Learning by Doing and Strategic Interaction in Environmental Policies in a Two-country Model

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Learning by Doing and Strategic Interaction in Environmental Policies in a Two-country Model*

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Abstract

This paper considers the impact of differences in endogenous technological change between two countries on global pollution emissions under international strategic interaction in environmental policies. A country that initially has a dirty technology (an environmentally lagging country) reduces more pollution emissions by imposing a higher rate of pollution reduction per unit of the emission, although it may generate larger total emissions. The more a country reduces pollutants, the more it learns how to produce in an environmentally friendly manner at low cost. The main finding is that an environmentally lagging country’s technology may continue to advance through a learning-by-doing effect until it exceeds the environmental friendliness of a leading country that initially had the cleanest technology. Whether a country eventually becomes an environmentally leading country depends on the country size and its awareness of environmental quality.

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1 Introduction

A strengthening of environmental regulations that are intended to reduce greenhouse gas emissions often induces environmentally friendly technological progress as a by-product (Newell et al. 1999, Popp 2002). While technological progress itself is good for the environment, it might ultimately increase the total pollution emissions in the world by triggering international interactions on environmental regulations such as so-called carbon leakage. Carbon leakage refers to the situation in which a reduction in carbon dioxide emissions by the home country with a strict climate policy causes an emissions increase in the foreign country that weakens its environmental restriction as a response, which might not only increase the total emissions in the short run but also slow the pace of climate policy-driven technological progress in the foreign country. As a result, the total emissions in the international economy might be higher also in the long run.

The present paper proposes a new two-country model that explicitly captures such an international and intertemporal interaction between environmental regulations that governs technological progress. We examine consequences of technological progress on pollution emissions in each country and the world.

For this purpose, we develop a two-country model in which the government in each country determines domestic environmental regulations and the firms respond to it by determining the scale of pollution reduction. There is a unique final good that generates transboundary pollution (greenhouse gas) as a by-product of production. In order to mitigate pollution damage, the national government requires each domestic firm to reduce its emissions, which induces environmentally friendly technological progress according to the finding by Newell et al. (1999) and Popp (2002). We follow Arrow (1962) and Romer (1986) by assuming technological progress results from learning by doing, which takes place as a by-product of the firms’ experiences on emission reduction.

A key mechanism in our model is that an environmental policy in one country induces domestic firms’ adoption of cleaner technologies, which may discourage the other country’s incentive for a stricter environmental policy. A less strict environmental policy in the other country should stifle the process of technological progress through learning by doing. In other words, the strategic interaction between countries might hamper long-term technological progress, which has a negative impact on the environment. To the best of our knowledge, this mechanism of international strategic interaction on environmental regulations is new to the literature on the environment and endogenous technological progress. The present paper could complement the existing studies by offering a new model incorporating that mechanism.

Using the model, we demonstrate that environmental leadership of a country may shift to the other along an equilibrium path. Environmental leadership is defined as the state whereby a country has the most environmentally friendly technology on pollution emissions. The intuition is the following. As each country is assumed to regulate its emissions to maximize individual welfare, a country that initially has a dirty technology (an environmentally lagging country) needs to reduce more pollution emissions. The environmentally lagging country tends to impose a higher rate of pollution reduction per

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1 More environmentally friendly technologies are widely recognized as a key component of the long-term strategy to mitigate global greenhouse gas emissions without compromising economic growth. As is well known, in order to control and limit climate change, long-term greenhouse gas emissions need to be reduced. See Organisation for Economic Co-operation and Development (OECD) (2012) and Intergovernmental Panel on Climate Change (IPCC) (2014).

2 This theoretical result seems consistent with empirical observations; see Section 4.2.
unit of the emission and reduce more pollution emissions, although it may generate a large amount of pollution emissions (i.e., implementing a weak environmental regulation in terms of emissions per unit of the good). This is realized as a result of international strategic interaction emerging in Nash equilibrium of the policy game. Consequently, learning-by-doing effects are large in the lagging country and its technology becomes environmentally friendly more rapidly than the other country that initially had a clean technology (an environmentally leading country). Thus, the lagging country’s environmental friendliness may continue to increase until it exceeds the leading country’s environmental friendliness. Each country’s friendliness converges to a certain level in the long run because the government ceases to implement environmental regulations when its technology is sufficiently clean. We can show that whether a country eventually becomes an environmentally leading country depends on country size and awareness of environmental quality.

The result that the leadership may endogenously fluctuate is not new in the context of price competition between firms. For instance, the important paper by Giovannetti (2001) considers a duopoly in which firms considering infinite technological adoption set prices with Bertrand competition in the product market. Using this model, Giovannetti identifies the conditions whereby firms alternate in adopting the new technology. He shows that demand conditions, such as price elasticities, play a role in determining whether such leapfrogging can be perpetual in Bertrand competition.\textsuperscript{3} In addition, some studies in the field of economic geography address both the theory of and empirical evidence for the possibility of leadership reversals between regions (for example, Quah 1996a, b).\textsuperscript{4} Different from the context of price competition, the present paper assumes that firms are perfectly competitive as in the standard endogenous growth model based on learning by doing (Romer 1986). Thus, there is no strategic interaction between firms but between international governments.\textsuperscript{5}

Another result we obtain from the model is that whether global pollution emissions decrease over time is ambiguous, despite the fact that environmental technology monotonically advances in both countries. More specifically, the amount of global pollution emissions converges to a constant level in the long run, which may be lower (higher) than the initial level of global emissions (i.e., the level in an early stage of adjustment under dirty technologies) when environmental leadership shifts between countries (does not shift) on an equilibrium path. In our model, the technology in the lagging country advances more rapidly than that in the leading country. This feature implies that technologies in the two countries advance considerably if both countries experience a state of environmental lagging for many periods. That is, both countries possess similarly clean technologies when the environmental leadership shifts internationally. This is why the long-term level of global pollution can become low in the case of leadership reversals.

Our results could suggest the importance of balanced technological change, while it should be safe to keep in mind that we are only seeing the learning-by-doing aspect of technological progress. Most of the world’s technological progress for the environment occurs in high-income countries (e.g., Lanjouw and Mody, 1996). Dechezleprêtre et al.

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\textsuperscript{3}See Athreye and Godley (2009), Giovannetti (2013), and Petrakos, Rodríguez-Pose, and Rovolis (2005) for more recent research.

\textsuperscript{4}Some papers in trade theory also address similar issues to this. See Furukawa (2015) for recent research.

\textsuperscript{5}See, for example, Hall (2008) and Harrington, Iskhakov, Rust, and Schjerning (2010) for research on dynamic strategic interaction between firms in the competitive process.
(2011) did find climate-friendly innovations in emerging economies, but these innovations are limited. While international transfers of climate-mitigation technologies occur mostly between developed countries, technology transfers from developed countries to emerging countries are few in number, but have been rising rapidly in recent years.\textsuperscript{6} We might need to accelerate international transfers to mitigate the imbalanced technological change between countries that could cause undesirable effects on the environment.

