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Abstract

This paper examines the extent to and the conditions under which resource misallocation negatively affects aggregate productivity in a model of heterogeneous firms to the highest degree. I analytically derive the minimum aggregate total factor productivity (TFP) under resource misallocation, when frictions are modeled as the taxes levied on a firm's output, and the range of these taxes is provided. I find that the lower limit of the minimum aggregate TFP is the TFP under perfect substitute goods and constant returns to scale technology. Further, the minimum aggregate TFP is achieved when the proportion of firms in the lowest tax level is small or when the TFP level of these firms is low.

Keywords: distortions; firm heterogeneity; misallocation; productivity; Ramsey problem

JEL classification: O11, O41

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1 Introduction

Cross-country differences in the aggregate total factor productivity (TFP) are one of the important sources for the income disparity between developed and underdeveloped countries. A large body of research proposes mechanisms that explain the differences in the aggregate TFP. As Restuccia and Rogerson (2007) point out, many of these mechanisms can be characterized as the theory of resource misallocation. This theory states that frictions due to various reasons prevent the efficient use of resources, resulting in a low aggregate TFP.

This paper poses the following questions: To what extent do resource misallocations affect the aggregate TFP? What kind of resource misallocation affects the aggregate TFP the most? This paper analytically addresses both these questions. There are two reasons for posing these questions. First, it is useful to know the applicability limit of the theory. Because there are infinite possibilities for resource misallocation between firms, the maximum effect of resource misallocation is not apparent. Second, the result provides information about the kind of resource misallocation mechanism researchers should focus on. While in the standard Ramsey problem, we analyze the conditions under which the maximum welfare is achieved, this paper analyzes the conditions under which the minimum aggregate TFP is achieved. In this sense, this paper inverses the standard Ramsey problem. Hence, I refer to this paper's analysis as an inverse Ramsey problem.

In order to answer the abovementioned questions, I develop a simple model of monopolistic competition with heterogeneous firms that draws heavily from previous works (Melitz, 2003, Restuccia and Rogerson, 2007, Hsieh and Klenow, 2007, and Alfaro, Charlton and Kanczuk, 2007). Following Restuccia and Rogerson (2007), frictions are described as the taxes levied on a firm's output. In this model, the differences in the taxes across firms result in resource misallocation and the loss of the aggregate TFP.[1]

[1] Although this model is static, we observe that the numerical value of the aggregate TFP is the same as that obtained in the dynamic model of Restuccia and Rogerson (2007).

Using the model, I address the abovementioned questions. I derive the minimum level of this aggregate TFP when the lower and upper bounds of the tax levels are provided, and obtain the conditions under the minimum aggregate TFP.[2] In the model, the higher the elasticity of substitution of goods and the firm's returns to scale are, the lower is the minimum aggregate TFP. The lower limit of the minimum aggregate TFP is the TFP under perfect substitute goods and constant returns to scale technology, where the minimum aggregate TFP relative to the TFP with no frictions is equal to the ratio of the gross maximum and minimum tax levels (the gross tax level implies $1 - \tau$, where τ is the taxes levied on a firm's output). The result suggests that researchers should focus on resource misallocation between firms or sectors that produce relatively substitutable goods.

Further, I find that the minimum aggregate TFP is achieved if the proportion of firms in the minimum tax level is small or if the TFP of these firms is low. Thus, resource misallocation is not necessarily related to the TFP levels of firms.[3] The result is consistent with the hypotheses that the aggregate TFP of underdeveloped countries is low because a small number of firms, such as state-owned enterprises, is protected by government policies or because the low TFP firms are protected by monopoly rights (Parente and Prescott, 1999) or by size-dependent policies (Guner, Ventura and Xu, 2008). However, this paper also reveals that to be consistent with data, the latter hypotheses might need some modifications, if goods are highly substitutive and the firm's returns to scale is high. On the other hand, the result suggests that the hypothesis that attributes the low aggregate TFP to the borrowing constraint of small firms might encounter difficulties when explaining the low aggregate TFP in underdeveloped countries. Moreover, I find that we need to maintain caution to apply

[2] I select the ratio of the (gross) lower and upper tax levels as the basis of plausibility. Since the differences in the (gross) taxes imply the differences in the factor input returns, a large difference in the lower and upper tax levels is implausible from the viewpoint of arbitrage. Under the criterion, we need to explain the differences in the aggregate TFP with a reasonable ratio of these taxes. Parente and Prescott (2005) developed a similar argument.

[3] Restuccia and Rogerson (2007) have also noted this point. I clarify that both the proportion and TFP of taxed firms quantitatively have the same effect on the aggregate TFP.

the lognormal approximation, which is widely used in the research.

