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Abstract

What is the optimal portfolio allocation when an agent is investing both for a firm and for himself? I address this question by solving a manager's decision problem under a specific executive compensation structure. Specifically, I study how flat wage and stock compensation affect the manager's investment decision. I show that the allocation is the same regardless of whether the manager is prohibited from trading the public shares of his own firm. Results from calibration show that the manager invests less in firm-specific technology and more in the aggregate stock market as the risk of the firm's project increases. More stock compensation discourages him from investing in the firm's risky technology, but encourages more risk-taking in terms of personal investment. In addition, I prove that flat wage, effectively as a riskless bond, hedges risk and leads to more risk-taking behavior both in firm investment and personal investment.

JEL classification: D9, G11, J33, M12

Keywords: Managerial incentives, Executive compensation, Corporate investment, Portfolio choice, Asset allocation, Dynamic optimization

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1 Introduction

Both the academic community and finance practitioners are interested in the problem of executive compensation. What sort of incentive structure best aligns what the manager wants to do and what he should do? In this paper I present a simple model that aims to tackle some aspects of this problem. Specifically, I ask the question if the manager makes two decisions: investing for the firm and investing for himself, what are the optimal portfolio holdings? I solve this portfolio choice problem in two distinct ways, and find the following results. First, having separate budget constraints for the firm and the manager's own wealth or having one budget constraint for the whole problem does not change the manager's optimal portfolio choice. This result hinges on the fact that the manager can invest in the risk-free asset both as part of the firm and as part of his own personal portfolio. Second, the optimal holdings in the firm's technology versus the aggregate market portfolio is sensitive to the capital shares. Specifically, if the firm's capital is a large portion of the total capital available for investment, the optimal number of shares invested in the risky technology will be relatively steady over time. On the other hand, if the manager's own wealth is a large portion of the total capital available for investment, the optimal number of shares invested in the aggregate market will be relatively constant through time. Third, the manager seeks to rebalance his portfolio. If he started with 30 percent in the risky technology and 30 percent in the aggregate market, and the market goes up while the risky technology stays still, he would want to sell some shares in the aggregate market and buy some more in the risky asset to maintain 30 percent of the total wealth in the risky technology and the aggregate market respectively. Lastly, as the flat wage the manager receives increases, he would want to invest more in both the risky technology and the aggregate stock market. The intuition is that the extra flat wage acts as an investment in some risk-free technology that makes the portfolio safer. The remainder of the paper is organized as follows. In Section 2 I introduce and set up the model. In Sections 3 and 4 I solve the model in closed form in two distinct ways. In Section 5 I present the main results from the model. In Section 6 I calibrate the model parameters. In Section 7 I discuss some empirical predictions of the model. Section 8 concludes.

2 Model Setup

Consider a firm with its manager in charge of the firm's investment decision between time $t \in [0, T]$. Meanwhile, the manager also invests his personal wealth within the same period. At any time between $[0, T]$, the capital of the firm at time t, denoted as K_t, could be allocated into the following two technologies:

1. A linear risky technology with the following instantaneous rate of return to investment:

$$dS_t = \mu dt + \sigma dW_t + \sigma_1 dW_{it} \tag{1}$$

   where the Brownian motion (BM) dW_t captures the aggregate risk and the BM dW_{it} captures the idiosyncratic risk associated with firm i.

2. A riskless storage technology with the real rate of return rdt.

To simplify analysis without much loss of generality, we assume that the firm only pays a liquidating dividend at time T, equal to the accumulated capital K_T. In other words, we assume the zero dividend policy[1] and that there is no fire sale at time T. Meanwhile, the manager's personal wealth, denoted as A_t, could be invested in either the aggregate stock market or a riskless asset:

1. The aggregate stock market is characterized by a total rate of return equal to:[2]

$$dS_t^M = \mu_M dt + \sigma_M dW_t \tag{2}$$

2. For ease of illustration, I assume the same real rate of return of the riskless asset as that of the riskless storage technology, denoted as $d\beta_t = rdt$.

As for the manager's compensation, he is paid nothing before T, but is awarded n shares of his own firm at time 0. In addition, he is promised a flat wage w at time T. Thus, the total compensation to the manager at time T is $w + nK_T$. In the next section, I allow the manager to trade the public shares of his own firm, the case of which should give the allocation that maximizes the manager's welfare. I solve this decision

[1] Imagine that the board is in charge of dividend payout.

[2] Note that I assume away all of the idiosyncratic risks in equilibrium.

problem using the Martingale approach as a benchmark. Then, in Section 4, I prohibit the manager from trading his own firm and study the distortion compared to the benchmark. I am particularly interested in how the portfolio allocation distortion changes with the manager's incentive structure, as described below.

