Optimal Privatization Policy under Private Leadership in Mixed Oligopolies

Lin, Ming Hsin and Matsumura, Toshihiro

Faculty of Economics, Osaka University of Economics, Institute of Social Science, the University of Tokyo

27 June 2017

Optimal Privatization Policy under Private Leadership in Mixed Oligopolies*

Ming Hsin Lin†
Faculty of Economics, Osaka University of Economics
and
Toshihiro Matsumura‡
Institute of Social Science, the University of Tokyo

June 27, 2017

Abstract

We discuss optimal privatization policies in mixed oligopolies in which a public firm is the Stackelberg follower (private leadership). We find that under constant marginal cost, the optimal degree of privatization is zero. When the marginal cost is increasing, however, the optimal degree is never zero, and full privatization can be optimal. These results suggest that the optimal privatization policy depends on the cost conditions. We also find that the optimal degree of privatization is substantially lower under private leadership than in the simultaneous-move model when there is no cost difference between public and private firms.

JEL classification numbers: H42, L13

Keywords: private leadership, mixed oligopoly, mixed ownership in public firms

*We acknowledge financial support from JSPS KAKENHI Grant Numbers (15K03347, 16K03681) and the Zengin Foundation for Studies on Economics and Finance. Needless to say, we are responsible for any remaining errors.

†Corresponding author. Faculty of Economics, Osaka University of Economics, 2-2-8, Osumi, Higashiyodogawa-ku, Osaka, 533-8533, Japan. E-mail: linmh@osaka-ue.ac.jp

‡E-mail: matsumur@iss.u-tokyo.ac.jp

1 Introduction

Japan had a typical mixed economy[1] in which public enterprises long played a leading role. For example, many believed that lending by public financial institutions such as the Development Bank of Japan had a pump-priming effect on private bank lending. Furthermore, public financing occupied an important position in Japan's financial markets for over 60 years since the 1940s, and public and private financial institutions still coexist.[2]

However, the Koizumi Cabinet (April 2001–September 2006) changed this by declaring that public firms should play a complementary role to private firms, with the latter leading the markets rather than the former. Consequently, major public institutions were substantially downsized (Matsumura and Ogawa, 2017).

We can describe this situation with Pal's (1998) mixed oligopoly model with private leadership.[3] He examined a model in which one public firm competes against private firms by adopting the observable delay game formulated by Hamilton and Slutsky (1990) and investigating the endogenous role in mixed oligopolies. He showed that private leadership (in which all private firms are Stackelberg leaders and one public firm is Stackelberg follower) yields greater welfare than public leadership does and that the private firms become the leaders and the public firm becomes the follower in equilibrium if the number of private firms is large. Although both public and private leadership equilibria exist in a mixed duopoly (i.e., the number of private firms is one), he suggested that private leadership is more natural and robust. Subsequently, many researchers proved his conclusion adequate. Matsumura and Ogawa (2010) showed that private leadership is risk-dominant in Pal's (1998) duopoly model. They also adopted Matsumura's (1998) partial privatization approach and showed the advantage of private leadership; that is, the range of the degree of privatization that yields a private leadership equilibrium is wider than that for the public leadership equilibrium. Capuano and De Feo (2010) introduced the shadow cost of public funds into Pal's (1998) duopoly model and showed that unless the shadow cost is too high, one of the following results holds: (i) the unique equilibrium is a private leadership equilibrium or (ii) both public and private leadership equilibria

[1] For a recent discussion of mixed oligopoly, see Ishida and Matsushima (2009), Nakamura (2015a,b), Chen (2017), and the works they cite.

[2] See Horiuchi and Sui (1993). Even before the 1940s, public financial institutions played an important role in Japan. In the international trade finance market, Yokohama Specie Bank competed with HSBC, a major foreign private bank, and obtained a major share of the market. The public sector worldwide has an important role in lending markets; see Bose et al. (2014).

[3] See Ino and Matsumura (2010) and Matsumura and Ogawa (2017).

exist and the private leadership equilibrium is risk-dominant.[4] Matsumura (2003a) used a different endogenous timing game (two-production period game) formulated by Saloner (1987) and Pal (1991) and showed that only the private leadership equilibrium is robust.[5] Ino and Matsumura (2010) showed that in the free entry market, welfare is greater in the private leadership model than in the public leadership and the simultaneous-move models. From these results, it is reasonable to focus on the private leadership model in mixed oligopolies.

The Koizumi Cabinet also planned to privatize most of the major public financial institutions. However, the subsequent cabinets in Japan postponed the privatization of public financial institutions such as the Development Bank of Japan and the Japan Finance Corporation, which play a complementary role to private banks, and keeps a 100 % share in them (Matsumura and Ogawa, 2017).[6] In this study, we investigate whether the government need not privatize public firms as long as these are followers and the private firms are leaders.