There is a growing body of literature that analyzes the effect of resource misallocation on the aggregate TFP using the general equilibrium model of heterogeneous firms. Guner et al. (2008), Restuccia and Rogerson (2007), and Jones (2008) theoretically analyze the effect of resource misallocation under several scenarios. While their papers first consider the scenarios of resource misallocation and then analyze their effects on the aggregate TFP, this paper first determines the lowest level of the aggregate TFP resulting from resource misallocation and then analyzes the scenario that achieves the lowest aggregate TFP. Hsieh and Klenow (2007) and Alfaro et al. (2007), among others, measure frictions on resource misallocation and calculate the effect of these frictions on the aggregate TFP. This paper's analysis will help analyze what kind of resource misallocation is important to their results.

The remainder of this paper is organized as follows. Section 2 introduces the model, and Section 3 defines the aggregate TFP. Given these settings, Section 4 solves the inverse Ramsey problem and analyzes the implication of the results. Finally, Section 5 presents the conclusions.

2 Model

I consider an economy where the final goods are produced from the intermediate goods by a constant elasticity of substitution (CES) function, the intermediate goods are produced by a constant proportion of monopolistically competitive firms using capital and labor, and the aggregate capital and labor supply is exogenously provided. In this model, frictions are modeled as taxes levied on the intermediate firm's output.

2.1 Final goods sector

Firms in the final goods sector produce final goods Y from intermediate goods $\{y_i\}$. Further, firms in the final goods sector are competitive and maximize the following problem:

$$\max_{\{y_i\}} Y(\{y_i\}) - \int p_i y_i di,$$

where

$$Y(\{y_i\}) = \left(\int y_i^{\rho} di \right)^{\frac{1}{\rho}},$$

and p_i is an intermediate good price. I assume that $0 < \rho \leq 1$.

The first-order conditions (FOCs) are as follows:

$$p_i = y_i^{\rho-1} Y^{1-\rho}, \tag{1}$$

$$Y = \int p_i y_i di. \tag{2}$$

2.2 Intermediate goods sector

Firms in the intermediate goods sector produce intermediate goods y_i from capital k_i and labor l_i. The profit maximization problem of a monopolistically competitive intermediate goods firm is as follows:

$$\max_{k_i, l_i} (1 - \tau_i) p_i y_i - r k_i - w l_i, \tag{3}$$

$$\text{s.t. } y_i = a_i k_i^{\alpha} l_i^{\gamma},$$

where p_i is given by (1), a_i is the firm's TFP, r and w are the factor costs of capital and labor, respectively. I assume that $0 < \alpha + \gamma \leq 1$, and that $\rho(\alpha + \gamma) < 1$. While, here, i corresponds to a firm that is the price setter for its output, we can instead consider a model in which i corresponds to a sector, and the firms in each sector are price takers. The final results do not change even if we adopt the latter setting.

From the FOCs, we obtain the following relation:

$$k_i = \frac{(1-\tau_i)}{r}\alpha\rho p_i y_i, \quad (4)$$
$$l_i = \frac{1}{(1+\tau_{li})w}\gamma\rho p_i y_i.$$

2.3 Resource constraints

The following resource constraints are satisfied:

$$\int k_i di = K, \int l_i di = L,$$

where K and L are the aggregate supply of capital and labor, respectively, which are exogenously provided.

2.4 Equilibrium allocation

Here, I derive the equilibrium allocation of Y. Substituting (4) into the resource constraint of capital, we obtain

$$\frac{1}{r} = \frac{K}{\int \alpha\rho p_i y_i \lambda_i di}$$

where $\lambda_i \equiv (1-\tau_i)$. Substituting this equation into (4) and rearranging, we obtain

$$k_i = \hat{\sigma}_i \hat{\lambda}_i K, \quad (5)$$

where $\hat{\sigma}_i \equiv p_i y_i / (\int p_i y_i di)$ and $\hat{\lambda}_i \equiv \lambda_i / (\int \hat{\sigma}_i \lambda_i di)$. In the same way, we can obtain

$$l_i = \hat{\sigma}_i \hat{\lambda}_i L. \quad (6)$$

By substituting the results arrived at, Y can be rewritten as follows:

$$Y = \left[\int a_i^{\rho} \hat{\sigma}_i^{\rho\theta} \hat{\lambda}_i^{\rho\theta} di \right]^{\frac{1}{\rho}} K^{\alpha} L^{\gamma},$$

where $\theta \equiv \alpha + \gamma$.

