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# Monetary Policy Analysis in a Closed Economy: A Dynamic Stochastic General Equilibrium Approach

Francis Vitek

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## Abstract

This paper develops and estimates a dynamic stochastic general equilibrium model of a closed economy which approximately accounts for the empirical evidence concerning the monetary transmission mechanism, as summarized by impulse response functions derived from an estimated structural vector autoregressive model, while dominating that structural vector autoregressive model in terms of predictive accuracy. The model features short run nominal price and wage rigidities generated by monopolistic competition and staggered reoptimization in output and labour markets. The resultant inertia in inflation and persistence in output is enhanced with other features such as habit persistence in consumption, adjustment costs in investment, and variable capital utilization. Cyclical components are modeled by linearizing equilibrium conditions around a stationary deterministic steady state equilibrium, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. Parameters and trend components are jointly estimated with a novel Bayesian full information maximum likelihood procedure.

*JEL Classification:* C11; C13; C32; E37; E52

*Keywords:* Monetary policy analysis; Dynamic stochastic general equilibrium model; Monetary transmission mechanism; Forecast performance evaluation

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## 1. Introduction

Estimated dynamic stochastic general equilibrium or DSGE models have recently emerged as quantitative monetary policy analysis tools. As extensions of real business cycle models, DSGE models explicitly specify the objectives and constraints faced by optimizing households and firms, which interact in an uncertain environment to determine equilibrium prices and quantities. The existence of short run nominal price and wage rigidities generated by monopolistic competition and staggered reoptimization in output and labour markets permits a cyclical

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stabilization role for monetary policy, which is generally implemented through control of the nominal interest rate according to a monetary policy rule. The persistence of the effects of monetary policy shocks on output and inflation is often enhanced with other features such as habit persistence in consumption, adjustment costs in investment, and variable capital utilization. Early examples of closed economy DSGE models incorporating some of these features include those of Yun (1996), Goodfriend and King (1997), Rotemberg and Woodford (1995, 1997), and McCallum and Nelson (1999), while recent examples of closed economy DSGE models incorporating all of these features include those of Christiano, Eichenbaum and Evans (2005), Altig, Christiano, Eichenbaum and Linde (2005), and Smets and Wouters (2003, 2005).

The economy is complex, and any model of it is necessarily misspecified to some extent. An operational substitute for the concept of a correctly specified model is the concept of an empirically adequate model. A model is empirically adequate if it approximately accounts for the existing empirical evidence in all measurable respects, which as discussed in Clements and Hendry (1998) does not require that it be correctly specified. As argued by Diebold and Mariano (1995), a necessary condition for empirical adequacy is predictive accuracy, which must be measured in relative terms. Quantitative monetary policy analysis should be based on empirically adequate models of the economy.

Thus far, empirical evaluations of DSGE models have generally focused on unconditional second moment and impulse response properties. While empirically valid unconditional second moment and impulse response properties are necessary conditions for empirical adequacy, they are not sufficient. Moreover, empirical evaluations of unconditional second moment properties are generally conditional on atheoretic estimates of trend components, while empirical evaluations of impulse response properties are generally conditional on controversial identifying restrictions. It follows that the empirical evaluation of predictive accuracy is a necessary precursor to a well informed judgment regarding the extent to which any DSGE model can and should contribute to quantitative monetary policy analysis.

Existing DSGE models featuring long run balanced growth driven by trend inflation, productivity growth, and population growth generally predict the existence of common deterministic or stochastic trends. Estimated DSGE models incorporating common deterministic trends include those of Ireland (1997) and Smets and Wouters (2005), while estimated DSGE models incorporating common stochastic trends include those of Altig, Christiano, Eichenbaum and Linde (2005) and An and Schorfheide (2006). However, as discussed in Clements and Hendry (1999) and Maddala and Kim (1998), intermittent structural breaks render such common deterministic or stochastic trends empirically inadequate representations of low frequency variation in observed macroeconomic variables. For this reason, it is common to remove trend components from observed macroeconomic variables with deterministic polynomial functions or

linear filters such as that described in Hodrick and Prescott (1997) prior to the conduct of estimation, inference and forecasting.

Decomposing observed macroeconomic variables into cyclical and trend components prior to the conduct of estimation, inference and forecasting reflects an emphasis on the predictions of DSGE models at business cycle frequencies. Since such decompositions are additive, given observed macroeconomic variables, predictions at business cycle frequencies imply predictions at lower frequencies. As argued by Harvey (1997), the removal of trend components from observed macroeconomic variables with atheoretic deterministic polynomial functions or linear filters ignores these predictions, potentially invalidating subsequent estimation, inference and forecasting. As an alternative, this paper proposes jointly modeling cyclical and trend components as unobserved components while imposing theoretical restrictions derived from the approximate multivariate linear rational expectations representation of a DSGE model.

The development of empirically adequate DSGE models for purposes of quantitative monetary policy analysis in a closed economy is currently an active area of research. Nevertheless, an estimated DSGE model of a closed economy which approximately accounts for the empirical evidence concerning the monetary transmission mechanism, as summarized by impulse response functions derived from an estimated structural vector autoregressive or SVAR model, while dominating that SVAR model in terms of predictive accuracy, has yet to be developed. This paper develops and estimates a DSGE model of a closed economy which satisfies these impulse response and predictive accuracy criteria. The model features short run nominal price and wage rigidities generated by monopolistic competition and staggered reoptimization in output and labour markets. The resultant inertia in inflation and persistence in output is enhanced with other features such as habit persistence in consumption, adjustment costs in investment, and variable capital utilization. Cyclical components are modeled by linearizing equilibrium conditions around a stationary deterministic steady state equilibrium, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. Parameters and trend components are jointly estimated with a novel Bayesian full information maximum likelihood procedure.

The organization of this paper is as follows. The next section develops a DSGE model of a closed economy. Estimation, inference and forecasting within the framework of a linear state space representation of an approximate unobserved components representation of this DSGE model are the subjects of section three. Finally, section four offers conclusions and recommendations for further research.

## 2. Model Development

Consider a closed economy consisting of households, firms, and a government. The government consists of a monetary authority and a fiscal authority.

### 2.1. The Utility Maximization Problem of the Representative Household

There exists a continuum of households indexed by  $i \in [0,1]$ . Households supply differentiated intermediate labour services, but are otherwise identical.

#### 2.1.1. Consumption and Saving Behaviour

The representative infinitely lived household has preferences defined over consumption  $C_{i,s}$  and labour supply  $L_{i,s}$  represented by intertemporal utility function

$$U_{i,t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_{i,s}, L_{i,s}), \quad (1)$$

where subjective discount factor  $\beta$  satisfies  $0 < \beta < 1$ . The intratemporal utility function is additively separable and represents external habit formation preferences in consumption,

$$u(C_{i,s}, L_{i,s}) = v_s^C \left[ \frac{(C_{i,s} - \alpha C_{s-1})^{1-1/\sigma}}{1-1/\sigma} - v^L \frac{(L_{i,s})^{1+1/\eta}}{1+1/\eta} \right], \quad (2)$$

where  $0 \leq \alpha < 1$ . This intratemporal utility function is strictly increasing with respect to consumption if and only if  $v_s^C > 0$ , and given this parameter restriction is strictly decreasing with respect to labour supply if and only if  $v^L > 0$ . Given these parameter restrictions, this intratemporal utility function is strictly concave if  $\sigma > 0$  and  $\eta > 0$ .

The representative household enters period  $s$  in possession of previously purchased nominal bonds  $B_{i,s}^P$  which yield interest at risk free rate  $i_{s-1}$ . It also holds a diversified portfolio of shares  $\{x_{i,j,s}\}_{j=0}^1$  in intermediate good firms which pay dividends  $\{IT_{j,s}\}_{j=0}^1$ . The representative household supplies differentiated intermediate labour service  $L_{i,s}$ , earning labour income at nominal wage  $W_{i,s}$ . Households pool their labour income, and the government levies a tax on pooled labour income at rate  $\tau_s$ . These sources of private wealth are summed in household dynamic budget constraint:

$$B_{i,s+1}^P + \int_{j=0}^1 V_{j,s} x_{i,j,s+1} dj = (1+i_{s-1})B_{i,s}^P + \int_{j=0}^1 (\Pi_{j,s} + V_{j,s}) x_{i,j,s} dj + (1-\tau_s) \int_{k=0}^1 W_{k,s} L_{k,s} dk - P_s C_{i,s}. \quad (3)$$

According to this dynamic budget constraint, at the end of period  $s$ , the representative household purchases bonds  $B_{i,s+1}^P$ , and a diversified portfolio of shares  $\{x_{i,j,s+1}\}_{j=0}^1$  at prices  $\{V_{j,s}\}_{j=0}^1$ . It also purchases final consumption good  $C_{i,s}$  at price  $P_s$ .

In period  $t$ , the representative household chooses state contingent sequences for consumption  $\{C_{i,s}\}_{s=t}^{\infty}$ , bond holdings  $\{B_{i,s+1}^P\}_{s=t}^{\infty}$ , and share holdings  $\{\{x_{i,j,s+1}\}_{j=0}^1\}_{s=t}^{\infty}$  to maximize intertemporal utility function (1) subject to dynamic budget constraint (3) and terminal nonnegativity constraints  $B_{i,T+1}^P \geq 0$  and  $x_{i,j,T+1} \geq 0$  for  $T \rightarrow \infty$ . In equilibrium, selected necessary first order conditions associated with this utility maximization problem may be stated as

$$u_C(C_t, L_{i,t}) = P_t \lambda_t, \quad (4)$$

$$\lambda_t = \beta(1+i_t)E_t \lambda_{t+1}, \quad (5)$$

$$V_{j,t} \lambda_t = \beta E_t (\Pi_{j,t+1} + V_{j,t+1}) \lambda_{t+1}, \quad (6)$$

where  $\lambda_{i,s}$  denotes the Lagrange multiplier associated with the period  $s$  household dynamic budget constraint. In equilibrium, necessary complementary slackness conditions associated with the terminal nonnegativity constraints may be stated as:

$$\lim_{T \rightarrow \infty} \frac{\beta^T \lambda_{t+T}}{\lambda_t} B_{t+T+1}^P = 0, \quad (7)$$

$$\lim_{T \rightarrow \infty} \frac{\beta^T \lambda_{t+T}}{\lambda_t} V_{j,t+T} x_{j,t+T+1} = 0. \quad (8)$$

Provided that the intertemporal utility function is bounded and strictly concave, together with all necessary first order conditions, these transversality conditions are sufficient for the unique utility maximizing state contingent intertemporal household allocation.

Combination of necessary first order conditions (4) and (5) yields intertemporal optimality condition

$$u_C(C_t, L_{i,t}) = \beta E_t (1+i_t) \frac{P_t}{P_{t+1}} u_C(C_{t+1}, L_{i,t+1}), \quad (9)$$

which ensures that at a utility maximum, the representative household cannot benefit from feasible intertemporal consumption reallocations.

### 2.1.2. Labour Supply and Wage Setting Behaviour

There exist a large number of perfectly competitive firms which combine differentiated intermediate labour services  $L_{i,t}$  supplied by households in a monopolistically competitive labour market to produce final labour service  $L_t$  according to constant elasticity of substitution production function

$$L_t = \left[ \int_{i=0}^1 (L_{i,t})^{\frac{\theta_t^L - 1}{\theta_t^L}} di \right]^{\frac{\theta_t^L}{\theta_t^L - 1}}, \quad (10)$$

where  $\theta_t^L > 1$ . The representative final labour service firm maximizes profits derived from production of the final labour service

$$\Pi_t^L = W_t L_t - \int_{i=0}^1 W_{i,t} L_{i,t} di, \quad (11)$$

with respect to inputs of intermediate labour services, subject to production function (10). The necessary first order conditions associated with this profit maximization problem yield intermediate labour service demand functions:

$$L_{i,t} = \left( \frac{W_{i,t}}{W_t} \right)^{-\theta_t^L} L_t. \quad (12)$$

Since the production function exhibits constant returns to scale, in competitive equilibrium the representative final labour service firm earns zero profit, implying aggregate wage index:

$$W_t = \left[ \int_{i=0}^1 (W_{i,t})^{1-\theta_t^L} di \right]^{\frac{1}{1-\theta_t^L}}. \quad (13)$$

As the wage elasticity of demand for intermediate labour services  $\theta_t^L$  increases, they become closer substitutes, and individual households have less market power.

In an extension of the model of nominal wage rigidity proposed by Erceg, Henderson and Levin (2000) motivated by Smets and Wouters (2003, 2005), each period a randomly selected fraction  $1-\omega^L$  of households adjust their wage optimally. The remaining fraction  $\omega^L$  of households adjust their wage to account for past inflation according to partial indexation rule

$$W_{i,t} = \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma^L} \left( \frac{\bar{P}_{t-1}}{\bar{P}_{t-2}} \right)^{1-\gamma^L} W_{i,t-1}, \quad (14)$$

where  $0 \leq \gamma^L \leq 1$ . Under this specification, although households adjust their wage every period, they infrequently adjust their wage optimally, and the interval between optimal wage adjustments is a random variable.

If the representative household can adjust its wage optimally in period  $t$ , then it does so to maximize intertemporal utility function (1) subject to dynamic budget constraint (3), intermediate labour service demand function (12), and the assumed form of nominal wage rigidity. Since all households that adjust their wage optimally in period  $t$  solve an identical utility maximization problem, in equilibrium they all choose a common wage  $W_t^*$  given by necessary first order condition:

$$\frac{W_t^*}{W_t} = - \frac{E_t \sum_{s=t}^{\infty} (\omega^L)^{s-t} \frac{\beta^{s-t} u_C(C_s, L_{i,s})}{u_C(C_t, L_{i,t})} \theta_s^L \frac{u_L(C_s, L_{i,s})}{u_C(C_s, L_{i,s})} \left[ \left( \frac{P_{t-1}}{P_{s-1}} \right)^{\gamma^L} \left( \frac{\bar{P}_{t-1}}{\bar{P}_{s-1}} \right)^{1-\gamma^L} \frac{W_s}{W_t} \right]^{\theta_s^L} \left( \frac{W_t^*}{W_t} \right)^{-\theta_s^L} L_s}{E_t \sum_{s=t}^{\infty} (\omega^L)^{s-t} \frac{\beta^{s-t} u_C(C_s, L_{i,s})}{u_C(C_t, L_{i,t})} (\theta_s^L - 1)(1-\tau_s) \frac{W_s}{P_s} \left[ \left( \frac{P_{t-1}}{P_{s-1}} \right)^{\gamma^L} \left( \frac{\bar{P}_{t-1}}{\bar{P}_{s-1}} \right)^{1-\gamma^L} \frac{W_s}{W_t} \right]^{\theta_s^L - 1} \left( \frac{W_t^*}{W_t} \right)^{-\theta_s^L} L_s}. \quad (15)$$

This necessary first order condition equates the expected present discounted value of the consumption benefit generated by an additional unit of labour supply to the expected present discounted value of its leisure cost. Aggregate wage index (13) equals an average of the wage set by the fraction  $1-\omega^L$  of households that adjust their wage optimally in period  $t$ , and the average of the wages set by the remaining fraction  $\omega^L$  of households that adjust their wage according to partial indexation rule (14):

$$W_t = \left\{ (1-\omega^L)(W_t^*)^{1-\theta_t^L} + \omega^L \left[ \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma^L} \left( \frac{\bar{P}_{t-1}}{\bar{P}_{t-2}} \right)^{1-\gamma^L} W_{t-1} \right]^{1-\theta_t^L} \right\}^{\frac{1}{1-\theta_t^L}}. \quad (16)$$

Since those households able to adjust their wage optimally in period  $t$  are selected randomly from among all households, the average wage set by the remaining households equals the value



of the aggregate wage index that prevailed during period  $t-1$ , rescaled to account for past inflation.

## 2.2. The Value Maximization Problem of the Representative Firm

There exists a continuum of intermediate good firms indexed by  $j \in [0,1]$ . Intermediate good firms supply differentiated intermediate output goods, but are otherwise identical. Entry into and exit from the monopolistically competitive intermediate output good sector is prohibited.

### 2.2.1. Employment and Investment Behaviour

The representative intermediate good firm sells shares  $\{x_{i,j,t+1}\}_{i=0}^1$  to households at price  $V_{j,t}$ . Recursive forward substitution for  $V_{j,t+s}$  with  $s > 0$  in necessary first order condition (6) applying the law of iterated expectations reveals that the post-dividend stock market value of the representative intermediate good firm equals the expected present discounted value of future dividend payments:

$$V_{j,t} = E_t \sum_{s=t+1}^{\infty} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \Pi_{j,s}. \quad (17)$$

Acting in the interests of its shareholders, the representative intermediate good firm maximizes its pre-dividend stock market value, equal to the expected present discounted value of current and future dividend payments:

$$\Pi_{j,t} + V_{j,t} = E_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \Pi_{j,s}. \quad (18)$$

The derivation of result (17) imposes transversality condition (8), which rules out self-fulfilling speculative asset price bubbles.

Shares entitle households to dividend payments equal to net profits  $\Pi_{j,s}$ , defined as after tax earnings less investment expenditures:

$$\Pi_{j,s} = (1 - \tau_s)(P_{j,s} Y_{j,s} - W_s L_{j,s}) - P_s I_s. \quad (19)$$

Earnings are defined as revenues derived from sales of differentiated intermediate output good  $Y_{j,s}$  at price  $P_{j,s}$  less expenditures on final labour service  $L_{j,s}$ . The government levies a tax on earnings at rate  $\tau_s$ , and negative dividend payments are a theoretical possibility.

The representative intermediate good firm utilizes capital  $K_s$  at rate  $u_{j,s}$  and rents final labour service  $L_{j,s}$  given labour augmenting technology coefficient  $A_s$  to produce differentiated intermediate output good  $Y_{j,s}$  according to constant elasticity of substitution production function

$$\mathcal{F}(u_{j,s}K_s, A_sL_{j,s}) = \left[ (\varphi)^{\frac{1}{\vartheta}} (u_{j,s}K_s)^{\frac{\vartheta-1}{\vartheta}} + (1-\varphi)^{\frac{1}{\vartheta}} (A_sL_{j,s})^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta-1}}, \quad (20)$$

where  $0 < \varphi < 1$ ,  $\vartheta > 0$  and  $A_s > 0$ . This constant elasticity of substitution production function exhibits constant returns to scale, and nests the production function proposed by Cobb and Douglas (1928) under constant returns to scale for  $\vartheta = 1$ .<sup>1</sup>

In utilizing capital to produce output, the representative intermediate good firm incurs a cost  $\mathcal{G}(u_{j,s}, K_s)$  denominated in terms of output:

$$Y_{j,s} = \mathcal{F}(u_{j,s}K_s, A_sL_{j,s}) - \mathcal{G}(u_{j,s}, K_s). \quad (21)$$

Following Christiano, Eichenbaum and Evans (2005), this capital utilization cost is increasing in the rate of capital utilization at an increasing rate,

$$\mathcal{G}(u_{j,s}, K_s) = \mu \left[ e^{\kappa(u_{j,s}-1)} - 1 \right] K_s, \quad (22)$$

where  $\mu > 0$  and  $\kappa > 0$ . In deterministic steady state equilibrium, the rate of capital utilization is normalized to one, and the cost of utilizing capital equals zero.

Capital is endogenous but not firm-specific, and the representative intermediate good firm enters period  $s$  with access to previously accumulated capital stock  $K_s$ , which subsequently evolves according to accumulation function

$$K_{s+1} = (1-\delta)K_s + \mathcal{H}(I_s, I_{s-1}), \quad (23)$$

where depreciation rate parameter  $\delta$  satisfies  $0 \leq \delta \leq 1$ . Following Christiano, Eichenbaum and Evans (2005), effective investment function  $\mathcal{H}(I_s, I_{s-1})$  incorporates convex adjustment costs,

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<sup>1</sup> Invoking L'Hospital's rule yields  $\lim_{\vartheta \rightarrow 1} \ln \mathcal{F}(u_{j,s}K_s, A_sL_{j,s}) = \varphi \ln(u_{j,s}K_s) + (1-\varphi) \ln(A_sL_{j,s}) - \varphi \ln \varphi - (1-\varphi) \ln(1-\varphi)$ , which implies that  $\lim_{\vartheta \rightarrow 1} \mathcal{F}(u_{j,s}K_s, A_sL_{j,s}) = \varphi^{-\varphi} (1-\varphi)^{-(1-\varphi)} (u_{j,s}K_s)^{\varphi} (A_sL_{j,s})^{1-\varphi}$ .

$$\mathcal{H}(I_s, I_{s-1}) = v_s^I \left[ 1 - \frac{\chi}{2} \left( \frac{I_s - I_{s-1}}{I_{s-1}} \right)^2 \right] I_s, \quad (24)$$

where  $\chi > 0$  and  $v_s^I > 0$ . In deterministic steady state equilibrium, these adjustment costs equal zero, and effective investment equals actual investment.

