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Comparative Advertising: The role of prices

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Abstract

In markets where firms sell similar goods to their competitors, firms may be able to free-ride off the costly price signalling of competitor firms by engaging in price comparative advertising. As the goods are similar, consumers can reason that if one good is high quality (revealed through price signalling) then so is the other. This paper models this phenomenon and finds that in equilibrium there will be firms price signalling as well as freeriding firms that signal through price comparative advertising. Welfare is strictly higher in markets where advertising firms are active relative to pure price signalling markets. In some cases advertising markets can be even more efficient than full information markets as advertisers surrender market power to avoid costly price signalling.

JEL Codes: D82, D83, M37

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1 Introduction

In many markets firms have more information regarding the quality of their goods than potential consumers. As a result firms with high quality goods can signal the true quality of their goods to consumers with prices and warranties being common instruments for this. In many of these markets some of this uncertainty is common to goods offered by different firms. An example is cable television where two providers may offer sets of channels with substantial overlap. An alternate example is package holidays where the utility from holidays to the same location from different but similar tour companies are likely to be similar. In instances like this it may be possible for firms to earn greater profits from free-riding on the costly signalling of other firms.

This paper investigates the potential for firms to engage in this kind of signal free-riding through comparative advertising. Comparative advertising occurs when a firm advertises by contrasting the price and features of its good as compared to those of rival firms. In the previous economic literature comparative advertising has been seen as directly informing consumers of the difference in vertical quality between two goods (Barigozzi et al. 2009) or of the difference in horizontal (seller specific) match utility a consumer would get if he bought a good from a competing firm as compared to buying from the advertising firm (Anderson & Renault 2009). The role of the disclosure of the *prices* of rivals in this context has received less attention.¹

There are many examples of comparative advertising however where the disclosure of price information is a major part of the message. A basic example is offered in the advertisements of Progressive Direct, an American auto insurance company that gives prospective consumers the prices offered by competitors for comparable insurance plans (Yu 2013). They also air advertisements promising “we compare our direct rates side by side to find you a great deal, even if its not with us”. Three, a major UK phone carrier, similarly advertises with a webpage that asks shoppers on their website to “see how our prices compare” with a price comparison of Three against the prices of all of the

¹To the best of my knowledge this is the first paper to examine the informational content of prices as a component of comparative advertising or the firm strategy of providing competitor prices to their consumers more generally.

other major phone carriers in the market (Three Mobile 2016). The online travel agent Skyscanner allows visitors to automatically replicate their flight searches on competing services such as Expedia and eDreams. Another example is provided by Book Depository, a leading UK online bookseller (Charlton 2009) which for every product sold presented a link to the corresponding Amazon page for that item.²

This paper examines a vertically differentiated market in which consumers cannot directly observe the quality of goods. In such a market when high quality firms have higher costs, there is a literature that shows these firms can separate themselves from low quality firms by proposing a high price, often above the monopoly price they would charge if their quality were known. This is called in the literature “price signalling” as the price transmits information about the underlying quality of the good. The problem for high quality firms in such markets is that the high price that enables signalling may result in lower profits than would be available by pricing at the (lower) monopoly price in a full information setting. In this setting we show that price comparative advertising has a clear role: By showing an identical product from a rival firm at a high price, a firm can signal to its customers that its product must also be of high quality, even if the firm itself does not charge such a high price. In doing so a firm may be able to increase profits by pricing closer to its monopoly price and freeriding on the rival firm’s price signal.³

Clearly the logic of this argument does not immediately carry over to equilibrium behaviour. If all firms revert to pricing at the lower level rather than price signalling, free-riding possibilities may be lost. One of the questions in this paper is whether this simple logic extends in some form to an equilibrium setting. Indeed, only some insights of the previous setting carry over once equilibrium forces come into play. In equilibrium there will be firms that engage in price comparative advertising and firms that remain pricing

²A further example is provided by Amazon itself which has a marketplace which allows competitors to compete against Amazon on Amazon’s website. Amazon also receives a portion of the revenues from these external sellers on their website. Whilst these revenues no doubt play some role in Amazon’s decision to operate this marketplace it is also true that this allows Amazon customers to compare Amazon’s prices with other vendors.

³The analogue in the traditional Spence (1973) signalling model of the labour market is that an uneducated worker goes to a firm with his educated identical twin. As both individuals are identical and this is observable the firm will reason that if one sibling is educated and has a high capability then so does the other.

at the signalling level. The advertising firms face increased price competition from the firms they advertise against and thus many will price at levels below the monopoly price. Thus while in normal price signalling markets asymmetric information has the effect of increasing prices, the equilibrium exhibiting price comparative advertising will have some firms pricing above and some below the monopoly level. Total welfare is improved by price comparative advertising relative to the asymmetric information equilibrium where no advertising is allowed. Firms do not earn higher profits however due to this additional price competition and the additional surplus is all accrued by consumers through lower prices. In some cases the welfare of an asymmetric information market with price comparative advertising can be greater than under full information as firms surrender market power in order to achieve more efficient signalling.

A number of extensions of the basic model are examined. When there is heterogeneity in marginal costs among high quality firms I find the intuitive result that it will be the lower cost firms that will be pricing lower while engaging in price comparative advertising. This result suggest another avenue through which price comparative advertising increases welfare as high price (and higher cost) advertisers lose sale quantity to low price (and lower cost) advertisers. An alternate extension is the special case where all high quality firms source their goods from a monopoly supplier.⁴ Here it was found that the possibility of advertising can induce the monopoly supplier to reduce the price they charge reselling firms in the hope of increasing the quantity they sell. These extensions all point to increased welfare from the use of price comparative advertising. These results on welfare are considerably more positive for comparative advertising than the previous literature that examined the comparative advertising of product attributes and found that total welfare could be decreased when there was a large vertical quality difference between rival firms (Anderson & Renault 2009).

Comparative advertising used to be relatively uncommon in developed countries but was legalised in the United States and Europe in 1979 (Federal Trade Commission 1979) and 1997 (Council of European Union 1997) respectively. Elsewhere in the world however

⁴For instance there is a monopoly supplier of Samsung smartphones and Iron Maiden albums.

comparative advertising is still subject to restrictions. In China and Hong Kong it is banned while in Japan it is allowed but seldom used due to it being perceived as impolite by Japanese consumers (Singh 2014). While comparative advertising has recently been legalised in Turkey (Gürkaynak et al. 2015) it remains banned in Saudi Arabia and Kuwait. In other countries such as Qatar the situation surrounding comparative advertising is simply ambiguous with no regulations or case law to determine its legality (Bradley 2014). The policy implications of this paper are clear: comparative advertising delivers lower prices to consumers, is welfare improving and should be supported and encouraged by governments and regulators.

This paper first provides an outline of the surrounding literature in section 2. The model is then presented in section 3 with welfare implications examined in section 4 and model extensions in section 5 before section 6 concludes.

2 Literature Review

This paper has strong links to two key literatures: the price signalling literature and the literature on comparative advertising.⁵ In addition this paper is related to the marketing literature on reference pricing as well as recent work on search deterrence.

An early paper to examine the economic consequences of consumers judging quality by price is that of Scitovszky (1944). Since that time, the advent of signalling theory has led to a number of papers applying price signalling to markets with imperfect information regarding product quality. A common mechanism for signalling is to have higher quality firms producing at a higher marginal cost than lower quality firms (Wolinsky 1983, Bagwell & Riordan 1991, Daughety & Reinganum 2008). In this way high quality firms have a higher optimal price than low quality firms which makes it comparatively less expensive for them to charge a high price, thus allowing signalling. This paper will use this feature of marginal cost increasing in product quality to model a market exhibiting price signalling.

Moving onto the comparative advertising literature, Barigozzi et al. (2009) argue that

⁵The comparative advertising literature can further be considered a branch of the informative advertising literature. See Renault (2016) for a survey.

firms engage in comparative advertising as a means of signalling the vertical quality of their good. By directly informing consumers of the vertical quality of their good compared to a competitor's good the firm opens itself up to litigation expenses if claims made about the comparison of goods are unreasonable. Only a firm with a high quality good would engage in this practice as low quality firms would face a high expected loss from litigation. In this way comparative advertising serves as a signal for good quality as well as a vehicle for direct disclosure of quality. While they do not consider the impact of comparative advertising on total welfare they conclude that it should be supported by regulators as it allows easier market access for new entrants. The authors note that this mechanism for comparative advertising is reliant on an effective court system which suggests comparative advertising would be less effective in countries with weaker institutions.

Another comparative advertising paper is that of Anderson & Renault (2009) who examine a duopolistic market where each firm sells a horizontally and vertically differentiated good. Vertical quality is readily apparent to consumers and so advertising focuses on horizontal differentiation rather than signalling vertical quality. Firms can engage in advertising by disclosing the match utility a consumer would get with their firm or comparative advertising by disclosing the match utility a consumer would get from both firms. They find that in cases where there is a small difference in vertical quality both firms will engage in advertising of their own match utility to benefit from the higher price generated by additional product differentiation. When there is a large quality differential however weaker firms will generally be the firms using comparative advertising to disclose both matches in order to increase their demand. They find in this setting that consumers and lower quality firms are better off when comparative advertising is allowed. So much damage is done to the higher quality firm profits however that total welfare falls as a result of comparative advertising. Related ideas are examined by Koessler & Renault (2012) and Celik (2014) who look at the conditions under which disclosure of quality attributes will occur.

This paper is different from the previous literature on comparative advertising in that it is the disclosure of *prices* of competitor firms rather than the direct disclosure

of horizontal or vertical product attributes that is important. Indeed to highlight this channel, the model I present will exhibit firms advertising against competing firms offering identical products with no horizontal differentiation existing.⁶ Before introducing this model however I will briefly mention two other literatures that are related to this paper.

