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Abstract Since the late 1960s, research in the field of general equilibrium theory has focused on economies in which spot markets for commodities coexist with some asset markets and trade takes place sequentially over time. The study of 'sequential economies' has developed along two paths inspired by Hicks's *Value and Capital*, which stress the dependence of agents' choices on their expectations of future prices. The first is temporary equilibrium theory, in which expectations are subjective. The second postulates that all agents exactly predict the future prices (sequential economies with perfect foresight). This paper points out that the inclusion of expectations among the determinants of equilibrium originates considerable analytical problems within each of the mentioned approaches derived from *Value and Capital*. On the basis of the studies of the 1970s and 1980s, it first illustrates the difficulties that arise in temporary equilibrium theory due to the subjective nature of individual forecasts. Then it moves on to examine sequential economies with perfect foresight. After illustrating the equilibrium notion on which the analysis of those economies relies, i.e. the 'equilibrium of plans, prices and price expectations' introduced by Radner (1972), it argues, in the light of recent contributions, that for plausible configurations of the economy the perfect foresight associated with Radner equilibria proves not only unrealistic but also theoretically dubious.

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JEL codes: B21; D46; D51; D84

1. Introduction

Since the late 1960s, research in the field of general equilibrium theory has focused on economies in which spot markets for commodities coexist with some asset markets and trade takes place sequentially over time. The study of 'sequential economies' was initially carried out along two paths inspired by Hicks's *Value and Capital*, which stress the dependence of agents' choices on their expectations about future prices. The first is temporary equilibrium theory, in which price expectations are assumed to be subjective and therefore likely to differ among agents. The second is characterized by the assumption that all agents exactly predict the prices that will rule in the future (sequential economies with perfect foresight). Temporary equilibrium theory declined in the mid-1980s, however, while research along the second path has developed up to our days in spite of the fact that the predictive capabilities attributed to agents are utterly unrealistic.

From the contributions put forward over the years it emerges that the inclusion of expectations among the determinants of equilibrium gives rise to considerable analytical problems within each of the above-mentioned approaches. The aim of this paper is to highlight this aspect in accessible and compact form. Section 2 thus illustrates, on the basis of the studies of the 1970s and 1980s, the difficulties that arise within the context of temporary equilibrium theory due to the subjective nature of individual forecasts. It then argues that those difficulties contribute to explaining why research in that field eventually declined. Section 3 moves on to address sequential economies with perfect foresight of future prices. After illustrating the equilibrium notion on which the analysis of those economies relies, namely the 'equilibrium of plans, prices and price expectations' introduced by Radner (1972), it argues, in the light of recent contributions, that for plausible configurations of the economy the perfect foresight associated with Radner equilibria proves not only unrealistic but also theoretically dubious (an example complementing the intuitive exposition is presented in Appendix 2). Finally, Section 4 recapitulates and briefly comments on the standpoint promoted by Hicks in *Value and Capital*. Discussion is conducted throughout the paper within the simplest analytical framework, that is in the absence of uncertainty and on the assumption that the asset markets in existence allow agents to freely reallocate their purchasing power across the different periods of time ('complete' asset markets).

2. Subjective expectations and temporary equilibrium in the studies of the 1970s and 1980s

Here the problems that expectations create in temporary equilibrium theory will be examined, as they emerge from the studies carried out in the field during the 1970s and 1980s. To simplify the exposition, it will be assumed that expectations are 'certain', in the sense that each agent expects a definite price system to obtain in the future with probability 1, and 'fixed', in the sense that price forecasts are assumed to depend exclusively on past prices and are therefore taken as given in the analysis.

Economies of pure exchange

As the initial step in our examination, consider a pure-exchange economy with H households (indexed $h = 1, \ldots, H$) and $N \geq 2$ non-storable goods (indexed $n = 1, \ldots, N$) that is active for two periods of time, 1 and 2. By assumption, in the first period there are N spot markets for commodities and a forward market for good 1. As will shortly be clarified, the latter market is the means of transferring purchasing power across time: to highlight this aspect, it will be referred to as a market for bonds, where a bond is a promise to deliver a unit of good 1 at the beginning of period 2. Finally, only the N spot markets for consumption goods are open in the second period. Given this market structure, let us now define the behaviour of agents in the first period, on the assumption that good 1 for spot delivery is the numéraire in terms of which prices are measured.

At the beginning of period 1, households observe the current prices, which are denoted by the non-negative vector $p = (p_1, q_1)$, where the sub-vector $p_1 = (p_{11}, \dots, p_{N1})$ such that $p_{11} = 1$ refers to the N spot markets and q_1 is the price of a bond. At the same time, households have definite expectations of the period 2 spot prices in terms of good 1, which, being subjective, will typically differ. The prices expected by the generic household h will accordingly be denoted by the non-negative vector $p_2^h = (p_{12}^h, \dots, p_{N2}^h)$ in which $p_{12}^h = 1$. Given the current and the expected prices, both the first period budget constraint and the expected second period budget constraint are determined for the generic household. However, each household will calculate that by trading appropriately on the bond market, it can purchase or sell commodities for future delivery as freely as in the presence of a complete system of forward markets. To clarify this point, suppose that household h thinks a unit of commodity n will exchange in period 2 for p_{n2}^h units of good 1. The household will then calculate that if it wishes to purchase in the present a unit of n for future delivery, it can obtain this result by buying p_{n2}^h bonds in the anticipation of exchanging the p_{n2}^h units of good 1 that it will receive in period 2 for the desired unit of n. Similarly, the household will calculate that if it wishes to sell in the present a unit of n delivered in 2, it can obtain this result by selling p_{n2}^h bonds in the anticipation of surrendering a unit of n in period 2 against p_{n2}^h units of good 1 and then using those units to honour the bonds supplied. According to the household, therefore, a unit of n delivered in 2 can be traded in the present at the price $q_n^h = q_1 \, p_{n2}^h$.

From the foregoing remark it follows that when markets open in period 1, the generic household h believes that it can in fact trade goods for current and future delivery subject to the single budget constraint

$$p_1 \, x_1^h + q^h \, x_2^h = p_1 \, \omega_1^h + q^h \, \omega_2^h \qquad [2.1]$$

where $q^h = q_1 \, p_2^h$ is the system of 'present prices' for commodities delivered in 2 as calculated by the household on the basis of its own expectations, and the non-negative vectors $x_t^h = (x_{1t}^h, \dots, x_{Nt}^h)$, $\omega_t^h = (\omega_{1t}^h, \dots, \omega_{Nt}^h)$ respectively denote a consumption bundle demanded by the household for period t and the household's commodity endowments in t ($t = 1, 2$). Under these circumstances, the household will determine its optimal action on period 1 markets in two steps. It will first choose a most preferred consumption stream $x^{h*} = (x_1^{h*}, x_2^{h*})$ among those complying with [2.1]. Then, in order to achieve that stream, the household will capitalise the expected value of future endowments by supplying the quantity of bonds $\bar{b}_1^h$ such that $q_1 \bar{b}_1^h = q^h \omega_2^h$ and use its total current wealth for purchasing $x_1^h *$ and the quantity of bonds $b_1^h * = p_2^h x_2^h *$ that it deems necessary for financing planned future consumption. In this setting, a temporary equilibrium for period 1 can be defined as a system of current prices, a constellation of individual expectations and a corresponding set of optimal actions $a^h * = (x_1^h *, \bar{b}_1^h, b_1^h *)$, $h = 1,\ldots, H$, such that the current spot markets and the market for bonds are simultaneously cleared.

Temporary equilibrium of the economy outlined here exists under the usual assumptions concerning households' characteristics such as non-satiation not only when expectations are 'fixed' but also when they depend continuously on the current prices. We shall refrain from substantiating these assertions since, as explained in Appendix 1, the model just presented is a particular specification of the one put forward by Arrow & Hahn (1971: 136-151). It will instead be pointed out that this initial model contains a hidden problem and is not robust.

Suppose the initial model is modified by assuming $N > 2$ and, more importantly, that *two* forward markets are open in the first period, say those for goods 1 and 2. As we shall now see, this slight increase in the number of forward markets creates a problem for temporary equilibrium theory.

