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Abstract

This study formulates a two-period model in which the government privatizes a state-owned public firm over multiple periods. We introduce the shadow cost of public funding (i.e., the excess burden of taxation). The government is concerned about both the total surplus and the revenue obtained from the privatization of the public firm. We find that the government may or may not increase the degree of privatization over time depending on the competitiveness of the product market and nationality of private competitors. The government increases the degree of privatization over time if the product market is competitive and the foreign ownership share in private firms is low. Although it adjusts its privatization policy over time, this harms welfare. In addition, this distortion in the ex-post incentive leads to too low a degree of privatization in the first period.

JEL classification H42, L33

Keywords timing of privatization, commitment, state-owned public enterprise; foreign competition

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1 Introduction

Although the privatization of state-owned public enterprises has been a global phenomenon for more than 50 years, many public and semipublic enterprises owned fully or partially by the public sector remain active. Further, while some public enterprises are traditional monopolists in natural monopoly markets, a considerable number of public and semipublic enterprises compete with private enterprises in a wide range of industries.[1] The optimal privatization policies in these industries have attracted extensive attention from economics researchers in such fields as industrial organization, public economics, financial economics, and development economics.

Specifically, the literature on mixed oligopolies has investigated the optimal privatization policy in different situations. Matsumura (1998) showed that the optimal degree of privatization is never zero unless full nationalization yields a public monopoly. Lin and Matsumura (2012) and Matsumura and Okamura (2015) found that the optimal degree of privatization increases with the number of private firms and decreases with the foreign ownership share in private firms. In free entry markets, Matsumura and Kanda (2005) showed that the optimal degree of privatization is zero when private competitors are domestic, while Cato and Matsumura (2012) found that it is strictly positive when they are foreign and increases with the foreign ownership share in private firms. In addition, Chen (2017) showed that the optimal degree of privatization is positive even in free entry markets if privatization improves production efficiency. Fujiwara (2007) showed a non-monotonic relationship between the degree of product differentiation and optimal degree of privatization. Cato and Matsumura (2015) discussed the relationship between the optimal trade and privatization policies, showing that a higher tariff rate reduces the optimal degree of privatization in free entry markets.

While these studies all assumed that a government privatizes a public firm as a one-time event, public enterprises are often privatized gradually over time. For example, the Japanese government

[1] Examples include the United States Postal Service, Deutsche Post AG, Areva, Nippon Telecom and Telecommunication (NTT), Japan Tobacco (JT), Volkswagen, Renault, Electricite de France, the Japan Postal Bank, Kampo, the Korea Development Bank, and the Korea Investment Corporation. For other examples of mixed oligopolies and recent developments in this field, see Ishibashi and Matsumura (2006), Ishida and Matsushima (2009), Chen (2017) and the works cited therein.

continued to sell shares in NTT (JT), which was a state-owned public monopolist until 1985, from 1986 to 2016 (to 2013). Moreover, it still holds a one-third share in both NTT and JT. In 2015, the Japanese government sold a minor share in the Japan Postal Bank, the largest bank in Japan, and Kampo, a major life insurance company, and it announced its plans to sell more shares in these enterprises in the near future again. Although these examples are cases in which the government gradually reduces the public ownership shares, examples of the opposite also exist. The French government increased its ownership of Renault, a leading automobile company, from 15% to 19.4% in 2015. Therefore, it is reasonable to assume that the government changes the degree of privatization of public enterprises over time.

In this study, we formulate a simple model to analyze the situation of gradual privatization. We present a two-period model in which the government can change the degree of privatization twice. In this setting, the government has an incentive to raise revenue from selling shares in the public firm because of the shadow cost of public funding (i.e., the excess burden of taxation).[2]

We find that the government has an incentive to change the degree of privatization in the second period. Whether it increases or decreases the degree of privatization depends on the number of private competitors (i.e., the competitiveness of the market) and nationality of those private competitors (i.e., foreign penetration in the domestic market). If the private competitors are domestic and manifold, the government sells additional shares in the public firm over time. On the contrary, if the private competitors are foreign, the government is likely to have an incentive to renationalize the public firm. However, these gradual adjustments in public ownership shares harm welfare because early stage partial privatization distorts the privatization policy in the later stage. Moreover, to mitigate this adjustment, the government chooses a suboptimal degree of privatization in the first period to reduce the future distortion in the privatization policy. We also find that once the government fully privatizes the public firm, it never renationalizes it.

[2]See Meade (1944) and Laffont and Tirole (1986) for more details on this concept. For its importance in the context of mixed oligopolies, see Capuano and De Feo (2010) and Matsumura and Tomaru (2013, 2015).

2 The Model

Consider a two-period model in which one domestic state-owned public firm, firm 0, competes against n private firms. Each period is indexed by $t(=1,2)$. We assume that every agent has the same discount factor $\delta \in (0,1)$.

At the beginning of the game, the government owns all the shares in firm 0 and sells them over two periods. The government sells α_1 shares at the beginning of period 1 and $\alpha_2 - \alpha_1$ shares at the beginning of period 2. α_t is a measure of the degree of privatization in period t. If $\alpha_2 - \alpha_1 < 0$, this implies that the government buys back the shares in firm 0 and renationalizes it.

Following the standard formulation in the literature on mixed oligopolies, we assume that firm 0 maximizes the weighted average of social welfare and its own profit and that the weight depends on α_t, whereas private firms maximize their own profits (Matsumura, 1998). Let W_t denote domestic social welfare and $\pi_{i,t}$ denote firm i's profit in period t.

The timeline is as follows. At the beginning of period 1, the government chooses α_1 to maximize $W_1+\delta W_2$. In period 1, given α_1, firms face Cournot competition. Firm 0 chooses its output to maximize $(1-\alpha_1)(W_1+\delta W_2)+\alpha_1(\pi_{0,1}+\delta\pi_{0,2})$ and firm i $(i=1,2,..n)$ chooses its output to maximize $\pi_{i,1}+\delta\pi_{i,2}$. At the beginning of period 2, the government chooses α_2 to maximize W_2. In period 2, given α_2, firms again face Cournot competition. Firm 0 chooses its output to maximize $(1-\alpha_2)W_2+\alpha_2\pi_{0,2}$ and firm i $(i=1,2,..n)$ chooses its output to maximize $\pi_{i,2}$.

