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# Fleet dynamics and overcapitalization under rational expectations* 

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#### Abstract

$\qquad$ When individual stay/exit decisions depend on the opportunity cost of exiting, capital is endogenously determined by the instruments used for stock rehabilitation. Using rational (forward looking) expectations, we quantify the overcapitalization associated with the use of input controls for stock rehabilitation in a general equilibrium dynamic framework where stay/exit decisions are endogenous. Using data from the Western Mediterranean, we show that the use of input controls generates a Spanish fleet around 14 percent larger than the one that would result from a non-distortionary instrument.


Keywords: Firm dynamics, Investment, General Equilibrium, Fisheries.
JEL codes: Q22; Q28

[^0]
## Introduction

As is well know, fisheries management based on input controls generates overcapitalization. Seminal papers, as Squires (1987) and Kompas et al. (2004), among others, show that policies based on input control lead to a fishery with more fishing firms with lower productivity, and overcapitalization. We quantify the overcapitalization associated with the use of input controls for stock rehabilitation in a general equilibrium dynamic framework where stay/exit decisions are endogenous.

We highlight the link between rational (forward looking) expectations and the resultant level of excess capacity. Our results show that: first, in contrast with model based on myopic expectations, as for example in Rust et al. (2016), vessels will enter the fishery even when the fish stock is lower than its steady state stock level. Second, the probabilities of entering, exiting and staying are endogenous to the policy instrument used for the stock recovery. Changes on instruments affect productivity, prices, fleet profitability and, therefore, entry and/or exit decisions along the transitional path to the new steady state.

Rational expectations on the transitional path have empirical implications. First, if we assume that vessels enter the fishery only when the fish stock is greater than its steady state stock, entry will not ever be observed along the fishery recovery process. Second, the model predicts that exit will be greater and faster when the fishery recovery policy is based on a non distortionary instrument. This finding is consistent with the fleet decrease observed in the Northeast Multispecies (Groundfish) U.S. fishery after the introduction of the New Management Plan in 2010 which reinforced the use of market instruments. ${ }^{1}$

Note that the beahavior observed when the use of market instruments is reinforced cannot be predicted by a probit/logit model. If we estimate the probabilities of entering, exiting and staying using historical data, as for example Tidd et al. (2011), exit can be underestimated if

[^1]the fishery suffers a structural change in the regulatory regime as the Northeast Multispecies (Groundfish) fishery did.

The next section (Section ) review the role of expectations in the entry/exit literature. Section develops a model and characterizes the equilibrium of it. Section discusses the case study, which is based on a Spanish fleet of the Mediterranean Sea. Section presents the results obtained, distinguishing between those which refer to the steady state and those for the transitional dynamics of the model. This is done under two policy instruments that, as explained above, allow the computation of the overcapitalization in the transition phase. A discussion and concluding remarks are presented in Section .

## Stay/exit decisions: an overview

Analysis of the dynamics of firms based on individual stay/exit decisions has received much less attention in economic literature than the analysis of optimal capacity investment paths under the assumption of a sole fleet owner. Indeed, in the spirit of Smith $(1968,1969)$ the literature has mostly focused on models in which capital is assumed to be equal to the number of vessels in fleets composed by homogeneous vessels. Stay/exit decisions have been modeled as an investment/disinvestment decision, and (usually) a sole fleet owner choses the optimal fleet size, or the capacity utilization under different assumptions on investment cost (Boyce, 1995; Nøstbakken, 2008; Sandal et al., 2007), stock dynamics (Botsford and Wainwright, 1985), stock uncertainty (Hannesson, 1987; Singh et al., 2006; Da-Rocha et al., 2014a) or the strategic effect of irreversible investment decisions in a strategic environment (Sumaila, 1995). For a summary of the literature see Nøstbakken et al. (2011).

Capital at the fishery level is closely related to the entry and exit decisions taken by firms, which are based on their expectations. In steady state solutions, these expectations can take different forms. As pointed out by Berck and Perloff (1984) how potential entrants to an
open-access fishery form their expectations determines the fishery's adjustment path to a steady state but not, necessarily, the steady state values themselves. For example, in Rust et al. (2016) steady state solutions have been obtained using myopic expectations (based on current values). The intuition is that vessels will enter the fishery only when the fish stock is greater than its steady state stock, while exiting will depend on the malleability of the capital based on the current fishing possibilities. However, in the transitional period to the steady state rational expectations are required in order to capture the capital dynamics of the firms. This is specially relevant when changes in fisheries management policies are assessed, specially on those which seek higher stock(s) levels. The consequence from the modeling side is that this entry-exit behavior has to be calculated endogenously.

By considering rational expectations we depart significantly from the capital dynamic literature. We assume that the fishery is operated by heterogeneous agents as in Terrebonne (1995) and in Heaps (2003) to relate (expected) future opportunity benefits for firms with the policy instruments. Like Weninger and Just (2002), we also assume that the abilities of individual firm follow a stochastic process and that there is a fixed operating cost that firms must incur if they want to remain in the fishery. These two assumptions generate firm dynamics over time. Therefore, in our environment, individual rational decisions are not only based on current profits and the whole transitional dynamic (induced by the instrument used to achieve the stock rehabilitation objective) must be computed to capture the behavior of firms and its consequences on economic variables. In fact in our model individual firms assess the expected value of remaining in the industry at each moment in time and compare it to the present discounted value of profits associated with exiting the industry. Based on this comparison, individual firms decide to stay in or exit the industry. The aggregate behavior of individual firms, and not the decision of a monopolistic fleet owner, determines the dynamics of capital in the industry. To do so a dynamic general equilibrium model is used in where prices are determined endogenously (Da-Rocha et al., 2017), unlike in partial equilibrium, where they are given or exogenously determined. This is important first, because it allows
the computation of the rational expectations and second, because prices are not based on estimations that cannot be adequate under a management/natural regimen shifts. In that sense the objective of this work is to establish the level of over-capacity on a fishery along the transitional path and to measure it, when entry-exit behavior is considered endogenous, using rational expectations.

In the literature it is well established that overcapacity is highly related to the management instrument used. In an input-based management system (effort regulated fisheries) can substitute regulated inputs by unregulated ones (Wilen, 1979) which could potentially decrease technical efficiency of the vessels (Kompas et al., 2004).To measure the over-capacity we follow Homans and Wilen (1997), and assume that the instruments chosen by managers to achieve the biological targets are exogenously determined. ${ }^{2}$ We compute fleet dynamics based on individual stay/exit decisions when managers use a non-distortionary instrument (market based instruments) and also when managers use input controls (the basic instruments in the command and control management approach currently used in many fisheries). By comparing them, we can offer a measure of the overcapacity. It is obtained that a management policy based on input controls generates less exit, a less productive fleet, and more overcapitalization. In particular, we show that an input controls policy leads to smaller vessels with lower yields and individual profits, and lower wages than a none non-distortionary one. The source of this overcapacity comes from the fact that the less productive vessels stay in the fishery and pay the idling cost to wait for better times, which reduces the average factor productivity of the fleet. The result of input controls is that more vessels with reduced efficiency are required to achieve the same biological targets, which implies an overcapitalized fleet. We also obtain that when individual stay/exit decisions depend on the opportunity cost of exiting (Ikiara and Odink, 1999), capital level is endogenously determined by the instruments used for stock rehabilitation.