The reference pricing strand of the marketing literature is related to the notion of price signalling. An example of reference pricing is a \$200 price crossed out and replaced with “\$100 for a limited time only!”. The idea is that this \$200 is suggestive of the quality of the good however the key problem with this strategy is credibility (Grewal & Compeau 2002, Compeau et al. 2002, Kan et al. 2013). Consumers will often doubt that the reference price is ever charged or is representative of the quality of the item. While this paper uses an economic signalling approach, its message might otherwise be motivated by external offers providing credible reference prices to consumers.

Finally the mechanism described by this paper represents an interesting contrast to the mechanism described by Armstrong & Zhou (2015) in a paper on search deterrence. That paper has a model where a shopper is presented with a good of known quality but has an unknown outside option. It is shown that where possible a seller can increase profits by committing to an exploding offer where the consumer has to buy before seeing the outside option. This is in contrast to the current paper where the additional external information can efficiently signal the quality of the current good and thereby increase seller profits. When the good is high quality, this provision of information is good for the selling firm as they can freeride on the price signalling of other firms.

3 The Model

There is a high quality good and a low quality good in the market each of which is sold by a unit mass of firms. The quality of a good is observable to firms but not to consumers. Firms selling low quality goods have a marginal cost of c_L while firms selling high quality

⁶Of course in the real world firms may advertise against competing products that are differentiated but share some common value component. In this case it is likely that comparative advertising plays the price-comparative role outlined in this paper along other roles of comparative advertising explored in the literature.

goods have a higher marginal cost of c_H with $c_H > c_L$. A fraction λ of firms sell high quality goods.

There is a unit mass of consumers, each of whom gets a utility equal to $Q - P_f$ for buying good $Q \in \{H, L\}$ from firm f at price P_f . Consumers have a heterogeneous outside option denoted by Ω_k for customer k . This outside option is logconcave distributed with a pdf denoted $\gamma(\Omega)$ and cdf denoted $\Gamma(\Omega)$.

The timing of this singleshoot game is as follows. A firm is approached by one random consumer. The firm offers that consumer a price and the consumer can either buy at that price or leave the firm in favour of their outside option. Denoting the consumer's perceived quality level by \hat{Q} , the condition for a consumer to buy is:

$$\hat{Q} - P_f \geq \Omega_k \tag{1}$$

Thus the maximum Ω_k consumer that will buy the good will have an Ω_k value of $\hat{Q} - P_f$ and hence from the firm's perspective the probability of a sale is given by $\Gamma(\hat{Q} - P_f)$.

Signalling equilibria are refined by the intuitive criterion (Cho & Kreps 1987). Beliefs are formalised by a function $\mu(P)$ that gives the believed probability of a good being high quality given a price of P . Finally as this paper examines the potential use of price comparative advertising⁷ as a signalling tool alongside price signalling, I will consider only fully separating equilibria.

3.1 Separating Equilibrium without price comparative advertising

In this section the fully separating equilibrium for this basic model lacking price comparative advertising will be examined. First defining the equilibrium concept:

Definition 1 PBNE (without advertising). *A pure strategy Perfect Bayesian Nash Equilibrium (PBNE) in this model without advertising will be described by low and high*

⁷To be defined in terms of the model in section 3.2.

firm pricing strategies P_L, P_H as well as a belief function $\mu(P)$, such that:

$$\pi_L(P_L) \geq \pi_L(P) \quad \forall \quad P \in \mathfrak{R}^+ \quad (2)$$

$$\pi_H(P_H) \geq \pi_H(P) \quad \forall \quad P \in \mathfrak{R}^+ \quad (3)$$

and the belief function $\mu(P)$ is derived in accordance with bayes rule and player strategies for all prices charged with positive probability in equilibrium.

As in a standard price signalling separating equilibrium we get the result that firms selling low quality goods will price at their monopoly price (to be denoted P_L) which maximises their profit:

$$P_L = \arg \max_{P \in \mathfrak{R}^+} (P - c_L)\Gamma(L - P) \quad (4)$$

With the corresponding profit being denoted $\pi_L^M = (P_L - c_L)\Gamma(L - P_L)$. The prices that high quality firms can charge to differentiate themselves from the low quality firms are those prices P^* that satisfy:

$$\pi_L^M \geq (P^* - c_L)\Gamma(H - P^*) \quad (5)$$

The price which makes this expression bind with equality (The Riley (1979) price) is denoted by P^S . With the intuitive criterion applied beliefs will satisfy $\mu(P^S) = 1$. A natural way to extend these beliefs over all prices is thus:

$$\mu(P) = \begin{cases} 1 & P \geq P^S \\ 0 & P < P^S \end{cases} \quad (6)$$

In any fully separating equilibrium, high quality firms will charge the maximum of P^S or the high firm's monopoly price (which will be denoted P_H^M) which maximises their profit in the absence of asymmetric information:

$$P_H^M = \arg \max_{P \in \mathfrak{R}^+} (P - c_H) \Gamma(H - P) \quad (7)$$

With the corresponding profit being denoted $\pi_H^M = (P_H^M - c_H) \Gamma(H - P_H^M)$. Henceforth we assume that this market is one where price signalling is costly and hence $P_H^M < P^S$. The profit of the high firm when signalling is denoted by π_H^S and can be expressed as:

$$\pi_H^S = (P^S - c_H) \Gamma(H - P^S) < \pi_H^M \quad (8)$$

To restrict attention to separating equilibria we assume that high firms prefer signalling to being mistaken for low firms. This condition is

$$\pi_H^S > (P - c_H) \Gamma(L - P) \quad \forall \quad P \in \mathfrak{R}^+ \quad (9)$$

Finally we can show that $P_H^M > P_L$ by first obtaining an expression for the optimal monopolist price from first order conditions of $\pi(P) = (P - c_Q) \Gamma(Q - P)$. This results in:

$$P = c_Q + \frac{\Gamma(Q - P)}{\gamma(Q - P)} \quad (10)$$

As $\Gamma(x)$ is logconcave there is a well-defined solution to this problem and $\frac{\Gamma(x)}{\gamma(x)}$ is a monotonically increasing function (Bagnoli & Bergstrom 2005). From this we can see that the solution price is increasing in cost and in perceived quality. Hence we will always have $P_H^M > P_L$.

Proposition 1. *Without the possibility of advertising, a PBNE exists where L and H firms price at P_L and P^S respectively and consumer belief formation is as per equation 6.*

Proof. Equations 4, 5 and 9 ensure that high and low firms cannot get higher profits from deviating. The belief function described in equation 6 is consistent with this equilibrium.

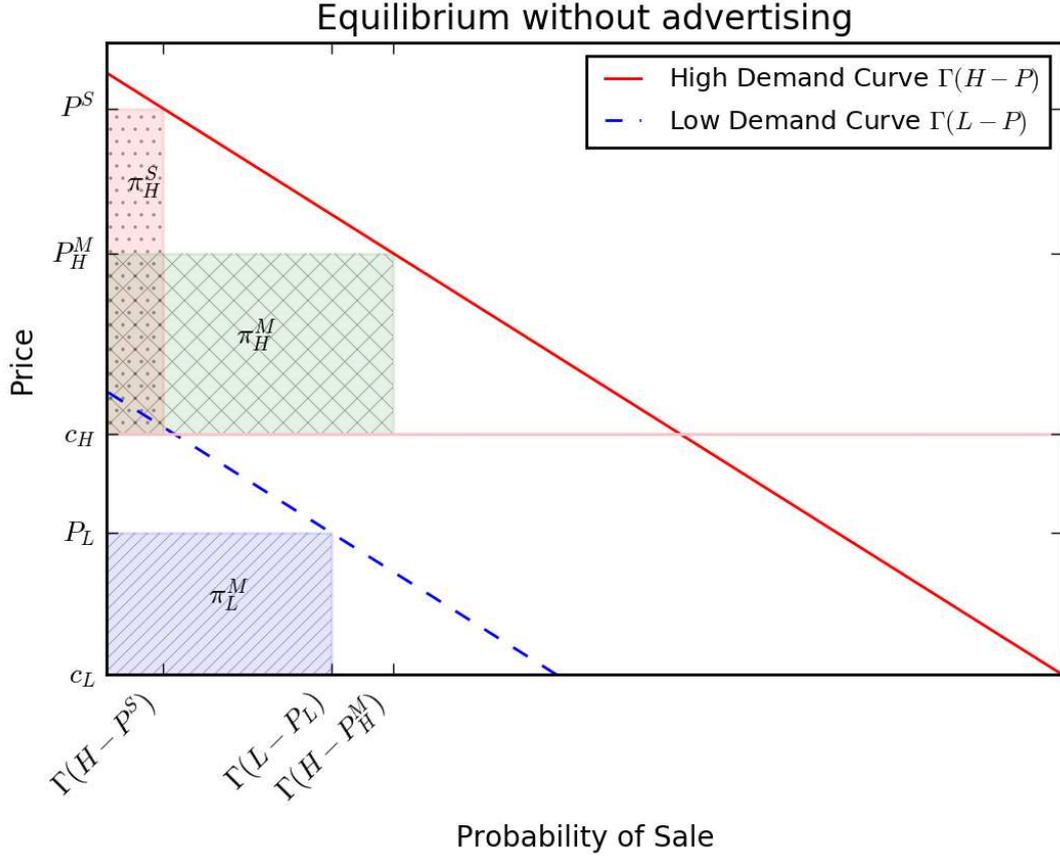


Figure 1: Profits in the no advertising fully separating equilibrium

□

The prices charged and profits earned by each firm in equilibrium along with the high firm full information profits (that are unattainable under asymmetric information) are shown in figure 1.