3 Trading Own Firm Allowed

Assume that the manager's utility is defined only on his final wealth. Specifically, I adopt the *CRRA* utility specification. Thus, the manager's problem is to choose how much the firm and he himself invest in different assets to maximize his own time-0 expected utility:

$$\max\ \mathbb{E}_0\left[e^{-\rho T}\frac{(A_T+w+nK_T)^{1-\gamma}}{1-\gamma}\right] \tag{3}$$

where ρ is the continuous-time discount rate and γ is the constant relative risk aversion, subject to the following dynamic budget constraints:

$$\begin{aligned}
\theta^o_{tP}\beta_t+\theta^o_{tA}S^M_t &= A_0+\int_0^t \theta^o_{uP}d\beta_u+\theta^o_{uA}dS^M_u && (4)\\
\theta^o_{tF}\beta_t+\theta^o_{tK}S_t &= K_0+\int_0^t \theta^o_{uF}d\beta_u+\theta^o_{uK}dS_u && (5)\\
\theta^o_{TP}\beta_T+\theta^o_{TA}S^M_T &= A_T && (6)\\
\theta^o_{TF}\beta_T+\theta^o_{TK}S_T &= K_T && (7)
\end{aligned}$$

where A_0 and K_0 are the given initial values (can treat as zeros), P denotes the shares invested in the risk-free asset in the manager's personal portfolio, F denotes the shares invested in the risk-free asset in the firm's portfolio, θ^o_{tA} is the shares invested in the aggregate stock market for the manager's personal portfolio, and θ^o_{tK} is the shares invested in the firm's linear risky technology. (4) and (5) are the self-financing conditions and (6) and (7) are the terminal values, where The (5) and (7) are imposed by the firm's technology, while (4) and (6) are imposed by the manager's own investment opportunities. In the most limited case, the manager is only able to invest in the aggregate stock market and the riskless asset. Limiting the manager's investment opportunity set in turn limits his maximized utility. If we allow the manager to also invest in the firm without restrictions - he is able to take long or short positions of any size in the firm - we can combine the above budget constraints. Giving the manager a larger investment opportunity set, I can look at the first-best optimal portfolio choice of the manager when he is least constrained. Adding (4) to $n \times$ (5) and (6) to $n \times$ (7), and setting $w = 0$ in the original problem for simplicity, the manager's decision problem turns into:

$$\max \ \mathbb{E}_0 \left[e^{-\rho T} \frac{(A_T + nK_T)^{1-\gamma}}{1-\gamma} \right] \tag{8}$$

subject to

$$\begin{aligned} (\theta^o_{tP} + n\theta^o_{tF})\beta_t + \theta^o_{tA} S^M_t + n\theta^o_{tK} S_t &= (A_0 + nK_0) + \int_0^t (\theta^o_{uP} + n\theta^o_{uF}) d\beta_u + \theta^o_{uA} dS^M_u + n\theta^o_{uK} dS_u \quad (9) \\ (\theta^o_{TP} + n\theta^o_{TF})\beta_T + \theta^o_{TA} S^M_T + n\theta^o_{TK} S_T &= A_T + nK_T \quad (10) \end{aligned}$$

Note that the left hand side of (9) is effectively the wealth of the manager at time t, denoted as $W_t \equiv (\theta^o_{tP} + n\theta^o_{tF})\beta_t + \theta^o_{tA} S^M_t + n\theta^o_{tK} S_t$, and I thus define the shares of wealth invested in the risky assets at time t as ϑ^o, respectively:

$$\vartheta^o_{tA} \equiv \frac{\theta^o_{tA} S^M_t}{W_t} \quad and \quad \vartheta^o_{tK} \equiv \frac{n\theta^o_{tK} S_t}{W_t}$$

which are effectively the optimal portfolio weights for the manager - the key variables of interest in this paper. Also, define the mean excess returns for the firm's risky technology and the aggregate stock market to be the vector $\boldsymbol{\lambda} \equiv [\lambda_A, \lambda_K] \equiv [\mu_M - r, \mu - r]$. Now, if I put the two BM's into the vector $dB_t \equiv [dW_t, dW_{it}]$, I can write the loadings on the BM's as $\boldsymbol{\sigma} \equiv \begin{bmatrix} \sigma_M & 0 \\ \sigma & \sigma_1 \end{bmatrix}$, and call the *market price of risk* $\nu \equiv \boldsymbol{\sigma}^{-1}\boldsymbol{\lambda} = \left[\frac{\lambda_A}{\sigma_M}, \frac{\sigma_M \lambda_K - \sigma\lambda_A}{\sigma_M \sigma_1}\right]$. I solve (8) in Appendix A1 through simple change of measures and obtain the main result of this section: the optimal portfolio weights of the total wealth invested in the stock market and the risky technology for the manager is given by

$$\boldsymbol{\vartheta}^o \equiv \begin{bmatrix} \vartheta^o_{tA} \\ \vartheta^o_{tK} \end{bmatrix} = \frac{1}{\gamma}(\boldsymbol{\sigma}\boldsymbol{\sigma}')^{-1}\boldsymbol{\lambda} = \frac{1}{\gamma} \begin{bmatrix} \frac{(\sigma^2+\sigma_1^2)\lambda_A - \sigma_M \sigma \lambda_K}{\sigma_M^2 \sigma_1^2} \\ \frac{-\sigma_M \sigma \lambda_A + \sigma_M^2 \lambda_K}{\sigma_M^2 \sigma_1^2} \end{bmatrix}. \tag{11}$$

