Government spending, GDP and exchange rate in Zero Lower Bound: measuring causality at multiple horizons

MAO TAKONGMO, Charles Olivier

1 July 2016

Government spending, GDP and exchange rate in Zero Lower Bound: measuring causality at multiple horizons*

Charles Olivier MAO TAKONGMO†
University of Ottawa

First version: July 2016;
This version: June 2, 2017

Abstract

This paper assesses the causality between government spending and gross domestic product in the United States at multiple horizons. We compare the Granger causality for normal periods (1959Q1 to 2006 Q4) with the causality for the ZLB period (2007Q1 to 2015Q4). We show that the Granger causality measures between government spending and GDP are very high and persistent in the ZLB period, but only if the exchange rate is not taken into account. When the exchange rate is taken into account, the Granger causality between government spending and GDP becomes very small and non-persistent.

Keywords: Zero Lower Bound; Causality Measures.

JEL classification: C01; C32, E62.

1 Introduction

This paper assesses the Granger causality between government spending and the gross domestic product (GDP) in the United States of America at multiple horizons. In addition, this paper analyses the effect of the real exchange rate on the causality measure during the Zero Lower Bound (ZLB) period. During the 2007 financial crisis and the recession that followed, the nominal interest rate reached its lower bound and remained at a very low level for a long period; in this paper, we define the period from 2007Q1 to 2015Q4 as the ZLB period. During that period, the Federal Reserve Bank lost its monetary policy, which consisted of lowering the nominal interest rate to increase the GDP. The government of the United States then started to increase government spending to increase the GDP.

Many researchers have shown, using theoretical macroeconomic models built in a closed economy, that the elasticity between government spending and GDP is very large when the nominal interest rate

*We thank Professor Jean-Marie Dufour and Professor Victoria Zinde Walsh for their useful discussions and comments. Earlier versions of this paper were presented at the university of Quebec at Montreal

†Replacement Assistant Professor, University of Ottawa, Centre interuniversitaire de recherche en analyse des organisations (CIRANO), Centre interuniversitaire sur le risque, les politiques économiques et l'emploi (CIRPÉE), University of Quebec at Montreal. Mailing address: Department of Economics, University of Ottawa, 120 University, Social Sciences Building, Room 9005, Ottawa, Ontario, Canada, K1N 6N5; e-mail: mao_takongmo.charles_olivier@uqam.ca Web page: https://sites.google.com/site/maotakongmocharles/

is binding (see for example, Christiano et al., 2011).Their results are due to the fact that there is no longer a crowding out effect via the interest rate in a closed economy.

Later, other researchers, using a macroeconomic theoretical model, discovered that the elasticity between government spending and GDP during the ZLB period is not large for open economies; its value is similar to what is usually obtained in a normal period (see for example, Mao Takongmo, 2017). The result in an open economy is due to appreciation of the real exchange rate after an increase in government spending. The author called this a crowding out effect via the exchange rate.

In this paper, we use the Granger causality measure proposed by Dufour & Taamouti (2010) (based on Dufour & Renault (1998)) to measure Granger causality during a normal period (1959Q1 to 2006Q4) compared with the same measure during the ZLB period (2007Q1 to 2015Q4). We put more emphasis on the role played by the real exchange rate. Our empirical results provide evidence that the Granger causality measures between government spending and GDP are very high and persistent in the ZLB period, but only if the exchange rate is not taken into account. When the exchange rate is taken into account, our measure of Granger causality between government spending and GDP becomes very small and non-persistent.

Many researchers have compared empirically the link between government spending and GDP. Using a vector auto regressive (VAR) method and annual panel data from 1951-2007 for 62 developed and developing countries, Karras (2012) showed that an increase in trade openness by 10 % of the GDP reduces the fiscal multiplier by 5 % . However, this research was done using data from a normal period. Ilzetzki et al. (2013) used panel data from 1960Q1 to 2007Q4 from 44 developed and developing countries and the structural VAR (SVAR) method to show that fiscal multipliers are usually lower for open economies. Ilzetzki et al. (2013) also did not cover the ZLB period. Zhang et al. (2016) used the methodology presented by Dufour & Taamouti (2010) to measure the causality between exchange rates and commodity prices. To our knowledge, this is the first time the Granger causality measure is used to assess the link between government spending and GDP at multiple horizons. Moreover, we discuss the role played by the real exchange rate.

The paper is organized as follows: section (2) presents the theoretical framework proposed by Dufour & Taamouti (2010); section (3) presents the data used and the results; and section (4) concludes.

2 Framework

A time series $\{X(t)\}$ causes another time series $\{Y(t)\}$, in the sense of Wiener (1956) and Granger (1969), if it is possible to better predict $\{Y(t)\}$ using all available information than using all available information without $\{X(t)\}$. Following Dufour & Renault (1998), Dufour & Taamouti (2010), and Song & Taamouti (2016), we define the concept of non-causality in terms of orthogonality conditions between sub-spaces of a Hilbert space of random variables with finite second moments. The notations here are those used by Song & Taamouti (2016).