In period  $t$ , the representative intermediate good firm chooses state contingent sequences for employment  $\{L_{j,s}\}_{s=t}^{\infty}$ , capital utilization  $\{u_{j,s}\}_{s=t}^{\infty}$ , investment  $\{I_s\}_{s=t}^{\infty}$ , and the capital stock  $\{K_{s+1}\}_{s=t}^{\infty}$  to maximize pre-dividend stock market value (18) subject to net production function (21), capital accumulation function (23), and terminal nonnegativity constraint  $K_{T+1} \geq 0$  for  $T \rightarrow \infty$ . In equilibrium, demand for the final labour service satisfies necessary first order condition

$$\mathcal{F}_{AL}(u_{j,t}K_t, A_tL_{j,t})\Phi_{j,t} = (1 - \tau_t) \frac{W_t}{P_t A_t}, \quad (25)$$

where  $P_s \Phi_{j,s}$  denotes the Lagrange multiplier associated with the period  $s$  production technology constraint. This necessary first order condition equates real marginal cost  $\Phi_{j,t}$  to the ratio of the after tax real wage to the marginal product of labour. In equilibrium, the rate of capital utilization satisfies necessary first order condition

$$\mathcal{F}_{uK}(u_{j,t}K_t, A_tL_{j,t}) = \frac{\mathcal{G}_u(u_{j,t}, K_t)}{K_t}, \quad (26)$$

which equates the marginal product of utilized capital to its marginal cost. In equilibrium, demand for the final investment good satisfies necessary first order condition

$$Q_t \mathcal{H}_1(I_t, I_{t-1}) + E_t \frac{\beta \lambda_{t+1}}{\lambda_t} Q_{t+1} \mathcal{H}_2(I_{t+1}, I_t) = P_t, \quad (27)$$

which equates the expected present discounted value of an additional unit of investment to its price, where  $Q_{j,s}$  denotes the Lagrange multiplier associated with the period  $s$  capital accumulation function. In equilibrium, this shadow price of capital satisfies necessary first order condition

$$Q_t = E_t \frac{\beta \lambda_{t+1}}{\lambda_t} \left\{ P_{t+1} \Phi_{j,t+1} \left[ u_{j,t+1} \mathcal{F}_{uK}(u_{j,t+1}K_{t+1}, A_{t+1}L_{j,t+1}) - \mathcal{G}_K(u_{j,t+1}, K_{t+1}) \right] + (1 - \delta) Q_{t+1} \right\}, \quad (28)$$

which equates it to the expected present discounted value of the sum of the future marginal cost of capital, and the future shadow price of capital net of depreciation. In equilibrium, the

necessary complementary slackness condition associated with the terminal nonnegativity constraint may be stated as:

$$\lim_{T \rightarrow \infty} \frac{\beta^T \lambda_{t+T}}{\lambda_t} Q_{t+T} K_{t+T+1} = 0. \quad (29)$$

Provided that the pre-dividend stock market value of the representative intermediate good firm is bounded and strictly concave, together with all necessary first order conditions, this transversality condition is sufficient for the unique value maximizing state contingent intertemporal firm allocation.

### 2.2.2. Output Supply and Price Setting Behaviour

There exist a large number of perfectly competitive firms which combine differentiated intermediate output goods  $Y_{j,t}$  supplied by intermediate good firms in a monopolistically competitive output market to produce final output good  $Y_t$  according to constant elasticity of substitution production function

$$Y_t = \left[ \int_{j=0}^1 (Y_{j,t})^{\frac{\theta_t^y - 1}{\theta_t^y}} dj \right]^{\frac{\theta_t^y}{\theta_t^y - 1}}, \quad (30)$$

where  $\theta_t^y > 1$ . The representative final output good firm maximizes profits derived from production of the final output good

$$\Pi_t = P_t Y_t - \int_{j=0}^1 P_{j,t} Y_{j,t} dj, \quad (31)$$

with respect to inputs of intermediate output goods, subject to production function (30). The necessary first order conditions associated with this profit maximization problem yield intermediate output good demand functions:

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta_t^y} Y_t. \quad (32)$$

Since the production function exhibits constant returns to scale, in competitive equilibrium the representative final output good firm earns zero profit, implying aggregate price index:

$$P_t = \left[ \int_{j=0}^1 (P_{j,t})^{1-\theta_t^Y} dj \right]^{\frac{1}{1-\theta_t^Y}}. \quad (33)$$

As the price elasticity of demand for intermediate output goods  $\theta_t^Y$  increases, they become closer substitutes, and individual intermediate good firms have less market power.

In an extension of the model of nominal price rigidity proposed by Calvo (1983) motivated by Smets and Wouters (2005), each period a randomly selected fraction  $1-\omega^Y$  of intermediate good firms adjust their price optimally. The remaining fraction  $\omega^Y$  of intermediate good firms adjust their price to account for past inflation according to partial indexation rule

$$P_{j,t} = \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma^Y} \left( \frac{\bar{P}_{t-1}}{\bar{P}_{t-2}} \right)^{1-\gamma^Y} P_{j,t-1}, \quad (34)$$

where  $0 \leq \gamma^Y \leq 1$ . Under this specification, optimal price adjustment opportunities arrive randomly, and the interval between optimal price adjustments is a random variable.

If the representative intermediate good firm can adjust its price optimally in period  $t$ , then it does so to maximize to maximize pre-dividend stock market value (18) subject to net production function (21), capital accumulation function (23), intermediate output good demand function (32), and the assumed form of nominal price rigidity. Since all intermediate good firms that adjust their price optimally in period  $t$  solve an identical value maximization problem, in equilibrium they all choose a common price  $P_t^*$  given by necessary first order condition:

$$\frac{P_t^*}{P_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} (\omega^Y)^{s-t} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \theta_s^Y \Phi_{j,s} \left[ \left( \frac{P_{t-1}}{P_{s-1}} \right)^{\gamma^Y} \left( \frac{\bar{P}_{t-1}}{\bar{P}_{s-1}} \right)^{1-\gamma^Y} \frac{P_s}{P_t} \right]^{\theta_s^Y} \left( \frac{P_t^*}{P_t} \right)^{-\theta_s^Y} P_s Y_s}{\mathbb{E}_t \sum_{s=t}^{\infty} (\omega^Y)^{s-t} \frac{\beta^{s-t} \lambda_s}{\lambda_t} (\theta_s^Y - 1)(1-\tau_s) \left[ \left( \frac{P_{t-1}}{P_{s-1}} \right)^{\gamma^Y} \left( \frac{\bar{P}_{t-1}}{\bar{P}_{s-1}} \right)^{1-\gamma^Y} \frac{P_s}{P_t} \right]^{\theta_s^Y - 1} \left( \frac{P_t^*}{P_t} \right)^{-\theta_s^Y} P_s Y_s}. \quad (35)$$

This necessary first order condition equates the expected present discounted value of the after tax revenue benefit generated by an additional unit of output supply to the expected present discounted value of its production cost. Aggregate price index (33) equals an average of the price set by the fraction  $1-\omega^Y$  of intermediate good firms that adjust their price optimally in period  $t$ , and the average of the prices set by the remaining fraction  $\omega^Y$  of intermediate good firms that adjust their price according to partial indexation rule (34):

$$P_t = \left\{ (1 - \omega^Y)(P_t^*)^{1-\theta_t^Y} + \omega^Y \left[ \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma^Y} \left( \frac{\bar{P}_{t-1}}{\bar{P}_{t-2}} \right)^{1-\gamma^Y} P_{t-1} \right]^{1-\theta_t^Y} \right\}^{\frac{1}{1-\theta_t^Y}}. \quad (36)$$

Since those intermediate good firms able to adjust their price optimally in period  $t$  are selected randomly from among all intermediate good firms, the average price set by the remaining intermediate good firms equals the value of the aggregate price index that prevailed during period  $t-1$ , rescaled to account for past inflation.

### 2.3. Monetary and Fiscal Policy

The government consists of a monetary authority and a fiscal authority. The monetary authority implements monetary policy, while the fiscal authority implements fiscal policy.

#### 2.3.1. The Monetary Authority

The monetary authority implements monetary policy through control of the nominal interest rate according to monetary policy rule

$$i_t - \bar{i}_t = \xi^\pi (\pi_t^P - \bar{\pi}_t^P) + \xi^Y (\ln Y_t - \ln \bar{Y}_t) + \nu_t^i, \quad (37)$$

where  $\xi^\pi > 1$  and  $\xi^Y > 0$ . As specified, the deviation of the nominal interest rate from its deterministic steady state equilibrium value is a linear increasing function of the contemporaneous deviation of inflation from its target value, and the contemporaneous proportional deviation of output from its deterministic steady state equilibrium value. Persistent departures from this monetary policy rule are captured by serially correlated monetary policy shock  $\nu_t^i$ .

#### 2.3.2. The Fiscal Authority

The fiscal authority implements fiscal policy through control of nominal government consumption and the tax rate applicable to the pooled labour income of households and the

earnings of intermediate good firms. In equilibrium, this distortionary tax collection framework corresponds to proportional output taxation.

The ratio of nominal government consumption to nominal output satisfies fiscal expenditure rule:

$$\ln \frac{P_t G_t}{P_t Y_t} - \ln \frac{\bar{P}_t \bar{G}_t}{\bar{P}_t \bar{Y}_t} = \nu_t^G. \quad (38)$$

Persistent departures from this fiscal expenditure rule are captured by serially correlated fiscal expenditure shock  $\nu_t^G$ .

The tax rate applicable to the pooled labour income of households and the earnings of intermediate good firms satisfies fiscal revenue rule

$$\ln \tau_t - \ln \bar{\tau}_t = \zeta^\tau \left[ \ln \left( -\frac{B_{t+1}^G}{P_t Y_t} \right) - \ln \left( -\frac{\bar{B}_{t+1}^G}{\bar{P}_t \bar{Y}_t} \right) \right] + \nu_t^\tau, \quad (39)$$

where  $\zeta^\tau > 0$ . As specified, the proportional deviation of the tax rate from its deterministic steady state equilibrium value is a linear increasing function of the contemporaneous proportional deviation of the ratio of net government debt to nominal output from its target value. This fiscal revenue rule is well defined only if the net government debt is positive. Persistent departures from this fiscal revenue rule are captured by serially correlated fiscal revenue shock  $\nu_t^\tau$ .

The fiscal authority enters period  $t$  holding previously purchased nominal bonds  $B_t^G$  which yield interest at risk free rate  $i_{t-1}$ . It also levies taxes on the pooled labour income of households and the earnings of intermediate good firms at rate  $\tau_t$ . These sources of public wealth are summed in government dynamic budget constraint:

$$B_{t+1}^G = (1 + i_{t-1})B_t^G + \tau_t \int_{i=0}^1 \int_{k=0}^1 W_{k,t} L_{k,t} dk di + \tau_t \int_{j=0}^1 (P_{j,t} Y_{j,t} - W_t L_{j,t}) dj - P_t G_t. \quad (40)$$

According to this dynamic budget constraint, at the end of period  $t$ , the fiscal authority purchases bonds  $B_{t+1}^G$ , and final government consumption good  $G_t$  at price  $P_t$ .

#### 2.4. Market Clearing Conditions

A rational expectations equilibrium in this DSGE model of a closed economy consists of state contingent intertemporal allocations for households and firms which solve their constrained optimization problems given prices and policy, together with a state contingent intertemporal allocation for the government which satisfies its policy rules and constraints given prices, with supporting prices such that all markets clear.

Let  $B_{t+1}$  denote the sum of private sector bond holdings and public sector bond holdings, which in equilibrium equals zero:

$$B_{t+1} = B_{t+1}^P + B_{t+1}^G = 0. \quad (41)$$

The imposition of equilibrium conditions on household dynamic budget constraint (3) reveals that the net increase in private sector asset holdings equals private saving less investment:

$$B_{t+1}^P - B_t^P = i_{t-1}B_t^P + (1 - \tau_t)P_tY_t - P_tC_t - P_tI_t. \quad (42)$$

The imposition of equilibrium conditions on government dynamic budget constraint (40) reveals that the net increase in public sector asset holdings equals public saving:

$$B_{t+1}^G - B_t^G = i_{t-1}B_t^G + \tau_tP_tY_t - P_tG_t. \quad (43)$$

Combination of these household and government dynamic budget constraints with bond market clearing condition (41) yields output market clearing condition:

$$Y_t = C_t + I_t + G_t. \quad (44)$$

In equilibrium, output is determined by the cumulative demands of households, firms, and the government.

#### 2.5. The Approximate Linear Model

Estimation, inference and forecasting are based on a linear state space representation of an approximate unobserved components representation of this DSGE model of a closed economy. Cyclical components are modeled by linearizing equilibrium conditions around a stationary deterministic steady state equilibrium which abstracts from long run balanced growth, while



trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path.

In what follows,  $E_t x_{t+s}$  denotes the rational expectation of variable  $x_{t+s}$ , conditional on information available at time  $t$ . Also,  $\hat{x}_t$  denotes the cyclical component of variable  $x_t$ , while  $\bar{x}_t$  denotes the trend component of variable  $x_t$ . Cyclical and trend components are additively separable, that is  $x_t = \hat{x}_t + \bar{x}_t$ .

### 2.5.1. Cyclical Components

The cyclical component of inflation depends on a linear combination of past and expected future cyclical components of inflation driven by the contemporaneous cyclical components of real marginal cost and the tax rate according to price Phillips curve:

$$\hat{\pi}_t^P = \frac{\gamma^Y}{1 + \gamma^Y \beta} \hat{\pi}_{t-1}^P + \frac{\beta}{1 + \gamma^Y \beta} E_t \hat{\pi}_{t+1}^P + \frac{(1 - \omega^Y)(1 - \omega^Y \beta)}{\omega^Y (1 + \gamma^Y \beta)} \left[ \ln \hat{\Phi}_t + \frac{\tau}{1 - \tau} \ln \hat{\tau}_t - \frac{1}{\theta^Y - 1} \ln \hat{\theta}_t^Y \right]. \quad (45)$$

The persistence of the cyclical component of inflation is increasing in indexation parameter  $\gamma^Y$ , while the sensitivity of the cyclical component of inflation to changes in the cyclical components of real marginal cost and the tax rate is decreasing in nominal rigidity parameter  $\omega^Y$  and indexation parameter  $\gamma^Y$ . This price Phillips curve is subject to price markup shocks.

The cyclical component of output depends on the contemporaneous cyclical components of utilized capital and effective labour according to approximate linear net production function

$$\ln \hat{Y}_t = \left( 1 - \frac{\theta^Y}{\theta^Y - 1} \frac{WL}{PY} \right) \ln(\hat{u}_t \hat{K}_t) + \frac{\theta^Y}{\theta^Y - 1} \frac{WL}{PY} \ln(\hat{A}_t \hat{L}_t), \quad (46)$$

where  $\frac{K}{Y} = \frac{\beta(1-\tau)}{1-\beta(1-\delta)} \left( \frac{\theta^Y - 1}{\theta^Y} - \frac{WL}{PY} \right)$ . This approximate linear net production function is subject to output technology shocks.

The cyclical component of consumption depends on a linear combination of past and expected future cyclical components of consumption driven by the contemporaneous cyclical component of the real interest rate according to approximate linear consumption Euler equation:

$$\ln \hat{C}_t = \frac{\alpha}{1 + \alpha} \ln \hat{C}_{t-1} + \frac{1}{1 + \alpha} E_t \ln \hat{C}_{t+1} - \sigma \frac{1 - \alpha}{1 + \alpha} \left[ \hat{r}_t + E_t \ln \frac{\hat{v}_{t+1}^C}{\hat{v}_t^C} \right]. \quad (47)$$

The persistence of the cyclical component of consumption is increasing in habit persistence parameter  $\alpha$ , while the sensitivity of the cyclical component of consumption to changes in the

cyclical component of the real interest rate is increasing in intertemporal elasticity of substitution parameter  $\sigma$  and decreasing in habit persistence parameter  $\alpha$ . This approximate linear consumption Euler equation is subject to preference shocks.

The cyclical component of investment depends on a linear combination of past and expected future cyclical components of investment driven by the contemporaneous cyclical component of the relative shadow price of capital according to approximate linear investment demand function:

$$\ln \hat{I}_t = \frac{1}{1+\beta} \ln \hat{I}_{t-1} + \frac{\beta}{1+\beta} E_t \ln \hat{I}_{t+1} + \frac{1}{\chi(1+\beta)} \ln \left( \hat{v}_t^I \frac{\hat{Q}_t}{\hat{P}_t} \right). \quad (48)$$

The sensitivity of the cyclical component of investment to changes in the cyclical component of the relative shadow price of capital is decreasing in investment adjustment cost parameter  $\chi$ . This approximate linear investment demand function is subject to investment technology shocks.

The cyclical component of the ratio of nominal government consumption to nominal output satisfies fiscal expenditure rule:

$$\ln \frac{\hat{P}_t \hat{G}_t}{\hat{P}_t \hat{Y}_t} = \hat{v}_t^G. \quad (49)$$

This fiscal expenditure rule supports convergence of the level of the ratio of net government debt to nominal output to its target value, and is subject to fiscal expenditure shocks.

The cyclical component of the real wage depends on a linear combination of past and expected future cyclical components of the real wage driven by the contemporaneous cyclical component of the deviation of the marginal rate of substitution between leisure and consumption from the after tax real wage according to wage Phillips curve:

$$\begin{aligned} \ln \frac{\hat{W}_t}{\hat{P}_t} = & \frac{1}{1+\beta} \ln \frac{\hat{W}_{t-1}}{\hat{P}_{t-1}} + \frac{\beta}{1+\beta} E_t \ln \frac{\hat{W}_{t+1}}{\hat{P}_{t+1}} + \frac{\gamma^L}{1+\beta} \hat{\pi}_{t-1}^P - \frac{1+\gamma^L \beta}{1+\beta} \hat{\pi}_t^P + \frac{\beta}{1+\beta} E_t \hat{\pi}_{t+1}^P \\ & + \frac{(1-\omega^L)(1-\omega^L \beta)}{\omega^L(1+\beta)} \left[ \frac{1}{\eta} \ln \hat{L}_t + \frac{1}{\sigma} \frac{\ln \hat{C}_t - \alpha \ln \hat{C}_{t-1}}{1-\alpha} + \frac{\tau}{1-\tau} \ln \hat{\tau}_t - \ln \frac{\hat{W}_t}{\hat{P}_t} - \frac{1}{\theta^L - 1} \ln \hat{\theta}_t^L \right]. \end{aligned} \quad (50)$$

Reflecting the existence of partial wage indexation, the cyclical component of the real wage also depends on past, contemporaneous, and expected future cyclical components of inflation. The sensitivity of the cyclical component of the real wage to changes in the cyclical component of inflation is increasing in indexation parameter  $\gamma^L$ , to changes in the cyclical component of the deviation of the marginal rate of substitution between leisure and consumption from the after tax real wage is decreasing in nominal rigidity parameter  $\omega^L$ , and to changes in the cyclical

component of employment is decreasing in elasticity of substitution parameter  $\eta$ . This wage Phillips curve is subject to wage markup shocks.

The cyclical component of real marginal cost depends on the contemporaneous cyclical component of the deviation of the after tax real wage from the marginal product of labour according to approximate linear implicit labour demand function:

$$\ln \hat{\Phi}_t = \ln \frac{\hat{W}_t}{\hat{P}_t \hat{A}_t} - \frac{\tau}{1-\tau} \ln \hat{\tau}_t - \frac{1}{\mathcal{G}} \left( 1 - \frac{\theta^y}{\theta^y - 1} \frac{WL}{PY} \right) \ln \frac{\hat{u}_t \hat{K}_t}{\hat{A}_t \hat{L}_t}. \quad (51)$$

The sensitivity of the cyclical component of real marginal cost to changes in the cyclical component of the ratio of utilized capital to effective labour is decreasing in elasticity of substitution parameter  $\mathcal{G}$ . This approximate linear implicit labour demand function is subject to output technology shocks.

The cyclical component of the relative shadow price of capital depends on the expected future cyclical component of the relative shadow price of capital, the contemporaneous cyclical component of the real interest rate, the expected future cyclical component of real marginal cost, and the expected future cyclical component of the marginal product of capital according to approximate linear investment Euler equation:

$$\begin{aligned} \ln \frac{\hat{Q}_t}{\hat{P}_t} &= \beta(1-\delta) E_t \ln \frac{\hat{Q}_{t+1}}{\hat{P}_{t+1}} - \hat{r}_t \\ &+ [1 - \beta(1-\delta)] E_t \ln \hat{\Phi}_{t+1} - \frac{1 - \beta(1-\delta)}{\mathcal{G}} \frac{\theta^y}{\theta^y - 1} \frac{WL}{PY} E_t \ln \frac{\hat{u}_{t+1} \hat{K}_{t+1}}{\hat{A}_{t+1} \hat{L}_{t+1}}. \end{aligned} \quad (52)$$

The sensitivity of the cyclical component of the relative shadow price of capital to changes in the cyclical component of the ratio of utilized capital to effective labour is decreasing in elasticity of substitution parameter  $\mathcal{G}$ . This approximate linear investment Euler equation is subject to output technology shocks.

The cyclical component of the rate of capital utilization depends on the contemporaneous cyclical component of the ratio of capital to effective labour according to approximate linear implicit capital utilization function:

$$\ln \hat{u}_t = - \frac{\theta^y}{\theta^y - 1} \frac{WL}{PY} \left( \kappa \mathcal{G} + \frac{\theta^y}{\theta^y - 1} \frac{WL}{PY} \right)^{-1} \ln \frac{\hat{K}_t}{\hat{A}_t \hat{L}_t}. \quad (53)$$

The sensitivity of the cyclical component of the rate of capital utilization to changes in the cyclical component of the ratio of capital to effective labour is decreasing in capital utilization

cost parameter  $\kappa$  and elasticity of substitution parameter  $\vartheta$ . This approximate linear implicit capital utilization function is subject to output technology shocks.

The cyclical component of the capital stock depends on the past cyclical component of the capital stock and the contemporaneous cyclical component of investment according to approximate linear capital accumulation function

$$\ln \hat{K}_{t+1} = (1 - \delta) \ln \hat{K}_t + \delta \ln(\hat{v}_t^I \hat{I}_t), \quad (54)$$

where  $\frac{I}{K} = \delta$ . This approximate linear capital accumulation function is subject to investment technology shocks.

The cyclical component of the nominal interest rate depends on the contemporaneous cyclical components of inflation and output according to monetary policy rule:

$$\hat{i}_t = \xi^\pi \hat{\pi}_t^P + \xi^Y \ln \hat{Y}_t + \hat{v}_t^i. \quad (55)$$

This monetary policy rule ensures convergence of the level of inflation to its target value, and is subject to monetary policy shocks. The cyclical component of the real interest rate satisfies  $\hat{r}_t = \hat{i}_t - E_t \hat{\pi}_{t+1}^P$ .