In each period, firms produce perfectly substitutable commodities for which the stationary inverse demand function is denoted by $p_t = p(Q_t)$, where p_t is the price and Q_t is the total output in period t. We assume that the function p is twice continuously differentiable and $p' < 0$ as long as $p > 0$. Firm 0's cost function is $c_0(q_{0,t})$, where $q_{0,t}$ is the output of firm 0 in period t. Each private firm i $(=1,\ldots,n)$ has an identical cost function, $c(q_{i,t})$, where $q_{i,t}$ is the output of private firm i in period t and $c(q_{i,t})$ is the cost.[3] We assume that the functions c_0 and c are twice continuously differentiable as well as the

[3]In this study, we allow a cost difference between public and private firms, although we do not allow a cost difference among private firms. While some readers might think that the public firm must be less efficient than the private firm, not all empirical studies support this view. See Megginson and Netter (2001) and Stiglitz (1988). In addition, Martin

interior solution in the output competition stages.

The profit of firm 0 in period t is given by $\pi_{0,t} = p(Q_t)q_{0,t} - c_0(q_{0,t})$ and that of firm i $(= 1, \ldots, n)$ in period t is given by $\pi_{i,t} = p(Q_t)q_{i,t} - c(q_{i,t})$. Domestic social welfare in period t is defined as

$$W_t = \int_0^{Q_t} p(q)dq - p(Q_t)Q_t + \pi_{0,t} + (1-\theta)\sum_{i=1}^{n} \pi_{i,t} + \lambda(D_t + R_t), \tag{1}$$

where $\lambda > 0$ is the additional social cost of public funding,[4] D_t is the revenue from firm 0's dividends, R_t is the revenue from privatization, and θ is the foreign ownership share in private firms. Private firms are foreign (domestic) when $\theta = 1$ $(\theta = 0)$.[5] The social cost of public funding is the deadweight loss from collecting a unit of tax (i.e., the excess burden of taxation). Thus, the government's revenue from firm 0 yields a λ welfare gain because it saves the excess burden of taxation in other markets.[6] We assume that $\lambda < 1$ for the tractability of our analysis.[7]

We assume that the financial market is perfect. That is, the government sells its shares in firm 0 at the fair value of the firm. The fair value of firm 0 in period 1, V_1, is equal to $\pi_{0,1} + \delta\pi_{0,2}$ and that in period 2, V_2, is equal to $\pi_{0,2}$. Therefore, at the beginning of period 1 (2), the government obtains $R_1 = \alpha_1 V_1$ $(R_2 = (\alpha_2 - \alpha_1)V_2)$. In addition, at the end of period t, the government obtains $D_t = (1 - \alpha_t)\pi_{0,t}$.

3 Equilibrium

We solve the game by backward induction. In the second stage in each period, each firm chooses its output simultaneously. Note that at the beginning of period t, the government has already sold firm 0's shares. Therefore, when firm 0 chooses $q_{0,t}$, R_t is given exogenously. In period 1, firm 0 maximizes

and Parker (1997) suggested that corporate performance can either increase or decrease after privatization, based on their study in the United Kingdom. See Matsumura and Matsushima (2004) for a theoretical discussion of the endogenous cost differences between public and private enterprises.

[4] $(1 + \lambda)$ is the so called marginal cost of public funding (MCF).

[5] For discussions on the nationality of private enterprises in mixed oligopolies, see the literature starting with Corneo and Jeanne (1994) and Fjell and Pal (1996). See also Pal and White (1998) and Bárcena-Ruiz and Garzón (2005a,b).

[6] See Matsumura and Tomaru (2013). Introducing the shadow cost of public funding λ is popular in many contexts, as used by the studies listed in footnote 2.

[7] According to Laffont (2005), λ is estimated to be around 0.3 in developed countries and thus this assumption is realistic in a developed country setting.

$(1-\alpha_1)(W_1+\delta W_2)+\alpha_1(\pi_{0,1}+\delta\pi_{0,2})$. However, because neither W_2 nor $\pi_{0,2}$ is affected by $q_{0,1}$, it chooses $q_{0,1}$ to maximize $(1-\alpha_1)W_1+\alpha_1\pi_{0,1}$. In period 2, firm 0 chooses $q_{0,2}$ to maximize $(1-\alpha_2)W_2+\alpha_2\pi_{0,2}$. By substituting $D_t=\alpha_t\pi_{0,t}$ into (1), we obtain the payoff of firm 0. The first-order condition of firm 0 in period t is

$$(1+(1-\alpha_t)^2\lambda)p+(1-(1-\alpha_t)(1-\theta)+(1-\alpha_t)^2\lambda)p'q_{0,t}-(1+(1-\alpha_t)^2\lambda)c_0'-(1-\alpha_t)\theta p'Q_t=0. \quad (2)$$

The first-order condition of private firm i ($i=1,\ldots,n$) is

$$p+p'q_{i,t}-c'=0. \quad (3)$$

We assume that the second-order conditions,

$$(1+(1-\alpha_t)^2\lambda)(p''q_{0,t}+2p'-c_0'')+(1-\alpha_t)(-p'-\theta p''Q_t-(1-\theta)p''q_{0,t})<0 \quad (4)$$

and

$$2p'+p''q_{i,t}-c''<0 \quad (5)$$

are satisfied. A sufficient but not necessary condition is that c_0'' and c'' are sufficiently large. We also assume

$$p'+p''q<0. \quad (6)$$

This assumption implies that the strategies of private firms in the quantity competition stage are strategic substitutes.[8] A sufficient but not necessary condition is $p''\leq 0$. These are standard assumptions in the literature.

Henceforth, we focus on the symmetric equilibrium wherein all private firms produce the same output level q (i.e., $q_{i,t}=q_{j,t}=q_t$ for all $i,j=1,\ldots,n$). Solving equations (2), (3), and (7) leads to the following equilibrium outputs in the third stage, given α and n:

$$Q_t=q_{0,t}+nq_t. \quad (7)$$

[8] We do not assume that the strategy of the public firm is a strategic substitute because it may be a strategic complement under plausible assumptions when private firms are foreign. See Matsumura (2003).

Let $q_0(\alpha_t)$, $q(\alpha_t)$, and $Q(\alpha_t) := q_0(\alpha_t) + nq(\alpha_t)$ be the equilibrium output of firm 0, that of each private firm, and the equilibrium total output given α_t. Note that these functions are not affected by time index t except for the effect through α_t.

Lemma 1 *$q_0(\alpha)$ and $Q(\alpha)$ are decreasing in α, and $q(\alpha)$ is increasing in α.*

Proof See the Appendix.

Lemma 1 is intuitive. A decrease in α makes the public firm, firm 0, more aggressive because it is more concerned about the consumer surplus. Although the objective of each private firm is not related to α, a decrease in α reduces the output of each private firm through the strategic interaction. Note that private firms' strategies are strategic substitutes. Then, the first direct effect dominates the second indirect strategic effect and thus a decrease in α increases the total output.