Let $L^2 \equiv L^2(\Omega, \mathcal{A}, Q)$ represent a Hilbert space of real random variables with finite second moments and mean zero, defined on a common probability space $(\Omega, \mathcal{A}, Q)$ with covariance as the inner product. As defined by Dufour & Renault (1998), the information available at time $t \subseteq \mathbb{Z}$ is a closed Hilbert subspace $I(t) \subseteq L^2$. The set of integers is denoted by $\mathbb{Z}$. We consider a set of non-decreasing sequences of information I with a starting point $\omega \in \mathbb{Z} \cup \{-\infty\}$. That information set I can be written as:

$$I = \{I(t) : t \in \mathbb{Z}, \, t > \omega\} \text{ with } t < t' \Rightarrow I(t) \subseteq I(t') \text{ for all } t > \omega \,, \tag{1}$$

where $I(t)$ is a Hilbert subspace of L^2, $\omega \in \mathbb{Z} \cup \{-\infty\}$ represents a "starting point" and $\mathbb{Z}$ is the set of integers. Using the notation of Dufour & Renault (1998), $X(\omega, \, t]$ and $Y(\omega, \, t]$ are information contained, respectively, in the variables X and Y up to time t. The information is added as follows:

$$I_X(t) = I(t) + X(\omega, \, t] \,, \tag{2}$$

$$I_{XY}(t) = I(t) + X(\omega,\, t] + Y(\omega,\, t] = I_X(t) + Y(\omega,\, t]. \tag{3}$$

$I_X(t)$ is information obtained by adding $X(\omega,\, t]$ to $I(t)$ and $I_{XY}(t)$ is the information obtained by adding $Y(\omega,\, t]$ to $I_X(t)$.

For horizon $h > 0$, $P[X(t+h)|\, B(t)]$ is the best forecast of $X(t+h)$ based on the information set $B(t)$, with $U[X(t+h)|\, B(t)] = X(t+h) - P[X(t+h)|\, andB(t)]$ the forecasting error. The variance-covariance matrix of the forecasting error is:

$$\Sigma[X(t+h)\,|\,B(t)] = \mathsf{E}\big\{U[X(t+h)\,|\,B(t)]\,U[X(t+h)\,|\,B_t]'\big\}. \tag{4}$$

2.1 Causality measures

Causalities may exist from X to Y or from Y to X. As Dufour & Taamouti (2010) pointed out, a statistical test cannot achieve that goal since it is only informative of the existence or non-existence of causality and statistical significance usually depends on available data and power. As McCloskey & Ziliak (1996) and Dufour & Taamouti (2010) argued, at a given level, a large effect may not be statistically significant and a statistically significant effect may not be relevant from an economic point of view. This is why the magnitude of forecasting improvement based on a loss function is preferred.

The causality measures proposed in Dufour & Taamouti (2010) for horizon $h > 0$ is based on the ratio of the restricted and unrestricted forecasting error. These causality measures are non-negative, cancel only when the causality does not exist, and increase with the strength of the causality. Definition 4.1 in Dufour & Taamouti (2010) defines the causality measure as follows.

Definition (*Causality Measures at horizon h*).

The function

$$C(Y \to X\,|\,I) = \ln\left[\frac{\det \Sigma[X(t+h)\,|\,I_X(t)]}{\det \Sigma[X(t+h)\,|\,I_{XY}(t)]}\right] \tag{5}$$

defines the *causality measure at horizon* h from Y to X, given I. Similarly, the function

$$C(X \to Y\,|\,I) = \ln\left[\frac{\det \Sigma[Y(t+h)\,|\,I_Y(t)]}{\det \Sigma[Y(t+h)\,|\,I_{XY}(t)]}\right]$$

defines the *causality measure* from X to Y, at horizon h given I.

For $m_1 = m_2 = 1$, definition 2.1 reduces to

$$C(Y \to X\,|\,I) = \ln\left[\frac{\sigma^2[X(t+h)\,|I_X(t)]}{\sigma^2[X(t+1)\,|\,I_{XY}(t)]}\right],\; C(X \to Y\,|\,I) = \ln\left[\frac{\sigma^2[Y(t+h)\,|I_Y(t)]}{\sigma^2[Y(t+h)\,|\,I_{XY}(t)]}\right].$$

$C(Y \to X\,|\,I)$ (resp. $C(X \to Y\,|\,I)$) measures the degree of the causal effect from Y to X (resp. X to Y) given I and the past of Y (resp. X).