By developing a two-country model of endogenous environmental regulations and learning by doing, this paper complements the literature on environmental regulations and endogenous technological change through research and development (R&D). Bovenberg and Smulders (1996) examined the link between tighter environmental policy and economic growth when the environmental R&D sector endogenously develops abatement technologies. Goulder and Mathai (2000) explored policy-induced technological change for the design of carbon-abatement policies when the channels of technological progress are based on R&D and learning-by-doing. Acemoglu et al. (2012) considered whether research can be directed to improving the productivity of clean and dirty intermediate goods sectors and showed that sustainable long-run growth can be achieved with temporary taxation of dirty innovation and production when the inputs are sufficiently substitutable. None of these studies developed a two-country model to study the strategic interaction of environmental policies between countries and its effects on technological progress and global pollution.\textsuperscript{7}

The rest of the article is organized as follows. Section 2 introduces our two-country model of environmental regulations and Section 3 considers a Nash equilibrium of the policy game. Section 4 explores each country’s environmental leadership in equilibrium. Section 5 investigates global pollution emissions. Section 6 concludes.

\section{Basic Model}

There are two countries, labeled by $i = A, B$. In the basic model, we keep the two countries as symmetric as possible. They differ only in initial environmental technological levels.\textsuperscript{8} There is a single consumption good, which is taken as the numeraire. The consumption good is produced by perfectly competitive firms in both countries. There are constant returns to scale, and the technology converts one unit of (effective) labor into one unit of a good. The (gross) marginal cost in country $i$ is thus equal to the wage rate, denoted as $w_i$. In the present model, time is discrete extending from $t = 0$ to $\infty$. Nevertheless, to simplify notation, we will drop the time index $t$, when it causes no confusion.

Industrial production emits pollution, which is treated as a global pure public bad. Assume that producing one unit of a good in country $i$ generates $\kappa_i > 0$ units of pollution. The variable $\kappa_i$ captures how harmful the production technology in country $i$ is to the environment. We model a country’s environmental technology by using $\kappa_i$, which may correspond to the amount of carbon dioxide (CO2) emissions per unit of GDP adjusted by

\textsuperscript{6}Popp (2012) provided a comprehensive review of the literature on environmentally friendly technological change and technology transfers.

\textsuperscript{7}In the literature on trade and the environment, the interaction of environmental policy interventions is investigated using a two-country model, but technologies are exogenously given to focus on the effects of trade liberalization. See, e.g., Copeland and Taylor (2004). Those studies focused on the interaction of environmental polices between countries but not on technological change.

\textsuperscript{8}In Section 4.3, we will investigate the roles of heterogeneity between the countries.
PPP in a commonly-used data set of the United Nations (the Millennium Development Goals (MDG) Indicators), given that we think of a single-good model.

In this paper, we use two different words concerning the environment. The first word is “awareness,” to which we relate parameter $\epsilon$. This captures how uncomfortable people feel about global pollutants. The second word is “friendliness,” inversely relating to $\kappa_i$. This captures to what extent the production technology of a country generates pollution emissions.

In this study, we highlight the government’s role in controlling emissions. In order to control the aggregate emission level, the national government of country $i$ requires each domestic firm to reduce its pollution by $100\tau_i\%$. In other words, firms in country $i$ are allowed to generate $\kappa_i(1 - \tau_i)$ of emissions for one unit of the good. We assume that every firm can reduce one unit of emission by hiring one unit of (effective) labor. The effective marginal cost for a firm to produce a unit of a good (with the inclusion of pollution reduction) is equal to $w_i(1 + \kappa_i\tau_i)$. We may refer to $\tau_i \in [0, 1]$ as the rate of pollution reduction per unit of the emission in country $i$.

In each country, there is a representative consumer who inelastically supplies $L/2$ units of (effective) labor. The consumer in country $i$ consumes $C_i$ units of the single consumption good and is endowed with the following utility function:

$$u_i = C_i - \epsilon (E_A + E_B)^2,$$

where $E_i$ is the flow of pollution emission generated by country $i$ and $\epsilon > 0$ denotes the degree of environmental awareness.

We treat pollution as a flow although most environmental problems are stock ones. The reason is as follows. First, if the depreciation rate of the pollution stock is high (e.g., the natural rate of removal of atmospheric pollution is high), the flow assumption may be a reasonable approximation (e.g., Schou, 2002; Grimaud and Tournemaine, 2007). Second, it simplifies the analysis without altering the main insight of our paper.

## 3 Equilibrium in a Two-country Model

In this section, we will characterize the short-run equilibrium of our model under given environmental technologies. Although our model is very simple, its equilibrium behavior appears to be complex because of strategic interaction between countries. To explain this, first, we will see the consumers’ and firms’ optimal activities in market equilibrium. Then, we will characterize the governments’ optimal environmental policy in a Nash equilibrium of the policy game played by the two countries.

### 3.1 Market Equilibrium

Assuming that firms of the two countries supply their products to the integrated world market, the effective marginal costs must be equated between the two countries. Thus we have $w_A(1 + \kappa_A\tau_A) = w_B(1 + \kappa_B\tau_B) = 1$. The equilibrium wages are obtained as

$$w_i = \frac{1}{1 + \kappa_i\tau_i}.$$
The labor market equilibrium conditions determine the equilibrium levels of national output equal to

\[ Y_i = \frac{L/2}{1 + \kappa_i \tau_i}. \]  

We thus obtain the indirect utility function as

\[ u_i = \frac{L/2}{1 + \kappa_i \tau_i} - \varepsilon \left( \sum_{i \in \{A, B\}} E_i \right)^2, \]

where the pollution is given by

\[ E_i = (1 - \tau_i) \frac{L \kappa_i / 2}{1 + \kappa_i \tau_i}. \]

for \( i = A \) and \( B \).