[^2]
## The Model

We consider a natural resource industry with heterogeneous firms. This industry is output constrained by a regulatory agency in order to rehabilitate a given stock.

There are two markets in the economy: final goods and labor (which is used to produce the final good) markets. Taking output price as the numeraire, we denote wages by $w(t)$. We assume that a continuum of identical households, which own the firms, consume the final good and supply labor by solving a consumption-leisure maximization problem.

We assume that firms, which have a finite lifespan, are heterogeneous. Let $g(z, t)$ be the measure of firms over time (i.e. the number of firms with productivity $z$ at time $t$ ). The decision rules of incumbent firms at time $t$ depend on $z$. We denote the optimal choices of output and labor as $y(z, t)$ and $l(z, t)$.

As in Weninger and Just (2002), we assume that the abilities of individual firms follow a stochastic process and that there is a fixed operating cost of $c_{f}$. That is, if a firm wants to remain active in the industry then it must pay the fixed cost. These two assumptions mean that individual firms change over time. At each particular moment in time some of them expand production, hiring staff, while others contract production, firing staff, and others exit the industry altogether.

The decision problem of incumbent firms produces two types of decision rules. There are continuous decision rules for the optimal choice of output $y(z, t)$ and labor $l(z, t)$, and there is a discrete decision rule for the optimal stay/exit decision.

Therefore, on one hand, we have endogenous exit. This decision depends on employment $l(z, t)$ and output $y(z, t)$ in each period. Conditional on the choices in each period, $l(z, t)$ and $y(z, t)$, the firm must assess the expected value of remaining in the industry and must compare it to the present discounted value of profits associated with exiting the industry
$S(t)$-a scrap value. On the other hand, a finite vessel lifespan implies depreciation. Finally, managers of fisheries allow entry when quota exceeds fleet capacity. Note that in contrast to the standard framework, the distribution of the productivity of firms is not exogenous. In our model it is endogenously determined by their decisions on exiting. Therefore, $g(z, t)$ changes over time.

We analyze the model in three steps. First we solve the individual problems of firms and households. This establishes the relationship between input controls and exit decisions. Then we specify the dynamics of the distribution of firms and the feasibility conditions. Finally, we define the equilibrium.

The problem of incumbent firms Let $\tau_{l}$ be a constraint on effort (in particular, $\tau_{l}$ is the maximum number of hours of labor per vessel). Conditional on this constraint, firms maximize profits subject to their available technology, $y=\sqrt{z l} .^{3}$ Thus, at time $t$, the intra-temporal profit maximization problem is

$$
\begin{aligned}
\max _{l(t), y(t)} y(t) & -w(t) l(t)-c_{f}, \\
\text { s.t. } \quad y(t) & =\sqrt{z l(t)}, \\
l(t) & \leq \tau_{l},
\end{aligned}
$$

where profits are defined as revenues $y(t)$ less labor costs $w(t) l(t)$ less the fixed operation cost $c_{f}$. Note that we assume that the behavior of fishermen is not affected by stock variability, -which is consistent with the findings of Ward and Sutinen (1994)- and that physical capital at vessel level is non-malleable (which means that we can normalize capital per vessel to one). Solving for the first order conditions of this problem, we find that labor demand, given

[^3]by
\[

l(t, z)=\left\{$$
\begin{array}{lll}
\frac{z}{4 w(t)^{2}} & \text { if } & z \leq z^{c}(t) \\
\tau_{l} & \text { if } & z>z^{c}(t)
\end{array}
$$\right.
\]

and profits, given by

$$
\pi(t, z)=\left\{\begin{array}{lll}
\pi(t) z-c_{f} & \text { if } & z \leq z^{c}(t) \\
\sqrt{z \tau_{l}}-w \tau_{l}-c f & \text { if } & z>z^{c}(t)
\end{array}\right.
$$

depend on the input constraint.

We assume that the productivity shock $z$ follows a stochastic process with a negative expected growth rate, $\mu$, i.e.

$$
d z=-\mu d t+\sigma_{z} d W
$$

where $\sigma_{z}$ is the per-unit time volatility, and $d W$ is the random increment to a Weiner process. The dynamic incumbents' problem is a stopping time problem defined by:

$$
\begin{aligned}
& v(z, t)=\max _{\tau} E_{0} \int_{0}^{\tau} \pi(z, t) e^{(\rho+\lambda) t} d t+S(t) e^{\rho t} \\
& \text { s.t. } d z=-\mu z d t+\sigma_{z} d w_{z}
\end{aligned}
$$

where $\lambda$ is the exogenous death rate of firms. ${ }^{4}$ Let $z$ be such that the firm does not exit. Then the following Hamilton-Jacobi-Bellman (HJB) equation holds

$$
(\rho+\lambda) v(z, t)=\pi(t) z-c_{f}+\mu z \partial_{z} v(z, t)+\frac{\sigma_{z}^{2}}{2} \partial_{z z} v(z, t)+\partial_{t} v(z, t)
$$

The value matching and smooth pasting conditions at the switching point $\underline{z}$ are $v(\underline{z}, t)=$ $S(t)$ and $v^{\prime}(\underline{z}, t)=0$, respectively. For $z$ lower than the exit threshold, $z \leq \underline{z}$, we have

[^4]$v(z, t)=S(t)$. The incumbent's problem can also be written as an Hamilton-Jacobi-Bellman (HJB) variational inequality, i.e.
\[

$$
\begin{equation*}
\min _{I_{\text {exit }}(z, t)}\left\{(\rho+\lambda) v(z, t)-\pi(t) z+c_{f}-\mu z \partial_{z} v(z, t)-\frac{\sigma_{z}^{2}}{2} \partial_{z z} v(z, t)-\partial_{t} v(z, t), v(z, t)-S(t)\right\} \tag{1}
\end{equation*}
$$

\]

where $I_{\text {exit }}(z, t)$ is an indicator function that summarizes the endogenous decision to exit.

Household problem Each representative household solves a static consumption-leisure maximization problem:

$$
\max _{C, L} \log C-e L,
$$

subject to the budget constraint $C=w(t) L+\Pi(t)$, where the right-hand side of the budget constraint is given by the wage income $w L$ and the total profits of operating firms, $\Pi .^{5}$ Notice that wages are determined by

$$
w(t)=e[w(t) L(t)+\Pi(t)] .
$$

Firm dynamics For prices to be calculated, the dynamics of firms must be computed. In our economy, the change over time in the measuring of firms is determined endogenously by entry/exit decisions made by firms themselves. Formally, $g(z, t)$ follows a Kolmogorov-Fokker-Planck (KFP) equation

$$
\begin{equation*}
\partial_{t} g(z, t)=-\partial_{z}[\mu z g(z, t)]+\frac{\sigma_{z}^{2}}{2} \partial_{z z} g(z, t)-\left(I_{\mathrm{exit}}(z, t)+\lambda\right) g(z, t)+g^{e}(z, t) \tag{2}
\end{equation*}
$$

[^5]$$
\left(I_{\text {exit }}(z, t)+\lambda\right) \quad g(z, t) \quad \text { exit and death }
$$

## Incumbents at time t



$$
1-\left(I_{\text {exit }}(z, t)+\lambda\right) g(z, t) \quad \text { survivors }
$$

$$
\downarrow
$$

Incumbents at time $\mathrm{t}+\mathrm{dt}$
$\uparrow$ entrants $\quad g^{e}(z, t)=g^{\mathrm{ss}}(z)$