The cyclical component of the tax rate depends on the contemporaneous cyclical component of the ratio of net government debt to nominal output according to fiscal revenue rule:

$$\ln \hat{\tau}_t = \zeta^\tau \ln \left( -\frac{\hat{B}_{t+1}^G}{\hat{P}_t \hat{Y}_t} \right) + \hat{v}_t^\tau. \quad (56)$$

This fiscal revenue rule ensures convergence of the level of the ratio of net government debt to nominal output to its target value, and is subject to fiscal revenue shocks.

The cyclical component of output depends on the contemporaneous cyclical components of consumption, investment, and government consumption according to approximate linear output market clearing condition

$$\ln \hat{Y}_t = \frac{C}{Y} \ln \hat{C}_t + \frac{I}{Y} \ln \hat{I}_t + \frac{G}{Y} \ln \hat{G}_t, \quad (57)$$

where  $\frac{C}{Y} + \frac{I}{Y} + \frac{G}{Y} = 1$ . In equilibrium, the cyclical component of output is determined by the cumulative demands of households, firms, and the government.

The cyclical component of the net government debt depends on the past cyclical component of the net government debt, the past cyclical component of the nominal interest rate, the contemporaneous cyclical component of tax revenues, and the contemporaneous cyclical

component of nominal government consumption according to approximate linear government dynamic budget constraint

$$\ln(-\hat{B}_{t+1}^G) = \frac{1}{\beta} \left[ \ln(-\hat{B}_t^G) + \hat{i}_{t-1} \right] + \left( \frac{B^G}{PY} \right)^{-1} \left[ \tau \ln(\hat{\tau}_t \hat{P}_t \hat{Y}_t) - \frac{G}{Y} \ln(\hat{P}_t \hat{G}_t) \right], \quad (58)$$

where  $\frac{B^G}{PY} = -\frac{\beta}{1-\beta} \left( \tau - \frac{G}{Y} \right)$ . This approximate linear government dynamic budget constraint is well defined only if the level of the net government debt is positive.

Variation in cyclical components is driven by eight exogenous stochastic processes. The cyclical components of the preference, output technology, investment technology, price markup, wage markup, monetary policy, fiscal expenditure, and fiscal revenue shocks follow stationary first order autoregressive processes:

$$\ln \hat{\nu}_t^C = \rho_{\nu^C} \ln \hat{\nu}_{t-1}^C + \varepsilon_t^{\nu^C}, \quad \varepsilon_t^{\nu^C} \sim \text{iid } \mathcal{N}(0, \sigma_{\nu^C}^2), \quad (59)$$

$$\ln \hat{A}_t = \rho_A \ln \hat{A}_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim \text{iid } \mathcal{N}(0, \sigma_A^2), \quad (60)$$

$$\ln \hat{\nu}_t^I = \rho_{\nu^I} \ln \hat{\nu}_{t-1}^I + \varepsilon_t^{\nu^I}, \quad \varepsilon_t^{\nu^I} \sim \text{iid } \mathcal{N}(0, \sigma_{\nu^I}^2), \quad (61)$$

$$\ln \hat{\theta}_t^Y = \rho_{\theta^Y} \ln \hat{\theta}_{t-1}^Y + \varepsilon_t^{\theta^Y}, \quad \varepsilon_t^{\theta^Y} \sim \text{iid } \mathcal{N}(0, \sigma_{\theta^Y}^2), \quad (62)$$

$$\ln \hat{\theta}_t^L = \rho_{\theta^L} \ln \hat{\theta}_{t-1}^L + \varepsilon_t^{\theta^L}, \quad \varepsilon_t^{\theta^L} \sim \text{iid } \mathcal{N}(0, \sigma_{\theta^L}^2), \quad (63)$$

$$\hat{\nu}_t^i = \rho_{\nu^i} \hat{\nu}_{t-1}^i + \varepsilon_t^{\nu^i}, \quad \varepsilon_t^{\nu^i} \sim \text{iid } \mathcal{N}(0, \sigma_{\nu^i}^2), \quad (64)$$

$$\hat{\nu}_t^G = \rho_{\nu^G} \hat{\nu}_{t-1}^G + \varepsilon_t^{\nu^G}, \quad \varepsilon_t^{\nu^G} \sim \text{iid } \mathcal{N}(0, \sigma_{\nu^G}^2), \quad (65)$$

$$\hat{\nu}_t^r = \rho_{\nu^r} \hat{\nu}_{t-1}^r + \varepsilon_t^{\nu^r}, \quad \varepsilon_t^{\nu^r} \sim \text{iid } \mathcal{N}(0, \sigma_{\nu^r}^2). \quad (66)$$

The innovations driving these exogenous stochastic processes are assumed to be independent, which combined with our distributional assumptions implies multivariate normality. In deterministic steady state equilibrium,  $\nu^C = \nu^I = 1$ .

### 2.5.2. Trend Components

The trend components of output, consumption, investment, and government consumption follow random walks with time varying drift  $g_t + n_t$ :

$$\ln \bar{Y}_t = g_t + n_t + \ln \bar{Y}_{t-1} + \varepsilon_t^{\bar{Y}}, \quad \varepsilon_t^{\bar{Y}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{Y}}^2), \quad (67)$$

$$\ln \bar{C}_t = g_t + n_t + \ln \bar{C}_{t-1} + \varepsilon_t^{\bar{C}}, \quad \varepsilon_t^{\bar{C}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{C}}^2), \quad (68)$$

$$\ln \bar{I}_t = g_t + n_t + \ln \bar{I}_{t-1} + \varepsilon_t^{\bar{I}}, \quad \varepsilon_t^{\bar{I}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{I}}^2), \quad (69)$$

$$\ln \bar{G}_t = g_t + n_t + \ln \bar{G}_{t-1} + \varepsilon_t^{\bar{G}}, \quad \varepsilon_t^{\bar{G}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{G}}^2). \quad (70)$$

It follows that the trend components of the ratios of consumption, investment, and government consumption to output follow random walks without drifts. This implies that along a balanced growth path, the levels of these great ratios are constant but state dependent.

The trend component of the price level follows a random walk with time varying drift  $\pi_t$ , the trend component of the nominal wage follows a random walk with time varying drift  $\pi_t + g_t$ , and the trend component of employment follows a random walk with time varying drift  $n_t$ :

$$\ln \bar{P}_t = \pi_t + \ln \bar{P}_{t-1} + \varepsilon_t^{\bar{P}}, \quad \varepsilon_t^{\bar{P}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{P}}^2), \quad (71)$$

$$\ln \bar{W}_t = \pi_t + g_t + \ln \bar{W}_{t-1} + \varepsilon_t^{\bar{W}}, \quad \varepsilon_t^{\bar{W}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{W}}^2), \quad (72)$$

$$\ln \bar{L}_t = n_t + \ln \bar{L}_{t-1} + \varepsilon_t^{\bar{L}}, \quad \varepsilon_t^{\bar{L}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{L}}^2). \quad (73)$$

It follows that the trend component of the income share of labour follows a random walk without drift. This implies that along a balanced growth path, the level of the income share of labour is constant but state dependent. The trend component of real marginal cost satisfies  $\ln \bar{\Phi}_t = \ln \Phi$ , while the trend component of the shadow price of capital satisfies  $\ln \bar{Q}_t = \ln \bar{P}_t$ . The trend component of the rate of capital utilization satisfies  $\ln \bar{u}_t = 0$ , while the trend component of the capital stock satisfies  $\ln \frac{\bar{K}_{t+1}}{\bar{Y}_t} = \ln \frac{K}{Y}$ .

The trend components of the nominal interest rate and tax rate follow random walks without drifts:

$$\bar{i}_t = \bar{i}_{t-1} + \varepsilon_t^{\bar{i}}, \quad \varepsilon_t^{\bar{i}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{i}}^2), \quad (74)$$

$$\ln \bar{\tau}_t = \ln \bar{\tau}_{t-1} + \varepsilon_t^{\bar{\tau}}, \quad \varepsilon_t^{\bar{\tau}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{\tau}}^2). \quad (75)$$

It follows that along a balanced growth path, the levels of the nominal interest rate and tax rate are constant but state dependent. The trend component of the real interest rate satisfies  $\bar{r}_t = \bar{i}_t - E_t \bar{\pi}_{t+1}^P$ , while the trend component of the net government debt satisfies  $\ln \left( -\frac{\bar{B}_{t+1}^G}{\bar{P}_t \bar{Y}_t} \right) = \ln \left( -\frac{B^G}{PY} \right)$ .

Long run balanced growth is driven by three common stochastic trends. Trend inflation, productivity growth, and population growth follow random walks without drifts:

$$\pi_t = \pi_{t-1} + \varepsilon_t^\pi, \quad \varepsilon_t^\pi \sim \text{iid } \mathcal{N}(0, \sigma_\pi^2), \quad (76)$$

$$g_t = g_{t-1} + \varepsilon_t^g, \quad \varepsilon_t^g \sim \text{iid } \mathcal{N}(0, \sigma_g^2), \quad (77)$$

$$n_t = n_{t-1} + \varepsilon_t^n, \quad \varepsilon_t^n \sim \text{iid } \mathcal{N}(0, \sigma_n^2). \quad (78)$$

All innovations driving variation in trend components are assumed to be independent, which combined with our distributional assumptions implies multivariate normality.

### 3. Estimation, Inference and Forecasting

Unobserved components models feature prominently in the empirical macroeconomics literature, while DSGE models are pervasive in the theoretical macroeconomics literature. The primary contribution of this paper is the joint modeling of cyclical and trend components as unobserved components while imposing theoretical restrictions derived from the approximate multivariate linear rational expectations representation of a DSGE model.

This merging of modeling paradigms drawn from the theoretical and empirical macroeconomics literatures confers a number of important benefits. First, the joint estimation of parameters and trend components ensures their mutual consistency, as estimates of parameters appropriately reflect estimates of trend components, and vice versa. As shown by Nelson and Kang (1981) and Harvey and Jaeger (1993), decomposing integrated observed nonpredetermined endogenous variables into cyclical and trend components with atheoretic deterministic polynomial functions or linear filters may induce spurious cyclical dynamics, invalidating subsequent estimation, inference and forecasting. Second, jointly modeling cyclical and trend components as unobserved components ensures stochastic nonsingularity of the resulting approximate linear state space representation of the DSGE model, as associated with each observed nonpredetermined endogenous variable is at least one exogenous stochastic process. As discussed in Ruge-Murcia (2003), stochastic nonsingularity requires that the number of observed nonpredetermined endogenous variables employed in full information maximum likelihood estimation of the approximate linear state space representation of a DSGE model not exceed the number of exogenous stochastic processes, with efficiency losses incurred if this constraint binds. Third, and of perhaps greatest practical importance, jointly modeling cyclical and trend components as unobserved components while ensuring the existence of a well defined

balanced growth path facilitates the generation of forecasts of the levels of nonpredetermined endogenous variables as opposed to their cyclical components, while ensuring that these forecasts satisfy the stability restrictions associated with balanced growth. These stability restrictions are necessary but not sufficient for full cointegration, as along a balanced growth path, great ratios are constant but state dependent, robustifying forecasts to intermittent structural breaks that occur within sample.

### 3.1. Estimation

The traditional econometric interpretation of macroeconomic models regards them as representations of the joint probability distribution of the data. Adopting this traditional econometric interpretation, Bayesian full information maximum likelihood estimation of a linear state space representation of an approximate unobserved components representation of this DSGE model of a closed economy, conditional on prior information concerning the values of parameters and trend components, facilitates an empirical evaluation of its impulse response and predictive accuracy properties.

#### 3.1.1. Estimation Procedure

Let  $\mathbf{x}_t$  denote a vector stochastic process consisting of the levels of  $N$  nonpredetermined endogenous variables, of which  $M$  are observed. The cyclical components of this vector stochastic process satisfy second order stochastic linear difference equation

$$\mathbf{A}_0 \hat{\mathbf{x}}_t = \mathbf{A}_1 \hat{\mathbf{x}}_{t-1} + \mathbf{A}_2 \mathbf{E}_t \hat{\mathbf{x}}_{t+1} + \mathbf{A}_3 \hat{\mathbf{v}}_t, \quad (79)$$

where vector stochastic process  $\hat{\mathbf{v}}_t$  consists of the cyclical components of  $K$  exogenous variables. This vector stochastic process satisfies stationary first order stochastic linear difference equation

$$\hat{\mathbf{v}}_t = \mathbf{B}_1 \hat{\mathbf{v}}_{t-1} + \boldsymbol{\varepsilon}_{1,t}, \quad (80)$$

where  $\boldsymbol{\varepsilon}_{1,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_1)$ . The trend components of vector stochastic process  $\mathbf{x}_t$  satisfy first order stochastic linear difference equation

$$\mathbf{C}_0 \bar{\mathbf{x}}_t = \mathbf{C}_1 + \mathbf{C}_2 \mathbf{u}_t + \mathbf{C}_3 \bar{\mathbf{x}}_{t-1} + \boldsymbol{\varepsilon}_{2,t}, \quad (81)$$



where  $\boldsymbol{\varepsilon}_{2,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_2)$ . Vector stochastic process  $\mathbf{u}_t$  consists of the levels of  $L$  common stochastic trends, and satisfies nonstationary first order stochastic linear difference equation

$$\mathbf{u}_t = \mathbf{u}_{t-1} + \boldsymbol{\varepsilon}_{3,t}, \quad (82)$$

where  $\boldsymbol{\varepsilon}_{3,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_3)$ . Cyclical and trend components are additively separable, that is  $\mathbf{x}_t = \hat{\mathbf{x}}_t + \bar{\mathbf{x}}_t$ .

If there exists a unique stationary solution to multivariate linear rational expectations model (79), then it may be expressed as:

$$\hat{\mathbf{x}}_t = \mathbf{D}_1 \hat{\mathbf{x}}_{t-1} + \mathbf{D}_2 \hat{\mathbf{v}}_t. \quad (83)$$

Consider the following real generalized Schur decomposition, where stable generalized eigenvalues are ordered first:

$$\begin{bmatrix} \mathbf{Q}_{1,1} & \mathbf{Q}_{1,2} \\ \mathbf{Q}_{2,1} & \mathbf{Q}_{2,2} \end{bmatrix} \begin{bmatrix} \mathbf{I}_N & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{Z}_{1,1} & \mathbf{Z}_{1,2} \\ \mathbf{Z}_{2,1} & \mathbf{Z}_{2,2} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{1,1} & \mathbf{S}_{1,2} \\ \mathbf{0} & \mathbf{S}_{2,2} \end{bmatrix}, \quad (84)$$

$$\begin{bmatrix} \mathbf{Q}_{1,1} & \mathbf{Q}_{1,2} \\ \mathbf{Q}_{2,1} & \mathbf{Q}_{2,2} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{I}_N \\ -\mathbf{A}_1 & \mathbf{A}_0 \end{bmatrix} \begin{bmatrix} \mathbf{Z}_{1,1} & \mathbf{Z}_{1,2} \\ \mathbf{Z}_{2,1} & \mathbf{Z}_{2,2} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{1,1} & \mathbf{T}_{1,2} \\ \mathbf{0} & \mathbf{T}_{2,2} \end{bmatrix}. \quad (85)$$

Following Klein (2000), matrices  $\mathbf{D}_1$  and  $\mathbf{D}_2$  may be expressed in terms of the results of this ordered real generalized Schur decomposition as

$$\mathbf{D}_1 = \mathbf{Z}_{2,1} \mathbf{Z}_{1,1}^{-1}, \quad (86)$$

$$\mathbf{D}_2 = (\mathbf{Z}_{2,2} - \mathbf{Z}_{2,1} \mathbf{Z}_{1,1}^{-1} \mathbf{Z}_{1,2}) \mathbf{R}, \quad (87)$$

where  $\text{vec}(\mathbf{R}) = (\mathbf{I}_K \otimes \mathbf{T}_{2,2} - \mathbf{B}_1 \otimes \mathbf{S}_{2,2})^{-1} \text{vec}(\mathbf{Q}_{2,2} \mathbf{A}_3)$ . This unique stationary solution exists if the number of unstable generalized eigenvalues equals  $N$ .

Let  $\mathbf{y}_t$  denote a vector stochastic process consisting of the levels of  $M$  observed nonpredetermined endogenous variables. Also, let  $\mathbf{z}_t$  denote a vector stochastic process consisting of the levels of  $N-M$  unobserved nonpredetermined endogenous variables, the cyclical components of  $N$  nonpredetermined endogenous variables, the trend components of  $N$  nonpredetermined endogenous variables, the cyclical components of  $K$  exogenous variables, and the levels of  $L$  common stochastic trends. Given unique stationary solution (83), these vector stochastic processes have linear state space representation

$$\mathbf{y}_t = \mathbf{F}_1 \mathbf{z}_t, \quad (88)$$

$$\mathbf{z}_t = \mathbf{G}_1 + \mathbf{G}_2 \mathbf{z}_{t-1} + \mathbf{G}_3 \boldsymbol{\varepsilon}_{4,t}, \quad (89)$$

where  $\boldsymbol{\varepsilon}_{4,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_4)$  and  $\mathbf{z}_0 \sim \mathcal{N}(\mathbf{z}_{0|0}, \mathbf{P}_{0|0})$ . Let  $\mathbf{w}_t$  denote a vector stochastic process consisting of preliminary estimates of the trend components of  $M$  observed nonpredetermined endogenous variables. Suppose that this vector stochastic process satisfies

$$\mathbf{w}_t = \mathbf{H}_1 \mathbf{z}_t + \boldsymbol{\varepsilon}_{5,t}, \quad (90)$$

where  $\boldsymbol{\varepsilon}_{5,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_5)$ . Conditional on known parameter values, this signal equation defines a set of stochastic restrictions on selected unobserved state variables. The signal and state innovation vectors are assumed to be independent, while the initial state vector is assumed to be independent from the signal and state innovation vectors, which combined with our distributional assumptions implies multivariate normality.

Conditional on the parameters associated with these signal and state equations, estimates of unobserved state vector  $\mathbf{z}_t$  and its mean squared error matrix  $\mathbf{P}_t$  may be calculated with the filter proposed by Vitek (2006a, 2006b), which adapts the filter due to Kalman (1960) to incorporate prior information. Given initial conditions  $\mathbf{z}_{0|0}$  and  $\mathbf{P}_{0|0}$ , estimates conditional on information available at time  $t-1$  satisfy prediction equations:

$$\mathbf{z}_{t|t-1} = \mathbf{G}_1 + \mathbf{G}_2 \mathbf{z}_{t-1|t-1}, \quad (91)$$

$$\mathbf{P}_{t|t-1} = \mathbf{G}_2 \mathbf{P}_{t-1|t-1} \mathbf{G}_2^\top + \mathbf{G}_3 \boldsymbol{\Sigma}_4 \mathbf{G}_3^\top, \quad (92)$$

$$\mathbf{y}_{t|t-1} = \mathbf{F}_1 \mathbf{z}_{t|t-1}, \quad (93)$$

$$\mathbf{Q}_{t|t-1} = \mathbf{F}_1 \mathbf{P}_{t|t-1} \mathbf{F}_1^\top, \quad (94)$$

$$\mathbf{w}_{t|t-1} = \mathbf{H}_1 \mathbf{z}_{t|t-1}, \quad (95)$$

$$\mathbf{R}_{t|t-1} = \mathbf{H}_1 \mathbf{P}_{t|t-1} \mathbf{H}_1^\top + \boldsymbol{\Sigma}_5. \quad (96)$$

Given these predictions, under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, estimates conditional on information available at time  $t$  satisfy updating equations

$$\mathbf{z}_{t|t} = \mathbf{z}_{t|t-1} + \mathbf{K}_{y_t} (\mathbf{y}_t - \mathbf{y}_{t|t-1}) + \mathbf{K}_{w_t} (\mathbf{w}_t - \mathbf{w}_{t|t-1}), \quad (97)$$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{K}_{y_t} \mathbf{F}_1 \mathbf{P}_{t|t-1} - \mathbf{K}_{w_t} \mathbf{H}_1 \mathbf{P}_{t|t-1}, \quad (98)$$

where  $\mathbf{K}_{y_t} = \mathbf{P}_{t|t-1} \mathbf{F}_1^\top \mathbf{Q}_{t|t-1}^{-1}$  and  $\mathbf{K}_{w_t} = \mathbf{P}_{t|t-1} \mathbf{H}_1^\top \mathbf{R}_{t|t-1}^{-1}$ . Given terminal conditions  $\mathbf{z}_{T|T}$  and  $\mathbf{P}_{T|T}$  obtained from the final evaluation of these prediction and updating equations, estimates conditional on information available at time  $T$  satisfy smoothing equations

$$\mathbf{z}_{t|T} = \mathbf{z}_{t|t} + \mathbf{J}_t (\mathbf{z}_{t+1|T} - \mathbf{z}_{t+1|t}), \quad (99)$$

$$\mathbf{P}_{t|T} = \mathbf{P}_{t|t} + \mathbf{J}_t (\mathbf{P}_{t+1|T} - \mathbf{P}_{t+1|t}) \mathbf{J}_t^\top, \quad (100)$$

where  $\mathbf{J}_t = \mathbf{P}_{t|t} \mathbf{G}_2^\top \mathbf{P}_{t+1|t}^{-1}$ . Under our distributional assumptions, these estimators of the unobserved state vector are mean squared error optimal.