We study the optimal degree of privatization in Stackelberg mixed oligopolies in which private firms are leaders.[7] While many studies investigated Stackelberg mixed oligopolies with private leadership, few discussed the optimal privatization policy.[8] In contrast, there is significant research on the optimal degree of privatization in Cournot mixed oligopolies. Matsumura (1998) showed that unless full nationalization of a public firm yields a public monopoly, the optimal degree of privatization in mixed duopolies is never zero under fairy general demand and cost conditions. Lin and Matsumura (2012) showed that the optimal degree of privatization increases with the number of private firms in two popular models of mixed oligopolies: the linear cost and quadratic cost models. Matsumura and Kanda (2005) showed that the optimal degree of privatization is zero in free entry markets, while

[4] If the shadow cost is high, the unique equilibrium is a Cournot equilibrium.

[5] However, this result depends on the assumption of quantity competition. Bárcena-Ruiz (2007) showed that the simultaneous-move outcome appears under price competition in the observable delay game. In addition, Matsumura (2003b) showed that even under quantity competition, public leadership can be a unique equilibrium if the private firm has only foreign owners. Tomaru and Saito (2010) introduced a production subsidy policy in a mixed duopoly and showed that under private leadership, the government minimizes the subsidy rate to achieve an efficient outcome. This may be another advantage of private leadership.

[6] In contrast, the government partially privatized the Postal Bank and Kampo, which compete with private banks and life-insurance companies.

[7] There are two reasons we focus on private leadership. One is that it is a natural outcome. First, as is discussed above, the private leadership in which all private firms produce first and then one public firm produces always constitutes an equilibrium and it is the unique equilibrium when the number of private firms exceeds three (Pal,1998, Jacques, 2004). Second, our research question is investigating whether or not the government need not privatize the public firm as long as it plays a complementary role as the follower. If the answer is yes, this fact rationalizes the current privatization policy in Japan.

[8] Matsumura and Ogawa (2010) and Bárcena-Ruiz and Garzón (2010) investigated endogenous timing in mixed duopolies under partial privatization, but did not discuss the optimal degree of privatization.

it is never zero if the number of firms is given exogenously.

In this study, we compare the privatization policy implications of private leadership and Cournot mixed oligopolies. We examine two popular models in the literature on mixed oligopolies: the linear cost and quadratic cost models. We find that when marginal cost is constant, the optimal degree of privatization is zero under private leadership, in contrast to the result in Cournot mixed oligopolies. This may support the recent Japanese privatization policy: the government should keep 100% share in the public firm as long as it plays a complementary role to the private sector. When the marginal cost is increasing, however, the optimal degree of privatization is never zero, and full privatization can be optimal. These results suggest that the optimal privatization policy depends on the cost conditions, and full nationalization is not always optimal, even under private leadership. Finally, we investigate the case with symmetric quadratic costs between public and private firms. We find that the optimal degree of privatization is substantially lower under private leadership than in the simultaneous-move model.

2 The Model

Consider a market in which one (partially privatized) domestic state-owned public firm, firm 0, competes against n private firms. Firm 0 maximizes the weighted average of domestic social surplus and its own profit, $\alpha\pi_0+(1-\alpha)W$, where $\alpha \in [0,1]$ indicates the degree of privatization. If $\alpha = 0$, firm 0 is fully nationalized. If $\alpha = 1$, it is fully privatized. All other firms (firm 1 ,..., firm n) maximize their own profits.[9]

Firms produce perfectly substitutable commodities for which $p(Q)$ denotes the inverse demand function, p is the price, and Q is total output. We assume that p is twice continuously differentiable and that $p' < 0$ and $p'' \leq 0$ as long as $p > 0$.[10] Firm 0's cost function is $c_0(q_0)$. All private firms have identical cost functions $c(q_i)$. We

[9]This is a standard model formulation of partial privatization in the literature on mixed oligopoly. We do not allow the government to nationalize both firms. As Merrill and Schneider (1966) pointed out, the most efficient outcome is achieved by the nationalization of all firms if nationalization does not change the firms' costs. The need to analyze mixed oligopolies lies in the fact that it is impossible or undesirable, for political or economic reasons, to nationalize an entire sector. For example, without a competitor, public firms may lose the incentive to improve their costs, resulting in a loss of social welfare. Thus, we neglect the possibility of nationalizing all firms.

[10]The assumption $p'' \leq 0$ is a sufficient but not necessary condition for our results. It guarantees the relevant stability and second-order conditions.

assume c_0', c_0'', c', and $c'' \geq 0$.

