Private and Public Health Investment Decisions

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Abstract. In recent years, there has been an explosive increase in the demand for health products and services by people all around the globe, and particularly in advanced economies. Aiming to enhance longevity but also improve quality of life, individual consumption of pharmaceutical products and services has risen exponentially since the early 1980s. This paper develops a model in which agents invest part of their resources in medical products and time in physical exercise to enhance their health status. In the first part of the paper, we study the steady state and transitional dynamics of the model with special emphasis on the effects of health decisions on aggregate outcomes. In the second part, we explore how public health policies may alter private economic decisions that promote healthier and more productive lives.

JEL Classification: O16, O24, O38, O43.

Keywords: Private and public health investments; endogenous discounting of the rate of time preference; neoclassical growth model; health policy.

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1. Introduction

Motivated by recent literature that finds significant economic returns from investment in health, this paper incorporates individual agents' health decisions in the neoclassical growth model. But unlike most of the existing literature that focuses attention on the effects of health status on economic development – and more specifically the link between mortality and growth in developing countries – this paper is concerned with health decisions by rational agents living in relatively wealthy/healthy societies. Although there is a clear increase in demand for health goods (e.g. medicines, vitamins, and vaccines), and services (e.g. surgery, psychotherapy, and health clubs), coupled with a lively debate regarding the relative merits of private vs. public provision of health in the US and Europe, there is surprisingly little research on how rational health investment behavior may affect economic aggregates and what, if any, should be the government's role in this process.

This is an important issue and in this paper we address it by introducing health decisions into an otherwise standard dynamic general equilibrium framework. We identify three channels whereby health status impinges on the aggregate economy. These include: (i) its direct impact on wellbeing and the consequences for consumption and leisure in utility, (ii) its impact on productivity, and (iii) its impact on the rate of time discount via longevity, as in Blanchard (1985) and the subsequent demographic literature. Once the long-run and transitional dynamic characteristics of the macroeconomic equilibrium are established, we address the key policy question, namely: What would be the effects of government health investment on productivity, growth, and welfare? Under-investment in health by individuals may require proactive government intervention to subsidize the purchase of medical goods and services as well as memberships to health clubs and other designated forms of physical activities and recreation. Our analysis, based on numerical simulations, assesses how government intervention may impact agents' health, productivity, and subsequently aggregate growth and wellbeing.

The policy experiments we perform suggest that there are significant welfare gains associated with movement from the current tax structure and rate of public health investment (which we take to be typical of a developed economy) to the first best optimum where the government controls all resources, including health, directly. For the benchmark calibration of our model, with a tax structure and other structural characteristics that approximate advanced economies like the US and UK, welfare would increase by 21.6% across steady states (measured in terms of equivalent variation of per capita consumption). Including the transitional path would further increase the welfare gains to 23.6%. Moreover, for a relatively modest increase in the productivity effect of health [channel (ii) above] the long-run welfare benefits increase even further to around 33%.[1]

However, a large proportion of these gains arise from the dramatic change in the tax structure necessary to achieve the first-best optimum. This is a general characteristic of the basic growth model, where in the absence of externalities associated with labor or consumption the optimal consumption tax must exactly offset the optimal labor income tax.[2] As is evident from Table 3 below, this is completely implausible and can be dismissed as impractical. Accordingly, to examine the potential of health investment to raise welfare we consider more modest (and more realistic) policy experiments. Specifically, we compare an increase in health investment expenditure of 1% of GDP under alternative modes of finance. For the benchmark parameterization, a lump-sum tax financed health investment yields more modest (but still quite significant) welfare gains of 3.70%, somewhat larger than the 3.22% increase obtained by employing a consumption tax. In the case where the investment is financed by a tax on capital income, the resulting decline in output and consumption is sufficient for the net effect to reduce welfare by 1.35%. But increasing the productive elasticity of health modestly enhances all these welfare gains, in which case even employing capital tax financing can generate a modest overall improvement in welfare.

These policy experiments highlight the fact that increasing public investment in health by 1%

[1] The increase in longevity associated with improvements in health tend to have a negative impact on per capita welfare, reflecting the fact that with people living longer, the output of the economy must be shared among a larger population.

[2] See e.g. Turnovsky (2000). As noted there this result is an intertemporal application of the Ramsey principle of optimal taxation; see Deaton (1981). If the utility function is multiplicatively separable in consumption and leisure, as we shall assume, then uniform taxation of leisure and consumption is optimal.

of GDP – which represents a substantial increase in the *level* of investment in health – with the economy's tax structure virtually unchanged nevertheless leaves the economy far from the social optimum. This raises the question of whether it is possible to realize a substantial portion of the potential welfare gains by combining the government investment in health with additional, moderate tax changes that move at least partially in the direction of the first-best policy. Thus, we find that 50% of the welfare gains can be realized by combining the 1% increase in health investment with a halving of the income tax rates from their respective 20% benchmark rates, compensated for by doubling the consumption tax. While these represent dramatic changes in the US tax structure, they are not implausible, and indeed reflect changes in the tax structure that have been advocated by conservative policy makers over the years.[3] Furthermore, eliminating the income tax entirely in favor of the consumption tax, while doubling investment in health (from 3% to 6% of GDP) would increase the welfare gains realized to more than 85% of the potential gains associated with the first best. In addition, we also find that the welfare gains obtained by increasing the rate of government investment in health are quite robust with respect to the initial tax structure.

There is an emerging interest in the potential effects of health on development (see, e.g. Strauss and Thomas (1998); Deaton (2003, 2004, 2009, 2013); Chakraborty (2004); Acemoglu and Johnson (2007); Birchenall (2007); Bloom and Canning (2005); Soares (2005); Lorentzen et al. (2006); Weil (2007); Birchenall and Soares (2009); Chakraborty, et al. (2013); Bhattacharya and Chakraborty (forthcoming)). This literature focuses primarily on investigating the hypothesis that health status (measured as positively related to life expectancy, or inversely related to mortality or diseases) is a key determinant in explaining cross-country income differences via its direct or indirect effect on individuals' productivity and savings behavior. Similar to this literature, in our work health status is an important determinant of productivity of the individuals. However, as our focus is on the potential effects of health investment on the productivity and welfare of agents living

[3] The Cato Institute in the United States is well known for advocating the elimination of the income tax in favor of the sales tax.

in relatively wealthy countries, our work also relates closely to a smaller set of theoretical contributions focusing on the effects of the decisions of individual agents to maximize, in addition to consumption, also their life expectancy (time horizon). These include Ehrlich and Chuma (1990), de la Croix and Licandro (1999), Aísa and Pueyo (2004a, 2004b), Finlay (2006), Agénor (2008, 2010) and Hosoya (2009). Our approach is most closely related to Blanchard's (1985) seminal contribution and more recently Faruqee (2003) in which agents' decisions impact the time discount rate.

The remainder of the paper is organized as follows. Section 2 discusses recent trends in health investment including both increased spending in medical goods and services and a very large increase in health club membership. Section 3 develops and describes in detail the basic elements of the model, and Section 4 studies the dynamic behavior and steady state for a decentralized economy. Section 5 extends the analysis to a centrally planned economy. Section 6 describes the explicit function forms and calibrations employed in the numerical simulations presented for the set of policy experiments in Section 7. Section 8 concludes, while technical details are relegated to the Appendix.

2. More Pills, More Therapy, More Health Clubs

Even a few decades ago the average citizen in a relatively wealthy nation like the US or the UK would not make active decisions regarding health issues, other than perhaps consulting the medical profession when necessary. This was partly due to lack of sufficient income to engage more actively in health investment decisions, such as those that would result in increasing one's lifespan and/or quality of life. Mostly, however, it was due to treating health and especially life expectancy as largely exogenous elements – functions of genes, luck, and perhaps the level of development in the community.

This passive outlook on how much control individuals have over their health seems to have reversed over the last few decades, where we now observe a huge emerging demand for health products and services. It is argued that this intriguing behavioral change is motivated from life saving medical technological advances that have contributed to the development of the mass production of new medical products at reasonable prices. Another reason for the drastic increase in health decisions is the structural transformation from agriculture and manufacturing to services. While agricultural production and manufacturing require physical labor, services primarily depend on human capital rather than on raw labor. Therefore, much of the physical activity necessary for a healthy existence, now need to be pursued outside one's work.

Recent studies confirm that benefits of regular physical activity include a reduced risk of premature mortality and reduced risks of coronary heart disease, diabetes, colon cancer, hypertension, and osteoporosis (United States Department of Health and Human Services, 1996). Regular physical activity also improves symptoms associated with musculoskeletal conditions and mental health conditions such as depression and anxiety. Physical activity, along with a healthy diet, plays an important role in the prevention of overweight and obesity, which have increased at alarming rates in the recent years.

In 1995, the Centers for Disease Control and Prevention (CDC) issued a public health recommendation that every US adult should accumulate 30 minutes or more of moderate-intensity physical activity on most, preferably all, days of the week. In 2007, the American Heart Association issued updated recommendations on the types and amounts of physical activity needed by healthy adults to improve and maintain health (Haskell et al, 2007). In October 2008, the US Department of Health and Human Services released guidance to help Americans age 6 and older improve their health through appropriate physical activity (United States Department of Health and Human Services, 2008).

The World Health Organization (WHO) reports that health expenditure in OECD countries increased on average by 24% during the period 1995-2012. The U.S. was at the top of the list of countries in terms of the percent of health expenditure to GDP of over 18% with France and the Netherlands distant second with 12%. Between 1999 and 2009, U.S. health care spending nearly doubled, climbing from \$1.3 trillion to \$2.5 trillion. Much of the health expenditure came from government albeit with diverse experiences across countries. The largest and smallest contributions of government to total health spending in 2012 among OECD countries were by Denmark (86%) and the U.S. (46%), respectively. It is worth noting that 11 OECD countries contributed more than 80% of total health spending. But individuals these days do not only invest in improving longevity but also quality of life. As is well known, there is a positive and very robust relationship between health spending and income levels (WHO, 2017).

The upper panel of Figure 1 shows this relationship at the cross-country level (this relationship has also been established at the individual/consumer level). The lower panel of Figure 1 depicts another well-established relationship (similar to the Preston curve (1975) relating life expectancy and per capital income) illustrating a strong and positive relationship between health spending and life expectancy early on, but a leveling off (indeed a flattening) of this relationship at older ages (over 80s). Combining the two figures implies that most of the health spending in developed countries is not aimed at increasing longevity but rather at improving quality of life.

Indeed, investing in preventive medicine and improved quality of life, especially in advanced economies, is clearly demonstrated by the drastic increase in the fitness and health clubs industry. According to the *International Health, Racquet and Sportsclub Association* (IHRSA) statistics, the number of health club or gym memberships has increased from 36.3 million in 2002 to approximately 58.5 million in January 2013.

The *Physical Activity Council's Participation Report* (2013) revealed that over 60% of Americans regularly participate in fitness sports as of 2012; this is the fifth consecutive year in which these numbers have remained stable at 60% or more. The *U.S. Bureau of Labor Statistics* suggests that the number of jobs in the health club industry is expected to increase by more than 23% over the next 10 years. This prediction was obtained by looking at how businesses and corporations are offering health club memberships to their employees as incentives. Consistent with these projections, the number of personal trainers has also risen incredibly over the past few years. According to the Labor Department, there were 231,500 personal trainers in America as of 2011 (Rampell, 2012). This goes to show that investing in preventive health and fitness is only showing potential for stronger growth in the US, with this phenomenon catching fast throughout the globe.