Before solving the two-period game formulated above, we consider the game in which the government chooses the degree of privatization in both periods, α_1 and α_2, in period 1 as a benchmark. In other words, in this benchmark game, the government can commit to α_2 in period 1. The government's problem is

$$\max_{\alpha_1,\alpha_2} \sum_{t=1}^{2} \delta^{t-1} \left(\int_0^{Q_t} p(q)dq - p(Q_t)Q_t + (1+\lambda)\pi_{0,t} + (1-\theta) \sum_{i=1}^{n} \pi_{i,t} \right).$$

Let α_1^{**} and α_2^{**} be the solutions to this problem. Because of the time invariance property of our model formulation, the first-order conditions for α_1 and α_2 are common and the common first-order condition is

$$\left(\frac{dq_0}{d\alpha}\right)(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)np'q) + n\left(\frac{dq}{d\alpha}\right)p'(\lambda q_0 - (\theta(Q-q_0) + (1-\theta)q)) = 0. \quad (8)$$

We assume that the second-order condition is satisfied.

We present a result on this optimal privatization policy.

Lemma 2 *(i) $\alpha_1^{**} = \alpha_2^{**}$. (ii) $\alpha_1^{**} = \alpha_2^{**} := \alpha^{**} = 0$ if and only if $\theta(Q(0) - q_0(0)) + (1-\theta)q(0) - \lambda q_0(0) \leq 0$. (iii) $\alpha^{**} < 1$ if $\theta = 1$ or $c_0(q) = c(q)$ for all q.*

Proof See the Appendix.

Lemma 2(i) implies that the ex-post change in the degree of privatization in period 2 is undesirable from the welfare viewpoint. Thus, committing not to change the degree of privatization improves welfare. However, this commitment may be difficult. The government may commit to not reducing public ownership in the future by enacting a law with a minimal public ownership share obligation. For example, the Japanese government must hold more than one-third of the shares in NTT and JT by law. However, there was a rule forcing the Japanese government to hold a two-thirds share in JT until 2012, which was subsequently reduced to one-third. Because the government can change the law, committing to not changing the public ownership share in the future is hard to implement.

We now solve the original privatization game over two periods. In the first stage of period 2, the government chooses α_2 to maximize W_2. By substituting $R_2 = (\alpha_2 - \alpha_1)\pi_{0,2}$ and $D_2 = \alpha_2 \pi_{0,2}$, we obtain the following first-order-condition for the interior solution:

$$\begin{aligned}\frac{dW_2}{d\alpha_2} \quad = \quad & \left(\frac{dq_0}{d\alpha}\right)(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)np'q) \\ & +n\left(\frac{dq}{d\alpha}\right)(-\theta(p'Q - p'q_0) - (1-\theta)p'q + \lambda p'q_0) \\ & -\lambda\alpha_1\left(\frac{dq_0}{d\alpha}\left(p + p'q_0 - c_0'\right) + n\frac{dq}{d\alpha}p'q_0\right) = 0. \end{aligned} \tag{9}$$

From (9), we see that the equilibrium α_2 of this subgame depends on α_1. Let $\alpha_2(\alpha_1)$ be the equilibrium degree of privatization in period 2 in this subgame. Substituting $\alpha_1 = 0$ into (9) and comparing it with (8) yield the following lemma.

Lemma 3 $\alpha_2^*(0) = \alpha^{**}$.

Lemma 3 implies that if the government holds all the shares of firm 0 at the beginning of period 2, there is no distortion. However, a positive degree of privatization in period 1 distorts the privatization policy in period 2. At the beginning of period 2, the government holds $(1 - \alpha_1)$ shares in firm 0. A one-unit increase in the profit of firm 0 in period 2 increases welfare by $(1 - \alpha_1)(1 + \lambda)$ units in period 2. Therefore, an increase in α_1 reduces the government's incentive to raise the profit in firm 0 at the cost of consumer surplus (i.e., the government puts a larger weight on the consumer surplus and puts a smaller weight on firm 0's profit). This yields the deviation of α_2 from α^{**} in period 2.

In the first stage in period 1, the government maximizes $W_1 + \delta W_2$ with respect to α_1. By substituting $R_1 = \alpha_1(\pi_{0,1} + \delta\pi_{0,2})$, $R_2 = (\alpha_2 - \alpha_1)\pi_{0,2}$, and $D_t = \alpha_t \pi_{0,t}$ into it, we obtain the following first-order-condition for the interior solution:

$$
\begin{aligned}
\frac{d(W_1 + \delta W_2)}{d\alpha_1} &= \Bigg[\Big(\frac{dq_0}{d\alpha}\Big)(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)np'q) \\
&\quad + n\Big(\frac{dq}{d\alpha}\Big)p'(\lambda q_0 - (\theta(Q - q_0) + (1-\theta)q))\Bigg]_{\alpha=\alpha_1} \\
&\quad + \delta\left[\frac{d\alpha_2}{d\alpha_1}\lambda\alpha_1\left(\frac{dq_0}{d\alpha}\left(p + p'q_0 - c_0'\right) + n\frac{dq}{d\alpha}p'q_0\right)\right]_{\alpha=\alpha_2(\alpha_1)} = 0. \qquad (10)
\end{aligned}
$$

We assume that the second-order condition is satisfied. Let α_1^* be the equilibrium degree of privatization in period 1. Define $\alpha_2^* := \alpha_2(\alpha_1^*)$.

In the static model (the model without period 2), the government chooses $\alpha_1 = \alpha^{**}$. However, a positive α_1 distorts its privatization policy in period 2. An increase in α_1 makes the government place less emphasis on the profit of firm 0 in period 2, and thus the profit of firm 0 decreases because of the distorted choice of α_2. Investors expect this privatization policy in period 2 and the stock price is then lower than that when the government can commit to the privatization policy in the future, thereby reducing the government's revenue from the privatization. To reduce the welfare loss from a future deviation from the optimal privatization policy, the government chooses a lower degree of privatization than the optimal one (α^{**}). This yields our main result, Proposition 1(i).

Proposition 1 *(i)* $\alpha_1^* \leq \alpha^{**}$. *(ii)* $\alpha_1^* = 0$ *if and only if* $\alpha^{**} = 0$. *(iii)* $\alpha_1^* = 1$ *if and only if* $\alpha^{**} = 1$. *(iv)* $\alpha_1^* = \alpha^{**} = 0$ *if and only if* $\theta(Q(0) - q_0(0)) + (1-\theta)q(0) - \lambda q_0(0) \leq 0$. *(v)* $\alpha_1^* < 1$ *if* $\theta = 1$ *or* $c_0(q) = c(q)$ *for all* q.

Proof See the Appendix.