The profit of firm 0 is given by $\pi_0 = p(Q)q_0 - c_0(q_0)$, and that of firm i $(= 1, \ldots, n)$ is $\pi_i = p(Q)q_i - c(q_i)$. We define social welfare as

$$W = \int_0^Q p(q)dq - pQ + \sum_{i=0}^{n} \pi_i. \tag{1}$$

Firm 0 maximizes $(1-\alpha)W + \alpha\pi_0$. In the full nationalization case ($i.e.$, $\alpha = 0$), firm 0 maximizes social welfare. In the full privatization case ($i.e.$, $\alpha = 1$), firm 0 maximizes its own profit. The degree of privatization is measured by α.

The game runs as follows. In the first stage, the government chooses $\alpha \in [0,1]$ to maximize W. In the second stage, after observing α, each firm i $(i = 1, 2, ...n)$ chooses q_i independently. In the third stage, after observing the private firms' output, firm 0 chooses q_0. Throughout this study, we use the subgame perfect Nash equilibrium as the equilibrium concept.

Before discussing the private leadership model, we present the results of a simultaneous-move game as a benchmark.[11] In the simultaneous-move model, the government chooses $\alpha \in [0,1]$, and after observing α, each firm i $(i = 0, 1, ...n)$ chooses q_i independently.

Result (Matsumura, 1998) *Suppose that all public and private firms choose their output simultaneously. Suppose that the output of each private firm is positive, even when $\alpha = 0$. Then, $\alpha = 0$ is never optimal.*

In the following sections, we adopt one model with linear costs (constant marginal costs) and another with linear demand and quadratic cost functions.

3 The model with constant marginal costs

In this section, we assume constant marginal costs. The marginal cost for firm 0 is z and that of each private firm is normalized to zero. We assume that $z > 0$. In other words, we assume that the public firm is less efficient than the private firm and z is a measure of the public firm's cost disadvantage.[12] Let p^p be the private Cournot

[11] Matsumura (1998) showed this result for duopolies and Matsumura and Kanda (2005) showed it for oligopolies.

[12] This is a very popular model in the literature on mixed oligopolies. See Pal (1998) and Matsumura (2003a). If $z \leq 0$, the public monopoly appears in equilibrium, and the equilibrium α is zero, regardless of the timing of production (simultaneous-move, public

equilibrium price in a game where firm 0 exits the market and n private firms face Cournot competition. We assume that $p^p > z$. Otherwise, firm 0 produces nothing, regardless of α, and thus any α is optimal.

In the third stage, firm 0 chooses q_0 given the private firms' output $Q_{-0} := \sum_{i=1}^{n} q_i$. Let Q^* be the output satisfying $p(Q^*) = z$. Obviously, if $Q^* - Q_{-0} \leq 0$, firm 0 chooses $q_0 = 0$, regardless of α. Suppose that $Q^* - Q_{-0} > 0$. The first-order condition of firm 0 is

$$p + \alpha p' q_0 - z = 0; \tag{2}$$

the second-order condition is satisfied. Let R_0 be the reaction function of firm 0 derived from (2). We obtain

$$R_0' = -\frac{p' + p''\alpha q_0}{p' + \alpha p' + \alpha p'' q_0} \in [-1, 0) \tag{3}$$

and $R_0' = -1$ if and only if $\alpha = 0$.

In the second stage, each firm i $(i = 1, 2, ..., n)$ chooses q_i independently. The first-order condition is

$$p + (1 + R_0')p' q_i = 0. \tag{4}$$

If $\alpha = 0$, $R_0' = -1$ as long as $Q^* - Q_{-0} \geq 0$. Thus, each private firm behaves as a price taker as long as $Q^* - Q_{-0} \geq 0$ because the price is independent of its output level. Therefore, $Q^* - Q_{-0}$ must be zero because a private firm has the incentive to increase its output otherwise. This is optimal for social welfare because more efficient firms produce all units, and obviously the private firms will never produce more for any α. Therefore, in the first stage, the government chooses $\alpha = 0$. This leads to the following Proposition.

Proposition 1 *Under private leadership, $\alpha = 0$ is optimal for social welfare (and thus the government chooses it) in the constant marginal cost model.*

This result is in sharp contrast to that in Matsumura's (1998) simultaneous-move model, in which $\alpha = 0$ is optimal only when the private firms produce nothing (public monopoly). In contrast, under private leadership (when the public firm is the Stackelberg follower), full nationalization is optimal. This result suggests that if

leadership, or private leadership models). To avoid such an obvious result, we assume $z > 0$.

the public firm complements the private sector, the government need not privatize it. We explain the intuition. In both simultaneous-move and private leadership models, if $\alpha = 0$, then the equilibrium price is equal to the public firm's marginal cost z. However, in the simultaneous-move model, each private firm chooses its output by considering that an increase in output reduces the price, given the other firms' output. Because strategies are strategic substitutes, the public firm's aggressive behavior caused by $\alpha = 0$ reduces the equilibrium output of each private firm, resulting in a welfare loss.[13]

In the private leadership model, each private firm recognizes that an increase in its output reduces the public firm's output. This strategic effect is stronger when the slope of the reaction curve of firm 0 is steeper. This strategic effect is the strongest when $\alpha = 0$, and thus, $\alpha = 0$ induces the most aggressive behavior (the largest output) in each private firm, in contrast to the simultaneous-move model. This leads to Proposition 1.