In summary, the health investment incorporated in our model is motivated by recent trends reflecting individuals trying to enhance their longevity and quality of life by purchasing medical goods and services and/or devoting some of their time to physical activities (thus in addition to work time, and leisure time, we also consider physical activity time). It is also motivated by the very large share of GDP allocated to public health expenditure, particularly in the US.

3. Baseline Model

The framework we develop extends and reformulates the basic neoclassical growth model in several directions, specifically by emphasizing the role of health as an additional productive sector (rather than as a homegrown activity). This sector produces health services using both a private input (labor) and a public input (infrastructure, i.e., medical equipment and hospitals). The use of health services directly enhances the welfare of individual agents and by enhancing longevity reduces their rate of time preference, making them more patient. In addition, the aggregate level of health services also enhances productive efficiency in the economy—an externality that is not internalized by firms and individuals. In light of this, the model allows for partial or universal subsidization of health costs, which is also a characteristic common to all advanced economies. Thus, our formulation is directed at capturing the role of health in a developed economy.[4] We begin by describing the model, emphasizing modifications and additions made to the standard neoclassical model to incorporate public and private health investments.

3.1 The Representative Consumer

We assume that all individuals are identical and that population grows at the constant rate n. The identical consumers in the economy maximize utility specified as a positive function of

[4] While the current model does not incorporate health insurance, it could be modified to address issues such as whether health costs ought to be shared between individuals and firms.

consumption (c), leisure (l), and health (h). In addition, as noted, since health increases longevity, the rate of time preference $\theta(h)$ is decreasing in health, i.e., $\theta_h(h) < 0$. Thus, agents maximize the concave intertemporal utility function

$$\int_0^\infty U\left(c,l,h\right)e^{-\int_0^t \theta(h(s))ds}dt\,, \qquad U_c > 0, U_l > 0, U_h > 0 \tag{1a}$$

subject to the budget constraint

$$\dot{k} = \left[(1-\tau_k)r - (n+\delta)\right]k + (1-\tau_w)w(1-l) - (1+\tau_c)c - p(1-s)h - T\,, \tag{1b}$$

where all quantities are in per capita terms: k is capital, r is return to capital, and δ is depreciation of capital, w is the wage rate, τ_k, τ_w, and τ_c are rates of capital, labor, and consumption taxes, and T denotes the lump-sum tax. Equation (1b) also shows that the agent purchases health services at a price p, which may be subsidized by the government at the rate s. These health services are broadly defined to include medical services, pills, and even subscriptions to health clubs. For simplicity, we identify the purchase of these health services as being identical to health itself.[5] In general, s is unrelated to any tax rate, but if, for example, $s = \tau_w$, this implies that individuals may exactly deduct their health costs from their tax paid on labor income. We also assume that the household may work either in the final output sector or in the health sector, with each sector paying the same wage.

To solve the consumer's optimization problem, it is convenient to define, $z(t)$, the cumulative rate of time preference over the time interval (0, t), by

$$z(t) \equiv \int_0^t \theta[h(s)]ds\,, \tag{2}$$

which implies

$$\dot{z}(t) \equiv \theta[h(t)]\,. \tag{3}$$

[5] More generally one would imagine that health is a positive concave function of resources devoted to health.

Thus, the agent's optimization problem can be re-expressed as to maximize

$$\int_0^\infty U\left(c,l,h\right)e^{-z(t)}dt\,, \tag{1a'}$$

subject to (1b) and (3).

Performing the optimization yields the following first order conditions:

$$U_c(c,l,h)=\lambda(1+\tau_c) \tag{4a}$$

$$U_l(c,l,h)=w(1-\tau_w)\lambda \tag{4b}$$

$$U_h(c,l,h)+\mu\theta_h = p\lambda(1-s) \tag{4c}$$

$$r(1-\tau_k)-n-\delta=\dot{z}(t)-\frac{\dot{\lambda}}{\lambda} \tag{4d}$$

$$-\frac{U(c,l,h)}{\mu}=\dot{z}(t)-\frac{\dot{\mu}}{\mu}\ , \tag{4e}$$

together with the transversality conditions

$$\lim_{t\to\infty}\lambda k e^{-z(t)}=\lim_{t\to\infty}\mu z e^{-z(t)}=0\,.$$

where λ,μ are the costate variables associated with the dynamic equations (1b) and (3), respectively.

The optimality conditions (4a) and (4b) equate the marginal utility of consumption and leisure to their respective tax-adjusted marginal costs expressed in utility units, and are standard. Equations (4d) and (4e) are arbitrage conditions. The first equates the rate of return on capital to the utility rate of return on consumption measured in terms of units of output, and is also standard. Integrating (4e) yields

$$\mu(t)=-\int_t^\infty U(s)e^{-[z(s)-z(t)]}ds\,,$$

indicating that $\mu(t)$ reflects the discounted losses in utility resulting from a higher discount rate. Hence, (4c) equates the marginal costs of health services (net of subsidy) to the benefits, which include the benefits from increased longevity, in addition to the direct utility benefits. By combining (4a) and (4c), we obtain

$$\frac{U_c(c,l,h)}{1+\tau_c} = \frac{U_h(c,l,h)+\mu\theta_h}{(1-s)p}$$

Thus, the optimal spending decision equates the tax-adjusted marginal utility of consumption to the subsidy-adjusted marginal utility of health, which as just noted, includes the direct increase in marginal utility plus the indirect marginal benefits, resulting from the increase in longevity.

3.2 The Production Process

Production in the economy takes place in two sectors: a conventional final output sector, with each firm owned by a private individual, and a health sector, owned by the government. The representative firm in the final output sector produces in accordance with the conventional production function

$$y = f(k,L,h), \qquad f_k > 0,\ f_L > 0,\ f_h > 0 \tag{5a}$$

which is homogeneous of degree one in k and labor, L. The firms choose k, L, to maximize profit:

$$f(k,L,h) - wL - rk \ . \tag{5b}$$

In doing so, they take the level of health (h) as given, which thus provides an externality. Factor returns are given by the usual marginal conditions

$$f_k(k,L,h) = r \tag{6a}$$

$$f_L(k,L,h) = w. \tag{6b}$$

Health services are produced in accordance with the production function

$$h = h(m,e), \qquad h_m > 0,\ h_e > 0 \tag{7a}$$

which is also homogeneous of degree one, in *m*, and *e*, where *m* is the per capita health infrastructure/capital provided by the government, while *e* is the labor employed in the health sector.[6] Thus (private) physical capital is specific to final goods production, while (public) health capital is specific to health services production. The health sector firm chooses employment, *e*, to maximize

$$ph(m,e) - we, \tag{7b}$$

leading to the optimality condition

$$ph_e(m,e) = w. \tag{7c}$$

The homogeneity of the health production function means that the government earns profit, $p(h - h_e e)$, which contributes to its revenue.

3.3 Government

Expressed in per capita terms, the government's budget constraint is

$$T = \dot{m} + nm + sph - \tau_k rk - \tau_w w(L+e) - \tau_c c - p(h - h_e e). \tag{8}$$

According to (8) current government expenditures include its increase in health capital, both per capita and for the growing population ($\dot{m} + nm$) and its subsidy to health expenditures (sph). Its revenues include the total tax collected ($\tau_k rk + \tau_w w(L+e) + \tau_c c$), as well as profit earned by the

[6] We recognize that this is a very stylized specification of the production function for health, but one that suffices for our purposes and is consistent with the dimensionality of the rest of the model. Basically, production functions for health begin by disaggregating labor (e.g. to physicians and paramedics), and some measure of medical capital but augment to include other factors like education, drug expenditures etc.; see e.g. Auster, Leveson, and Sarachek (1969) for an early seminal formulation.

health sector, $p(h-h_e e)$. To the extent that these items are not balanced it finances the difference with lump-sum taxes. Since we are focusing on a growing economy, we assume that the government devotes a fraction, *g*, of aggregate output to augment the aggregate stock of public health capital. Thus, in per-capita terms, we have

$$\dot{m} = gf(k,L,h) - nm \,, \tag{9}$$

which using (7a)-(7b) enables us to rewrite the government budget constraint (8) in the form

$$T = gf(k,L,h) + sph - \tau_k f_k k - \tau_w f_L (L+e) - \tau_c c - p(h-h_e e) \,. \tag{8'}$$

3.4 Market Clearance

Labor is assumed to be both fully employed and enjoys free mobility across sectors, implying:

$$L + e = 1 - l \,. \tag{10}$$

Combining the consumers' budget constraint, (1b), and government's (per-capita) budget constraint, (8'), recalling (7) and utilizing (10) yields the final goods market clearing condition

$$\dot{k} = (1-g) f(k,L,h) - c - (n+\delta)k \,, \tag{11}$$

which determines the evolution of the capital in the economy.

As noted in the introduction, there is a significant variance across countries in the extent to which health is publicly or privately provided. This model is a hybrid in that the government provides the capital employed in the health sector, while the labor is private individuals.[7] Thus, the total expenditure on health is $gf + ph$.

[7] This is a contrast to the growth model of Agénor (2008, 2013), for example, in which health is fully provided by the government.

4. Solving the Baseline Model

To solve the system for the macrodynamic equilibrium, we reduce it to a core dynamic system. It is evident that as we have set out the model, it is already specified in terms of stationary variables. This is because we assume zero growth in factor productivities in both sectors. Thus, the growth in the aggregates of this economy along the balanced growth path is driven solely by the growth of population.

To describe the core dynamic system that governs the evolution of the economy, we first equate (7b) and (7c) to obtain

$$ph_e(m,e) = f_L(k,L,h). \tag{12}$$

Using (12) and (7a-7b) to eliminate *r*, *w*, and *p* from (4b-4c), we can reduce the short-run equilibrium conditions to (4a), (5b), (10), and

$$U_l(c,l,h) = \lambda(1-\tau_w) f_L(k,L,h) \tag{4b'}$$

$$\left(\frac{1-s}{1-\tau_w}\right) U_l(c,l,h) = \left[U_h(c,l,h) + \mu\theta_h(h)\right] h_e(m,e). \tag{4c'}$$

Equations (4b') and (4c') imply that time should be allocated such that the marginal utility of leisure, the tax-adjusted marginal utility of wages foregone in the final output sector, and the marginal utility of wages foregone in the health sector should all be equalized.

These 5 equations determine c, h, l, L, e as functions of m, k, λ, μ the dynamics of which are driven by the system consisting of (11), (9), and

$$\frac{\dot{\lambda}}{\lambda} = \theta(h) + n + \delta - f_k(k,L,h)(1-\tau_k) \tag{4d'}$$

$$\frac{\dot{\mu}}{\mu} = \theta(h) + \frac{U(c,l,h)}{\mu}\ , \tag{4e'}$$

given the time paths of the exogenous policy variables $g, \tau_k, \tau_w, \tau_c$, and s, with lump-sum taxes adjusting residually to satisfy the government's budget constraint, (8').

