Corporate social responsibility and privatization policy in a mixed oligopoly

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Abstract  
This article formulates a mixed oligopoly in which a public firm competes with two private firms that may adopt corporate social responsibility (CSR). We investigate the optimal privatization policy and find that, depending on the magnitude of CSR, the optimality of either nationalization or full privatization can hold. In particular, we show that the optimal degree of privatization is decreasing in the magnitude of CSR and thus nationalization can be optimal if they have homogeneous objectives. Under significant heterogeneity of the objectives among firms, however, the optimal degree of privatization is non-monotone with the magnitude of CSR, but full privatization can be optimal. This result suggests that the optimal privatization policy depends on both the magnitude of CSR and the heterogeneity of the objectives among private firms.  

JEL Classification: L13; D45; H23  

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1. Introduction

The trend of privatization has been increasing worldwide since the 1980s. Nevertheless, public and semi-public (partially privatized) firms are still important player in the global economic arena and highly concentrated in a few sectors with large portions of the world’s resources.\(^1\) In particular, public firms successfully compete with their private counterparts in a wide range of industries such as health care, education, financing (e.g., banking and insurance), transportation (e.g., railways and airlines), telecommunications, electricity, power generation, natural gas, and other energy-related industries. Given that it has become a common theme, privatization in such industries has attracted extensive policy attention from economics researchers in developed, developing, and transitional economies such as Eastern Europe, Latin America, and Asia including China.\(^2\)

Early analysis of mixed oligopolies dates back to at least Merrill and Schneider (1966), although it is only more recently that the literature in this field has become richer and more diverse. The literature on mixed oligopolies adopted the partial privatization model first formulated by Matsumura (1998), who discussed the optimal degree of privatization in mixed oligopolies. Since then, models of partial privatization have been analyzed within a rapidly growing literature and numerous recent works showed that various factors affect the

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1 Referring to report by the OECD, Kowalski et al. (2013) noted that of the 2,000 largest public companies in the world, more than 10% are either public firms or have significant government ownership; these government-associated companies’ sales are equivalent to approximately 6% of the global GDP. More than half (in terms of value) of all public firms in OECD countries are significant players in energy-related industries.

2 China has adopted privatization policy to reform its public firms since 1978 and thus, Chinese economists have paid considerable attention to the privatization and liberalization policies in social market economies.
optimal degree of privatization. However, most papers assumed that private enterprises having homogeneous objectives maximize their own profits.

On the other hand, corporate social responsibility (CSR) has now become the global mainstream business strategy. According to KPMG (2008, 2013), nearly 80% of the 250 largest companies worldwide issued CSR reports in 2008 and more than 30% (71% and 90%) of companies in the US (the UK and Japan, respectively) adopted CSR in 2013. While a large number of firms in the world issue various CSR statements/activities, many of them belong to the industries characterized as mixed oligopolies in which CSR-firms compete with public firms. It represents that the heterogeneity of objectives among private firms emerges as an important research topic in the literature.

The recent topic on CSR has received increasing attention from broad research in both empirical and theoretical analyses in the areas of business and economics. For example, some theoretical papers including Goering (2012, 2014), Kopel and Brand (2012), Brand and Grothe (2013, 2015), Nakamura (2014), Chang, et al. (2014), Kopel (2015), Lambertini and Tampieri (2015) and Bian, et al. (2016) analyzed different models of oligopolies where profit-maximizing private firms compete with other private firms that adopt CSR activities. In particular, they utilized a model in which the private firm adopts consumer surplus as a proxy of its own CSR concerns. Then, a CSR-related incentive combines both profitability

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4 Matsumura and Okamura (2015) investigated a model in which private firms are concerned with other private firms' profits. However, they did not consider the heterogeneity of the objectives among the private firms.

5 For the intensive discussions on the economics of CSR, see Kitzmueller and Shimshack (2012), Crifo and Forget (2012) and Liu, et al. (2015).
and consumer surplus, and thus the objective of CSR-firm is a convex combination of consumers’ surplus and its own profits.

Taking account of the fact that CSR is a growing trend in many industries including those characterized by mixed oligopolies, in which the investigation of privatization policies is significant, it is restrictive to assume that private firms have homogeneous objectives, wherein they maximize only their own profits. Therefore, it is urgent to examine the interactions between CSR activities of the private firms and optimal privatization policies on the public firm. In this research line, we adopt the formal approach of CSR and investigate how the heterogeneity of the objectives among the private firms affects the optimal privatization policy in a mixed oligopoly in which a public firm competes with CSR-firms.