To illustrate the nature of the problem, assume that the price system ruling on forward markets at the beginning of period 1 is $\bar{q} = (\bar{q}_1, \bar{q}_2)$. Assume further that the generic household *h* believes that the future price of good 2 will be $\bar{p}_{22}^h > (\bar{q}_2 / \bar{q}_1)$. In these circumstances, household *h* has a strong incentive to trade on forward markets for speculative purposes. Suppose, for example, that the household buys forward a unit of good 2 and simultaneously sells forward $(\bar{q}_2 / \bar{q}_1)$ units of good 1. The total cost of this operation is zero under the postulated price conditions. On the other hand, household *h* will calculate that in period 2 it will be able to exchange the unit of good 2 that it will then receive for $\bar{p}_{22}^h$ units of good 1, thereby reaping a profit equal to $\bar{p}_{22}^h - (\bar{q}_2 / \bar{q}_1)$ units of the numéraire. Household *h* will thus conclude that forward markets provide an opportunity for profitable arbitrage operations and, on the usual assumption of non-satiation, it will tend to increase without limit the quantity of good 2 for future delivery demanded in the present and financed by selling forward good 1. This means that the household's optimal action is not determined, however, and temporary equilibrium cannot therefore exist.

A symmetric argument shows that the household's optimal action is not determined in the opposite case either, namely when $\bar{p}_{22}^h < (\bar{q}_2 / \bar{q}_1)$. It thus emerges that a system of first period prices can support temporary equilibrium of the modified exchange economy only if at those prices the no-arbitrage condition $p_{22}^{h} = (q_2/q_1)$ holds for each h, which in turn requires that all households have exactly the same expectation of the future price of good 2. Since expectations are subjective, however, one cannot expect the required coinciding of forecasts to be fulfilled. Thus, in the case of fixed expectations, it is enough for only two households to disagree over the future price of good 2 to violate the no-arbitrage condition and prevent the existence of temporary equilibrium. But even assuming the expected prices to be functions of current prices, it is perfectly possible that two or more households disagree over the future price of good 2 at any system of current prices, thereby giving rise to the same negative result.

The problem that perceived arbitrage opportunities create for the existence of temporary equilibrium was pointed out by Green (1973). Green showed that the problem is reduced when expectations are not 'certain' but take the form of probability distributions of future prices. At the same time, he made it clear that the difficulty is not entirely ruled out under the latter formulation, as there must still be substantial overlapping of individual expectations in order to prevent unlimited arbitrage. This problem was tackled during the 1980s but no accepted solution emerged.[1]

Economies with production

We shall now move on to address the problems that conflicting expectations create in the treatment of production. This will be done by transforming the initial model of the previous subsection into a model with production.

Let us modify the initial model as follows. First, assume that the N commodities also include goods and services susceptible of being used in production. Second, assume that a given number F of firms (indexed $f = 1, \dots, F$) are active in the economy. Third, assume that the ownership of each firm is divided among households at the beginning of period 1 in accordance with a given allocation of 'ownership shares'. Finally, assume that F markets for the shares of

[1] Milne (1980) suggested that the problem could be solved by introducing bounds to short-sales on forward markets that reflect the lenders' subjective evaluation of the feasibility of the future deliveries promised by borrowers. Milne's approach, however, relies on the awkward assumption that each potential lender has full information concerning the whole set of transactions planned by each candidate borrower. Moreover, Stahl (1983) pointed out that the existence of temporary equilibrium is problematic when one moves from the two-household economy of Milne's illustrative example to economies with a larger number of traders. An alternative route was explored in Stahl (1985). This paper assumes that forward trades are carried out with the mediation of an institution, the Clearinghouse, which acts as a third-party warrantor for every contract and restricts every household to a set of trades such that the household will be solvent for all the future price systems that the Clearinghouse itself regards as possible. In this setting, and assuming 'probabilistic' expectations, temporary equilibrium of pure exchange exists even in the absence of overlapping expectations. A weakness of this contribution is that there is no reason why a single clearinghouse should be active in the economy. On the other hand, in a paper published when research in the field had already declined, Stahl himself showed that with many clearinghouses only the existence of 'approximate' temporary equilibrium can be proved (Stahl, 1995). Intuitively, an approximate equilibrium differs from a true equilibrium owing to the presence of discrepancies between demand and supply, which tend to become negligible vis-à-vis the size of markets *if* the number of agents is large enough. For rigorous treatment of this notion, cf. Arrow & Hahn (1971: Ch. 7).

ownership in firms are open in the first period besides the *N* spot markets and the market for bonds specified in terms of good 1. Having thus altered the model, we shall now go on to analyse the behaviour of agents in period 1. As before we shall take the consumption good listed as 'good 1' as numéraire.

We assume that production develops in cycles. A production plan of the generic firm *f* will accordingly be denoted by the vector $y^f = (y_1^f, y_2^f)$, where $y_1^f \in \Re_-^N$ denotes first period inputs (negative numbers) and $y_2^f \in \Re_+^N$ the corresponding outputs emerging at the beginning of period 2. Due to the cyclical nature of production, the economy is endowed at the beginning of period 1 with stocks of commodities derived from the firms' previous activity. We assume that these stocks are entirely included in the households' endowments and that, for this reason, firms must finance their input expenditure entirely by issuing bonds. Moreover, we assume that each firm is run by a manager who selects the production plan as follows.

At the beginning of period 1, the manager of the generic firm *f* observes the current prices $p = (p_1, q_1)$ and expects the non-negative price vector $p_2^f = (p_{12}^f, \ldots, p_{N2}^f)$, in which $p_{12}^f = 1$, to obtain on future markets. In evaluating a hypothetical plan y^f, the manager therefore calculates that the firm would have to issue a number of bonds b^f such that $q_1 \, b^f = - \, p_1 \, y_1^f$ in order to cover the input cost and would thus have to repay $b^f = -(1/q_1)\, p_1 \, y_1^f$ units of numéraire in period 2. According to the manager's own expectations, the plan would therefore yield the amount of profits $\pi_2^f = p_2^f \, y_2^f + (1/q_1)\, p_1 \, y_1^f$ in period 2. Considering that π_2^f units of numéraire delivered in period 2 can be traded in the present on the bond market at the total price $\pi_1^f = q_1 \, \pi_2^f$, it may be said, however, that the present value of the expected profits is $\pi_1^f = q^f \, y_2^f + p_1 \, y_1^f$, where $q^f = q_1 \, p_2^f$. Adopting the latter formulation, it is assumed that the manager chooses a technically feasible plan $y^{f*} = (y_1^{f*}, y_2^{f*})$ that maximises π_1^f. This choice in turn identifies the supply of bonds $b^{f*} = -(1/q_1)\, p_1 \, y_1^{f*}$ and thus determines the optimal action $a^{f*} = (y_1^{f*}, b^{f*})$ taken by the firm on current markets.

Let us now consider households. By assumption, the generic household *h* is endowed at the beginning of period 1 with given 'shares of ownership' in firms. This share endowment is denoted by the vector $\bar{\theta}^h = (\bar{\theta}_1^h, \ldots, \bar{\theta}_F^h)$ and it is assumed $\bar{\theta}^h \geq 0$ for all *h*, $\Sigma_h \bar{\theta}_f^h = 1$ for all *f*. The postulated behaviour of firms implies that the possession of an ownership share in a firm throughout period 1 entitles the holder to the same proportion of the firm's future profits. When the firms' plans are announced, however, households will estimate the associated profits according to their subjective expectations, and since the latter will typically differ, households will find it advantageous to trade shares on the corresponding F markets. Taking this aspect into account, let us now define the behaviour of households *after* the announcement of the plans selected by managers.