The long-run equilibrium (steady state) of the system is described by the above set of equations when dynamic variables are time-invariant, i.e., by setting $\dot{k} = \dot{m} = \dot{\lambda} = \dot{\mu} = 0$. This yields:

$$U_c(\tilde{c}, \tilde{l}, \tilde{h}) = \tilde{\lambda}(1+\tau_c) \tag{13a}$$

$$U_l(\tilde{c}, \tilde{l}, \tilde{h}) = \tilde{\lambda}(1-\tau_w) f_L(\tilde{k}, \tilde{L}, h) \tag{13b}$$

$$\left(\frac{1-s}{1-\tau_w}\right) U_l(\tilde{c}, \tilde{l}, \tilde{h}) = \left[U_h(\tilde{c}, \tilde{l}, \tilde{h}) + \tilde{\mu}\theta_h(\tilde{h})\right] h_e(\tilde{m}, \tilde{e}) \tag{13c}$$

$$\tilde{h} = h(\tilde{m}, \tilde{e}) \tag{13d}$$

$$\tilde{L} + \tilde{l} + \tilde{e} = 1 \tag{13e}$$

$$(1-g) f(\tilde{k}, \tilde{L}, \tilde{h}) = \tilde{c} + (n+\delta)\tilde{k} \tag{13f}$$

$$g f(\tilde{k}, \tilde{L}, \tilde{h}) = n\tilde{m} \tag{13g}$$

$$f_k(\tilde{k}, \tilde{L}, \tilde{h})(1-\tau_k) = \theta(\tilde{h}) + n + \delta \tag{13h}$$

$$\tilde{\mu}\theta(\tilde{h}) + U(\tilde{c}, \tilde{l}, \tilde{h}) = 0\,. \tag{13i}$$

These equations then can be solved for $(\tilde{c}, \tilde{l}, \tilde{L}, \tilde{e}, \tilde{h}, \tilde{k}, \tilde{m}, \tilde{\lambda}, \tilde{\mu})$, where the tilde denotes the steady state value of the corresponding variable. In particular, (13h) corresponds to the modified golden rule equilibrium of the standard neoclassical growth model. As in that model, increases in capital-labor ratio arise from factors that raise marginal product of capital (such as growth in total factor productivity). While this also applies to h in this model (as long as $f_{kh} > 0$), this equation shows that, by reducing the rate of time preference and stimulating savings, an increase in health status has an additional long-run stimulative effect on the capital-labor ratio.

From these solutions one can derive the equilibrium relative price of health services from the relationship $\tilde{p} = f_L(\tilde{k}, \tilde{L}, \tilde{h}) / h_e(\tilde{m}, \tilde{e})$. The steady-state government budget constraint is (in per capita terms) given by

$$\tilde{T} = g\tilde{f} - \tau_k \tilde{f}_k \tilde{k} - \tau_w \tilde{f}_L(\tilde{L} + \tilde{e}) - \tau_c \tilde{c} + s\tilde{p}\tilde{h} - \tilde{p}(\tilde{h} - h_e \tilde{e}). \tag{13j}$$

To solve for the transitional dynamics, we substitute the solutions for *c, h, l, e*, and *L* obtained from (4a), (4b'), (4c'), (7a), and (10) into the dynamic equations (11), (9), (4d'), and (4e'). Linearizing the resulting expressions around the steady state (13) yields the local dynamics. The partial derivatives required for this process are reported in Appendix A.1, enabling us to approximate the transitional dynamics by the following linearized system

$$\begin{pmatrix} \dot{k} \\ \dot{m} \\ \dot{\lambda} \\ \dot{\mu} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} k - \tilde{k} \\ m - \tilde{m} \\ \lambda - \tilde{\lambda} \\ \mu - \tilde{\mu} \end{pmatrix}, \tag{14}$$

where the expressions for the coefficients in (14) can be found in Appendix A.1. We assume that this system has saddlepoint structure, yielding a unique stable transitional path, an assumption that is unambiguously supported by the simulations that we discuss below.[8]

5. The Social Planner's Problem

By its very nature, health has many of the characteristics of a public good. In terms of our formal model, it appears as an externality in the production of the physical good as the health status of the worker is a non-market or non-remunerated input into the production function. It is thus natural to determine the socially optimal resource allocation in such an economy, and to investigate the extent to which the government in a decentralized economy can use the fiscal instruments at its

[8] Specifically, there are two positive and two negative eigenvalues, corresponding to the two "jump variables" λ, μ and the two sluggish variables, k, m

disposal to replicate the first-best outcome, as would be chosen by a central planner.

Specifically, we assume that the social/central planner chooses the resource allocations directly, namely, c, l, e, L, h, g , and the rates of accumulation, $\dot{k}, \dot{z}, \dot{m}$, to maximize the representative agent's intertemporal utility (1a) subject to the resource constraints, (3), (7a), (9), (10), and (11). Consolidating (9) and (11), the Hamiltonian summarizing the planner's optimization problem is

$$U(c,l,h)e^{-z(t)} + \lambda e^{-z(t)}\left[f(k,L,h) - c - (n+\delta)k - nm - \dot{k} - \dot{m}\right] + \mu e^{-z(t)}\left[\theta(h) - \dot{z}\right]$$

$$+\nu_1 e^{-z(t)}\left[1 - l - L - e\right] + \nu_2 e^{-z(t)}\left[h(m,e) - h\right] .$$

The resulting optimality conditions are respectively:

$$U_c(c,l,h) = \lambda \tag{15a}$$

$$U_l(c,l,h) = \lambda f_L(k,L,h) = \nu_1 \tag{15b}$$

$$-\nu_1 + \nu_2 h_e = 0 \tag{15c}$$

$$U_h(c,l,h) + \lambda f_h + \mu\theta_h(h) = \nu_2 \tag{15d}$$

$$\theta(h) - \frac{\dot{\lambda}}{\lambda} = f_k - (n+\delta) = h_m \frac{\nu_2}{\lambda} - n \tag{16a}$$

$$\theta(h) - \frac{\dot{\mu}}{\mu} = -\frac{U(c,l,h)}{\mu}. \tag{16b}$$

The second equality in (16a) asserts that the rate of return on consumption, expressed in units of health equals the rate of return on investing in health. The latter equals the utility generated per unit of production, divided by cost less the growing population that needs to be supported.

From these equations, the macroeconomic equilibrium in the centrally planned economy can be reduced to the following static equations:

$$U_c(c,l,h) = \lambda \tag{15a}$$

$$U_l(c,l,h) = \lambda f_L(k,L,h) \tag{15b}$$

$$U_l(c,l,h) = \left[U_h(c,l,h) + \lambda f_h + \mu\theta_h(h)\right] h_e(m,e) \tag{15d'}$$

$$\frac{f_k(k,L,h) - \delta}{f(k,L,h)} = \frac{h_m(m,e)}{h_e(m,e)} \tag{16a'}$$

$$h = h(m,e) \tag{7a}$$

$$l + L + e = 1\,. \tag{10}$$

together with the dynamic equations:

$$\dot{k} = (1-g)f(k,L,h) - c - (n+\delta)k \tag{11}$$

$$\dot{m} = gf(k,L,h) - nm \tag{9}$$

$$\frac{\dot{\lambda}}{\lambda} = \theta(h) + n + \delta - f_k \tag{16a''}$$

$$\frac{\dot{\mu}}{\mu} = \theta(h) + \frac{U(c,l,h)}{\mu}\,. \tag{16b'}$$

It is important to note that this macrodynamic equilibrium implicitly generates a time path for the optimal government health investment, *g*. To determine this, we view the set of static equations (15a), (15b), (15d'), (16a'), (7a) and (10) as a subsystem that can be solved for c, l, L, e, h, μ as functions of k, m, λ. In particular, consider $\mu = \mu(k,m,\lambda)$. Taking the time derivative of this expression yields

$$\dot{\mu} = \mu_k \dot{k} + \mu_m \dot{m} + \mu_\lambda \dot{\lambda}\,,$$

implying that the four dynamic equations (11), (9), (16a), (16b') are not all independent. Substituting for these equations yields

$$\mu\theta(h) + U = \mu_k\left[(1-g)f - c - (n+\delta)k\right] + \mu_m\left[gf - nm\right] + \mu_\lambda\lambda\left[(\theta+n+\delta) - f_k\right].$$

This equation can now be solved explicitly for the time path of g, which through this relationship is also of the form

$$g = g(k, m, \lambda). \tag{17}$$

The steady state of the centrally planned economy, denoted by bars, consists of

$$U_c(\bar{c}, \bar{l}, \bar{h}) = \bar{\lambda} \tag{18a}$$

$$U_l(\bar{c}, \bar{l}, \bar{h}) = \bar{\lambda} f_L(\bar{k}, \bar{L}, \bar{h}) \tag{18b}$$

$$U_l(\bar{c}, \bar{l}, \bar{h}) = \left[U_h(\bar{c}, \bar{l}, \bar{h}) + \bar{\lambda}\bar{f}_h + \bar{\mu}\theta_h(\bar{h})\right] h_e(\bar{m}, \bar{e}) \tag{18c}$$

$$\frac{f_k(\bar{k}, \bar{L}, \bar{h}) - \delta}{f(\bar{k}, \bar{L}, \bar{h})} = \frac{h_m(\bar{m}, \bar{e})}{h_e(\bar{m}, \bar{e})} \tag{18d}$$

$$\bar{h} = h(\bar{m}, \bar{e}) \tag{18e}$$

$$\bar{l} + \bar{L} + \bar{e} = 1 \tag{18f}$$

$$(1-\bar{g}) f(\bar{k}, \bar{L}, \bar{h}) = \bar{c} + (n+\delta)\bar{k} \tag{18g}$$

$$\bar{g} f(\bar{k}, \bar{L}, \bar{h}) = n\bar{m} \tag{18h}$$

$$f_k(\bar{k}, \bar{L}, \bar{h}) = [\theta(\bar{h}) + n + \delta] \tag{18i}$$

$$\bar{\mu}\theta(\bar{h}) + U(\bar{c}, \bar{l}, \bar{h}) = 0. \tag{18j}$$

These ten equations determine the steady-state values of $\bar{c}, \bar{l}, \bar{L}, \bar{e}, \bar{h}, \bar{k}, \bar{m}, \bar{\lambda}, \bar{\mu}, \bar{g}$.

5.1 Replication of the Steady-state of the First-best Equilibrium

We now turn to the question posed earlier: can government use its instruments (tax and subsidy rates) to replicate the first-best outcome in the decentralized, market equilibrium? For convenience, we focus on the steady state. To achieve this, we need to determine if we can find $\tau_w, \tau_c, \tau_k, s$ so that the allocation of (13) mimics that of (18). Our objective is to replicate the first-best allocation of real quantities. To do this we eliminate shadow values and prices, which only serve as signals for resource allocation.

Proposition 1 (*Replication of the Steady State of the First-Best Equilibrium*). *The first-best optimum will be attained if and only if the following three conditions hold:*

$$\tau_k = 0 \tag{19a}$$

$$\tau_w + \tau_c = 0 \tag{19b}$$

$$\frac{1-s}{1-\tau_w} = \frac{U_h - U\theta_h\theta^{-1}}{U_h - U\theta_h\theta^{-1} + U_l f_h (f_L)^{-1}} = 1 - \frac{f_h}{f_L} h_e < 1 . \tag{19c}$$

Proof: See Appendix A.2.