### 3.2 Optimal Policy Equilibrium

The government in each country, say \( i \), controls their pollution reduction rate \( \tau_i \) as an environmental policy tool so as to maximize utility, given their foreign policy \( \tau_j \). A reaction function is defined as \( \tau_i^*(\tau_j^*) = \arg \max_{\tau_i \in [0, 1]} u_i \). Solving this nonlinear optimization problem with (4) derives the reaction function as

\[
\tau_i^*(\tau_j^*) = \begin{cases} 
0 & \text{if } \varepsilon L \leq \frac{1}{1 + \kappa_i} \left( \kappa_i + \frac{\tau_j^* (1 - \tau_j^*)}{1 + \kappa_j \tau_j^*} \right)^{-1} \\
\varepsilon L & \text{otherwise} \\
1 & \text{if } \varepsilon L \geq \frac{1}{1 + \kappa_i} \left( \kappa_j (1 - \tau_j^*) \right)^{-1} 
\end{cases},
\]

where

\[
e_i = \frac{\varepsilon L - \frac{1}{1 + \kappa_i} \left( \frac{1}{1 + \kappa_i} - \frac{\kappa_j (1 - \tau_j^*)}{1 + \kappa_j \tau_j^*} \varepsilon L \right)}{\varepsilon L + \left( \frac{1}{1 + \kappa_i} - \frac{\kappa_j (1 - \tau_j^*)}{1 + \kappa_j \tau_j^*} \varepsilon L \right)}. \]

Equation (6) suggests a possibility of so-called carbon leakage. Since \( \tau_i^*(\tau_j^*) \) is globally a decreasing function in \( \tau_j^* \), one country would prefer a lower rate of pollution reduction when the other country takes a higher pollution reduction rate. Thus, it is less likely for both countries to take a very high rate of reduction at the same time. At the aggregate level, this would imply the possibility of carbon leakage.

Next let us think of a Nash equilibrium in the policy game played between the two governments. Denote as \( (\tau_A^*, \tau_B^*) \) a pair of strategies taken in the Nash equilibrium. This equilibrium pair of policies can be calculated as a solution to the system consisting of the two optimal policy equations: \( \tau_A^* = \tau_A^*(\tau_B^*) \) and \( \tau_B^* = \tau_B^*(\tau_A^*) \). To derive the equilibrium policies, first, it is useful to note two basic facts. First, \( (\tau_A^*, \tau_B^*) = (1, 1) \) and \( (\tau_A^*, \tau_B^*) = (e_A, e_B) \) cannot be Nash equilibria.\(^{10}\) Second, if the world pollution level

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\(^{10}\) The proof is as follows. Substituting \( e_j \) into \( e_i \) results in \( \frac{\tau_j^*}{\kappa_i} + 1 \left( \frac{\kappa_i}{2(1 + \kappa_i)} - \frac{\kappa_j}{2(1 + \kappa_j)} \right) = 0 \). This does not hold in general because \( \tau_i^* > 0 \).
\( \kappa_A + \kappa_B \) is sufficiently low, both countries do not adopt any pollution reduction policy:

\[
(\tau_A^*, \tau_B^*) = (0, 0) \text{ if } \kappa_A + \kappa_B < \min_{i \in \{A, B\}} \left\{ \frac{1}{\varepsilon L (1 + \kappa_i)} \right\}.
\]  

(8)

By using (6) and (7), we can easily obtain the equilibrium pair of the policy in the following; see Appendix A for detailed derivations. Define \( \hat{\kappa} \) such that \( \hat{\kappa} = 1/\left( \varepsilon L (1 + \hat{\kappa}) \right) \). With \( i \neq j \), the policy pair is characterized by

\[
(\tau_i^*, \tau_j^*) = \begin{cases} 
(p_i, 0) & \text{if } \min \left\{ \kappa_i, \frac{1}{\varepsilon L (1 + \kappa_i)} \right\} > \frac{1}{\varepsilon L (1 + \kappa_i)} - \kappa_i \\
(1, 0) & \text{if } \hat{\kappa} > \kappa_j > \frac{1}{\varepsilon L (1 + \kappa_i)} \\
(1, q_j) & \text{if } \kappa_i > \kappa_j \geq \hat{\kappa}
\end{cases},
\]

(9)

where we define two functions in \( t \), \( p_i \) and \( q_j \), that satisfy \( 0 < q_i < p_j < 1 \). Formal definitions of these two functions are

\[
p_i \equiv \frac{\varepsilon L - \frac{1}{\kappa_i} \left( \frac{1}{1 + \kappa_i} - \varepsilon L \kappa_i \right)}{\varepsilon L + \left( \frac{1}{1 + \kappa_i} - \varepsilon L \kappa_i \right)} \quad \text{and} \quad q_j \equiv \frac{\varepsilon L - \frac{1}{\kappa_j} \left( \frac{1}{1 + \kappa_j} - \varepsilon L \kappa_j \right)}{\varepsilon L + \left( \frac{1}{1 + \kappa_j} - \varepsilon L \kappa_j \right)}.
\]

(10)

By using (8) and (9) with (10), Figure 1 relates the environmental technologies of both countries, \((\kappa_A, \kappa_B)\), to their equilibrium pollution reduction rates, \((\tau_A^*, \tau_B^*)\), in (8) and (9). These complex equations and figures simply imply that the country that has a dirtier technology (larger \( \kappa_i \)) is more willing to impose a higher rate of pollution reduction per unit of the emission (larger \( \tau_i \)). We can formally prove our main result.

**Theorem 1** A country with a less environmentally friendly technology tends to implement a higher rate of pollution reduction per unit of the emission generated in the country in equilibrium; \( \tau_i^* > \tau_j^* \) if \( \kappa_i > \kappa_j \).

Theorem 1 implies that the government of a country with dirtier technologies would prefer to reduce more pollutants in percentage terms. This results from international strategic interactions emerging in a Nash equilibrium of the policy game. This will deliver the results on environmental leadership and global pollution as shown in the next two sections.

One may think that the implication of Theorem 1 seems inconsistent with recent empirical literature showing that new technologies lower the cost of regulation and increase the willingness to regulate (e.g., Carrion-Flores and Innes, 2010; Lovely and Popp, 2011). However, we believe that these two seemingly opposite views are not necessarily inconsistent but just seeing two different aspects of environmental technology. On the one hand, the empirical literature focuses on a cost reduction of environmental regulation driven by new technologies, which encourages the willingness to regulate as shown. On the other hand, we are currently seeing new technologies to reduce the potential amount of pollution emissions, which can be expressed by a decrease in \( \kappa_i \). The reduction of potential pollution directly decreases the need to regulate itself, which would subsequently weaken the willingness for a country to regulate pollution emissions. Our model captures this aspect of environmental technology, which is essentially consistent with the way the empirical literature considers.
Note that the amount of pollution emissions per unit of the good within a country, \( i \), i.e., \( \kappa_i (1 - \tau_i^*) \), can be greater when the technology of country \( i \) is dirtier (less environmentally friendly), i.e., \( \kappa_i \) is higher, although it is accompanied by a higher reduction rate \( \tau_i^* \) per unit of the emission. This suggests that, in our model, a country with a less environmentally friendly technology tends to implement a weaker environmental regulation in terms of emissions per unit of the good, which is consistent with the empirical literature mentioned above.

4 Technological Leadership in the Environment

In this section, we will introduce a learning-by-doing process through which the environmental technology in either country advances. To begin with, we define environmental technological leadership as the state whereby a given country has the most environmentally friendly technology among all countries. Thus, we refer to a country that has a lower \( \kappa_i \) as an environmentally leading country. A country with a higher \( \kappa_i \) is called an environmentally lagging country. Without loss of generality, we assume \( \kappa_A < \kappa_B \) holds in period 0 (initial period); \( \kappa_A(0) < \kappa_B(0) \). Country \( A \) is initially an environmentally leading country.