In order to obtain the equilibrium allocation of Y, I derive the equilibrium allocations of $\hat{\sigma}_i$ and $\hat{\lambda}_i$. Appendix A shows:

$$\hat{\sigma}_i = \frac{a_i^{\kappa\rho} \lambda_i^{\kappa\rho\theta}}{W}, \tag{7}$$

where $\kappa \equiv 1/(1 - \rho\theta)$ and

$$W = \int a_i^{\kappa\rho} \lambda_i^{\kappa\rho\theta} di.$$

Using (7), the denominator of $\hat{\lambda}_i$ is written as follows:

$$\int \hat{\sigma}_i \lambda_i di = \frac{Z}{W},$$

where

$$Z = \int a_i^{\kappa\rho} \lambda_i^{\kappa} di.$$

By using the derived $\hat{\sigma}_i$ and $\hat{\lambda}_i$, we finally obtain the equilibrium allocation of Y as follows:[4]

$$Y = \frac{W^{\frac{1}{\rho}}}{Z^{\theta}} K^{\alpha} L^{\gamma}. \tag{8}$$

[4]This is a slightly extended version of the one obtained in Alfaro et al. (2007).

3 Aggregate TFP

I define the aggregate TFP A as follows:

$$A \equiv \frac{Y}{K^{\alpha} L^{\gamma}}.$$

Subsequently, the aggregate TFP in equilibrium is given by:

$$A = \frac{W^{\frac{1}{\rho}}}{Z^{\theta}}. \tag{9}$$

This equation can be rewritten as follows:

$$A = A^* N,$$

where

$$\begin{aligned} A^* &\equiv \left(\int a^{\kappa\rho} di \right)^{\frac{1}{\rho} - \theta}, \\ N &\equiv \left(\int \frac{a_i^{\kappa\rho}}{\int a_i^{\kappa\rho} di} \nu_i^{\rho} di \right)^{\frac{1}{\rho}} \Bigg/ \left(\int \frac{a_i^{\kappa\rho}}{\int a_i^{\kappa\rho} di} \nu_i^{\frac{1}{\theta}} di \right)^{\theta}, \end{aligned}$$

and $\nu_i \equiv \lambda_i^{\kappa\theta}$. A^* is the aggregate TFP level when there is no friction. I refer to N as the relative TFP because it corresponds to the aggregate TFP relative to the TFP with no frictions. Since

$$dH_i \equiv \frac{a_i^{\kappa\rho}}{\int a_i^{\kappa\rho} di} di$$

can be considered as a distribution, N can be further revised as follows:

$$N = \left(\int \nu_i^{\rho} dH_i \right)^{\frac{1}{\rho}} \Bigg/ \left(\int \nu_i^{\frac{1}{\theta}} dH_i \right)^{\theta}.$$

We can confirm $N \leq 1$ from the property of power means, because $1/\rho > \theta$.

In the following sections, I analyze how N can be lowered by resource misallocation. Moreover, I only consider the case wherein the number of tax levels is finite. Subsequently, N can be rewritten as follows (here, I slightly modify the notations):

$$N = \left(\sum_i h_i \nu_i^\rho\right)^{\frac{1}{\rho}} \Bigg/ \left(\sum_i h_i \nu_i^{\frac{1}{\theta}}\right)^\theta, \tag{10}$$

where h_i is the proportion of firms in the same tax level, adjusted by the firm's TFP

$$h_i \equiv \int_{j:\{\nu_j=\nu_i\}} \frac{a_j^{\kappa\rho}}{\int a_j^{\kappa\rho} dj} dj. \tag{11}$$

Obviously, $\sum_i h_i = 1$.

4 Inverse Ramsey Problem

4.1 Derivation of the minimum relative TFP

This section derives the minimum relative TFP, $N_{\min}$, when the gross minimum tax level $\lambda_s \equiv (1 - \tau_s)$ and the gross maximum tax level $\lambda_t \equiv (1 - \tau_t)$ are exogenously provided.[5] Here, I use the subscript s for the variables with the minimum tax level, and subscript t for those with the maximum tax level.

Owing to the following proposition, we only need to consider the distribution of λ_s and λ_t (the proof is presented in Appendix B).

Proposition 1. $N_{\min}$ *is achieved under the following condition:* $h_s + h_t = 1$.

Then, the inverse Ramsey problem is as follows:

$$\begin{aligned} N_{\min} = \min_{h_s} \quad & N \\ \text{s.t. } & N = (h_s\nu_s^\rho + h_t\nu_t^\rho)^{\frac{1}{\rho}} \Big/ \left(h_s\nu_s^{\frac{1}{\theta}} + h_t\nu_t^{\frac{1}{\theta}}\right)^\theta, \\ & h_s + h_t = 1. \end{aligned} \tag{12}$$

[5] As will be revealed later, we only need to determine the ratio of λ_s and λ_t in order to derive $N_{\min}$.