Setting the long-run optimal tax on capital, $\hat{\tau}_k = 0$, is consistent with the celebrated proposition introduced by Chamley (1986) and Judd (1985). However, here a different mechanism is at play. The Chamley-Judd proposition was based on the assumption that government expenditure was fixed in *levels*. If instead it were set as an optimal fraction of income, $\hat{g}$ say, then the corresponding optimal tax on capital would be set correspondingly as $\hat{\tau}_k = \hat{g}$. By contrast, in the present context, if the fraction of government expenditure, *g*, is set arbitrarily, then one can show $\hat{\tau}_k = g(1-\phi/\lambda)$ where ϕ is the shadow value of public capital (health).[9] With private capital, *k*, being costless to transfer to public capital, *m*, (and vice versa), the optimal policy is to set the allocation between the two capital goods so as to equate their respective shadow prices, implying

[9] This is evident from the aggregate resource constraint: $f(k,L,h) - c - (n+\delta)k - nm = \dot{k} + \dot{m}$.

$\hat{\tau}_k = 0$.

The condition (19b) is also well known.[10] Since there are no imperfections on either the labor market or the final goods market, the distortions introduced by taxing labor income and consumption must be exactly offsetting. A given tax rate on labor income must be exactly offset by an identical subsidy on consumption and vice versa. An important consequence of this constraint is that the ability to tax consumption introduces flexibility with respect to taxing income from labor. This is important in generating the tax revenues to finance its expenditures. With, τ_c and τ_w working against each other in terms of revenue generation, the effective tax base for the government is the difference between the labor income and consumption. Given the difference between the two is almost surely less than one-fifth of GDP for a typical country, raising realistic amount of revenue (even without considering subsidies for health good) would require very high tax rates. Moreover, the tax/subsidy rate would be highly sensitive to the revenue needs.

The equation (19c) is new and arises specifically through the introduction of health. It implies that a necessary condition to replicate the first best optimum is that $s > \tau_w$; that is the subsidy to health costs must exceed the rate of income tax (on labor income). The reason is as follows. The purpose of the subsidy is to offset the undersupply of health services in the market equilibrium arising from externality in final goods production. However, a tax on labor income by reducing reward to working, and hence reducing overall labor supply and production in both sectors, counteracts the effectiveness of subsidy. s must exceed τ_w to address the equilibrium under provisioning of health services. Finally (19c), intuitively implies that the higher the indirect marginal benefit of labor through employment in health sector $(f_h h_e)$ in terms of output relative to its direct benefit through employment in goods sector, the higher is the subsidy.

Substituting (19a-19c) into the government budget constraint (13j) and using the steady-state conditions (13) and (18) one can show the following tradeoffs between the optimal tax on labor

[10] Similar conditions to (19a) and (19b) are obtained by Turnovsky (2000) in a macro growth model that abstracts from the health sector.

income, $\hat{\tau}_w$, the health subsidy, $\hat{s}$, and lump-sum taxes, $\hat{T}$, that will ensure that the steady-state equilibrium in the decentralized economy will replicate the first-best optimum. To simplify the expressions and facilitate the interpretation we shall assume that production in the final output and health sectors are specified by the following Cobb-Douglas functions that we shall employ in subsequent simulations:

$$y = Ak^{\alpha} L^{1-\alpha} h^{\beta} \tag{20a}$$

$$h = Bm^{\varphi} e^{1-\varphi} . \tag{20b}$$

Substituting the optimal tax rates (19a)-(19c) into the government budget constraint (8'), yields

$$\hat{\tau}_w + \frac{\hat{T}/y}{\hat{g} + \beta - \left(\dfrac{\varphi}{1-\varphi}\right)\dfrac{\overline{e}(1-\alpha)}{\overline{L}} - \left(\dfrac{\theta(h)}{\theta(h)+n+\delta}\right)\alpha} = \frac{\hat{g} + \beta - \left(\dfrac{\varphi}{1-\varphi}\right)\dfrac{\overline{e}(1-\alpha)}{\overline{L}}}{\hat{g} + \beta - \left(\dfrac{\varphi}{1-\varphi}\right)\dfrac{\overline{e}(1-\alpha)}{\overline{L}} - \left(\dfrac{\theta(h)}{\theta(h)+n+\delta}\right)\alpha} \tag{21a}$$

$$\hat{s} = \hat{\tau}_w + (1-\hat{\tau}_w)\beta \frac{(1-\varphi)\overline{L}}{(1-\alpha)\overline{e}} . \tag{21b}$$

Two special cases naturally arise. The first is to set $\hat{\tau}_w = \hat{\tau}_c = 0$, in which case (21a) and (21b) reduce to

$$\hat{T}/y = \hat{g} + \beta - \left(\frac{\varphi}{1-\varphi}\right)\frac{\overline{e}(1-\alpha)}{\overline{L}} \tag{22a}$$

$$\hat{s} = \beta \frac{(1-\varphi)\overline{L}}{(1-\alpha)\overline{e}} . \tag{22b}$$

Thus, one option for achieving the first best optimum is to subsidize health in accordance with (22b) and to finance all expenditures with lump-sum taxes. The health subsidy increases with the importance of health, β, in the production of final output and labor productivity in the health sector, $(1-\varphi)/\overline{e}$, but decreases with labor productivity in the final output sector, $(1-\alpha)/\overline{L}$.

In the more plausible case where lump-sum taxation is unavailable, the optimal policy is:

$$-\hat{\tau}_c = \hat{\tau}_w = \frac{\hat{g} + \beta - \left(\frac{\varphi}{1-\varphi}\right)\frac{\bar{e}(1-\alpha)}{\bar{L}}}{\hat{g} + \beta - \left(\frac{\varphi}{1-\varphi}\right)\frac{\bar{e}(1-\alpha)}{\bar{L}} - \left(\frac{\theta(h)}{\theta(h)+n+\delta}\right)\alpha} \equiv \tau^*, \tag{23}$$

with $\hat{s}$ set correspondingly by (21b). In order to be feasible $0 < \hat{\tau}_w < 1$, which leads to two cases:

(i) $\hat{g} + \beta < \left(\frac{\varphi}{1-\varphi}\right)\frac{\bar{e}(1-\alpha)}{\bar{L}}$ which implies $0 < \hat{\tau}_w = -\hat{\tau}_c < 1$,

(ii) $\left(\frac{\varphi}{1-\varphi}\right)\frac{\bar{e}(1-\alpha)}{\bar{L}} < \hat{g} + \beta < \left(\frac{\varphi}{1-\varphi}\right)\frac{\bar{e}(1-\alpha)}{\bar{L}} + \left(\frac{\theta(h)}{\theta(h)+n+\delta}\right)\alpha$ which implies $\hat{\tau}_w = -\hat{\tau}_c < 0$. [11]

Case (i) immediately ensures that $\hat{s} > 0$. Case (ii) implies $\hat{s} > 0$ if and only if $\tau^* > -\beta(1-\varphi)\bar{L}\left[(1-\alpha)\bar{e} - \beta(1-\varphi)\bar{L}\right]^{-1}$, a condition which all of our simulations satisfy. Of these two cases, (ii) is the more plausible suggesting that for most parameterizations replication of the first best optimum may be attained by taxing consumption, while subsidizing labor income and health costs. However, as our simulations below illustrate the tax and subsidy rates involved may be unrealistically high and therefore attainment of the first optimum may in fact be impractical.

Further intuition underlying the optimal tax policies summarized by (22) and (23) is obtained by realizing that $\varphi(1-\varphi)^{-1}\bar{e}(1-\alpha)(\bar{L})^{-1}$ equals the profit per unit of output earned by the government from providing health, π_h/y.[12] Thus, (22a) asserts that if the profit earned by the government from running health is more than sufficient to cover its expenditure commitments (investment plus health subsidy) it can finance a general subsidy $T/y < 0$. Alternatively, it becomes feasible to tax labor income and subsidize consumption in accordance with (23). An example of this

[11] The third condition $\varphi(1-\varphi)^{-1}\bar{e}(1-\alpha)\bar{L}^{-1} + \theta(h)[\theta(h)+n+\delta]^{-1}\alpha < \hat{g} + \beta$ implies $\hat{\tau}_w > 1$ in which case the first best optimum cannot be replicated by distortionary taxes alone.

[12] See this, recall that profit earned from providing health equals $p(h - h_e e) \equiv \pi_h$. Substituting for p, $p(h-h_e e)/y = \left(f_L/(yh_e)\right)(h - h_e e)$. For the Cobb-Douglas functions (20a), (20b), this simplifies further to $\varphi(1-\varphi)^{-1}\bar{e}(1-\alpha)(\bar{L})^{-1}$.

is provided by in our numerical simulations. The other extreme case identified in footnote 11, which would require $\hat{\tau}_w > 1$ and is therefore infeasible, corresponds to the situation where the profit from providing health is extremely small.

6. Numerical Simulations

This section lays the groundwork for further analysis of the model using numerical methods. To this end, it begins by specifying the functional forms and the values of various parameters of the model to match salient relevant features of a typical advanced Western economy. This is followed by the computation of the long-run, social optimally allocations, in particular, government expenditure on health and the assessment of the feasibility of its decentralization using various taxes and subsidies. As we shall see, this exercise verifies our conjecture from the previous section about the infeasibility of the required optimal taxes and subsidies. It also forms the basis of policy experiments implementing more practical tax policies to get closer to the optimal government spending on health in the next section—with the idea of capturing at least some of the gains associated with increased investment in health to address the health externality in the production process.

In addition to the two Cobb-Douglas production functions specified in (20a) and (20b), we specify utility by the conventional constant elasticity function

$$U = \frac{1}{\gamma}\left(cl^{\eta}h^{\omega}\right)^{\gamma}, \tag{20c}$$

and the rate of time preference by the exponential function

$$\theta(h) = \theta e^{-\sigma h}. \tag{20d}$$

The functional form of the rate of time preference, $\theta(h)$, is motivated by the non-linear empirical relationship between life-expectancy and health expenditure illustrated in Figure 1 (upper panel), together with the fact that increasing life expectancy is reflected as a decreasing rate of time discount; see e.g. Blanchard (1985), Boucekkine, de la Croix, and Licandro (2002). Together these suggest that increasing health expenditures will reduce the rate of time preference, doing so at a decreasing rate.

6.1 The Parameter Values

While many key parameters, particularly those pertaining to aggregate consumption, preferences, and aggregate output, are well documented, direct information on other relevant parameters and measures—particularly related to health—is less available, although other available information provides helpful guidance as to their plausible magnitudes. Table 1 reports the assigned parameter values for what we shall characterize as the baseline scenario.

Referring to Table 1, we assume final sector production function with the productive elasticity of private capital, $\alpha = 0.36$, which is fairly standard. The elasticity of health, $\beta = 0.10$ is based on the comprehensive evidence provided by Bom and Ligthart (2014) whose meta-regression analysis yields an estimate of the elasticity of public capital in aggregate output to be of the order of 0.10.[13] Assuming that health capital is a constant fraction of the overall public infrastructure, this suggests $\beta = 0.10$ as a reasonable benchmark, although we also consider $\beta = 0.15$, as part of the sensitivity analysis. With respect to utility, the intertemporal elasticity of substitution 0.4 is conventional, as is the elasticity of leisure. The rate of population growth $n = 0.01$ and the rate of depreciation $\delta = 0.05$ are non-controversial. Finally, as a benchmark the two income tax rates are set at 20%, as is the subsidy to health expenditures, while the consumption tax rate is set at the lower rate of 10%, rates which are typical of a generic developed economy.