We show that, depending on the magnitude of CSR, either nationalization or full privatization can be the optimal policy. This result is strikingly contrast to the previous literature in mixed oligopoly. In fact, by focusing on different aspects of economic phenomena, many researches have analyzed the optimal partial privatization, which is firstly shown by Matsumura (1998) under moderate conditions in a homogeneous mixed duopoly, and proved that the optimality of partial privatization is strikingly robust.\footnote{As exceptional works, Matsumura and Kanda (2005) and Xu, et al. (2017) provided the rationale on full nationalization policy in a free entry mixed oligopoly market. However, Cato and Matsumura (2012, 2015) showed that partial privatization is always optimal when the competitors are foreign.}

The main contribution of our paper is to provide different results by investigating the optimal privatization policy in a mixed oligopoly with the CSR-firms. In particular, we find that the optimal degree of privatization is decreasing in the magnitude of CSR in private firms and thus nationalization can be optimal if they have homogeneous objectives. If there is significant heterogeneity of the objectives among private firms, however, the optimal degree of privatization is non-monotone with respect to the magnitude of CSR but full.
privatization can be optimal. This result suggests that the optimal privatization policy depends on both the degree of CSR and the homogeneity of objectives among private firms. This highlights the role of homogeneity of objectives among private firms in choosing an optimal privatization policy and its effects on the non-monotonicity in privatization policies.

This paper is organized as follows. Section 2 constructs the basic model of mixed oligopolies with CSR. In section 3, we analyze the model and examine two specific cases where (i) both private firms have the homogenous CSR and (ii) one firm is a profit-maximizer while the other firm adopts CSR. The final section concludes the paper.

2. Model

We consider a mixed triopoly in which one public firm and two private firms that might engage in CSR activities produce homogenous products. The inverse demand function is given by $P = A - Q$, where $Q = q_0 + \sum_{i=1}^{2} q_i$ denotes market outputs, and $q_0$ and $q_i$ are the outputs of the public firm and private firm $i$ ($i = 1, 2$), respectively. Each firm’s cost function is assumed to be quadratic, $c_i(q_i) = \frac{1}{2} q_i^2$. Then, social welfare is defined as

$W = CS + \sum_{i=0}^{2} \pi_i$ where $CS = \frac{Q^2}{2}$ is the consumer surplus and $\pi_i = Pq_i - c_i(q_i)$ is each firm’s profit.

The payoff of public firm 0 is given by $U_0 = (1-\theta)W + \theta\pi_0$, where $\theta$ ($0 \leq \theta \leq 1$) represents the degree of privatization, which is determined by the welfare-maximizing

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7 The model with the linear demand and quadratic cost functions is considered as a standard and is popularly used in the literature on mixed oligopolies to rule out the uninteresting case of a public monopoly. De Fraja and Delbono (1989) and Matsumura and Okamura (2015) provided the economic rationale behind this formulation.
benevolence. That is, $\theta = 0$ indicates nationalization, $\theta = 1$ indicates full privatization, and $\theta \in (0,1)$ indicates partial privatization.

On the other hand, the payoff of the private firm is given by $U_i = \pi_i + \alpha_i CS$ where $\alpha_i$ ($\alpha_i \geq 0$) represents the degree of CSR, which is exogenously given. That is, CSR implies that the private firm is interested in consumers’ welfare in addition to its profit. Thus, when a private firm engaged in CSR or altruistic concern places a weight on consumer surplus in its objective function, it is analogous to assuming that the firm places a higher weight on output. Here, $\alpha_i = 0$ indicates a pure profit-maximizing private firm. We assume that $0 \leq \alpha_1 + \alpha_2 < 2$ to assure the interior solutions (i.e., all firms produce positive outputs) in the equilibrium.

The two-stage game runs as follows: In the first stage, the government determines the optimal level of privatization, $\theta$, to maximize social welfare. In the second stage, after observing $\theta$, each firm chooses its output level $q_0$ and $q_i$ at the same time.

### 3. Analysis

In the second stage, assuming interior solutions, we have the following first-order conditions:

\[
\frac{\partial U_0}{\partial q_0} = A - (2 + \theta)q_0 - q_1 - q_2 = 0
\]  

(1)

\[
\frac{\partial U_i}{\partial q_i} = A - q_0 - 3q_1 - q_2 + \alpha_i(q_0 + q_1 + q_2) = 0
\]  

(2)

\[
\frac{\partial U_2}{\partial q_2} = A - q_0 - q_1 - 3q_2 + \alpha_2(q_0 + q_1 + q_2) = 0
\]  

(3)
The second-order conditions are satisfied. From (2) or (3), we obtain the reaction function of the private firm. It is noteworthy that when \( \alpha_i < 1 \), the reaction curve has a negative slope, which implies that outputs are strategic substitutes. Also, as \( \alpha_i \) approaches 1, the slope becomes less steep (and thus firm \( i \)'s best reply is less sensitive to the rivals’ output). When \( \alpha_i > 1 \), however, the reaction curve has a positive slope, which implies that outputs are strategic complements if the magnitude of CSR is high enough. It will affect the optimal choices of privatization policies.