Let  $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbb{R}^J$  denote a  $J$  dimensional vector containing the parameters associated with the signal and state equations of this linear state space model. The Bayesian full information maximum likelihood estimator of this parameter vector has posterior density function

$$f(\boldsymbol{\theta} | \mathcal{I}_T) \propto f(\mathcal{I}_T | \boldsymbol{\theta}) f(\boldsymbol{\theta}), \quad (101)$$

where  $\mathcal{I}_t = \{\{\mathbf{y}_s\}_{s=1}^t, \{\mathbf{w}_s\}_{s=1}^t\}$ . Under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, conditional density function  $f(\mathcal{I}_T | \boldsymbol{\theta})$  satisfies:

$$f(\mathcal{I}_T | \boldsymbol{\theta}) = \prod_{t=1}^T f(\mathbf{y}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta}) \cdot \prod_{t=1}^T f(\mathbf{w}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta}). \quad (102)$$

Under our distributional assumptions, conditional density functions  $f(\mathbf{y}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta})$  and  $f(\mathbf{w}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta})$  satisfy:

$$f(\mathbf{y}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta}) = (2\pi)^{-\frac{M}{2}} |\mathbf{Q}_{t|t-1}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\mathbf{y}_t - \mathbf{y}_{t|t-1})^\top \mathbf{Q}_{t|t-1}^{-1} (\mathbf{y}_t - \mathbf{y}_{t|t-1})\right\}, \quad (103)$$

$$f(\mathbf{w}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta}) = (2\pi)^{-\frac{M}{2}} |\mathbf{R}_{t|t-1}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\mathbf{w}_t - \mathbf{w}_{t|t-1})^\top \mathbf{R}_{t|t-1}^{-1} (\mathbf{w}_t - \mathbf{w}_{t|t-1})\right\}. \quad (104)$$

Prior information concerning parameter vector  $\boldsymbol{\theta}$  is summarized by a multivariate normal prior distribution having mean vector  $\boldsymbol{\theta}_1$  and covariance matrix  $\boldsymbol{\Omega}$ :

$$f(\boldsymbol{\theta}) = (2\pi)^{-\frac{J}{2}} |\boldsymbol{\Omega}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\theta}_1)^\top \boldsymbol{\Omega}^{-1} (\boldsymbol{\theta} - \boldsymbol{\theta}_1)\right\}. \quad (105)$$

Independent priors are represented by a diagonal covariance matrix, under which diffuse priors are represented by infinite variances.

Inference on the parameters is based on an asymptotic normal approximation to the posterior distribution around its mode. Under regularity conditions stated in Geweke (2005), posterior mode  $\hat{\boldsymbol{\theta}}_T$  satisfies

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{d} \mathcal{N}(\mathbf{0}, -\mathcal{H}_0^{-1}), \quad (106)$$

where  $\boldsymbol{\theta}_0 \in \boldsymbol{\Theta}$  denotes the pseudotrue parameter vector. Following Engle and Watson (1981), Hessian  $\mathcal{H}_0$  may be estimated by

$$\begin{aligned} \hat{\mathcal{H}}_T &= \frac{1}{T} \sum_{t=1}^T \mathbb{E}_{t-1} \left[ \nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^{\top} \ln f(\mathbf{y}_t | \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T) \right] + \frac{1}{T} \sum_{t=1}^T \mathbb{E}_{t-1} \left[ \nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^{\top} \ln f(\mathbf{w}_t | \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T) \right] \\ &\quad + \frac{1}{T} \nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^{\top} \ln f(\hat{\boldsymbol{\theta}}_T), \end{aligned} \quad (107)$$

where  $\mathbb{E}_{t-1} \left[ \nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^{\top} \ln f(\mathbf{y}_t | \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T) \right] = -\nabla_{\boldsymbol{\theta}} \mathbf{y}_{t|t-1}^{\top} \mathbf{Q}_{t|t-1}^{-1} \nabla_{\boldsymbol{\theta}} \mathbf{y}_{t|t-1} - \frac{1}{2} \nabla_{\boldsymbol{\theta}} \mathbf{Q}_{t|t-1}^{\top} (\mathbf{Q}_{t|t-1}^{-1} \otimes \mathbf{Q}_{t|t-1}^{-1}) \nabla_{\boldsymbol{\theta}} \mathbf{Q}_{t|t-1}$ ,  
 $\mathbb{E}_{t-1} \left[ \nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^{\top} \ln f(\mathbf{w}_t | \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T) \right] = -\nabla_{\boldsymbol{\theta}} \mathbf{w}_{t|t-1}^{\top} \mathbf{R}_{t|t-1}^{-1} \nabla_{\boldsymbol{\theta}} \mathbf{w}_{t|t-1} - \frac{1}{2} \nabla_{\boldsymbol{\theta}} \mathbf{R}_{t|t-1}^{\top} (\mathbf{R}_{t|t-1}^{-1} \otimes \mathbf{R}_{t|t-1}^{-1}) \nabla_{\boldsymbol{\theta}} \mathbf{R}_{t|t-1}$ , and  
 $\nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^{\top} \ln f(\hat{\boldsymbol{\theta}}_T) = -\boldsymbol{\Omega}^{-1}$ .

### 3.1.2. Estimation Results

The set of parameters associated with this DSGE model of a closed economy is partitioned into two subsets. The first subset is calibrated to approximately match long run averages of functions of observed nonpredetermined endogenous variables where possible, and estimates derived from existing microeconomic studies where necessary. The second subset is estimated by Bayesian full information maximum likelihood, conditional on prior information concerning the values of parameters and trend components.

Subjective discount factor  $\beta$  is restricted to equal 0.99, implying an annualized deterministic steady state equilibrium real interest rate of approximately 0.04. In deterministic steady state equilibrium, the output price markup  $\frac{\theta^y}{\theta^y - 1}$  and wage markup  $\frac{\theta^l}{\theta^l - 1}$  are restricted to equal 1.15. Depreciation rate parameter  $\delta$  is restricted to equal 0.015, implying an annualized deterministic steady state equilibrium depreciation rate of approximately 0.06. The deterministic steady state equilibrium income share of labour  $\frac{WL}{PY}$  is restricted to equal 0.50. In deterministic steady state equilibrium, the ratio of government consumption to output  $\frac{G}{Y}$  is restricted to equal 0.20, while the tax rate  $\tau$  is restricted to equal 0.22.

Table 1. Deterministic steady state equilibrium values of great ratios

| Ratio | Value  | Ratio    | Value   |
|-------|--------|----------|---------|
| $C/Y$ | 0.6277 | $WL/PY$  | 0.5000  |
| $I/Y$ | 0.1723 | $K/Y$    | 2.8710  |
| $G/Y$ | 0.2000 | $B^G/PY$ | -0.4950 |

*Note:* Deterministic steady state equilibrium values are reported at an annual frequency based on calibrated parameter values.

Bayesian full information maximum likelihood estimation of the remaining parameters of this DSGE model of a closed economy is based on the levels of nine observed nonpredetermined endogenous variables for the United States described in Appendix A. Those parameters associated exclusively with the conditional variance function are estimated conditional on diffuse priors. Initial conditions for the cyclical components of exogenous variables are given by their unconditional means and variances, while the initial values of all other state variables are treated as parameters, and are calibrated to match functions of preliminary estimates of trend components calculated with the linear filter described in Hodrick and Prescott (1997). The posterior mode is calculated by numerically maximizing the logarithm of the posterior density kernel with a modified steepest ascent algorithm. Estimation results pertaining to the period 1964Q3 through 2005Q3 are reported in Appendix B. The sufficient condition for the existence of a unique stationary rational expectations equilibrium due to Klein (2000) is satisfied in a neighbourhood around the posterior mode, while the Hessian is not nearly singular at the posterior mode, suggesting that the approximate linear state space representation of this DSGE model of a closed economy is locally identified.

The prior mean of indexation parameter  $\gamma^Y$  is 0.75, implying considerable output price inflation inertia, while the prior mean of nominal rigidity parameter  $\omega^Y$  implies an average duration of output price contracts of two years. The prior mean of capital utilization cost parameter  $\kappa$  is 0.10, while the prior mean of elasticity of substitution parameter  $\varrho$  is 0.75, implying that utilized capital and effective labour are moderately close complements in production. The prior mean of habit persistence parameter  $\alpha$  is 0.95, while the prior mean of intertemporal elasticity of substitution parameter  $\sigma$  is 2.75, implying that consumption exhibits considerable persistence and moderate sensitivity to real interest rate changes. The prior mean of investment adjustment cost parameter  $\chi$  is 5.75, implying moderate sensitivity of investment to changes in the relative shadow price of capital. The prior mean of indexation parameter  $\gamma^L$  is 0.75, implying considerable sensitivity of the real wage to changes in inflation, while the prior mean of nominal rigidity parameter  $\omega^L$  implies an average duration of wage contracts of two years. The prior mean of elasticity of substitution parameter  $\eta$  is 2.00, implying considerable insensitivity of the real wage to changes in employment. The prior mean of the inflation

response coefficient  $\xi^\pi$  in the monetary policy rule is 1.50, while the prior mean of the output response coefficient  $\xi^y$  is 0.125, ensuring convergence of the level of inflation to its target value. The prior mean of the net government debt response coefficient  $\zeta^\tau$  in the fiscal revenue rule is 1.00, ensuring convergence of the level of the ratio of net government debt to nominal output to its target value. All autoregressive parameters  $\rho$  have prior means of 0.85, implying considerable persistence of shocks driving variation in cyclical components.

The posterior modes of these structural parameters are all close to their prior means, reflecting the imposition of tight independent priors to ensure the existence of a unique stationary rational expectations equilibrium. The estimated variances of shocks driving variation in cyclical components are all well within the range of estimates reported in the existing literature, after accounting for data rescaling. The estimated variances of shocks driving variation in trend components are relatively high, indicating that the majority of variation in the levels of observed nonpredetermined endogenous variables is accounted for by variation in trend components.

Prior information concerning the values of trend components is generated by fitting fourth order deterministic polynomial functions to the levels of all observed nonpredetermined endogenous variables by ordinary least squares. Stochastic restrictions on the trend components of all observed nonpredetermined endogenous variables are derived from the fitted values associated with these ordinary least squares regressions, with innovation variances set proportional to estimated prediction variances assuming known parameters. All stochastic restrictions are independent, represented by a diagonal covariance matrix, and are harmonized, represented by a common factor of proportionality. Reflecting moderate confidence in these preliminary trend component estimates, this common factor of proportionality is set equal to one.

Predicted, filtered and smoothed estimates of the cyclical and trend components of observed nonpredetermined endogenous variables are plotted together with confidence intervals in Appendix B. These confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. The predicted estimates are conditional on past information, the filtered estimates are conditional on past and present information, and the smoothed estimates are conditional on past, present and future information. Visual inspection reveals close agreement with the conventional dating of business cycle expansions and recessions.

Theoretical autocovariances are plotted together with confidence intervals versus empirical autocovariances for selected nonpredetermined endogenous variables in Appendix B. These confidence intervals assume a multivariate normally distributed state innovation vector and known parameters. Visual inspection reveals the existence of numerous statistically significant differences between the theoretical and empirical autocovariances. These differences to some extent reflect the atheoretic removal of trend components from observed nonpredetermined

endogenous variables with the linear filter described in Hodrick and Prescott (1997) prior to the calculation of empirical autocovariances.

### 3.2. Inference

Whether this estimated DSGE model approximately accounts for the empirical evidence concerning the monetary transmission mechanism in a closed economy is determined by comparing its impulse responses to a monetary policy shock with impulse responses derived from an estimated SVAR model.

#### 3.2.1. Empirical Impulse Response Analysis

Consider the following SVAR model of the monetary transmission mechanism in a closed economy

$$\mathbf{A}_0 \mathbf{y}_t = \boldsymbol{\mu}(t) + \sum_{i=1}^p \mathbf{A}_i \mathbf{y}_{t-i} + \mathbf{B} \boldsymbol{\varepsilon}_t, \quad (108)$$

where  $\boldsymbol{\mu}(t)$  denotes a fourth order deterministic polynomial function and  $\boldsymbol{\varepsilon}_t \sim \text{iid } \mathcal{N}(\mathbf{0}, \mathbf{I})$ . Vector stochastic process  $\mathbf{y}_t$  consists of inflation  $\pi_t^p$ , output  $\ln Y_t$ , consumption  $\ln C_t$ , investment  $\ln I_t$ , and nominal interest rate  $i_t$ . The diagonal elements of parameter matrix  $\mathbf{A}_0$  are normalized to one, while the off diagonal elements of positive definite parameter matrix  $\mathbf{B}$  are restricted to equal zero, thus associating with each equation a unique endogenous variable, and with each endogenous variable a unique structural innovation.

This SVAR model is identified by imposing restrictions on the timing of the effects of a monetary policy shock and on the information set of the monetary authority. In particular, prices and quantities are restricted to not respond instantaneously to a monetary policy shock, while the monetary authority can respond instantaneously to changes in these variables.

This SVAR model of the monetary transmission mechanism in a closed economy is estimated by full information maximum likelihood over the period 1964Q3 through 2005Q3. As discussed in Hamilton (1994), in the absence of model misspecification, this full information maximum likelihood estimator is consistent and asymptotically normal, irrespective of the cointegration rank and validity of the conditional multivariate normality assumption. The lag order is selected to minimize multivariate extensions of the model selection criterion functions of Akaike (1974), Schwarz (1978), and Hannan and Quinn (1979) subject to an upper bound equal

to the seasonal frequency. On the basis of the model selection criterion function due to Schwarz (1978), a lag order of one is selected.

Table 2. Model selection criterion function values

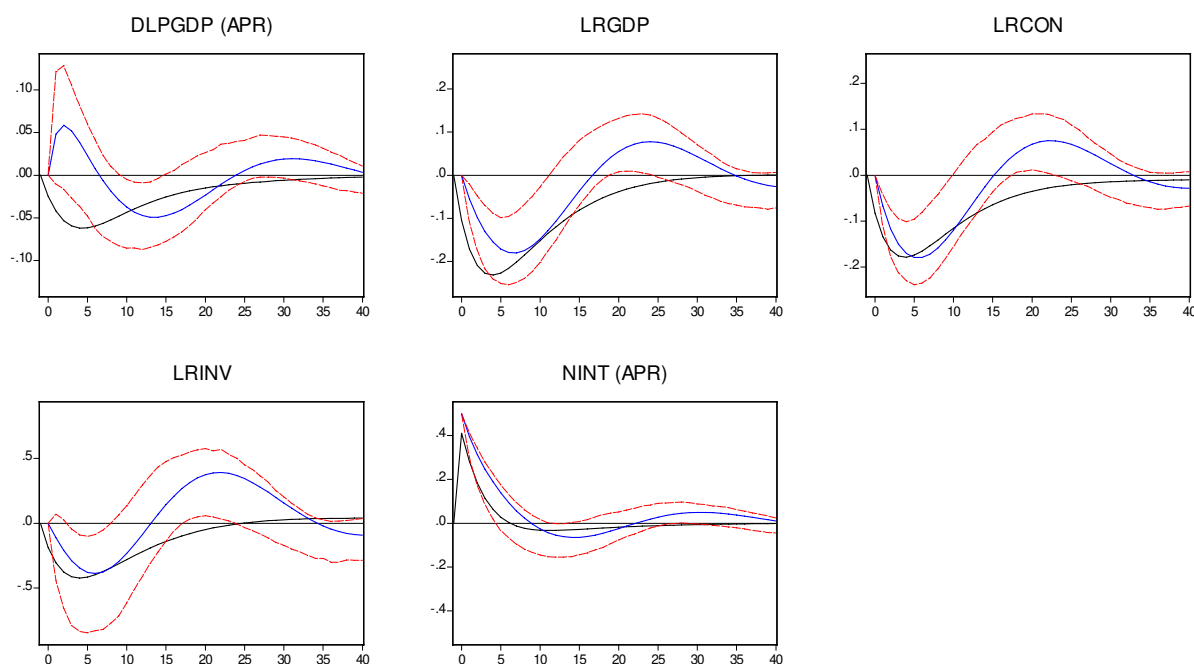
| $p$ | $AIC(p)$  | $SC(p)$   | $HQ(p)$   |
|-----|-----------|-----------|-----------|
| 1   | -37.9810  | -36.7422* | -37.4780  |
| 2   | -38.3645  | -36.6491  | -37.6680* |
| 3   | -38.3927* | -36.2009  | -37.5028  |
| 4   | -38.3500  | -35.6817  | -37.2666  |

Note: Minimized values of model selection criterion functions are indicated by \*.

Theoretical impulse responses to a monetary policy shock are plotted versus empirical impulse responses in Figure 1. Following a monetary policy shock, the nominal interest rate exhibits an immediate increase followed by a gradual decline. These nominal interest rate dynamics induce persistent and generally statistically significant hump shaped negative responses of inflation, output, consumption and investment, with peak effects realized after approximately one to two years. These results are qualitatively consistent with those of SVAR analyses of the monetary transmission mechanism in closed economies such as Sims and Zha (1995), Gordon and Leeper (1994), Leeper, Sims and Zha (1996), and Christiano, Eichenbaum and Evans (1998, 2005).



Figure 1. Theoretical versus empirical impulse responses to a monetary policy shock



*Note:* Theoretical impulse responses to a 50 basis point monetary policy shock are represented by black lines, while blue lines depict empirical impulse responses to a 50 basis point monetary policy shock. Asymmetric 95% confidence intervals are calculated with a nonparametric bootstrap simulation with 999 replications.

Visual inspection reveals that the theoretical impulse responses to a monetary policy shock generally lie within confidence intervals associated with the corresponding empirical impulse responses, suggesting that this estimated DSGE model approximately accounts for the empirical evidence concerning the monetary transmission mechanism in a closed economy. However, these confidence intervals are rather wide, indicating that considerable uncertainty surrounds this empirical evidence.

### 3.2.2. Theoretical Impulse Response Analysis

In a closed economy, business cycles are generated by interactions among a variety of nominal and real shocks. Theoretical impulse responses and forecast error variance decompositions to preference, output technology, investment technology, price markup, wage markup, monetary policy, fiscal expenditure, and fiscal revenue shocks are plotted in Appendix B.

Following an output technology shock, there arise persistent hump shaped positive responses of output, consumption, and investment. Inflation exhibits a persistent hump shaped decline in

response to a reduction in real marginal cost. The nominal and real interest rates exhibit persistent hump shaped declines in response to a reduction in inflation, mitigated by an increase in output.

Following a monetary policy shock, the nominal and real interest rates exhibit immediate increases followed by gradual declines, inducing persistent hump shaped negative responses of output, consumption, and investment. Inflation exhibits a persistent hump shaped decline in response to a reduction in real marginal cost.

Following a domestic fiscal expenditure shock, there arise immediate positive responses of output and government consumption, together with persistent hump shaped negative responses of consumption and investment. Inflation rises in response to an increase in real marginal cost. The nominal and real interest rates exhibit immediate increases followed by gradual declines.

### *3.3. Forecasting*

While it is desirable that forecasts be unbiased and efficient, the practical value of any forecasting model depends on its relative predictive accuracy. In the absence of a well defined mapping between forecast errors and their costs, relative predictive accuracy is generally assessed with mean squared prediction error based measures. As discussed in Clements and Hendry (1998), mean squared prediction error based measures are noninvariant to nonsingular, scale preserving linear transformations, even though linear models are. It follows that mean squared prediction error based comparisons may yield conflicting rankings across models, depending on the variable transformations examined.

To compare the dynamic out of sample forecasting performance of the DSGE and SVAR models, forty quarters of observations are retained to evaluate forecasts one through eight quarters ahead, generated conditional on parameters estimated using information available at the forecast origin. The models are compared on the basis of mean squared prediction errors in levels, ordinary differences, and seasonal differences. The DSGE model is not recursively estimated as the forecast origin rolls forward due to the high computational cost of such a procedure, while the SVAR model is. Presumably, recursively estimating the DSGE model would improve its predictive accuracy.

Mean squared prediction error differentials are plotted together with confidence intervals accounting for contemporaneous and serial correlation of forecast errors in Appendix B. If these mean squared prediction error differentials are negative then the forecasting performance of the DSGE model dominates that of the SVAR model, while if positive then the DSGE model is dominated by the SVAR model in terms of predictive accuracy. The null hypothesis of equal

squared prediction errors is rejected by the predictive accuracy test of Diebold and Mariano (1995) if and only if these confidence intervals exclude zero. The asymptotic variance of the average loss differential is estimated by a weighted sum of the autocovariances of the loss differential, employing the weighting function proposed by Newey and West (1987). Visual inspection reveals that these mean squared prediction error differentials are generally negative, suggesting that the DSGE model dominates the SVAR model in terms of forecasting performance, in spite of a considerable informational disadvantage. However, these mean squared prediction error differentials are rarely statistically significant at conventional levels, indicating that considerable uncertainty surrounds these predictive accuracy comparisons.

Dynamic out of sample forecasts of levels, ordinary differences, and seasonal differences are plotted together with confidence intervals versus realized outcomes in Appendix B. These confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Visual inspection reveals that the realized outcomes generally lie within their associated confidence intervals, suggesting that forecast failure is absent. However, these confidence intervals are rather wide, indicating that considerable uncertainty surrounds the point forecasts.

#### **4. Conclusion**

This paper develops and estimates a DSGE model of a closed economy which approximately accounts for the empirical evidence concerning the monetary transmission mechanism, as summarized by impulse response functions derived from an estimated SVAR model, while dominating that SVAR model in terms of predictive accuracy. Cyclical components are modeled by linearizing equilibrium conditions around a stationary deterministic steady state equilibrium which abstracts from long run balanced growth, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. This estimated DSGE model consolidates much existing theoretical and empirical knowledge concerning the monetary transmission mechanism in a closed economy, provides a framework for a progressive research strategy, and suggests partial explanations for its own deficiencies.

In an open economy, the monetary transmission mechanism features both interest rate and exchange rate channels, while the monetary authority must react to a variety of nominal and real shocks originating both domestically and abroad. The extension of this DSGE model of a closed economy to an open economy framework remains an objective for future research.