One feature of the model is its flexibility in being able to reflect different economies with alternative health structures. In this regard, it is easy to adapt the model to approximate and contrast a "European health system" with a "US" system. The reason why we find this distinction to be

[13] As Bom and Ligthart (2014) document, empirical estimates on the productive elasticity of public capital, e, are far ranging. In their comprehensive study, they summarize 578 estimates and find the average productive elasticity of public capital to be around 0.19, while their meta-regression analysis yields an estimate of around 0.10.

useful is that these two major economies differ substantially with respect to the relative sizes of their health sectors, the roles paid by public vs. private financing, and more generally with regard to their underlying tax structures.

With respect to health expenditures, over the period 2010-2014, total health expenditures as fractions of GDP in the US and Europe have averaged around 17% and 10%, respectively. In addition, around 48% of health in the US has been publically provided, while in Europe it is around 77%. Combining these ratios, we see that around 8.2% of US GDP is allocated to public health, while in Europe it is around 7.7%. These ratios suggest a substantially different production function for health in the US than in Europe. Setting the productive elasticity of capital in the health production function at φ = 0.40 for the US and 0.60 for Europe, enables us to replicate these ratios remarkably closely. In our parameterization, we set the productive elasticity $\varphi = 0.50$, yielding the share of publically provided health somewhere between that of these two economies.[14]

Table 2 reports the corresponding benchmark steady-state equilibrium for our chosen parameters. In addition to the health ratios, we note that all of the implied ratios appear to be plausible and to reflect the structure of a modern advanced economy.

6.2 The Long-Run Social Optimum

Table 3 characterizes the tax structures and expenditures that enable the decentralized economy to achieve the first best allocation as set out in Section 5.1. We present results for two values of $\beta = 0.10, 0.15$. For each value of β, we consider three values of $\sigma = 0, 3, 6$, which parameterizes the effect of health on longevity and, thus, on the discount rate. With respect to the configuration of government policy we consider the two alternatives summarized in (22) and (23).

[14] We have also conducted preliminary simulations assuming tax rates and health expenditures more explicitly representative of the US and European economies, respectively. Most of the results are similar to those presented here. One difference worth noting is that with τ_w being substantially higher than τ_k in Europe, financing an increase in health investment by 1 percentage point using τ_w leads to a reduction in welfare analogous to that obtained using τ_k in the present analysis (and for the US parameterization as well).

In the first, where the government has only distortionary taxes available $T/f = 0, \hat{\tau}_k = 0, \hat{\tau}_w = -\hat{\tau}_c$, the second where all distortionary taxes are set to zero and only lump-sum taxation is employed. All other parameters are set at their benchmark levels identified in Table 1.

Table 3 highlights several elements pertaining to the optimal policy. First, in all cases the economy would experience dramatic improvements in long-run consumption, capital stock and output, if they were in steady state corresponding to the first best optimum. We have also reported changes in welfare, measured in terms of equivalent variations in per capital consumption. Thus, W_{SS} suggests about 22%-24% increase in welfare across steady states for $\beta = 0.10$, which increases to over 33% if the productivity of health in producing final output is increased to $\beta = 0.15$. The corresponding measure, W_{INT} approximates the present value of the gains, taking into account the entire transitional path. These too are significant.

However, attainment of the first optimum is really not practical. For example, to achieve the first-best optimum in the benchmark case $(\beta = 0.10, \sigma = 3)$ would require taxing consumption and subsidizing labor income at 48.6%, while subsidizing health expenditure at 51.5%. Alternatively, it could be attained by imposing a lump-sum tax of 4.3% while subsidizing health at 67.4%. While these are in principle feasible policies, they are dramatically different from any currently existing tax structure and would involve a revolution in tax policy to implement. Other optimal policies are equally, or even more, extreme and one over-riding characteristic is the sensitivity of the optimal tax structure to variations in σ, which impacts the rate of time preference. This is somewhat surprising since the implied impact on $\theta(h)$ is relatively small. For example, increasing σ from 0 to 3 reduces $\theta(h)$ from 0.045 to 0.039. One final observation, we see that when $\sigma = 0$, the government makes a sufficient profit on providing health care that it can achieve the optimum by taxing labor at 11.2%, subsidizing general consumption at that same rate, and healthcare at 41.6%.

7. Policy Experiments

The above discussion of the potential to attain the first best optimum is useful in terms of providing a benchmark, but for reasons just alluded to regarding the necessarily massive adjustments of tax policy to implement, is of little more than academic interest. In this section, we focus on more pragmatic policy issues.

7.1 Tax Policy and Welfare Benefits of Increased Government Health Investment

It is clear that the tax rates can generate sufficient revenues for the governments to increase their rate of investment in health if they so choose. Thus, we consider the impact of increasing the rate of investment, *g*, by one percentage point from its base level of 0.03 to 0.04. We compare the financing of this increase under 5 different ways: (i) Lump sum taxes (bond financing), (ii) increase in τ_w, (iii) increase in τ_k, (iv) increase in τ_c, (v) reduction in subsidy to health expenditure, s. In cases (ii) to (v) the taxes are adjusted so as to maintain the government deficit at its initial level. Table 4 reports the long-run responses for the baseline parameterization ($\sigma = 3, \beta = 0.10$), as well as for the case where the productive elasticity of health is increased to 0.15. All changes are across steady states. In particular, the change in welfare reported in the final column, is measured in terms of equivalent variations in steady-state per capita consumption, but does not reflect welfare changes incurred during the transition.

The following features in Table 4 can be identified. Unsurprisingly, the overall ranking places lump-sum tax financing best from a welfare standpoint. This is followed in turn by consumption tax financing, labor income tax, reduction of the health subsidy, with the worst being to finance by a tax on capital income. In the case of $\beta = 0.10$, financing the investment in health by taxing capital leads to a contraction in activity and reduces welfare by 1.35%. Financing by a tax on labor income reduces per capita consumption, but the offsetting increase in health ensures an overall increase in welfare. The same ranking obtains if the productive elasticity of health is increased to $\beta = 0.15$. In all cases the welfare gains are enhanced, and even taxing capital, although it is still associated with an albeit minor decline in activity, nevertheless yields an overall modest welfare improvement. In terms of quantities, the rankings are generally similar, with the exception that in most cases financing by reducing the subsidy has a greater effect than does financing by a tax on wage income. Finally, in all cases we see that increasing investment by the government in health leads to a dramatic reduction in the relative price of health services, thereby reducing health costs to the private sector.

7.2 Transitional Dynamics

Figure 2 illustrates the transitional dynamics followed by key variables for the baseline parameterization in response to an increase in government investment in health under the five alternative modes of finance. With the exception of health capital, all quantities converge rather rapidly to their respective new steady-state equilibria. Also, the magnitudes of the relative responses are remarkably uniform along the transition.

The striking outlier in these adjustments is the case of financing by taxing capital, which is associated to a long-run decline in economic activity and a loss in welfare. On impact, it is actually associated with an increase in consumption. This is because to finance the public investment with a tax on private capital requires a large increase in the tax rate, τ_k, leading to an immediate substantial reduction in the rate of private investment, which in turn reduces the productivity of labor, reducing the wage rate, inducing workers to supply less labor in both sectors, instead allocating more time to leisure and with it consumption. With health capital being augmented only slowly, health production declines in the short run. Over time, as the private capital stock continues to decline consumption declines. In contrast, the accumulation of health capital brought about by government investment brings about a reversal in the decline of health, which eventually increases to above its initial level.

7.3 Tax Policy and Welfare Benefits of Health Investment: Further Analysis

As Table 3 highlights, the increase in economic activity and the resulting welfare improvement associated with the long-run social optimum are substantial. Comparing our preferred parameterization

($\beta = 0.10, \sigma = 3$) we see that the welfare gains are in excess of 20% as measured by equivalent variation in consumption units. But as the table also underscores and as already noted, the corresponding tax rates required to achieve these gains are simply unrealistic from a practical point of view. In contrast, the policy experiments summarized in Table 4, starting from the existing tax structure and raising g from 3% to 4%, generate decidedly modest welfare gains. This raises the question of whether it is possible to realize a substantial portion of the benefits associated with the social optimum by combining government investment in health with more moderate and feasible tax changes that move at least partially in the direction of the first-best policy. This issue is addressed in Table 5, which focuses only on comparisons across steady states.

Row 1 (of Panel 1) of the Table 5 summarizes the percentage gains in output, capital, consumption and welfare associated with the first best optimum. The remaining panels in Table 5 consider realistic tax changes that move successively closer to the optimal tax structure. In particular, in each panel the first row with g=0.03 shows the pure change in tax policy without changing the rate of health investment. Successive reductions in the income tax, compensated by appropriate increases in the consumption tax lead to successively larger welfare gains. Thus, for example, the penultimate panel shows the elimination of capital income tax, as advocated since Chamley (1986) and Judd (1985), coupled with the reduction in subsidy and labor tax rate to 10% from 20%, compensated by an increase in the consumption tax to 22.4% can capture more than 50% of the welfare gain relative to the optimum (11.8% vs 21.6%). Eliminating the tax on labor income entirely and increasing the consumption tax further to 29.6% increases the welfare gain to 13.5%. And while these represent dramatic changes in the US tax structure, they are not implausible, and indeed reflect changes in the tax structure that have been advocated by conservative policy makers over the years.

Subsequent rows introduce increases in health investment in conjunction with the corresponding tax policy. Combining tax and health investment policies can further enhance the welfare gains. Thus for example, halving the income tax rates τ_w, τ_k from 20% to 10%, while doubling the consumption tax τ_c from 10% to 20.7%, and simultaneously increasing health investment from 3% to 4% captures approximately half the welfare benefits associated with the social optimum (10.7% vs. 21.6%). Moreover, as health investment increases further from 3% to 6% and the income tax is increasingly reduced in favor of a consumption tax it is possible to take the welfare gains to above 85% of those associated with the social optimum, as the final panel indicates.

One further observation, the results in Table 5 demonstrate the robustness of the welfare gains from increases in government investment in health with respect to initial tax structure. From the perspective of health investment policy, this is an important result. Specifically, in Table 5, in all panels, which represent very different tax structures, an increase in g from 3% to 6% results in an additional welfare gain of 5.5% to 6.5%.

7.4 Tax-Health Tradeoffs

Our numerical analysis has so far been restricted primarily to the baseline calibration, $\beta = 0.10, \sigma = 3$. However, there is in fact a tradeoff between the level of the economy's health and the tax rates necessary to support its level of expenditures. This is illustrated in Table 6 where, for each of the three taxes we present the rates required to finance an increase in government investment in health from 3% of GDP to 4% of GDP. The table presents results for a grid over which the productivity of health varies from $\beta = 0.05$ (low productivity) to $\beta = 0.20$ (high productivity) and its impact on longevity varies from $\sigma = 0$ (no impact) to $\sigma = 6$ (strong impact). The combination corresponding to the baseline summarized in Table 4 ($\beta = 0.10, \sigma = 3$) is indicated in bold face.

Focusing initially on panel A of the table indicates that in the benchmark situation to finance an increase in the share of government investment in GDP by 1 percentage point using a tax on labor income would require τ_w to increase from 0.200 to 0.220, resulting in an increase in welfare of 2.48%. If the productive elasticity of health were increased from $\beta = 0.10$ to $\beta = 0.15$, this could be financed by a lower tax rate, τ_w, namely 0.215, yielding a larger welfare gain of 3.89%. And if health productivity were higher still, $\beta = 0.20$, the required tax rate could be reduced further to

0.207. There is clearly a tradeoff between (i) the productive elasticity of health in producing final output and the tax rate necessary to finance any increase in government health investment. This tradeoff applies to all three tax rates.