From the FOC, we obtain h_s, which achieves $N_{\min}$, $h_{s,\min}$ as follows:[6]

$$h_{s,\min} = \frac{1}{1-\rho\theta}\left(\frac{\rho\theta}{\nu^{\rho}-1} - \frac{1}{\nu^{\frac{1}{\theta}}-1}\right),$$

where $\nu \equiv \nu_s/\nu_t$. By substituting this equation into (12), we obtain $N_{\min}$ as follows:

$$N_{\min} = \left[\left(\frac{1-\mu}{1-\rho\theta}\right)^{1-\rho\theta}\left(\frac{\mu}{\rho\theta}\right)^{\rho\theta}\right]^{\frac{1}{\rho}} \tag{13}$$

where

$$\mu \equiv \frac{\nu^{\rho}-1}{\nu^{\frac{1}{\theta}}-1} = \frac{\lambda^{\frac{\rho\theta}{1-\rho\theta}}-1}{\lambda^{\frac{1}{1-\rho\theta}}-1}, \quad \lambda \equiv \lambda_s/\lambda_t.$$

$N_{\min}$ has the following limit values:

$$N_{\min} \xrightarrow[\rho\to 0]{} e^{\theta}\lambda^{-\frac{\theta}{\lambda-1}}\left(\frac{\ln\lambda}{\lambda-1}\right)^{\theta}, \tag{14}$$

$$\xrightarrow[\rho\theta\to 1]{} \frac{1}{\lambda}. \tag{15}$$

4.2 Analysis of the result

This section analyzes the results obtained in the previous section, when $\lambda \equiv (1-\tau_s)/(1-\tau_t)$ is between one and ten.[7]

Figure 1 plots the minimum relative TFP $N_{\min}$ for the following three cases using (10), (14), and (15): *(i)* $\rho \to 0$ and $\theta = 1$, *(ii)* $\rho = 1$ and $\theta = 0.9$, and *(iii)* $\rho\theta \to 1$. The parameter values of the first case are similar to those in Restuccia,

[6]Appendix C proves that the second-order condition is positive (i.e., N obtained is the local minimum). Since N under the implicit corner solutions ($h_s = 0$ and $h_s = 1$) is equal to unity and coincides with the no fraction level, the N that satisfies the FOC is the global minimum.

[7]The upper bound of ten is not unusual due to the following reason. The firm's maximization problem in (3) can be rewritten as

$$\max_{k_i,l_i} p_i y_i - \frac{r}{1-\tau_i}k_i - \frac{w}{1-\tau_i}l_i.$$

Thus, the tax on output can be interpreted as frictions on factor prices. The value of ten for λ corresponds to, for example, the rental rate variation between 3% to 30%, which I think is reasonable as the upper bound.

Yang and Zhu (2008) and Hayashi and Prescott (2006).[8] The parameter values of the second case are the same as those in Restuccia and Rogerson (2007). The third case corresponds to Parente and Prescott (1999). The second and third cases can generate a large loss of the aggregate TFP caused by resource misallocation, while the first case has relatively a low ability. One might infer from Figure 1 that $N_{\min}$ lowers as $\rho\theta$ increases. This inference is correct (for an explanation, see Appendix D). The result is analogous to the implication of the standard Ramsey problem that tax on goods with elastic demand highly distorts welfare.

An interesting point is that the correlation of the firm's TFP and tax level is not required to generate the above results. Although the firm's TFP enters into h_s, h_s can be changed arbitrarily by changing the proportion of firms. This result is particularly interesting when $N_{\min}$ converges to the Parente and Prescott (1999) case, because only at the limit, the proportion of firms does not affect the aggregate TFP.

Another interesting point is the discrepancy between the analysis in this paper and the lognormal approximation used in the literature.[9] If we assume that the distribution of the firm's TFP and tax is approximated by a joint lognormal distribution, from (9), the aggregate TFP can be approximated as follows:

$$A \simeq \exp\left\{\mu_{\ln a} + \frac{1}{1-\rho\theta}\left(\rho\sigma^2_{\ln a} - \theta\sigma^2_{\ln\lambda}\right)\right\},$$

where $\mu_{\ln a}$ is the mean of $\ln a_i$, and $\sigma^2_{\ln a}$ and $\sigma^2_{\ln\lambda}$ are the variances of $\ln a_i$ and $\ln\lambda_i$. Suppose that $\sigma^2_{\ln a} = 0$ and $\sigma^2_{\ln\lambda} > 0$. Then, as $\rho\theta$ converges to unity, the aggregate TFP converges to zero, even if the variance of taxes is considerably small. The result stems from a characteristic of the lognormal distribution that its domain is unbounded. Our result suggests that caution is required when the lognormal approximation is applied.

[8] All of the papers cited here pertain to the theory of resource misallocation.

[9] See, for example, Manuelli (2003), Hsieh and Klenow (2007), and Jones (2008).

Next, I examine the composition of firms under the minimum relative TFP. I plot the h_s under the minimum relative TFP, $h_{s,\min}$, in Figure 2. We find that $h_{s,\min}$ is small. This is because the maximum effect of frictions on the relative TFP increases as h_s decreases, in the following manner:[10]

$$\lim_{\lambda \to \infty} N = h_s^{\frac{1}{\rho} - \theta}. \tag{16}$$

(On the other hand, $h_{s,\min}$ does not become zero, because the effect of frictions when λ is small decreases as h_s decreases. Figure 3 illustrates the trade-off between these two effects.) Moreover, $h_{s,\min}$ decreases as $\rho\theta$ increases. This is because, as (16) suggests, the maximum effect of the frictions lowers as $\rho\theta$ increases. In order to compensate for it, h_s should be lower.