Figure 1: Firm dynamics
where entry, when allowed, is given by the distribution $g^{e}(z, t)$.
Notice that $N(t)=\int_{\underline{z}(t)}^{\infty} g(z) d z$ represents the number of firms. Therefore, $N(t)$ is equal to capital in period $t$ in Clark et al. (1979). Therefore, investment in "capital", satisfies $N(t+d t)=N(t)+I(t)$. Thus, investment is

$$
I(t)=\int_{\underline{z}(t)}^{\infty}\left[g^{e}(z, t)-\left(I_{\mathrm{exit}}(z, t)+\lambda\right) g(z, t)\right] d z
$$

Feasibility conditions To close the model we need to define feasibility conditions. The household budget constraint implies that the final output market is in equilibrium. That is,

$$
C=w L+\Pi \Rightarrow C=\int_{\underline{z}(t)}^{\infty} y(z, t) g(z, t) d z-c_{f} N(t)
$$

where $c_{f} N(t)$ is the value of output allocated to produce the fixed operating cost. ${ }^{6}$ The

[^6]manager of the fishery sets the input control such that the individual decisions given by
\[

y(t, z)=\left\{$$
\begin{array}{lll}
y(t, z)^{*}=\frac{z}{2 w(t)}- & \text { if } & z \leq z^{c}(t) \\
y(t, z)^{c}=\sqrt{z \tau_{l}} & \text { if } & z>z^{c}(t)
\end{array}
$$\right.
\]

satisfy the quota path, $Q(t)$. Therefore, feasibility conditions in the labor and output markets are given respectively by

$$
\begin{align*}
\int_{\underline{z}(t)}^{\infty} l(z, t) g(z, t) d z & =L(t),  \tag{3}\\
\int_{\underline{z}(t)}^{z^{c}(t)} y^{*}(t) g(z, t) d z+\int_{z^{c}(t)}^{\infty} y(t, z)^{c} g(z, t) d z & =Q(t) . \tag{4}
\end{align*}
$$

Note that, given $Q(t)$, equations (3-4) jointly determine $w(t)$ and $\underline{z}(t)$. Moreover, after some manipulation, we can write the wage as a function of $e, Q$ and the mass of firms, $N(t)$, i.e

$$
w(t)=e\left[Q(t)-c_{f} N(t)\right] .
$$

## Definition of equilibrium

Given an output restriction, $Q(t)$, and an input control $\tau_{l}$, an equilibrium is a measure of firms $g(z, t)$, wages $w(t)$, value functions of incumbents $v(z, t)$, individual decision rules $l(z, t), y(z, t)$ and a threshold $\underline{z}(t)$, such that:
i) (Firm optimization) Given prices $w(t)$, the exit rule, $I_{\text {exit }}(z, t)$ and $v(z, t)$ solve incumbent problem, equation (1), and $l(z, t), y(z, t)$, are optimal policy functions.
ii) (Firm measure) $g(z, t)$ satisfies the Kolmogorov-Fokker-Planck equation (2).
ii) (Market clearing-feasibility) Given individual decision rules, and the firm measure function, $w(t)$ and $\underline{z}(t)$, solve equations (3-4).

Steady State The economy can be represented by the following system of equations

$$
\begin{array}{r}
\min _{I_{\text {exit }}(z)}\left\{\rho v(z)-\pi(z)+c_{f}-\mu z \partial_{z} v(z)-\frac{\sigma_{z}^{2}}{2} \partial_{z z} v(z), v(z)-S\right\} \\
-\partial_{z}[\mu z g(z)]+\frac{\sigma_{z}^{2}}{2} \partial_{z z} g(z)-\left(I_{\text {exit }}(z)+\lambda\right) g(z, t)+g^{e}(z, t)=0 \\
\int_{\underline{z}}^{\infty} g(z) d z=N \\
\int_{\underline{z}}^{z^{c}} y^{*} g(z) d z+\int_{z^{c}}^{\infty} y(z)^{c} g(z) d z=Q \\
e\left[Q-c_{f} M\right]=w
\end{array}
$$

Finally, note that in a stationary equilibrium $g^{e}(z)=g(z)$ and $I=0$.

## Case study

We apply the model to assess the impact of input controls on the Spanish demersal fleet in the Mediterranean Sea. Our data comes from the Expert Working Group of the Multiannual Plan for demersal fisheries in the Western Mediterranean drawn up by the Scientific, Technical and Economic Committee for Fisheries (STECF, 2016b).

According to STECF (2016a), in 2014 the fleet potentially targeting demersal fisheries covered by the Multiannual Plan included around 9,000 vessels with a combined gross tonnage of $56,331 \mathrm{GT}$ and engine power of $473,615 \mathrm{~kW}$. They accounted for 932,798 days at sea, and the estimated employment in these fisheries was 14,119 jobs, corresponding to 10,717 full time equivalent jobs.

The main species caught by demersal fisheries in the Western Mediterranean are hake, red mullet, blue whiting, monkfish, deep-water rose shrimp, giant red shrimp, blue and red shrimp and Norway lobster. In 2014 landings of European hake, red mullet, and deep water rose shrimp amounted to 10,000 metric tones, accounting for about $€ 69$ million (which is

Table 1: Species and references points caught by Spanish demersal fishing vessels in the Mediterranean Sea

| GSA | 3A code | Scientific name | Ref year | FMSY | Fcurr/FMSY |
| :--- | :--- | :--- | :---: | :---: | :---: |
| 17 | HKE | Merluccius merluccius | 2014 | 0.39 | 3.59 |
| 1 | ARA | Aristeus antennatus | 2014 | 0.41 | 3.41 |
| 1 | ANK | Lophius budegassa | 2013 | 0.16 | 1.56 |
| 1 | MUT | Mullus barbatus | 2013 | 0.27 | 4.85 |
| 1 | DPS | Parapenaeus longirostris | 2012 | 0.26 | 1.65 |
| 5 | ARA | Aristeus antennatus | 2013 | 0.24 | 1.75 |
| 5 | ANK | Lophius budegassa | 2013 | 0.08 | 10.50 |
| 5 | MUT | Mullus barbatus | 2012 | 0.14 | 6.64 |
| 5 | DPS | Parapenaeus longirostris | 2012 | 0.62 | 1.24 |
| 6 | ANK | Lophius budegassa | 2013 | 0.14 | 6.50 |
| 6 | MUT | Mullus barbatus | 2013 | 0.45 | 3.27 |
| 6 | DPS | Parapenaeus longirostris | 2012 | 0.27 | 5.19 |
| 7 | ANK | Lophius budegassa | 2011 | 0.29 | 3.34 |
| 7 | MUT | Mullus barbatus | 2013 | 0.14 | 3.21 |
| Source: STECF $(2016 b)$ |  |  |  |  |  |

around $25 \%$ of the overall demersal output). The leading species, in both volume and value, is hake, followed by red mullet and deep water rose shrimp. Hake, in Geographical Sub Areas 1-7, is principally targeted by Spanish vessels (which land 58 percent of the total). The average price of the red mullet, deep water rose shrimp, and hake landed by Spanish vessels are $€ 5.92 / \mathrm{kg}$, $€ 16.15 / \mathrm{kg}$, and $€ 6.68 / \mathrm{kg}$, respectively.

We consider a stock rehabilitation policy associated with a reduction in the fishing mortality level from the statu quo to the maximum sustainable yield fishing mortality level. Table 1 provides the details of the reduction in fishing mortality for each of the 14 different stocks considered by (STECF, 2016b).