As regards trading on share markets, analysis is drastically simplified by assuming that the shares of each firm are automatically transferred to the household (or group of households) expecting the highest profits from the firm's plan, at a price exactly equal to the present value of those expected profits. Let us now take the moment in which those transfers of shares have just been carried out and households have to pay for them. In such a situation, the generic household h will calculate that by reallocating purchasing power through the bond market, in the present it can buy consumption goods for current and future delivery subject to the single budget constraint

$$p_1\, x_1^h + q^h\, x_2^h + \sum_f \theta_f^h\, v^f = p_1\, \omega_1^h + q^h \omega_2^h + \sum_f \bar{\theta}_f^h\, v^f + \sum_f \theta_f^h\, (q^h\, y_2^f * + p_1\, y_1^f *) \quad [2.2]$$

where $q^h = q_1\, p_2^h$ as before denotes the household's system of 'present prices' for commodities delivered in 2, θ_f^h is the share in firm f transferred to the household, v^f is the price for 100% of firm f's shares – or market value of the firm for short – and the bracketed term is the present value of the profits that the household expects from firm f's plan. It should be noted, however, that the working of share markets as postulated implies that θ_f^h is strictly positive only if $q^h\, y_2^f * + p_1\, y_1^f *$ $= v^f$ and must otherwise be zero. As a consequence, constraint [2.2] can be written more concisely as

$$p_1\, x_1^h + q^h\, x_2^h = p_1\, \omega_1^h + q^h \omega_2^h + \sum_f \bar{\theta}_f^h\, v^f \quad [2.2']$$

Comparison of budget constraints [2.2'] and [2.1] shows that once the firms' plans have been announced and the assumed share transfers have taken place, households are fundamentally in the same position as in the initial model of the previous subsection. Accordingly it is assumed that the generic household h will determine its optimal action on first-period markets in a similar way. More precisely, it will first select a most preferred consumption stream $x^h * = (x_1^h *,\ x_2^h *)$ from among those compatible with [2.2']. Then, in order to attain that stream, the household will capitalise its expected future wealth (inclusive of the expected profits from firms) by supplying the quantity of bonds $\bar{b}_1^h$ such that $q_1 \bar{b}_1^h = q^h \omega_2^h + \Sigma_f \theta_f^h (q^h\, y_2^f * + p_1\, y_1^f *)$, and use its total current wealth partly to pay for the share transfers and partly to buy the bundle $x_1^h *$ and the quantity of bonds $b_1^h * = p_2^h x_2^h *$ that it considers necessary for financing desired future consumption. With this determination of the household's optimal action $a^h * = (x_1^h *, \bar{b}_1^h, b_1^h *)$, the treatment of agents' behaviour is complete. Since by assumption share markets are automatically cleared, a temporary equilibrium of the economy under consideration can be defined as a system of current prices, a constellation of individual expectations and a corresponding set of optimal actions on the part of firms and households such that the *N* spot markets and the market for bonds are simultaneously cleared.

As explained in Appendix 1, the model with production just outlined is essentially that put forward by Arrow & Hahn (1971: 136-151). We can therefore take advantage of the results obtained by those authors, who prove that temporary equilibrium exists not only in the case of fixed expectations but also when the expected prices depend on current prices. Having clarified this point, we shall now go on to discuss the assumptions on production decisions made in the model. It will be argued that they are more problematic than they may appear.

Let us examine the position of households in the Arrow-Hahn model as presented above. Budget constraint [2.2'] shows that any household holding an initial share in the generic firm *f* will favour the choice of the production plan that maximises the firm's market value v^f. In the Arrow-Hahn model, however, the manager of the firm selects the plan to which he individually attaches the greatest present value, so that he does not try in general to act in the interest of the firm's initial owners. An unsatisfactory feature of the model is therefore that the criterion of choice attributed to managers has no clear rationale. It will now be seen that this shortcoming is the symptom of an authentic analytical problem.

Suppose for the sake of argument that the manager of the generic firm, in an effort to serve the interests of the initial owners, forms a definite opinion as regards the production plan that will generate the highest market value of the firm and announces precisely that project. Since the manager's opinion is subjective, the firm's initial owners may happen to have a different view and wish to alter the manager's decision. What is more, the initial owners may well have conflicting opinions as regards which plan will ensure maximisation of the firm's market value, and in that case a sort of social choice problem would arise within the constituency of the firm's owners. While this problem could be tackled in principle by assuming that some institutional rule leading to a definite decision is at work within the firm, the fact that a variety of such rules can be conceived of (e.g.

different voting schemes) makes it hard to see how that assumption should be precisely specified. It should be noted that the problem would not arise in the special case of *uniform* price expectations.[2]

On the other hand, it is possible to adopt a pragmatic attitude and argue that the assumption that managers choose production plans according to their own evaluation of future receipts provides a realistic representation of where control over firms resides (e.g. Bliss, 1976: 194–195). This attitude may explain why that assumption has been commonly adopted in temporary equilibrium models with production. As discussion of a further shortcoming of the Arrow-Hahn model will presently show, however, the assumption of production plans autonomously chosen by managers is not easily accommodated in a temporary equilibrium framework.

The aspect we shall now discuss involves the financing of the plans selected by managers. As has been seen, the Arrow-Hahn model assumes that firms finance such plans by selling bonds on a single market where the securities issued by different agents are traded at the same price and therefore treated as perfect substitutes. It is highly doubtful, however, that rational households would be generally willing to trade on such a market. A simple example will clarify this point.

Consider an economy with two firms and assume that the manager of each firm selects a plan that maximises the present value of profits calculated on the basis of his personal expectations. Then assume that when the manager of firm 1 announces the chosen project, all the other agents expect that the price of planned output will be so low as to generate negative profits in the second period. Finally, assume that all households expect positive profits from the plan announced by firm 2. In such circumstances, the entire ownership of firm 1 would be transferred to the firm's manager and the following situation would occur on the bond market. Except for the manager of firm 1, all households would calculate that firm 1 is going to issue bonds that cannot be repaid out of the firm's future receipts – and since they do not know whether the future wealth of the firm's new owner will be sufficient to guarantee repayment - those households would have to regard the bonds floated by firm 1 as risky assets. At the same time, they would regard the bonds issued by firm 2 as perfectly safe. In this situation it is not reasonable to suppose, as the Arrow-Hahn model implicitly does, that households would be disposed to purchase bonds on a single market where risky securities cannot be distinguished from safe ones. It should be noted that the problematic situation

[2] In that particular case, agents would unanimously agree as regards the profits obtainable from any production plan. Under the assumption made in the text as regards share prices, the criterion of choice that Arrow & Hahn attribute to managers would thus lead to the selection of plans that do maximize the firms' market values. The managers' decisions could accordingly be justified *ex post* as those best serving the owners' interest. If it is further assumed that the coincidence of individual forecasts is common knowledge, the owners of each firm would unanimously agree as regards which plan leads to maximisation of the firm's market value and the criterion of choice attributed to managers would be fully justified *ex ante*.

just described may also arise if the model is modified by assuming that managers endeavour to maximise the market value of their respective firms.[3]

C. Bliss (1976, 1983) perceived that the assumption of a single capital market on which agents can freely borrow is problematic in a temporary equilibrium framework and in the Arrow-Hahn model tried to introduce conditions ensuring that the bonds of the different firms could be regarded as equally safe. Thus he proposed a constrained version in which managers can only choose plans that turn out to be profitable on the basis of a system of 'reference prices' for period 2, which in turn should reflect the expectations of one external actor capable of guaranteeing the repayment of the firms' debts or, alternatively, a 'market view' concerning future prices. However, both interpretations of the reference prices present drawbacks that were pointed out by Sato (1976: 203-204) and admitted by the author himself (Bliss 1976: 199-200; 1983: 149). Grandmont & Laroque (1976) suggested instead an alternative representation of the financing of production, which basically consists of assuming that the stocks of produced commodities available at the initial date constitute the endowments of firms and that managers finance the chosen plans out of the value of those stocks. This alternative route dismisses the fact that firms do normally borrow. What is more, reflection shows that in the presence of conflicting expectations it is not sound to presume that managers can freely dispose of the 'initial wealth' of firms.[4] The financing of production,

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[3] For example, consider an economy with two firms, A and B, that can produce two different qualities of wine by employing grape must as the only input. Assume that each firm can produce any combination of wines by operating two independent processes defined by the functions $y_{12} = (-y_1)^{1/2}$ for type 1 wine and $y_{22} = 2(-y_1)^{1/2}$ for type 2 wine, where y_1 (a negative number) denotes the quantity of must employed and y_{i2} the output of wine of type i (i = 1, 2). Assume further that there are four households in the economy characterized by the following fixed expectations. Household 1, which includes only the manager of firm A, expects that the price for wine 1 will be $\hat{p}_{12} > 0$ and that the price for wine 2 will be zero. Household 2 has the same expectations as household 1. Household 3, which includes only the manager of firm B, expects that the price for wine 1 will be zero and that the price for wine 2 will be $\overline{p}_{22} = 1/2\ \hat{p}_{12}$. Finally, household 4 has the same expectations as household 3. Considering the assumption made in the model as regards share prices, we see that at any given positive price for must there are always two distinct production plans ensuring maximisation of the market value of the generic firm. The first involves producing only wine 1 in the quantity that maximises the present value of profits calculated at the price expected for that good by households 1 and 2. The second involves producing only wine 2 in the quantity that maximises the present value of profits calculated at the price expected for that wine by households 3 and 4. Now assume that managers seek to maximise the market value of their respective firms and that if two or more plans ensuring this result are identified, each manager chooses the one that he thinks will yield the highest profits (reasonable behaviour). Moreover, assume for the sake of argument that both managers can correctly predict how individual households will evaluate any feasible production plan. On these assumptions, the manager of A will announce the plan that involves producing only wine 1, while the manager of B will announce the plan that involves producing only wine 2. On the other hand, every household will calculate that one of these plans will yield positive profits while the other is bound to bring about losses. The announcement of production plans will thus signal to households that risky bonds may coexist with safe ones in the overall supply.