4.3 What kind of resource misallocation should be focused on?

The results in the previous section suggest that in order to understand the differences in aggregate TFP between developed and underdeveloped countries, it is important to focus on resource misallocation between firms or sectors that produce relatively substitutable goods.

It is also important to explore the resource misallocations that are consistent with small h_s. The hypothesis that a small proportion of firms, for example, state-owned enterprises, is selectively protected by the government policies is consistent with small h_s. The hypothesis that low TFP firms are protected is also consistent with small h_s. Table 1 reports the h_i of firms (referred to as plants in their paper) classified by the TFP levels (instead of the same tax level) in the U.S., which is calculated from Table 2 in Restuccia and Rogerson

[10](16) also achieves the lower bound of Restuccia and Rogerson's (2007) numerical experiment. For example, in their uncorrelated case, wherein the frictions were uncorrelated with the firm's TFPs, h_s corresponds to 0.5. Then, the lower bound of the relative TFP given by (16) is $(1/2)^{0.1} \approx 0.93$, which is close to their lowest value.

(2007).[11] The h_i of firms with the lowest TFP is 0.04, although such firms constitute more than half of all firms. Hence, if firms with the lowest TFP is protected, it would considerably lower the aggregate TFP. However, it should also be noted that $h_{s,\min}$ with high $\rho\theta$ and relatively high λ is smaller than 0.04, for example, $h_{s,\min}$ at $\rho\theta = 0.9$ and $\lambda = 2$ is less than 0.01 (see Figure 4, which plots the limits of $\rho\theta$ above which $h_{s,\min}$ falls below 0.04). Thus, even if we focus on resource misallocation with respect to the low TFP firms, it is important to explore the possibility that some of the low TFP firms are selectively protected.

On the other hand, it might be difficult to explain the large differences in the aggregate TFP by means of the borrowing constraint of small firms. This is because these small firms belong to $(1 - h_{s,\min})$ of firms, while as observed in Table 1, the h_i of small firms is marginal.

5 Conclusion

This paper analytically examines the extent to and the conditions under which resource misallocation negatively affects the aggregate TFP to the highest degree, when frictions are modeled as the taxes levied on a firm's output. The implications derived from the analysis would be effective in researching the mechanisms of resource misallocation that explain the differences in the aggregate TFP of developed and underdeveloped countries.

There are several important issues that still need to be addressed in future research. First, while I derive the minimum aggregate TFP when the lower and upper tax levels are provided, other specifications on the constraint of frictions might be possible. Second, I abstract from fixed costs. Qualitatively, under the fixed costs, higher frictions on the lower TFP firms (higher frictions implies

[11] Using (6), the h_i is measured as

$$\begin{aligned} h_i &= \frac{g_i a_i^{\kappa\rho}}{\sum_i g_i a_i^{\kappa\rho}} \\ &= \frac{g_i l_i}{\sum_i g_i l_i}, \end{aligned}$$

where g_i is the fraction of i firms, and l_i is firm i's labor input under the assumption that the U.S. is an economy with no frictions. Note that the measured h_i does not depend on ρ and θ.

higher taxes in this paper's model) can discourage these firms from operation and entry, which results in lowering the aggregate TFP. Thus, lower frictions on a small proportion of relatively high TFP firms negatively affect the aggregate TFP the most. In order to quantitatively analyze this effect, assumptions on the fixed costs and the distribution of firms that are not arbitrary are required. Finally, as emphasized in Jones (2008), complementarity among material inputs could magnify the resource misallocation effect.

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Appendix

A Derivation of $\hat{\sigma}_i$

By using (1) and (2), $\hat{\sigma}_i$ can be written as follows:

$$\begin{aligned}\hat{\sigma}_i &= \frac{y_i^\rho}{Y^\rho} \\ &= \frac{a_i^\rho \hat{\sigma}_i^{\rho\theta} \lambda_i^{\rho\theta}}{\int a_i^\rho \hat{\sigma}_i^{\rho\theta} \lambda_i^{\rho\theta} di},\end{aligned}$$

where $\theta \equiv \alpha + \gamma$. By rewriting this equation, we obtain

$$\hat{\sigma}_i = \frac{a_i^{\kappa\rho} \lambda_i^{\kappa\rho\theta}}{W},$$

where $\kappa \equiv 1/(1-\rho\theta)$ and W is defined as

$$W \equiv \left(\int a_i^{\rho} \hat{\sigma}_i^{\rho\theta} \lambda_i^{\rho\theta} di \right)^{\kappa}.$$

W can be further extended as follows:

$$W = \left(\int a_i^{\rho} \lambda_i^{\rho\theta} \left(\frac{a_i^{\kappa\rho} \lambda_{ii}^{\kappa\rho\theta}}{W} \right)^{\rho\theta} di \right)^{\kappa}.$$

By rearranging W, we thus obtain

$$W = \int a_i^{\kappa\rho} \lambda_i^{\kappa\rho\theta} di.$$

Using this result, $\hat{\sigma}_i$ can be expressed by exogenous variables.