Solving these first-order conditions simultaneously, we derive the equilibrium outputs as follows:

\[
q_0^* (\theta) = \frac{A(2-\alpha_i-\alpha_j)}{6 + 4\theta - (1+\theta)(\alpha_i + \alpha_j)}
\]

\[
q_i^* (\theta) = \frac{A[(1+\theta)(2-\alpha_j) + (3+\theta)\alpha_i]}{2[6 + 4\theta - (1+\theta)(\alpha_i + \alpha_j)]}
\]

\[
q_2^* (\theta) = \frac{A[(1+\theta)(2-\alpha_i) + (3+\theta)\alpha_j]}{2[6 + 4\theta - (1+\theta)(\alpha_i + \alpha_j)]}
\]

\[
Q^* (\theta) = \frac{2A(2+\theta)}{6 + 4\theta - (1+\theta)(\alpha_i + \alpha_j)}
\]

Note that CSR increases total industry outputs. Thus, when a CSR-firm places a higher weight on consumer surplus, it increases more outputs.

After necessary calculations on the profits and consumer surplus, we obtain the following welfare \( W^* (\theta) \) as a function of \( \theta \) and \( \alpha_i \).

\[
W^* (\theta) = \frac{A^2}{4[6 + 4\theta - (1+\theta)(\alpha_i + \alpha_j)]^2} \left[ 20 - (\alpha_i - \alpha_j)^2 - 8(\alpha_i + \alpha_j) \right] \theta^2 + \left[ 72 - 4(\alpha_i - \alpha_j)^2 - 28(\alpha_i + \alpha_j) \right] \theta + 52 - 7(\alpha_i^2 + \alpha_j^2) + 2\alpha_i\alpha_j - 12(\alpha_i + \alpha_j)
\]
In the first stage, when the government decides the optimal degree of privatization, we should consider the corner solutions because of the constraint \( 0 \leq \theta \leq 1 \). Then, the analysis of maximizing welfare provides the following three cases. The first case is \( \frac{\partial W^*(0)}{\partial \theta} \leq 0 \), which implies that the optimum is \( \theta^* = 0 \), that is, nationalization. The second case is \( \frac{\partial W^*(1)}{\partial \theta} \geq 0 \), which implies that the optimum is \( \theta^* = 1 \), that is, full privatization. The final case refers to partial privatization, which implies that the optimum is \( \theta^* \in (0,1) \). Then, it should satisfy the following first-order condition: \( \frac{\partial W^*(\theta)}{\partial \theta} = 0 \). In that case, we obtain the following optimal degree of privatization in the interior solutions:

\[
\theta^* = \frac{B(\alpha_i, \alpha_j)}{D(\alpha_i, \alpha_j)},
\]

where \( B = 5(\alpha_i^2 + \alpha_j^2) - 8(\alpha_i + \alpha_j) + 2\alpha_i\alpha_j + 4 \) and \( D = 12 - 6(\alpha_i + \alpha_j) - (\alpha_i - \alpha_j)^2 > 0 \). Then, we obtain the following proposition.\(^8\)

**Proposition 1** The optimal degree of privatization \( \theta^* \) depends on \( \alpha_i \) and \( \alpha_j \).

(i) \( \theta^* = 0 \) (Nationalization) if \( B \leq 0 \).

(ii) \( \theta^* = 1 \) (Full Privatization) if \( B \geq D \).

(iii) \( 0 < \theta^* < 1 \) (Partial Privatization) if \( D > B > 0 \).

[ Fig. 1. The Optimal Privatization \( \alpha_i, \alpha_j > 0 \) and \( 0 \leq \alpha_i + \alpha_j < 2 \). ]

Fig. 1 shows the optimal privatization policies when both private firms engage in CSR activities. It indicates that the shaded regions represent nationalization, the hatched regions

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\(^8\) The proofs of propositions are provided in the Appendix.
represent full privatization, and non-shaded regions represent partial privatization. Note that partial privatization is not the optimum when either $\alpha_i$ is very large or both $\alpha_i$ and $\alpha_j$ are not too small.

Next, we continue the analysis for two special, but interesting, cases.

3.1. When both private firms have the homogenous CSR: $0 \leq \alpha_1 = \alpha_2 < 1$

We consider the homogenous case that both CSR-firms have the same degree of CSR, that is, $0 \leq \alpha_1 = \alpha_2 = \alpha < 1$. Then, from the first-order conditions (1) ~ (3) in the second-stage, the reaction functions of each firm are as follows:

$$q_0 = \frac{A - q_1 - q_2}{2 + \theta}, \quad q_1 = \frac{A - (1 - \alpha)q_0 - (1 - \alpha)q_2}{3 - \alpha}, \quad q_2 = \frac{A - (1 - \alpha)q_0 - (1 - \alpha)q_1}{3 - \alpha}$$

Note that the strategies of all firms are strategic substitutes. As $\alpha$ increases, however, the output of the private firm becomes less sensitive to its rival’s output.