3.2 Separating Equilibrium with price comparative advertising

In this section firms are allowed to engage in price comparative advertising at no cost. As motivation for this, consider that a high quality firm deviates from the no advertising signalling equilibrium described in the preceding section. They charge the monopolist price P_H^M . At this price however the consumer cannot tell if the good is of high or low quality. The firm can show the offer of another firm offering this same (high quality) good. This other price will be P^S as all other firms selling the high quality good charge this price. The consumer knows that both goods are the same and so will be convinced

that the good is high quality after seeing this other price. The firm will earn $\pi_H^M > \pi_H^S$ and hence this is a profitable deviation. This section considers how the possibility of this profitable deviation changes the equilibrium outlined in proposition 1.

It is assumed that when the consumer enters the market and meets a firm, that firm can costlessly show the consumer an offer from a competing firm that sells the same good.⁸ As both offers concern the same good if one is high quality then so is the other. Henceforth the term *advertiser* will be used to describe a firm that provides an external offer. Firms that do not provide an external offer will be referred to as *non-advertisers*.

The assumed timing is that each firm decides on their own price and whether or not to show a competitors price simultaneously. A key assumption is made that advertising is undirected - firms cannot condition their advertising on the price offered by the firm they advertise against. This assumption can be simply motivated by considering firms to commit separately and simultaneously to their strategy of prices and advertising.⁹

For simplicity an advertising firm only shows one price from an external firm. A consumer observes a price from the external firm but not the advertising strategy of this external firm. A consumer at this point can decide to buy from either the advertising firm or the external firm (at no extra cost). The tie-breaking rule is adopted that when both firms price at an equal amount is that it is assumed that the consumer will buy from each with 50% probability.

The beliefs of a consumer who has seen one price will be represented by the function $\mu(P)$ while the beliefs of consumer who has been shown two prices will be given by a function taking two arguments, $\mu(P, P_E)$. The convention will be maintained that the first price is that of the advertising firm and the second price is the price from an external firm. This paper will examine a fully separating mixed pricing PBNE. The equilibrium price domain of type $Q \in \{L, H\}$ firms with strategy $s \in \{A, N\}$ for advertising and

⁸There is an assumption here prohibiting a high/low firm from showing an offer from a low/high firm. This can be justified by the presence of many goods in the real world (while in the model there is one high and one low quality good). Thus a consumer shown an unrelated good cannot use this information to infer information about the good at hand and there is no incentive for firms to show these unrelated goods to the consumer.

⁹This assumption of undirected price comparison seems to be a good representation of progressive direct; three; skyscanner and book depository from the introduction.

non-advertising will be denoted by the set $D_{Q,s}$.

Definition 2 PBNE (with advertising). *A fully separating PBNE in this model with advertising will be described by pricing strategies and equilibrium profits denoted $\hat{\pi}_L, \hat{\pi}_H$, such that:*

$$\pi_{H,N}(P) = \hat{\pi}_H \geq \pi_{H,N}(P') \quad \forall \quad P \in D_{H,N}, P' \in \mathfrak{R}^+ \setminus D_{H,N} \quad (11)$$

$$\pi_{H,A}(P) = \hat{\pi}_H \geq \pi_{H,A}(P') \quad \forall \quad P \in D_{H,A}, P' \in \mathfrak{R}^+ \setminus D_{H,A} \quad (12)$$

$$\pi_{L,N}(P) = \hat{\pi}_L \geq \pi_{L,N}(P') \quad \forall \quad P \in D_{L,N}, P' \in \mathfrak{R}^+ \setminus D_{L,N} \quad (13)$$

$$\pi_{L,A}(P) = \hat{\pi}_L \geq \pi_{L,A}(P') \quad \forall \quad P \in D_{L,A}, P' \in \mathfrak{R}^+ \setminus D_{L,A} \quad (14)$$

The belief functions $\mu(P), \mu(P, P_E)$ are in accordance with the intuitive criterion, Bayes rule and player strategies for all information sets reached with positive probability in equilibrium.

The logic behind simple price signalling for a firm that does not advertise still applies and hence $\mu(P)$ will be as set in equation 6. Beliefs when advertising is undertaken shall be in accordance with Bayes rule at all points within the domain of prices charged in equilibrium. Thus:

$$\mu(P_A, P_E) = \frac{\lambda \text{Prob.}(P_A, P_E | \text{Good is H})}{\lambda \text{Prob.}(P_A, P_E | \text{Good is H}) + (1 - \lambda) \text{Prob.}(P_A, P_E | \text{Good is L})} \quad (15)$$

As we are restricting attention to fully separating equilibria we restriction attention to the case where there is no price point $P^S > P > P_L$ that is charged by a positive mass of high firms and low firms.

Lemma 1. *In any fully separating equilibrium, no low firms offer a price $P > P_L$.*

Proof. We can first show there will be no advertisers pricing above P_L . Consider a putative equilibrium where there were advertisers pricing above P_L with price dispersion. The highest pricing advertiser would make no sales and be better off monopolising at P_L .

Consider a putative equilibria where there are low quality advertisers at a masspoint above P_L . As there are no high firms at this price (by the restriction to fully separating

equilibria), these firms will be seen as being of low quality. Hence beliefs cannot worsen from undercutting and there is a profitable deviation for a firm to undercut this masspoint. For low firms that do not advertise, as pricing at P_L dominates pricing higher given the beliefs in equation 6 there will be no low firms pricing above P_L either. \square

This lemma makes it possible for a high firm to signal high quality by advertising whilst pricing at less than P^S but more than P_L . This leads to the first proposition which states that in any equilibrium there will exist advertising firms.

Proposition 2. *In any equilibrium there will be a positive mass of high firm advertisers.*

Proof. In the event of all mass of high firms being non-advertisers at P^S , consider a high firm's option of deviating to price at P_H^M and advertising against another firm selling the same good. If the customer accepted this firm was high quality the deviating firm would be better off. By contrast a low firm attempting to emulate high quality by doing the same would be worse off even if they were believed to be high quality as they would lose the sale to another low firm (that would be pricing at P_L). Thus this is a profitable deviation as the high firm could use the intuitive criterion to price closer to their monopoly price whilst convincing consumers of their high quality. As this profitable deviation remains whilst there is a zero measure of advertisers we get the proposition. \square

One implication of this proposition is that the equilibrium described in Proposition 1 is no longer an equilibrium where advertising is allowed. Whilst beliefs of $\mu(P, P_E) = 0 \forall P, P_E$ could support this equilibrium such beliefs would not be supported by the intuitive criterion.

Lemma 2. *At all prices $P > P_L$, the equilibrium price distributions of high quality advertisers is atomless.*

Proof. If there were an atom at a price exceeding marginal cost then one of those firms could undercut the others. With a similar intuitive criterion argument as presented in the proof of proposition 2, the undercutting firm could convince consumers of their high quality and hence could get a discontinuous jump in expected profits. If there were an

atom at a price equal to marginal cost (Bertrand 1883) then profits are zero and the firms at this price would be better off monopolising to earn π_H^S . \square

This lemma is similar in spirit to Varian (1980, Proposition 3) or Stahl (1989, Lemma 1) and reflects the fact that if there were a mass of firms offering a certain price then one of those firms could get a discontinuous jump in expected profits by undercutting the others. Note that non-advertising firms are monopolists and hence there is no similar restriction on mass points in the pricing distribution of these firms. This lemma is truncated to prices above P_L to reflect the possibility of adverse beliefs for advertisers pricing at or below P_L which would prevent undercutting.

We have now established that no low firms will price above P_L . The next two lemmas show that all low firms will in fact not be advertising and will be charging a price of P_L .

Lemma 3. *If the equilibrium price distribution of high firm advertisers is atomless at prices $P \leq P_L$ then in equilibrium there will be a mass of non-advertising low firms offering a price of P_L .*

Proof. For low firms that do not advertise, Pricing at P_L dominates any other price.

If all firms were advertising with price dispersion the firm with the highest price would get no profit and hence would be better off not advertising whilst setting a price of P_L . If all low firms were advertising at a certain price less than P_L then from equation 15 (and the lack of high firm masspoints below P_L) they would be considered low quality and would be better off not advertising at a price of P_L . \square

Lemma 4. *If the equilibrium price distribution of high firm advertisers is atomless at prices $P \leq P_L$ then in equilibrium all low firm will be non-advertisers at a price of P_L .*

Proof. Lemma 1 shows that no low firm will price at $P > P_L$ and not-advertising at P_L dominates not-advertising at any other price. Thus it suffices to show no low firms will advertise at any price $P \leq P_L$. Considering the possibility of advertising at a price $P \leq P_L$ the possible price distributions are a continuous distribution of prices, a masspoint of low firm advertisers at certain discrete prices or a distribution with both of these features. We shall show that these distributions are not sustainable in equilibrium.

We can first show that no low firm advertiser masspoints can survive in equilibrium at prices less than P_L . If there were an advertiser masspoint at a price $P < P_L$ then from equation 15 (and the lack of high firm masspoints below P_L) they would be considered low quality and would earn strictly higher profits undercutting other firms in this masspoint. Thus no masspoints of low firm advertisers can survive in equilibrium.