2.2 Causality measure based on VARMA models in term of impulse response functions

Consider three second-order stationary time series variables, $X(t)$, $Y(t)$, and $S(t)$. Let $W(t) = (X(t)', Y(t)', S(t)')' \in L^2$. Assume that $W(t)$ has the following VARMA (p,q) representation:

$$\Phi(L)W(t) = \Theta(L)u(t) \tag{6}$$

where, $m = m_1 + m_2 + m_3$

$$\Phi(L) = \begin{pmatrix} \varphi_{XX}(L) & \varphi_{XY}(L) & \varphi_{XS}(L) \\ \varphi_{YX}(L) & \varphi_{YY}(L) & \varphi_{YS}(L) \\ \varphi_{SX}(L) & \varphi_{SY}(L) & \varphi_{SS}(L) \end{pmatrix}; \qquad \Theta(L) = \begin{pmatrix} \theta_{XX}(L) & \theta_{XY}(L) & \theta_{XS}(L) \\ \theta_{YX}(L) & \theta_{YY}(L) & \theta_{YS}(L) \\ \theta_{SX}(L) & \theta_{SY}(L) & \theta_{SS}(L) \end{pmatrix}$$

$$\varphi_{ll}(L) = I_{ml} - \sum_{i=1}^{p} \varphi_{lli} L^i, \qquad \varphi_{lk}(L) = -\sum_{i=1}^{p} \varphi_{lki} L^i,$$

$$\theta_{ll}(L) = I_{ml} - \sum_{j=1}^{q} \theta_{llj} L^j, \qquad \theta_{lk}(L) = \sum_{i=1}^{q} \theta_{lkj} L^j,$$

for $l \neq k$ and $l, k = X, Y, S$,

$E[u(t)] = 0, E[u(t)u(s)'] = \begin{cases} \Sigma_u & \text{for s=t} \\ 0 & \text{for s}\neq\text{t} \end{cases}$

Σ_u is a symmetric positive definite matrix and u(t) is assumed orthogonal to $\{W(s),\ s \leq t\}$. If the process $W(t)$ is stationary, it has a $MA(\infty)$ representation, which can been written as follows:

$$W(t) = \Psi(L)u(t), \tag{7}$$

$$\Psi(L) = \Phi(L)^{-1}\Theta(L) = \sum_{j=0}^{\infty} \Psi_j L^j = \sum_{j=0}^{\infty} \begin{pmatrix} \Psi_{XXj}(L) & \Psi_{XYj}(L) & \Psi_{XSj}(L) \\ \Psi_{YXj}(L) & \Psi_{YYj}(L) & \Psi_{YFj}(L) \\ \Psi_{SXj}(L) & \Psi_{SYj}(L) & \Psi_{SFj}(L) \end{pmatrix} \tag{8}$$

where $\Psi_0 = I_m$.

A vector moving average, $VMAR(\infty)$, representation of the unconstrained model is written as

$$W(t+h) = \Psi(L)u(t+h) = \sum_{j=0}^{\infty} \Psi_j L^j u(t+h)$$

and the forecasting error of $W(t+h)$ is

$$U(W(t+h) \mid I_W(t)) = W(t+h) - E[W((t+h) \mid I_W(t))] = \sum_{j=0}^{h-1} \Psi_j u(t+h-j).$$

The covariance matrix of $W(t+h)$ is

$$\Sigma[W(t+h) \mid I_W(t)] = \sum_{j=0}^{h-1} \Psi_j \Sigma_u \Psi_j'$$

The variance-covariance matrix of the unconstrained forecast error of $X(t+h)$ is

$$\Sigma[X(t+h) \mid I_W(t)] = \sum_{j=0}^{h-1} J_1 \Psi_j \Sigma_u \Psi_j' J_1'$$

and the variance-covariance matrix of the unconstrained forecast error of $Y(t+h)$ is

$$\Sigma[Y(t+h) \mid I_W(t)] = \sum_{j=0}^{h-1} J_2 \Psi_j \Sigma_u \Psi_j' J_2'$$

where $J_1 = (I_{m1}, 0, 0)$, and $J_2 = (0, I_{m2}, 0)$

Similarly, the constrained model is

$$W_0(t) = \sum_{i=0}^{\infty} \overline{\Psi}_j L^j \epsilon(t)$$

, the forecasting error of the constrained model $W_0(t+h)$ is

$$U_0\left(W_0(t+h) \mid I_{W_0}(t)\right) = W_0(t+h) - E\left[W_0\left((t+h) \mid I_{W_0}(t)\right)\right] = \sum_{j=0}^{h-1} \overline{\Psi}_j \epsilon(t+h-j)$$

and its variance-covariance is

$$\Sigma\left[W_0(t+h) \mid I_{W_0}(t)\right] = \sum_{j=0}^{h-1} \overline{\Psi}_j \Sigma_\epsilon \overline{\Psi}_j^{'}$$

The variance-covariance of the constrained forecast error of $X(t+h)$ is

$$\Sigma\left[X(t+h) \mid I_{W_0}(t)\right] = \sum_{j=0}^{h-1} J_0 \overline{\Psi}_j \Sigma_\epsilon \overline{\Psi}_j^{'} J_0$$

with $J_0 = (I_{m1}, 0)$.

