}}{\kappa} \int_{\tilde{\lambda}}^{\infty} (\ln \lambda)\, \lambda^{-\frac{1+\kappa}{\kappa}} d\lambda,$$

where the second equality uses (12) and (B2). Given that

$$\lambda^{-\frac{1+\kappa}{\kappa}} = \frac{d}{d\lambda} \frac{\lambda^{1-\frac{1+\kappa}{\kappa}}}{1-\frac{1+\kappa}{\kappa}},$$

we have

$$\int_0^1 \ln \lambda(\omega) d\omega = \frac{\tilde{\lambda}^{1/\kappa}}{\kappa} \int_{\tilde{\lambda}}^{\infty} (\ln \lambda) \left[\frac{d}{d\lambda} \left(\frac{\lambda^{1-\frac{1+\kappa}{\kappa}}}{1-\frac{1+\kappa}{\kappa}} \right) \right] d\lambda.$$

Applying integration by parts, we calculate

$$\frac{\tilde{\lambda}^{1/\kappa}}{\kappa} \int_{\tilde{\lambda}}^{\infty} (\ln \lambda) \left[\frac{d}{d\lambda} \left(\frac{\lambda^{1-\frac{1+\kappa}{\kappa}}}{1-\frac{1+\kappa}{\kappa}} \right) \right] d\lambda = \frac{\tilde{\lambda}^{1/\kappa}}{\kappa} \left\{ \left| \frac{\lambda^{1-\frac{1+\kappa}{\kappa}}}{1-\frac{1+\kappa}{\kappa}} \ln \lambda \right|_{\tilde{\lambda}}^{\infty} - \int_{\tilde{\lambda}}^{\infty} \frac{\lambda^{-\frac{1+\kappa}{\kappa}}}{1-\frac{1+\kappa}{\kappa}} d\lambda \right\}.$$

From

$$\left| \frac{\lambda^{1-\frac{1+\kappa}{\kappa}}}{1-\frac{1+\kappa}{\kappa}} \ln \lambda \right|_{\tilde{\lambda}}^{\infty} = \kappa \tilde{\lambda}^{-\frac{1}{\kappa}} \ln \tilde{\lambda} \text{ and } \int_{\tilde{\lambda}}^{\infty} \frac{\lambda^{-\frac{1+\kappa}{\kappa}}}{1-\frac{1+\kappa}{\kappa}} d\lambda = -\kappa^2 \tilde{\lambda}^{-\frac{1}{\kappa}},$$

we have $\int_0^1 \ln \lambda(\omega) d\omega = \ln \tilde{\lambda} + \kappa$.