4.1 Learning by Doing and Technological Progress

In order to incorporate the basic idea that environmental regulations induce environmentally friendly technological progress (Newell et al. (1999) and Popp (2002)), we consider the learning by doing setting à la Arrow (1962) and Romer (1986);\(^{11}\) however, if we thought of a more general and realistic setting as in Young’s (1991) bounded learning by doing model, our main result would not change qualitatively. We believe that using such a simple setting is beneficial for us to elaborate our main story.

The key assumption is that a country that reduces more pollutants learns how to produce in a more environmentally friendly manner. Specifically, we suppose that the pollution level of a technology in country \( i \) in period \( t + 1 \), \( \kappa_i(t + 1) \), is determined by the cumulative stock of past experiences on reducing pollutants:

\[
\kappa_i(t + 1) = \bar{\kappa}_i - \sum_{s=0}^{t} \eta(\tau_i^*(s)\kappa_i(s)Y_i(s)),
\]

where \( \tau_i^*(s)\kappa_i(s)Y_i(s) \) is the pollution reduction made by country \( i \) in period \( s \) and \( \eta \) is a learning-by-doing function that maps the pollution reduction country \( i \) does in a period, \( s \), to how much country \( i \) will learn to produce environmentally friendly from its experience on reducing pollutants. \( \bar{\kappa}_i \) denotes an initial (period 0) pollution level in country \( i \), which is exogenously given.

We put two natural assumptions on the learning-by-doing function \( \eta \). (a) \( \eta(0) = 0 \); there is no advance in a country if there is no reduction. (b) \( \eta(z') > \eta(z) \) for \( z' > z \) for any \( z, z' > 0 \); a country that reduces more pollutants learns more on how to produce in an environmentally friendly manner. It can be easily verified that, in equilibrium,\(^{11}\) See also Furukawa (2007) for learning by doing in an innovation-based growth model, which is considered in much the same fashion as Arrow’s original paper.
\( \tau_i^*(s) \kappa_i(s) Y_i(s) \) monotonically increases with \( \tau_i^*(s) \), which plays a key role in showing the following lemma.

**Lemma 1** The international environmental friendliness \((\kappa_A(t), \kappa_B(t))\) have seven different phases as shown in Figure 2.\(^{12}\)

**Proof.** For the sake of explanation, by (11), we can derive the following expression in terms of a flow:

\[
\kappa_i(t + 1) - \kappa_i(t) = -\eta(\tau_i^*(t) \kappa_i(t) Y_i(t)). \tag{12}
\]

Together with Figure 1, (12) implies that there are three typical patterns of the direction in which \((\kappa_A(t), \kappa_B(t))\) moves over time, depending on the international pair of pollution reduction rates \((\tau_A^*(t), \tau_B^*(t))\).

First, in the region of \((\tau_A^*(t), \tau_B^*(t)) = (0, 0)\), there are no technological advances by assumption (a). Here \((\kappa_A(t), \kappa_B(t))\) never moves and is stable. Second, in the regions of \((\tau_A^*(t), \tau_B^*(t)) = (p_A, 0)\) and \((1, 0)\) \((\tau_A^*(t), \tau_B^*(t)) = (0, p_B)\) and \((0, 1)\), only country \(A\) \((B)\) engages in the abatement activity. Therefore, only \(\kappa_A(t)\) \((\kappa_B(t))\) decreases over time by assumption (a). This fact is indicated by the left arrow (down arrow) within those regions. Third, in the region of \((\tau_A^*(t), \tau_B^*(t)) = (1, q_B)\) \((\tau_A^*(t), \tau_B^*(t)) = (q_A, 1)\), both countries make the environmental investment. As the pollution reduction rate in country \(A\) \((B)\) is higher, i.e., \(\tau_A^*(t) > \tau_B^*(t)\) \((\tau_A^*(t) < \tau_B^*(t))\), the pollution level in country \(A\) \((B)\), i.e., \(\kappa_A(t)\) \((\kappa_B(t))\), decreases more sharply than the pollution level in country \(B\) \((A)\), i.e., \(\kappa_B(t)\) \((\kappa_A(t))\) through learning by doing. This comes from the assumption (b) and the equilibrium property that \(\tau_i^*(s) \kappa_i(s) Y_i(s)\) monotonically increases with \(\tau_i^*(s)\). This is indicated by the long left arrow and the shorter down arrow for the region of \((1, q_B)\) and the long down arrow and the shorter left arrow for the region of \((q_A, 1)\). All seven phases are characterized, proving the lemma. \(\blacksquare\)

By means of the phase diagram in Figure 2, we can determine the direction in which international environmental friendliness, \((\kappa_A(t), \kappa_B(t))\), advances over time and roughly trace a path for any initial point. A typical trajectory, starting from point \(K_0\), is illustrated by dotted arrows in Figure 2.

### 4.2 Environmental Leadership

Take an example path starting from \(K_0\) in Figure 2, in which \(\kappa_A(0) < \kappa_B(0)\). Along an equilibrium path from \(K_0\), as can be shown by using the phase diagram, environmental leadership may shift between the two countries. At first, country \(A\) is the leader with lower \(\kappa_A(t)\) and it retains its environmental leadership in the subsequent periods \(1 \rightarrow 4\). Along the equilibrium path, the environmental leadership internationally shifts in period 5; country \(B\) becomes a new environmental leader.

We can formally identify this possibility of environmental leadership's shift. Recall that by (9) and Figure 1, the equilibrium environmental policy pair is \((\tau_A^*(0), \tau_B^*(0)) = (0, 0), (0, p_B(0))\), \((0, 1)\), or \((q_A(0), 1)\). Define a new threshold value \(\tilde{\kappa}\) such that \(2\tilde{\kappa} = 1/(\varepsilon L (1 + \tilde{\kappa}))\) (see Figure 3). If an initial point exists in the blue region in Figure 3 (a),

\(^{12}\)In Figure 2, \(\kappa_A(t)\) \((\kappa_B(t))\) is measured along the horizontal (vertical) axis, and the time index \(t\) is omitted for simplicity.
like point $k_0$, the environmental friendliness pair $(\kappa_A(t), \kappa_B(t))$ will eventually fall below the 45 degree line. The blue region is characterized by

$$\kappa_B(0) > \kappa_A(0) \in (\hat{k}, \tilde{k}).$$

(13)

See Figure 3 (b), in which the red region corresponds to

$$\kappa_B(0) > \kappa_A(0) \geq \hat{k}.$$

(14)

If the pair $(\kappa_A(t), \kappa_B(t))$ exists such as $k'_0$ in the red region in Figure 3 (b), it may eventually either fall below the 45 degree line or move to the blue region of (13). This is guaranteed by assuming that the extent of technological progress that takes place within a period is not too large, i.e., there exists some $\delta > 0$ such that $\eta(\cdot, t) < \delta$. Given this assumption, if (14) holds, we can show that environmental leadership will eventually shift internationally.