Solving these first-order conditions simultaneously, we derive the equilibrium outputs as follows:

$$q_0 = \frac{A(1 - \alpha)}{3 + 2\theta - \alpha - \alpha\theta}, \quad q_1 = \frac{A(1 + \alpha + \theta)}{2(3 + 2\theta - \alpha - \alpha\theta)}, \quad q_2 = \frac{A(1 + \alpha + \theta)}{2(3 + 2\theta - \alpha - \alpha\theta)}$$

and $Q = \frac{A(2 + \theta)}{3 + 2\theta - \alpha - \alpha\theta}$.

Again, CSR increases total industry outputs. Some comparative static effects with respect to $\alpha$ and $\theta$ are as follows:

$$\frac{\partial q_0}{\partial \alpha} < 0, \quad \frac{\partial q_0}{\partial \theta} < 0, \quad \frac{\partial q_1}{\partial \alpha} > 0, \quad \frac{\partial q_1}{\partial \theta} > 0, \quad \frac{\partial q_2}{\partial \alpha} > 0, \quad \frac{\partial q_2}{\partial \theta} > 0, \quad \frac{\partial Q}{\partial \alpha} > 0, \quad \frac{\partial Q}{\partial \theta} < 0.$$

9
These comparative static results show that an increase in the degree of CSR induces CSR-firms to produce more outputs, which in turn causes public firm to reduce its own output but increase the industry outputs. Meanwhile, an increase in the degree of privatization induces public firm to fewer outputs, which in turn, causes CSR firms to increase outputs, but reduces the industry outputs.

The profits of the public firm and CSR firms are as follows:

\[
\pi_0 = \frac{A^3(1-\alpha)^2(1+2\theta)}{2(3+2\theta-\alpha-\alpha\theta)^2}, \quad \pi_1 = \pi_2 = \frac{A^3(1+\alpha)(3+3\theta-5\alpha-4\alpha\theta)}{8(3+2\theta-\alpha-\alpha\theta)^2}.
\]  

(8)

Then, we obtain the following welfare as a function of \( \alpha \) and \( \theta \):

\[
W = \frac{A^2 \left[ (5-4\alpha)\theta^2 + 2(9-7\alpha)\theta + 13-6\alpha-3\alpha^2 \right]}{4(3+2\theta-\alpha-\alpha\theta)^2}.
\]  

(9)

Note that the welfare is decreasing (increasing) in \( \alpha \) when the degree of CSR is high (low). That is, \( \frac{\partial W}{\partial \alpha} \leq 0 \) when \( \alpha > \frac{\theta^2 + \theta + 2}{2\theta^2 + 5\theta + 6} = \hat{\theta} \in (0,1) \). This result shows that a higher degree of CSR is not always beneficial to society.

In the first stage, the optimal degree of privatization is provided by the following proposition.

**Proposition 2.** Suppose that \( 0 \leq \alpha_1 = \alpha_2 = \alpha < 1 \). The optimal degree of privatization is

\[
\theta^* = \max \left\{ \frac{1}{3} - \alpha, 0 \right\}.
\]
Proposition 2 implies that (i) the optimal degree of privatization is non-increasing in \( \alpha \), (ii) full privatization is never optimal, (iii) partial privatization is optimal when \( \alpha < \frac{1}{3} \), and (iv) full nationalization is optimal when \( \alpha > \frac{1}{3} \).

We explain the intuition behind this result. An increase in \( \theta \) reduces the output of the public firm, increases that of each CSR-firm, and lowers the total output. In other words, an increase in \( \theta \) induces production substitutions from the public firm to the private CSR-firms. This improves welfare as long as the marginal cost of the public firm is larger (i.e., the public firm produces more).\(^9\) When \( \alpha \) is large, marginal costs of the CSR-firms are higher (i.e., each CSR-firm produces more than the public firm), and thus, privatization never improves welfare. This is why full nationalization is optimal when \( \alpha \) is large. This result is in sharp contrast to that of Matsumura (1998) who showed that full nationalization is never optimal in mixed oligopolies.

The smaller the value of \( \alpha \) is, however, the higher the improvement in welfare because of the production substitution. This is why the optimal degree of privatization is non-increasing in \( \alpha \). Consequently, full privatization is never optimal because partial privatization is optimal when \( \alpha = 0 \), as Matsumura (1998) showed.