4 Alternative model: Quadratic costs and linear demand

In this section, we analyze another popular model in the literature. The inverse demand function is $p(Q) = a - Q$, where a is a positive constant, p is the price, and Q is the total output. Let the cost function of firm 0 be $c_0(q_0) = (k_0/2)q_0^2$ and that of firm i ($i = 1, 2, ..., n$) be $c_i(q_i) = (k/2)q_i^2$, where $k_0, k > 0$.[14] We restrict our attention to a symmetric equilibrium in which all private firms produce the same output.[15]

Given Q_{-0}, firm 0 maximizes $\alpha\pi_0 + (1 - \alpha)W$. The first-order condition is

$$p + \alpha p' q_0 - k_0 q_0 = a - Q_{-0} - (1 + \alpha + k_0)q_0 = 0; \tag{5}$$

[13] A smaller α makes the public firm more aggressive because it is more concerned with total social surplus, including consumer surplus, and it thus has more incentive to reduce the price-cost margin through a larger output.

[14] Models with linear demand and a quadratic cost function are popular in this field. See Matsumura and Shimizu (2010) and the works they cite. We allow both the case where the public firm is less efficient than private firms ($k_0 > k$) and the case where the public firm is as efficient as private firms ($k_0 = k$). Whether public firms are less efficient than private firms is a controversial issue. Some empirical works support the former view and other works support the latter. Thus, we assume $k_0 \geq k$ to allow both cases. Moreover, we intentionally avoid assuming $k_0 = k$ and discuss more general costs than De Fraja and Delbono (1989). If we assume $k_0 = k$, it may not be clear that the contrasting results from Propositions 1 and 2 depend on whether the public firm is less efficient or equally efficient, rather than whether the marginal costs are increasing or constant. We use our model formulation to avoid a possible misunderstanding.

[15] We can show that no asymmetric equilibrium exits in this model.

the second-order condition is satisfied. From (5), we obtain firm 0's reaction function:

$$R_0(Q_{-0}, \alpha) = \frac{a - Q_{-0}}{1 + \alpha + k_0}. \tag{6}$$

In the first stage, each firm i ($i = 1, 2, ..., n$) choses q_i independently. The first-order condition is

$$p + (1 + R_0')p'q_i - kq_i = a - Q - \Big(k + \frac{\alpha + k_0}{1 + \alpha + k_0}\Big)q_i = 0; \tag{7}$$

the second-order condition is satisfied.

Let $q^*(\alpha)$ be the equilibrium output of each private firm, $q_0^*(\alpha)$ be that of the public firm (firm 0), and $Q^*(\alpha) := nq^* + q_0^*$ be the equilibrium total output. From (6), (7), and the equation $Q^* = nq^* + q_0^*$, we obtain

$$q^*(\alpha) = \frac{(\alpha + k_0)a}{(n+1)(\alpha + k_0) + (1 + \alpha + k_0)k}. \tag{8}$$

$$q_0^*(\alpha) = a\frac{\alpha + k_0 + (1 + \alpha + k_0)k}{(1 + \alpha + k_0)((n+1)(\alpha + k_0) + (1 + \alpha + k_0)k)}. \tag{9}$$

From these equations, we obtain the following result.

Lemma 1 *$q_0^*(\alpha)$ and $Q^*(\alpha)$ is decreasing in α, and q^* is increasing in α.*

Proof See the Appendix.

These are common results for the simultaneous-move model. An increase in α reduces the optimal output for firm 0 given the output of private firms because firm 0 is less concerned with consumer surplus (Matsumura, 1998). This effect reduces q_0^*, and through the strategic interaction, it increases q^*. An increase in α has another effect under private leadership: it makes the slope of the reaction curve of firm 0 less steep because the public firm is less concerned with consumer surplus, and thus has less incentive to make up the private firms' insufficient supply. This effect makes the private leaders less aggressive, and through the strategic interaction, increases the output of firm 0. The former effect dominates the latter under the linear demand and quadratic cost model. Therefore, q_0^* is decreasing in α and q^* is increasing in α.[16]

[16]The effect on the slope of the reaction curve crucially depends on the property of firm 0's cost function because an increase in α makes firm 0 less aggressive given the other firms' output. This reduces the marginal cost of firm 0 when marginal cost is increasing, and this makes firm 0 more aggressive. Consequently, the reaction function effect mentioned above is partially canceled when marginal cost is increasing, and thus, this effect is weakened. This is why the reaction function effect is weaker in the increasing marginal cost model than in the constant marginal cost model.

Let $W^*(\alpha)$ be the welfare when $q_0 = q_0^*$ and $q_1 = q_2 = ... = q_n = q^*$. We obtain the following Proposition.