But there is also a tradeoff that applies in the opposite direction between the tax rate and the impact of health on longevity and on the rate of time preference. Focusing again on τ_w and the benchmark parameterization, we see that if σ were increased to 6, the 1 percentage point investment in government health would require a slightly higher tax rate 0.222 leading to a substantially reduced increase in welfare of only 0.45% per capita. Intuitively, the reduced rate of time preference leads to a slight increase in leisure met by a reduction in employment in both sectors, necessitating an increase in the tax rate to finance the increase in government investment.

Looking across the three panels we see the following. First, in the case where $\sigma = 0$, so that health has no impact on the rate of time preference, if the productive elasticity of health is sufficiently large ($\beta = 0.15, \beta = 0.20$), it is possible to finance the increase in government investment out of the higher productivity and thereby reduce the corresponding tax rate. This is true for each of the three tax rates. Second, the required capital income tax financing is extremely sensitive to the productive elasticity of health, particularly if $\sigma = 0$. In that case, it ranges from a high tax rate, $\tau_k = 0.309$ with the largest welfare loss of 3.81% to a low tax rate $\tau_k = 0.089$ and the largest welfare gain of 18.7%. Third, variations in the consumption tax across the grid are modest and, in all cases, they yield a welfare gain. Fourth, financing with a labor income tax generally yields a welfare gain, expect if $\beta = 0.05, \sigma = 6$. However, in all cases financing by a labor income tax is always dominated from a welfare enhancing standpoint by either of the other two alternatives.

8. Conclusion

This paper has incorporated individual agents' health decisions into the neoclassical growth model to study the effects of government health investment subsidies on productivity, growth, and welfare. The paper has first examined the steady state and transitional dynamic properties of the model focusing on the effects of health decisions on aggregate outcomes. Next, it has considered how public health policies may alter private economic decisions.

The analysis shows that there is a significant welfare upside to government interventions to subsidize the consumption of medical goods and services and also subsidize physical activities and recreation. The policy experiments we perform show that there are significant welfare gains associated with movement from the current tax structure and public health investment to the optimal levels identified by the model. We also show that this result is robust to other initial tax structures that are closer to the first best equilibrium generated by the model. The tradeoff we have identified between health status and the tax rate leads to an important, but little discussed, policy implication. It suggests that one of the benefits of a healthier more productive labor force is that any specified level of government services can be supported by a lower tax rate, enabling consumers to enjoy a higher level of welfare.

Finally, the model we have presented is highly stylized. Future work should aim at calibrating existing national and regional health systems and investment plans (e.g. contrasting US and Europe) to better identify the potential aggregate output benefits from government health subsidies.

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Appendix

A.1 The Linearized Core Dynamic System

To derive the transitional dynamics, we linearize the macrodynamic equilibrium around the steady state (13). As stated in the text, we first solve equations (4a), (4b'), (4c'), (5b), and (10) for *c, h, l, L*, and *e* in terms of $k, m, \lambda,$ and μ. The solution is of the form: $c = c(k, m, \lambda, \mu; \tau_w, \tau_c, s)$, $h = h(k, m, \lambda, \mu; \tau_w, \tau_c, s)$, $l = l(k, m, \lambda, \mu; \tau_w, \tau_c, s)$ $L = L(k, m, \lambda, \mu; \tau_w, \tau_c, s)$, $e = e(k, m, \lambda, \mu; \tau_w, \tau_c, s)$, where the partial derivatives are obtained from the following system:

$$\begin{pmatrix} U_{cc} & U_{ch} & U_{cl} & 0 & 0 \\ U_{lc} & U_{lh} - \lambda(1-\tau_w) f_{Lh} & U_{ll} & -\lambda(1-\tau_w) f_{LL} & 0 \\ \left(\frac{1-s}{1-\tau_w}\right) U_{lc} - U_{hc} h_e & \left(\frac{1-s}{1-\tau_w}\right) U_{lh} - \left[U_{hh} + \mu\theta_{hh}\right] h_e & \left(\frac{1-s}{1-\tau_w}\right) U_{ll} - U_{hl} h_e & 0 & -\left[U_h + \mu\theta_h\right] h_{ee} \\ 0 & 1 & 0 & 0 & -h_e \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} dc \\ dh \\ dl \\ dL \\ de \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & (1+\tau_c) & 0 \\ \lambda(1-\tau_w) f_{Lk} & 0 & (1-\tau_w) f_L & 0 \\ 0 & \left[U_h + \mu\theta_h\right] h_{em} & s_e & \theta_h h_e \\ 0 & h_m & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} dk \\ dm \\ d\lambda \\ d\mu \end{pmatrix} + \begin{pmatrix} 0 & \lambda & 0 \\ -\lambda f_L & 0 & 0 \\ -\frac{1-s}{(1-\tau_w)^2} U_l & 0 & \frac{1}{(1-\tau_w)} U_l \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} d\tau_w \\ d\tau_c \\ ds \end{pmatrix} \quad \text{(A.1)}$$

Linearizing the dynamic system (11), (9), (4d'), and (4e') around the steady state (13) yields

$$\begin{pmatrix} \dot{k} \\ \dot{m} \\ \dot{\lambda} \\ \dot{\mu} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} k - \tilde{k} \\ m - \tilde{m} \\ \lambda - \tilde{\lambda} \\ \mu - \tilde{\mu} \end{pmatrix} \quad \text{(A.2)}$$

where

$$a_{11} \equiv (1-g)\left[f_k + f_L L_k + f_h h_k\right] - c_k - (n+\delta) \qquad a_{21} \equiv g\left[f_k + f_L L_k + f_h h_k\right]$$
$$a_{12} \equiv (1-g)\left[f_L L_m + f_h h_m\right] - c_m \qquad a_{22} \equiv g\left[f_L L_m + f_h h_m\right] - n$$
$$a_{13} \equiv (1-g)\left[f_L L_\lambda + f_h h_\lambda\right] - c_\lambda \qquad a_{23} \equiv g\left[f_L L_\lambda + f_h h_\lambda\right]$$
$$a_{14} \equiv (1-g)\left[f_L L_\mu + f_h h_\mu\right] - c_\mu \qquad a_{24} \equiv g\left[f_L L_\mu + f_h h_\mu\right]$$

$$a_{31} \equiv \lambda\left[\theta_h h_k - \left(f_{kk} + f_{kL} L_k + f_{kh} h_k\right)(1-\tau_k)\right] \qquad a_{41} \equiv \mu\theta_h h_k + U_c c_k + U_l l_k + U_h h_k$$
$$a_{32} \equiv \lambda\left[\theta_h h_m - \left(f_{kL} L_m + f_{kh} h_m\right)(1-\tau_k)\right] \qquad a_{42} \equiv \mu\theta_h h_m + U_c c_m + U_l l_m + U_h h_m$$
$$a_{33} \equiv \lambda\left[\theta_h h_\lambda - \left(f_{kL} L_\lambda + f_{kh} h_\lambda\right)(1-\tau_k)\right] \qquad a_{43} \equiv \mu\theta_h h_\lambda + U_c c_\lambda + U_l l_\lambda + U_h h_\lambda$$
$$a_{34} \equiv \lambda\left[\theta_h h_\mu - \left(f_{kL} L_\mu + f_{kh} h_\mu\right)(1-\tau_k)\right] \qquad a_{44} \equiv \theta(h) + \mu\theta_h h_\mu + U_c c_\mu + U_l l_\mu + U_h h_\mu$$

and the partial derivatives are derived from (A.1). For the functional forms employed in the numerical simulations, (A.1) becomes:

$$\begin{pmatrix} \frac{\gamma-1}{c} & \frac{\omega\gamma}{h} & \frac{\eta\gamma}{l} & 0 & 0 \\ \frac{\gamma}{c} & \frac{(\omega\gamma-\beta)}{h} & \frac{(\eta\gamma-\beta)}{l} & \frac{\alpha}{L} & 0 \\ \frac{\gamma}{c}\left(1-\frac{\Gamma}{\Phi}\right) & \left(\frac{\omega\gamma}{h} - \frac{(\omega\gamma-1)}{h}\frac{\Gamma}{\Phi} - \sigma\frac{\Gamma}{\Phi}\right) & \left(\frac{\gamma\eta-1}{l} - \frac{\gamma\eta}{l}\frac{\Gamma}{\Phi}\right) & 0 & \frac{\varphi}{e} \\ 0 & \frac{1}{h} & 0 & 0 & -\frac{(1-\varphi)h}{e} \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} dc \\ dh \\ dl \\ dL \\ de \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & \frac{1}{\lambda} & 0 \\ \frac{\alpha}{k} & 0 & \frac{1}{\lambda} & 0 \\ 0 & \frac{\varphi}{m} & 0 & -\frac{1}{\mu}\frac{\Delta}{\Phi} \\ 0 & \frac{\varphi}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} dk \\ dm \\ d\lambda \\ d\mu \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{1+\tau_c} & 0 \\ -\frac{1}{1-\tau_w} & 0 & \frac{1}{1-s} \\ 0 & 0 & \lambda \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} d\tau_w \\ d\tau_c \\ ds \end{pmatrix} \qquad \text{(A.1')}$$

where $\Gamma \equiv \omega c^{\gamma} l^{\eta\gamma} h^{\omega\gamma-1}$; $\Delta = \mu\theta\sigma Exp(-\sigma h)$; $\Phi \equiv (\Gamma - \Delta)$. Applying these partial derivatives, the corresponding dynamic system (A.2) employed in the simulations has the appropriate saddlepoint structure, yielding a unique stable transitional dynamic time path.

A.2 Proof of Proposition 1

We begin by eliminating shadow values, $\tilde{\lambda}, \tilde{\mu}$ from the steady-state allocations for the decentralized/market equilibrium described by (13). This leads to

$$\frac{U_l(\tilde{c},\tilde{l},\tilde{h})}{U_c(\tilde{c},\tilde{l},\tilde{h})}=\left(\frac{1-\tau_w}{1+\tau_c}\right)f_L(\tilde{k},\tilde{L},\tilde{h}) \tag{A.3a}$$

$$\left(\frac{1-s}{1-\tau_w}\right)U_l(\tilde{c},\tilde{l},\tilde{h})=\left[U_h(\tilde{c},\tilde{l},\tilde{h})-\frac{U(\tilde{c},\tilde{l},\tilde{h})\theta_h(\tilde{h})}{\theta(\tilde{h})}\right]h_e(\tilde{m},\tilde{e}) \tag{A.3b)}$$

$$\tilde{h}=h(\tilde{m},\tilde{e}) \tag{A.3c)}$$

$$\tilde{L}+\tilde{l}+\tilde{e}=1 \tag{A.3d)}$$

$$(1-g)f(\tilde{k},\tilde{L},\tilde{h})=\tilde{c}+(n+\delta)\tilde{k} \tag{A.3e}$$

$$gf(\tilde{k},\tilde{L},\tilde{h})=n\tilde{m} \tag{A.3f)}$$

$$f_k(\tilde{k},\tilde{L},\tilde{h})(1-\tau_k)=\theta(\tilde{h})+n+\delta \tag{A.3g)}$$

These seven equations now determine the equilibrium allocation of the quantity variables $\tilde{c},\tilde{l},\tilde{L},\tilde{e},\widetilde{H},\tilde{k},\widetilde{m}$ for given $\tau_k,\tau_w,\tau_c,s,$ and g.