## **Acknowledgements**

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## **Appendix A. Description of the Data Set**

The data set consists of quarterly seasonally adjusted observations on nine macroeconomic variables for the United States over the period 1964Q1 through 2005Q3. All aggregate prices and quantities are expenditure based. Model consistent employment is derived from observed nominal labour income and a nominal wage index, while a model consistent tax rate is derived from observed nominal output and disposable income. The nominal interest rate is measured by the federal funds rate expressed as a period average. All data was extracted from the FRED database maintained by the Federal Reserve Bank of Saint Louis.

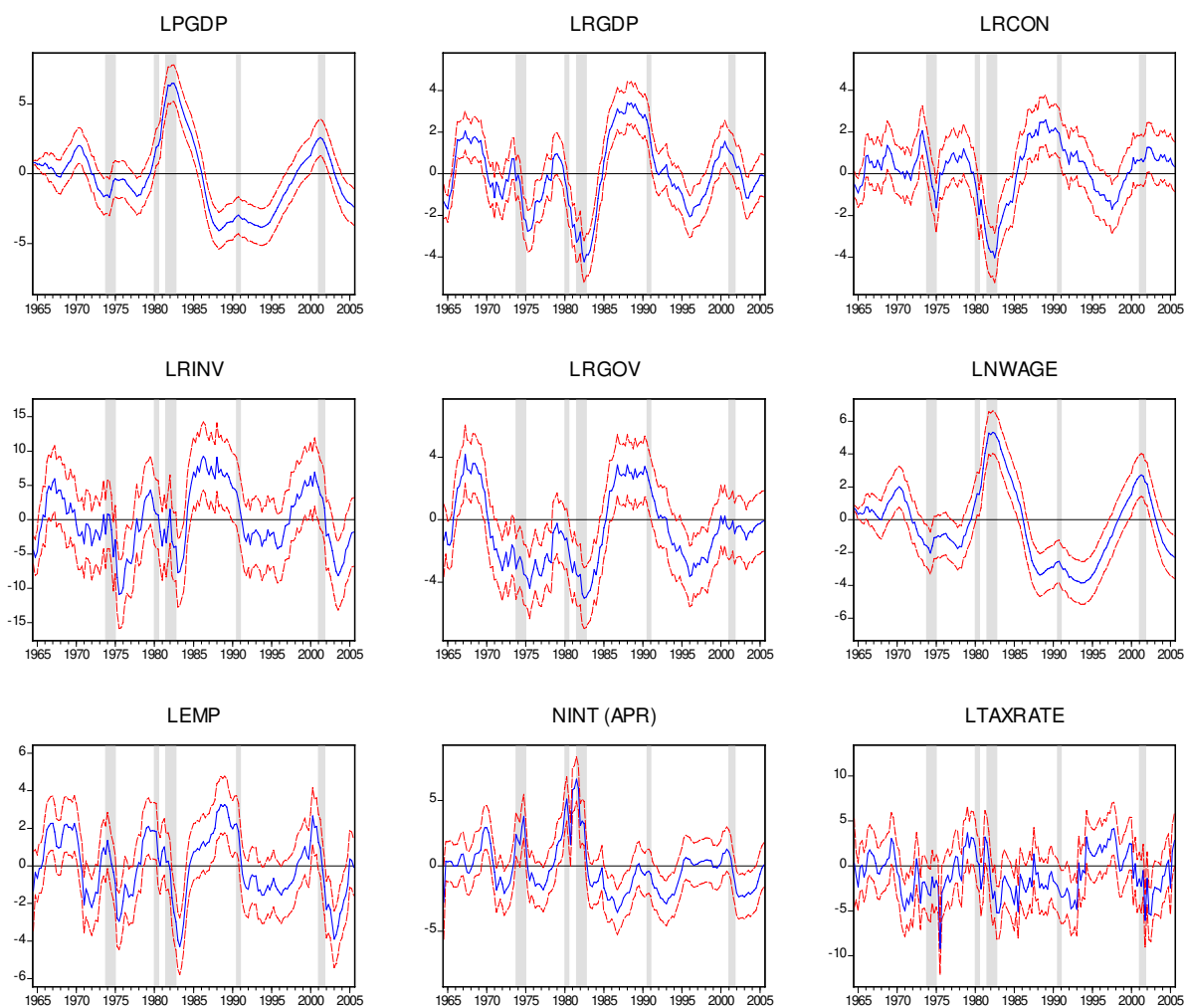
## Appendix B. Tables and Figures

Table 3. Bayesian full information maximum likelihood estimation results

| Parameter             | Prior Distribution |                | Posterior Distribution |                |
|-----------------------|--------------------|----------------|------------------------|----------------|
|                       | Mean               | Standard Error | Mode                   | Standard Error |
| $\alpha$              | 0.950000           | 0.004750       | 0.920650               | 0.004191       |
| $\chi$                | 5.750000           | 0.028750       | 5.734400               | 0.028746       |
| $\eta$                | 2.000000           | 0.010000       | 2.002200               | 0.009997       |
| $\kappa$              | 0.100000           | 0.000500       | 0.100010               | 0.000500       |
| $\sigma$              | 2.750000           | 0.013750       | 2.760200               | 0.013739       |
| $\vartheta$           | 0.750000           | 0.003750       | 0.750060               | 0.003750       |
| $\gamma^Y$            | 0.750000           | 0.003750       | 0.750070               | 0.003740       |
| $\gamma^L$            | 0.750000           | 0.003750       | 0.750110               | 0.003748       |
| $\omega^Y$            | 0.875000           | 0.004375       | 0.885070               | 0.003827       |
| $\omega^L$            | 0.875000           | 0.004375       | 0.877780               | 0.004293       |
| $\xi^\pi$             | 1.500000           | 0.007500       | 1.502600               | 0.007489       |
| $\xi^Y$               | 0.125000           | 0.000625       | 0.124550               | 0.000625       |
| $\xi^r$               | 1.000000           | 0.005000       | 0.996870               | 0.004998       |
| $\rho_{v^c}$          | 0.850000           | 0.004250       | 0.858400               | 0.004228       |
| $\rho_A$              | 0.850000           | 0.004250       | 0.849330               | 0.004125       |
| $\rho_{v^i}$          | 0.850000           | 0.004250       | 0.851330               | 0.004229       |
| $\rho_{\theta^p}$     | 0.850000           | 0.004250       | 0.850260               | 0.004250       |
| $\rho_{\theta^b}$     | 0.850000           | 0.004250       | 0.850260               | 0.004250       |
| $\rho_{v^j}$          | 0.850000           | 0.004250       | 0.858540               | 0.004076       |
| $\rho_{v^g}$          | 0.850000           | 0.004250       | 0.854440               | 0.004210       |
| $\rho_{v^f}$          | 0.850000           | 0.004250       | 0.852430               | 0.004196       |
| $\sigma_{v^c}^2$      | —                  | $\infty$       | 0.188090               | 0.063808       |
| $\sigma_A^2$          | —                  | $\infty$       | 0.325890               | 0.042217       |
| $\sigma_{v^i}^2$      | —                  | $\infty$       | 0.595370               | 0.146370       |
| $\sigma_{\theta^p}^2$ | —                  | $\infty$       | 0.249700               | 1.872700       |
| $\sigma_{\theta^b}^2$ | —                  | $\infty$       | 0.248840               | 3.732000       |
| $\sigma_{v^j}^2$      | —                  | $\infty$       | 0.044990               | 0.005345       |
| $\sigma_{v^g}^2$      | —                  | $\infty$       | 0.238210               | 0.033062       |
| $\sigma_{v^f}^2$      | —                  | $\infty$       | 2.122900               | 0.228260       |
| $\sigma_{\bar{P}}^2$  | —                  | $\infty$       | 0.230620               | 0.025737       |
| $\sigma_{\bar{Y}}^2$  | —                  | $\infty$       | 0.096784               | 0.011592       |
| $\sigma_{\bar{C}}^2$  | —                  | $\infty$       | 0.088447               | 0.009935       |
| $\sigma_{\bar{I}}^2$  | —                  | $\infty$       | 3.041200               | 0.340350       |
| $\sigma_{\bar{G}}^2$  | —                  | $\infty$       | 0.196850               | 0.038862       |
| $\sigma_{\bar{W}}^2$  | —                  | $\infty$       | 0.165930               | 0.018368       |
| $\sigma_{\bar{L}}^2$  | —                  | $\infty$       | 0.063471               | 0.016259       |
| $\sigma_{\bar{T}}^2$  | —                  | $\infty$       | 0.002277               | 0.000487       |
| $\sigma_{\bar{r}}^2$  | —                  | $\infty$       | 0.066754               | 0.019213       |
| $\sigma_\pi^2$        | —                  | $\infty$       | 0.000129               | 0.000038       |
| $\sigma_g^2$          | —                  | $\infty$       | 0.000015               | 0.000009       |
| $\sigma_n^2$          | —                  | $\infty$       | 0.000015               | 0.000014       |

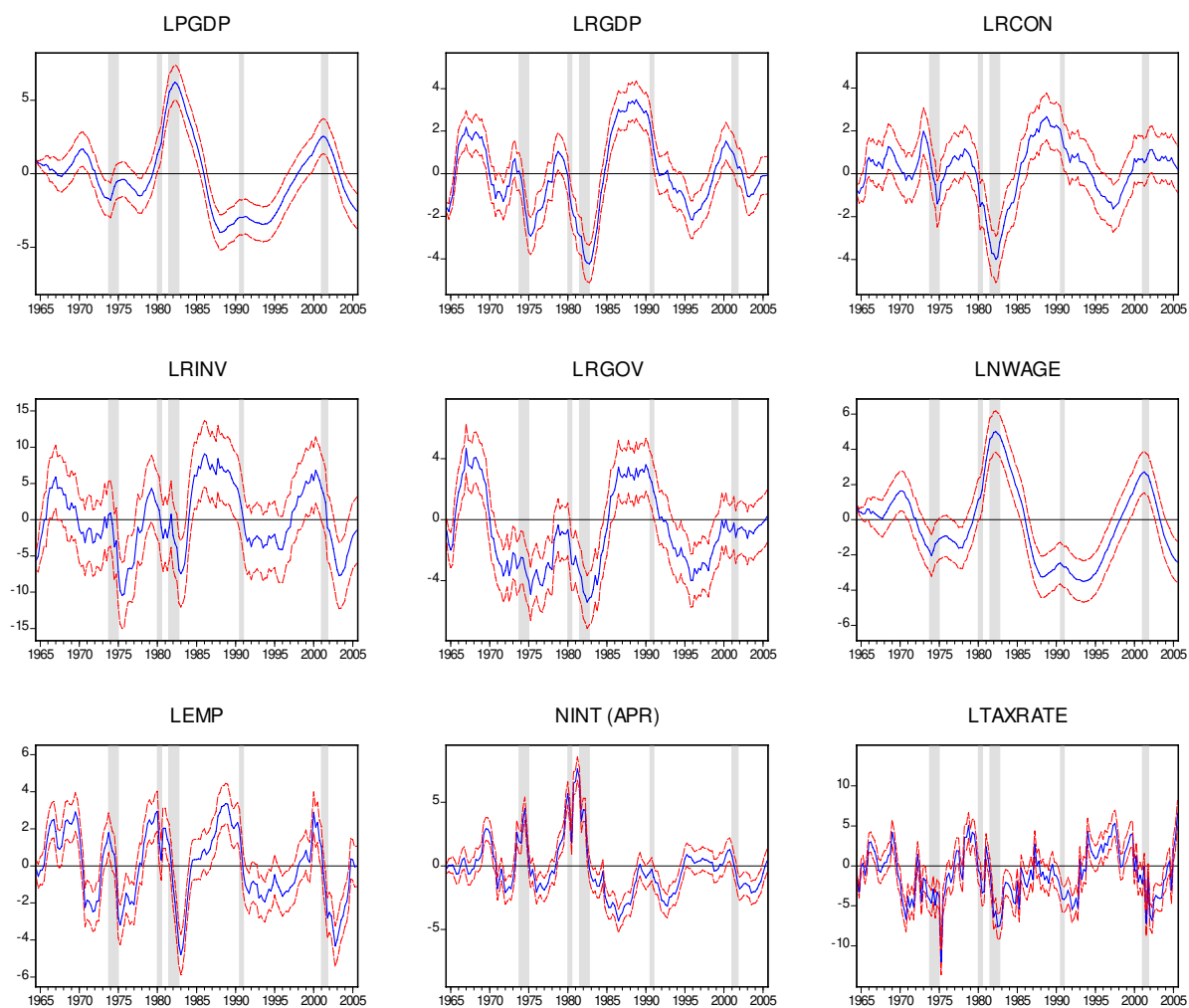
Note: All observed nonpredetermined endogenous variables are rescaled by a factor of 100.

Figure 2. Predicted cyclical components of observed nonpredetermined endogenous variables



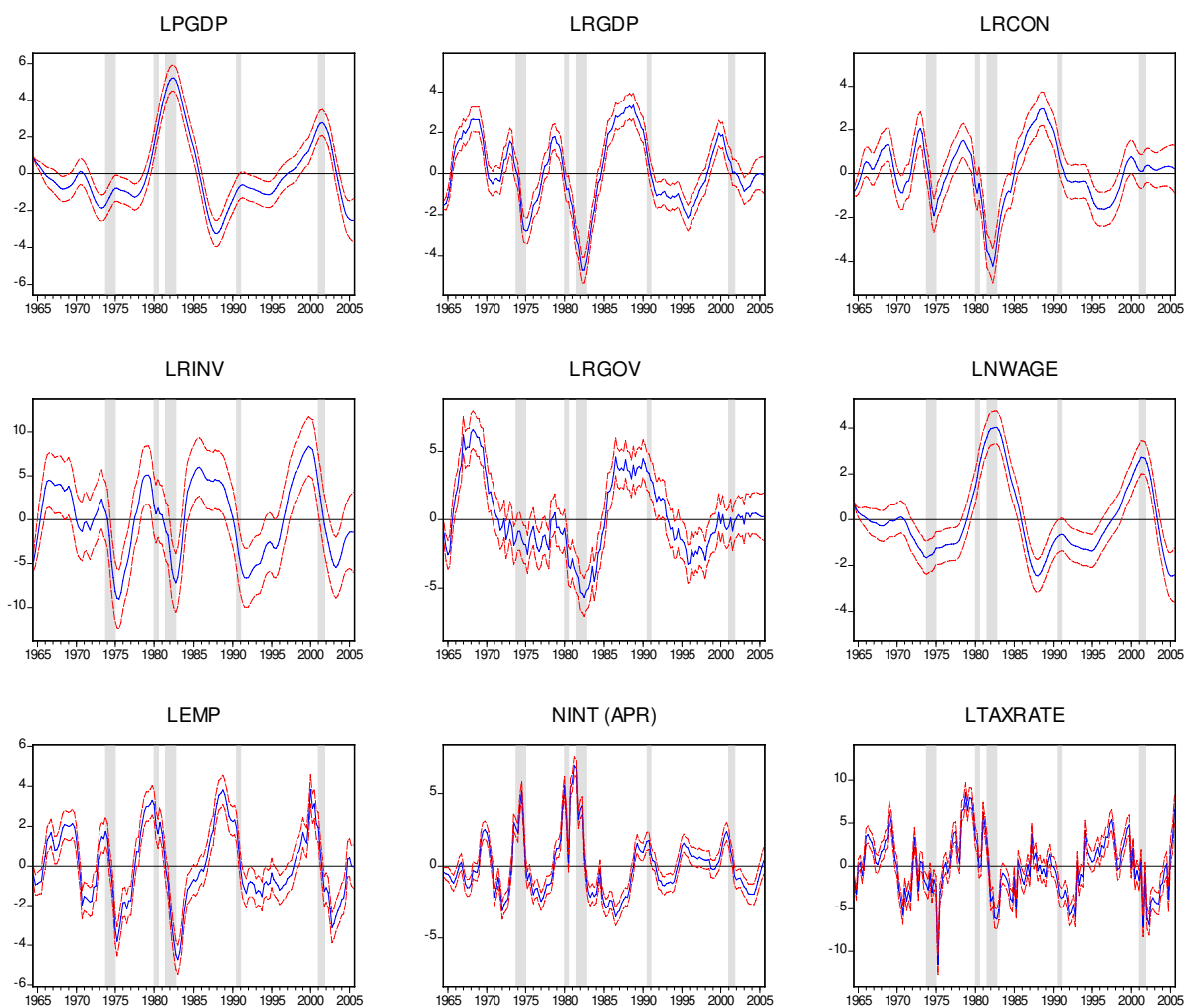
*Note:* Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

Figure 3. Filtered cyclical components of observed nonpredetermined endogenous variables



*Note:* Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

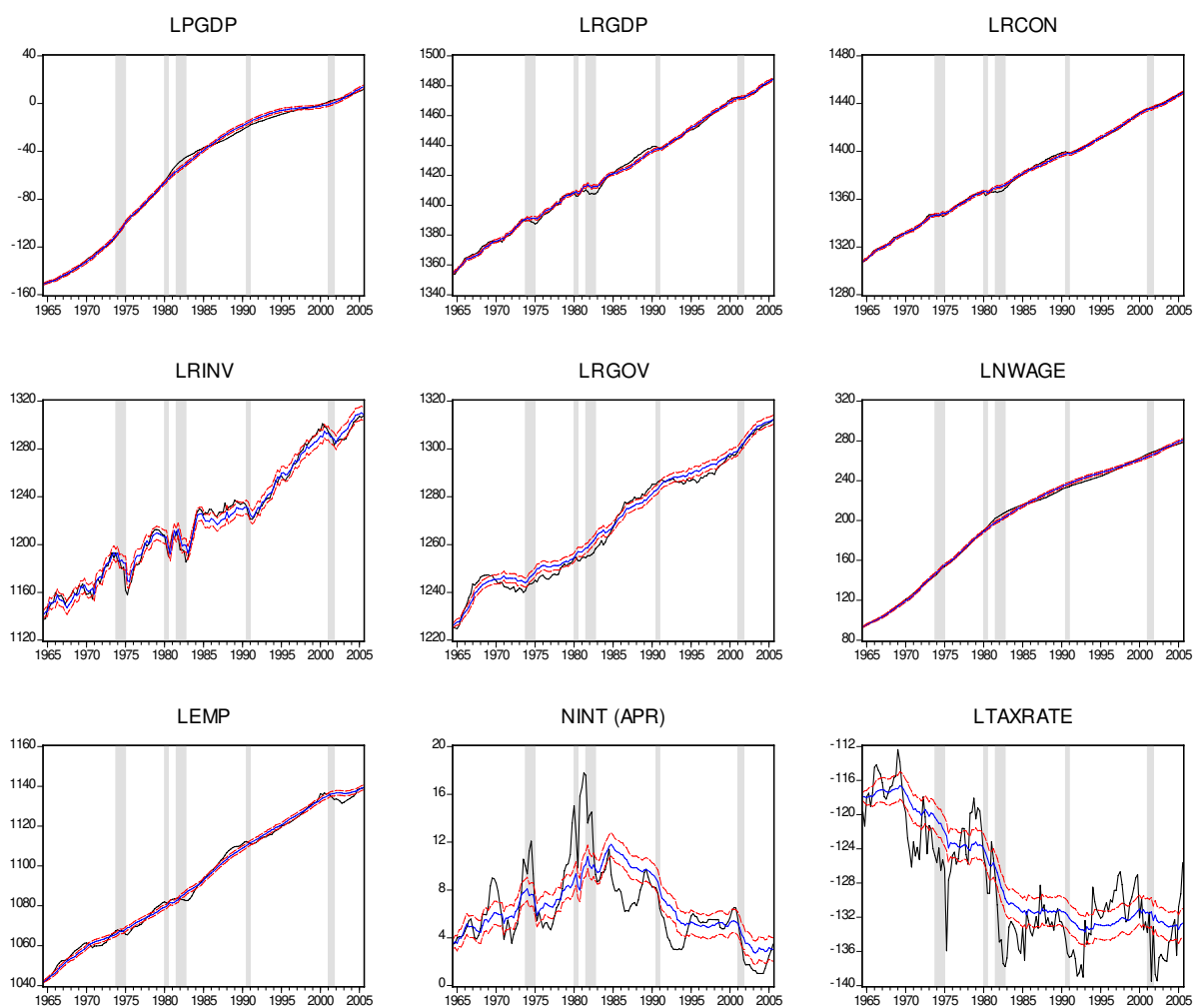
Figure 4. Smoothed cyclical components of observed nonpredetermined endogenous variables