Proposition 1(i) states that the government, when it has the opportunity to adjust the degree of privatization in the future, may choose a lower degree of privatization but never a degree larger than the optimal one. For example, the Japanese government sold small stakes in the Japan Postal Bank, Kampo, NTT, and Japan Rail (JR) East, West, and Central during the first stage of privatization, and

Proposition 1(i) may thus explain these actions well.

We now briefly discuss the other results in Proposition 1. Proposition 1(ii) states that the equilibrium degree of privatization is strictly positive if the optimal degree of privatization is positive. On one hand, a marginal increase in α_1 from zero yields the distortion in period 2; however, this is by the second order because of the envelope theorem. When $\alpha_1 = 0$, the government chooses the optimal α_2 at the equilibrium and there is no distortion. On the other hand, the welfare-improving effect of an increase in α_1 from zero has a first-order effect as long as $\alpha^{**} > 0$. Therefore, $\alpha_1^* > 0$ as long as $\alpha^{**} > 0$.

Matsumura (1998) investigated the case with $\theta = \lambda = 0$ and showed that the full nationalization of the public firm is never optimal unless full nationalization yields a public monopoly. Proposition 1(iv) states that full nationalization can be optimal if λ is large, even when $\theta = 0$. An increase in α reduces the output of the public firm and increases that of private firms, while it may also decrease the profit of the public firm. When λ is large, it may harm welfare. This is why full nationalization can be optimal.

Proposition 1(iii) states that if $\alpha^{**} = 1$, the government sells all its shares during the first stage of privatization. Indeed, the Japanese government sold all its shares in the state-owned J-Power in 2004, which may support our result.

Proposition 1(v) states that if the private competitors are foreign, or the public firm is as efficient as private firms, it is never optimal to fully privatize the public firm. Note that $\theta = 1$ or $c_0(q) = c(q)$ is a sufficient, but very far from necessary condition for $\alpha_1^* < 1$. Even if $\theta < 1$ and/or the public firm is less efficient than private firms, α_1^* can be strictly lower than one.

Lemma 3 and Proposition 1(ii) imply the following result.

Lemma 4 If $\alpha^{**} = 0$, then $\alpha_1^* = \alpha_2^* = 0$.

Lemma 4 states that if the optimal degree of privatization is zero in the static model, there is no additional distortion in the dynamic model. However, when $\alpha^{**} > 0$, the privatization policy in period 2 is distorted. As discussed above, the government places less weight on the profit of firm 0 in period 2, and the resulting profit of firm 2 drops below the optimal level. Under these conditions, if an increase in α decreases (increases) the profit of the public firm, the government chooses too high (low) a degree of privatization in period 2.

We speculate that whether an increase in α decreases the profit of the public firm depends on the foreign ownership share in private firms and number of private firms. When the foreign ownership share in private firms is high, the aggressive behavior of firm 0 is more beneficial. Therefore, in the static model, the government chooses a lower degree of privatization (Lin and Matsumura, 2012), and the public firm is more aggressive than profit-maximizing at the equilibrium. Under these conditions, firm 0's profit may be increasing in α, and the government has an incentive to reduce α in period 2 to raise the consumer surplus at the cost of firm 0's profit. However, the profit motive for the government is weaker in period 2.

Suppose that private firms are domestic. The government is concerned about both the profits of these private firms and the consumer surplus as well as firm 0's profit. An increase in α decreases q_0 and increases q_i for $i = 1, 2, ..., n$. Because firm 0's price-cost margin is lower than each private firm's as long as $\alpha < 1$, the above production substitution from the public to private firms improves welfare.[9] When this welfare-improving production substitution effect is strong, the government chooses α above the profit-maximizing level.[10] The welfare effect is stronger when n is larger. Thus, we naturally expect that the government reduces the public ownership share in firm 0 in period 2 when n is larger.

Unfortunately, we fail to derive clear results or prove the above speculation under general demand and cost functions. Hence, we adopt linear demand and quadratic cost functions, which are popular in the literature on mixed oligopolies to present a clear-cut result.[11]

Proposition 2 *Suppose that $p(Q) = a - Q$ and $c_0(q) = c(q) = q^2/2$. (i) $\alpha_2^* > \alpha^{**}$ if and only if*

$$\theta < \theta(n) := \frac{n^2 - 8}{3n(n+4)},$$

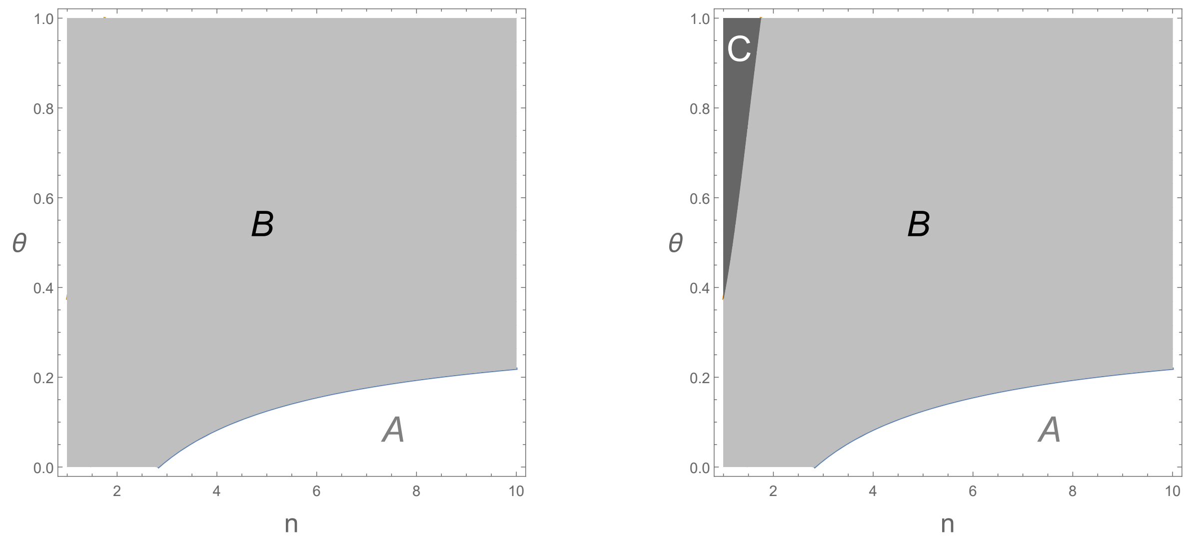
[9] For a discussion of welfare-improving production substitution, see Lahiri and Ono (1988). See Matsumura (1998) in the context of mixed oligopolies.

[10] The profit-maximizing level of α is not one because of the strategic effect discussed in the literature on delegation games. See Vickers (1985).