In order to compute the output constraints faced by the Spanish fleet associated with the stock rehabilitation policy, we use the value added path generated by the age structured models for each species (see Appendix ). Table 2 provides prices.

Table 2: Species and Prices of Spanish demersal fisheries in the Mediterranean Sea

|  |  | Species |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | hake | red mullet | DW Red <br> Shrimp | Monk fish | blue and red <br> shrimp |  |
| country | GSA | HKE | MUT | DPS | ANK | ARA |  |
| Spain | 1 |  | x | x | x | x |  |
| Spain | 5 |  | x | x | x | x |  |
| Spain | 6 |  | x | x | x | x |  |
| France /Spain | 7 | x | x |  |  |  |  |
|  |  | HKE | MUT | DPS | ANK | ARA |  |
| Share of each Species | $1-7$ | 0.58 | 1.00 | 1.00 | 1.00 | 1.00 |  |
|  |  | HKE | MUT | DPS | ANK | ARA |  |
| Prices of each Species | $1-7$ | 6.68 | 5.93 | 16.15 | $=$ HKE | =DPS |  |

## Calibration

Table 3: Calibration

| Parameter | Value | Statistic |  |
| :--- | ---: | ---: | :--- |
| $Q$ | 1 | TAC | Normalization |
| $e$ | 1.5339 | utility parameter | $\mathrm{L}=1 / 3$ |
| $\rho$ | 0.04 | discount rate | Da-Rocha et al. (2014b) |
| $\lambda$ | 0.04 | vessel lifespan | 25 years |
| $\mu$ | -0.04 | Productivity Drift | Weninger and Just (2002) |
| $\sigma^{2}$ | 0.01 | Productivity Drift | Da-Rocha and Sempere (2016) |
| $S$ | 0 | Scrap value | No decommissioning scheme |
| $c_{f}$ | 0.2403 | fixed cost | STECF (2016b) |

We select the values of $\mu$ from Weninger and Just (2002) and $\sigma^{2}$ from Da-Rocha et al. (2014b). Given this stochastic process, it is necessary to calibrate six parameters $Q, \lambda, S$, $c_{f}, e$ and $\rho$. We start by selecting a value of the annual interest rate $\rho=0.04$ which is standard in the macroeconomic literature. ${ }^{7}$ We set $Q=1$. We consider a vessel life span of 25 years $(\lambda=0.04)$. We assume that there are no decommissioning schemes, $S=0$. We use

[^7]data from structure and economic performance estimates by member states' fleets operating in the Mediterranean \& Black Sea regions in 2014 to compute the fixed cost. ${ }^{8}$ Finally, we calibrate utility parameter $e$ by solving the model when the economy is non-distorted in order to match a labor supply of $1 / 3$. This is a standard normalization in macroeconomic literature.

## Results

This section is divided into two sub-sections. The first one presents the main results regarding the steady state solution of the model. The second presents the results obtained from the analysis of the transitional dynamics implied by stock rehabilitation policies leading to a situation in which all stocks are on their maximum sustainable yield fishing mortality level.

## Steady State

The mass of vessels, $N(t)=\int_{\underline{z}(t)}^{\infty} g(z) d z$ represents the number of "standardized" firms (fishing vessels). Firms operate capital (the vessel) and stay active if they find it optimal to pay the idling cost, $c_{f}$. Note that the marginal firm (the least efficient active vessel) is indifferent between paying the idling cost and exiting the market. This marginal firm makes negative instantaneous profits, i.e. $\pi(\underline{z}, t)-c_{f}=-\frac{\sigma^{2}}{2} \partial_{z z} v(\underline{z}, t)<0$, and the total expected value of operating the vessel is zero. ${ }^{9}$

To assess the macroeconomic and welfare implications of effort controls (changes in $\tau_{l}$ ), the model generates the optimal response in three (management) variables: (1) average catch per unit effort (CPUE) per day at sea and per vessel, Total Factor Productivity

[^8]

Figure 2: General Equilibrium effects of different levels of control on days in the Steady State
$T F P=E[y(z) / l(z)] ;(2)$ average days at sea per vessel, $E[l(z)]$; and (3) the number of vessels, $N(t) .{ }^{10}$

Effort controls -i.e., the days-at-sea scheme- change all the three management variables at the same time. First notice that effort controls mean a lower wage. The intuition of this result is as follows. If effort controls are in place more vessels are active for the same quota. Notice too that having more vessels means higher operating cost, $c_{f} N$, and remember from the household problem in Section that higher operating costs mean a lower consumption level $C=\left[Q-c_{f} M\right]$. This lower consumption level increases the marginal utility of labor. Therefore, in equilibrium, wages have to decrease so the following equation holds:

$$
\partial_{C} U(C) w(t)=-\partial_{L} U(L) \Rightarrow \frac{w(t)}{C(t)}=e(t)
$$

[^9]Note that for the new wage rate (induced by the effort control), the labor supply is lower (the graph at the top left in Figure 2 illustrates this). Lower wages result in changes in nominal effort composition. On the one hand, the demand for labor for each vessel is reduced, i.e. effort control is active and $E[l(z)]$ is lower for each vessel (each vessel spends fewer days at sea). On the other hand, lower wages induce some vessels (that would otherwise exit) to stay, as $\underline{z}$ decreases and the average productivity of the fleet, $f(z, t)$ decreases. Therefore an increment in fleet size is compatible with fewer days at sea in total $L(t)=N(t) E[l(z)]$ and lower effort per vessel $E[l(z)]$ generated by effort controls. The graph at the top right in Figure 2 illustrates this last effect.

Summarizing, effort controls give rise to fleets with more vessels. Productivity of vessels $(\mathrm{TFP}=E[y(z) / l(z)])$ is reduced, vessels stay at sea for fewer days (lower $E[l(z)])$, and total catches per vessel, $E[y(z)]$, are lower. As a result, both profits per vessel and the value of each vessel $(E[\pi(z)]$ and $E[v(z)]$, respectively) are lower.

Table 4 shows the steady state associated with different levels of effort control $\tau_{l}$ (measured as the $\%$ of $z$ constrained). This table illustrates what was argued in the previous paragraphs with some more precise details. For instance, the line for fleet size shows how it increases monotonically with more restrictive output controls, and the next line show how this effect is accompanied by a monotonic reduction in the wage rate. The following lines show a decrease in total factor productivity, employment per vessel, profits per vessel, and value per vessel. The next lines show the values of several economic variables of interest for policy-makers and their sensitivity to different degrees of input controls.