4 Trading Own Firm Prohibited

Now, I consider the case in which the manager is prohibited from trading the public shares of the firm he works for. As before the manager's utility is defined only on his total wealth, so his optimization problem reads:

$$\max_{\vartheta^c_{tA},\vartheta^c_{tK}} \quad \mathbb{E}_0\left[e^{-\rho T}\frac{\left(A_T+w+nK_T\right)^{1-\gamma}}{1-\gamma}\right] \tag{12}$$

$$s.t. \qquad dA_t = \quad \left(A_t r + A_t\vartheta^c_{tA}\lambda_A\right)dt + A_t\vartheta^c_{tA}\sigma_M dW_t \tag{13}$$

$$dK_t = \quad \left(K_t r + K_t\vartheta^c_{tK}\lambda_K\right)dt + K_t\vartheta^c_{tK}\left(\sigma dW_t + \sigma_1 dW_{it}\right) \tag{14}$$

where $\vartheta^c_{tA} \equiv \frac{\theta^c_{tA}S^M_t}{A_t}$ denotes the fraction of the manager's total personal wealth invested in the aggregate stock market, $\vartheta^c_{tK} \equiv \frac{\theta^c_{tK}S_t}{K_t}$ denotes the fraction of the firm's capital invested in the risky technology, θ^c_{tA} and θ^c_{tK} are the counterparts of θ^o_{tA} and θ^o_{tK} defined in the last section, and λ_A and λ_K are also defined as before. The "c" superscripts distinguish this problem as a more constrained optimization compared to the one before denoted by the superscripts "o" where the manager was allowed to trade on his own firm's public shares. Thus, I can write a Bellman equation as:

$$\sup_{\vartheta^c_{tA},\vartheta^c_{tK}} D\,J\left(A_t,K_t,t\right)=0 \tag{15}$$

where

$$\begin{aligned} D\,J\left(A_t,K_t,t\right) \quad = \quad & J_t + J_A\left[A_t\left(r+\vartheta^c_{tA}\lambda_A\right)\right] + J_K\left[K_t\left(r+\vartheta^c_{tK}\lambda_K\right)\right] \\ & +\frac{1}{2}J_{AA}\left[A_t^2{\vartheta^c_{tA}}^2\sigma_M^2\right] + \frac{1}{2}J_{KK}\left[K_t^2{\vartheta^c_{tK}}^2\left(\sigma^2+\sigma_1^2\right)\right] \\ & +J_{AK}\left[A_t\vartheta^c_{tA}K_t\vartheta^c_{tK}\sigma_M\sigma\right] \end{aligned}$$

with boundary condition

$$J\left(A_T,K_T,T\right) = \frac{\left(A_T+w+nK_T\right)^{1-\gamma}}{1-\gamma} \tag{16}$$

I solve this Bellman equation in Appendix A2 and obtain the following optimal portfolio weights of the total personal wealth invested in the stock market and of the firm's capital invested in the risky technology for the manager:

$$\boldsymbol{\vartheta}^c \equiv \begin{bmatrix} \vartheta^c_{tA} \\ \vartheta^c_{tK} \end{bmatrix} = \begin{bmatrix} \frac{A_t+we^{-r(T-t)}+nK_t}{\gamma A_t}\left(\frac{\lambda_A\left(\sigma^2+\sigma_1^2\right)}{\sigma_1^2\sigma_M^2} - \frac{\lambda_K}{\sigma_1^2}\frac{\sigma}{\sigma_M}\right) \\ \frac{A_t+we^{-r(T-t)}+nK_t}{\gamma n K_t}\left(\frac{\lambda_K}{\sigma_1^2} - \frac{\lambda_A}{\sigma_1^2}\frac{\sigma}{\sigma_M}\right) \end{bmatrix}. \tag{17}$$

5 Main Results

In this section I present the main results of the paper with some key comparative statics. When the manager can trade his own firm's public shares and thus has a larger investment opportunity set as in Section 3, his optimal portfolio weights are:

$$\begin{aligned} \vartheta^o_{tA} = \frac{\theta^o_{tA}S^M_t}{W_t} &= \frac{1}{\gamma}\left(\frac{\lambda_A}{\sigma_1^2}\frac{\sigma^2+\sigma_1^2}{\sigma_M^2} - \frac{\lambda_K}{\sigma_1^2}\frac{\sigma}{\sigma_M}\right) \equiv \varphi_a \\ \vartheta^o_{tK} = \frac{n\theta^o_{tK}S_t}{W_t} &= \frac{1}{\gamma}\left(\frac{\lambda_K}{\sigma_1^2} - \frac{\lambda_A}{\sigma_1^2}\frac{\sigma}{\sigma_M}\right) \equiv \varphi_k \end{aligned}$$