Under a VARMA representation in (6) and invertibility, Theorem 5.1 in Dufour & Taamouti (2010) shows that the causality measure from Y to X at horizon $h \geq 1$, in terms of reduced-form impulse responses, is:

$$C(Y \xrightarrow{h} X \mid I) = \ln\left[\frac{\det\left(\sum_{j=0}^{h-1} J_0 \overline{\Psi}_j \Sigma_\epsilon \overline{\Psi}_j^{'} J_0\right)}{\det\left(\sum_{j=0}^{h-1} J_1 \Psi_j \Sigma_u \Psi_j^{'} J_1^{'}\right)}\right] \tag{9}$$

with $J_1 = (I_{m1}, 0, 0)$ and $J_0 = (I_{m1}, 0)$.

If S is not taken into account in the measure (i.e., $m_3 = 0$), equation (9) is a measure of unconditional causality from Y to X.

2.3 Estimation and inference

Assume that for each variable we have T observations. Also assume that our $ARMA(p,q)$ model is equivalent to a VAR model with infinite order, $\Phi(L)W(t) = u(t)$, that can be approximated by a VAR representation with finite order. That order $k(T)$ may depend on the sample size. The finite order VAR representation can be written as

$$\Phi_k(L)W(t) = u(t) \tag{10}$$

The order of the process is chosen using the Akaike information criteria (AIC). The ordinary least squares (OLS) estimators of unknown parameters of the $VAR\left[k(T)\right]$ model are estimated. These estimates are used to compute the impulse response estimators $\widehat{\Psi}_k$ of the unrestricted model. The same methodology is used in the estimation of the impulse response estimators of the restricted model $\widehat{\overline{\Psi}}_k$. The OLS estimators of the variance-covariance matrix of the forecasting error of the unrestricted model $\widehat{\Sigma}_{u,k}$ and restricted model $\widehat{\Sigma}_{\epsilon,k}$ are used as estimators of a corresponding unknown variance-covariance matrix of forecasting errors. Those estimators replace the unknown parameters in the causality measure to obtain an estimator of the causality measure.

An estimator of the causality measure from Y to X at horizon $h \geq 1$, in terms of reduced-form impulse responses, is:

$$\widehat{C}(Y \xrightarrow{h} X \mid I) = \ln \left[\frac{\det \left(\sum_{j=0}^{h-1} J_0 \widehat{\widehat{\Psi}}_{j,k} \widehat{\Sigma}_{\epsilon,k} \widehat{\widehat{\Psi}}'_{jk} \; J_0 \right)}{\det \left(\sum_{j=0}^{h-1} J_1 \widehat{\Psi}_{j,k} \widehat{\Sigma}_{u,k} \widehat{\Psi}'_{jk} J'_1 \right)} \right]$$

with $J_1 = (I_{m1}, 0, 0)$ and $J_0 = (I_{m1}, 0)$.

Proposition 8.1 of Dufour & Taamouti (2010) shows that, under some regularity conditions,

$$\sqrt{T} \left[\widehat{C} \left(X \xrightarrow{h} Y|I \right) - C \left(X \xrightarrow{h} Y|I \right) \right] \xrightarrow{d} N(0, \Sigma_c(h))$$

where

$$\Sigma_c(h) = \frac{\partial C \left(X \xrightarrow{h} Y|I \right)}{\partial \delta} \Sigma_\delta \frac{\partial C \left(X \xrightarrow{h} Y|I \right)'}{\partial \delta}$$

$\delta = [\text{vec}(\Phi)', \text{vech}(\Sigma_u)']$ and Σ_δ is the asymptotic variance-covariance matrix of $\widehat{\delta}$.

2.4 Bootstrap

The bootstrap method to build confident intervals is proposed by Dufour & Taamouti (2010).

1. Let $W = (X', Y', S')'$. Estimate the $VAR(p)$ model, $W(t+h) = \Phi(L)W(t) + \epsilon(t+h)$ and save the OLS estimator of the parameters $\widehat{\Phi}(L)$ and the residual $\widehat{\epsilon}(t+h)$. Let $\{\epsilon^*(t) = (\epsilon^*_1(t+h), ...\epsilon^*_N(t+h))\}$ denote the bootstrap sample from $\left\{ \widehat{\epsilon}(t+h) = W(t+h) - \widehat{\Phi}(L)W(t) \right\}$

2. . Generate the bootstrap panel data using the following bootstrap data-generating process:

$$W^*(t+h) = \widehat{\Phi}(L)W(t) + \epsilon^*(t+h)$$

3. Estimate the $VAR(p)$ model, $W^*(t+h) = \Phi(L)^* W^*(t) + \xi(t+h)$, and save the bootstrap OLS estimator $\widetilde{\Phi}^*(L)$ and bootstrap residual $\widetilde{\epsilon}^*(t+h)$.

4. Let $W_0 = (X', S')'$. Estimate the constrained model, $W^*_0(t+h) = \Phi_0(L)^* W^*_0(t) + \epsilon^*(t+h)$ using the bootstrap sample W^*.