Table 5 shows the steady state associated with different level of output constraints with and without effort control. The first part of the table shows values for the variables of interest for different levels of $Q$ and a $25 \%$ constraint on effort. The second part shows values for the same variables and levels for $Q$ but for unconstrained effort. The table enables two types of comparison to be made: One shows that for the same $Q$ different levels of constraint on

Table 4: Effects of different levels of control on days

|  |  | $\mathrm{Q}=1$ |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Control on Inputs | $\tau_{l}(\%$ of $z$ constrained $)$ | 0.000 | 0.187 | 0.375 | 0.562 | 0.750 |
| Fleet Size | $M$ | 0.132 | 0.133 | 0.138 | 0.149 | 0.175 |
| wage | $w$ | 1.485 | 1.485 | 1.483 | 1.479 | 1.469 |
|  | $E[y(z) / l(z)]$ | data per vessel |  |  |  |  |
| TFP | $E[l(z)]$ | 2.971 | 2.970 | 2.966 | 2.958 | 2.939 |
| Employment per vessel | $E[y(z)]$ | 7.583 | 2.505 | 2.348 | 2.047 | 1.536 |
| Yield per vessel | $E[\pi(z)]$ | 3.551 | 3.550 | 7.254 | 3.532 | 6.724 |
| 5.756 | 3.210 |  |  |  |  |  |
| Profits per vessel | $E[v(z)]$ | 28.778 | 28.785 | 28.764 | 28.502 | 27.202 |
| Wealth per vessel | $E[y(z)]$ | Inequality: Gini Coeff. |  |  |  |  |
|  | $E[v(z)]$ | 0.573 | 0.573 | 0.573 | 0.573 | 0.573 |
| Revenues |  | 0.624 | 0.624 | 0.623 | 0.620 | 0.606 |
| Wealth | $c_{f} M$ | Aggregate Accounts |  |  |  |  |
|  | $Q-c_{f} M$ | 0.032 | 0.032 | 0.033 | 0.036 | 0.042 |
| Operating Cost | 0.968 | 0.968 | 0.967 | 0.964 | 0.958 |  |
| Consumption | 0.500 | 0.495 | 0.480 | 0.450 | 0.395 |  |
| Remuneration of employees | $w L$ | 0.468 | 0.473 | 0.487 | 0.514 | 0.563 |
| Gross operating surplus | $\Pi$ | Days at the see |  |  |  |  |
|  |  | 0.337 | 0.334 | 0.324 | 0.304 | 0.269 |
| Total days at sea | $L$ | 1.000 | 0.991 | 0.961 | 0.904 | 0.799 |
| Impact of effort control | $L / L^{*}$ |  |  |  |  |  |

Table 5: Effects of different output constraints

| Statistic | Q | $\tau_{l}(\%$ of $z$ constrained $)=0.25$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1.000 | 1.352 | 1.704 | 2.056 | 2.407 |
| Fleet Size wage | M | 0.175 | 0.268 | 0.365 | 0.485 | 0.606 |
|  | $w$ | 1.469 | 1.844 | 2.217 | 2.582 | 2.946 |
|  |  | data per vessel |  |  |  |  |
| TFP | $E[y(z) / l(z)]$ | 3.717 | 4.670 | 5.625 | 6.556 | 7.495 |
| Employment per vessel | $E[l(z)]$ | 1.536 | 1.014 | 0.747 | 0.566 | 0.455 |
| Yield per vessel | $E[y(z)]$ | 5.710 | 4.733 | 4.202 | 3.710 | 3.412 |
| Profits per vessel | $E[\pi(z)]$ | 3.213 | 2.624 | 2.306 | 2.009 | 1.831 |
| Wealth per vessel | $E[v(z)]$ | 27.202 | 22.097 | 19.310 | 16.744 | 15.183 |
|  |  | Inequality: Gini Coeff. |  |  |  |  |
| Revenues | $E[y(z)]$ | 0.573 | 0.558 | 0.533 | 0.522 | 0.503 |
| Wealth | $E[v(z)]$ | 0.606 | 0.600 | 0.581 | 0.577 | 0.563 |
|  |  | Aggregate Accounts |  |  |  |  |
| Operating Cost | $c_{f} M$ | 0.042 | 0.064 | 0.088 | 0.117 | 0.145 |
| Consumption | $Q-c_{f} M$ | 0.958 | 1.202 | 1.445 | 1.683 | 1.921 |
| Remuneration of employees | $w L$ | 0.395 | 0.500 | 0.604 | 0.709 | 0.812 |
| Gross operating surplus | $\Pi$ | 0.563 | 0.702 | 0.841 | 0.974 | 1.109 |
|  |  | Welfare |  |  |  |  |
| Total employees | $L$ | 0.269 | 0.271 | 0.273 | 0.275 | 0.276 |
| Employment constraint | $L$ | 1.000 | 1.008 | 1.013 | 1.021 | 1.025 |
|  |  | $\tau_{l}(\%$ of $z$ constrained $)=0.00$ |  |  |  |  |
| Fleet Size wage | M | 0.132 | 0.202 | 0.275 | 0.366 | 0.457 |
|  | $w$ | 1.485 | 1.868 | 2.250 | 2.626 | 3.001 |
|  |  | data per vessel |  |  |  |  |
| TFP | $E[y(z) / l(z)]$ | 2.971 | 3.737 | 4.501 | 5.251 | 6.002 |
| Employment per vessel | $E[l(z)]$ | 2.553 | 1.680 | 1.238 | 0.936 | 0.753 |
| Yield per vessel | $E[y(z)]$ | 7.583 | 6.277 | 5.573 | 4.915 | 4.521 |
| Profits per vessel | $E[\pi(z)]$ | 3.551 | 2.898 | 2.546 | 2.217 | 2.020 |
| Wealth per vessel | $E[v(z)]$ | 28.778 | 23.350 | 20.393 | 17.663 | 16.008 |
|  |  | Inequality: Gini Coeff. |  |  |  |  |
| Revenues | $E[y(z)]$ | 0.573 | 0.558 | 0.533 | 0.522 | 0.503 |
| Wealth | $E[v(z)]$ | 0.624 | 0.619 | 0.601 | 0.598 | 0.584 |
|  |  | Aggregate Accounts |  |  |  |  |
| Operating Cost | $c_{f} M$ | 0.032 | 0.048 | 0.066 | 0.088 | 0.110 |
| Consumption | $Q-c_{f} M$ | 0.968 | 1.218 | 1.467 | 1.712 | 1.956 |
| Remuneration of employees | $w L$ | 0.500 | 0.633 | 0.767 | 0.900 | 1.033 |
| Gross operating surplus | $\Pi$ | 0.468 | 0.585 | 0.700 | 0.812 | 0.923 |
|  |  | Welfare |  |  |  |  |
| Total employees | $L$ | 0.337 | 0.339 | 0.341 | 0.343 | 0.344 |
| Employment constraint | $L$ | 1.000 | 1.007 | 1.012 | 1.018 | 1.023 |

effort result in different values of the variables. The other shows that for the same level of effort constraint different $Q_{s}$ mean different values of the variables. Some regularities can be observed. For instance, a larger $Q$ implies larger fleet size, higher wage rate, higher TFP, lower profits, and lower employment per vessel for all levels of constraints on effort. On the other hand, for the same $Q$, more constraints on effort means larger fleet size, lower wage rate, lower TFP, lower profits, and lower employment per vessel.