The monotone result in Proposition 2, however, depends on the assumption of homogenous objectives among the CSR-firms. Suppose that \( \alpha_2 = \frac{1}{2} \) in Fig. 1, for example. Then, we find that when \( \alpha_1 \) is close to zero, partial privatization is optimal. As \( \alpha_1 \) increases, the optimal privatization policy switches to full nationalization, then returns to

\(^9\) For an excellent explanation of the welfare-improving production substitution effect, see Lahiri and Ono (1988).
partial privatization, and finally switches to full privatization. This implies that the optimal degree of privatization can be non-monotone if we allow payoff heterogeneity between the CSR-firms. In the next subsection, we investigate the simplest and thus solvable case to illustrate this point.

3.2. When one firm is a profit-maximizer: \( 0 = \alpha_2 < \alpha_i < 2 \)

We consider a heterogeneous case in which one firm, say firm 2, is a pure profit-maximizer and thus it does not participate in CSR activities, that is, \( 0 = \alpha_2 < \alpha_i < 2 \). Then, from the first order conditions (1) ~ (3) in the second-stage, the reaction functions of each firm are as follows:

\[
q_0 = \frac{A - q_1 - q_2}{2 + \theta}, \quad q_i = \frac{A - (1 - \alpha_i)q_0 - (1 - \alpha_i)q_2}{3 - \alpha_i}, \quad q_2 = \frac{A - q_0 - q_1}{3}. \quad (10)
\]

Note that from the reaction function of CSR-firm 1, the rival’s output is a strategic substitute to the CSR-firm if \( 0 < \alpha_i < 1 \), while it is strategic complement if \( 1 < \alpha_i < 2 \). Furthermore, as \( \alpha_i \) increases, the output of CSR-firm 1 is less sensitive to its rival’s output when \( 0 < \alpha_i < 1 \), while it is more sensitive when \( 1 < \alpha_i < 2 \). It represents that there exists a counter effect of CSR-firm, depending on the magnitude of CSR.

Solving these reaction functions simultaneously, we derive the equilibrium outputs as follows:

\[
q_0 = \frac{A(2 - \alpha_i)}{6 + 4\theta - \alpha_i - \alpha_i\theta}, \quad q_i = \frac{A(2 + 2\theta + \alpha_i\theta + 3\alpha_i)}{2(6 + 4\theta - \alpha_i - \alpha_i\theta)}, \quad q_2 = \frac{A(2 + 2\theta - \alpha_i - \alpha_i\theta)}{2(6 + 4\theta - \alpha_i - \alpha_i\theta)},
\]

and \( Q = \frac{2A(2 + \theta)}{6 + 4\theta - \alpha_i - \alpha_i\theta}. \quad (11) \)
Again, CSR increases total industry outputs. Some comparative static effects with respect to $\alpha_i$ and $\theta$ are as follows:

$$\frac{\partial q_0}{\partial \alpha_i} < 0, \frac{\partial q_0}{\partial \theta} < 0, \frac{\partial q_1}{\partial \alpha_i} > 0, \frac{\partial q_1}{\partial \theta} > 0, \frac{\partial q_2}{\partial \alpha_i} < 0, \frac{\partial q_2}{\partial \theta} > 0, \frac{\partial Q}{\partial \alpha_i} > 0, \frac{\partial Q}{\partial \theta} < 0.$$

These comparative static results show that an increase in the degree of CSR induces CSR-firm to produce more outputs, but both public and private firms reduce their outputs. However, it decreases the price of goods because the industry output increases. Meanwhile, an increase in the degree of privatization induces public firm to produce fewer outputs, but other private firms increase outputs, which in turn reduces the industry outputs.

Then, this gives the following respective profits of each firm:

$$\pi_0 = \frac{A^2(2 - \alpha_i)^2(1 + 2\theta)}{2(6 + 4\theta - \alpha_i - \alpha_i\theta)^2},$$

$$\pi_1 = \frac{A^2[(2 + 2\theta + 3\alpha_i + \theta\alpha_i)(6 + 6\theta - 7\alpha_i - 5\theta\alpha_i)]}{8(6 + 4\theta - \alpha_i - \alpha_i\theta)^2}$$

and

$$\pi_2 = \frac{3[A^2(1 + \theta)^2(2 - \alpha_i)]}{8(6 + 4\theta - \alpha_i - \alpha_i\theta)^2} \quad (12)$$

Then, we obtain the following welfare as a function of $\alpha_i$ and $\theta$:

$$W = \frac{A^2(2 - \alpha_i)[(10 + \alpha_i)\theta^2 + 4(9 + \alpha_i)\theta + 26 + 7\alpha_i]}{4(6 + 4\theta - \alpha_i - \alpha_i\theta)^2} \quad (13)$$

Note that the welfare is decreasing (increasing) in $\alpha_i$ when the magnitude of CSR is high (low). That is, $\frac{\partial W}{\partial \alpha_i} < 0$ when $\alpha_i \geq \frac{\theta^2 + \theta + 2}{2(\theta^2 + 3\theta + 3)} \equiv \bar{\alpha}_i \in (0,1)$. This result indicates that a
higher magnitude of CSR is not always beneficial to the society. In particular, if the magnitude of CSR is sufficiently high, the output of CSR-firm is too large for social welfare because the larger output of CSR-firm reduces the other firms' outputs. Because the marginal cost of the CSR-firm is high due to its larger output, this production substitution from non-CSR firms to the CSR-firm is not beneficial to the society.