where φ_k and φ_a need to be calibrated. When the manager cannot trade his own firm's public shares and thus has a constrained investment opportunity set as in Section 4, his optimal portfolio weights are:

$$\begin{aligned} \vartheta^c_{tA} = \frac{\theta^c_{tA}S^M_t}{A_t} = \frac{W_t}{A_t}\frac{\theta^c_{tA}S^M_t}{W_t} &= \frac{A_t+w_t+nK_t}{A_t}\frac{1}{\gamma}\left(\frac{\lambda_A}{\sigma_1^2}\frac{\sigma^2+\sigma_1^2}{\sigma_M^2} - \frac{\lambda_K}{\sigma_1^2}\frac{\sigma}{\sigma_M}\right) = \frac{W_t}{A_t}\varphi_a \\ \vartheta^c_{tK} = \frac{\theta^c_{tK}S_t}{K_t} = \frac{W_t}{nK_t}\frac{n\theta^c_{tK}S_t}{W_t} &= \frac{A_t+w_t+nK_t}{nK_t}\frac{1}{\gamma}\left(\frac{\lambda_K}{\sigma_1^2} - \frac{\lambda_A}{\sigma_1^2}\frac{\sigma}{\sigma_M}\right) = \frac{W_t}{nK_t}\varphi_k \end{aligned}$$

where $W_t = A_t + w_t + nK_t$ and $w_t = we^{-r(T-t)}$. Therefore, the two solutions are the same - they give the same portfolio allocations of the manager's total wealth W_t.[3]

Proposition 1. ***(Budget Equivalence)*** *Separate budget constraints and combined budget constraints yield the same solution to the manager's optimization problem.*

The intuition is simple: the concern for non-additivity comes from the possibility of violation of the self-financing condition by taking short positions in either investments, but notice that in this setup we have the same riskless investment technology for both the manager and the firm, then we can always add up

[3]This suggests that we can effectively add up the dynamic budget constraints of the two investment opportunities for the manager's problem.

the budget constraints given that we can always hedge (take long/short positions) using the same riskless asset respectively and still stay in the self-financing strategies.

Next, define the manager's firm-investment share of wealth, private-investment share of wealth, and discounted-wage share of wealth respectively as

$$s_{k,t} \equiv \frac{nK_t}{W_t},\ s_{a,t} \equiv \frac{A_t}{W_t},\ and\ s_{w,t} \equiv \frac{w_t}{W_t}$$

Therefore, I can write the two optimal investment shares as: *Fraction of Firm Capital in Linear Risky Technology* (the Benchmark, see Figure 1): $\vartheta_{tK}^c = \frac{\varphi_k}{s_{k,t}}$, or say the optimal risky investment share of firm capital. *Fraction of Personal Wealth in Aggregate Stock Market* (the Benchmark, see Figure 2): $\vartheta_{tA}^c = \frac{\varphi_a}{s_{a,t}}$, or say the optimal risky investment share of personal wealth.

Proposition 2. ***(Asymptotic Myopia)*** *If the manager's wealth from firm investment (nK_t) is large enough, the optimal risky investment share of firm capital will be relatively constant. On the other hand, if the manager's wealth from private investment (A_t) is large enough, then the optimal risky investment share of personal wealth will be relatively constant.*

To see this, notice that

$$\vartheta_{tK}^c = \frac{\varphi_k}{s_{k,t}} = \frac{A_t + w_t + nK_t}{nK_t}\varphi_k \overset{nK_t \to \infty}{\longrightarrow} \varphi_k \ \ and \ \ \vartheta_{tA}^c = \frac{\varphi_a}{s_{a,t}} = \frac{A_t + w_t + nK_t}{A_t}\varphi_a \overset{A_t \to \infty}{\longrightarrow} \varphi_a$$

When we look at the direct effect of wealth shares on optimal investments, we have

$$\frac{\partial \vartheta_{tK}^c}{\partial s_{k,t}} = -\frac{\varphi_k}{s_{k,t}^2} \gtreqless 0 \ \ and \ \ \frac{\partial \vartheta_{tA}^c}{\partial s_{a,t}} = -\frac{\varphi_a}{s_{a,t}^2} \gtreqless 0$$

so the signs of φ_k and φ_a are crucial for the following key results.