[4] For example, assume that the manager of the generic firm, guided by his personal evaluation of future receipts, chooses a plan that involves using the whole of the firm's initial wealth to meet input costs. Assume further that when that project is announced, all the other households anticipate that the firm's planned output will have negligible value in the future. In these circumstances, it is reasonable to presume that the firm's market value would be very close to zero.

therefore, remained an unsettled issue in the temporary equilibrium literature of the 1970s and 1980s.

The decline of temporary equilibrium theory

From the studies carried out in the field it thus emerges that the subjective expectations of agents give rise to considerable problems for temporary equilibrium theory, even when analysis is developed within the simplest framework and focused exclusively on the existence of temporary equilibrium in the initial period. In particular, we have seen that in the typical case in which agents hold conflicting views as regards future prices, difficulties arise not only in the determination of households' behaviour, but also as regards the formation of production decisions within firms and the financing of production. These difficulties help to explain why research in the field of temporary equilibrium theory declined about thirty years ago. Moreover, they contribute to explaining why general equilibrium theorists have since chosen to study sequential economies under the assumption that agents can perfectly foresee the future prices, which obviously rules out divergences in individual expectations. Attention will now be focused on the studies developed along this second path.

3. Sequential economies with perfect foresight

This second path was opened up by Arrow (1953) and Radner (1972), who analysed an 'intermediate world' (cf. Guesnerie & Jaffray, 1974) between that of the Arrow-Debreu model, in which all the commodities for present and future delivery are traded at the initial date, and that of temporary equilibrium. If we continue to assume a two-period economy, at the beginning of the first period, only the spot markets for commodities and some asset markets are open, and agents are thus forced to form expectations as regards the spot prices of the second period in order to select their intertemporal plans as in the temporary equilibrium model. However, in this 'intermediate world', analysis is focused on situations in which agents hold common price expectations that, moreover, lead to mutually consistent individual plans over the whole time sequence of markets. In this setting

Assume that this is indeed the case and consider the position of the firm's initial owners. Apart from the negligible price they could receive from the sale of their shares in the firm, these owners would calculate that the manager's decision requires them to give up some of their potential period 1 wealth (corresponding to the value of the firm's endowment) in order to finance a project that they regard as a sheer waste of resources. At the same time, each owner would calculate that he would be better off if the firm were instructed to close down, as then he could regain his share of the firm's wealth and improve his consumption. All the initial owners would thus prefer the firm not to engage in production, and in the presence of this unanimously preferred option it is paradoxical to suppose that they would passively agree to finance the manager's plan.

Radner put forward the notion of 'equilibrium of plans, prices and price expectations,' which has since traditionally been taken as an equilibrium in which agents perfectly foresee the future prices.

In the following, a reconstruction will be made of the origin and content of equilibrium of plans, prices and price expectations, or Radner equilibrium for short. Then it will be seen how, for plausible configurations of the economy, the perfect foresight usually associated with Radner equilibrium proves theoretically doubtful, because at the beginning of the second period the price vector initially expected by agents may emerge as just one element of a continuum of equilibrium price systems.

The origin and content of Radner equilibrium

As Radner (1982; 1991) explicitly recognizes, the source of his sequential equilibrium is Hicks's 'equilibrium over time'.

In chapter X of *Value and Capital*, Hicks considers an economy that is active for two weeks, with markets opening on both the first and the second Monday. Hicks stresses that, whilst the equilibrium on the first Monday depends on the expectations about the prices that will rule in the second week, the equilibrium in the second week depends in turn on the decisions taken by agents on the first Monday, as the prices realized on the second Monday stem also from the quantity of resources left over at the end of the first week as the result of those previous decisions. This interconnection provides the possibility to think of a unique equilibrium over time of the economy, rather than reasoning in terms of two separate temporary equilibria. In Hicks's words:

> In determining the system of prices established on the first Monday, we shall also have determined with it the system of plans which will govern the distribution of resources during the following week. If we suppose these plans to be carried out, then they determine the quantity of resources which will be left over at the end of the week, to serve as the basis for the decisions which have to be taken on the second Monday. On that second Monday a new system of prices has to be set up, which may differ more or less from the system of prices which was established on the first.
>
> The wider sense of Equilibrium—Equilibrium over Time, as we may call it, to distinguish it from the Temporary Equilibrium which must rule within any current week—suggests itself when we start to compare the price-situations at any two dates. [Hicks, 1946: 131-2].

According to Hicks (1946: 132)[5] what characterizes equilibrium over time 'is the condition that the prices realized on the second Monday are the same as those which were previously *expected* to rule at that date'.[6] More precisely,

[5] As known, Hicks (1933) already adverts to a general equilibrium model with perfect foresight when referring to Hayek (1928). On this occasion, he defines a perfect foresight model as being one with a set of current and expected prices such that both the current and the future markets clear.

> In equilibrium, the change in prices which occurs is that which was expected. If tastes and resources also remain what they were expected to remain, then in equilibrium nothing has occurred to disturb the plans laid down on the first Monday. So far as can be seen, no one has made any mistakes, and plans can continue to be executed without any revision. [Hicks, 1946: 132].

To find these 'correct' expected prices, Hicks (1946: 135) points out that forward trading is a device through which 'expectations and plans can be (at least partially) coordinated', while in a pure 'spot economy' this coordination is left to chance. Thus he introduces a pure 'futures economy'—essentially, an Arrow-Debreu economy without uncertainty—in which 'everything was fixed up in advance' on the markets open at the initial date and hence 'not only would current demands and supplies be matched, but also planned demands and supplies' (Hicks, 1946: 136). In Hicks's view, the theoretical use of the economy with complete forward markets lies in the fact that '[b]y examining what system of prices would be fixed up in a futures economy, we can find out what system of prices would maintain equilibrium over time' (Hicks, 1946: 140) under the assumption of perfect foresight (cf. also Hicks, 1965: 75).

Now, with complete asset markets, Radner equilibrium may be seen as an extension of the Hicksian notion of 'equilibrium over time',[7] taking uncertainty into account and also (under certain hypotheses) the possibility of introducing rational expectations (cf. Radner, 1974: 459; 1982: 932).[8] Radner assumes in fact that 'at the beginning of time all agents have available a (common) forecast of the *equilibrium* spot prices that will prevail at every future date and event' (1982: 928, emphasis added), and defines the equilibrium of plans, prices and price expectations (EPP&PE hereafter) as 'a set of prices on the current market**,** a set of common expectations for the future, and a consistent set of individual plans, one for each agent, such that, given the current prices and price-expectations, each individual agent's plan is optimal for him, subject to an appropriate sequence of budget constraints' (Radner, 1982: 932).

To give an example, for the exchange economy of Section 2, an EPP&PE can be defined as a system of first period prices for commodities and bonds $p^*=(p_1^*, q_1^*)$, a (common) system of

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[6] This implies common expectations since, as Hicks specifies, "[i]f one person expects the price of a particular commodity to fall between this Monday and the next, and another person expects it to rise; then they cannot both be right" (Hicks, 1946: 133).

[7] Indeed Radner (1991: 438-40) viewed his equilibrium of plans, prices and price expectations as 'closest in spirit', but not identical to, Hicks's equilibrium over time, also because he includes the possibility that asset markets may not be complete, and thus the equilibrium need not exist, may not be determinate and may not be optimal even under the standard assumptions about preferences and technology. It is beyond the scope of the present paper to discuss the literature on general equilibrium with incomplete asset markets. We can refer the reader to Magill & Shafer (1991); Magill & Quinzii (1996); Borglin & Tvede (2006).