B Proof of Proposition 1

I prove Proposition 1 by contradiction.

Suppose that there are n tax levels between λ_s and λ_t with positive h_i. Subsequently, $\nu_s > \nu_1, \ldots, \nu_i, \ldots \nu_n > \nu_t$, where $\nu_i \equiv \lambda_i^{\kappa\theta}$. The following conditions should be satisfied:

$$\frac{\partial \ln N}{\partial \nu_i} = 0, \text{ for all } \nu_i \text{ between } \nu_s \text{ and } \nu_t.$$

If these conditions are not satisfied, N can be lowered by changing λ_i between λ_s and λ_t. $\partial \ln N / \partial \nu_i$ is given by

$$\frac{\partial \ln N}{\partial \nu_i} = \frac{h_i}{\nu_i} \left(\frac{1}{h_i + \sum_{m \neq i} h_m \left(\frac{\nu_m}{\nu_i} \right)^{\rho}} - \frac{1}{h_i + \sum_{m \neq i} h_m \left(\frac{\nu_m}{\nu_i} \right)^{\frac{1}{\theta}}} \right) = 0. \quad (17)$$

From this condition, we obtain

$$\nu_i^{\rho-\frac{1}{\theta}} = \frac{\sum_m h_m \nu_m^{\rho}}{\sum_m h_m \nu_m^{\frac{1}{\theta}}}.$$

Since this condition holds for any ν_j between ν_s and ν_t, $\nu_i = \nu_j$. Thus, we only need to consider the case wherein there is one ν_i between ν_s and ν_t.

Next, I examine the second-order condition (SOC) of $\ln N$ when (17) is satisfied. I refer to the denominator of the first term in the parenthesis in (17) as R, and the second term as T. Then,

$$\begin{aligned}\frac{\partial^2 \ln N}{\partial \nu_i^2} &= -\frac{h_i}{\nu_i^2}\left(\frac{1}{R} - \frac{1}{T}\right) + \frac{h_i}{\nu_i}\left(\frac{\rho}{\nu_i}\frac{R-h_i}{R^2} - \frac{1}{\theta\nu_i}\frac{T-h_i}{T^2}\right) \\ &= \frac{\theta h_i}{\nu_i^2}\frac{h_s\left(\frac{\nu_s}{\nu_i}\right)^{\rho} + h_t\left(\frac{\nu_t}{\nu_i}\right)^{\rho}}{R^2}(\rho\theta - 1) \le 0.\end{aligned}$$

Equality holds only if $h_s = h_t = 0$. Then, the maximum of N is achieved. Otherwise, N becomes the local maximum. Both cases contradict the assumption that N is the minimum.

C Second-order condition of N

I demonstrate that the SOC of the problem provided in (12) is positive for $\lambda > 1$. Note that, here, I use $\ln N$ instead of N.

The FOC is given by

$$\frac{\partial \ln N}{\partial h_s} = \frac{1}{\rho}\frac{r}{R} - \theta\frac{t}{T} = 0,$$

where $r \equiv \nu_s^{\rho} - \nu_t^{\rho}$, $R \equiv h_s\nu_s^{\rho} + h_t\nu_t^{\rho}$, $t \equiv \nu_s^{1/\theta} - \nu_t^{1/\theta}$, and $T \equiv h_s\nu_s^{1/\theta} + h_t\nu_t^{1/\theta}$.

The SOC when the FOC is satisfied is

$$\begin{aligned}\frac{\partial^2 \ln N}{\partial h_s^2} &= -\frac{1}{\rho}\left(\frac{r}{R}\right)^2 + \theta\left(\frac{t}{T}\right)^2 \\ &= \theta\left(\frac{t}{T}\right)^2 (1-\rho\theta) > 0.\end{aligned}$$

D $N_{\min}$ lowers as $\rho\theta \to 1$

Figure 5 displays $N_{\min}$ powered by $1/\theta$, over the ranges of $\rho\theta$ and λ. In this figure, for any λ, $N_{\min}^{1/\theta}$ lowers as $\rho\theta$ increases. The shape of the figure is preserved for $N_{\min}$. Thus, for any given θ, $N_{\min}$ also lowers as $\rho\theta$ increases (i.e., ρ increases). In addition, for any given $\rho\theta$, $N_{\min}$ lowers as θ increases. Therefore, $N_{\min}$ lowers as ρ and θ increase.