Taking into account (13) and (14) with Theorem 1, we have the following.

**Proposition 1** The environmental leadership of a country may be temporary. Suppose $\kappa_A(0) < \kappa_B(0)$. If the extent of technological progress taking place within a period is not too large, so long as

$$\kappa_A(0) > \hat{k},$$

(15)

the environmental leadership initially retained by country A will eventually shift to the initial lagging country B.

To explain why such a reversal of environmental leadership can take place under (15), let us review our result on a step-by-step basis. Initially, country A is an environmentally leading country with $\kappa_A(0) < \kappa_B(0)$. As the environmentally lagging country B is more polluting, it requires domestic firms to reduce pollutants more by setting a higher rate of pollution reduction, i.e., $\tau_B^*(0) > \tau_A^*(0)$ (Theorem 1). Recall that this does not necessarily imply the lagging country B taking a stricter environmental regulation because it can generate a larger amount of pollution emissions. Through the learning-by-doing process, the lagging country B’s technology thus becomes environmentally friendly more rapidly than the leading country A’s technology does. If the technology of the leading country A were initially environmentally friendly enough to satisfy $\kappa_A(0) < \tilde{k}$, the world economy would get to the equilibrium without any pollution reduction ($\tau^*_1(t) = 0$). However, as the leading country A’s technology is initially not very environmentally friendly ($\kappa_A(0) > \tilde{k}$), the lagging country’s friendliness continues to increase until it exceeds the leading country’s. Therefore, if (15) holds, the environmental leadership eventually shifts internationally.

In a nutshell, the environmentally lagging country may learn to produce in an environmentally friendly way faster than the leading country since the lagging country reduces more pollution emissions by setting a higher pollution reduction rate, which enhances learning by doing. This creates a possibility of the shift of environmental leadership between countries.

What happens after that? The answer to this question is that the leadership shift may occur once again. See Figure 2, in which $K_5$ moves horizontally in the subsequent

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13If a step of technological progress was very large, $(\kappa_A(t), \kappa_B(t))$ might immediately jump into the grey region of $(0, 0)$. 

period 6. Imagine that \( K_5 \) crosses the 45 degree line, so the technological leadership shifts internationally again in period 6. However, in the long run, the world economy’s friendliness pair \((\kappa_A(t), \kappa_B(t))\) eventually converges to the grey region in Figure 2, in which \((\tau_A^*(t), \tau_B^*(t)) = (0, 0)\) and \((\kappa_A(t), \kappa_B(t))\) stays constant.

Denote by \((\kappa_A^*, \kappa_B^*)\) the point that \((\kappa_A(t), \kappa_B(t))\) finally reaches in the grey region. Whether \(\kappa_A^* > \kappa_B^*\) or \(\kappa_A^* < \kappa_B^*\) is not determinate, depending in a complex fashion on the initial friendliness levels \((\kappa_A(0), \kappa_B(0))\). That is, which country ultimately becomes an environmentally leading country is indeterminate. This indeterminacy essentially comes from the symmetry between the countries (which differ only in \(i(t)\)). In any case, our message here is that the environmental leadership retained by a country at some point of time might be intrinsically impermanent.

Our theoretical result seems consistent with empirical observations. A transition of the key variable in our model, \(\kappa_i(t)\), which indicates the amount of emissions form producing one unit of a good (environmental friendliness of technology) in country \(i\), may correspond to that of CO2 emissions per 1 US dollar (USD) GDP adjusted by PPP, provided that composition of economic activity is constant in the country. According to the Millennium Development Goals (MDG) Indicators of the United Nations, many countries including developed and emerging economies reduced their CO2 emissions per 1 USD GDP (PPP) since the 1990s.\(^{14}\) For instance, emissions per 1 USD GDP in 1991 were 0.430kg in Germany and 0.325kg in Japan but those in 2010 were 0.272kg in Germany and 0.297kg in Japan.\(^{15}\) Emissions per 1 USD GDP in Poland were 1.176kg in 1990 and 0.479kg in 2010, while those in Bulgaria were 1.155kg in 1990 and 0.515kg in 2010. Emissions per 1 USD GDP in 1992 were 1.730kg in China and 1.408kg in Russia, while those in 2000 were 1.011kg in China and 1.237kg in Russia. The values are reversed again between them because in 2010, they were 0.908kg in China and 0.863kg in Russia. Thus, our result might explain that a country with initially high emissions per 1 USD GDP (PPP) reduces the emissions significantly compared with other countries with initially low emissions per 1 USD GDP (PPP). Although this explanation is not more than just a suggestive interpretation of our result, one would think that our analysis on a path of \((\kappa_A(t), \kappa_B(t))\) is relevant to one of the well known indicators (the MDG indicators).

### 4.3 Which Country Prevails? The Role of Country Heterogeneity

So long as countries are essentially identical, in the analysis above, which country prevails is not determined. A fundamental question arises as to which country becomes the ultimate environmental leader in the long run. In this subsection, we will give an answer to this question by allowing for country heterogeneity.

Suppose that one country is relatively aware of environmental quality, say country \(A\), and the other has a large amount of effective labor (i.e., population times their labor productivity), say country \(B\). Denote as \(L_i\) and \(\varepsilon_i\) the effective labor and environmental awareness of country \(i\), where \(i = A, B\). Then, \(\varepsilon_A \geq \varepsilon_B\) and \(L_A \leq L_B\). Equilibrium optimal policies are shown in Figure 4. (See Appendix B for mathematical details.)

Figure 4 (a), by setting \(\varepsilon_A = \varepsilon_B\) and \(L_A < L_B\), shows how the difference in international effective labor sizes affects the equilibrium policies. Define \(\hat{k}_i\) such that

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\(^{15}\) In the United Kingdom, emissions per 1 USD GDP were 0.443kg in 1991 and 0.242kg in 2010.
\( \hat{\kappa}_i = 1/(\varepsilon_i L_i (1 + \hat{\kappa}_i)) \). Because \( \hat{\kappa}_B \) is lower than \( \hat{\kappa}_A \) in this case, the stable region \((0, 0)\) is twisted with a rightward bias. In fact, as \( L_B \) increases, \( \hat{\kappa}_B \) decreases and \( \hat{\kappa}_A \) increases. Therefore, when country \( B \)'s effective labor \( L_B \) is very large, \( A \) is lower than \( B \) in this case, the stable region \((0, 0)\) is twisted with a rightward bias. In fact, as \( L_B \) increases, \( \hat{\kappa}_B \) decreases and \( \hat{\kappa}_A \) increases. Given that the world economy eventually moves into the stable region \((0, 0)\), we can say that a county with large effective labor is more likely to eventually obtain the environmental leadership \((\kappa_A > \kappa_B)\).