*Note:* Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

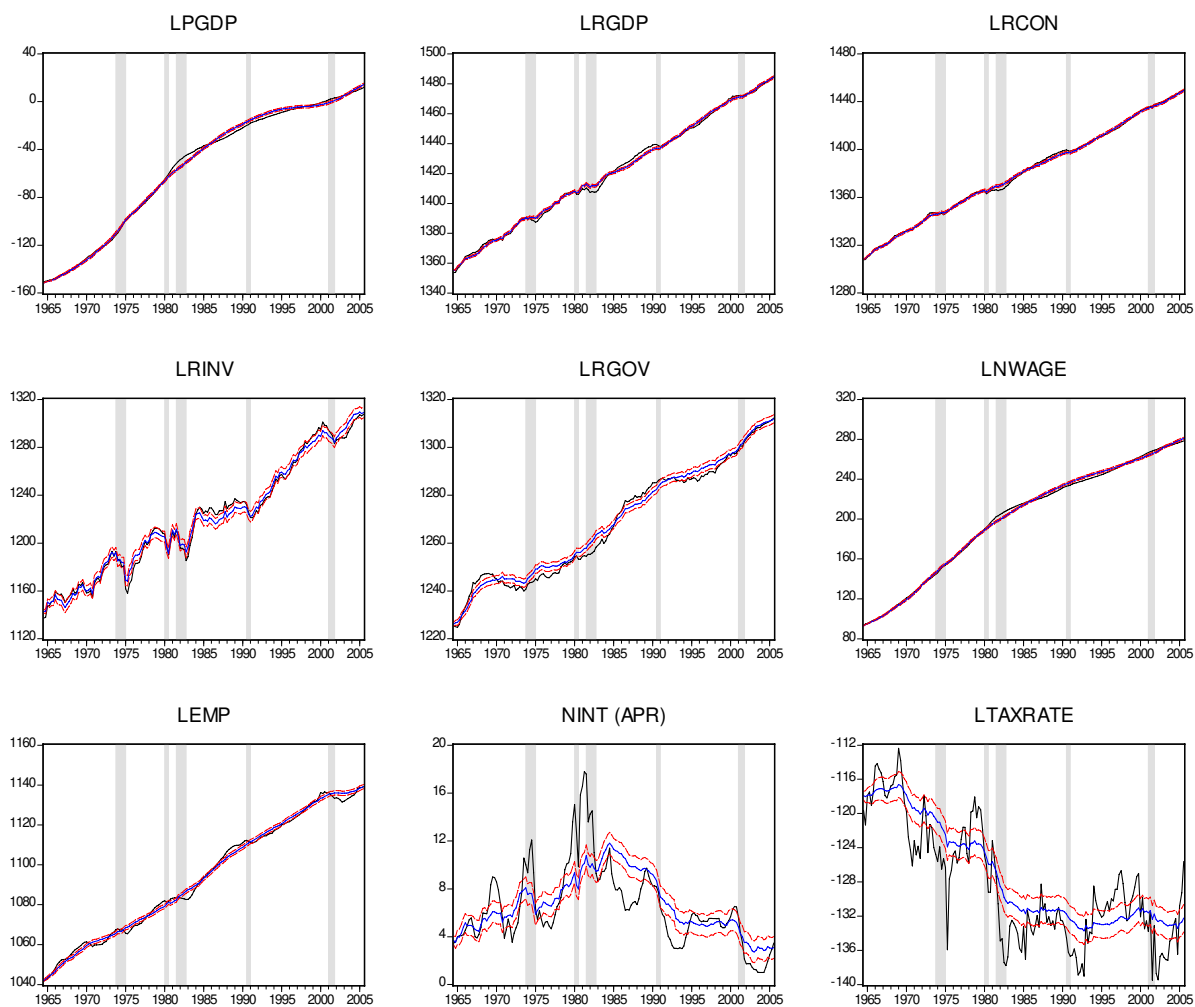


Figure 5. Predicted trend components of observed nonpredetermined endogenous variables



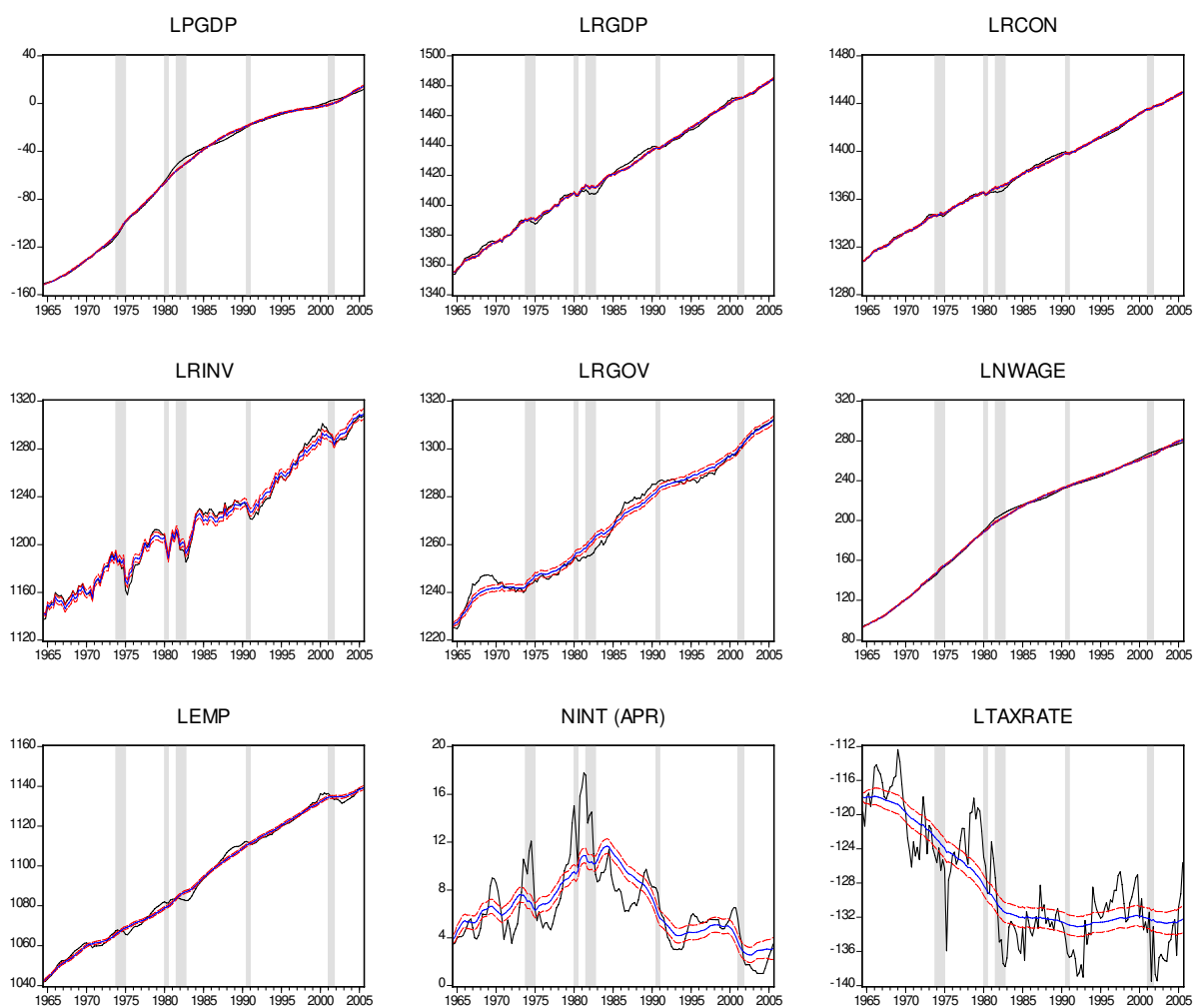
*Note:* Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

Figure 6. Filtered trend components of observed nonpredetermined endogenous variables



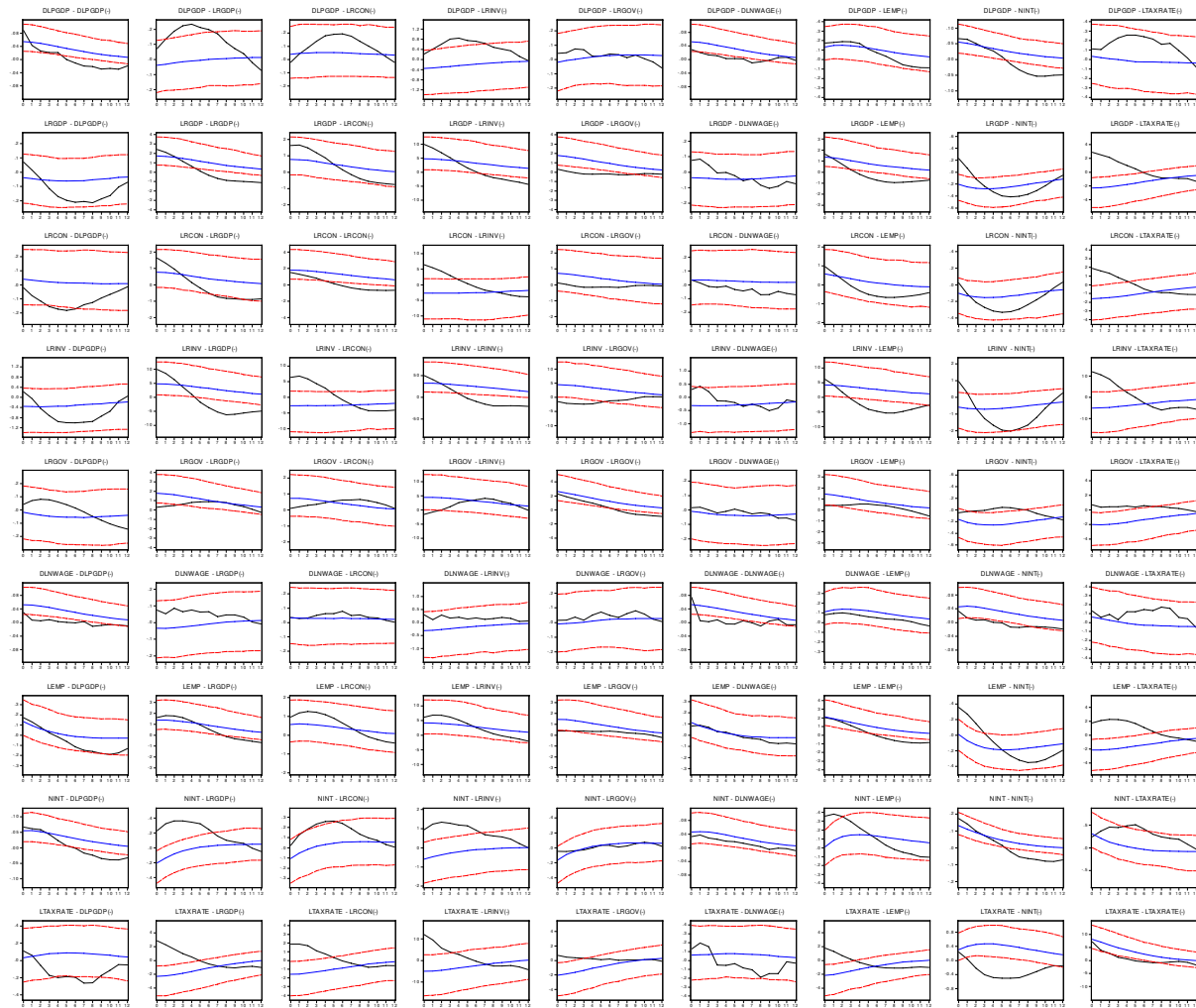
*Note:* Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

Figure 7. Smoothed trend components of observed nonpredetermined endogenous variables



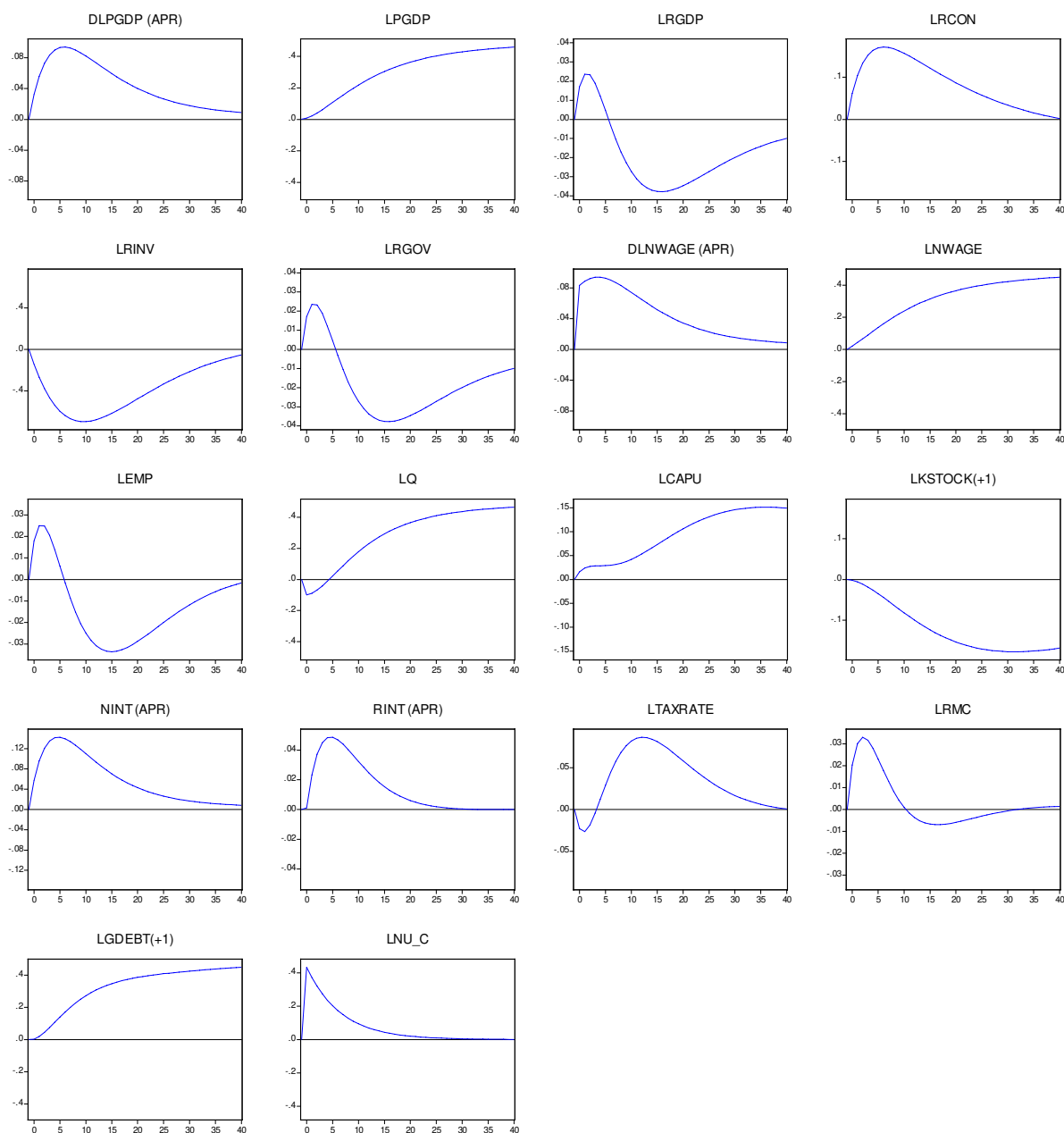
*Note:* Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

Figure 8. Theoretical versus empirical autocovariances



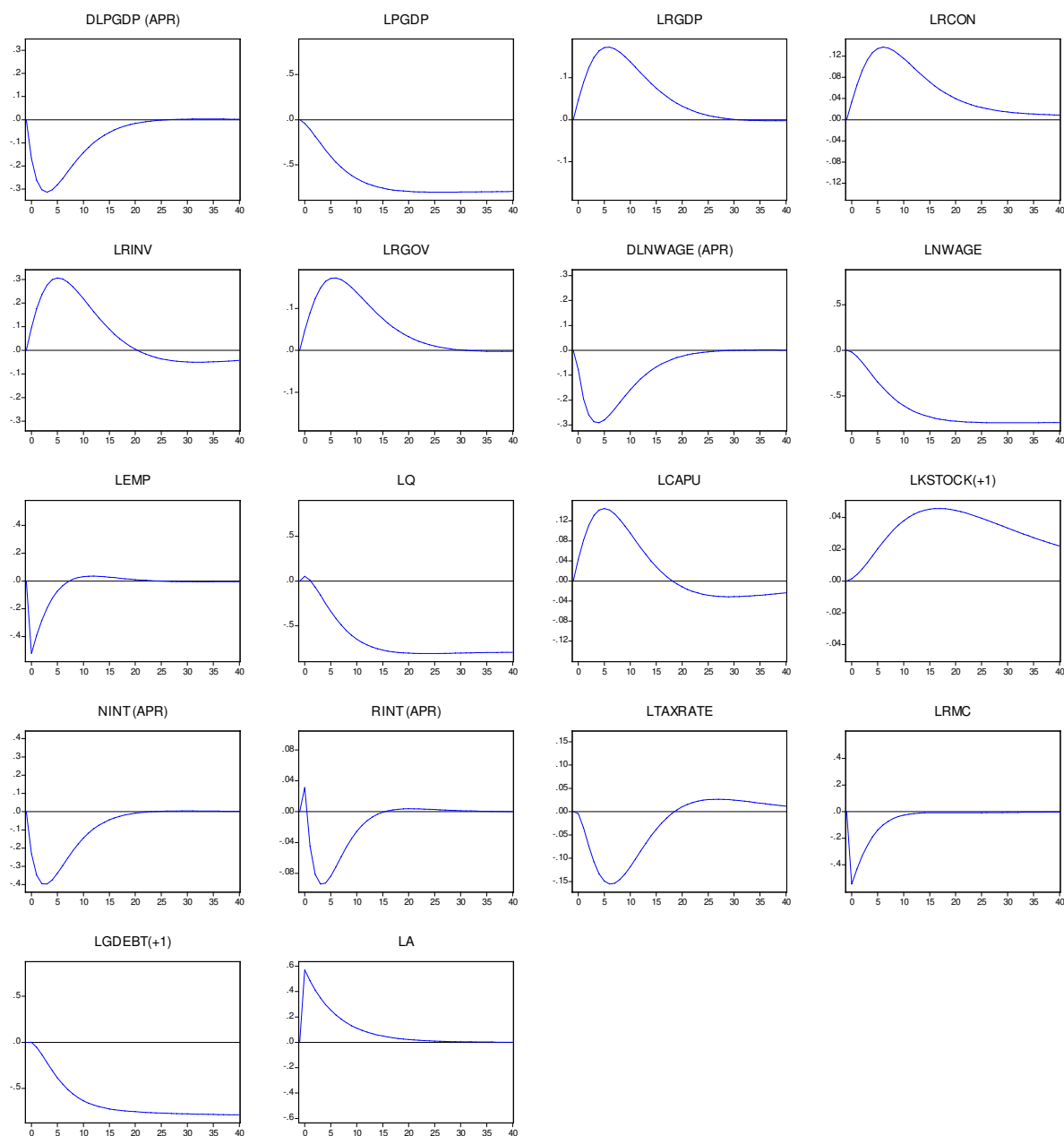
*Note:* Empirical autocovariances are represented by black lines, while blue lines depict theoretical autocovariances. Asymmetric 95% confidence intervals are calculated with a Monte Carlo simulation with 999 replications for  $2T$  periods, discarding the first  $T$  simulated observations to eliminate dependence on initial conditions, where  $T$  denotes the observed sample size.

Figure 9. Theoretical impulse responses to a preference shock



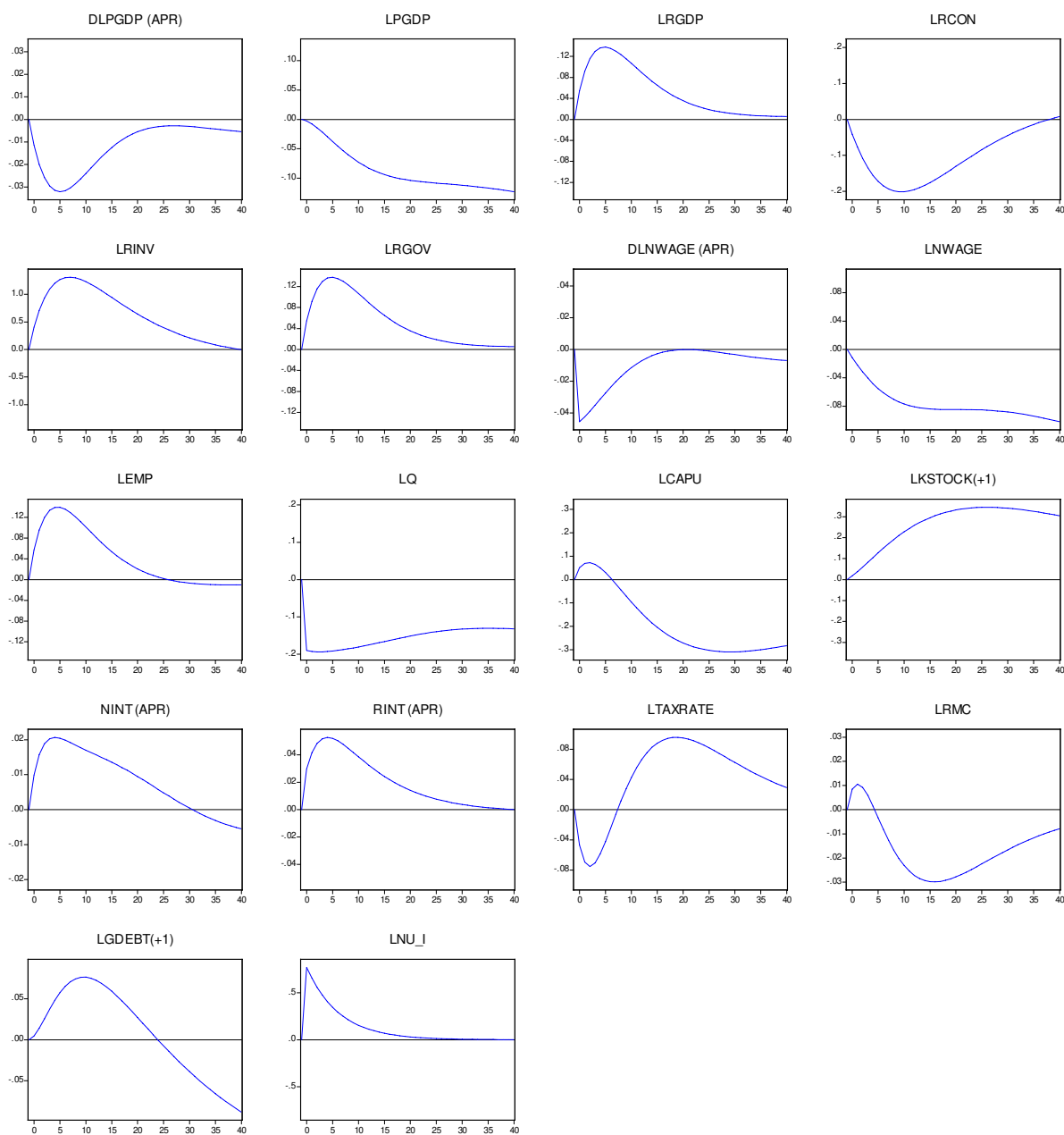
*Note:* Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 10. Theoretical impulse responses to an output technology shock