5. Calculate, at step j, the bootstrap causality measures at horizon h, denoted by $\widetilde{C}^{(j)*} \left(X \xrightarrow{h} Y|I \right)$

6. Repeat steps 2 through 5 B times, and save the bootstrap distribution of the measure of causality.

Proposition 8.2 of Dufour & Taamouti (2010) shows that under some regularities conditions,

$$\sqrt{T} \left[\widetilde{C}^* \left(X \xrightarrow{h} Y|I \right) - \widehat{C} \left(X \xrightarrow{h} Y|I \right) \right] \xrightarrow{d} N(0, \Sigma_c(h))$$

where $\Sigma_c(h)$ is defined in the previous subsection.

3 Data and results

Our main objective is to assess empirically the causality measure between government spending and real GDP in the case of ZLB and analyze that causality measure when the exchange rate is taken into account in a ZLB period. The data are quarterly observations from 1959Q1 through 2015Q4 United States macroeconomic time series from McCracken & Ng (2015). This is an updated version of Stock & Watson (2012). The main source of data is the Federal Reserve Bank of St. Louis database. Outliers have been removed and the series have been transformed by the authors to induce stationarity. We use data from the panel data created by those authors to facilitate the replication of this paper. The US real GDP is expressed in billions of chained 2009 dollars, as is the real government consumption. We also use the exchange rate between Canada and the United States. All three times series have been transformed by the authors by applying the first difference of the log in the original data. Dickey-Fuller tests on each of those data (not presented in this paper) suggest that the first difference of each variable is stationary.

For simplicity, let G define stationary transformation of real government spending, GDP the stationary transformation of real GDP, and S the stationary transformation of exchange rate. In our VAR representation in (10), $W(t) = (GDP(t)', G(t)', S(t)')'$ for conditional causality, and $W(t) = (GDP(t)', G(t)')'$ for unconditional causality. The value k that represents the order of the VAR is chosen using AIC information criteria.

3.1 Empirical results

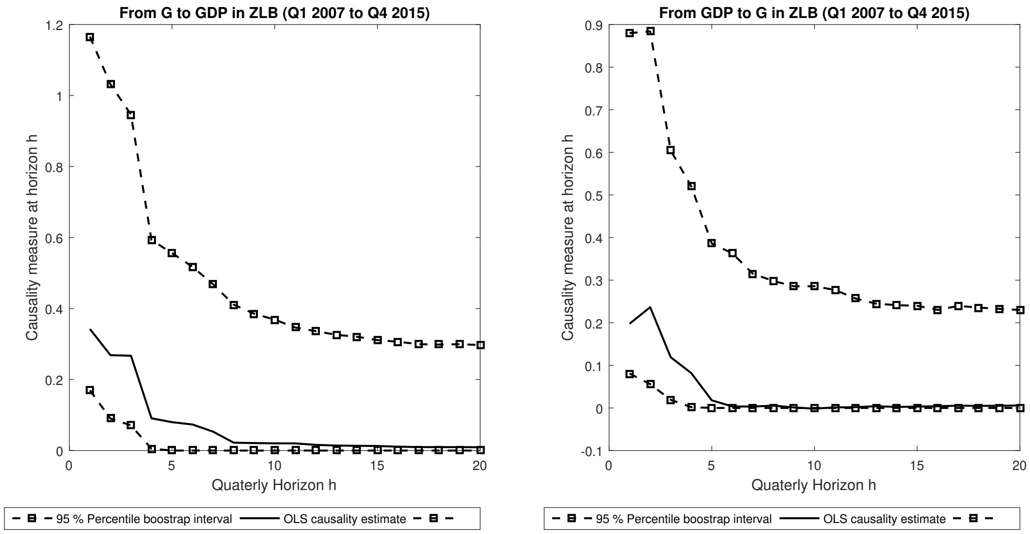
Figure (1) presents the unconditional causality measure between government spending (G) and real gross domestic product in a ZLB period. This is the causality measure between the two variables when the exchange rate is not taken into account. Data used to assess that period span from the first quarter of 2007 to the fourth quarter of 2015. The left-hand side presents the unconditional causality measure from government spending to GDP and the right-hand side presents the opposite. Figure (2) presents the same information in the period when the ZLB is not binding. Evidence in figure (1) shows that when the real exchange rate is not taken into account the unconditional causality measures between the two variables in ZLB are high and last longer (up to order h= 5 period-when the bootstrap confidence lower value is not zero), compared to unconditional causality measures when not in ZLB (see in figure2) where the value is approximately half.

In ZLB, when the exchange rate is taken into account (see figure 3), the causality measures are lower than in ZLB when the exchange rate is not taken into account. Additionally, the causality lasts less time (no longer than h=1 period ahead - when the bootstrap confidence lower value starts to take the value of zero). This suggests that the exchange rate may be a channel that cancels the causality between government spending and gross domestic product.