Now consider a putative equilibria where there is no atom of high firm advertisers at P_L but a mass of low firm non-advertisers at P_L (as per lemma 3) and a distribution of low firm advertisers on the domain $[\bar{P}, \underline{P}]$ with $P_L \geq \bar{P} > \underline{P}$. The total mass of low firm advertisers is some number $0 < \nabla < 1$. Consider the advertiser pricing at \bar{P} . It matches with a firm pricing at P_L with probability $(1 - \nabla)$ and matches with another (lower priced) advertiser with probability ∇ . When $P_A = \bar{P}$, $P_E = P_L$ are subbed into the beliefs equation 15 the numerator of this expression $P(\bar{P}, P_L | \text{Good is H})$ will be zero as there is no mass of high firms charging P_L . The denominator is nonzero as there is a mass of low firms offering this price. Thus the advertiser at \bar{P} is recognised as being low quality and earns:

$$\pi_{L,A}(\bar{P}) = (1 - \nabla)\bar{P}\Gamma(L - \bar{P}) < \pi_{L,M}(P_L) \quad (16)$$

Thus the advertiser would be strictly better off being a non-advertiser at P_L .

In addition note that due to the assumed tiebreaking rule an advertiser at P_L is worse off than a non-advertiser at the same price. Thus if the equilibrium price distribution of high firm advertisers is atomless at a price $P \leq P_L$ there can be no low firm advertisers in equilibrium. \square

We have now established that if the equilibrium price distribution of high firm advertisers is atomless at a price $P \leq P_L$ then all low firms will not advertise while setting a price of P_L . We can now shift attention to examining the behaviour of high firms. Assuming an atomless high firm pricing distribution, As no low firm will ever price at a price other than P_L high firms will be able to use the intuitive criterion to price at any level that represents a profitable deviation for them and still be recognised as high quality. We

can thus write the profit function for high firm advertisers when the price distribution of high firm advertisers is atomless:

$$\pi_{H, \text{Advertiser}}(P) = (P - c_H)\Gamma(H - P) [(1 - \eta) + \eta G(P) + \eta G(P)] \quad (17)$$

Where $G(P)$ is the survival function of the prices charged by high quality advertisers and η is the proportion of high quality advertisers. The terms in the square brackets account for the probabilities of matching with a non-advertiser, matching with an advertiser and an external advertiser matching with the firm respectively.

Now denoting $g(P)$ to be the pdf of the advertiser price distribution (which is defined when the advertiser pricing distribution is atomless) we can show that the pricing distribution of high firms will be gapless:

Lemma 5. *If the equilibrium price distribution of high firm advertisers is atomless then there are no equilibria exhibiting gaps of positive measure in the price distribution of advertising high firms. That is for any $P, P+\epsilon$ with $\epsilon > 0$ and with $g(P) > 0, g(P+\epsilon) > 0$ then $\int_{\epsilon}^{P+\epsilon} g(p)dp > 0$.*

Proof. Note from Lemma 4 that atomlessness in equilibrium implies low firms never advertise and an advertiser will always be recognised as high quality. Consider the case if such a gap did form between prices P and $P + \epsilon$ with $\epsilon > 0$. Consider in particular the firm pricing at P . This firm could increase its price to $P + \epsilon$ which would increase $(P - c_H)\Gamma(H - P)$ whilst not changing $[(1 - \eta) + \eta G(P) + \eta G(P)]$. Thus there is a profitable deviation. \square

We can now find that there will be high firm non-advertisers and in equilibrium all high firms will earn the same profits they would make price signalling at P^S . Intuitively this occurs because the possibility of not advertising while charging P^S puts a lower bound on how much Bertrand competition among advertisers can reduce their profits.

Lemma 6. *If the equilibrium price distribution of high firm advertisers is atomless then there is a positive mass of high firms selling at the signalling price P^S and not providing additional offers to consumers.*

Proof. If all mass of high firms were advertising in an atomless distribution then the top pricing firm would make no profits and be strictly better off offering a price of P^S without advertising. \square

Corollary 7. *If the equilibrium price distribution of high firm advertisers is atomless then in equilibrium all high firms earn profits of π_H^S .*

Proof. Immediate from lemma 6, proposition 2 and definition 2. \square

We can now use the profit function from equation 17 and the equilibrium profit from Corollary 7 to find the domain of high firm advertiser pricing and the proportion of advertisers.

Lemma 8. *If the equilibrium price distribution of high firm advertisers is atomless, The bottom pricing advertiser will charge P_B where P_B is the solution to:*

$$(P_B - c_H)\Gamma(H - P_B) = \frac{\pi_H^S}{1 + \eta} \quad (18)$$

Proof. Substituting equilibrium profit and that for the bottom pricing firm $G(P_B) = 1$ into equation 17 yields this expression. \square

Lemma 9. *If the equilibrium price distribution of high firm advertisers is atomless then the top pricing advertiser will charge P_H^M .*

Proof. This comes from equation 17. In the event the top pricing firm had a price less than P_H^M they could raise their price and increase $(P - c_H)\Gamma(H - P)$ whilst the fraction lost to other firms $[(1 - \eta) + 2\eta G(P)]$ stayed the same. \square

Lemma 10. *If the equilibrium price distribution of high firms is atomless, the proportion of advertising firms in equilibrium is:*

$$\eta = 1 - \frac{\pi_H^S}{\pi_H^M} \quad (19)$$

Proof. To see this consider equation 17 for the advertiser offering a price of P_H^M . As $G(P_H^M) = 0$, their profit is $\pi_H^M(1 - \eta)$ which from lemma 7 must be equal to π_H^S in

expectation for a firm to price at this level. The lemma follows immediately from this equality. \square

Now now split the analysis into two different cases. The first is where $P_B > P_L$ and hence the atomlessness of the high firm advertiser pricing distribution is assured from lemma 2. The second is the complementary case where atomlessness is not assured.

3.2.1 Case A: $P_B > P_L$

From Lemma 8 all high firm advertisers will price at least at $P_B > P_L$. Thus from lemma 2 the advertiser price distribution will be atomless and all of the preceding results are applicable.

Proposition 3. *There will exist a PBNE for this game. All low firms will price at P_L and earn $\pi_L = (P_L - c_L)\Gamma(L - P_L)$ whilst all high firms will earn π_H^S as defined by equation 8. A proportion η as defined by equation 19 of high firms will advertise with a pdf of prices as given by:*

$$g(P) = \frac{\pi_H^S}{2\eta} \left[\frac{\Gamma(H - P) - (P - c_H)\gamma(H - P)}{(P - c_H)^2\Gamma(H - P)^2} \right] \quad P_B \leq P \leq P_H^M \quad (20)$$

A proportion $1 - \eta$ of high firms will not advertise and will set a price of P^S .

Beliefs in this PBNE will satisfy:

$$\mu(P) = \begin{cases} 1 & P \geq P^S \\ 0 & P < P^S \end{cases}$$

$$\mu(P, P_E) = \begin{cases} 1 & \text{for } P > P_L \quad \text{and} \quad \forall P_E \\ 0 & \text{for } P \leq P_L \quad \text{and} \quad \forall P_E \end{cases}$$

Proof. This pricing distribution can be obtained by noting that in equilibrium the price

distribution $G(P)$ must satisfy:

$$\begin{aligned}
(P - c_H)\Gamma(H - P) [1 - \eta + 2\eta G(P)] &= \pi_H^S \\
[1 - \eta + 2\eta G(P)] &= \frac{\pi_H^S}{(P - c_H)\Gamma(H - P)} \\
G(P) &= \frac{1}{2\eta} \left[\frac{\pi_H^S}{(P - c_H)\Gamma(H - P)} - 1 + \eta \right] \quad P_B \leq P \leq P_H^M
\end{aligned} \tag{21}$$

The associated pdf can be found by differentiating the survival function and multiplying by negative one. Equation 21 is a valid survival function being decreasing in price with endpoints of $G(P_H^M) = 0$ and $G(P_B) = 1$, thus this price distribution is feasible.

These beliefs will be satisfied in this equilibria as all high firms price at more than P_L and all low firms price at P_L . There is no profitable deviations for high firms who earn π_H^S at any point in the pricing domain and cannot earn higher profits outside this domain. Likewise low firms cannot profitably deviate. \square

From its construction we can note that the equilibrium pricing distribution in proposition 3 is unique in the class of fully separating equilibria.¹⁰ This equilibrium can be seen in figure 2 where the advertising support line shows the prices and corresponding sale quantities of advertising firms. At prices close to P_M^H the advertising support line sits beneath the high firm demand curve, $\Gamma(H - P)$, as they lose a share of their customers to the firms they advertise against. At lower prices it sits above the non-advertiser demand curve as they retain most of their customers and also gain customers from competing advertising firms. While the advertisers at P_H^M retain a proportion of $1 - \eta$ of the firms they encounter, those pricing closer to c_H retain or win a greater total proportion of $1 + \eta$ firms. This demonstrates the demand shifting taking place with the lowest pricing advertisers selling $\frac{(1+\eta)\Gamma(H-P_B)}{(1-\eta)\Gamma(H-P_H^M)}$ times more goods than the highest pricing advertiser. All advertisers earn the same equilibrium profits however with expected profit the same as for the price signalling non-advertiser firms.

¹⁰Although this equilibrium pricing distribution could be sustained by different out of equilibrium beliefs.