[8] As Grandmont (1988) points out, in this case, the expectation functions of the agents are appropriately chosen to lead to perfect foresight on the assumption that traders know beforehand the true structure of the system.

expected prices p_2^e and a corresponding set of first period optimal actions $a^h{*} = (x_1^h{*},\ \bar{b}_1^h,\ b_1^h{*})$ and planned future consumptions $x_2^h{*}$, h = 1, 2, …, H, such that not only the current markets for commodities and bonds are simultaneously cleared as in temporary equilibrium, but also the future spot markets would clear at the expected prices.

For economies with complete asset markets, it has been proved that the EPP&PE can be derived from the Arrow-Debreu equilibrium based on the same set of preferences, technology and endowments (cf., for example, Mas-Colell, Whinston & Green, 1995: 694-698; cf. also Grandmont, 1977: 535). To grasp how this can be done along the lines suggested by Hicks,[9] let us continue to focus, for the sake of simplicity, on the two-period exchange economy with N commodities in each period and no uncertainty.

As the first step, assume that forward markets for every commodity are open at the beginning of period 1. Taking commodity 1 for spot delivery as numéraire, prices can accordingly be denoted by a vector $\tilde{p} = (p_1, q)$, in which the sub-vector $p_1 = (p_{11}, p_{21}, \ldots, p_{N1})$, with $p_{11} = 1$, refers to spot markets and $q = (q_1, q_2, \ldots, q_N)$ to forward markets. Within this context, an Arrow-Debreu equilibrium is a system of prices $\tilde{p}{*} = (p_1{*}, q{*})$ and a set of consumption streams $x^h{*} = \left(x_1^h{*}, x_2^h{*}\right)$, h = 1, 2, …, H, such that: (a) for each h, $x^h{*}$ maximizes utility subject to the intertemporal budget constraint $p_1{*}x_1^h + q{*}x_2^h = p_1{*}\omega_1^h + q{*}\omega_2^h$; and (b) for each commodity, total demand equals total supply at each date.

Now consider the case in which only the N spot markets are open in the first period together with the market for a bond that promises to pay one unit of commodity 1 in period 2. As in Section 2, assume that the generic household capitalises the expected value of future endowments by supplying the quantity of bonds $\bar{b}_1^h$ and demands the quantity b_1^h that it deems necessary for financing planned future consumption.

Following Radner (1982: 928), let us posit, in each period, good 1 as the numéraire. Given the Arrow-Debreu equilibrium price vector of the first step $\tilde{p}{*} = (p_1{*}, q{*})$, an EPP&PE for the sequential economy can be constructed by taking: (1) the vector $p{*} = (p_1{*}, q_1{*})$, as the first period prices; (2) the vector $p_2^e = (1/q_1{*})q{*}$ as the spot prices unanimously expected for period 2, expressed in terms of good 1 delivered in 2.

Considering that the bond market in existence allows for transfers of purchasing power across periods, it can be readily seen that, at the prices ($p{*}$, p_2^e), each household will be in exactly

[9] This task is narrower than showing the equivalence between an Arrow-Debreu economy and a Radner economy. As for that equivalence, cf. for example Arrow (1953) and Guesnerie & Jaffray (1974).

the same position as in the Arrow-Debreu equilibrium of the first step,[10] and hence the consumption streams $x^h* = (x_1^h*, x_2^h*)$, h = 1, 2, …, H, included in that equilibrium will be optimal for households also in the sequential framework under examination. Moreover, since those plans are part of an Arrow-Debreu equilibrium, they will be consistent in the sense that total demand will equal total supply for each commodity and each period. Finally, in view of market clearing on commodity markets, it is easily shown that also the bond market will clear at the prices (p*, p_2^e).[11] It can therefore be concluded that the price system (p*, p_2^e), the corresponding individual transactions on the bond market and the consumption streams $x^h* = (x_1^h*, x_2^h*)$, h = 1, 2, …, H, constitute an EPP&PE of the sequential economy.

Radner equilibrium with complete asset markets and 'second period indeterminacy'

A feature traditionally attributed to Radner equilibrium is that the prices initially expected by agents will actually be realized when markets open in the future periods. Albeit not strictly following from the definition of EPP&PE, this property is suggested by the parallel that Radner himself draws with Hicks's equilibrium over time and, as pointed out by Drèze & Herings (2008: 445), it is usually taken for granted in general equilibrium literature (for example, McKenzie, 1989: 20; Mas-Colell, Whinston & Green, 1995: 696; cf. also Radner, 1989: 312; 1991: 438). This is indeed the reason why Radner equilibria are commonly regarded as sequential equilibria with perfect foresight.

However, some recent contributions showing the possibility of equilibrium multiplicity, or even indeterminacy, for the economy generated as the continuation of a Radner equilibrium, make doubts arise about this idea of perfect foresight. To clarify the point, here we shall rely on the 'sequential indeterminacy' argument developed by M. Mandler in various papers.[12]

Mandler (1995; 1999) examines a two-period production economy with no uncertainty, in which all firms have access to a technology composed of a finite number of linear activities and households can transfer purchasing power across periods by trading on forward markets for capital goods.[13]

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[10] Similarly to what has been argued for the initial model of Section 2, the generic household h will choose its consumption stream subject to the single budget constraint $p_1 * x_1^h + q_1 * p_2^e x_2^h \leq p_1 * \omega_1^h + q_1 * p_2^e \omega_2^h$, in which the prices to be paid in the present for commodities delivered in period 2 are $q_1 * p_2^e = q_1 * (1/q_1*) \cdot q* = q*$.

[11] By assumption, the generic household supplies the quantity of bonds $\bar{b}_1^h$ such that $q_1 * \bar{b}_1^h = q_1 * p_2^e \omega_2^h$ and demands the quantity of bonds b_1^h* such that b_1^h* = $p_2^e x_2^h*$. On these assumptions, $\sum_h b_1^h * - \sum_h \bar{b}_1^h = p_2^e \sum_h (x_2^h * - \omega_2^h)$. From the fact that the second period commodity markets clear at the prices p_2^e it then follows that $\sum_h b_1^h * - \sum_h \bar{b}_1^h = 0$.

[12] As regards the possibility of multiple equilibria for the continuation of a Radner equilibrium, cf. the example presented by Drèze & Herings (2008) for the case of pure exchange.

[13] Mandler writes that 'agents in the first period will buy or sell rights to the outputs of the activities that produce the second period factors and then in the second period receive or deliver the second-period factors they contracted for in

In the first period there are N_1 activities that employ the initial endowments in order to produce L_1 consumption goods to be delivered in that period and K_1 pure capital goods that will be ready for delivery at the beginning of period 2.[14] Thus, the capital goods are obtained by means of intertemporal production activities which use first period endowments as inputs and yield second period goods as outputs. In the second period, N_2 activities produce L_2 consumption goods by employing the endowments available at the beginning of the period, which include the K_1 capital goods stemming for the processes activated in period 1.

On the markets in existence at the beginning of the first period, agents can trade: i) inputs initially available; ii) L_1 first-period consumption goods; iii) K_1 capital goods to be delivered in period 2. Consumption goods for future delivery cannot therefore be traded in period 1. However, as said above, households can transfer their purchasing power by trading the capital goods as assets. Thus, when markets re-open in period 2, households wish to exchange the pure capital goods in which they invested their savings for consumption goods.

For this two-period economy Mandler shows that, even if there is a unique EPP&PE, typically there is a continuum of price vectors satisfying the equilibrium conditions for this second-period market re-opening. As Mandler (1999: 700) stresses, since the second-period endowment of capital goods springs from the initial intertemporal equilibrium plans, they are perfectly adjusted to their employments if those plans are carried on.[15] In other terms, if the initial intertemporal plans are carried on in period 2, then the possibility of a second-period equilibrium with excess supplies and zero prices of capital goods is ruled out notwithstanding they are inelastically supplied and technology is made up of linear production methods. As a result, if the number of factors inelastically supplied is greater than the number of processes in use, then the market-clearing conditions for these factors do not contribute enough to the price determination.[16] Therefore, in this case, the price vectors that represent – with the same aggregate production plan – a second-period

the first period and use their income to trade for consumption'(Mandler, 2008: 2042). Since the number of capital goods is $K_1 \geq 1$, asset markets are complete.