Proposition 2 $\partial W^*(\alpha)/\partial\alpha|_{\alpha=0} > 0$ *regardless of* k_0 *and* k.

Proof See the Appendix.

Proposition 2 implies that full nationalization of firm 0 is never optimal, a sharp contrast to Proposition 1. We now explain the intuition behind Proposition 2. Suppose that $\alpha = 0$. Then, firm 0 chooses q_0 to maximize welfare given q^*. In other words, the price must be equal to firm 0's marginal cost. On the other hand, each private firm chooses q to maximize its own profit, and as expected, q is too small for welfare maximization (i.e., the price is higher than the marginal cost of each private firm). Suppose that α increases slightly. The increase in α reduces q_0^* and increases q^*. Since q_0^* is optimal for welfare when $\alpha = 0$, the decrease in q_0^* reduces welfare but has a second-order effect on welfare (envelope theorem). In other words, $\partial W/\partial q_0 = 0$.[17]

Finally, we consider the case in which public and private firms have the same cost function (i.e., $k_0 = k = 1$). The welfare function under private leadership is

$$W(\alpha) = \frac{(1+\alpha)(n^2(1+\alpha)(2+\alpha)^2 + 2(3+2\alpha)^2 + n(2+\alpha)^2(5+3\alpha))}{2(2+\alpha)^2(3+n+2\alpha+n\alpha)^2}. \tag{10}$$

The first-order condition is[18]

$$\frac{\partial W(\alpha)}{\partial\alpha} = -\frac{\alpha(3+2\alpha)^3 + n(-4+3\alpha+15\alpha^2+12\alpha^3+3\alpha^4)}{(2+\alpha)^3(3+n+2\alpha+n\alpha)^3} = 0. \tag{11}$$

The second-order condition

$$\frac{-2(1-\alpha)(3+2\alpha)^4 + 3n^2(-14-22\alpha-4\alpha^2+12\alpha^3+9\alpha^4+2\alpha^5) + n(-156-240\alpha+200\alpha^3+135\alpha^4+28\alpha^5)}{(2+\alpha)^4(3+n+2\alpha+n\alpha)^4} < 0$$

[17] This principle can apply to the Cournot model (simultaneous quantity choice model) but not to the public leadership model. When the public firm is the Stackelberg leader, the choice of q_0 directly affects the choice of q and the public firm chooses q_0 considering this strategic effect. In contrast, because q^* is too small for welfare when $\alpha = 0$, the increase in q^* improves welfare and has a first-order effect on welfare. Therefore, the latter welfare-improving effect dominates the former welfare-reducing effect. This yields Proposition 2. Note that this principle cannot apply to a model with constant marginal cost. In this case, we find the corner solution (i.e., $q_0 = 0$ in equilibrium) under private leadership. Moreover, an increase in α does not increase each private firm's output. Thus, our discussion above cannot apply to the constant marginal cost model.

[18] This occurs because $\partial W/\partial\alpha > 0$ when $\alpha = 0$ and $\partial W/\partial\alpha < 0$ when $\alpha = 1$, and the solution is interior (i.e., partial privatization is optimal).

is satisfied for $n \geq 1$ and $\alpha \in [0, 1]$. Because the denominator in (11) is positive, the first-order condition is satisfied if and only if the numerator in (11) is zero. Solving this equation, we have two imaginary number solutions, one negative real number solution, and one positive real number solution. The last solution is the optimal degree of privatization. The lower curve in Figure 1 describes the relationship between the number of private firms and the optimal degree of privatization.[19]

****

Insert Figure 1 here

****

We now compare this solution to that in the simultaneous-move model. The welfare function in the simultaneous-move model is

$$W(\alpha) = \frac{(1+\alpha)(8 + n^2(1+\alpha) + n(7+3\alpha))}{2(4 + n + 2\alpha + n\alpha)^2}. \tag{12}$$

The first-order condition is[20]

$$\frac{\partial W(\alpha)}{\partial \alpha} = -\frac{2(n(-1+\alpha) + 4\alpha)}{(4 + n + 2\alpha + n\alpha)^3} = 0. \tag{13}$$

The second-order condition

$$-4\frac{n^2(2-\alpha) + n(7-6\alpha) + 8(1-\alpha)}{(4 + n + 2\alpha + n\alpha)^4} < 0 \tag{14}$$

is satisfied for $n \geq 1$ and $\alpha \in [0, 1]$. Because the denominator in (13) is positive, the first-order condition is satisfied if and only if the numerator in (13) is zero. Solving this equation, we obtain the following solution.

$$\alpha = \frac{n}{4+n}. \tag{15}$$

The upper curve in Figure 1 describes the relationship between the number of private firms and the optimal degree of privatization.