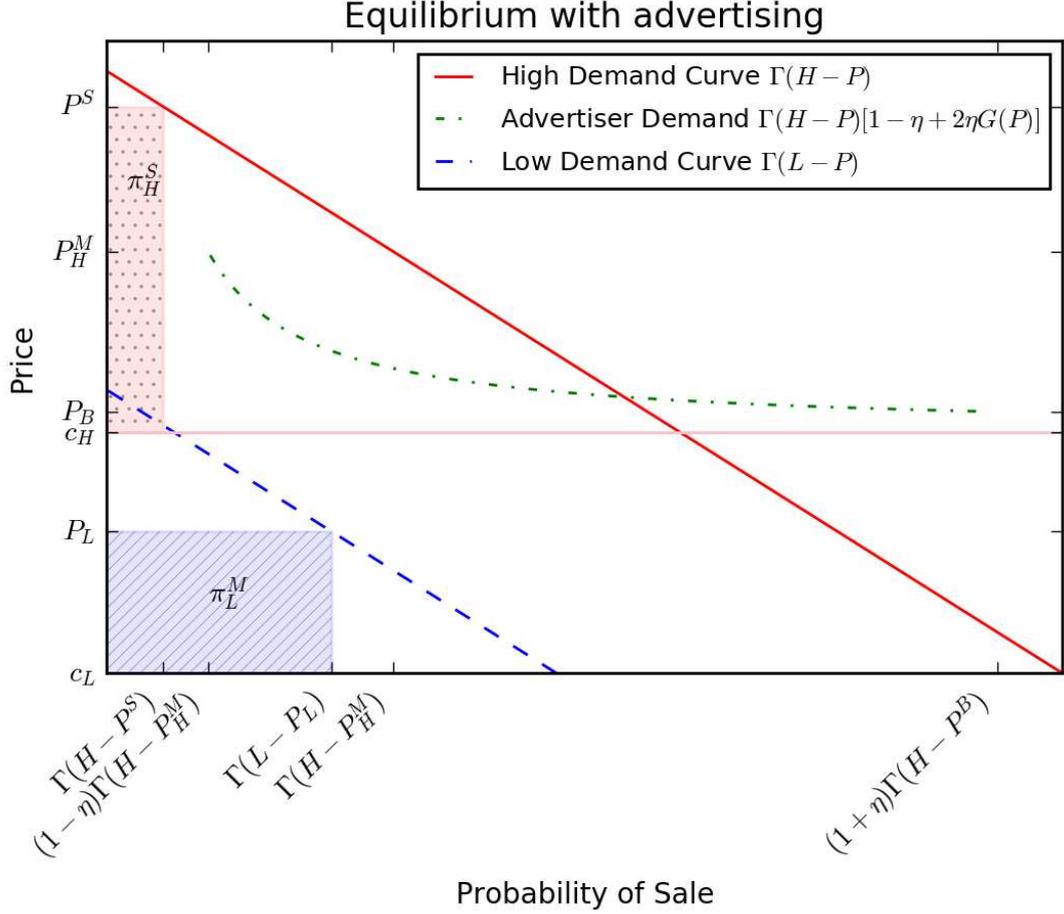


Figure 2: Equilibrium with advertising

3.2.2 Case B: $P_B \leq P_L$

Considering the case when $P_B < P_L$ is more problematic as lemma 2 cannot be used to obtain atomlessness of the pricing distribution. The equilibrium explored in this section will be one where the price distribution of high firms is guessed to be atomless at all prices (this guess will be verified later on). As a result of this however no claim can be made as to the uniqueness of the resulting equilibrium among the class of fully separating equilibria. Note that this assumption of atomlessness at any price is sufficient such that all of the previous supporting lemmas hold. In particular this conjecture results in an equilibrium closely related to the equilibrium expressed in proposition 3. This equilibrium will have the same functions to describe firm pricing decisions however P_L is cut out of this distribution

Proposition 4. *There will exist a PBNE for this game. All low firms will price at P_L*

and earn $\pi_L = (P_L - c_L)\Gamma(L - P_L)$ whilst all high firms will earn π_H^S as defined by equation 8. A proportion η as defined by equation 8 of high firms will advertise with a pdf of prices given by:

$$g(P) = \begin{cases} \frac{\pi_H^S}{2\eta} \left[\frac{\Gamma(H-P) - (P-c_H)\gamma(H-P)}{(P-c_H)^2\Gamma(H-P)^2} \right] & \text{for } P_B \leq P \leq P_H^M, P \neq P_L \\ 0 & \text{for } P = P_L \end{cases} \quad (22)$$

A proportion $1 - \eta$ of high firms will not advertise and will set a price of P^S .

Beliefs in this PBNE will satisfy:

$$\mu(P) = \begin{cases} 1 & P \geq P^S \\ 0 & P < P^S \end{cases} \quad (23)$$

$$\mu(P, P_E) = \begin{cases} 1 & \text{for } P > P_L \quad \text{and} \quad \forall P_E \\ 1 & \text{for } P \leq P_L \quad \text{and} \quad P_E \neq P_L \\ 0 & \text{for } P \leq P_L \quad \text{and} \quad P_E = P_L \end{cases} \quad (24)$$

Proof. The proof of this proposition largely follows that of proposition 3. The key difference is the omission of P_L (a zero measure set) from the advertiser pricing function. The pricing distribution is atomless (which verifies the atomless guess) and all of the previous lemmas hold. Thus low firms will all not advertise while setting a price of P_L as per lemma 4 and beliefs for advertisers are the same as in proposition 3 for any set price. \square

While this is an equilibrium with beliefs that are robust to the intuitive criterion it relies on discontinuous beliefs at a certain point. This may be less credible as a model for some markets than the previous case where $P_B > P_L$. For instance this equilibrium may not be robust if the marginal cost of low firms is continuously heterogeneous in some interval as there would then be a continuum of P_L prices.

4 Producer and Consumer Surplus

The total surplus generated in a separating equilibria when a consumer visits a firm is $Q - c_Q$ if that consumers buys the good and is that consumer's outside option otherwise. The problem for the consumer on the other hand is to choose from the maximum of $Q - P_f$ and Ω_k . Clearly this implies that the closer is price to marginal cost the greater surplus generated in the market.

In order to evaluate the impact of advertising we can note that the low firms do not change their price from the full information case or the no advertising case (presented in proposition 1) and so welfare is unchanged for consumers visiting low firms. When consumers visiting high firms are considered, equilibrium advertiser prices are less than both the full information monopoly price or the signalling price. Specifically we can write the following expressions for the surplus generated from consumers visiting high firms in the full information (FI) equilibrium, pure price signalling (PS) equilibrium as well as the surplus from a single advertiser (SA) and surplus from the advertising equilibrium (AE) respectively:

$$S_{\text{FI}} = (H - c_H)\Gamma(H - P_H^M) + \int_{H-P_H^M}^{\bar{\Omega}} \Omega\gamma(\Omega)d\Omega \quad (25)$$

$$S_{\text{PS}} = (H - c_H)\Gamma(H - P^S) + \int_{H-P^S}^{\bar{\Omega}} \Omega\gamma(\Omega)d\Omega \quad (26)$$

$$S_{\text{SA}} = (1 - \eta) \int_{P_B}^{P_H^M} \left[(H - c_H)\Gamma(H - P) + \int_{H-P}^{\bar{\Omega}} \Omega\gamma(\Omega)d\Omega \right] g(P)dP \quad (27)$$

$$+ \eta \int_{P_B}^{P_H^M} \left[(H - c_H)\Gamma(H - P) + \int_{H-P}^{\bar{\Omega}} \Omega\gamma(\Omega)d\Omega \right] \bar{g}(P)dP$$

$$S_{\text{AE}} = (1 - \eta)S_{\text{PS}} + \eta S_{\text{SA}} \quad (28)$$

Where the distribution $\bar{g}(P)$ is the pdf of the first order statistic from two draws from the advertiser pricing distribution. This arises when a consumer has prices from two advertisers and will pick the lower price.

Here it can be seen that the surplus generated by a single advertiser is higher than the surplus generated by a price signalling firm or a firm in the full information equilibrium

as the price an advertiser offers is always lower. This implies that when the fraction of advertisers is sufficiently high, it is possible for the advertising equilibrium to deliver greater surplus than the corresponding full information equilibrium. As the above expressions are analytically intractable this is shown numerically but first stating these implications in a proposition.

Proposition 5. *In any asymmetric information market, surplus is always weakly greater in the fully separating equilibrium exhibiting advertising relative to the fully separating equilibrium where no advertising is allowed. In some cases surplus can be higher in asymmetric information markets with advertising than in the corresponding full information markets.*

Proof. Proof for the first statement is provided by the fact consumers always choose the maximum of Ω_k and $H - P_f$, whilst surplus is maximised by them taking the maximum of Ω_k and $H - c_H$. The price distribution in the price signalling case weakly stochastically dominates the price distribution where advertising occurs and hence delivers weakly lesser surplus (strictly if $\pi_H^S < \pi_H^M$). The proof of the second statement is by example 1. \square

Example 1. *We define a uniform distribution of outside options in the space $[0, 1]$. Thus we have $\Gamma(x) = x$ in this domain. We assume that $H = 1$, $c_L = 0$ and $L = \frac{8}{17}$.¹¹ We can use these to get the following expressions for monopoly prices and profits in terms of c_H :*

$$\begin{aligned} P_L &= \frac{4}{17} & P_H^M &= \frac{1 + c_H}{2} & P^S &= \frac{16}{17} \\ \pi_L &= \frac{16}{289} & \pi_H^M &= \left(\frac{1 - c_H}{2}\right)^2 & \pi_H^S &= \frac{1}{17}\left(\frac{16}{17} - c_H\right) \end{aligned}$$

From these prices and profits expressions for the proportion of advertisers and the bottom

¹¹The values given H , c_L are chosen so that the demand curve is downward sloping across all feasible prices. The value given L is chosen as it is approximately halfway up the interval and exploits the pythagorean triple $(8, 15, 17)$ to get a rational signalling price. In general a higher L value leads to full information delivering higher surplus for all c_H whilst lower L values lead to the advertising equilibrium being more efficient for all c_H . Figures 1 and 2 were constructed with these parameters and $c_H = 0.4$.

price can be obtained:

$$\eta = 1 - \frac{\pi_H^S}{\pi_H^M} \quad P_B = \frac{1 + c_H}{2} - \frac{\sqrt{(1 - c_H)^2 - 4\frac{\pi_H^S}{1+\eta}}}{2}$$

Finally we can write the survival function and pdf of the advertiser pricing distribution as well as an expression for $\bar{g}(P)$.¹²

$$G(P) = \frac{\pi_H^S}{2\eta(P - c_H)(1 - P)} + \frac{1}{2} - \frac{1}{2\eta} \quad g(P) = \frac{\pi_H^S(1 + c_H - 2P)}{2\eta(P - c_H)^2(1 - P)^2}$$

$$\bar{g}(P) = 2G(P)g(P)$$

Now examining the bounds of feasible c_H values we focus on the case of a separating equilibria with costly signalling. Hence the maximum c_H value we consider is $\frac{15}{17}$ as at this cost level the signalling price is equal to the monopoly price for the high firm. The lower limit c_H value is zero as by construction it is at a marginal cost of c_L when a firm is indifferent to selling with an expected value of L at the low monopoly price or H at the signalling price.