[11] The pioneering work of De Fraja and Delbono (1989) adopted this setting. See also Matsumura and Shimizu (2010) and the works cited therein.

and $\theta(n)$ is increasing in n. (ii) $\alpha_1^* = \alpha_2^* = \alpha^{**} = 0$ if and only if $g(n, \lambda, \theta) := (n-1)\theta(2+\lambda) + 2(1-\lambda^2) - \lambda\theta - n\theta^2 \leq 0$. (iii) $g(n, \lambda, \theta) \leq 0$ only if $n < 2$, and $g(n, \lambda, \theta)$ is decreasing in both λ and θ for $n < 2$.

Proof See the Appendix.



$\lambda = 1/2$ $\lambda = 7/8$

Figure 1: The comparison between α_2^* and α^{**}. The left figure is the case where $\lambda = 1/2$, and the right figure is the case where $\lambda = 7/8$. The region A shows the area where $\alpha_2^* > \alpha^{**}$, the region B shows the area where $\alpha_2^* < \alpha^{**}$, and the region C shows the area where $\alpha_2^* = \alpha^{**} = 0$.

Figure 1 indicates that over-privatization ($\alpha_2^* > \alpha^{**}$) is more likely to take place when the number of private firms is high and the foreign ownership share in private firms is low. Furthermore, as the right-hand figure in Figure 1 shows, full nationalization may be realized under duopoly when λ and θ are large.

In regions A and B, $\alpha_1^* < \alpha^{**}$. In region C as well as on the border between region A and region B, $\alpha_1^* = \alpha^{**}$. Thus, this example illustrates that for a fairly wide range of parameters, the government chooses a degree of privatization below the optimal one during the early stage of privatization to reduce distortion in the later stage.

Proposition 2 also suggests the possible danger of using a duopoly model in which one public firm competes against one private firm. If we consider a duopoly model, we may conclude that over-privatization never takes place. However, Proposition 2(i) suggests that this is possible if the number of private firms exceeds three. Moreover, if we consider only a duopoly model, we may conclude that full nationalization takes place for a reasonable value of λ. However, Proposition 2(ii) states that full nationalization does not take place if the number of private firms exceeds two. Proposition 2 thus suggests that we should carefully check the robustness of the results in mixed oligopolies.

We now discuss another implication of Proposition 2. Suppose that $\alpha^{**} > 0$. Because $\alpha_1^* < \alpha^{**}$ (Proposition 1(i)), $\alpha_2^* > \alpha^{**}$ is a sufficient (but not necessary) condition for $\alpha_1^* < \alpha_2^*$. Thus, Proposition 2 implies the following lemma.

Lemma 5 *Under the linear demand and quadratic cost specified in Proposition 2,* $\alpha_2^* > \alpha_1^*$ *if* $\theta < \theta(n)$.

As discussed in the Introduction, the Japanese government has often increased the degree of privatization over time, such as in the cases of the Japan Postal Bank, Kampo, NTT, and JT. Our result may explain such a gradual privatization process over time.

Finally, we present a result including the welfare implications of full privatization. In the model discussed in Proposition 2, $\alpha^{**} < 1$ (Proposition 1(v)). However, under more general cost conditions, $\alpha^{**} = 1$ can hold. We find that once the government chooses full privatization in period 1, it never renationalizes in period 2. However, the government chooses full privatization in period 2 even when $\alpha^{**} < 1$.

Proposition 3 *(i) If* $\alpha^{**} = 1$*, then* $\alpha_1^* = \alpha_2^* = 1$*. (ii) Even if* $\alpha^{**} < 1$*,* α_2^* *can be one.*

Proof See the Appendix.

Proposition 3(i) states that if full privatization is optimal, the government fully privatizes the public firm during the early stage (in period 1) and never renationalizes it in the later stage (in period 2). When the public firm is fully privatized in period 1, a marginal reduction in α in period 2 makes the public firm more aggressive and this increases firm 0's profit.[12] Thus, if $\alpha_1 = 1$ is optimal, then $\alpha_2 = 1$ must be optimal because the government's profit motive is weaker in period 2.

Indeed, the renationalization of firms that are fully privatized is exceptional. For example, the Japanese government fully privatized Japan Airlines (JAL) in 1987, KDDI in 1998, J-Power and JR East in 2002, JR West in 2004, JR Central in 2006, and JR Kyushu in 2016. Similarly, the Korean government fully privatized Korean Air in 1969 and POSCO in 2000. Except for JAL, these firms were not renationalized, even partially. Although JAL was renationalized in 2010 because it faced bankruptcy, the government fully privatized it again in 2012.

Our result, however, depends on the stationarity assumption. In other words, the inverse demand function, cost functions, and number of private firms do not change over time. If these conditions do change over time, the optimal policy also changes over time. Further, political reasons explain why the degree of privatization changes over time. For example, if the ruling party changes from right to left, it is plausible that fully privatized firms might be renationalized.[13] Our result suggests that without such changes in circumstances, the renationalization of fully privatized firms is exceptional, although the public ownership share in partially privatized firms may often be adjusted.

Proposition 3(ii) states that even if the government fully privatizes a firm, this does not imply that full privatization is optimal. As discussed above, the government has a distorted incentive to sell its remaining shares in a partially privatized firm but this may harm welfare.

4 Concluding Remarks

In this study, we formulate a two-period model of privatization and investigate the welfare implications of privatization policies across multiple periods. We find that the government changes its privatization policy over time even when external circumstances (e.g., demand and cost conditions) remain

[12] A decrease in α increases q_0 and decreases q_i for $(i = 1, 2, ..., n)$. The former reduces firm 0's profit but $\alpha = 1$ by the second order (from the envelope theorem). The latter increases firm 0's profit by the first order. Therefore, the latter effect dominates the former.

[13] From 1945 to 1977, the Labour government in the United Kingdom repeatedly renationalized several major enterprises that had been privatized by the Conservative government. Similar fluctuations in privatization policies have been observed in France, too.

unchanged. In the later stage, the government is less concerned about the public firm's profit as it raises the stock price and increases the revenue from the firm because it has already sold some of its shares in the public firm to the private sector, and this yields the distortion.

We show that whether the government increases or decreases the public ownership share in the public firm depends on the competitiveness of the product market and nationality of private competitors. When private firms are domestic, the government has an ex-post incentive to increase the degree of privatization when the number of private firms is above some threshold. When private firms are foreign, the government has an ex-post incentive to decrease the degree of privatization. These changes in the degree of privatization are undesirable in terms of social welfare. Hence, the desirable policy depends on the number and nationality of private firms. When private firms are domestic and their number is above some threshold, it is important to commit not to further privatize semipublic firms. When private firms are foreign, on the contrary, it is important to commit not to renationalize the public firm.