Firm size	Small	Medium	Large
Share of firms	0.51	0.47	0.02
Average employment	4.2	64.8	1042.0
h_i	0.04	0.57	0.39

Table 1: Distribution of firms. Notes: These numbers are obtained and calculated from Table 2 of Restuccia and Rogerson (2007). h_i is the proportion of firms with the same TFP level, adjusted by their TFP, and is calculated in a manner similar to (11) (here, h_i is for firms with the same TFP level instead of the same tax level). For the calculation of h_i, see footnote 11.

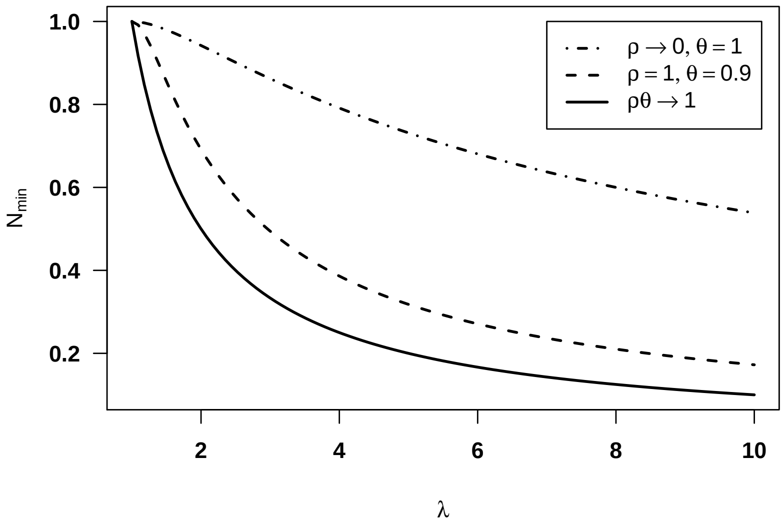

Figure 1: The minimum relative TFP, $N_{\min}$, under different parameter values. Notes: ρ is the parameter on the substitutability of goods. θ is the firm's returns to scale. λ is the ratio of the gross lowest and highest tax levels, $(1-\tau_s)/(1-\tau_t)$.

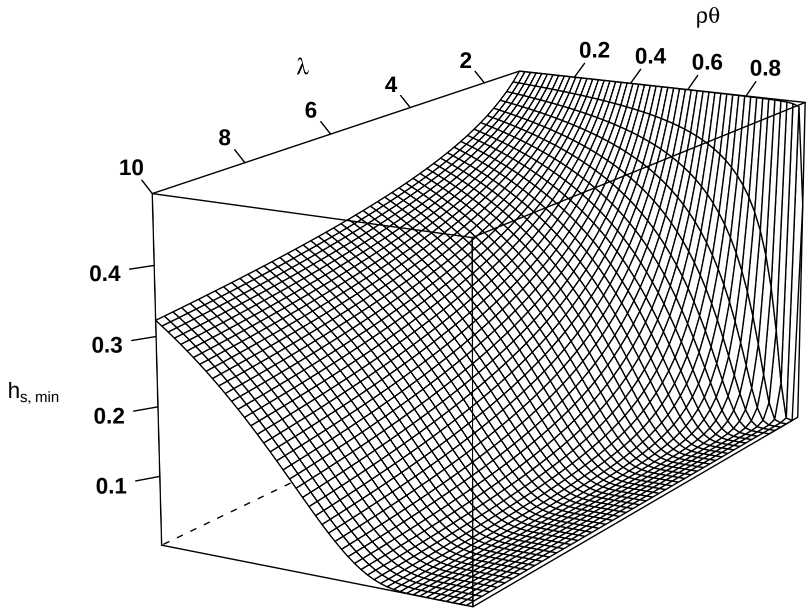

Figure 2: Proportion of firms with the lowest tax level, adjusted by the firm's TFP, $h_{s,\min}$ that generates the minimum relative TFP, $N_{\min}$, under a range of parameter values. Notes: ρ is the parameter on the substitutability of goods. θ is the firm's returns to scale. λ is the ratio of the gross lowest and highest tax levels, $(1 - \tau_s)/(1 - \tau_t)$.