For example, if $\varphi_k > 0$ and $\varphi_a > 0$, then it means that when the share of total wealth coming from firm investment increases, the manager optimally chooses to reduce the investment of firm capital K_t in the risky technology, meanwhile, when the share of total wealth coming from private investment increases, the manager optimally chooses to reduce the investment of personal wealth A_t in the stock market. We notice that when $\vartheta_{tK}^c = \frac{\varphi_k}{s_{k,t}} \lesseqgtr 0$, then $\frac{\partial \vartheta_{tK}^c}{\partial s_{k,t}} \gtreqless 0$, and when $\vartheta_{tA}^c = \frac{\varphi_a}{s_{a,t}} \lesseqgtr 0$, then $\frac{\partial \vartheta_{tA}^c}{\partial s_{a,t}} \gtreqless 0$. So, when the manager takes a long position in the risky investment for firm capital ($\vartheta^c_{tK} > 0$), as the share of his total wealth coming from firm investment increases ($s_{k,t} \uparrow$), he reduces the risky investment for firm capital ($\vartheta^c_{tK} \downarrow$). On the other hand, when he takes a short position in the risky investment for firm capital ($\vartheta^c_{tK} < 0$), as the share of his total wealth coming from firm investment increases ($s_{k,t} \uparrow$), he raises the risky investment for firm capital ($\vartheta^c_{tK} \uparrow$). Yet, in either case, the magnitude of the risky asset investment of firm capital unambiguously decreases ($|\vartheta^c_{tK}| \downarrow$). The same logic applies to the risky asset investment of personal wealth (ϑ^c_{tA}). Thus, we have the following result.

Proposition 3. ***(Relative-wealth Rebalancing)*** *Regardless of taking long or short positions, the manager will reduce the magnitude (in absolute value) of firm capital investment in the risky technology, if his firm-investment share of wealth increases. Similarly, he will reduce the magnitude of personal wealth investment in the stock market, if his private-investment share of wealth increases.*

Economically, since the manager has no control over the firm's level of available capital K_t, the changes in his share of wealth coming from firm investment effectively come from the variation in his executive compensation scheme n. In empirical analysis below, I evaluate the relative-wealth rebalancing effect by doing comparative statics of the optimal investment strategy with respect to n. Figures 3 and 4 numerically confirm *Proposition 3* by displaying uniformly downward shifts of the time-series of optimal weights as the number of corporate shares awarded to the manager (n) increases. This is intuitive in the sense that as the executive compensation gets larger, the incentive to pursue risky investments becomes less for the manager. What would be interesting here is that when the firm's risky investment technology is actually the aggregate market portfolio (e.g. the firm is a mutual fund), and thus $\lambda_K = \lambda_A$ and $\sigma = \sigma_M$, then we would have $\vartheta^c_{tK} = 0$, $\vartheta^c_{tA} = \frac{1}{s_{a,t}} \frac{\lambda_A}{\gamma \sigma_M^2}$ and thus $\frac{\partial \vartheta^c_{tK}}{\partial s_{k,t}} = 0$, $\frac{\partial \vartheta^c_{tA}}{\partial s_{a,t}} = -\frac{1}{s_{a,t}^2} \frac{\lambda_A}{\gamma \sigma_M^2} < 0$. In this case, the manager's optimization problem collapses back to a typical power utility investor's myopic portfolio choice and we would obtain the standard investment rule.

Next, notice the effect of terminal flat wage on optimal investment shares:

$$\frac{\partial \vartheta^c_{tK}}{\partial w} = \frac{\partial \vartheta^c_{tK}}{\partial s_{k,t}} \frac{\partial s_{k,t}}{\partial w_t} \frac{\partial w_t}{\partial w} = \frac{\varphi_k}{s_{k,t}^2} \frac{n K_t}{(A_t + w_t + n K_t)^2} e^{-r(T-t)} \quad \gtreqless \quad 0$$
$$\frac{\partial \vartheta^c_{tA}}{\partial w} = \frac{\partial \vartheta^c_{tA}}{\partial s_{a,t}} \frac{\partial s_{a,t}}{\partial w_t} \frac{\partial w_t}{\partial w} = \frac{\varphi_a}{s_{a,t}^2} \frac{A_t}{(A_t + w_t + n K_t)^2} e^{-r(T-t)} \quad \gtreqless \quad 0$$

which only depend on the signs of φ_k and φ_a. This is in similar spirit as *Proposition 3*. Note that when $\vartheta^c_{tK} = \frac{\varphi_k}{s_{k,t}} \gtreqless 0$, then $\frac{\partial \vartheta^c_{tK}}{\partial w} \gtreqless 0$, and when $\vartheta^c_{tA} = \frac{\varphi_a}{s_{a,t}} \gtreqless 0$, then $\frac{\partial \vartheta^c_{tA}}{\partial w} \gtreqless 0$. So, when the manager takes a long position in the risky investment for firm capital ($\vartheta^c_{tK} > 0$), as the terminal wage increases ($w \uparrow$), he raises the risky asset investment of firm capital ($\vartheta^c_{tK} \uparrow$). On the other hand, when he takes a short position in the risky investment for firm capital ($\vartheta^c_{tK} < 0$), as the terminal wage increases ($w \uparrow$), he reduces the risky asset investment of firm capital ($\vartheta^c_{tK} \downarrow$). Yet, in either case, the magnitude of the risky asset investment of firm capital unambiguously increases ($|\vartheta^c_{tK}| \uparrow$). The same logic applies to the risky asset investment of personal wealth (ϑ^c_{tA}). Thus, we have the following result.