Let us add one caveat at this point. According to our analysis, an ex-post change in the degree of privatization harms social welfare. This observation depends on the assumption that the environment is fixed. If demand or supply conditions change, an ex-post change in the degree of privatization becomes desirable. Thus, we must also consider whether further privatization is an adaptation to the changing environment or simply opportunistic behavior by the government.

In this study, we assume that firms face quantity competition. As Matsumura and Ogawa (2012) showed, if firms can choose whether they compete on price or quantity, they choose price competition in contrast to the private market (Singh and Vives, 1984).[14] Thus, an analysis of price competition remains for future research.

Moreover, we assume that the number of private firms is given exogenously. Owing to recent deregulation and liberalization, entry restrictions in mixed oligopolies have significantly weakened. As a result, private enterprises have newly entered many mixed oligopolies such as the banking, insurance, telecommunications, and transportation industries and the literature on mixed oligopolies has inten-

[14] For the oligopoly version in mixed oligopolies, see Haraguchi and Matsumura (2016).

sively investigated the optimal privatization policy in free entry markets.[15] However, it is commonly assumed that governments privatize public firms only once. Dynamic privatization policies are important in this context, too. Extending our analysis to free entry markets remains another promising avenue for future research.

[15] See Matsumura and Kanda (2005). For recent discussions on this topic, see Chen (2017) and the works cited therein.

Appendix

In the following proofs, we suppress the arguments of the functions.

Proof of Lemma 1

Let

$$H := \begin{pmatrix} A & 0 & B \\ 0 & p' - c'' & p' + p''q \\ -1 & -n & 1 \end{pmatrix},$$

$$A := (1 - (1-\alpha)(1-\theta) + (1-\alpha)^2\lambda)p' - (1 + (1-\alpha)^2\lambda)c_0'' < 0,$$

and

$$B := (1 + (1-\alpha)^2\lambda)(p''q_0 + p') + (1-\alpha)(-p' - \theta p''Q - (1-\theta)p''q_0).$$

By differentiating (2), (3), and (7), we obtain

$$H \begin{pmatrix} dq_0 \\ dq_1 \\ dQ \end{pmatrix} = -\begin{pmatrix} \frac{(1-\alpha)^2\lambda - 1}{1+(1-\alpha)^2\lambda} p'(-\theta Q - (1-\theta)q_0) \\ 0 \\ 0 \end{pmatrix} d\alpha, \tag{11}$$

By applying Cramer's rule to (11), we obtain

$$\frac{dq_0}{d\alpha} = -\frac{\frac{(1-\alpha)^2\lambda-1}{1+(1-\alpha)^2\lambda}p'(-\theta Q - (1-\theta)q_0)(p' - c'' + n(p' + p''q))}{|H|}, \tag{12}$$

$$\frac{dq}{d\alpha} = \frac{\frac{(1-\alpha)^2\lambda-1}{1+(1-\alpha)^2\lambda}p'(-\theta Q - (1-\theta)q_0)(p' + p''q)}{|H|}, \tag{13}$$

$$\frac{dQ}{d\alpha} = -\frac{\frac{(1-\alpha)^2\lambda-1}{1+(1-\alpha)^2\lambda}p'(-\theta Q - (1-\theta)q_0)(p' - c'')}{|H|}, \tag{14}$$

where $|H| = (p' - c'')(A+B) + nA(p' + p''q)$. From the second-order condition of q_0, we obtain $A+B < 0$. From (6), we obtain $(p' + p''q) < 0$. Thus, $|H| > 0$. Because $\lambda < 1$, we obtain $(1-\alpha)^2\lambda - 1 < 0$. Under these conditions, (12) and (14) are negative, and (13) is positive. Q.E.D.

Proof of Lemma 2

(i) This is immediately derived from the time invariance property of our model formulation.

(ii) Define $\mathcal{W}$ by

$$\mathcal{W} := \sum_{t=1}^{2} \delta^{t-1} \left(\int_0^{Q_t} p(q)dq - p(Q_t)Q_t + (1+\lambda)\pi_{0,t} + (1-\theta)\sum_{i=1}^{n} \pi_{i,t} \right) \text{ s.t. } \alpha_1 = \alpha_2 = \alpha$$

We obtain

$$\frac{\partial \mathcal{W}}{\partial \alpha_t}\bigg|_{\alpha=0} = (1+\delta)\left[n\Big(\frac{dq}{d\alpha}\Big)(\lambda p' q_0 - (\theta p'(Q-q_0) + (1-\theta)p'q))\right]_{\alpha=0}. \tag{15}$$

$\alpha^{**} = 0$ if (15) is nonpositive, and it is nonpositive only if $\theta(Q(0) - q_0(0)) + (1-\theta)q(0) - \lambda q_0(0)$ is nonpositive.

(iii) By substituting $\theta = 1$ into (10) and using (2), we obtain

$$\frac{\partial \mathcal{W}}{\partial \alpha_t}\bigg|_{\alpha_t=1} = \left[\Big(\frac{dQ}{d\alpha}\Big)(-p'Q) + \Big(n\frac{dq}{d\alpha}\Big)p'(1+\lambda)q_0\right]_{\alpha=1}. \tag{16}$$

Because $dQ/d\alpha < 0$, $dq/d\alpha > 0$ (Lemma 1), $p' < 0$, and $q_0 > 0$, (16) is negative and thus $\alpha^{**} < 1$.

Suppose that $c_0 = c$. Then, $q_0 = q$ at $\alpha = 1$, and thus

$$\frac{\partial \mathcal{W}}{\partial \alpha_t}\bigg|_{\alpha_t=1} = \left[-\frac{dQ}{d\alpha}(1+n\theta)p'q + n\frac{dq}{d\alpha}(\lambda+\theta)p'q\right]_{\alpha=1} < 0. \tag{17}$$

Therefore, $\alpha^{**} < 1$. Q.E.D.

To prove Proposition 1, we present a supplementary lemma.

Lemma 6 *For $\alpha_2(\alpha_1) \in (0,1)$, $\alpha_2'(\alpha_1)$ is positive (negative, zero) if and only if*

$$\left[\frac{dq_0}{d\alpha}\left(p + p'q_0 - c_0'\right) + n\frac{dq}{d\alpha}p'q_0\right]_{\alpha=\alpha_2(\alpha_1)}$$

is negative (positive, zero).