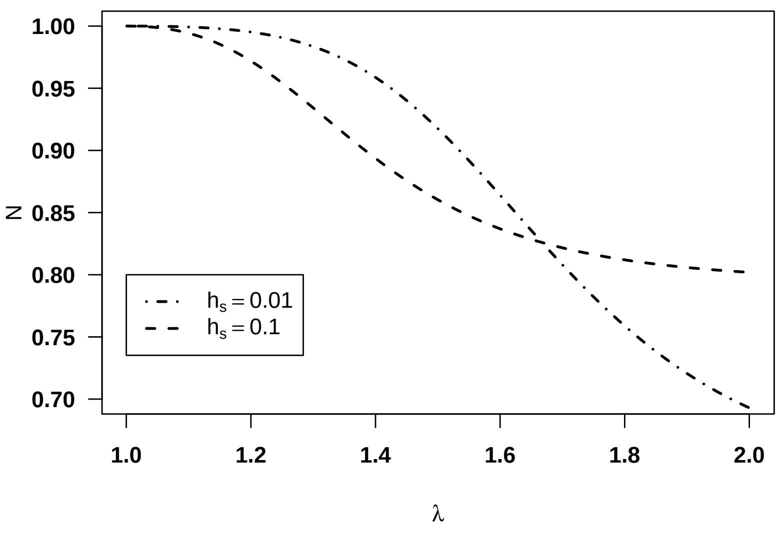

Figure 3: The relative TFP, N, under two different h_s. Notes: h_s is the proportion of firms with the lowest tax level, adjusted by the firm's TFP. λ is the ratio of the gross lowest and highest taxes, $(1 - \tau_s)/(1 - \tau_t)$.

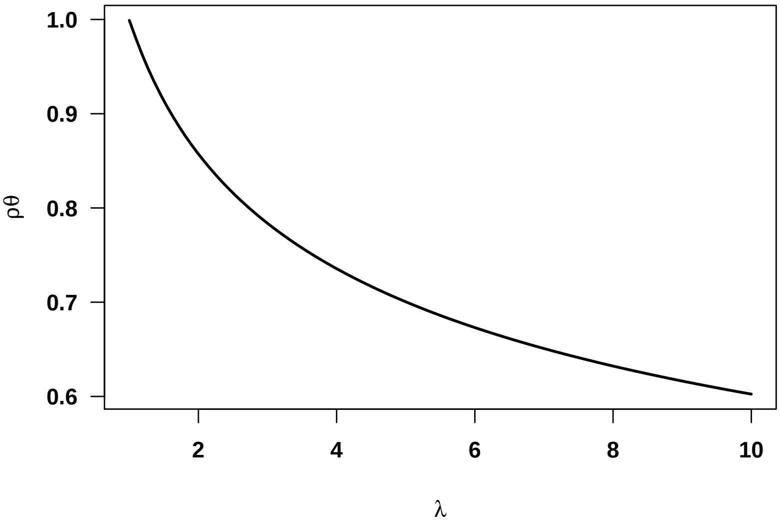

Figure 4: The limit of $\rho\theta$ above which $h_{s,\min}$ that generates $N_{\min}$ falls below 0.04, for each λ. Notes: ρ is the parameter on the substitutability of goods. θ is the firm's returns to scale. λ is the ratio of the gross lowest and highest tax levels, $(1-\tau_s)/(1-\tau_t)$. For example, for $\lambda = 2$, $\rho\theta = 0.86$, which implies that with this λ and $\rho\theta > 0.86$, $h_{s,\min}$ becomes less than 0.04.

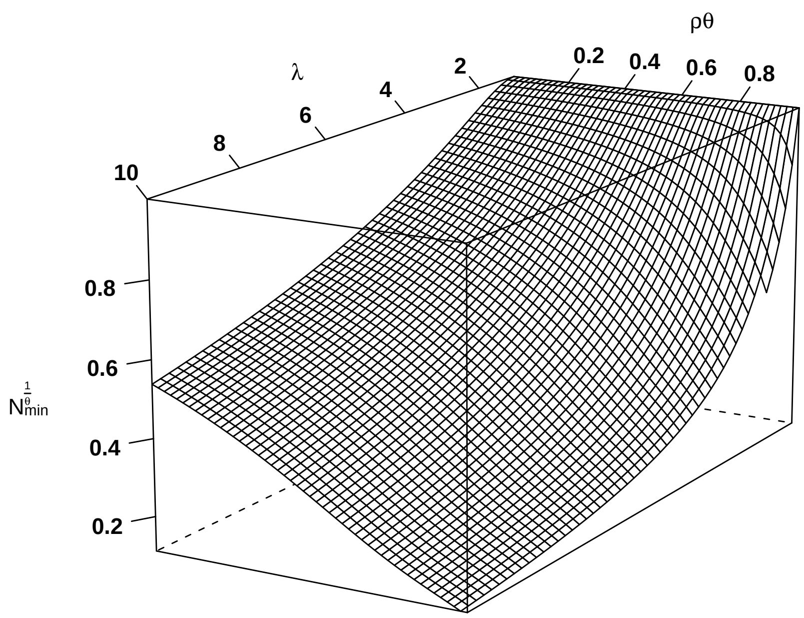

Figure 5: The minimum relative TFP powered by $1/\theta$, $N_{\min}^{1/\theta}$ under a range of parameter values. Notes: ρ is the parameter on the substitutability of goods. θ is the firm's returns to scale. λ is the ratio of the gross lowest and highest taxes, $(1-\tau_s)/(1-\tau_t)$.