In the first stage, the optimal degree of privatization is provided by the following proposition.

**Proposition 3.** Suppose that $0 < \alpha_2 < \alpha_1 < 2$. (i) When $0 < \alpha_1 < \frac{4}{3}$, the optimal degree of privatization is

$$\theta^* = \frac{5\alpha_1^2 - 8\alpha_i + 4}{12 - (6 + \alpha_i)\alpha_i}.$$  
Thus, partial privatization is optimal and it takes the U-shape in $\alpha_i$. (ii) Otherwise, full privatization ($\theta^* = 1$) is optimal.

[ Fig. 2. The Optimal Privatization when $0 = \alpha_2 < \alpha_i < 2$. ]

Proposition 3 implies that nationalization is never optimal under the heterogeneous case where one firm is a pure profit-maximizer but the other firm engages in CSR. For an expositional purpose, Fig.2 shows the optimal privatization when firm 1 is CSR-firm and firm 2 is a pure-profit-maximizer. It shows that as $\alpha_i$ increases, the optimal degree of privatization gradually decreases until $\alpha_i$ reaches $\hat{\alpha}$, and then increases toward 1 until $\alpha_i$ reaches $\overline{\alpha} = \frac{4}{3}$.

Propositions 3 is in sharp contrast to Proposition 2. When private firms' objectives are homogenous, full privatization is never optimal and the optimal degree of privatization is non-increasing in $\alpha$ (Proposition 2). In contrast, when one firm is concerned with CSR
and the other does not, full privatization can be optimal and the optimal degree of privatization is non-monotone (Proposition 3). We explain the intuition behind these results.\textsuperscript{10}

Suppose that $\alpha_1 < 1$. Because both private firms follow strategic substitution, an increase in $\theta$ induces production substitution from the public firm to the private firms and it improves welfare as long as the marginal costs of the private firms are lower than those of the public firm (i.e., the public firm produces more than the private firms). At the same time, an increase in $\theta$ reduces total output as well as welfare. The optimal degree of privatization is determined by these trade-offs between production substitution effect and total output effect. An increase in $\alpha_1$ makes the reaction curve of firm 1 less steep, and thus, firm 1’s output is less sensitive to firm 0’s. Therefore, a slight increase in $\alpha_1$ from zero weakens the welfare-improving production substitution effect, and thus reduces the optimal degree of privatization. When $\alpha_1$ becomes larger, the output of firm 2 is smaller and production substitution from firm 0 to firm 2 improves welfare more significantly. Thus, when $\alpha_1$ reaches the critical value, the optimal degree of privatization increases in $\alpha_1$ to stimulate the production of firm 2.

On the other hand, suppose that $\alpha_1 > 1$. Then, firm 1’s reaction curve has a positive slope, and its output is increasing in $\theta$. At this stage, firm 1’s production is too high from the viewpoint of social welfare. Although an increase in $\theta$ reduces total output and it induces welfare loss, it increase the output of firm 2 and reduces the output of firm 1, and

\textsuperscript{10} A rigorous explanation is provided in the Appendix B.
both improve welfare. Therefore, an increase in $\alpha_1$ further improves welfare. When $\alpha_1 \geq \frac{4}{3}$, this effect is so strong that the optimal degree of privatization is one.

4. Concluding remarks

This paper investigates a mixed oligopoly in which one public firm competes with two private firms that may adopt CSR. This approach links two existing lines of related works. The first line comprises the literature on optimal privatization policies in a mixed oligopoly, and the second line refers to the literature on CSR.

We find that, depending on the magnitude of CSR, full nationalization and partial or full privatization can be optimal privatization policies. In particular, we show that if private firms adopt a homogenous level of CSR, the optimal degree of privatization is non-increasing in the magnitude of CSR and full nationalization can be optimal whereas full privatization is never optimal. In contrast, if only one firm adopts CSR and the other is a profit-maximizer, the optimal degree of privatization is non-monotone with respect to the magnitude of CSR and full privatization can be optimal, whereas full nationalization is never optimal. Therefore, our results suggest that heterogeneity of objectives among private firms may play a crucial role in determining the optimal privatization policy.