Figure (4) presents the causality measure between exchange rate and gross domestic product in ZLB, conditional on government spending. The figure shows direct causality between exchange rate and gross domestic product and vice versa, up to horizon two. Figure (5) presents the causality measure between exchange rate and government spending in ZLB, conditional on the gross domestic product. The figure shows that the exchange rate causes government spending up to horizon 2 and government spending causes exchange rate up to horizon 3.

Since unconditional causalities between government spending and gross domestic product are high in ZLB and the conditional causalities (when we control for exchange rate) between the two are low, given that the exchange rate causes the GDP and vice versa and that the exchange rate also causes the government spending and vice versa, we can conclude that the real exchange rate is the channel through which government spending causes the gross domestic product. In our case, it turns out that the causalities between exchange rate and GDP as well as between exchange rate and government spending

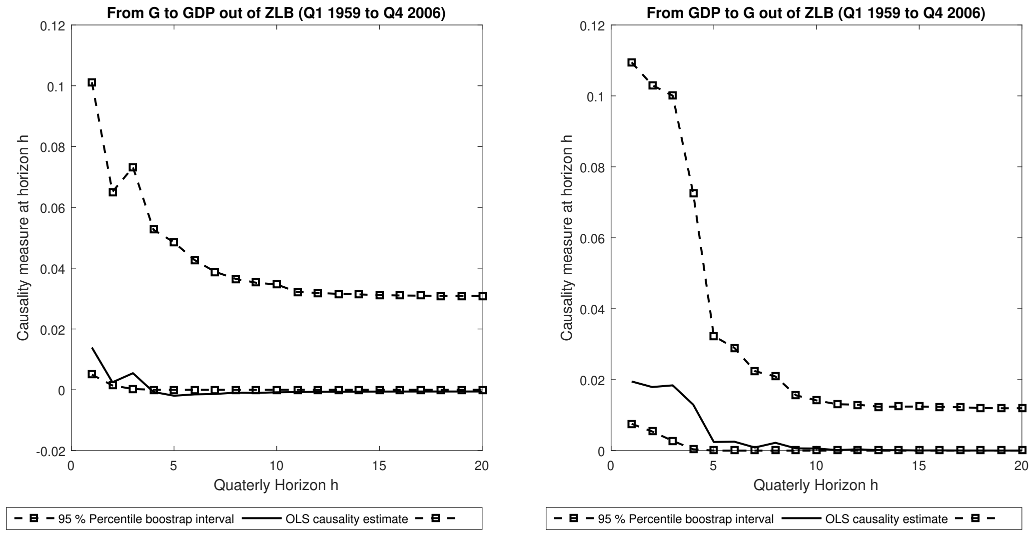
Figure 1: Unconditional causality measure between government spending and real gross domestic product in ZLB period (Q1 2007 to Q4 2015)- when the exchange rate is not taken into account



Note: These two figures present the causality measure between government spending and real gross domestic product in a ZLB period (Q1 2007 to Q4 2015) when the real exchange rate is not taken into account. The left-hand side presents the causality measure from government spending to GDP and the right-hand side the opposite. This figure offers evidence that when the real exchange rate is not taken into account the causality measures between the two variables are high and last longer, up to order h= 5 period, when the bootstrap confidence lower value is not zero. The model is $W(t) = \Phi_{0,k} + \sum_{i=1}^{k} \Phi_{i,k} W(t-i) + u(t)$ where $W(t) = (GDP(t)', G(t)')'$

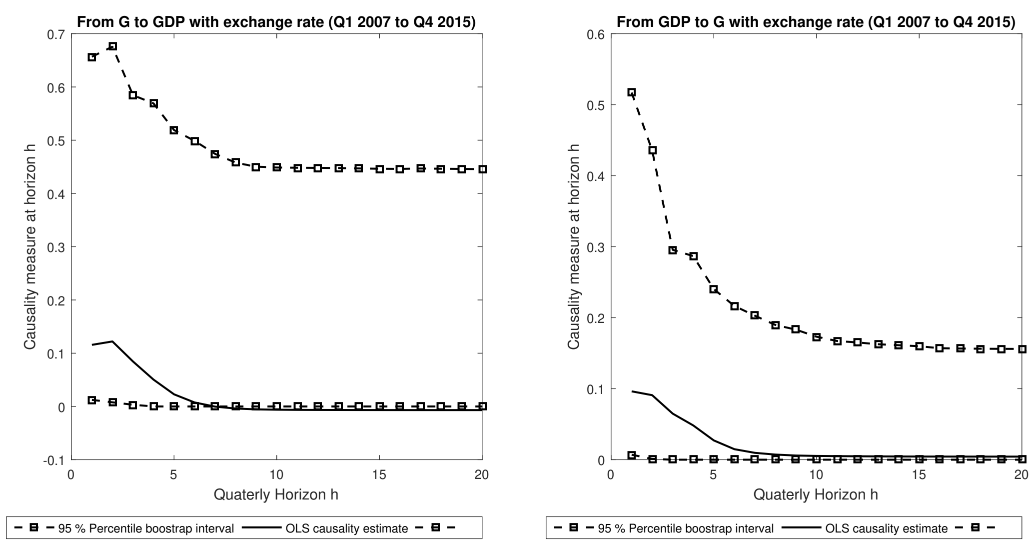
crowd out the direct causality between government spending and gross domestic product.