The corresponding equations for the centralized/planner's problem are:

$$\frac{U_l(\overline{c},\overline{l},\overline{h})}{U_c(\overline{c},\overline{l},\overline{h})}=f_L(\overline{k},\overline{L},\overline{h}) \tag{A.4a}$$

$$U_l(\overline{c},\overline{l},\overline{h})=\left[U_h(\overline{c},\overline{l},\overline{h})-\frac{U(\overline{c},\overline{l},\overline{h})\theta_h(\overline{h})}{\theta(\overline{h})}+U_c(\overline{c},\overline{l},\overline{h})\overline{f}_h\right]h_e(\overline{m},\overline{e}) \tag{A.4b)}$$

$$\frac{f_k(\overline{k},\overline{L},\overline{h})-\delta}{f(\overline{k},\overline{L},\overline{h})}=\frac{h_m(\overline{m},\overline{e})}{h_e(\overline{m},\overline{e})} \tag{A.4c)}$$

$$\overline{h}=h(\overline{m},\overline{e}) \tag{A.4d}$$

$$\overline{l}+\overline{L}+\overline{e}=1 \tag{A.4e)}$$

$$(1-\overline{g})f(\overline{k},\overline{L},\overline{h})=\overline{c}+(n+\delta)\overline{k} \tag{A.4e)}$$

$$\overline{g}f(\overline{k},\overline{L},\overline{h}) = n\overline{m} \quad \text{(A.4f))}$$

$$f_k(\overline{k},\overline{L},\overline{h}) = [\theta(\overline{h}) + n + \delta] \quad \text{(A.4g))}$$

Note that four of the seven equations governing the two allocations are identical. The first-best optimum will be attained if and only if the following three conditions hold:

$$\tau_k = 0 \quad \text{(A.5a)}$$

$$\frac{1-\tau_w}{1+\tau_c} = 1 \quad \text{(A.5b)}$$

$$\frac{1-s}{1-\tau_w} = \frac{U_h - U(\overline{c},\overline{l},\overline{h})\theta_h(\overline{h})\cdot(\theta(\overline{h}))^{-1}}{U_h - U(\overline{c},\overline{l},\overline{h})\theta_h(\overline{h})\cdot(\theta(\overline{h}))^{-1} + U_l f_h \cdot (f_L)^{-1}} \quad \text{(A.5c)}$$

To further simplify (A.5c), from (A.4a) and (A.4b), we have

$$U_l = \left(U_h - U(\overline{c},\overline{l},\overline{h})\theta_h(\overline{h})\cdot(\theta(\overline{h}))^{-1} + U_l f_h \cdot (f_L)^{-1}\right) h_e(\overline{m},\overline{e})$$

and hence

$$U_l(\overline{c},\overline{l},\overline{h}) = \frac{\left(U_h - U(\overline{c},\overline{l},\overline{h})\theta_h(\overline{h})\cdot(\theta(\overline{h})^{-1}\right) h_e(\overline{m},\overline{e})}{1 - f_h \cdot (f_L)^{-1} h_e(\overline{m},\overline{e})}$$

and using these in (A.5c) implies

$$\frac{1-s}{1-\tau_w} = \frac{\left(U_h - U\theta_h \cdot \theta^{-1}\right) h_e}{U_l} = 1 - \frac{f_h}{f_L} h_e < 1 \quad \text{(A.5c')}$$

Table 1. Baseline parameter values

Utility	$\gamma = -1.5$ (i.e. IES 0.4), $\eta = 1.5, \omega = 0.25$
Final Output	$A = 1,\ \alpha = 0.36,\ \beta = 0.10\ (0.15)$
Health Production	$B = 0.5, \varphi = 0.5$
Rate of Time Preference	$\theta = 0.045, \sigma = 3\ (0, 6)^{*}$
Government policy parameters	$g = 0.03,\ \tau_k = 0.20, \tau_w = 0.20,\ \tau_c = 0.10,\ s = 0.20$
Population growth rate	$n = 0.01$
Depreciation rate	$\delta = 0.05$

Note: * These values yield an equilibrium rate of time preference that varies approximately between 0.045-0.033.

Table 2. Equilibrium ratios

Consumption-GDP ratio	0.71
Capital-output ratio	3.01
Allocation of time to leisure	0.702
Allocation of labor to final output production	0.276
Allocation of labor to health production	0.0227
Equilibrium rate of time discount	0.0358
Public health as percentage of GDP	7.5%
Public health as percentage of total health	61.1%
Total health as a percentage of GDP	12.3%

Table 3. Policies to attain the long-run social optimum

	Gains from attaining long-run optimum						$\frac{T}{f}=0,\ \tau_k=0$		$\tau_k=\tau_w=\tau_c=0$	
	$\frac{\Delta y}{y}$	$\frac{\Delta k}{k}$	$\frac{\Delta c}{c}$	ΔW_{SS}	ΔW_{INT}	$\bar{g}$	$\hat{\tau}_w=-\hat{\tau}_c$	$\hat{s}$	$\frac{T}{f}$	$\hat{s}$
$\beta=.10;\ \sigma=0$	49.2%	86.4%	42.1%	24.3%	21.8%	0.0265	0.1118	0.4161	-0.0194	0.3425
$\boldsymbol{\beta}$ = .10; $\boldsymbol{\sigma}$ = 3	**48.6%**	**89.2%**	**41.8%**	**21.6%**	**23.6%**	**0.0168**	**-0.4861**	**0.5153**	**0.0426**	**0.6738**
$\beta=.10;\ \sigma=6$	49.4%	91.6%	42.9%	23.2%	19.3%	0.0120	-1.0722	1.0505	0.0632	1.0244
$\beta=.15;\ \sigma=0$	57.5%	96.8%	49.2%	33.4%	25.7%	0.0310	-0.0736	0.3989	0.0106	0.4401
$\beta=.15;\ \sigma=3$	59.6%	107.1%	50.7%	33.4%	25.1%	0.0209	-1.7707	0.5480	0.0813	0.8369
$\beta=.15;\ \sigma=6$	61.0%	111.2%	52.2%	35.5%	21.3%	0.0153	-8.7428	3.4710	0.1055	1.2536

Table 4. Macroeconomic and welfare effects of increase in government investment

A. Baseline ($\sigma = 3, \beta = 0.10$)

	$\boldsymbol{l}$	$\boldsymbol{L}$	$\boldsymbol{e}$	$\boldsymbol{y}$	$\boldsymbol{k}$	$\boldsymbol{c}$	$\boldsymbol{h}$	$\boldsymbol{p}$	$\frac{\boldsymbol{ph+mn}}{\boldsymbol{y+ph}}$	ΔW_{SS}
Baseline	**0.7016**	**0.2757**	**0.0227**	**0.3427**	**1.0304**	**0.2706**	**0.0764**	**0.4730**	**0.1226**	-
Δ(T/y)=0.0175	0.7010 -0.06%pts	0.2795 0.38%pts	0.0196 -0.31%pts	0.3535 3.15%	1.0708 3.92%	0.2751 1.66%	0.0831 8.77%	0.3808 -19.5%	0.1189 -3.02%	3.70%
$\Delta\tau_c = 0.0138$	0.7033 0.17%pts	0.2770 0.13%pts	0.0197 -0.30%pts	0.3502 2.19%	1.0607 2.94%	0.2725 0.70%	0.0830 8.64%	0.3837 -18.9%	0.1200 -2.12%	3.22%
$\Delta\tau_w = 0.0199$	0.7060 0.44%pts	0.2744 -013%pts	0.0196 -0.31%pts	0.3464 1.08%	1.0484 1.75%	0.2696 -0.37%	0.0823 7.72%	0.3841 -18.8%	0.1203 -1.88%	2.48%
$\Delta\tau_k = 0.0748$	0.7047 0.31%pts	0.2749 -0.08%pts	0.0204 -0.23%pts	0.3278 -4.35%	0.8986 -12.8%	0.2607 -3.66%	0.0817 6.94%	0.3803 -19.6%	0.1231 0.41%	-1.35%
$\Delta s = -0.1874$	0.7026 0.10%pts	0.2798 0.41%pts	0.0176 -0.51%pts	0.3497 2.04%	1.0540 2.29%	0.2725 0.70%	0.0785 2.75%	0.3593 -24.0%	0.1117 -8.89%	1.61%

B. Alternative (with $\beta = 0.15$)

	$\boldsymbol{l}$	$\boldsymbol{L}$	$\boldsymbol{e}$	$\boldsymbol{y}$	$\boldsymbol{k}$	$\boldsymbol{c}$	$\boldsymbol{h}$	$\boldsymbol{p}$	$\frac{\boldsymbol{ph+mn}}{\boldsymbol{y+ph}}$	$\frac{\boldsymbol{\Delta W}}{\boldsymbol{W}}$
Baseline	**0.7002**	**0.2748**	**0.0249**	**0.2744**	**0.8206**	**0.2169**	**0.0716**	**0.4447**	**0.1309**	**-**
Δ(T/y)=0.0178	0.6997 -0.05%pts	0.2787 0.39%pts	0.0216 -0.33%pts	0.2856 4.08%	0.8609 4.91%	0.2226 2.63%	0.0786 9.78%	0.3611 -18.8%	0.1268 -3.13%	4.88%
$\Delta\tau_c = 0.0099$	0.7014 0.12%pts	0.2768 0.20%pts	0.0217 -0.32%pts	0.2836 3.35%	0.8549 4.18%	0.2210 1.89%	0.0785 9.64%	0.3630 -18.4%	0.1277 -2.45%	4.52%
$\Delta\tau_w = 0.0147$	0.7034 0.32%pts	0.2750 0.02%pts	0.0216 -0.33%pts	0.2812 2.48%	0.8469 3.20%	0.2191 1.01%	0.0780 8.94%	0.3631 -18.4%	0.1278 -2.37%	3.89%
$\Delta\tau_k = 0.0237$	0.7024 0.22%pts	0.2753 0.05%pts	0.0222 -0.27%pts	0.2701 -0.08%	0.7576 -1.57%	0.2138 -1.43%	0.0775 8.24%	0.3603 -19.0%	0.1300 -0.69%	1.02%
$\Delta s = -0.1426$	0.7011 0.09%pts	0.2790 0.42%pts	0.0199 -0.50%pts	0.2820 2.77%	0.8465 3.16%	0.2199 1.38%	0.0749 4.61%	0.3436 -22.7%	0.1203 -8.10%	2.73%

Table 5. Welfare Gains of various policies in the baseline scenario

		$\frac{\Delta y}{y}$	$\frac{\Delta k}{k}$	$\frac{\Delta c}{c}$	ΔW_{SS}
	First best optimum	48.6%%	89.2%	41.8%	21.6%
g=0.03	$s=\tau_w=\tau_k=0.20, \tau_c=0.10$ (Baseline)	0.00%	0.00%	0.00%	0.00%
g=0.04	$s=\tau_w=\tau_k=0.20, \tau_c=0.114$	2.19%	2.94%	0.72%	3.22%
g=0.06	$s=\tau_w=\tau_k=0.20, \tau_c=0.144$	4.94%	6.74%	0.54%	6.23%
g=0.03	$s=\tau_w=\tau_k=0.15, \tau_c=0.142$	6.70%	13.5%	5.14%	3.98%
g=0.04	$s=\tau_w=\tau_k=0.15, \tau_c=0.158$	8.93%	16.7%	5.77%	7.18%
g=0.06	$s=\tau_w=\tau_k=0.15, \tau_c=0.191$	11.7%	20.9%	5.39%	10.1%
g=0.03	$s=\tau_w=\tau_k=0.10, \tau_c=0.189$	13.1%	27.5%	9.77%	7.52%
g=0.04	$s=\tau_w=\tau_k=0.10, \tau_c=0.207$	15.3%	31.0%	10.3%	10.7%
g=0.06	$s=\tau_w=\tau_k=0.10, \tau_c=0.243$	18.2%	35.5%	9.73%	13.4%
g=0.03	$s=\tau_w=0.10, \tau_k=0, \tau_c=0.224$	20.6%	51.4%	13.6%	11.8%
g=0.04	$s=\tau_w=0.10, \tau_k=0, \tau_c=0.243$	23.0%	55.5%	14.0%	14.9%
g=0.06	$s=\tau_w=0.10, \tau_k=0, \tau_c=0.281$	25.9%	60.7%	13.2%	17.4%
g=0.03	$s=\tau_w=\tau_k=0, \tau_c=0.296$	25.0%	56.9%	17.7%	13.5%
g=0.04	$s=\tau_w=\tau_k=0, \tau_c=0.317$	27.3%	61.0%	18.0%	16.6%
g=0.06	$s=\tau_w=\tau_k=0, \tau_c=0.359$	30.3%	66.3%	17.1%	19.1%

Table 6.