There remains future research. First, although we adopt a specific CSR approach, the basic principle should be applied to various model formulations. For example, if we adopt the CSR approach investigate by Ghosh and Mitra (2014) and Matsumura and Ogawa (2014), in which the rival’s profit is included in the CSR-firm’s objective, or the payoff interdependent approach discussed by Matsumura, et al. (2013) and Matsumura and Okamura (2015), we should check whether the principle similar to the one adopted in this paper can be applied.
Second, the triopoly model is the simplest model to discuss the heterogeneity of objectives among the CSR-firms. Even in this simplest model, mathematical calculations are complicated and messy, which implies that it is difficult to treat a more general oligopoly model. However, as Haraguchi and Matsumura (2016) suggested, the model with more than two private firms may yields a different implication in mixed oligopolies. Although examining a more general model is tough work, it may yield a new insight into this field. This remains for future research.

Finally, we examine homogeneous products under quantity competition. However, undertaking CSR initiatives may affect other activities such as emission abatement activities or fair procurement. For example, Bian, et al. (2016) compared price and quantity competition under CSR incentives, and Hirose, et al. (2016, 2017) examined environmental corporate social responsibility under price competition. Extending our analysis to the endogenous choice between price and quantity also remains for future research.

Appendix A: Proofs

(i) **Proof of Proposition 1.**

We check the three cases. (i) The condition \( \frac{\partial W^*(0)}{\partial \theta} \leq 0 \) yields \( B \leq 0 \). (ii) The condition \( \frac{\partial W^*(1)}{\partial \theta} \geq 0 \) yields \( B \geq D \). (iii) When \( D > B > 0 \), partial privatization is derived from the optimality condition \( \frac{\partial W^*(\theta)}{\partial \theta} = 0 \) in (5). ■

(ii) **Proof of Proposition 2.**
Using the same procedures in Proposition 1, we show that condition \( \frac{\partial W(1)}{\partial \theta} \geq 0 \) never holds. Condition \( \frac{\partial W(0)}{\partial \theta} \leq 0 \) shows that \( \alpha \geq \frac{1}{3} \). Otherwise, the first-order condition for an interior solution should satisfy the following condition:

\[
\frac{\partial W}{\partial \theta} = \frac{A^2 (1 - \alpha)(1 - 3\alpha - 3\theta)}{2(3 + 2\theta - \alpha - \alpha \theta)^3} = 0.
\]

The second-order condition, \( \frac{\partial^2 W(\theta^*)}{\partial \theta^2} \leq 0 \), is satisfied. Then, the optimal degree of privatization is \( \theta^* = \frac{1}{3} - \alpha \).

(iii) Proof of Proposition 3.

Proof. Using the same procedures in Proposition 1, we show that condition \( \frac{\partial W(0)}{\partial \theta} \leq 0 \) never holds. First, condition \( \frac{\partial W(1)}{\partial \theta} \geq 0 \) shows that \( \alpha \geq \frac{4}{3} \). Otherwise, the first-order condition for an interior solution should satisfy the following condition:

\[
\frac{\partial W}{\partial \theta} = \frac{A^2 (2 - \alpha)[(\alpha_i^2 + 6\alpha_i - 12)\theta + 5\alpha_i^2 - 8\alpha_i + 4]}{2(6 + 4\theta - \alpha_i - \alpha_i \theta)^3} = 0.
\]

(A1)

The second-order condition, \( \frac{\partial^2 W}{\partial \theta^2} \leq 0 \), is satisfied.

From (15) we obtain

\[
\theta^* = \frac{5\alpha_i^2 - 8\alpha_i + 4}{12 - (6 + \alpha_i)\alpha_i}.
\]

(A2)
The optimal solution is partial privatization, where $0 < \theta^* < 1$ if $0 < \alpha_1 < \frac{4}{3}$. Further, if $0 < \alpha_1 < \frac{4}{3}$, we obtain

$$\frac{\partial \theta^*}{\partial \alpha_1} = -\frac{2(19\alpha_1^2 - 64\alpha_1 + 36)}{(\alpha_1^2 + 6\alpha_1 - 12)^2} < 0$$

when $\alpha_1 < \hat{\alpha} = \frac{32 - 2\sqrt{85}}{19} = 0.713$.

However, if $\alpha_1 \geq \frac{4}{3}$, $\theta^* = 1$ and it is invariant with $\alpha_1$. ■

Appendix B: Rigorous Explanations on Proposition 3

We present a more detailed explanation. We examine the sign of $\frac{\partial \theta^*}{\partial \alpha_1}$. Let $W_\theta(\alpha_1)$ satisfy the optimal condition in (A1). Then, from the implicit function theorem and the second-order condition of optimality, the sign of $\frac{\partial \theta^*}{\partial \alpha_1}$ is the same as that of $\frac{\partial W_\theta}{\partial \alpha_1}$. Then, we obtain the following relations:

$$\frac{\partial W_\theta}{\partial \alpha_1} > 0 \text{ if } \alpha_1 > \theta_w = \frac{2\theta^2 + 13\theta + 22 - \sqrt{28\theta^4 + 164\theta^3 + 333\theta^2 + 260\theta + 52}}{2(2\theta^2 + 8\theta + 9)}.$$

The result confirms Proposition 3 in that the relation between $\alpha_1$ and $\theta^*$ is U-shaped, depending on the degree of CSR, $\alpha_1$. Using this fact, we provide the economic explanation with regard to Proposition 3.