Figure 2: Unconditional causality measure between government spending and real gross domestic product out of ZLB period (Q1 1959 to Q4 2006), when the exchange rate is not taken into account



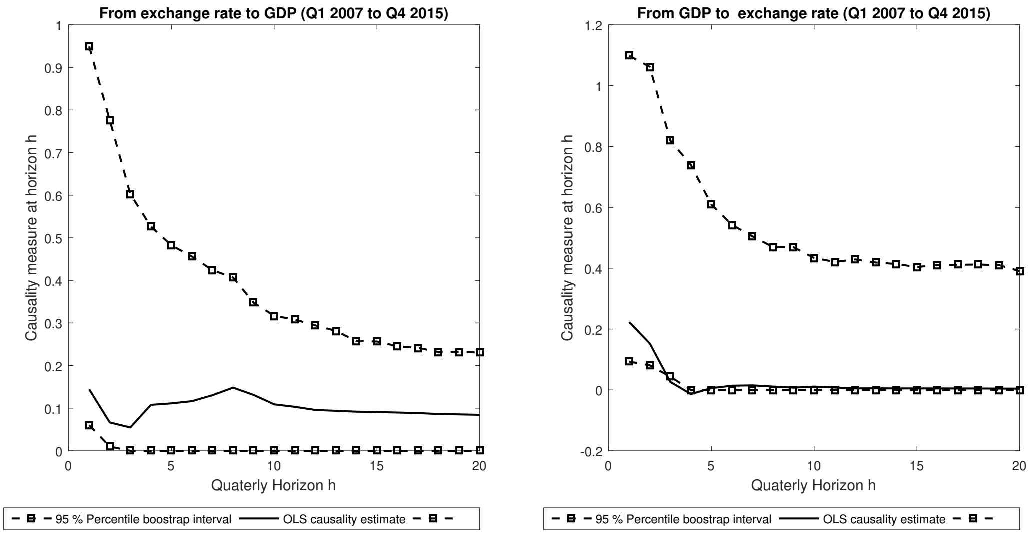
Note: These two figures present the causality measure between government spending and real gross domestic product out of a ZLB period (Q1 1959 to Q4 2006), when the real exchange rate is not taken into account. The left-hand side presents the causality measure from government spending to GDP and the right-hand side the opposite. This figure shows evidence that when the real exchange rate is not taken into account, the causality measure between government spending and GDP is lower than in the case of ZLB. The model is $W(t) = \Phi_{0,k} + \sum_{i=1}^{k} \Phi_{i,k} W(t-i) + u(t)$ where $W(t) = (GDP(t)', G(t)')'$

Figure 3: Conditional causality measure between government spending and real gross domestic product in ZLB period (Q1 2007 to Q4 2015)- when the exchange is taken into account



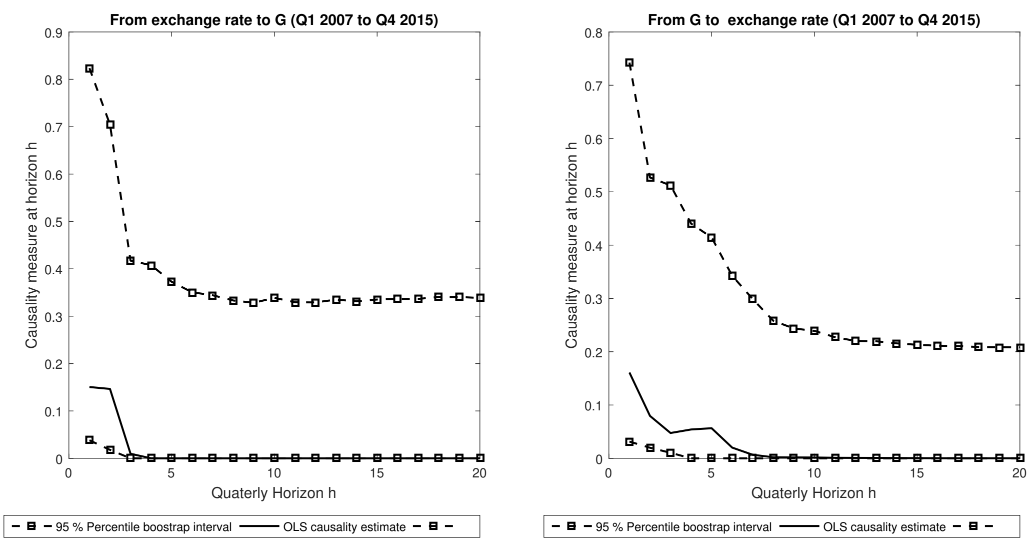
Note: These two figures present the causality measures in a ZLB period (Q1 2007 to Q4 2015) when the real exchange rate is taken into account. This is evidence that when the real exchange rate is taken into account, the causality measure is lower that when it is not taken into account. It also does not last long, no longer than h=1 period ahead, when the bootstrap confidence lower value starts to take the value of zero. The model is $W(t) = \Phi_{0,k} + \sum_{i=1}^{k} \Phi_{i,k} W(t-i) + u(t)$ where $W(t) = (GDP(t)', G(t)', S(t)')'$