## Transitions

This section focuses on characterizing the transition dynamics caused by stock rehabilitation policies leading the fishery from a given statu quo to a stationary situation where all stocks are at their maximum sustainable yield fishing mortality levels (Fmsy). Our characterization strategy follows two steps. First, we set a drastic reduction of Fmsy for all species in the fishery and compute the value of landings (VA) using the age structured mode (see Appendix 1). Second, given the VA for the Spanish fleet associated with a reduction to Fmsy for all species, we compute the transition dynamics associated with the non-distortionary instrument $\tau(t)^{11}$ that drive the fishery from the statu quo (the VA associated with fishing mortality in the statu quo) to the stationary solution where fishing mortality is equal to Fmsy for all species. Formally, we assume that the non-distortionary instrument is such that the VA target in each period is implemented. That is:

$$
\pi(t)=(1-\tau(t)) y(t) \Rightarrow Q(t)=\int_{\underline{z}(t)}^{z^{c}(t)} y^{*}(t) g(z, t) d z+\int_{z^{c}(t)}^{\infty} y(t, z)^{c} g(z, t) d z
$$

We assume that tax revenue is returned to households in the form of a non-distortionary lump sum transfer. Tax revenue is $T(t)=\int_{\underline{z}(t)}^{\infty} \tau(t) y(t) g(z, t) d z .{ }^{12}$

[^10]The transitional dynamics are described by the following system of equations

$$
\begin{array}{r}
\min _{I_{\mathrm{exit}}(z, t)}\left\{\rho v(z, t)-\pi(t) z+c_{f}-\mu z \partial_{z} v(z, t)-\frac{\sigma_{z}^{2}}{2} \partial_{z z} v(z, t)-\partial_{t} v(z, t), \quad v(z, t)-S(t)\right\}, \\
-\partial_{z}[\mu z g(z, t)]+\frac{\sigma_{z}^{2}}{2} \partial_{z z} g(z, t)-\left(I_{\mathrm{exit}}(z, t)+\lambda\right) g(z, t)+g^{e}(z, t)=\partial_{t} g(z, t), \\
\int_{\underline{z}(t)}^{\infty} g(z, t) d z=N(t) \\
\int_{\underline{z}(t)}^{z^{c}(t)} y^{*}(t) g(z, t) d z+\int_{z^{c}(t)}^{\infty} y(t, z)^{c} g(z, t) d z=Q(t), \\
e\left[Q(t)-c_{f} N(t)\right]=w(t) \\
\operatorname{Exit}(t)=\int_{\underline{z}(t)}^{\infty} I_{\text {exit }}(z, t) g(z, t) d z
\end{array}
$$

We solve this system using the following algorithm. First, we compute the stationary value functions, $v(z \mid Q)$, and fleet distributions, $g(z \mid Q)$, associated with the status quo, $Q_{0}=1$, and the stock rehabilitation, $Q_{T}=2.407$. Second, we guess a function $\tau(t) \forall t=1, . ., T$, then we follow this iterative procedure:

1. Given $w(t)$, compute $v(z, t)$ by solving the HJB equation (5) with terminal condition $v\left(z \mid Q_{T}\right)$ and also compute $I_{\text {exit }}(z, t)$;
2. Given $I_{\text {exit }}(z, t)$, compute $g(z, t)$ by solving the KFP equation (5) using $g\left(z \mid Q_{0}\right)$ as the initial conditions;
3. Given $g(z, t)$, calculate $w^{1}(t)$ using equation (5) and update $w(t)$. Stop when $w^{1}(t)$ is sufficiently close to $w(t)$.
4. Given $w(t)$, compute $Q(t)$. Allow entry if $Q(t)$ is lower than the VA path associated with the stock rehabilitation policy. Stop when $Q(t)$ is sufficiently close to the VA path. Otherwise update $\tau(t)$

We compute two transitions. The first is computed when input controls $\tau_{l}$ are used. This is related to a stationary constraint (\% of $z$ constrained) of 25 percent. The second is
computed for the case of no input controls. Note that throughout the transition the fraction of $z$ constrained is endogenous (it is a function of $w(t)$ ). That is, CPUE is given by

$$
\frac{y(z, t)}{l(z, t)}=\left\{\begin{array}{ccc}
2 w(t) & \text { if } & z \leq z^{c}(t)  \tag{5}\\
\sqrt{\frac{z}{\tau_{l}(t)}} & \text { if } & z>z^{c}(t)
\end{array}\right.
$$



Figure 3: Entry- exit behaviour and total capital without effort controls (panels a and b) and with effort controls (panels c and d)

In our model, capital dynamics is associated with the fleet size dynamics caused by the existence of heterogeneous agents and endogenous entry/exit. In a stationary solution, capital is constant, i.e. $N(t)=N(t+d t)=N$ and $I(t)=0$.

On the stock rehabilitation path, capital dynamics depends on the use (or not) of input controls. Figure 3(a) shows that without effort controls some firms exit in the first few months and there is entry of firms at the end of the period considered. Therefore, without input controls exit of firms produces a reduction in capital in the fishery. The figure shows how, starting from the status quo number of vessels (normalized to 1 ), some vessels exit in the first four months, then the size of the fleet is stable for several months until the stock has recovered enough and entry is allowed. Entry occurs at a constant rate in the last few months. Figure 3(b) shows the capital dynamics. First there is a drop in capital then it remains constant, and then finally it rises when the stock increases enough.

However, Figure 3(c) shows that when input controls are used there are no exits and no reduction in capital. This figure shows that for this type of policy the number of vessels remains constant for more than a year. Then, once the stock of fish has recovered enough, entry is allowed. Then entry takes place at a constant rate until the final period. Figure 3(d) shows that the capital remains constant until it starts to rise at a constant rate.

These findings can be summarized by computing the excess of capacity associated with the use of input controls. We compute excess of capacity associated with the distortionary policy as the difference in each period between capital in the fishery regulated with input controls and capital in the fishery regulated with a non-distortionary instrument. In more precise terms, we first compute the measure of firms along the transitional path under the two policy regimes and once we those measures are obtained we compute the sufficient differences, as shown in Figures 4(b) and 4(a).

Figure 4(b) shows that excess of capacity, measured as the difference between measures $g(z, t)$ associated with the different policies for each $z$ and $t$, is positive for each $z$ and $t$. This difference is larger for low productivity levels (i.e. for $z$ closer to zero). This implies that excess capacity is also associated with lower average levels of productivity as it is relatively more concentrated in vessels with low productivity.


Figure 4: Excess capacity associated with the use of input controls, measured in terms of increased average productivity of firms if non-distortionary instruments were used (panel a) and the increased number of vessels when input controls are used (panel b).

Figure 4(a) represents the difference between the number of vessels (in percentages) associated with a regulatory policy based in input controls and the number of vessels associated to policy based on non-distortionary instruments for each moment in time. We refer to this number as "the excess of the fleet". The figure shows that the excess of the fleet is always positive. It is increasing in the early periods and it may be close to 16 percent for some periods. Later it remains positive and stabilizes at about 14 percent.

## Discussion and conclusions

In a given fishery, already overcapitalized, a policy of input controls makes the problem even worse as the excess of capital is always positive with respect to that resulting from other less distortionary policies .Our results seem to be supported by the empirical evidence provided by the Spanish fleet. The management system in the Mediterranean is mainly based on effort restrictions (limitations on average days at sea and other measures of time per vessel), so our results mean that we should expect more overcapitalization in fleets operating in the Mediterranean than in fleets operating in the Atlantic. Figure 5 shows the status of the Spanish fleets. The long-term economic profitability of vessels as measured by Return on Fixed Tangible Assets (ROFTA, calculated as Net profit/Capital Value) is plotted on the y-axis and the Sustainable Harvest Indicator (SHI) on the x-axis. SHI measures how much a fleet segment depends on overexploited stocks at levels above MSY for its revenues. SHI values greater than 1.2 indicate that fleets are operating under biological imbalance.