When $\alpha_1$ increases, $q_1$ increases but both $q_2$ and $q_0$ decrease. Then, we have

$$\left|\frac{\partial q_1}{\partial \alpha_1}\right| > \left|\frac{\partial q_0}{\partial \alpha_1}\right|.$$ Again, a higher degree of CSR activities by firm 1 leads to an increase in the total industry output, and thus, the production substitution effect is beneficial to consumer.
surplus. However, when \( \alpha_1 \) increases, firm 1 has to pay a higher cost while its rival firms can save their costs. This points to the existence of the cost reallocation effect. Therefore, the welfare effect also depends on the trade-off between the production substitution effect and the cost reallocation effect. In particular, if \( \alpha_1 < \theta_w \), the relation between \( \alpha_1 \) and \( \theta^* \) is negative, and thus the production substitution effect is dominated by the cost reallocation effect. However, the production substitution effect dominates the cost reallocation effect if \( \alpha_1 < \tilde{\theta}_1 \), while the production substitution effect is outweighed by the cost reallocation effect if \( \alpha_1 > \tilde{\theta}_1 \) in Fig. 2.

Therefore, when deciding the optimal degree of privatization, the welfare effect depends on \( \alpha_1 \). On the one hand, when \( \alpha_1 < \tilde{\theta}_1 \), and thus when the public firm produces more output than the other private firms, welfare increases as \( \alpha_1 \) increases because the production substitution effect dominates cost reallocation effect. Thus, the government decreases the degree of privatization in order to increase the outputs of firm 0 and the industry, that is, \( \frac{\partial q_0}{\partial \theta} > \frac{\partial q_{-0}}{\partial \theta} \). This will decrease the marginal output substitution effect but increase the marginal cost reallocation effect, so that the two marginal benefits will be the same at a lower level of privatization.

On the other hand, when \( \alpha_1 > \tilde{\theta}_1 \) and thus the CSR-firm 1 produces more output than the other private and public firms, welfare decreases as \( \alpha_1 \) increases because the production substitution effect is outweighed by cost reallocation effect. In that case, we have three cases.
First, when $\hat{\alpha}_1 < \alpha < \hat{\alpha}$, at which $\hat{\alpha}$ satisfies $\theta^* = \theta_w$, welfare decreases as $\alpha_1$ increases. In that case, the output difference between CSR-firm 1 and the public firm is relatively small. Thus, the government still decreases the degree of privatization in order to increase the outputs of firm 0 and the industry, that is, $\frac{\partial q_0}{\partial \theta} > \frac{\partial q_{-1}}{\partial \theta}$. This will increase the marginal production substitution effect, which is larger than the increment of the marginal cost reallocation effect. Thus, the two marginal benefits will be the same at a lower level of privatization.

Second, however, when $\hat{\alpha} < \alpha_1 < \bar{\alpha}$, welfare increases as $\alpha_1$ increases. In that case, the output difference between CSR-firm 1 and the public firm is relatively large. Then, the government increases the degree of privatization in order to decrease the output of firm 0, which induces an increase in other firms’ outputs but a decrease in industry outputs, that is, $\frac{\partial q_{-1}}{\partial \theta} < \frac{\partial q_0}{\partial \theta}$. This will decrease marginal benefit of the production substitution effect. In that case, however, the CSR-firm is less sensitive to the change in the public firm, and thus, it will decrease the marginal cost reallocation effect even further. Thus, the two marginal benefits will be the same at a higher degree of privatization. Note that if $1 < \alpha_1 < 2$, the output of the public firm is a complement, and as $\alpha_1$ increases, the output of CSR-firm 1 becomes more sensitive. Thus, the positive relations between $\alpha_1$ and $\theta^*$ becomes steeper, as $\alpha_1$ increases.

Finally, when $\alpha_1 \geq \bar{\alpha}$, the government chooses $\theta^* = 1$, namely full privatization, which means that it cannot increase the degree of privatization any further. Therefore, $\theta^*$ takes a U-shape in $\alpha_1$. 

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References


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[ Fig. 1. The Optimal Privatization ($\alpha_1, \alpha_2 > 0$ and $0 \leq \alpha_1 + \alpha_2 < 2$). ]
[Fig. 2. The Optimal Privatization when \(0 = \alpha_2 < \alpha_1 < 2\).]