[19] Presenting the solution explicitly requires a rather complicated and messy exposition of about one page. We believe that presenting it is not useful for most readers and thus we omit it. It is available from the authors upon request.

[20] Because $\partial W/\partial \alpha > 0$ when $\alpha = 0$ and $\partial W/\partial \alpha < 0$ when $\alpha = 1$, so the solution is interior (i.e., partial privatization is optimal).

From Figure 1, we find that

(a) the optimal degree of privatization is increasing with the number of the private firms in both the private leadership and simultaneous-move models;

(b) the optimal degree of privatization is lower in the private leadership model than that in the simultaneous-move model; and

(c) the optimal degree of privatization is less sensitive to the number of private firms in the private leadership model than in the simultaneous-move model.

In addition, we can show that the optimal degree of privatization converges to one as $n \to \infty$ in the simultaneous-move model, while the upper bound of the optimal degree of privatization is less than 0.4 (about 0.378663) under private leadership.

These findings contain rich implications. First, the more private firms exist in the market, the more the government should privatize the public firm, even under private leadership. Second, the government should privatize less if the public firm is the follower. Third, the optimal ownership share in the public firm can be close to zero in the simultaneous-move model when the number of the private firms is sufficiently large, while the government should hold a substantial share in the public firm if the public firm is the follower.

We now explain the intuition behind these results. We first address the property (a) in the simultaneous-move model above. Lin and Matsumura (2012) and Matsumura and Okamura (2015) showed that the optimal degree of privatization is increasing with the number of private firms in the simultaneous-move model. As we stated above, a larger α induces production substitution from the public firm to the private firms. As long as the marginal cost is higher in the public firm than in each private firm (i.e., the public firm produces more than the private firms do), the production substitution improves production efficiency in the industry and thus improves welfare. We call this the "production-substitution effect." At the same time, an increase in α reduces total output, and thus reduces welfare. We call this the "total output effect." When the number of private firms is larger, the output of each private firm is smaller, and thus the marginal cost for each private firm is smaller. Therefore, the higher the number of private firms, the stronger the welfare improving production-substitution effect is. When the number of private firms is larger, the total output is larger, and thus the welfare loss caused by a reduction in total output is smaller. These two effects yield the above first property (a), and a similar principle can apply to the private leadership model.

However, under private leadership, another effect exits. An increase in α reduces the slope of the public firm's reaction curve because the public firm is less concerned with consumer surplus, and thus has less incentive to make up for the private firms' insufficient supply. This weakens the leaders' strategic behavior and makes the private firms less aggressive, resulting in a welfare loss. Therefore, the government chooses a smaller degree of privatization when the public firm is the follower in order to stimulate the private leaders' production. This leads to the second property (b) mentioned above. We call this the "reaction function effect."

The reaction function effect is more significant when the number of leaders is larger. As the number of private firms increases, the positive production-substitution effect strengthens, the negative total production effect weakens, and the negative reaction function effect increases. The first and second effects have the opposite direction of the third effect, and they are canceled out. The first and second effects work in both the simultaneous-move and private leadership models, whereas the third effect works in the private leadership model only. Therefore, under private leadership, the optimal degree of privatization is less sensitive to the number of the private firms, and the government should hold a substantial share in the private firm due to the third effect, even when the number of private firms is large. This leads the third property (c) mentioned above.

5 Concluding remarks

In this study, we discuss the optimal degree of privatization in mixed Stackelberg oligopolies in which the private firms are leaders. We find that the optimal degree of privatization is zero if the marginal cost is constant. This may support the current Japanese privatization policies. However, this result never holds if firms have quadratic costs. Our results suggest that we should carefully check the policy implications of privatization policy in mixed oligopolies, even if we adopt a popular model from the literature.

In this study, we assumed that the degree of privatization does not affect firms' production costs. However, it is possible that privatization can improve production efficiency, which then affects the resulting welfare. Incorporating this effect into mixed Stackelberg models is quite difficult, and we failed to derive clear-cut results. This task remains for future research.

Appendix

Proof of Lemma 1 From (8) and (9), we obtain

$$\begin{aligned}
\frac{\partial q^*}{\partial \alpha} &= \frac{ak}{((1+n)(k_0+\alpha)+k(1+k_0+\alpha))^2} > 0, \\
\frac{\partial q_0^*}{\partial \alpha} &= -a\frac{((1+n)(k_0+\alpha)^2+k^2(1+k_0+\alpha)^2+k(1+k_0+\alpha)(n+k_0(2+n)+(2+n)\alpha)}{(1+k_0+\alpha)^2((1+n)(k_0+\alpha)+k(1+k_0+\alpha))^2} < 0, \\
\frac{\partial Q^*}{\partial \alpha} &= n\frac{\partial q^*}{\partial \alpha}+\frac{\partial q_0^*}{\partial \alpha} \\
&= -a\frac{((1+n)(k_0+\alpha)^2+k^2(1+k_0+\alpha)^2+2k(k_0+k_0^2+\alpha+2k_0\alpha+\alpha^2)}{(1+k_0+\alpha)^2((1+n)(k_0+\alpha)+k(1+k_0+\alpha))^2} < 0. \ \textbf{Q.E.D.}
\end{aligned}$$