Figure 5 shows that Mediterranean fleets do indeed operate under lower ROFTA and greater biological imbalance than Atlantic fleets. This suggests (more) overcapitalization in the Mediterranean fleets than in the Atlantic ones. In fact this is consistent with the conclusion obtained from the results. In the modeling framework presented, in the steady state equilibrium, a fishing days limitation policy leads to smaller vessels with lower yields, lower individual profits, and lower wages. The lower wages allow less productive vessels (that would otherwise exit) to stay in the fishery, reducing the average productivity of the fleet. The result of input controls is that more vessels are required to achieve the same biological targets, which means an overcapitalized fleet.

We have also characterized the transition dynamics caused by stock rehabilitation policies leading the fishery from a given statu quo to a stationary situation where all stocks are at their maximum sustainable yield fishing mortality levels. We show that on the stock


Figure 5: Return on Fixed Tangible Assets (ROFTA) and Sustainable Harvest Indicator (SHI) for Spanish Atlantic Spanish fleet segments (black) and Spanish Mediterranean Spanish fleet segments (red). where PGP, DFN, PSM and HOK, represent different fleet segments. Source: MAAMMA (2014).
rehabilitation path capital depends on the use (or not) of input controls. We conclude that without input controls firms exit on the transition path and the stock of capital in the fishery is reduced. However, when input controls are used as no firms exit on the transition path and the capital is not reduced. Under rational expectations, the less productive vessels stay in the fishery and pay the idling cost to wait for better times. In addition, we show that the excess of capacity associated with input controls also produces lower average levels of productivity as it is relatively more concentrated in vessels with low productivity. Furthermore, the excess of the fleet associated with this type of policy is always positive.

From the policy perspective we relax the assumption that fishing firms' form expectations of the productivity to current capital based only on the current level of the biomass in deciding the level of capital investment or disinvestment. In this case rational expectations discount future fishing possibilities and hence the decision of entry or exit is based on future evolution of the fishery. This is an extremely relevant modeling characteristic when stock rebuilding
policies are assessed given that firms' behaviour can be influenced by the "glistering" future coming from the policy. Furthermore, this characteristic is not constrained to the fishery policy itself, but it has to be considered when dealing with natural regimen shifts that can be caused by, for example, climate change.

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## Appendix

## Age-structured Stock dynamics

For each species we use an age-structured model (see Figure 6) to assess the impact of each fishing mortality, $F(t)$, trajectory to Fmsy (see Figure 7) on landings generated by the transitional dynamics of the stocks, $n(a, t)$ (see Figure 8$)$. Let $n(a, t)$ be the number of fish of age $a$ at time $t$. As in Botsford and Wainwright (1985), the conservation law is described by the following McKendrick-von Foerster partial differential equation: ${ }^{13}$

$$
\begin{equation*}
\frac{\partial n(a, t)}{\partial t}=-\frac{\partial n(a, t)}{\partial a}-[m(a)+p(a) F(t)] n(a, t) \tag{6}
\end{equation*}
$$

Equation (6) shows that the rate of change in the number of fish in a given age interval, $\frac{\partial n(a, t)}{\partial t}$, is equal to the net rate of departure less the rate of deaths. Given all fish ages, the net rate of departure is equal to $\frac{\partial n(a, t)}{\partial a}$. The rate of deaths at age $a$ is proportional to the number of fish of age $a$, i.e. $[m(a)+p(a) F(t)] n(a, t)$. Recruitment and maximum age occurs as boundary conditions. We assume that fish die at age $A$, and that there is constant recruitment i.e $n(0, t)=1$ and $n(A, t)=0 .{ }^{14}$ For a given $F(t)$ trajectory, catches at age $a$ are equal to $p(a) F(t) n(a, t)$, therefore $Q(t)$, is equal to

$$
Q(t)=\left(\int_{0}^{A} \omega(a) p(a) n(a, t) d a\right) F(t)
$$

[^11]Figure 6: Age Structured Models



(d) MUT 3


(g) ANK 2

(j) DPS 1


(e) MUT 4

(h) ARA 1



(f) ANK 1

(i) ARA 2

(k) DPS 2
(1) DPS 3

Figure 7: Targets


Figure 8: Equilibrium Distributions by age

(a) HKE

(d) MUT 3

(g) ANK 2

(j) DPS 1

(b) MUT 1

(e) MUT 4

(h) ARA 1

(k) DPS 2

(c) MUT 2

(f) ANK 1

(i) ARA 2

(l) DPS 3

## Finite difference method

Following Achdou et al. (2014) Achdou et al. (2015) we use a finite difference method and approximate the fuctions $v(z, t)$ and $g(z, t)$ (equations 1 and 2). We use the shorthand notation $v_{i}^{n}=v\left(z_{i}, t_{n}\right)$ and $g_{i}^{n}=g\left(z_{i}, t_{n}\right)$.

Linear Complementarity Problems (LCP). We approximate (1)

$$
\rho v_{i}^{n}=\pi_{i}^{n}+\left[\mu_{i}\right]^{+}\left(\frac{v_{i+1}^{n}-v_{i}^{n}}{\Delta z}\right)+\left[\mu_{i}\right]^{-}\left(\frac{v_{i}^{n}-v_{i-1}^{n}}{\Delta z}\right)+\frac{\sigma_{z}^{2}}{2}\left(\frac{v_{i+1}^{n}-2 v_{i}^{n}+v_{i-1}^{n}}{\Delta z^{2}}\right)+\left(\frac{v_{i}^{n+1}-v_{i}^{n}}{\Delta t}\right),
$$

where $\left[\mu_{i}\right]^{+}=\max \left\{\mu_{i}, 0\right\}$ and $\left[\mu_{i}\right]^{-}=\min \left\{\mu_{i}, 0\right\}$. Therefore, collecting terms, we have

$$
\begin{aligned}
\rho v_{i}^{n} & =\pi_{i}^{n}+a_{i} v_{i-1}^{n}+b_{i} v_{i}^{n}+c_{i} v_{i+1}^{n}+\left(\frac{v_{i}^{n+1}-v_{i}^{n}}{\Delta t}\right), \text { where } \\
a_{i} & =-\frac{\min \left\{\mu_{i}, 0\right\}}{\Delta z}+\frac{\sigma_{z}^{2}}{2 \Delta z^{2}}, \\
b_{i} & =-\frac{\max \left\{\mu_{i}, 0\right\}}{\Delta z}+\frac{\min \left\{\mu_{i}, 0\right\}}{\Delta z}-\frac{\sigma_{z}^{2}}{\Delta z^{2}}, \\
c_{i} & =\frac{\max \left\{\mu_{i}, 0\right\}}{\Delta z}+\frac{\sigma_{z}^{2}}{2 \Delta z^{2}} .
\end{aligned}
$$

Note that $a_{i}+b_{i}+c_{i}=0$. Thus, equation (7) in matrix form

$$
\rho v^{n}=\pi^{n}+\mathbf{A} v^{n}+\frac{1}{\Delta t}\left(v^{n+1}-v^{n}\right),
$$

where (for $\mathrm{i}=1,2,3,4$ )

$$
\mathbf{A}=\left[\begin{array}{cccc}
b_{1} & c_{1} & 0 & 0 \\
a_{2} & b_{2} & c_{2} & 0 \\
0 & a_{3} & b_{3} & c_{3} \\
0 & 0 & a_{4} & \hat{b}_{4}
\end{array}\right]
$$

Boundary conditions: from $\partial_{z} v(\infty, t)=0$ we have, $v_{I}^{n}=v_{I+1}^{n}$, then $\hat{b}_{I}=b_{I}+\frac{\sigma_{z}^{2}}{2 \Delta z^{2}}$ such that $a_{I}+b_{I}=0$.