Tax-health tradeoff in financing increase in government investment from 0.03 to 0.04

A. Labor income tax

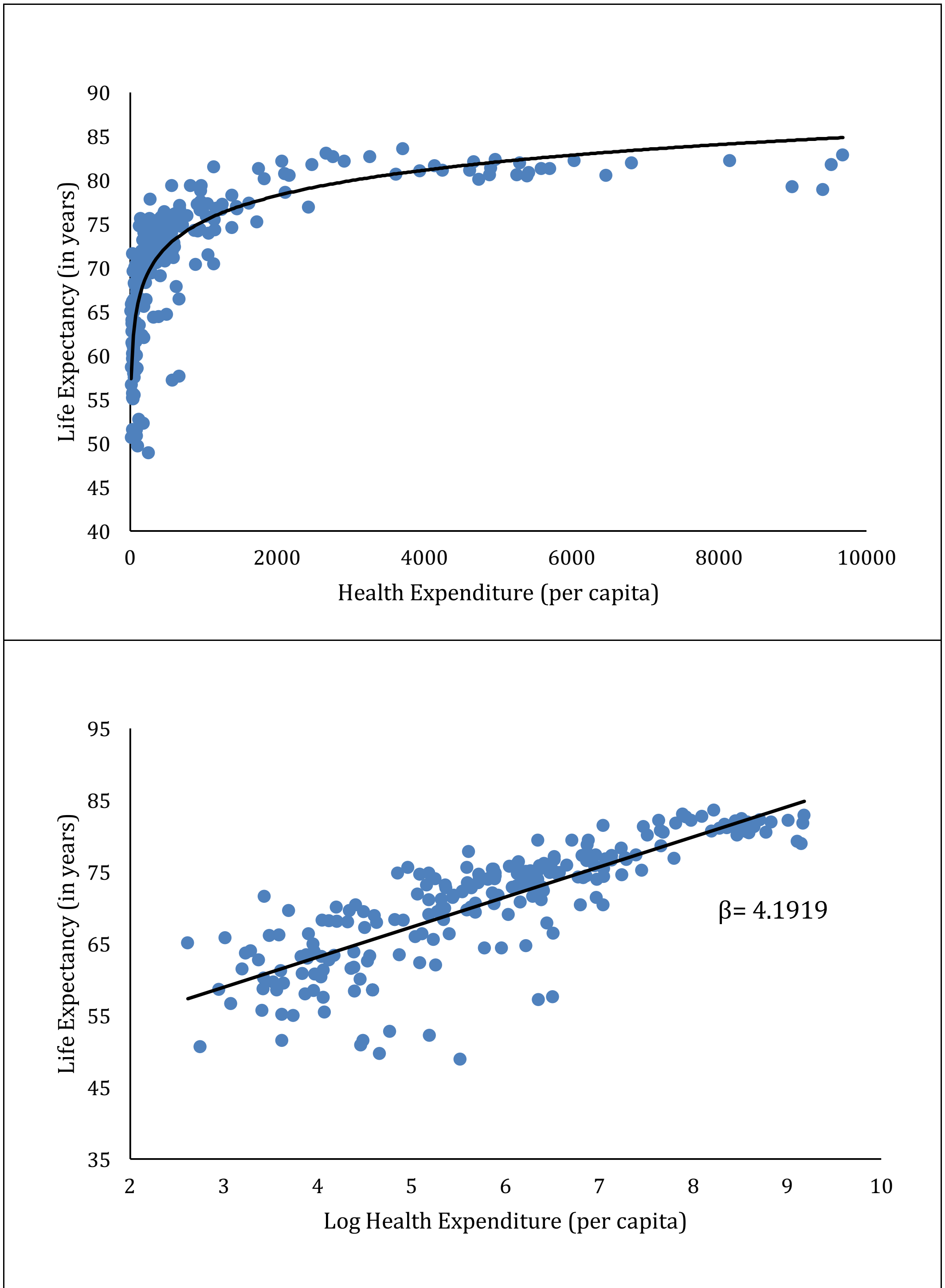
	$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.15$		$\beta = 0.20$	
	τ_w	ΔW_{SS}	τ_w	ΔW_{SS}	τ_w	ΔW_{SS}	τ_w	ΔW_{SS}
$\sigma = 0$	0.211	3.69%	0.201	5.76%	0.189	8.35%	0.175	11.5%
$\sigma = 3$	0.224	1.36%	**0.220**	**2.48%**	0.215	3.89%	0.207	5.68%
$\sigma = 6$	0.224	-0.20%	0.222	0.45%	0.220	1.29%	0.217	2.40%

B. Capital income tax

	$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.15$		$\beta = 0.20$	
	τ_k	ΔW_{SS}	τ_k	ΔW_{SS}	τ_k	ΔW_{SS}	τ_k	ΔW_{SS}
$\sigma = 0$	0.309	-3.81%	0.204	5.52%	0.143	12.0%	0.089	18.7%
$\sigma = 3$	0.290	-3.12%	**0.275**	**-1.35%**	0.255	1.02%	0.228	4.14%
$\sigma = 6$	0.276	-3.45%	0.272	-2.70%	0.265	-1.64%	0.254	-0.13%

C. Consumption tax

	$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.15$		$\beta = 0.20$	
	τ_c	ΔW_{SS}	τ_c	ΔW_{SS}	τ_c	ΔW_{SS}	τ_c	ΔW_{SS}
$\sigma = 0$	0.107	3.99%	0.100	5.79%	0.093	7.83%	0.085	13.2%
$\sigma = 3$	0.117	2.13%	**0.114**	**3.22%**	0.110	4.52%	0.105	6.07%
$\sigma = 6$	0.116	0.41%	0.115	1.08%	0.114	1.94%	0.111	3.01%



Source: World Bank WDI, and authors estimates

Fig 1. Per Capita Health Expenditures and Life Expectancy (2015)

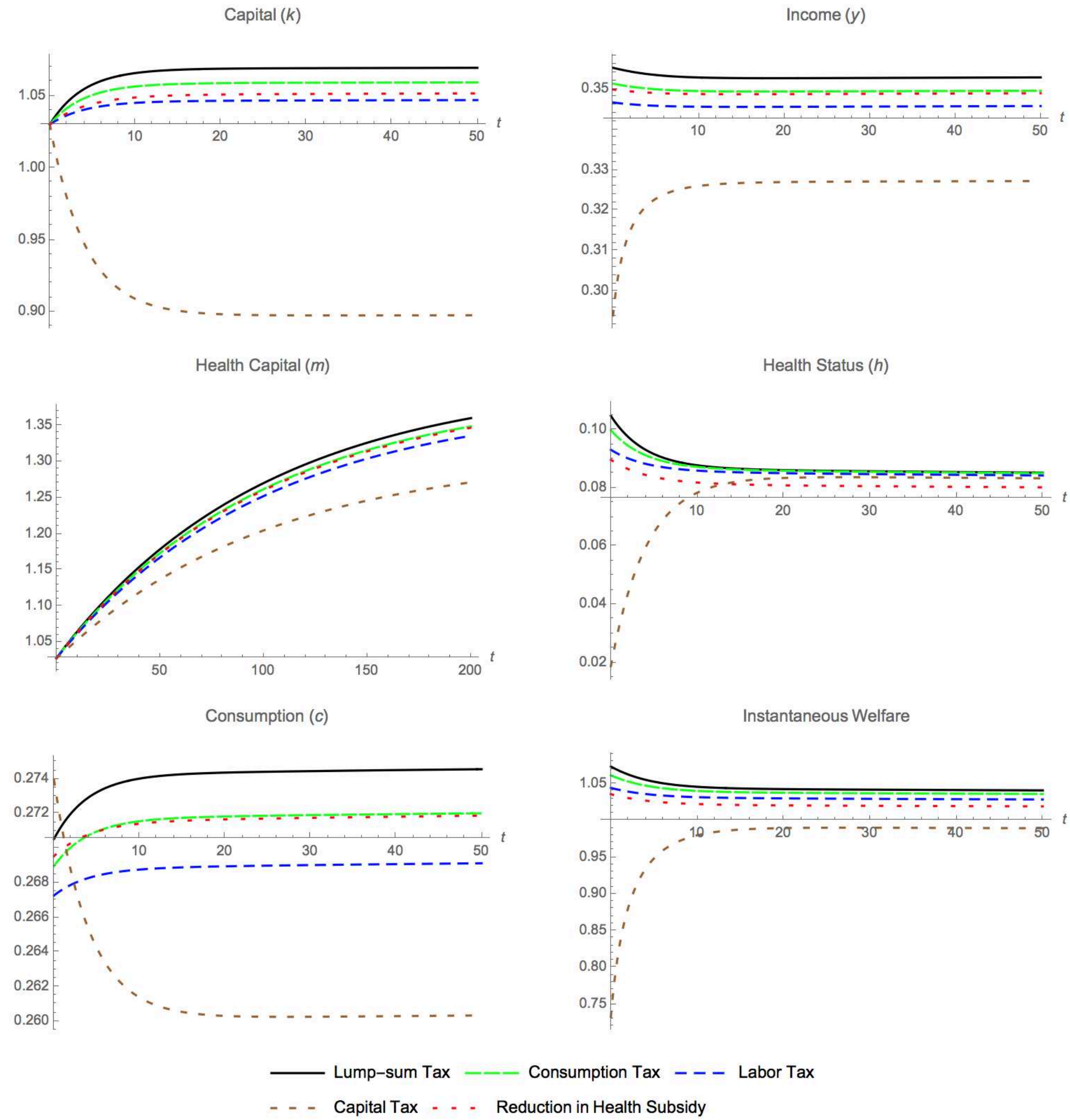


Fig 2. Dynamic Response to Increase in Public Investment in Health