Proof of Lemma 6 By applying the implicit function theorem to (9), we obtain

$$\frac{d\alpha_2}{d\alpha_1} = -\frac{\frac{\partial^2 W_2}{\partial \alpha_1 \partial \alpha_2}}{\frac{\partial^2 W_2}{\partial \alpha_2^2}}. \tag{18}$$

Because the denominator in (18) is negative, (18) is positive (negative, zero) if and only if

$$\frac{\partial^2 W_2}{\partial \alpha_1 \partial \alpha_2} = -\lambda\left(\left[\frac{dq_0}{d\alpha}\left(p + p'q_0 - c_0'\right) + n\frac{dq}{d\alpha}p'q_0\right]_{\alpha=\alpha_2}\right)$$

is positive (negative, zero). Q.E.D.

Proof of Proposition 1

(i) First, we show that if $\alpha^{**} < 1$, then $d(W_1+\delta W_2)/d\alpha_1$ is nonpositive at $\alpha_1 = \alpha^{**}$, and thus, $\alpha_1^* \le \alpha^{**}$.

$$\left.\frac{d(W_1+\delta W_2)}{d\alpha_1}\right|_{\alpha_1=\alpha^{**}} = \Bigg[\left(\frac{dq_0}{d\alpha}\right)(-p'Q + (1+\lambda)(p+p'q_0-c_0') + (1-\theta)np'q) \tag{19}$$

$$+n\left(\frac{dq}{d\alpha}\right)p'(\lambda q_0 - (\theta(Q-q_0)+(1-\theta)q))\Bigg]_{\alpha=\alpha^{**}} \tag{20}$$

$$+\delta\left[\frac{d\alpha_2}{d\alpha_1}\lambda\alpha_1\left(\frac{dq_0}{d\alpha}\left(p+p'q_0-c_0'\right)+n\frac{dq}{d\alpha}p'q_0\right)\right]_{\alpha=\alpha_2(\alpha^{**})} \tag{21}$$

Suppose that $\alpha^{**} \in (0,1)$. From (8), (19)+ (20) is zero. Lemma 6 implies that (21) is nonpositive. Therefore, $d(W_1+\delta W_2)/d\alpha_1$ is nonpositive at $\alpha_1 = \alpha^{**} \in (0,1)$.

Suppose that $\alpha^{**} = 0$. (19)+ (20) is nonpositive. (21) is zero when $\alpha_1 = \alpha^{**} = 0$. Thus, $d(W_1 + \delta W_2)/d\alpha_1$ is nonpositive at $\alpha_1 = \alpha^{**} = 0$.

Finally, we show that if $\alpha^{**} = 1$, then $\alpha_1^* = 1$. First, we show that if $\alpha^{**} = 1$, $\alpha_2(\alpha_1) = 1$ for any $\alpha_1 \in [0,1]$.

$$\left.\frac{dW_2}{d\alpha_2}\right|_{\alpha_2=1} = \Bigg[\left(\frac{dq_0}{d\alpha}\right)(-p'Q+(1+\lambda)(p+p'q_0-c_0')+(1-\theta)np'q) \tag{22}$$

$$+\ n\left(\frac{dq}{d\alpha}\right)(-\theta(p'Q-p'q_0)-(1-\theta)p'q+\lambda p'q_0) \tag{23}$$

$$-\ \lambda\alpha_1\left(\frac{dq_0}{d\alpha}\left(p+p'q_0-c_0'\right)+n\frac{dq}{d\alpha}p'q_0\right)\Bigg]_{\alpha=1} \tag{24}$$

When $\alpha^{**} = 1$, (22)+ (23) is nonnegative. Furthermore, (24) is also nonnegative for any $\alpha \in [0,1]$. Under these conditions, $dW_2/d\alpha_2$ is nonnegative at $\alpha_2 = 1$ for any $\alpha_1 \in [0,1]$, and thus $\alpha_2(\alpha_1) = 1$ for any $\alpha_1 \in [0,1]$.

From the discussion above, we find that $\alpha_2'(\alpha_1) = 0$ for all $\alpha_1 \in [0,1]$ when $\alpha^{**} = 1$. By substituting $d\alpha_2/d\alpha_1 = 0$ into (10) and comparing it with (8), we obtain $\alpha_1^* = 1$.

(ii) Proposition 1(i) implies that if $\alpha^{**} = 0$, then $\alpha_1^* = 0$. We show that $\alpha_1^* = 0$ only if $\alpha^{**} = 0$.

$$\left.\frac{dW_1+\delta W_2}{d\alpha}\right|_{\alpha=0} = \left[n\left(\frac{dq}{d\alpha}\right)(\lambda p'q_0 - (\theta p'(Q-q_0)+(1-\theta)p'q))\right]_{\alpha=0}. \tag{25}$$

$\alpha_1^* = 0$ if (25) is nonpositive and it is nonpositive only if $\theta(Q(0)-q_0(0)) + (1-\theta)q(0) - \lambda q_0(0)$ is nonpositive. From Lemma 2(ii) we find that $\alpha_1^* = 0$ only if $\alpha^{**} = 0$.

(iii) Proposition 1(i) implies that $\alpha_1^* = 1$ only if $\alpha^{**} = 1$ since $\alpha_1^* \leq \alpha^{**}$. In the Proof of Proposition 1(i), we showed that if $\alpha^{**} = 1$, then $\alpha_1^* = \alpha_2^* = 1$.

(iv) Lemma 2(ii) and Proposition 1(i) imply Proposition 1(iv).

(v) Lemma 2(iii) and Proposition 1(i) imply Proposition 1(v). Q.E.D.

Proof of Proposition 2

From the first-order condition of each private firm in the quantity competition stage, we obtain $q = (a - q_0)/(n + 2)$ and $Q = (na + 2q_0)/(n + 2)$. Thus, we obtain

$$\frac{dq}{d\alpha} = \frac{1}{n+2}\frac{dq_0}{d\alpha}.$$

From Lemma 2(iii) we obtain $\alpha^{**} \neq 1$. First, we consider the case where $\alpha^{**} \in (0, 1)$. Note that $\alpha_1^* > 0$ when $\alpha^{**} > 0$ (Proposition 1(ii)). The first-order condition for the interior solution for α^{**} is

$$\begin{aligned}
\left.\frac{\partial \mathcal{W}}{\partial \alpha_t}\right|_{\alpha_t=\alpha^{**}} &= \left(\frac{dq_0}{d\alpha}\right)(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)np'q) + n\left(\frac{dq}{d\alpha}\right)p'(\lambda q_0 - (\theta(Q - q_0) + (1-\theta)q)) \\
&= \frac{dq_0}{d\alpha}\left(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)np'q - \frac{n}{n+2}p'(\lambda q_0 - (\theta(Q - q_0) + (1-\theta)q))\right) \\
&= \frac{dq_0}{d\alpha}p'\left(-Q + (1+\lambda)(a - Q - 2q_0) + (1-\theta)nq - \frac{n}{n+2}(\lambda q_0 - \theta(Q - q_0) - (1-\theta)q)\right) \\
&= -\frac{dq_0}{d\alpha}\Bigg(-\frac{na + 2q_0}{n+2} + (1+\lambda)\left(\frac{2a - (2n+6)q_0}{n+2}\right) + (1-\theta)\frac{n}{n+2}(a - q_0) \\
&\qquad -\frac{n}{n+2}\left(\lambda q_0 - \theta\frac{n(a - q_0)}{n+2} - (1-\theta)\frac{a - q_0}{n+2}\right)\Bigg) = 0.
\end{aligned}$$