Proposition 4. ***(Labor Income Hedging)*** *A higher terminal flat wage increases the magnitudes (in absolute values) of risky asset investments of both firm capital and personal wealth.*

In empirical analysis below, I evaluate the labor income hedging effect by doing comparative statics of the optimal investment strategy with respect to w. Figures 5 and 6 numerically confirm *Proposition 4* by displaying uniformly upward shifts of the time-series of optimal weights as the terminal flat wage (w) increases. This is intuitive in the sense that we can view the exogenous flat wage as an extra source of riskless investment for the manager. Such an additional "risk-free bond" in the background provides an extra hedging channel and the absolute shares in risky assets will therefore rise for both investments.

Corollary 1. *A higher share of total wealth coming from the discounted flat wage increases the magnitudes (in absolute values) of risky asset investments of both firm capital and personal wealth.*

To see this, note that

$$\frac{\partial \vartheta^c_{tK}}{\partial s_{w,t}} = \frac{\partial \vartheta^c_{tK}}{\partial s_{k,t}} \frac{\partial s_{k,t}}{\partial w_t} \frac{1}{\frac{\partial s_{w,t}}{\partial w_t}} = \frac{\varphi_k}{s^2_{k,t}} \frac{nK_t}{A_t + nK_t} \gtreqless 0 \quad \textit{and} \quad \frac{\partial \vartheta^c_{tA}}{\partial s_{w,t}} = \frac{\partial \vartheta^c_{tA}}{\partial s_{a,t}} \frac{\partial s_{a,t}}{\partial w_t} \frac{1}{\frac{\partial s_{w,t}}{\partial w_t}} = \frac{\varphi_a}{s^2_{a,t}} \frac{A_t}{A_t + nK_t} \gtreqless 0$$

which again only depend on the signs of φ_k and φ_a, and is directly related to *Proposition 4*. Also, we can see that as nK_t or A_t goes to infinity, we would have $\frac{\partial \vartheta^c_{tK}}{\partial s_{w,t}} = \varphi_k$ or $\frac{\partial \vartheta^c_{tA}}{\partial s_{w,t}} = \varphi_a$, constant.

Lastly, I turn to the comparative statics of optimal investment shares with respect to return parameters:

$$\frac{\partial \vartheta^c_{tK}}{\partial \lambda_K} = \underbrace{\frac{\partial \frac{1}{s_{k,t}}}{\partial \lambda_K} \varphi_k}_{?} + \underbrace{\frac{\partial \varphi_k}{\partial \lambda_K} \frac{1}{s_{k,t}}}_{+} \quad \textit{and} \quad \frac{\partial \vartheta^c_{tA}}{\partial \lambda_A} = \underbrace{\frac{\partial \frac{1}{s_{a,t}}}{\partial \lambda_A} \varphi_a}_{?} + \underbrace{\frac{\partial \varphi_a}{\partial \lambda_A} \frac{1}{s_{a,t}}}_{+}$$

which require calibration to sign and see their magnitudes, so do the partials with respect to volatility parameters $\frac{\partial \vartheta^c_{tK}}{\partial \sigma}$, $\frac{\partial \vartheta^c_{tK}}{\partial \sigma_1}$, $\frac{\partial \vartheta^c_{tK}}{\partial \sigma_M}$, $\frac{\partial \vartheta^c_{tA}}{\partial \sigma}$, $\frac{\partial \vartheta^c_{tA}}{\partial \sigma_1}$, *and* $\frac{\partial \vartheta^c_{tA}}{\partial \sigma_M}$. In the following, I want to emphasize the role of σ_1, i.e. the idiosyncratic riskiness of the firm's risky investment technology.

Conjecture 1. ***(Impact of Idiosyncratic Risk)*** *I conjecture that as the level of idiosyncratic risk in firm's linear technology (σ_1) increases, the optimal share of firm capital invested in the risky technology decreases while the optimal share of personal wealth invested in the market portfolio increases.*

In empirical analysis below, I evaluate the impact of idiosyncratic risk by doing comparative statics of the optimal investment strategy with respect to σ_1. Figures 7 and 8 numerically confirm *Conjecture 1* by displaying that as σ_1 increases, 1). the time-series of optimal weights of firm capital invested in the linear risky technology uniformly shift downward; 2). the time-series of optimal weights of personal wealth invested in the market portfolio uniformly shift upward. This is intuitive in the sense that as the idiosyncratic risk component becomes larger and larger for the firm's linear technology, the manager will optimally choose to reduce the investment of firm capital in this risky technology and invest more in the riskless asset. And as personal wealth can be invested in the aggregate stock market that has no idiosyncratic risk, the higher Sharpe ratio in this case will induce the manager to optimally invest more of his personal wealth in the market portfolio and less in the riskless asset, possibly taking on leverage positions.