[14] Since the endowments of the first period are arbitrarily given, 'free disposal' activities are included in order to get rid of the possible excess supply associated with a zero price of some of the inputs.

[15] As Mandler (1995: 425) points out, in the Radner equilibrium capital goods prices adjust to ensure that agents "do not waste resources by saving so much capital that it becomes superfluous in the second period." Therefore, for the second-period economy, the argument of Debreu (1970) of the generic determinacy of the equilibrium cannot apply.

[16] Let us say that the second-period endowments are made up of $K_1 + 1$ factors: capital goods and labour. Assuming that there are N_2 possible production activities, with $N_2 \leq K_1$, and the activity levels y_2 are those consistent with the initial EPP&PE, then the $K_1 + 1$ market-clearing conditions on factor markets do not provide any contribution to price determination, since no factor is in excess supply so to force its price to zero. In this case, the burden of determining $L_2 + K_1$ equilibrium relative prices – associated with the vector of activity levels y_2 – rests completely on the residual equilibrium conditions. They are L_2 market-clearing conditions for the consumption goods and N_2 zero-profit conditions of the production activities, but only $L_2 + N_2 - 1$ of them can be independent because of Walras's law. Therefore, since $K_1 \geq N_2$, the residual system has more unknowns to determine than equations.

equilibrium will generally be indeterminate.[17] The point is clarified in details by an example in Appendix 2.

As far as the traditional interpretation of Radner equilibrium is concerned, the indeterminacy of equilibrium prices in the second period matters because it entails that the price expectations initially held by agents will generically not be realized.[18] Even assuming, for the sake of argument, that a 'smart auctioneer' capable of calling equilibrium prices only is at work, in the presence of a continuum of equilibrium price vectors, the probability that he may call out precisely the prices expected at the initial date is negligible. The notion of a sequential economy with perfect foresight of future prices thus appears theoretically doubtful in the case under discussion. As pointed out by Kehoe & Levine (1990: 224), indeterminacy "undermines the concept of perfect foresight equilibrium. The agents in the model, like the modeller, cannot use the model itself to make determinate predictions about the future."

4. Conclusions

In this paper we addressed the studies on sequential economies that have been performed since the 1970s along paths inspired by Hicks's *Value and Capital*. In particular, attention was focused on the analytical problems that the inclusion of price expectations among the determinants of equilibrium originates in those paths.

In Section 2 we thus illustrated the difficulties that arise in temporary equilibrium theory in view of the fact that price expectations typically differ among agents. Then we argued that the difficulties engendered by conflicting expectations help to explain why temporary equilibrium theory declined in the 1980s and why general equilibrium theorists have since chosen to study sequential economies under the assumption of perfect foresight of future prices. In Section 3 we moved on to examine the latter approach. We first illustrated the equilibrium notion commonly adopted in the studies of sequential economies with perfect foresight, that is the 'equilibrium of plans, prices and price expectations' introduced by Radner (1972) as a development of Hicks's 'equilibrium over time'. Then we pointed out, in the light of recent work by M. Mandler that for

[17] As clarified by Fratini & Levrero (2011), the result is due to the fact, with the re-opening of the market in the second period, the link that in the in the EPP&PE ties the prices for the service of the capital goods with their production costs is broken, so that the former are free to take also values independent from the latter.

[18] More recently, Mandler himself seems to underline this point when stating that if there is price indeterminacy in the second-period economy rational agents would "predict that an investment in an activity producing a second-period factor will not except by chance earn the rate of return anticipated in the first period of a sequential-trading equilibrium […]. The predictions of the general equilibrium model thus become untenable when agents trade repeatedly through time and factor-price indeterminacy is present […]" (Mandler, 2008: 2043). Notice also that, albeit price determinacy holds with differentiable technology (cf. Mandler, 1999: 701), the presence of specialized inputs always limit factor substitution possibilities. The possibility of sequential indeterminacy cannot therefore be easily discarded.

plausible configurations of the economy the perfect foresight associated with Radner equilibrium can hardly be sustained. In particular, we argued that even if in the initial period both the current and the expected prices conform to Radner equilibrium, situations of indeterminacy of equilibrium prices can arise as time unfolds that make it hard to believe that the initial expectations—understood in the usual way—may be realized.

Summing up, in the paper we assessed the developments of the original Hicksian ideas about the possible treatment of price expectations by jointly examining the results obtained in different periods within two parallel streams of literature. By this combined analysis we were able to conclude that the project of grounding the supply-and-demand analysis of value on temporary equilibria or sequential equilibria with perfect foresight – which Hicks promoted in *Value and Capital* in opposition to the long-period analyses of the early marginalist authors – still encounters substantial difficulties precisely in view of the central role assigned to the price expectations of agents.

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Appendix 1

Here we shall clarify the relationship between the original temporary equilibrium model with production of Arrow & Hahn (1971) and the two models presented in the first and second part of Section 2. To start with the model with production of the second part, even though all the assumptions made are either borrowed from the original Arrow-Hahn model or compatible with it, there are two differences in the formulation adopted. As readers can check, in the original model prices are expressed in terms of a fictitious currency ('bancors') and the unit bond is a promise to pay a unit of that currency in period 2. These differences are, however, immaterial. Consider a version of the original model in which all agents expect that the future price of good 1 in terms of 'bancors' will be equal to 1. In these circumstances, the market for bonds paying in 'bancors' becomes the same thing as a market for bonds specified in terms of good 1. This version of the original model therefore coincides with the model of the second part of Section 2 except for the numéraire. Since the behaviour of agents in the Arrow-Hahn model is independent of the numéraire adopted, however, we can safely choose good 1 as numéraire. Having thus established that the model of the second part is simply a version of the original model of Arrow & Hahn, we now note that further specification of that version yields precisely the initial model of the first part. Assume there is just one firm in the economy, which can only operate *free disposal* processes. Under this particular specification, which is compatible with Arrow & Hahn's formal model, the single firm in existence will remain totally inactive in period 1 at every non-negative vector of current prices. As a result, the particular 'production economy' under consideration coincides in fact with the initial exchange economy of Section 2.

Appendix 2

In this appendix an example will be presented in which, notwithstanding the uniqueness of Radner equilibrium for a sequential economy with complete asset markets, a continuum of equilibrium price vectors arises for the spot markets active in the final period. This will be done by means of a simplified model inspired by the contributions of M. Mandler aimed at showing the (generic) possibility of 'second period indeterminacy' in models with many capital goods and fixed-coefficient production methods.

In the economy that we shall consider there are three different kinds of commodities: two circulating capital goods, labeled *a* and *b*, and a pure consumption good, labeled *c*. Since we assume two possible dates of delivery – period 1 and period 2 – once commodities are distinguished by dates, there are six commodities.

Since a Radner equilibrium with complete asset markets can always be derived from an Arrow-Debreu equilibrium, we shall start by considering the latter and assume that markets open at the beginning of period 1 only and that there are as many markets as dated commodities. We shall then go on to the sequential model whose Radner equilibrium, or EPP&PE, is obtained from the previously determined Arrow-Debreu equilibrium. Finally, we shall show that, when markets open again in the second period, a continuum of equilibrium price vectors emerges.

The Arrow-Debreu equilibrium

In the Arrow-Debreu setting, the markets for the six dated commodities are assumed to be open simultaneously at the beginning of period 1. A price vector on these markets is therefore a (row) vector with non-negative entries $\tilde{p} = (p_{a1}, p_{b1}, p_{c1}, q_a, q_b, q_c)$. Taking the consumption good delivered in period 1 as numéraire, we have the set of normalized price vectors $P \equiv \{\tilde{p} \in \Re^6_+ : p_{c1} = 1\}$.

As for the production side, we assume that firms have access to the same technological possibilities and that these are expressed by linear production activities. For the sake of simplicity, joint production and alternative methods of production are ruled out.

The production activity of the consumption good is an 'instantaneous' process, in the sense that good *c* delivered in period *t* (*t* =1, 2) is obtained by employing the capital goods *a* and *b* in the same period. The production activities of the capital goods, instead, are 'intertemporal' processes: goods *a* and *b* delivered in period 2 are obtained by employing *a* and *b* in period 1.

Goods *a* and *b* delivered in period 1—since they are the outputs of processes decided outside the time frame considered in the model—are taken as arbitrarily given endowments. Since these endowments may not be in the proper proportion, 'free disposal' activities will be included for them—i.e. activities that employ goods *a* and *b* as inputs in period 1and give no output—in order to get rid of the possible excess supply associated with a zero price.