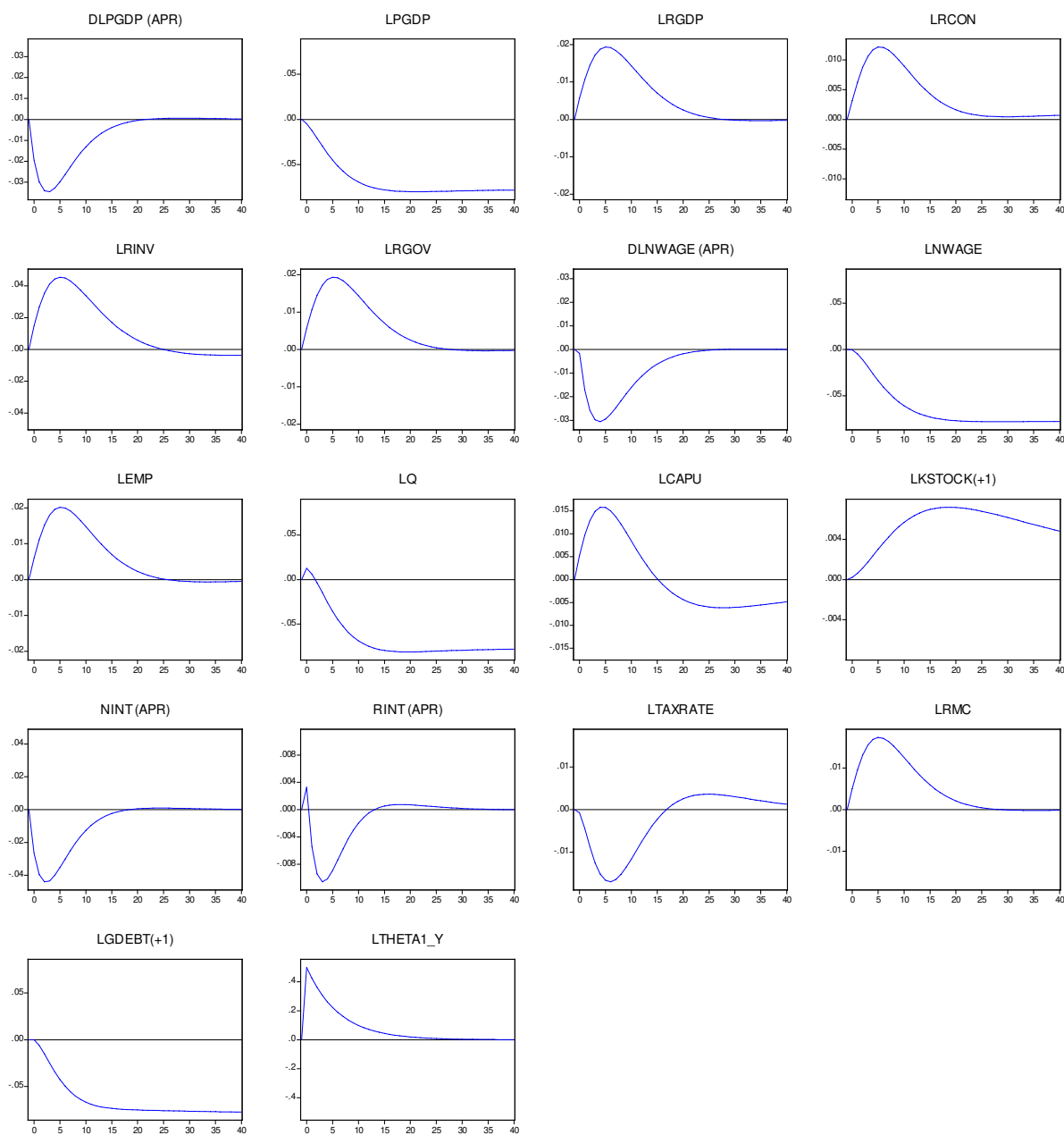
*Note:* Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 11. Theoretical impulse responses to an investment technology shock



*Note:* Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

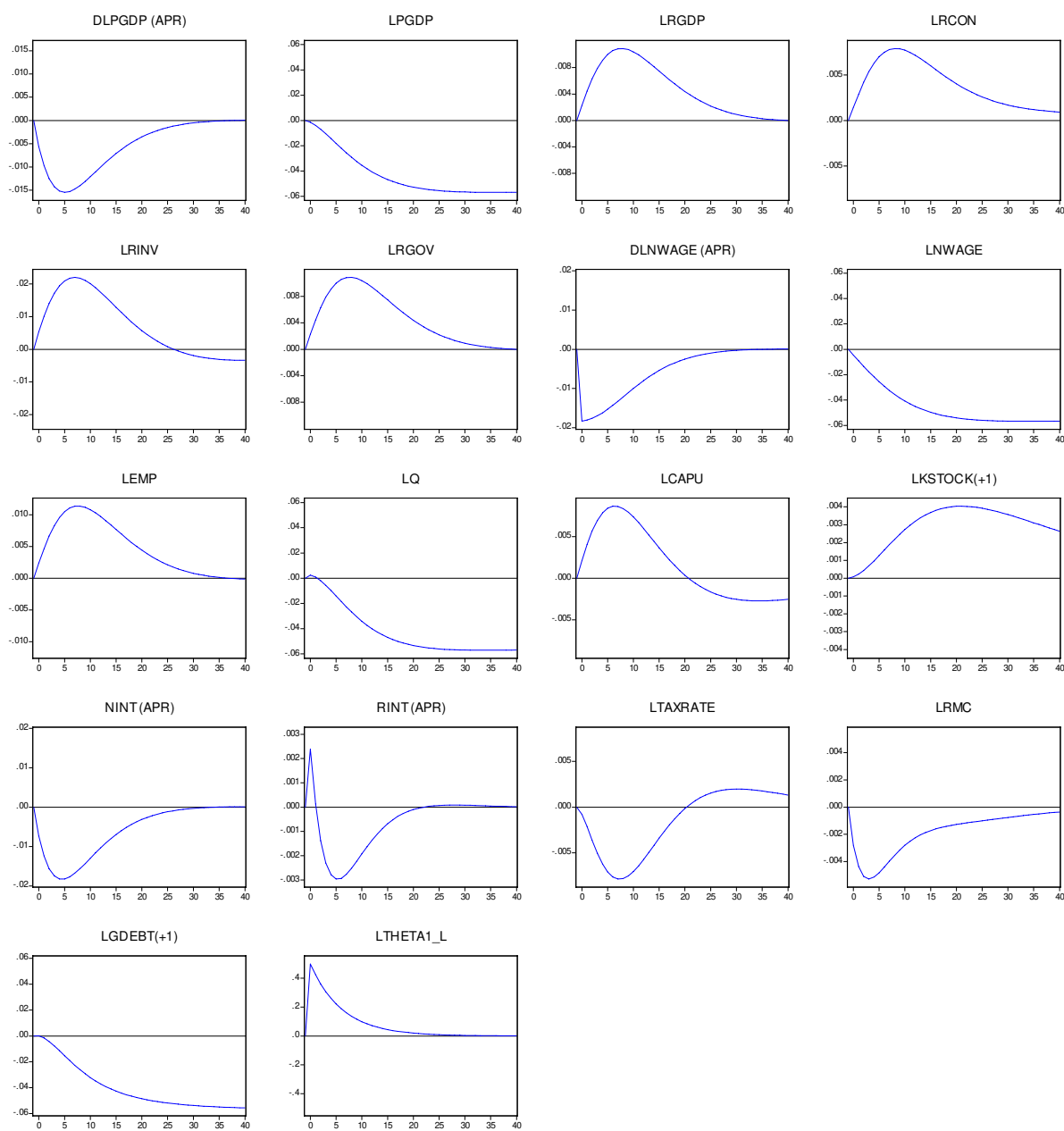
Figure 12. Theoretical impulse responses to a price markup shock



*Note:* Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

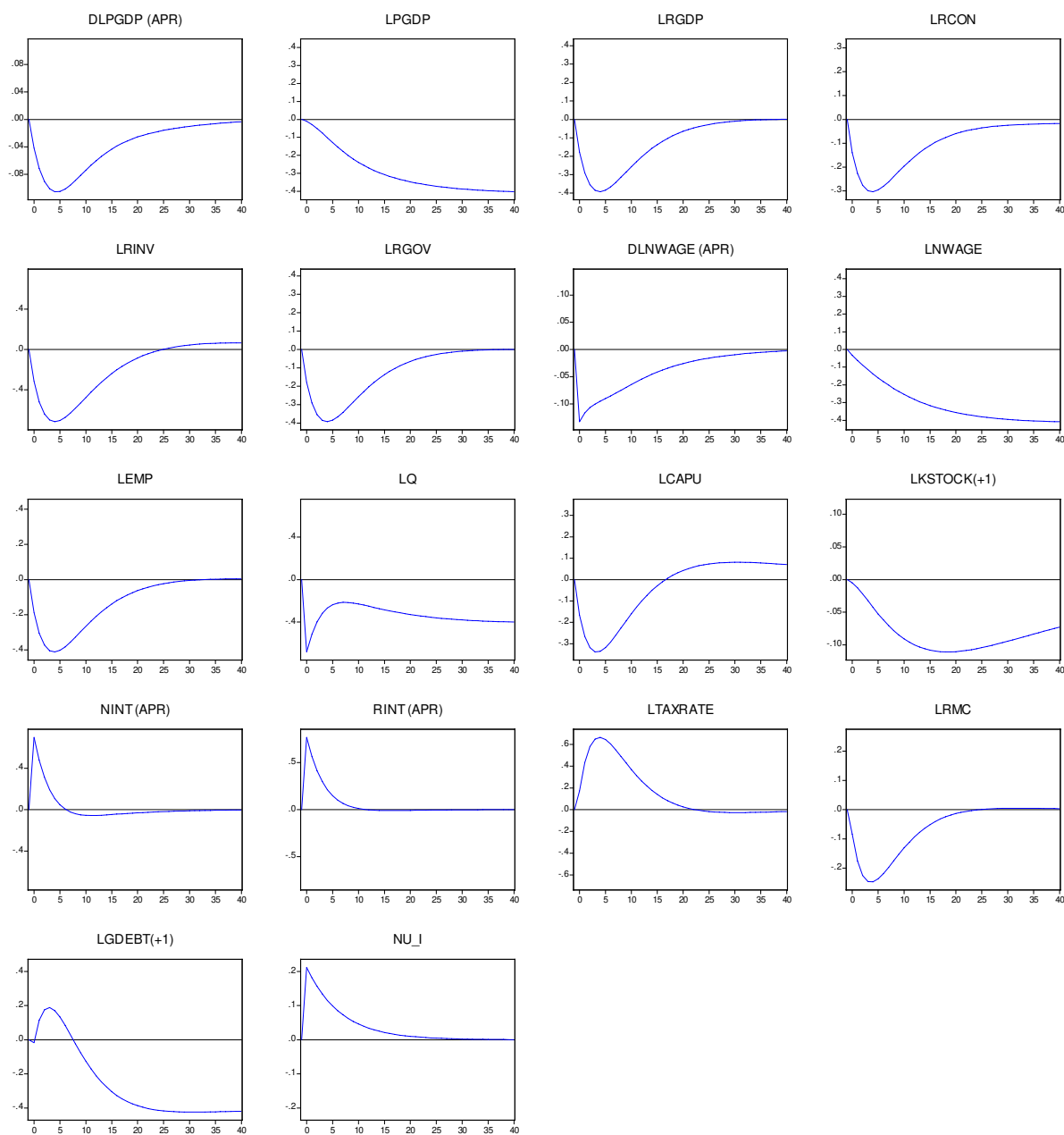


Figure 13. Theoretical impulse responses to a wage markup shock



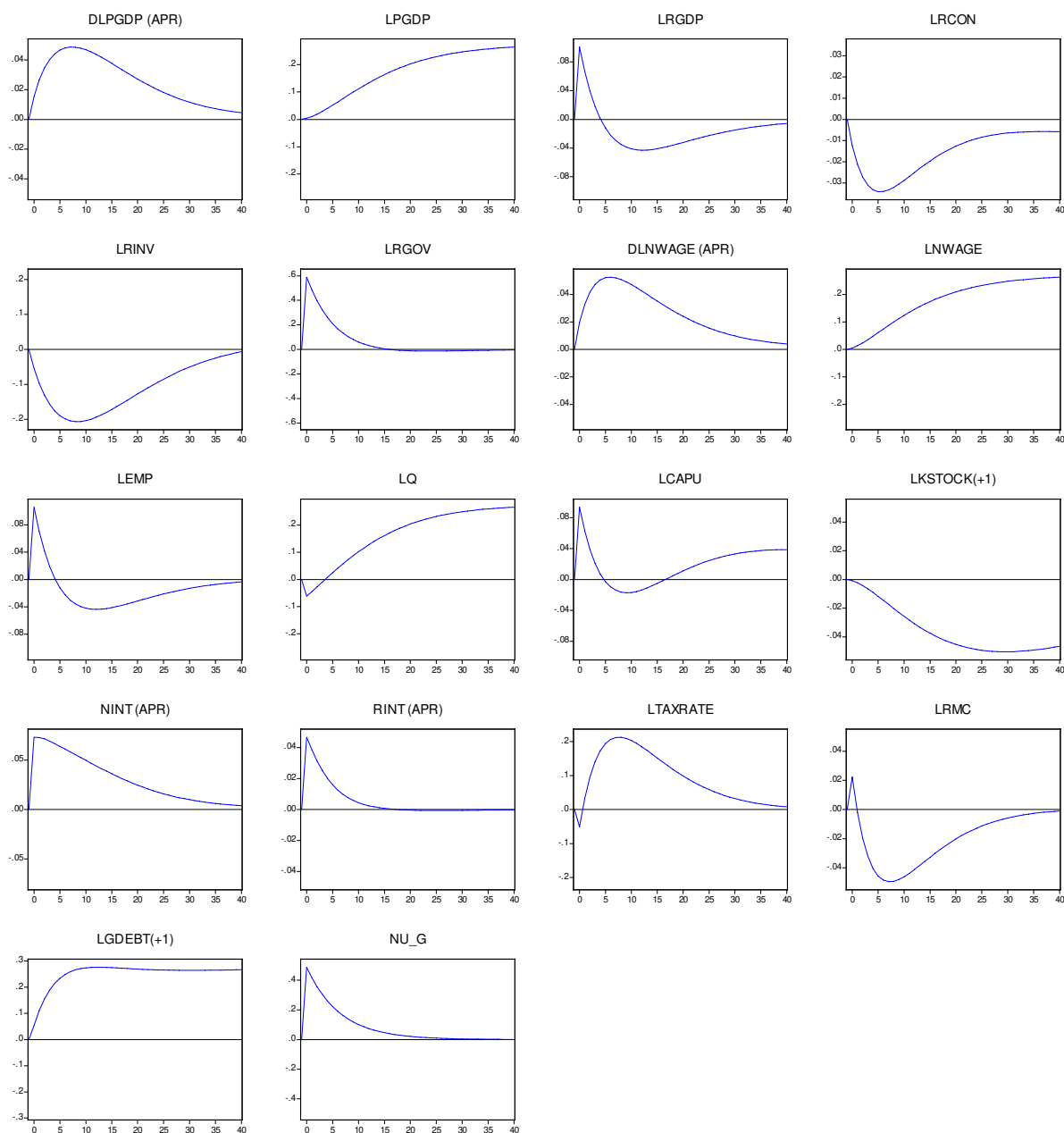
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 14. Theoretical impulse responses to a monetary policy shock



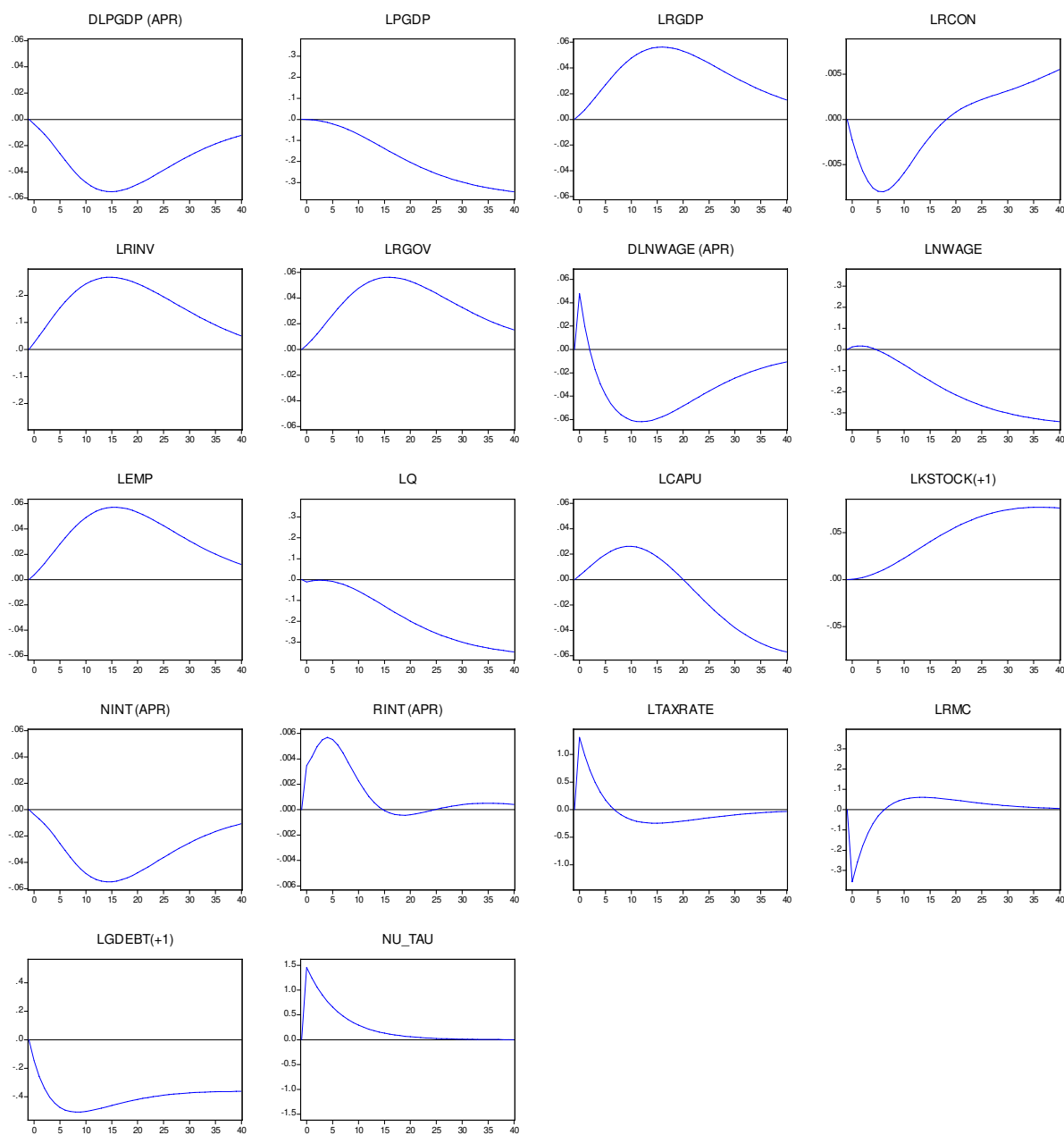
*Note:* Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 15. Theoretical impulse responses to a fiscal expenditure shock



*Note:* Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 16. Theoretical impulse responses to a fiscal revenue shock



*Note:* Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 17. Theoretical forecast error variance decompositions