5.1 Representative Investor

The representative investor's preferences are reflected by the state price density in equilibrium. Notice that in this case the firm under consideration has measure zero compared to the size of the market and the state price density depends only on the aggregate market dynamics dS^M_t. Thus, in construction of the market price of risk, I take $dB_t = dW_t$ and I can write the loadings on the BM as $\boldsymbol{\sigma} = [\sigma_M, \sigma]$. Then, I can define the market price of risk as $\nu \equiv \frac{\lambda_K}{\sigma} = \frac{\lambda_A}{\sigma_M}$, and the Radon-Nikodym derivative as $\xi_t \equiv e^{-\nu B_t - \frac{t}{2}\nu^2}$. So, the state price density is $\pi_t \equiv e^{-rt}\xi_t = e^{-(r+\frac{\nu^2}{2})t - \nu B_t}$.

Notice that there are still two sources of investment in this case, the aggregate stock market portfolio and the firm's linear risky technology, but the two investment opportunities are exposed to the same Brownian risk and thus admit the same Sharpe ratio. As the representative investor (RI) can invest in either source and the firm is so small, he can just invest in the market portfolio (effectively investing in the firm's risky technology as well). Thus, the RI's optimization problem collapses to a standard power utility (over final wealth) investor's portfolio choice problem with only one asset - the market portfolio, with return $dS_t^M = \mu_M dt + \sigma_M dW_t$. Therefore, it is straightforward that the RI's optimal investment strategy will be different from that of the manager, who invests in both the market portfolio and the firm's risky technology, and will be the myopic rule: $\vartheta_{tK}^{RI} = \frac{n\theta_{tK}^{RI} S_t}{W_t} = 0$, $\vartheta_{tA}^{RI} = \frac{\theta_{tA}^{RI} S_t^M}{W_t} = \frac{\lambda_A}{\gamma\sigma_M^2} = \frac{\mu_M - r}{\gamma\sigma_M^2}$. Compared to the manager's optimal choice, the RI's optimal investment in the market portfolio does not depend on the idiosyncratic risk from the firm's risky technology (σ_1) nor on the aggregate risk exposure of this technology (σ), because now that the firm is so small compared to the market, the idiosyncratic risk is subsumed and the aggregate risk can be just accounted for by the market exposure.

6 Calibration

To illustrate the model, I calibrate some key parameters to put numbers on the portfolio choice problem. I obtain daily equity returns data from The Center for Research in Security Prices (CRSP) from September 2009 through December 2014 for IBM ("the firm") and the S&P 500 value-weighted index ("the aggregate stock market"). I also obtain the 3-month Treasury bill from CRSP to use as the risk-free rate. Since there is no continuous returns data, I use one-day returns as a proxy for dS_t^M and dS_t. To find the drift and volatility of the aggregate stock market, I simply take the unconditional mean and standard deviation of the S&P 500 value-weighted index: $\mathbb{E}\left(vwretd\right) = \mu_M$ and $Var\left(vwretd\right) = \sigma_M^2$, where $vwretd$ is the daily S&P 500 value-weighted returns. For the firm's technology, I take the unconditional mean of IBM's returns as the drift: $\mathbb{E}\left(ret\right) = \mu$, where ret is the daily IBM returns. Running a market model of IBM's returns on the aggregate market gives me the market beta of IBM, the covariance of IBM's returns with the market returns: $\beta_{IBM} = \frac{Cov\left(dS, dS^M\right)}{Var\left(dS^M\right)} = \frac{\sigma_M\sigma}{\sigma_M^2} = \frac{\sigma}{\sigma_M}$ and $\sigma = \beta_{IBM}\sigma_M$. I have σ_M from earlier and β_{IBM} can be obtained from the time-series regression: $ret_t = \alpha + \beta_{IBM} vwretd_t + \varepsilon_t$. By definition, the idiosyncratic risk of the firm is uncorrelated with the aggregate risk. The total variance of the linear risky technology is just the sum of the variance of the part exposed to aggregate risk and the variance of the idiosyncratic part: $Var\left(ret\right) = \sigma^2 + \sigma_1^2$. Thus, σ_1 can be obtained as $\sigma_1 = \sqrt{Var\left(ret\right) - \sigma^2}$. For the risk-free rate, I take the time-series average of the yield on the 3-month T-bills, adjusting them into daily returns. The calibrated parameters are as follows:

μ	0.000563	σ	0.0120	σ_M	0.0164
μ_M	0.000210	σ_1	0.0102	r	0.000000833

Notice that σ, σ_1 and σ_M all have the same magnitude. It should be pointed out that the above calibration is fine only when IBM has only one project going on, which unlikely holds in reality. Alternatively, I amplify the idiosyncratic volatility to $5\sigma_1$, implicitly assuming that there are 25 independent ongoing R&D projects in IBM (an assumption for simplicity without much loss of generality). Table 1 below describes all the parameters that I use in the calibration. Parameters of mean returns and volatilities are obtained from the above estimation. Following standard literature in dynamic portfolio choice, I set the relative risk aversion coefficient $\gamma = 10$. Unfortunately, I don't have good enough empirical evidence on the value of flat wage and thus I set $w = 1$ as a starting point. Later on, I will show the comparative statics results as w changes. As for another important parameter n, the number of corporate shares rewarded to the manager, I will also show results as n changes from 2% to 20%.