On the foregoing assumptions, there are six activities in total: the production activity of good *c* delivered in 1; the production activities of goods *a*, *b*, and *c* delivered in 2; two free disposal activities for *a* and *b* delivered in 1. Technical coefficients (at a unit scale) are organized in the 'activity matrix' *M*, which has as many rows as commodities and as many columns as activities:[19]

[19] The meaning of the entry m_{ij}—i.e. the entry on the *i*-th row and *j*-th column—depends on its sign:
- if $m_{ij} > 0$, then it is the quantity of commodity "*i*" produced by the activity "*j*";
- if $m_{ij} < 0$, then it is the quantity of commodity "*i*" employed by the activity "*j*";
- if $m_{ij} = 0$, then commodity "*i*" is neither an output nor an input for activity "*j*".

$$M = \begin{bmatrix} -a_c & -a_a & -a_b & 0 & -1 & 0 \\ -b_c & -b_a & -b_b & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -a_c & 0 & 0 \\ 0 & 0 & 1 & -b_c & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad \text{[a.1]}$$

In the above matrix, the first three rows refer to goods *a*, *b*, *c* delivered in 1 and the last three rows to goods *a*, *b*, *c* delivered in 2. The columns, instead, represent the activities. In particular, the first column is made up of the coefficients for the production of good *c* delivered in 1; the second and the third column refer to the production of *a* and *b* delivered in 2; the fourth to the production of *c* delivered in 2; the last two columns to the free disposal activities.

As for the activity levels, for the generic firm f they are denoted by a (column) vector of non-negative quantities $y^f = (y^f_{c1}, y^f_{a2}, y^f_{b2}, y^f_{c2}, u^f_{a1}, u^f_{b1})$; as a consequence, $M \cdot y^f$ is a technically feasible production plan for the firm. Given a price vector $\tilde{p} \in P$, each firm chooses its activity levels so as to maximise the amount of profits $\tilde{p} \cdot M \cdot y^f$. The aggregate vector of activity levels is $y = \Sigma_f \, y^f$ and the corresponding aggregate profits are $\tilde{p} \cdot M \cdot y$.

If prices are such that an activity yields strictly positive profits at the unit scale, the profit maximising activity level is clearly indefinite. Only price vectors entailing $\tilde{p} \cdot M \leq 0$ are thus compatible with equilibrium and, for those prices, every vector of activity levels bringing about zero profits is optimal.

As regards the consumption side, for a generic household h let us take as given: i) a utility function $u^h = u^h(x^h_{c1}, x^h_{c2})$, where x^h_{ct} is a quantity of c delivered in period t; ii) the (strictly positive) first period endowments of capital goods $\omega^h_1 = (\omega^h_{1a}, \omega^h_{1b})$; iii) the endowment of shares in the profits of firms $\bar{\theta}^h = (\bar{\theta}^h_1, \ldots, \bar{\theta}^h_F)$. For the sake of simplicity, we assume that the generic household h has the same share in every firm, that is $\bar{\theta}^h_f = \bar{\theta}^h$ for $f = 1, \ldots, F$. As for the utility function, besides the customary properties, it is assumed that: a) it is monotonically increasing, that is the consumption good is desired without satiation in each period; b) it tends to its minimum—or to $-\infty$ if minimum does not exist—when x^h_{c1} and/or x^h_{c2} tend to zero, that is the consumption of c is necessary in each period.[20]

[20] Assumption (b) is fulfilled, for example, in the case of Cobb-Douglas utility functions.

As usual, for any price vector $\tilde{p} \in P$ and aggregate vector of activity levels y, the demand for consumption goods $x_c^h(\tilde{p}, y) = [x_{c1}^h(\tilde{p}, y), x_{c2}^h(\tilde{p}, y)]$ is determined by maximizing utility subject to the budget constraint:

$$x_{c1}^h + q_c x_{c2}^h = p_{a1}\omega_{a1}^h + p_{b1}\omega_{b1}^h + \bar{\theta}^h \tilde{p} \cdot M \cdot y. \qquad [a.2]$$

Accordingly, $x_c(\tilde{p}, y) = \Sigma_h \; x_c^h(\tilde{p}, y)$ denotes the aggregate demand for consumption goods and $\omega_1 = \Sigma_h \; \omega_1^h$ the aggregate supply of capital goods in period 1.

For the Arrow-Debreu economy we are dealing with, a price vector $\tilde{p} \in P$, a set of vectors of activity levels $\{y^f\}_{f=1}^F$ and a set of consumption plans $\{x_c^h(p, y)\}_{h=1}^H$ constitute an equilibrium if and only if:

$$a_c y_{c1} + a_a y_{a2} + a_b y_{b2} + u_{a1} = \omega_{a1} \qquad [a.3.1]$$

$$b_c y_{c1} + b_a y_{a2} + b_b y_{b2} + u_{b1} = \omega_{b1} \qquad [a.3.2]$$

$$x_{c1}(\tilde{p}, y) = y_{c1} \qquad [a.3.3]$$

$$a_c y_{c2} = y_{a2} \qquad [a.3.4]$$

$$b_c y_{c2} = y_{b2} \qquad [a.3.5]$$

$$x_{c2}(\tilde{p}, y) = y_{c2} \qquad [a.3.6]$$

$$1 = p_{a1} a_c + p_{b1} b_c \qquad [a.4.1]$$

$$q_a = p_{a1} a_a + p_{b1} b_a \qquad [a.4.2]$$

$$q_b = p_{a1} a_b + p_{b1} b_b \qquad [a.4.3]$$

$$q_c = q_a a_c + q_b b_c \qquad [a.4.4]$$

$$p_{a1} \geq 0, \text{ with } p_{a1} u_{a1} = 0 \qquad [a.4.5]$$

$$p_{b1} \geq 0, \text{ with } p_{b1} u_{b1} = 0 \qquad [a.4.6]$$

It is clear enough that [a.3.1] – [a.3.6] are the market clearing conditions for goods *a*, *b* and *c* delivered in 1 and 2. As for the conditions [a.4.1] – [a.4.6], it has already been pointed out that only price vectors such that $\tilde{p} \cdot M \leq 0$ are compatible with equilibrium and that the firms' optimal production plans must accordingly bring about zero profits in equilibrium, i.e. they must be such that $\tilde{p} \cdot M \cdot y = 0$. Considering that y_{c1} and y_{c2} must be strictly positive in equilibrium as good *c* is necessary in both periods, this implies that all four *stricto sensu* production activities must yield zero profits at the unit scale. Equations [a.4.1] – [a.4.4] express precisely this requirement. The free disposal activities, instead, can satisfy the zero profit condition by a null activity level, and must do so in the case of strictly positive price (conditions [a.4.5] – [a.4.6]).

Finally, we assume that the 'data' of the economy are such that a unique Arrow-Debreu equilibrium exists: $\tilde{p}^*, \{y^f{}^*\}_{f=1}^F$, $\{x_c^{h*}\}_{h=1}^H$.[21]

The equilibrium of plans, prices and price expectations

As mentioned in Section 3, once an Arrow-Debreu equilibrium has been determined, a Radner equilibrium for a sequential economy with complete asset markets (characterized by the same preferences, endowments and technology) can be derived from it. This is exactly what we shall do now in order to construct our example.

Let us consider an economy with the same 'data' as in the previous subsection, but in which there is at least one missing forward market at the beginning of period 1. More precisely, let us assume that at the beginning of period 1 there exist spot markets for the three goods and forward markets for the capital goods *a* and *b*, but no forward market for good *c*. As a consequence, the spot markets for *a*, *b*, *c* will also be active in period 2. Due to the presence of complete asset markets, however, agents in the first period can freely allocate their purchasing power over time.

At the beginning of period 1, agents face the price vector for the five markets in existence and unanimously anticipate the three prices that will prevail on period 2 markets. As for the first price system, it is assumed that the prices ruling in period 1 form the vector $p = [p_{a1}*, p_{b1}*, 1, q_a*, q_b*]$, whose components exactly coincide with the first five components of the Arrow-Debreu equilibrium price vector $\tilde{p}^*$ of the previous subsection.