It can be seen in figure 3 that when c_H is low it implies that π_H^M is high compared to π^S and thus there are many advertisers in the market. Intense competition between these firms depresses prices closer to marginal cost. This results in surplus in the advertising equilibrium being higher than the full information equilibrium.

On the other hand when c_H is high π_H^M is not much higher than π^S and thus there are fewer advertisers in the market with less intense competition between them. In this case surplus is higher in the full information equilibrium than the advertising equilibrium.

¹²For discussion on finding the pdf of an order statistic see for instance Blitzstein & Hwang (2015, Theorem 8.6.4)

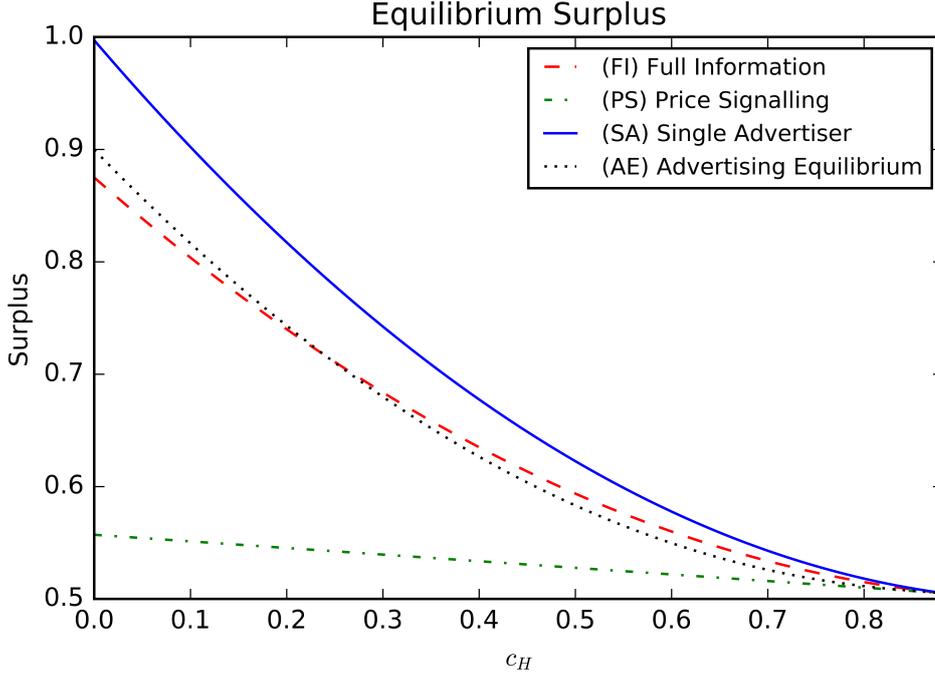


Figure 3: Surplus generated in each equilibrium case

5 Extensions

5.1 Cost heterogeneity in high firms

As noted in Shelegia (2012) even small differences in marginal cost can lead to firms randomising over different ranges of prices in a mixed strategy pricing equilibria such as those presented in this paper. This observation may have welfare implications in this paper as advertising results in a shift of quantity from higher pricing advertisers to lower price advertisers. If low cost firms are also low price firms this means that advertising can boost aggregate surplus by awarding larger quantity to lower cost firms.

This section examines this possibility by augmenting the model of section 3 with high firms with heterogeneous costs. Rather than having a homogeneous group of high firms we assume half are α firms with a marginal cost of c_α and half are β firms with a higher marginal cost of $c_\beta > c_\alpha$. Both types of firms sell high quality goods and a firm cannot selectively advertise against one of the two classes of firms. In all other aspects the model is unchanged. As the analysis for the most part follows that performed in section 3 it is deferred for appendix A but two key implications are here stated.

Proposition 6. *In any fully separating equilibrium exhibiting advertising:*

(a) *All α firms will price equal or less than all β firms.*

(b) *β firms will never earn more than their signalling profits however α firms may earn more.*

Proof. See appendix A. □

The intuition for the point (a) is that α firms have a lower monopoly price which means price signalling is relatively more expensive for them as compared to β firms. This leads them to be more likely to advertise and more likely to offer lower prices while advertising. For a simple example of point (b) consider a case where c_α is low enough such that the monopoly price of α firms, P_α^M , is less than the marginal cost of β firms, c_β . In this case an α firm could price at P_α^M and advertise losing at most half of their consumers to other α firms and still gain some customers from advertising against β firms. In some cases this fraction of the monopoly profit will exceed the signalling profit.

Thus in the presence of cost heterogeneity, advertising can add to market efficiency as it shifts demand away from higher cost firms towards lower cost firms. Firms with lower costs will position themselves as lower pricing firms within the advertising equilibrium. Thus there is an efficiency gain from demand being shifted towards firms with a lower marginal cost.

5.2 Monopolist supplier for high quality good

Now we consider the special case where the common good sold in the market is provided by a single supplier firm that behaves as a monopolist and produces the product costlessly. This will often be the case where the product is copyrighted or patented. We will refer to these supplying firms as suppliers and the firms that buy from the suppliers as merchants. We assume full information exists between suppliers and merchants but consumers do not know the quality level of a good unless it is signalled to them.

In analysing this case we will first write an expression for the c_H level which equalises the monopoly price of high firms (which does depend on c_H) with the signalling price

(which does not depend on c_H). This cost is denoted c_{Sig} and is defined as the cost level which solves the following equation.

$$P^S = \arg \max_{P \in \mathbb{R}^+} (P - c_{\text{Sig}}) \Gamma(H - P) \quad (29)$$

where P^S is as defined in section 3.1. We can now write the profit function for the supplier as a function of the price they charge merchants. In this expression we will write the advertiser price pdf as $g_c(P)$, the bottom price as $P_B(c)$ and the monopoly price as $P_H^M(c)$ reflecting the fact that these are affected by c :

$$\pi_{\text{Supplier}}(c) = \begin{cases} c\Gamma(H - P_H^M(c)) & c > c_{\text{Sig}} \\ c\Gamma(H - P^S) & c \leq c_{\text{Sig}}, \quad \text{without advertising} \\ c \left[(1 - \eta)\Gamma(H - P^S) + \eta \int_{P_B(c)}^{P_H^M(c)} \Gamma(H - P) g_c(P) dP \right] & c \leq c_{\text{Sig}}, \quad \text{with advertising} \end{cases} \quad (30)$$

The benchmark price signalling model without advertising is first considered. We get the result that suppliers will always price at least c_{Sig} such that $P_H^M \geq P^S$.

Proposition 7. *In markets where advertising is not allowed the price charged by suppliers to merchants will never be less than c_{Sig}*

Proof. Given that merchants will need to charge at least the signalling price to ensure low quality firms will not emulate high quality, all firms will price at P^S for all levels of c below a critical level to be denoted c_{Sig} . As the price charged by merchants is the minimum of the signalling and their monopoly price, this critical level c_{Sig} is such that these prices are equal as defined in equation 29.

Therefore in the absence of advertising a supplier will never charge less than c_{Sig} as they would sell the same quantity of $\Gamma(H - P^S)$ whilst earning a lower price. \square

The intuition here is that the supplier profit strictly increases as c increases until c reaches c_{Sig} . This is because merchants charge P^S at all of these cost levels and hence as the supplier increases c the price the supplier receives increases whilst the quantity stays

constant.

If advertising is allowed when $c \geq c_{\text{Sig}}$ then $P^S \leq P_H^M$ and hence no advertising will be undertaken.¹³ On the other hand if advertising is allowed when $c < c_{\text{Sig}}$ then $P^S > P_H^M$ and hence advertising will be undertaken. This observation raises the following possibility:

Proposition 8. *In some markets where advertising is allowed it may be optimal for suppliers to reduce their price below c_{Sig} and thus induce advertising to increase their sale quantity.*

Proof. Proof is by example 2. □

Example 2. *This example follows on from example 1 except for c_H now being endogenously determined by a monopolist supplier who produces the good costlessly. All other parameters and the outside option distribution are identical. We can note that in this case setting $P_H^M = P^S$ obtains $c_{\text{Sig}} = \frac{15}{17}$:*

The demand curve faced by the monopolist supplier in the advertising and no advertising case along with areas representing optimal profits are shown in figure 4.¹⁴ It can be seen that where no advertising is allowed the monopolist supplier prices at c_{Sig} . When advertising is allowed however a lower c of approximately 0.41 is set as the greater sales volume obtainable is sufficient to make up for the loss in margin.