**Remark 1** A country that has a large amount of effective labor tends to eventually become an environmental leader in the long run.

The implication of Remark 1 is as follows. A large amount of effective labor implies huge potential pollution emissions. Thus, the government of country \( B \) tends to implement a higher rate of pollution reduction for a longer time that promotes the technological progress as a by-product in the long-term. Therefore, given its large effective labor, country \( B \) may tend to obtain environmental leadership eventually, even if it is initially an environmentally lagging country.

Heterogeneity of environmental awareness, \( \varepsilon_A > \varepsilon_B \), determines which country finally retains the environmental leadership. See Figure 4 (b), with the definition of \( \pi \) where \( 2\pi (1 + \pi) = 1/\varepsilon_A \), which means \( \pi = \pi(\varepsilon_A) \) with \( \pi'(\varepsilon_A) < 0 \). Starting from any point in the red-box region (where \( \kappa_B(t) < \pi \) and \( \kappa_B(t) < \kappa_A(t) \)), \( \kappa_B(t) < \kappa_A(t) \) holds in the long run. Outside the red-box region, any path eventually converges to a state with \( \kappa_B(t) > \kappa_A(t) \), where country \( A \) is the leading country. As, by \( \pi'(\varepsilon_A) < 0 \), the red-box region becomes smaller as \( \varepsilon_A \) increases, we have the following statement.

**Remark 2** A country that has greater awareness of environmental quality tends to become an environmental leader in the long run.

The implication of Remark 2 is straightforward. Given its greater environmental awareness \( \varepsilon_A > \varepsilon_B \), country \( A \) is more likely to adopt a higher pollution reduction rate, abating more emissions. It follows that the learning-by-doing effect works more actively in country \( A \), which would advance environmental technology in country \( A \) faster (decreasing \( \kappa_A(t) \) faster than \( \kappa_B(t) \)).

## 5 Global Pollution

In this section, we investigate how global pollution changes over time. In doing so, we assume that the two countries differ only in their technological friendliness in the initial period; \( \kappa_A(0) < \kappa_B(0) \). Using (5), (9), and (10), we will elaborate how global pollution, \( E(t) = E_A(t) + E_B(t) \), changes over time in each stage of environmental development.

**Stage I:** Consider an earlier stage of environmental technology development, in which both countries set a positive rate of pollution reduction, \( (\tau_A(t), \tau_B(t)) = (q_A(t), 1) \). As shown in the phase diagram in Figure 2, environmental technology advances in both countries; both \( \kappa_A(t) \) and \( \kappa_B(t) \) decreases over time. By (5), (9), and (10), we have

\[
E(t) = \frac{1}{2 \varepsilon (1 + \kappa_A(t))} \equiv e'_A(t) \text{ as } \frac{1}{\varepsilon L(1+\kappa_A(t))} < \kappa_A(t). \tag{16}
\]
We find that, during this early stage (stage I), global pollution increases as environmental technologies in the leading country advance. That is, $E(t)$ increases as $\kappa_A(t)$ decreases.

**Stage II:** The second stage is with $(\tau_A^*(t), \tau_B^*(t)) = (0, 1)$, where technological progress takes place only for the lagging country. Only $\kappa_B(t)$ decreases over time. Global emissions in this case can be calculated as

$$E(t) = \frac{\kappa_A(t)L}{2} \equiv e_A^2(t)$$

where technological progress is reasonably large.

While the leading country generates a constant amount of pollution, the lagging country reduces all of its pollution emissions. Therefore, it is clear that global pollution is kept constant. That is, $E(t)$ never changes while $\kappa_B(t)$ decreases over time.

A fundamental question is whether global pollution rises or declines in the period of regime switching from stages I to II. The answer is not clear and global pollution depends on the extent of technological progress that takes place within that period. Suppose that regime switching from stages I to II occurs from periods $t$ to $t+1$. If the extent of technological progress in the leading country, i.e., $\eta_A(t)$, is reasonably large, global pollution is reduced with this regime switching, $E(t+1) < E(t)$.\(^{16}\)

**Stage III:** In a more advanced stage of environmental technology development with $(\tau_A^*(t), \tau_B^*(t)) = (0, p_B(t))$, as in stage II, only $\kappa_B(t)$ decreases over time. We can obtain

$$E(t) = \frac{1}{2\varepsilon(1+\kappa_B(t))} \equiv e(t)$$

as $\kappa_A(t) < \frac{1}{eL(1+\kappa_B(t))} < \kappa_A(t) + \kappa_B(t)$. \(^{17}\)

Global emissions start to increase again. In a regime switch from stages II to III, global pollution necessarily increases.\(^{17}\)

**Stage IV:** Finally, if both countries have a sufficiently clean technology such that if $\kappa_A(t) + \kappa_B(t) < \frac{1}{eL(1+\kappa_B(t))}$, they do not need pollution reduction; $(\tau_A^*(t), \tau_B^*(t)) = (0, 0)$.\(^{16}\)

In this case, global pollution is given by

$$E(t) = \frac{(\kappa_A(t) + \kappa_B(t))L}{2} \equiv e^3(t)$$

where technological progress for the lagging country, $\eta_B(t)$, is reasonably large.

We have shown the following proposition from the above analysis.

**Proposition 2** The global pollution $E(t)$ may fluctuate over time in the process of environmental technological progress but finally converges to the constant level in the long-run steady state.

Proposition 2 shows that the level of global pollution emissions becomes constant in the long-run. Our theoretical result might be consistent with empirical observations. According to the International Energy Agency (IEA), global energy-related CO2 emissions

\(^{16}\)To verify this, consider a numerical example with $L = 0.5$ and $\varepsilon = 0.5$. Assume $(\kappa_A(t), \kappa_B(t)) = (1.75, 4.5)$ and $(\kappa_A(t+1), \kappa_B(t+1)) = (1.3, 5)$. Then, regime switching occurs from $t$ to $t+1$, noting (16) and (17). Furthermore, $E(t) = 0.36364$ declines to $E(t+1) = 0.25$.

\(^{17}\)Suppose that the world goes from stages II to III in periods $t+1$ to $t+2$. By (17) and (18), noting $\kappa_A(t+1) = \kappa_A(t+2)$ in stage II, we can easily verify $E(t+1) < E(t+2)$.
were flat in 2014-2016 although the world economy grew for that period.\textsuperscript{18} Energy is an indispensable input for production and emissions from the energy sector is one of the primary sources of greenhouse gas emissions. It reports that flat emissions came from expanding renewable power generation, shifting from coal to natural gas, and improvements in energy efficiency, because of technological progress and environmental policies. In our model, production of the good will increase over time because environmental technology advances through learning-by-doing effects and we can save labor for abatement activity and use more labor for production. Although this explanation is a suggestive interpretation of our result, our analysis on the long-run level of global pollution emissions may be relevant to flat CO2 emissions from the global energy sector.