Proof of Proposition 2 Consider the welfare function $W(q_0^*(\alpha), q^*(\alpha))$.

$$\frac{\partial W}{\partial \alpha} = (p-c_0')\frac{\partial q_0^*}{\partial \alpha} + n(p-c')\frac{\partial q^*}{\partial \alpha}, \tag{16}$$

where $c_0'(=k_0q_0)$ and $c'(=kq)$ are the marginal costs for public and private firms, respectively. From the first-order condition of firm 0, we obtain $p-c_0'=0$ when $\alpha=0$. Thus, the first term in (16) is zero. From the first-order condition of each private firm i, we obtain $p-c'=-p'(1+R_0')q_i>0$ because $R_0'=-1/(1+k_0)>-1$. Because $\partial q^*/\partial\alpha>0$ (Lemma 1), (16) is positive when $\alpha=0$. This implies Proposition 2. **Q.E.D.**

References

Bárcena-Ruiz, Juan Carlos. 2007. Endogenous timing in a mixed duopoly: price competition. *Journal of Economics* 91(3):263–72.

Bárcena-Ruiz, Juan Carlos, and Marìa Begoña Garzón. 2010. Endogenous timing in a mixed oligopoly with semipublic firms. *Portuguese Economic Journal* 9(2):97–113 .

Bose, Arup, Debashis Pal, and David E. M. Sappington. 2014. The impact of public ownership in the lending sector. *Canadian Journal of Economics* 47(4):1282–311.

Capuano, Carlo, and Giuseppe De Feo. 2010. Privatization in oligopoly: the impact of the shadow cost of public funds. *Rivista Italiana Degli Economisti* 15(2):175–208.

Chen, Tai-Liang. 2017. Privatization and efficiency: a mixed oligopoly approach. *Journal of Economics* 120(3):251–68.

De Fraja, Gianni, and Flavio Delbono. 1989. Alternative strategies of a public enterprise in oligopoly. *Oxford Economic Papers* 41(2):302–11.

Hamilton, Jonathan H., and Steven M. Slutsky. 1990. Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. *Games and Economic Behavior* 2(1): 29–46.

Horiuchi, Akiyoshi, and Qing-Yuan Sui. 1993. Influence of the Japan Development Bank loans on corporate investment behavior. *Journal of the Japanese and International Economies* 7(4):441–65.

Ino, Hiroaki, and Toshihiro Matsumura. 2010. What role should public enterprises play in free-entry markets? *Journal of Economics* 101(3):213–30.

Ishida, Junichiro and Noriaki Matsushima. 2009. Should civil servants be restricted in wage bargaining? A mixed-duopoly approach. *Journal of Public Economics* 93(3–4):634–46.

Jacques, Armel. 2004. Endogenous timing in a mixed oligopoly: a forgotten equilibrium. *Economics Letters* 83(2):147–8.

Lin, Ming Hsin, and Toshihiro Matsumura. 2012. Presence of foreign investors in privatized firms and privatization policy. *Journal of Economics* 107(1):71–80.

Matsumura, Toshihiro. 1998. Partial privatization in mixed duopoly. *Journal of Public Economics* 70(3):473–83.

Matsumura, Toshihiro. 2003a. Endogenous role in mixed markets: a two production period model. *Southern Economic Journal* 70(2):403–13.

Matsumura, Toshihiro. 2003b. Stackelberg mixed duopoly with a foreign competitor. *Bulletin of Economic Research* 55(3):275–87.

Matsumura, Toshihiro. and Osamu Kanda. 2005. Mixed oligopoly at free entry markets. *Journal of Economics* 84(1):27–48.

Matsumura, Toshihiro, and Akira Ogawa. 2010. On the robustness of private leadership in mixed duopoly. *Australian Economic Papers* 49(2):149–60.

Matsumura, Toshihiro, and Akira Ogawa. 2017. Inefficient but robust public leadership. forthcoming in *Journal of Industry, Competition and Trade.*

Matsumura, Toshihiro, and Makoto Okamura. 2015. Competition and privatization policies revisited: The payoff interdependence approach. *Journal of Economics* 116(2):137–50.

Matsumura, Toshihiro, and Daisuke Shimizu. 2010. Privatization waves. *Manchester School* 78(6):609–25.

Merrill, William, and Norman Schneider. 1966. Government firms in oligopoly industries: a short-run analysis. *Quarterly Journal of Economics* 80(3):400–12.

Nakamura, Yasuhiko. 2015a. Endogenous choice of strategic contracts in a mixed duopoly with bargaining over managerial delegation contracts. Australian Economic Papers 54(2): 121–34.