As regards the expected period 2 prices, they are assumed to be expressed in terms of good *c* delivered in that period. Accordingly, let $p_{a2}{}^* = q_a{}^*/q_c{}^*$ and $p_{b2}{}^* = q_b{}^*/q_c{}^*$, the price vector unanimously expected by agents is $p_2^e = [p_{a2}*, p_{b2}*, 1]$. Therefore, agents base their intertemporal decisions in period 1 on the prices *p* ruling on the markets in existence and on their common price expectations p_2^e.

The presence of complete asset markets implies that, at the price system (p, p_2^e) derived from the Arrow-Debreu equilibrium prices $\tilde{p}^*$, the optimal consumption and production plans of

[21] As is well-known, some restrictions on the aggregate households' decisions—such as gross substitutability or the satisfaction of the weak axiom of revealed preference—ensure the uniqueness of Arrow-Debreu equilibrium. None of these restrictions is strictly required here, however, since they are sufficient but not necessary conditions for uniqueness.

agents are the same as in the Arrow-Debreu equilibrium.[22] Hence, there is a set of consumption plans $\{x_c^h*\}_{h=1}^H$, a set of production levels $\{y^f*\}_{f=1}^F$ and they are mutually consistent.

The only novelty concerns the asset markets. Firms cover the costs of the intertemporal activities by selling forward the output of goods *a* and *b* they will deliver in period 2. On the other hand, by buying on first period markets appropriate quantities of goods *a* and *b* for delivery in period 2, households provide themselves with the purchasing power required for financing the desired future consumption of *c*.[23]

It should be noted, however, that at the price system $(p,\ p_2^e)$ the two assets are perfect substitutes for households, which get $1/q_c*$ units of good *c* delivered in 2 for each unit of numéraire—i.e. good *c* delivered in 1—invested, independently of the kind of asset chosen. Hence, without loss of generality, the ratio of the aggregate demands for assets may be assumed to coincide with the ratio of the produced quantities, i.e. / = $y_{a2}*/y_{b2}*$. Moreover, as for the levels, on the one hand, because of the equality between the total value of the demand for assets in terms of good *c* delivered in 2 and the total optimal consumption of that good we have: $p_{a2}*\Sigma_h x_{a2}^h*+p_{b2}*\Sigma_h x_{b2}^h*=x_{c2}*$. On the other hand, since p_2^e is based on the Arrow-Debreu equilibrium prices, equilibrium conditions [a.3.4], [a.3.5] and [a.4.4] imply $p_{a2}*y_{a2}*+p_{b2}*y_{b2}*=y_{c2}*$. Therefore, given that $x_{c2}*=y_{c2}*$, the asset markets clear too, that is $\Sigma_h x_{a2}^h*=y_{a2}*$ and $\Sigma_h x_{b2}^h*=y_{b2}*$.

Concluding, at the ruling prices *p* and expected prices p_2^e, the plans $\{y^f*\}_{f=1}^F$, $\{x_c^h*\}_{h=1}^H$ and $\{(x_{a2}^h*, x_{b2}^h*)\}_{h=1}^H$ are optimal and mutually consistent. We thus have an equilibrium of plans, prices and price expectations.

The second period equilibrium

In the sequential economy, markets open again at the beginning of period 2. At that time, firms deliver the capital goods that households bought forward as assets and, moreover, they choose the levels of the production activities to be carried out in the second period.

[22] It was seen in the previous sections that with complete asset markets, households' decisions are subject to the same budget constraint as in the Arrow-Debreu model. Moreover, as for the production side, since it assumed that the production activity of good *c* is an 'instantaneous' process and that goods *a* and *b* delivered in period 2 can also be traded on the markets open in period 1, then the sequential structure of the economy does not entrain particular consequences for firms' decisions.

[23] Although the presence of more than one asset in principle entails the unbounded arbitrage problem discussed in Section 2, here the problem does not arise because, in view of our assumptions, the relative price of the two assets on first period markets equals the one agents expect will emerge on period 2 markets, that is $q_{a2}*/q_{b2}* = p_{a2}*/p_{b2}*$. In other words, the no-arbitrage condition mentioned in Section 2 is fulfilled by assumption.

Since period 2 is the terminal period of the economy, the only production process taking place in it is the instantaneous production of consumption good *c* by means of goods *a* and *b*. The technical coefficients characterizing that process are those listed in the fourth column of the matrix *M* and the activity level of the generic firm *f* will be denoted by y_{c2}^{f}. As before: $y_{c2} = \Sigma_f y_{2c}^{f}$.

At any given second period price vector $p_2 = (p_{a2}, p_{b2}, p_{c2}) \in P_2$, with $P_2 \equiv \{p_2 \in \Re_+^3 : p_{c2} = 1\}$, each firm wishes to maximize its amount of profits. So, as before, price vectors involving strictly positive profits at the unit scale are incompatible with second period equilibrium. When these prices are ruled out, we must have $[1-(p_{a2}a_c + p_{b2}b_c)]y_{c2}^{f} = 0$ for $f = 1, 2, \dots, F$.

As for households, they are initially endowed with the quantities of goods *a* and *b* bought forward in period 1, and wish to sell them in order to buy the consumption good *c*. Therefore, given a price vector $p_2 \in P_2$, an aggregate production level y_{c2} and the quantities of assets $x_{a2}^{h}*$ and $x_{b2}^{h}*$ bought in the previous period, the budget constraint of the generic household in period 2 is

$$x_{c2}^{h} = p_{a2}x_{a2}^{h}* + p_{b2}x_{b2}^{h}* + \bar{\theta}^{h}[1-(p_{a2}a_c + p_{b2}b_c)]y_{c2}. \qquad [a.5]$$

Moreover, since there is just one consumption good, the quantity of that good demanded by the generic household results directly from the budget constraint. Hence, from equation [a.5] we get $x_{c2}^{h} = x_{c2}^{h}(p_2, y_{c2})$. Then, by aggregating, we have: $x_{c2} = \Sigma_h x_{c2}^{h}$.

For the second period economy under consideration, a price vector $p_2 \in P_2$, a set of activity levels $\{y_{c2}^{f}\}_{f=1}^{F}$ and a set of consumption demands $\{x_{c2}^{h}\}_{h=1}^{H}$ are an equilibrium if and only if:

$$a_c y_{c2} = y_{a2}* \qquad [a.6.1]$$

$$b_c y_{c2} = y_{b2}* \qquad [a.6.2]$$

$$x_{c2}(p_2, y_{c2}) = y_{c2} \qquad [a.6.3]$$

$$1 = p_{a2}a_c + p_{b2}b_c. \qquad [a.7]$$

Equations [a.6.1]-[a.6.3] are the market clearing conditions for the three goods and [a.7] is the zero profit condition. The system is made up of four equations, but only three of them are independent. In particular, by aggregating the budget constraints [a.5] and considering that $\Sigma_h x_{a2}^{h}* = y_{a2}*$ and $\Sigma_h x_{b2}^{h}* = y_{b2}*$, we get $x_{c2} = p_{a1}y_{a2}* + p_{b2}y_{b2}* + [1-(p_{a2}a_c + p_{b2}b_c)]y_{c2}$. Therefore, equilibrium conditions [a.6.1], [a.6.2] imply that $x_{c2} = y_{c2}$.

Now, the production level y_{c2} *—namely the one included in the intertemporal production plans $\{y^f *\}_{f=1}^F$—must satisfy the equilibrium conditions [a.6.1]-[a.6.2] independently of the prices p_{a2} and p_{b2}. This follows very easily from the fact that the production plans $\{y^f *\}_{f=1}^F$ satisfy the Arrow-Debreu equilibrium conditions [a.3.4] and [a.3.5]. As a consequence, when the aggregate production of good *c* delivered in 2 is set equal to y_{c2} *, the burden of the determination of p_{a2} and p_{b2} rests entirely on the equation [a.7],[24] so that a continuum of (non-negative) equilibrium price systems exists in the second period since one equation cannot exactly determine two unknowns.[25]

We thus see that, in the sequential economy under consideration, the price vector p_2^e unanimously expected at the beginning of period 1 is just one element of a continuum of equilibrium price vectors.

[24] As we have proved above, when equations [a.6.1] and [a.6.2] are satisfied, equation [a.6.3] is satisfied too.
[25] In other terms, because of the assumption that the intertemporal production plans are carried on, two equilibrium conditions are automatically satisfied, namely [a.6.1] and [a.6.2], but just one unknown is determined, y_{c2}, thus the remaining system becomes under-determinate.