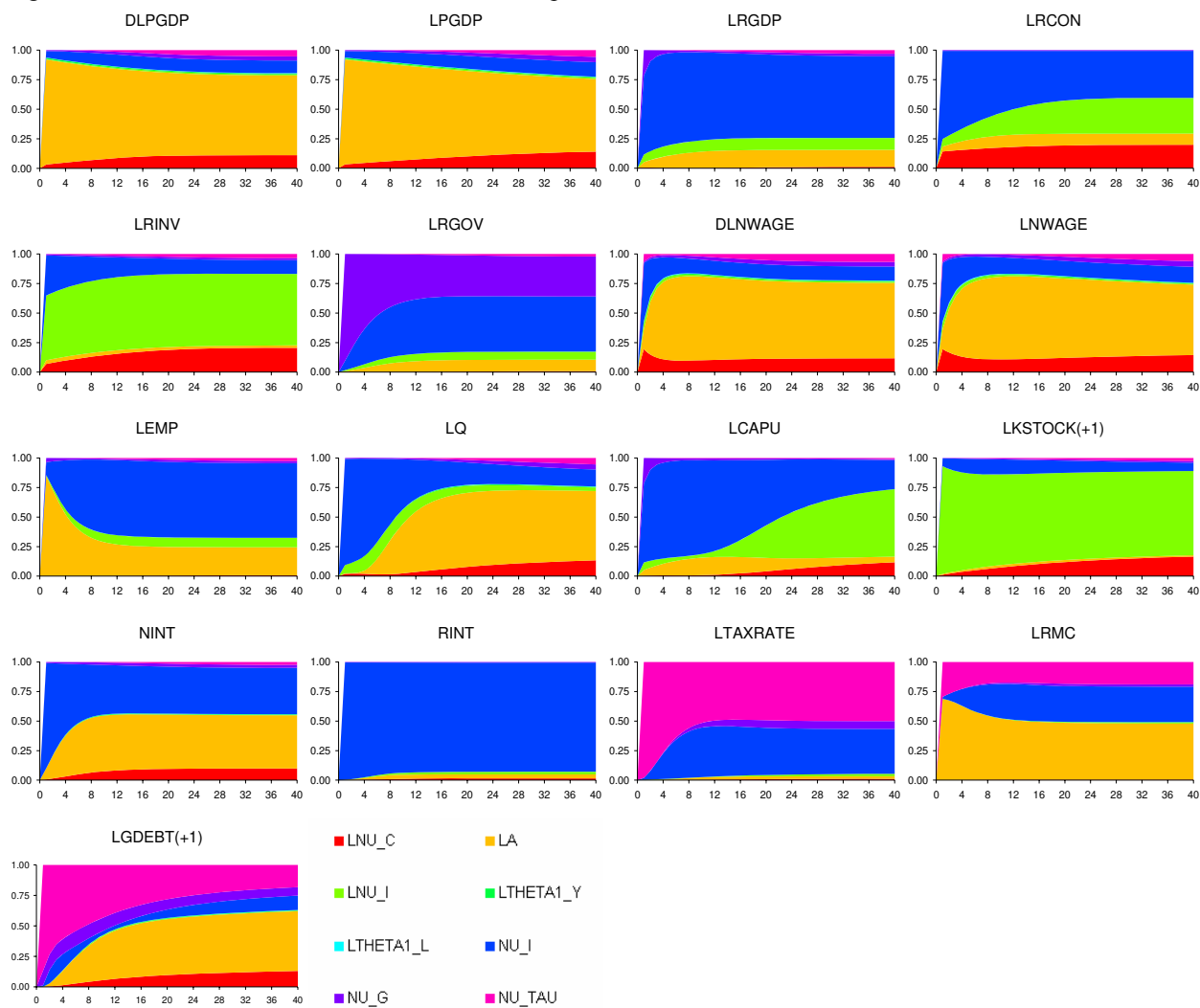
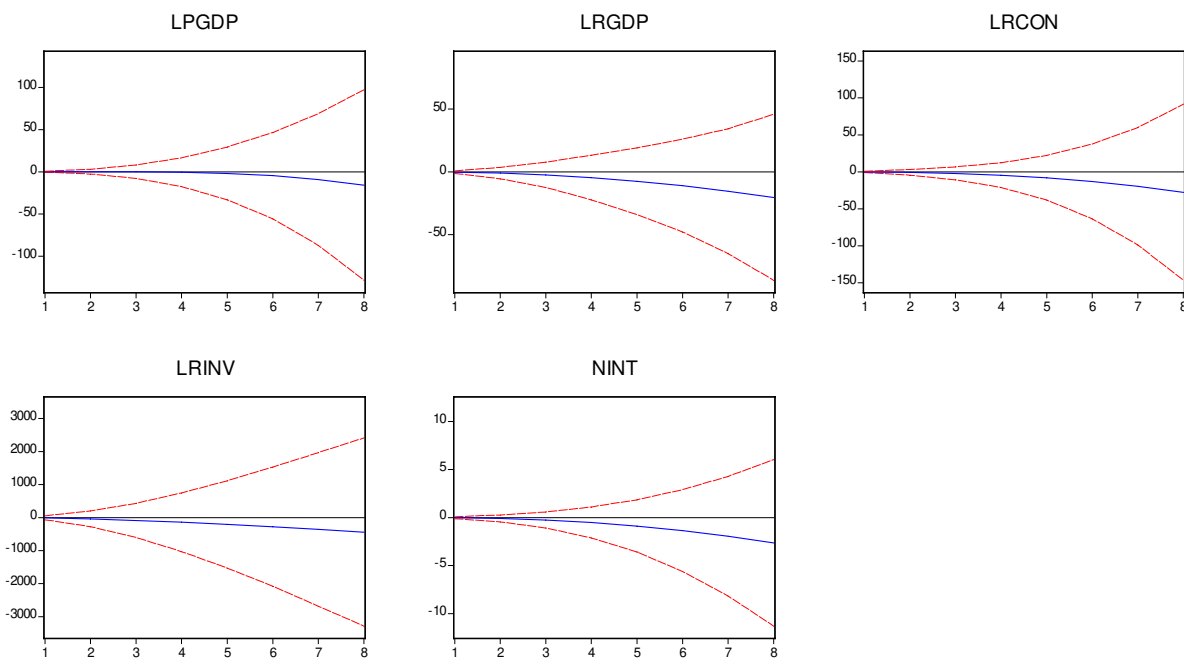
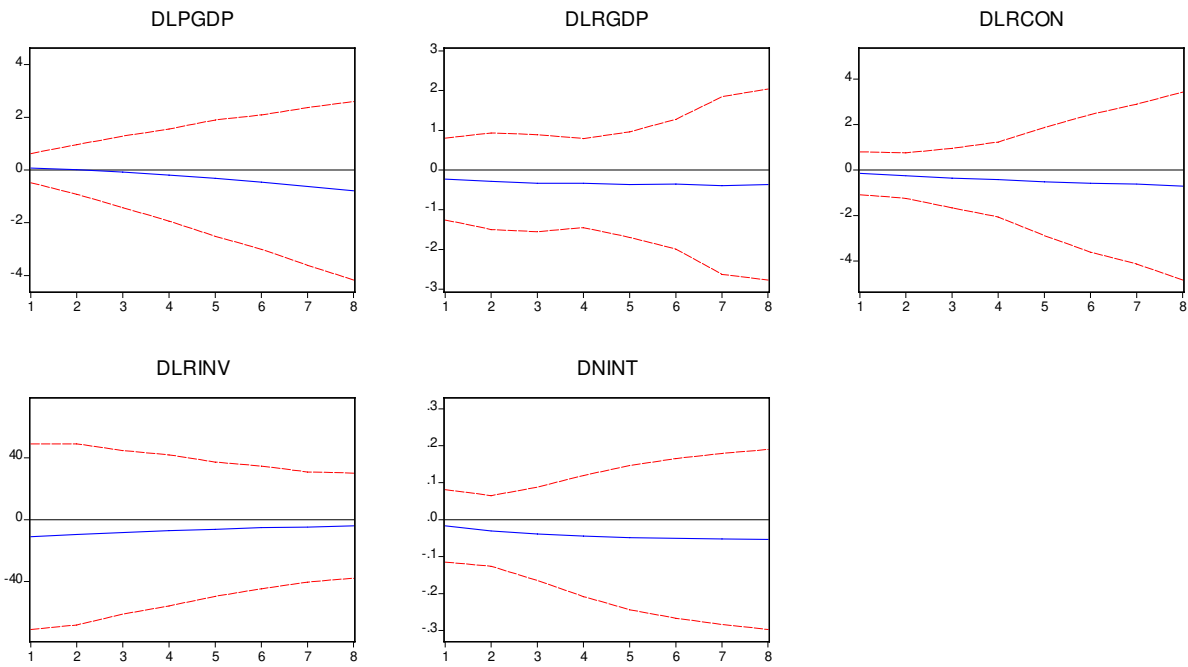


Figure 18. Mean squared prediction error differentials for levels



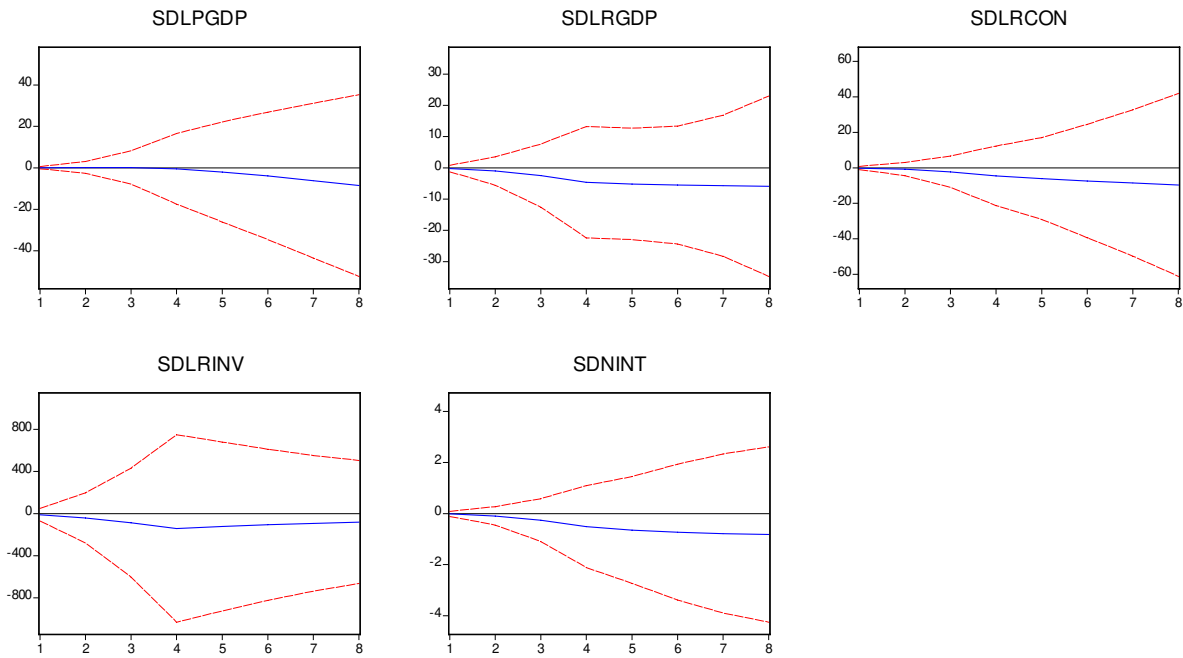
*Note:* Mean squared prediction error differentials are defined as the mean squared prediction error for the DSGE model less that for the SVAR model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

Figure 19. Mean squared prediction error differentials for ordinary differences



*Note:* Mean squared prediction error differentials are defined as the mean squared prediction error for the DSGE model less that for the SVAR model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

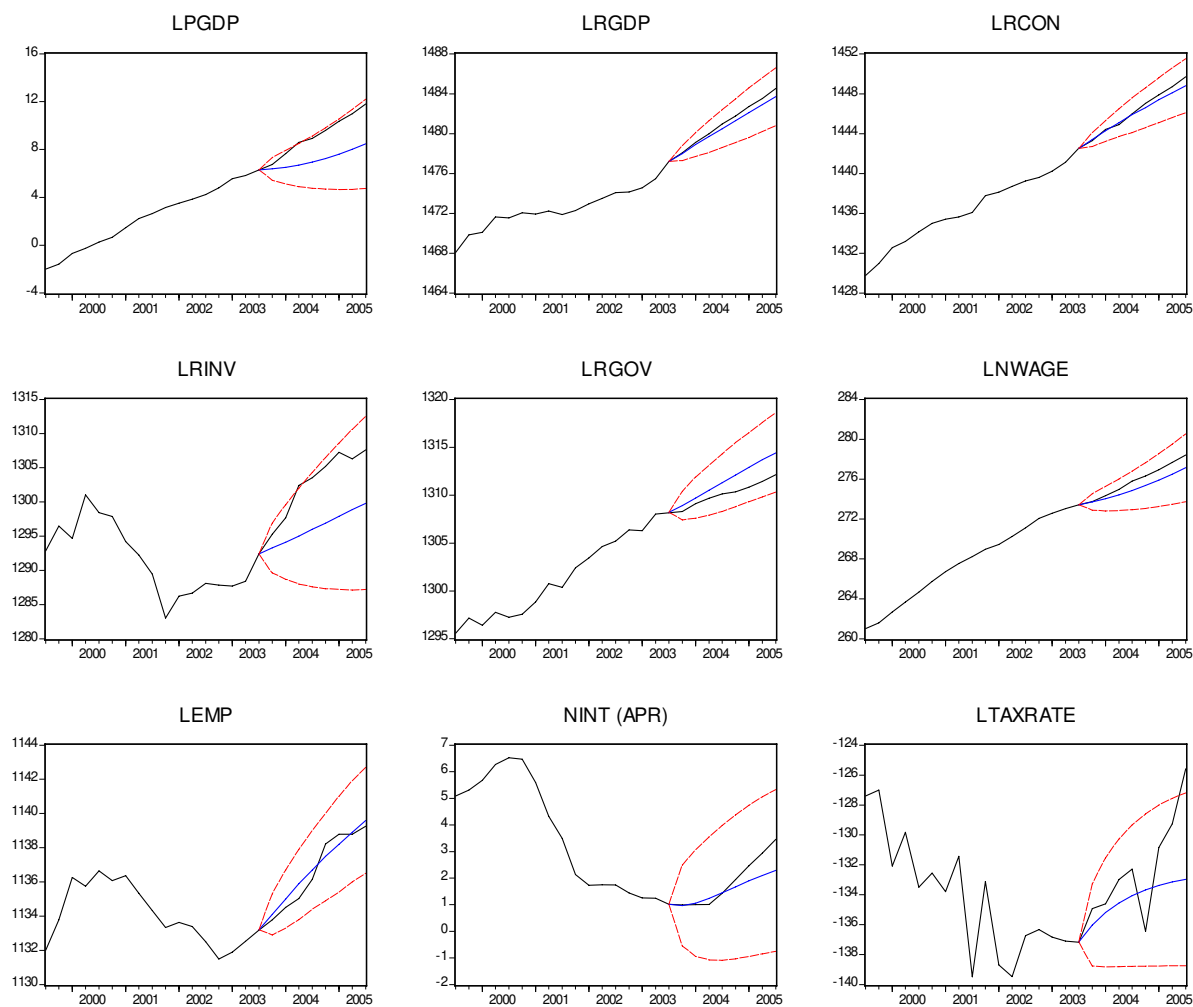
Figure 20. Mean squared prediction error differentials for seasonal differences



*Note:* Mean squared prediction error differentials are defined as the mean squared prediction error for the DSGE model less that for the SVAR model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

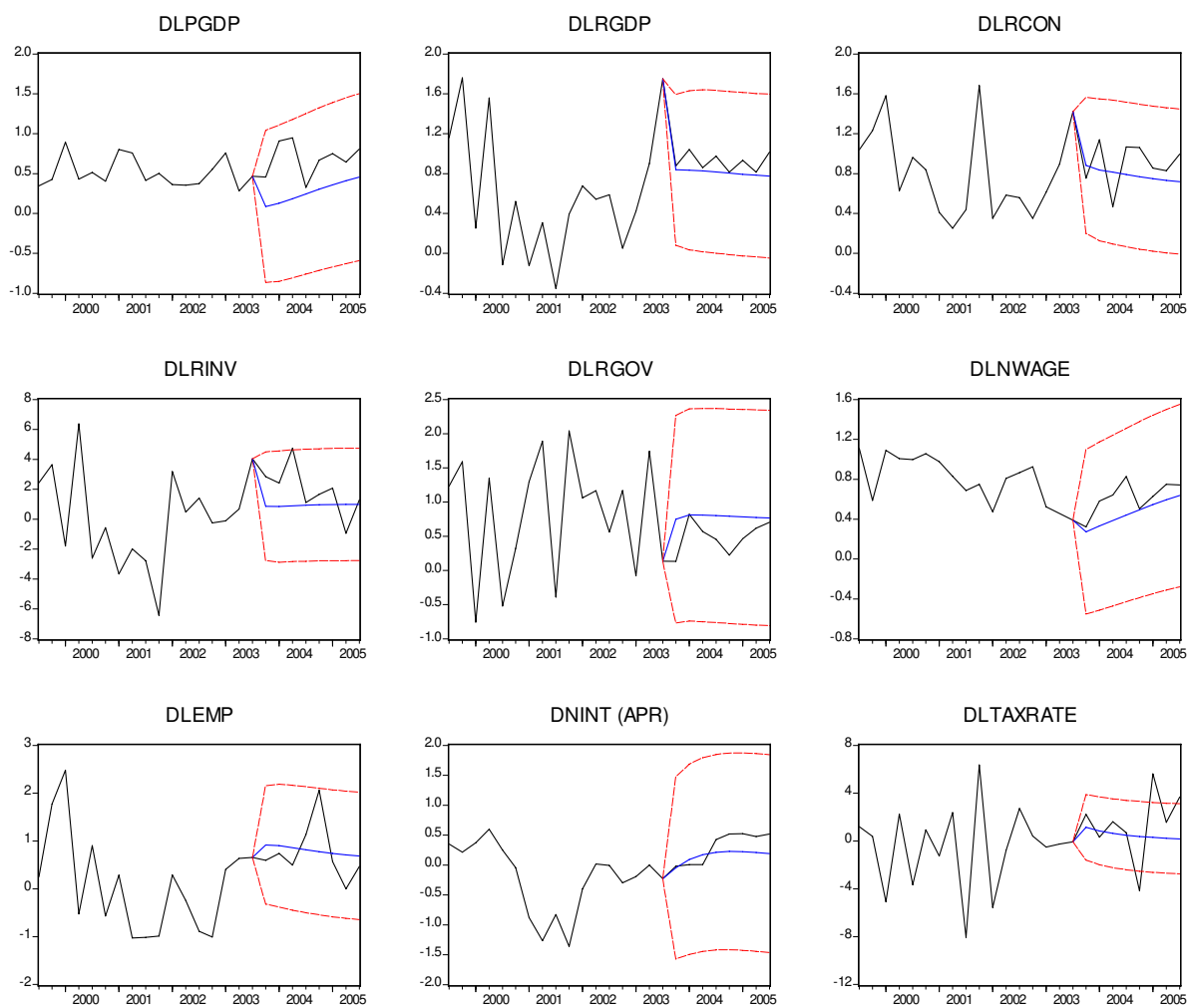


Figure 21. Dynamic forecasts of levels of observed nonpredetermined endogenous variables



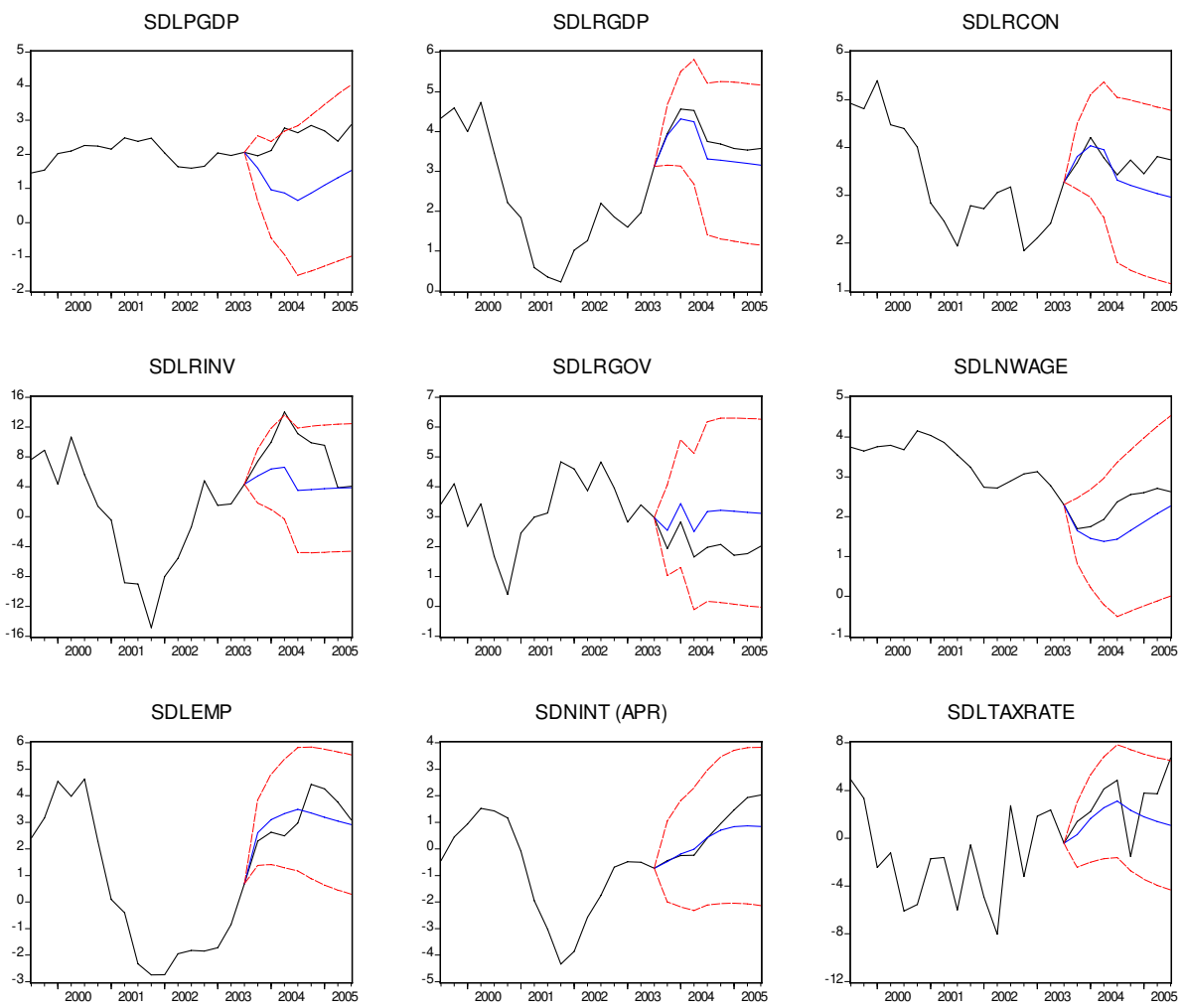
*Note:* Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters.

Figure 22. Dynamic forecasts of ordinary differences of observed nonpredetermined endogenous variables



*Note:* Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters.

Figure 23. Dynamic forecasts of seasonal differences of observed nonpredetermined endogenous variables



Note: Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters.

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