To solve equation (1) we follow Huang and Pang (1988). They show that the variational inequality problem (the discretized version of equation (1) )

$$
\min _{I_{\mathrm{exit}}(z, t)}\left\{\rho v^{n}-\pi^{n}-\mathbf{A} v^{n}-\frac{1}{\Delta t}\left(v^{n+1}-v^{n}\right), v^{n}-S^{n}\right\},
$$

can be formulated as Linear Complementarity Problems (LCP), i.e

$$
\begin{aligned}
\left(v^{n}-S^{n}\right) \perp\left(\mathbf{B}\left(v^{n}-S^{n}\right)+\mathbf{q}^{m+1}\right) & =0 \\
\left(v^{n}-S^{n}\right) & \geq 0 \\
\mathbf{B}\left(v^{n}-S^{n}\right)+\mathbf{q}^{m+1} & \geq 0
\end{aligned}
$$

where $\mathbf{B}=\left(\rho+\frac{1}{\Delta t}\right) \mathbf{I}-\mathbf{A}$ and $\mathbf{q}^{m+1}=\mathbf{B} S^{n}-\pi^{n}-\frac{1}{\Delta t} v^{n+1} .{ }^{15}$
Kolmogorov Forward equation. We approximate the KFP equation (2) using the following approximation for $\partial_{z}[\mu z v(z, t)]$

$$
\partial_{z}[\mu z v(z, t)] \simeq\left[\left(\frac{\left[\mu_{i}\right]^{+} g_{i}^{n}-\left[\mu_{i-1}\right]^{+} g_{i-1}^{n}}{\Delta z}\right)+\left[\left(\frac{\left[\mu_{i+1}\right]^{-} g_{i+1}^{n}-\left[\mu_{i-}\right]^{-} g_{i}^{n}}{\Delta z}\right)\right] .\right.
$$

Therefore, we have

$$
\begin{aligned}
\left(\frac{g_{i}^{n+1}-g_{i}^{n}}{\Delta t}\right) & =c_{i-1} g_{i-1}^{n}+b_{i} g_{i}^{n}+a_{i+1} g_{i+1}^{n}-I_{i}^{n} g_{i}^{n}+\delta^{n}, \text { where } \\
a_{i+1} & =-\frac{\min \left\{\mu_{i+1}, 0\right\}}{\Delta z}+\frac{\sigma_{z}^{2}}{2 \Delta z^{2}} \\
b_{i} & =-\frac{\max \left\{\mu_{i}, 0\right\}}{\Delta z}+\frac{\min \left\{\mu_{i}, 0\right\}}{\Delta z}-\frac{\sigma_{z}^{2}}{\Delta z^{2}} \\
c_{i-1} & =\frac{\max \left\{\mu_{i-1}, 0\right\}}{\Delta z}+\frac{\sigma_{z}^{2}}{2 \Delta z^{2}}
\end{aligned}
$$

[^12]Note that (7) in matrix form

$$
\frac{1}{\Delta t}\left(g^{n+1}-g^{n}\right)=\mathbf{A}^{T} g^{n}
$$

where (for $\mathrm{i}=1,2,3,4$ )

$$
\mathbf{A}^{T}=\left[\begin{array}{cccc}
b_{1} & a_{2} & 0 & 0 \\
c_{1} & b_{2} & a_{3} & 0 \\
0 & c_{2} & b_{3} & a_{4} \\
0 & 0 & c_{3} & b_{4}
\end{array}\right]
$$

and $g^{n}=I_{\text {exit }}^{n} g^{n}$. Finally density is computed as $f_{i}=\frac{g_{i}}{\sum_{i=1}^{I} g_{i} \Delta z}$, where $\sum_{i=1}^{I} g_{i} \Delta z$ is the mass of firms.


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[^1]:    ${ }^{1}$ Da-Rocha et al. (2017) report that after remaining stable from 2007 to 2009 , the number of active vessels decreased by $32 \%$ in 2010 in the Northeast Multispecies (Groundfish) U.S. fishery.

[^2]:    ${ }^{2}$ For an analysis of the optimal combination of instruments under stock uncertainty see Da-Rocha and Gutiérrez (2012).

[^3]:    ${ }^{3}$ Our technology is in accordance with the fifty-fifty rule, i.e. $50 \%$ of net revenues are accounted for by payments to crew members.

[^4]:    ${ }^{4}$ The death rate of firms is equal to the inverse of the vessel lifetime.

[^5]:    ${ }^{5}$ Controls on inputs/outputs per vessel generate unemployment and (potentially) introduce heterogeneity in households. We apply a convenient technical devise developed by Hansen (1985) and Rogerson (1988) to simplify the problem and use the representative household framework to solve the problem. That is, we assume the existence of a lottery such that each household has the same probability $p_{n}$ of being selected to work. Therefore, in expected terms, each household will work $p_{n} L$ hours. Note that the rules of this lottery imply that there is perfect insurance in the sense that every household gets paid whether it works or not. Hence, they will have identical consumption, i.e. $C=w L+\Pi$. Under these conditions, the utility function associated with the lottery is quasilinear in labor.

[^6]:    ${ }^{6}$ Note that $C$ is equal to
    $w L+\Pi=\int_{\underline{z}(t)}^{\infty} w(t) l(t) g(z, t) d z+\int_{\underline{z}(t)}^{\infty}\left(y(t)-w(t) l(t)-c_{f}\right) g(z, t) d z=\int_{\underline{z}(t)}^{\infty} y(z, t) g(z, t) d z-c_{f} N(t)$.

[^7]:    ${ }^{7}$ See, for instance Restuccia and Rogerson (2008).

[^8]:    ${ }^{8}$ See Table 4.3 of the STECF (2016b).
    ${ }^{9}$ If the marginal active firm decides to leave the market, it obtains the value $v(\underline{z})=S=0$. From the smooth pasting condition and stationarity, $\left(\partial_{z} v(\underline{z}, t)=\partial_{t} v(\underline{z}, t)=0\right)$ we have equation (1) $-\pi(\underline{z}, t)+c_{f}+$ $\frac{\sigma^{2}}{2} \partial_{z z} v(\underline{z}, t)=0$.

[^9]:    ${ }^{10}$ Given that

    $$
    Y(t)=N(t) \int_{\underline{z}(t)}^{\infty}\left(\frac{y(z, t)}{l(z, t)}\right) l(z, t) f(z, t) d z .
    $$

[^10]:    ${ }^{11} \tau(t)$ can be interpreted as a tax rate or as the price of an ITQ in a system of fully tradable individual quotas.
    ${ }^{12}$ Then $C=w(t) L(t)+\Pi(t)+T(t)=Q(t)-c_{f} N(t)$.

[^11]:    ${ }^{13}$ See Von-Foerster (1959) and McKendrick (1926).
    ${ }^{14} \mathrm{~A}$ Stock Recruitment relationship can be assumed. In that case, each period, the number of fish at age zero is given by $n(0, t)=\Psi\left(\int_{0}^{A} \omega(a) \mu(a) n(a, t) d a\right)$, where, $\int_{0}^{A} \omega(a) \mu(a) n(a, t) d a$ is the SSB. See Da-Rocha et al. (2012).

[^12]:    15 Matlab provides Yuval Tassa's Newton-based LCP solver, downloadable from http://www.mathworks . com/matlabcentral/fileexchange/20952.