However, whether the long-run level of global pollution emissions is lower than the initial level is not clear in general; it can be either higher or lower potentially. We will think of two typical numerical examples:\textsuperscript{19} see Appendix C for details.

Figure 5 (a) illustrates the first example (a).\textsuperscript{20} In this case, the reversal of environmental leadership does not take place, and global emissions fluctuates and eventually increases up to the level higher than the initial level. This implies that the level of global pollution $E(t)$ may increase over time, despite the fact that environmental technology monotonically advances in both countries.

The intuition behind the result is as follows. Changes in pollution can be decomposed into two fundamental forces: scale and technique effects. As shown in Grossman and Krueger (1993), the scale effect measures the increase in pollution that would be generated if the economy was simply scaled up, holding all else constant; the technique effect captures reduction in pollution caused by a fall in emissions intensity, holding all else constant. In our model, a higher pollution reduction rate in the early stage of environmental technology development, accompanied by a larger amount of pollution reduction, induces rapid technological progress (through learning by doing), which reduces pollution (the technique effect). As technological progress enables a country to employ more labor in production of the good, which causes an increase in pollution (the scale effect). Example (a) suggests that the scale effect in some cases may play a dominant role, where environmental technology advances, but emissions also increase.\textsuperscript{21}

Figure 5 (b) describes the second example (b).\textsuperscript{22} In this example, the environmental leadership shifts between countries, where global pollution fluctuates at first, but finally declines to the lowest level. This implies that the long-run amount of global pollution can be lower than the initial amount. This is essentially because, in our model, the technology in the lagging country advances more rapidly than that in the leading country. Technologies in the two countries advance considerably and similarly if both countries experience the state of a lagging country for more periods. This implies that technological progress may be more balanced between the two countries as an international reversal of environmental leadership occurs more frequently.


\textsuperscript{19}In both examples, we set $\varepsilon = L = 0.5$ and take $(\kappa_A(t-1), \kappa_B(t-1)) = (2.5, 7.5)$ as an initial state.

\textsuperscript{20}We consider the following specific learning-by-doing function: $(\eta_i(t), \eta_k(t)) = (2, 0.75)$ if $\tau_i^*(t) > \tau_k^*(t) > 0$ and $(2, 0)$ if $\tau_i^*(t) > \tau_k^*(t) = 0$, which is consistent with assumptions (a) and (b).

\textsuperscript{21}Given that the scale effect dominates, an increase in global pollution over time implies that production increases over time. We can easily verify that in terms of utility, the increase in the output dominates the increase in pollution.

\textsuperscript{22}We consider the following learning-by-doing function: $(\eta_i(t), \eta_k(t)) = (1, 0.3)$ if $\tau_i^*(t) > \tau_k^*(t) > 0$ and $(1, 0)$ if $\tau_i^*(t) > \tau_k^*(t) = 0$, which is consistent with assumptions (a) and (b).
Remark 3 The long-run level of global pollution emissions can be either lower or higher than the initial level. In equilibrium where the environmental leadership shifts internationally (does not shift internationally), the long-run global pollution emissions may tend to be lower (higher) than their initial level.

6 Concluding Remarks

In this paper, we constructed a simple two-country model with global pollution and technological progress induced by learning by doing. We characterized the structure of equilibria and the environmental policies that achieve technological progress. Long-term global emissions and the dynamic path of environmental friendliness are related to the initial environmental friendliness, environmental awareness, and learning process between countries. Our findings underscore the importance of considering the implications of technological progress in a multicountry framework.

We have built a simple general equilibrium model to shed some light on the issue of environmental regulations and their effects on the learning by doing process and global emissions. It is certainly worthwhile to build alternative models to more deeply understand the mechanism in our paper. The following are in particular worth mentioning and have been left for future research. First, our analysis does not consider dynamic optimization because we treat pollution as a flow to derive clear-cut results. However, it is interesting to investigate the issue when pollution is a stock variable. Second, technological progress might be reinforced if the national government considers not only negative externalities caused by pollution, but also positive externalities of learning-by-doing. Third, the channel for knowledge growth could be by R&D investments as well as learning by doing. Last, there is no terms-of-trade effect because we have used a one-good model. Environmental regulations are affected by terms-of-trade effects, which could change the long-term pace of technological progress.

References


Appendix A

We will show the derivations for (9). Assume $\kappa_i > \kappa_j$. By substituting $(\tau_i^*, \tau_j^*) = (e_i, 0)$ and $(\tau_A, \tau_B) = (1, e_j)$ into (6) and (7), we have

$$e_i = \frac{\varepsilon L - \frac{1}{\kappa_i} \left( 1 + \frac{1}{\kappa_i} - \varepsilon L \kappa_j \right)}{\varepsilon L + \left( \frac{1}{1 + \kappa_i} - \varepsilon L \kappa_j \right)}$$

and

$$e_j = \frac{\varepsilon L - \frac{1}{\kappa_j} \frac{1}{1 + \kappa_j} \varepsilon L}{\varepsilon L + \frac{1}{1 + \kappa_j}},$$

respectively. With (A1) and (A2), noting $0 \leq e_i \leq 1$ and $0 \leq e_j \leq 1$ would imply (9), given the definitions of $p_i$ and $q_i$.

Appendix B

We will show the derivations for Figure 4 (the case with heterogeneous countries). The reaction function becomes

$$\tau_i^* (\tau_j^*) = \begin{cases} 
0 & \text{if } \varepsilon_i \leq \frac{1}{1 + \kappa_i} \left( \kappa_i L_i + \frac{\kappa_j (1 - \tau_j^*)}{1 + \kappa_j} L_j \right)^{-1} \\
e_i & \text{otherwise} \\
1 & \text{if } \varepsilon_i \geq \frac{1}{1 + \kappa_i} \left( \kappa_j (1 - \tau_j^*) \frac{1}{1 + \kappa_j} L_j \right)^{-1}
\end{cases},$$

where

$$e_i = \frac{\varepsilon_i L_i - \frac{1}{\kappa_i} \left( 1 + \frac{1}{\kappa_i} - \frac{\kappa_j (1 - \tau_j^*)}{1 + \kappa_j} \varepsilon_i L_j \right) \varepsilon_i L_j}{\varepsilon_i L_i + \left( \frac{1}{1 + \kappa_i} - \frac{\kappa_j (1 - \tau_j^*)}{1 + \kappa_j} \varepsilon_i L_j \right) \varepsilon_i L_j}.$$  