6 Conclusion

This paper has examined comparative advertising from the perspective of disclosing differences in prices. While at first glance this strategy seems counter-intuitive as it would result in greater pricing competition it is found that it can act as an alternative method of signalling quality than price signalling.

In the fully separating equilibria that this paper presents some firms will remain price signalling whilst other firms will lower their price closer to the monopoly level and instead

¹³From equation 29 we have $\pi_H^S = \pi_H^M$ when $c = c_{\text{Sig}}$. As c_{Sig} increases above this level the signalling profit drops below the monopoly profit. Hence there is no incentive to drop price and advertise.

¹⁴Note that equations for these demand curves are as described in equation 30 (once the c term giving the margin per item is removed)

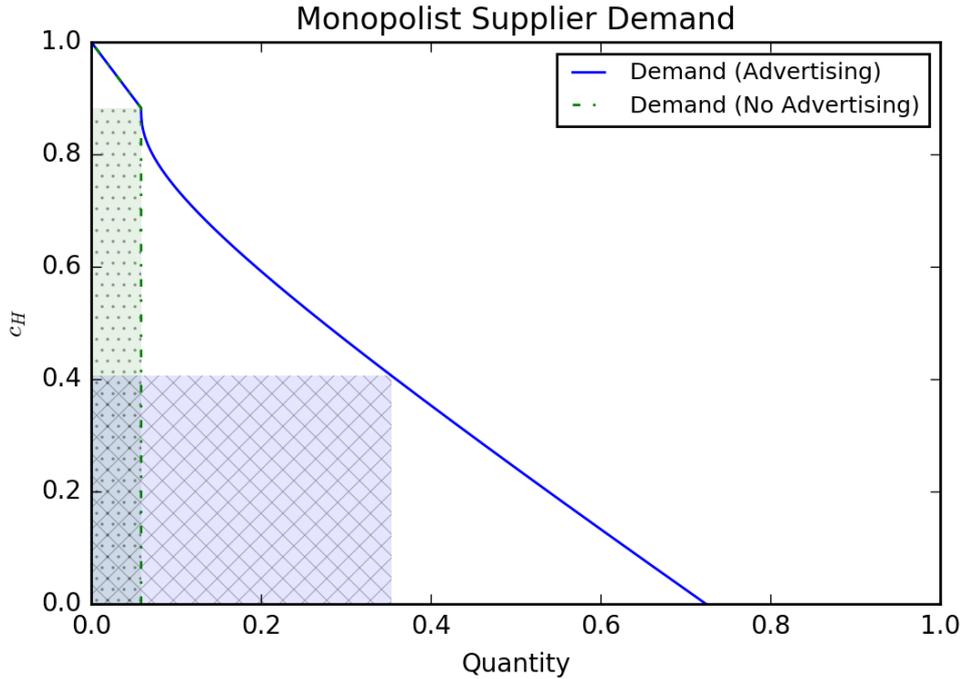


Figure 4: Monopolist supplier demand curves and profits

signal by price comparative advertising against rival firms. While an advertiser may be able to achieve greater sales quantity by reducing their price, they also face competition from the firm they advertise against. Advertising has the effect of decreasing the equilibrium distribution of prices offered in the asymmetric information setting which increases consumer surplus. Firm profits on the other hand do not change from the case where no advertising is allowed. While firms do manage to price closer to their monopoly price any additional profits are lost through increased competition with other advertising firms.

A number of extensions were examined including the possibility of high quality firms having heterogeneous marginal costs. In this case advertising can play a role in shifting demand from higher marginal cost firms to lower cost firms. The case of a monopoly supplier who provides goods to many reselling firms was also examined. It is found that advertising can result in suppliers reducing the price they charge reselling firms. As a large fraction of reselling firms no longer need to price signal (at a high price with low sale quantity) the supplier can reduce their price to incentivise advertising and boost sales volume. If the increase in quantity makes up for the decrease in margin then this can be more profitable.

These implications for welfare are substantially more clearcut and supportive of comparative advertising than previous papers that model it as the disclosure of differences in product features. This may indicate that the form of comparative advertising matters from a policy perspective. Anderson & Renault (2009) find that comparative advertising of horizontal good attributes can deteriorate total welfare when there is a sufficiently large quality gap between rival firms. By contrast this paper implies that price comparative advertising will increase total welfare and in addition no agent's surplus is decreased by the legalisation of comparative advertising. In the basic model this increase in welfare comes entirely from lower prices to consumers and more efficient signalling for firms. When the extensions are considered however there are additional vehicles for surplus to increase including the shifting of quantity to lower cost producers of the high quality good and the inducing of a monopolist supplier into decreasing the price they charge reselling firms. All of these insights present the clear and unambiguous implication that price comparative advertising is beneficial for welfare and should be supported by legislators and regulators.

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Appendices

A Heterogeneity Main Results

Following on from the extension to the model introduced in section 5.1, equilibrium in this context can be defined as:

Definition 3. *A fully separating PBNE in this model will be described by pricing strategies and equilibrium profits $\hat{\pi}_L$, $\hat{\pi}_\alpha$, $\hat{\pi}_\beta$, such that:*

$$\pi_{L,N}(P) = \hat{\pi}_L \geq \pi_{L,N}(P') \quad \forall \quad P \in D_{L,N}, P' \in \mathfrak{R}^+ \setminus D_{L,N} \quad (\text{A.1})$$

$$\pi_{L,A}(P) = \hat{\pi}_L \geq \pi_{L,A}(P') \quad \forall \quad P \in D_{L,A}, P' \in \mathfrak{R}^+ \setminus D_{L,A} \quad (\text{A.2})$$

$$\pi_{\alpha,N}(P) = \hat{\pi}_\alpha \geq \pi_{\alpha,N}(P') \quad \forall \quad P \in D_{\alpha,N}, P' \in \mathfrak{R}^+ \setminus D_{\alpha,N} \quad (\text{A.3})$$

$$\pi_{\alpha,A}(P) = \hat{\pi}_\alpha \geq \pi_{\alpha,A}(P') \quad \forall \quad P \in D_{\alpha,A}, P' \in \mathfrak{R}^+ \setminus D_{\alpha,A} \quad (\text{A.4})$$

$$\pi_{\beta,N}(P) = \hat{\pi}_\beta \geq \pi_{\beta,N}(P') \quad \forall \quad P \in D_{\beta,N}, P' \in \mathfrak{R}^+ \setminus D_{\beta,N} \quad (\text{A.5})$$

$$\pi_{\beta,A}(P) = \hat{\pi}_\beta \geq \pi_{\beta,A}(P') \quad \forall \quad P \in D_{\beta,A}, P' \in \mathfrak{R}^+ \setminus D_{\beta,A} \quad (\text{A.6})$$

The belief functions $\mu(P)$ and $\mu(P, P_E)$ are derived in accordance with Bayes rule and player strategies for all information sets reached with positive probability in equilibrium.

It can be seen that all of the results from lemma 1 to lemma 6 carry over without modification to this case. Thus all low firms will not advertise while pricing at P_L and all advertisers will be believed as being of high quality. It can be noted that the signalling price, P^S of both firms will be identical and as described in equation 5. Profits at the signalling price will be denoted by π_α^S and π_β^S . The profit of a type $d \in \{\alpha, \beta\}$ advertiser can be written as:

$$\pi_{d,A}(P) = (P - c_d)\Gamma(H - P)[1 - \eta + 2\eta G(P)] \quad (\text{A.7})$$

Where η is the proportion of all firms that are advertising and $G(P)$ is the advertiser price distribution. For brevity in some proofs this will be written as:

$$\pi_{d,A}(P) = (P - c_d)Q(P) \quad (\text{A.8})$$

with $Q(P) = \Gamma(H - P) [1 - \eta + 2\eta G(P)]$.

The monopoly prices of α and β firms are denoted P_α^M , P_β^M respectively with the corresponding profits denoted π_α^M and π_β^M . To ensure that from lemma 2 the advertiser price distribution is atomless we assume that the bottom pricing advertiser will price above P_L . This condition will be formalised later on.

Lemma 11. *No equilibrium exhibits an α firm not advertising while a β firm does advertises.*

Proof. To see this consider the case where a β firm advertises at some price $P \in D_{\beta,A}$ while an α firm is not advertising. For the β firm we must have for some advertising price P :

$$\begin{aligned} (P - c_\beta)Q(P) &\geq (P^S - c_\beta)\Gamma(H - P^S) \\ (P - c_\alpha)Q(P) + (c_\alpha - c_\beta)Q(P) &\geq (P^S - c_\alpha)\Gamma(H - P^S) + (c_\alpha - c_\beta)\Gamma(H - P^S) \\ (P - c_\alpha)Q(P) - (P^S - c_\alpha)\Gamma(H - P^S) &\geq (c_\alpha - c_\beta) [\Gamma(H - P^S) - Q(P)] \end{aligned}$$

Note that as $c_\alpha < c_\beta$ and $\Gamma(H - P^S) < Q(P)$ the right hand side is positive. This putative case also implies for α firms:

$$\begin{aligned} (P^S - c_\alpha)\Gamma(H - P^S) &\geq (P - c_\alpha)Q(P) \\ (P - c_\alpha)Q(P) - (P^S - c_\alpha)\Gamma(H - P^S) &\leq 0 \end{aligned}$$

A contradiction. Thus no equilibrium exhibits an α firm not advertising while a β firm does advertise. \square

Corollary 12. *In any equilibrium there will be a positive mass of α firms advertising.*

Proof. From proposition 2 there must be a positive mass of advertisers. From lemma 11 these advertisers cannot all be β firms unless all firms are advertisers which would contradict lemma 6. \square

Lemma 13. *In any equilibrium:*

1. A positive mass of β firms will not advertise while setting a price at P^S ;
2. β firms earn π_β^S in equilibrium. Thus $\hat{\pi}_\beta = \pi_\beta^S$.