From this, we obtain

$$q_0(\alpha^{**}) = \frac{2(1+\lambda) + \frac{2n}{n+2}\theta - \frac{n}{n+2}(1-\theta)}{(1+\lambda)(n+6) - 2 + \frac{2n}{n+2}\theta - \frac{n}{n+2}(1-\theta)}a.$$

By substituting it into W_2, we obtain

$$\begin{aligned}
\left.\frac{dW_2}{d\alpha_2}\right|_{\alpha_2=\alpha^{**}} &= -\lambda\alpha_1^*\left(\frac{dq_0}{d\alpha}(p+p'q_0-c_0')+n\frac{dq}{d\alpha}p'q_0\right)\\
&= -\lambda\alpha_1^*\frac{dq_0}{d\alpha}\left(a-Q-2q_0+\frac{n}{n+2}q_0\right)\\
&= -\lambda\alpha_1^*\frac{dq_0}{d\alpha}\frac{a}{n+2}\left(2-(n+6)\frac{2(1+\lambda)+\frac{2n}{n+2}\theta-\frac{n}{n+2}(1-\theta)}{(1+\lambda)(n+6)-2+\frac{2n}{n+2}\theta-\frac{n}{n+2}(1-\theta)}\right)\\
&= -\lambda\alpha_1^*\frac{dq_0}{d\alpha}\frac{a}{n+2}\frac{n^2-8-3n(n+4)\theta}{\left((1+\lambda)(n+6)-2+\frac{2n}{n+2}\theta-\frac{n}{n+2}(1-\theta)\right)(n+2)}.
\end{aligned}$$

This is positive if and only if $n^2-8-3n(n+4)\theta>0$ (or equivalently $\theta<\theta(n)$). Thus, $\alpha_2^*>\alpha^{**}$ if $\theta<\theta(n)$. $\theta'(n)=4(5n^2+12n+8)/(3n(n+4))^2>0$.

Next, we consider the case in which $\alpha^{**}=0$. In this case $\alpha_1^{**}=\alpha_2^{**}=0$. From Lemma 2, we find that $\alpha^{**}=0$ if and only if $\theta(Q(0)-q_0(0))+(1-\theta)q(0)-\lambda q_0(0)\le 0$. This holds if and only if $g(n,\lambda,\theta):=(n-1)\theta(2+\lambda)+2(1-\lambda^2)-\lambda\theta-n\theta^2\le 0$.

Because $\partial g(n,\lambda,\theta)/\partial n=\theta(2+\lambda-\theta)\ge 0$ and $g(2,\lambda,\theta)=2\theta(1-\theta)+2(1-\lambda^2)>0$, $g(n,\lambda,\theta)<0$ only if $n<2$.

For $n<2$, we find that $\partial g(n,\lambda,\theta)/\partial\lambda=(n-1)\theta-4\lambda-\theta<0$ and that $\partial g(n,\lambda,\theta)/\theta=(n-2)\lambda-2<0$. Q.E.D.

Proof of Proposition 3

(i) We have already shown this in the proof of Proposition 1(i).

(ii)We present an example in which $\alpha_2^*=1$ and $\alpha^{**}<1$. Suppose that $\theta=0$, $p(Q)=a-Q$, $c_0(q)=k_0q$, $c(q)=kq$ (constant marginal costs), and $k_0>k$. We normalize $k=0$. Suppose that $n=1$ and $\lambda=1/2$.

In this specification, $q=(a-q_0)/2, Q=q+q_0$, and q_0 satisfies the first-order condition

$$\left(1+\frac{1}{2}(1-\alpha)^2\right)\frac{a-q_0}{2}-\left(\alpha+\frac{1}{2}(1-\alpha)^2\right)q_0-\left(1+\frac{1}{2}(1-\alpha)^2\right)k_0=0,$$

which yields

$$q_0(\alpha)=\frac{\left(1+\frac{1}{2}(1-\alpha)^2\right)\left(\frac{a}{2}-k_0\right)}{\frac{1}{2}+\alpha+\frac{3}{4}(1-\alpha)^2}.$$

By substituting this into W_2, we obtain

$$\begin{aligned}
\left.\frac{dW_2}{d\alpha_2}\right|_{\alpha_2=1} &= \left(\frac{dq_0}{d\alpha}\right)(-p'q_0) + \left(\frac{dq}{d\alpha}\right)(-p'q + \lambda p'q_0) - \lambda\alpha_1\left(\frac{dq}{d\alpha}p'q_0\right) \\
&= \frac{dq_0}{d\alpha}p'\left(-q_0 + \frac{1}{2}\left(q - \lambda q_0 + \lambda\alpha_1 q_0\right)\right) \\
&= \frac{dq_0}{d\alpha}p'\left(\frac{a}{4} - \frac{1}{4}q_0 - q_0 + \frac{1}{2}\left(-\lambda q_0 + \lambda\alpha_1 q_0\right)\right) \\
&= \frac{dq_0}{d\alpha}\frac{p'}{4}\left(a - (6-\alpha_1)\frac{a-2k_0}{3}\right). \qquad (26)
\end{aligned}$$

From Lemma 3, we find that $\alpha^{**} = 1$ if and only if (26) at $\alpha_1 = 0$ is nonnegative. Thus, $\alpha_2^* = 1$ if and only if $k_0 \geq a/4$. $\alpha_2^* = 1$ if and only if (26) at $\alpha_1 = \alpha_1^*$ is nonnegative. Therefore, $\alpha_2^* = 1$ if and only if $k_0 \geq y := a(3-\alpha_1^*)/(12-2\alpha_1^*)$.

Suppose that $k_0 = a/4 - \varepsilon$, where ε is positive and sufficiently small. Then $\alpha^{**} < 1$. From Proposition 1, we obtain $\alpha_1^* > 0$. Because y is decreasing in α_1^* and $y = a/4$ when $\alpha_1^* = 0$, $y < a/4$. Therefore, $a/4 - \varepsilon > y$, and thus, $\alpha_2^* = 1$. This example yields $\alpha^{**} < 1$ and $\alpha_2^* = 1$. Q.E.D.

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