Define $\hat{\kappa}_i$ such that $\hat{\kappa}_i \equiv \frac{1}{\varepsilon_i L_i (1 + \kappa_i)} L_i$. Then, using (B1) and (B2), the equilibrium policy pair goes to

$$(\tau_i^*, \tau_j^*) = \begin{cases} 
(0, 0) & \text{if } \kappa_A L_A + \kappa_B L_B \leq \min_{i \in \{A, B\}} \left\{ \frac{1}{\varepsilon_i (1 + \kappa_i)} \right\} \\
(p_i, 0) & \text{if } \min \left\{ \frac{\varepsilon_i L_i}{\varepsilon_j}, \left( \kappa_i + \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_i} \right) \frac{1}{\varepsilon_i L_j (1 + \kappa_i)} \right\} > \kappa_j \geq \frac{1}{\varepsilon_i L_j (1 + \kappa_j)} - \frac{L_i}{L_j} \\
(1, 0) & \text{if } \hat{\kappa}_j > \kappa_j \geq \frac{1}{\varepsilon_i L_j (1 + \kappa_i)} \left( \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_i} \right) \\
(1, q_j) & \text{if } \varepsilon_i L_i \left( \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_i} \right) \left( \frac{1}{1 + \kappa_j} \right) \kappa_j > \kappa_j \geq \hat{\kappa}_j
\end{cases},$$

where

$$p_i = \frac{\varepsilon_i L_i - \frac{1}{\kappa_i} \left( 1 + \frac{1}{\kappa_i} - \varepsilon_i \varepsilon_i L_j \right)}{\varepsilon_i L_i + \left( \frac{1}{1 + \kappa_i} - \varepsilon_i \varepsilon_i L_j \right) L_j} \quad \text{and} \quad q_i = \frac{\varepsilon_i L_i - \frac{1}{\kappa_i} \frac{1}{1 + \kappa_i}}{\varepsilon_i L_i + \frac{1}{1 + \kappa_i}}.$$ 

It is straightforward to illustrate Figure 4 by using the above equilibrium conditions.

Appendix C

18
We will explain numerical calculations for Remark 3 in detail. In both examples, we think of \((\kappa_A(t-1),\kappa_B(t-1)) = (2.5, 7.5)\) as an initial point, which ensures stage I for country A as a leading country, noting (16). Set \(\varepsilon = L = 0.5\). Then, we calculate \(E(t-1) = \frac{1}{1+2.5} \approx 0.28571\).

Example (a): Technological progress follows \((\eta_i(t), \eta_k(t)) = (2, 0.75)\) if \(\tau_i^*(t) > \tau_k^*(t) > 0\) with \((2, 0)\) if \(\tau_i^*(t) > \tau_k^*(t) = 0\), which is consistent with the learning rules that we assume.

As \((\kappa_A(t-1), \kappa_B(t-1)) = (2.5, 7.5)\), the world is in stage I by (16), and \(E(t-1) = \frac{1}{1+2.5} \approx 0.28571\). Given the values of \(\eta_i(t)\) assumed, it goes to \((\kappa_A(t), \kappa_B(t)) = (1.75, 5.5)\). By (16), the world is also in stage I and we have \(E(t) = \frac{1}{1+1.75} \approx 0.36364\). In the subsequent period \(t+1\), it becomes \((\kappa_A(t+1), \kappa_B(t+1)) = (1, 3.5)\). Noting (17), the world shifts to stage II in period \(t+1\). We can calculate \(E(t+1) = 0.25\). Next, \((\kappa_A(t+2), \kappa_B(t+2)) = (1, 1.5)\) satisfies the inequality condition in (18), so it is in stage III and \(E(t+2) = \frac{1}{1+1.5} = 0.4\). Finally, it goes to \((\kappa_A(t+3), \kappa_B(t+3)) = (1, 0.5)\), which satisfies (19). In period \(t+3\), the world moves to the terminal stage IV and we can calculate \(E(t+3) = \frac{1}{4} = 0.375\), which is higher than the initial level \(E(t-1) \approx 0.28571\).

Example (b): Technological progress follows \((\eta_i(t), \eta_k(t)) = (1, 0.3)\) if \(\tau_i^*(t) > \tau_k^*(t) > 0\) and \((1, 0)\) if \(\tau_i^*(t) > \tau_k^*(t) = 0\), which is consistent with the learning rules that we assume.

Through the assumed process of technological progress, stage I continues in periods \(t\) to \(t+2: (\kappa_A(t), \kappa_B(t)) = (2.2, 6.5)\), \((\kappa_A(t+1), \kappa_B(t+1)) = (1.9, 5.5)\), and \((\kappa_A(t+2), \kappa_B(t+2)) = (1.6, 4.5)\) while \(E(t) = \frac{1}{1+2.2} \approx 0.3125\), \(E(t+1) = \frac{1}{1+1.9} \approx 0.34483\), and \(E(t+2) = \frac{1}{1+1.6} \approx 0.38462\). In periods \(t+3\) and \(t+4\), it goes to \((1.3, 3.5)\) and then \((1.3, 2.5)\), in which case the world is in stage II noting (17). Then, \(E(t+3) = E(t+4) = \frac{1.3}{4} = 0.325\). Next, \((\kappa_A(t+5), \kappa_B(t+5)) = (1.3, 1.5)\), which satisfies (18). It is stage III and \(E(t+5) = \frac{1}{1+1.5} = 0.4\). In period \(t+6\), it goes to \((1.3, 0.5)\), in which country B is a new leading country. An analogous inequality to that in (18), \(\kappa_B(t) < \frac{1}{\varepsilon L(1+\kappa_A(t))} < \kappa_A(t) + \kappa_B(t)\), is satisfied, so that the world is in stage III, \(E(t+6) = \frac{1}{1+1.3} = 0.43478\). Finally, it goes to \((\kappa_A(t+7), \kappa_B(t+7)) = (0.3, 0.5)\), in which the leadership internationally shifts again. Country A regains the leadership and it satisfies (19), stage IV. Then, we calculate \(E(t+7) = \frac{0.3}{4} = 0.2\), which is lower than the initial level \(E(t-1) \approx 0.28571\).
Figure 1: Seven regions of equilibrium policy on a $\kappa_A - \kappa_B$ plane
Figure 2: Phase diagram
Figure 3 (a): Environmental leapfrogging
Figure 3(b): Environmental leapfrogging
Figure 4 (a): $\varepsilon_A = \varepsilon_B$ and $L_A < L_B$
\[ \kappa_B = \frac{\varepsilon_A}{\varepsilon_B} \left( \kappa_A + \frac{\varepsilon_A - \varepsilon_B}{\varepsilon_B} \right) \]

Figure 4 (b): \( \varepsilon_A > \varepsilon_B \) and \( L_A = L_B \)
Figure 5 (a): Global pollution without environmental leapfrogging

Figure 5 (b): Global pollution with environmental leapfrogging