Proof. From lemma 6 in any equilibrium there exists a positive mass of α non-advertisers and/or β non-advertisers and one or both of the following equalities will hold:

$$\pi_\alpha = \pi_\alpha^S \qquad \pi_\beta = \pi_\beta^S \qquad (\text{A.9})$$

Application of lemma 11 shows that no equilibrium with only α advertisers can exist. Thus in equilibrium there must be a positive mass of β firms not advertising while setting a price of P^S . As a result of this all β firms must earn π_β^S . \square

Proposition 9. *In any equilibrium all α firms will price lower than all β advertisers.*

Proof. First considering the case for advertisers. Consider a putative equilibrium where β firms weakly prefer pricing at P than pricing at P' and α firms weakly prefer pricing at P' than P with $P < P'$. Then for β firms:

$$(P - c_\beta)Q(P) \geq (P' - c_\beta)Q(P') \qquad (\text{A.10})$$

$$(P - c_\alpha)Q(P) + (c_\alpha - c_\beta)Q(P) \geq (P' - c_\alpha)Q(P') + (c_\alpha - c_\beta)Q(P') \qquad (\text{A.11})$$

$$(P - c_\alpha)Q(P) - (P' - c_\alpha)Q(P') \geq (c_\alpha - c_\beta)[Q(P') - Q(P)] \qquad (\text{A.12})$$

Note that as $c_\beta > c_\alpha$ and $Q(P) > Q(P')$ the right hand side is positive. Now consider the case of the α firm:

$$(P' - c_\alpha)Q(P') \geq (P - c_\alpha)Q(P) \qquad (\text{A.13})$$

$$(P - c_\alpha)Q(P) - (P' - c_\alpha)Q(P') \leq 0 \qquad (\text{A.14})$$

A contradiction. Hence there is no equilibrium where α firms advertise at a price higher than β firms. Now considering non-advertisers from lemma 11 there can never be α non-advertisers whilst there are β firms advertising. \square

At this point we introduce the bottom advertising price (analogously to P_B in equation 18).

Lemma 14. *The bottom price will be $P_{B,\alpha}$ which will be defined by:*

$$(P_{B,\alpha} - c_\alpha)\Gamma(H - P_{B,\alpha}) = \frac{\pi_\alpha}{1 + \eta} \quad (\text{A.15})$$

Proof. From proposition 9 the bottom pricing advertiser will be an α . The bottom price is the lowest price that delivers this firm the equilibrium profit level for α firms. \square

The condition for all advertisers to price more than P_L is thus $P_{B,\alpha} > P_L$. From this point onwards we focus on proving the existence of equilibrium in the special case where there are some β firms advertising.¹⁵

Lemma 15. *If $\pi_\beta^M \geq 2\pi_\beta^S$ then there will be a positive mass of β advertisers in equilibrium.*

Proof. If this did not hold then a β firm could price at P_β^M and would win against all other β firms to earn profits of at least $\frac{\pi_\beta^M}{2}$. With the assumption $\pi_\beta^M \geq 2\pi_\beta^S$ this is strictly more than the profits attainable by not advertising with a price of P^S . \square

Lemma 16. *If $\pi_\beta^M \geq 2\pi_\beta^S$ the top advertiser price will be P_β^M*

Proof. The top pricing advertiser is a β firm. With similar arguments to lemma 9 they will charge their monopoly price. \square

Lemma 17. *If $\pi_\beta^M \geq 2\pi_\beta^S$ then η is given by:*

$$\eta = 1 - \frac{\pi_\beta^S}{\pi_\beta^M} \quad (\text{A.16})$$

¹⁵Whilst there is in principle no impediment to analysing the alternate case where all β firms (and potentially some α firms) monopolise, this restriction is in order to show the notable result where α firms earn above signalling profits as discussed in section 5.1.

Proof. The top pricing β firm charges P_β^M where $G(P_\beta^M) = 0$ and must earn π_β^S . The expression follows immediately from substituting these factors into equation A.7. \square

Lemma 18. *If $\pi_\beta^M \geq 2\pi_\beta^S$ then there is a unique price charged by both types of advertisers $\bar{P} \equiv D_{\beta,A} \cap D_{\alpha,A}$ which is given by:*

$$\bar{P} = \frac{c_\beta \hat{\pi}_\alpha - c_\alpha \hat{\pi}_\beta}{\hat{\pi}_\alpha - \hat{\pi}_\beta} \quad (\text{A.17})$$

Proof. At the unique price \bar{P} where there are advertisers from both types of firm the profits are:

$$\pi_{\beta,A}(\bar{P}) = (\bar{P} - c_\beta)\Gamma(H - \bar{P})[1 - \eta + 2\eta G(\bar{P})] \quad (\text{A.18})$$

$$\pi_{\alpha,A}(\bar{P}) = (\bar{P} - c_\alpha)\Gamma(H - \bar{P})[1 - \eta + 2\eta G(\bar{P})] \quad (\text{A.19})$$

And thus:

$$\frac{\pi_\beta(\bar{P})}{\pi_\alpha(\bar{P})} = \frac{\bar{P} - c_\beta}{\bar{P} - c_\alpha} \quad (\text{A.20})$$

Rearranging this equation and noting at this point they make their equilibrium profits yields the lemma. \square

Proposition 10. *If $\pi_\beta^M \geq 2\pi_\beta^S$ then α firms will earn more than their signalling profits in equilibrium.*

Proof. First recounting equation A.20 and noting that at \bar{P} both high firm types earn their equilibrium profits.

$$\frac{\hat{\pi}_\beta}{\hat{\pi}_\alpha} = \frac{\bar{P} - c_\beta}{\bar{P} - c_\alpha} \quad (\text{A.21})$$

Now examining signalling profits:

$$\pi_\beta^S = (P^S - c_\beta)\Gamma(H - P^S) \quad (\text{A.22})$$

$$\pi_\alpha^S = (P^S - c_\alpha)\Gamma(H - P^S) \quad (\text{A.23})$$

And thus:

$$\frac{\pi_\beta^S}{\pi_\alpha^S} = \frac{P^S - c_\beta}{P^S - c_\alpha} \quad (\text{A.24})$$

As $\frac{P-c_\beta}{P-c_\alpha}$ is a monotonic function for of P in the region $[c_\beta, 1]$.

$$\frac{\hat{\pi}_\beta}{\hat{\pi}_\alpha} < \frac{\pi_\beta^S}{\pi_\alpha^S} \quad (\text{A.25})$$

And substituting in $\hat{\pi}_\beta = \pi_\beta^S$

$$\pi_\alpha^S < \hat{\pi}_\alpha \quad (\text{A.26})$$

So profits above signalling profits are made. □

At this point all of the results presented in section 5.1 have been shown to hold. The last remaining task is to show that an equilibrium will exist.

The profit functions for α and β firms are:

$$\pi_{\beta,A}(\bar{P}) = (\bar{P} - c_\beta)\Gamma(H - \bar{P})[1 - \eta + 2\eta G(\bar{P})] \quad (\text{A.27})$$

$$\pi_{\alpha,A}(\bar{P}) = (\bar{P} - c_\alpha)\Gamma(H - \bar{P})[1 - \eta + 2\eta G(\bar{P})] \quad (\text{A.28})$$

And after rearranging to get the required $G(P)$:

$$G(P) = \begin{cases} \frac{1}{2\eta} \left[\frac{\pi_\beta}{(P-c_\beta)\Gamma(H-P)} - 1 + \eta \right] & \text{for } \bar{P} < P \leq P_\beta^M \\ \frac{1}{2\eta} \left[\frac{\pi_\alpha}{(P-c_\alpha)\Gamma(H-P)} - 1 + \eta \right] & \text{for } P_{\alpha,B} \leq P \leq \bar{P} \end{cases} \quad (\text{A.29})$$

Proposition 11. *If $\pi_\beta^M \geq 2\pi_\beta^S$ then the equilibrium described by a proportion η of β firms (as described in equation A.16) monopolising at P^S and all other firms advertising at prices described by the survival functions in equation A.29 and the beliefs described by*

$$\mu(P) = \begin{cases} 1 & P \geq P^S \\ 0 & P < P^S \end{cases} \quad (\text{A.30})$$

$$\mu(P, P_E) = \begin{cases} 1 & \text{for } P > P_L \quad \text{and} \quad \forall P_E \\ 0 & \text{for } P \leq P_L \quad \text{and} \quad \forall P_E \end{cases} \quad (\text{A.31})$$

is a PBNE.

Proof. Similar arguments as were made in lemma 4 show that these beliefs will be robust in equilibrium and no low firms will attempt to emulate high quality.

The $G(P)$ function described in equation A.29 is feasible, being decreasing in price and ranges between 0 and 1 when price changes from P_β^M to $P_{\alpha,B}$.

The case of low firms is unchanged to that described in proposition 3 with them unable to convincingly advertise. Hence there is no profitable deviation for these firms.

All α firms earn the same profit at any price $P \in D_{\alpha,A} \equiv [P_{\alpha,B}, \bar{P}]$. From proposition 10 they earn more than their signalling profit. From proposition 9 they also earn more than is possible at any point in $D_{\beta,A}$ and hence there are no profitable deviations for these firms.

All β firms earn the same profit at any price $P \in D_{\beta,A} \equiv [\bar{P}, P_\beta^M]$. From proposition 13 they earn their signalling profit. From lemma 9 they also earn more than is possible at any point in $D_{\alpha,A}$ and hence there are no profitable deviations for these firms. \square