Figure 4: Conditional causality between exchange rate and gross domestic product in ZLB period (Q1 2007 to Q4 2015)- when the government spending is taken into account



Note: These figures present the causality measure between exchange rate and gross domestic product in a ZLB period (Q1 2007 to Q4 2015) when the government spending is taken into account. The left-hand side presents the causality measure from exchange rate to GDP and the right-hand side the opposite. This figure shows evidence that when government spending is not taken into account, the causality measures from exchange rate to GDP and from GDP to exchange rate are both statistically significant from 0 until two periods. The model is $W(t) = \Phi_{0,k} + \sum_{i=1}^{k} \Phi_{i,k} W(t-i) + u(t)$ where $W(t) = (GDP(t)', G(t)', S(t)')'$

Figure 5: Conditional causality between exchange rate and government spending in ZLB period (Q1 2007 to Q4 2015)- when gross domestic product is taken into account



Note: These figures present the causality measure between exchange rate and government spending in a ZLB period (Q1 2007 to Q4 2015) when the gross domestic product is taken into account. The left-hand side presents the causality measure from exchange rate to government spending and the right-hand side the opposite. This figure shows evidence that when the GDP is taken into account, the causality measures from exchange rate to government spending last longer than 2 periods and the causality measure from government spending to exchange rate lasts about 6 periods. $W(t) = \Phi_{0,k} + \sum_{i=1}^{k} \Phi_{i,k} W(t-i) + u(t)$ where $W(t) = (GDP(t)', G(t)', S(t)')'$

4 Conclusion

The aim of this paper is to assess, for the United States of America, the Granger causality measure between government spending and real GDP and to compare the value obtained in ZLB to that obtained out of ZLB. We used quarterly data from the period 1959Q1 to 2015Q4 for the United States. Variables used include the real GDP, government spending, and the real exchange rate between the United States and Canada. The Granger causality measure proposed by Dufour & Taamouti (2010) is used to compare the measure of causality for a normal period (1959Q1 to 2006Q4) with the measure of causality obtained for ZLB periods (2007Q1 to 2015Q4). We emphasize the role played by the real exchange rate. Our results present evidence that in ZLB, the Granger causality measure is stronger and more persistent if the exchange rate is not taken into account but becomes lower and does not last very long if the real exchange rate is taken into account.

References

Christiano, L., Eichenbaum, M., & Rebelo, S. (2011). When is the government spending multiplier large? *Journal of Political Economy*, *119*(1), 78–121.

Dufour, J.-M., & Renault, E. (1998). Short run and long run causality in time series: theory. *Econometrica*, 1099–1125.

Dufour, J.-M., & Taamouti, A. (2010). Short and long run causality measures: Theory and inference. *Journal of Econometrics*, *154*(1), 42–58.

Granger, C. W. (1969). Investigating causal relations by econometric models and cross-spectral methods. *Econometrica: Journal of the Econometric Society*, 424–438.

Ilzetzki, E., Mendoza, E. G., & Végh, C. A. (2013). How big (small?) are fiscal multipliers? *Journal of monetary economics*, *60*(2), 239–254.

Karras, G. (2012). Trade openness and the effectiveness of fiscal policy: some empirical evidence. *International Review of Economics*, *59*(3), 303–313.

Mao Takongmo, C. O. (2017). Government-spending multipliers and the zero lower bound in an open economy. *Review of International Economics, https:/doi.org/10.1111/roie.12295*.

McCloskey, D. N., & Ziliak, S. T. (1996). The standard error of regressions. *Journal of Economic Literature*, *34*(1), 97–114.

McCracken, M. W., & Ng, S. (2015). Fred-md: A monthly database for macroeconomic research. *Journal of Business & Economic Statistics*(just-accepted).

Song, X., & Taamouti, A. (2016). Measuring nonlinear granger causality in mean. *Journal of Business & Economic Statistics*(just-accepted), 1–37.

Stock, J. H., & Watson, M. W. (2012). Disentangling the channels of the 2007-09 recession. *Brookings Papers on Economic Activity*, 120–157.

Wiener, N. (1956). The theory of prediction. *Modern mathematics for engineers*, *1*, 125–139.

Zhang, H. J., Dufour, J.-M., & Galbraith, J. W. (2016). Exchange rates and commodity prices: measuring causality at multiple horizons. *Journal of Empirical Finance*, *36*, 100–120.