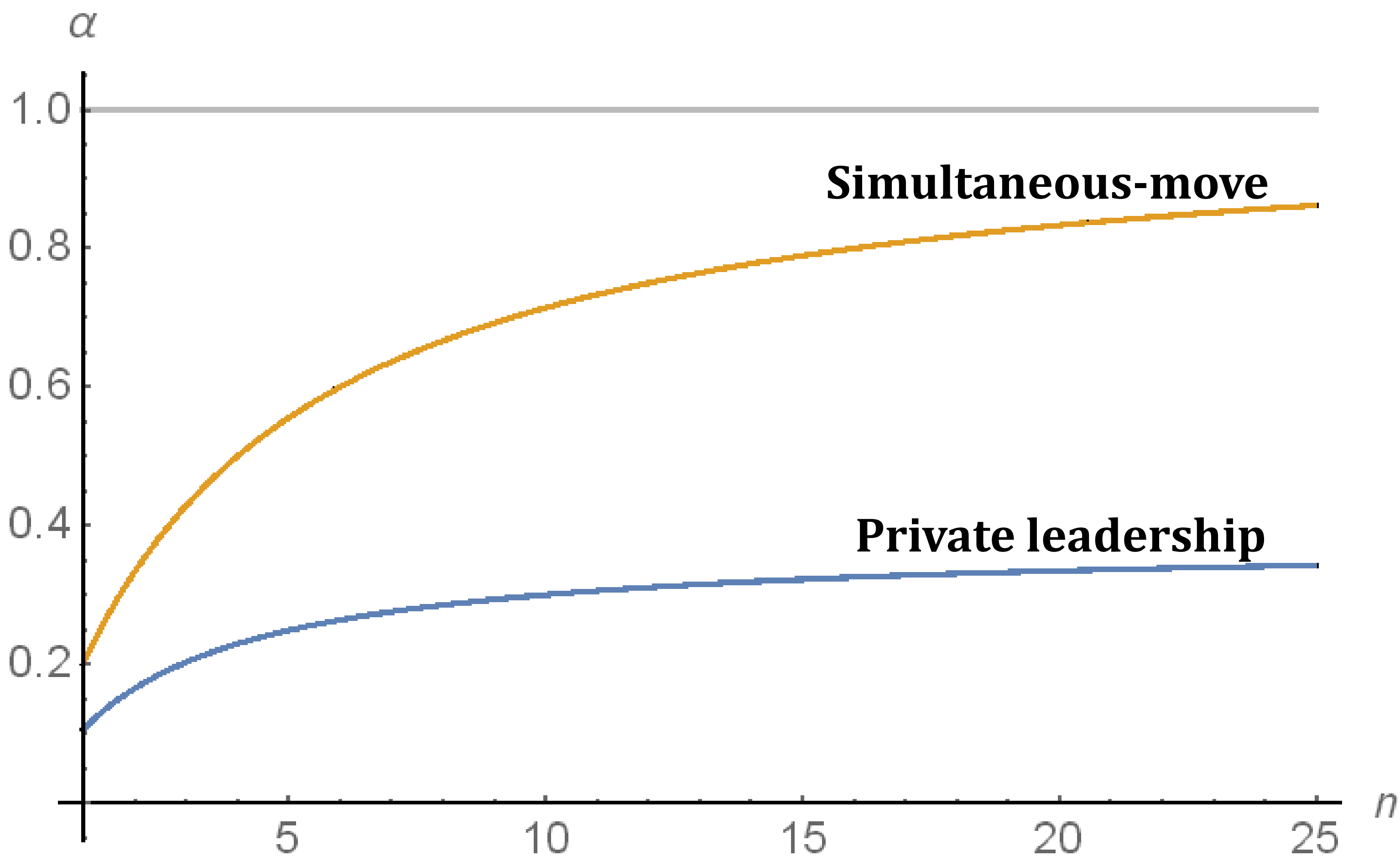
Nakamura, Yasuhiko. 2015b. Endogenous choice of strategic incentives in a mixed duopoly with a new managerial delegation contract for the public firm. International Review of Economics and Finance 35: 262–77

Pal, Debashis. 1991. Cournot duopoly with two production periods and cost differentials. *Journal of Economic Theory* 55(2):441–8.

Pal, Debashis. 1998. Endogenous timing in a mixed oligopoly. *Economics Letters* 61(2):181–5.

Saloner, Garth. 1987. Cournot duopoly with two production periods. *Journal of Economic Theory* 42(1):183–7.

Tomaru, Yoshihiro and Masayuki Saito. 2010. Mixed duopoly, privatization, and subsidization in an endogenous timing framework. *Manchester School* 78(1):41–59.



α
1.0
0.8
0.6
0.4
0.2
Simultaneous-move
Private leadership
5
10
15
20
25
n