Table 1: Parameters

	Parameter	**Choice**
T	Number of days from Sep 2009 to Dec 2014	1342
μ	Mean return of risky technology	0.000563
σ	Technology exposure to aggregate risk	0.0120
σ_1	Technology exposure to idiosyncratic risk	0.0510
μ_M	Mean return of market portfolio	0.000210
σ_M	Aggregate stock market volatility	0.0164
r	Risk-free rate	0.000000833
γ	Risk aversion coefficient	10
w	Flat wage	1
A_0	Personal initial wealth	1
K_0	Initial firm capital	5
n	Shares paid to the manager	2%

7 Empirical Predictions

In this section, I briefly discuss some empirical predictions that can be made based on the main theoretical results from Section 5. From *Proposition 2*, if the manager is at the beginning of his tenure and has not yet accumulated much personal wealth, then the compensation from the firm's capital will be a large part of the manager's overall budget constraint. Then, investment in the firm's risky technology will be less volatile compared to a similar firm yet managed by a senior manager who has already accumulated a large amount of personal wealth compared to the firm's capital. In other words, similar firms' investments in similar projects should look different for managers with differential personal wealth. In an ideal empirical setting, we would have two identical firms with identical risky projects and total capital, and two managers with different personal wealth levels. We would expect to see the firm that has manager with the higher wealth level invest in a more consistent manner in the risky project, compared to the firm that has manager with the lower wealth level. For the manager with larger personal wealth, we should see his personal portfolio have a more constant share in the aggregate stock market than the manager with smaller personal wealth. Next, from *Proposition 3*, we should see a manager rebalancing his portfolio. Specifically, if the firm's risky technology has a period of sufficiently high returns relative to the aggregate stock market, so that the fraction of total personal wealth invested in this risky technology has gone up, we should see the manager sell some shares in the firm's risky technology and buy more shares in the aggregate stock market. Similarly, if the aggregate market return goes up sufficient high relatively to the return on the risky technology, so that the fraction invested in the aggregate stock market goes up, we should see the manager sell some shares of the aggregate stock market and invest more in the firm's risky technology. We can easily test this if we could see the portfolio holdings of the manager in practice. Finally, from *Proposition 4*, we should see differential effects on two very similar firms with similar managers, who yet get paid different flat wages. In an ideal empirical setting, we would have two almost identical firms with managers who have similar levels of wealth, but one firm pays its manager a higher flat wage compared to the other. In that case, the manager who gets paid a higher flat wage would invest more in both the risky technology of his firm and the aggregate stock market.

8 Conclusion

In this paper, I set out to explore the question of optimal investment behavior of a firm's manager in the scenario where he is investing both for the firm and for himself, and he derives utility from his final wealth that comes from three sources: executive compensation in the form of the firm's capital shares, return from private investment, and a terminal flat wage. First, I have shown that the allocation is the same regardless of whether the manager is prohibited from trading the public shares of his own firm or not. In the non-negligible presence of such individual firm that the manager is attached to, he will optimally deviate from a representative investor even in a complete markets setting. I have also shown that there will be important effects such as relative-wealth rebalancing and labor income hedging, which illustrate the empirical importance of both the executive compensation scheme and the flat wage structure on incentivizing the manager to carry out first-best investment strategies for the firm. Specifically, more stock compensation discourages the manager from investing in the firm's risky technology, but encourages more risk-taking in terms of his private investment in the aggregate stock market. In addition, flag wage, effectively as a risk-free bond, hedges against inter-temporal risk and leads to more risk-taking behavior both in firm investment and private investment. It is important to notice that the idiosyncratic risk component σ_1 in the firm's risky technology plays a significant role in affecting the optimal investment strategy. Results from calibration show that the manager invests less in the firm's risky technology and more in the aggregate stock market as the risk of firm R&D increases. The larger is such individual risk, the more we expect the manager to allocate his personal wealth to the aggregate stock market and the less we expect him to allocate the firm's capital to the risky technology. All these findings above are important for empirical evaluation and incentive provision for firms in consideration of executive compensation structure. More research can be done in this line of inquiry such as to perform cross-section analysis with different return structures from various firms to gauge relative significance of the effects shown in this paper with different firm characteristics. Also, it might be useful to delve deeper into the R&D structure of firms to more accurately measure returns on projects to be used in the risky technology investment, which otherwise would be less convincing since we were implicitly restricting the firms' properties and types of business when we used the equity returns as a proxy for the risky technology investment.

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Appendix A. Derivations and Proofs

A1. Solution of Problem (8)

Proof. I use the Radon-Nikodym derivative ξ, defined as $\xi_t \equiv e^{-\nu' B_t - \frac{t}{2}\nu' \upsilon}$, to change measures from risk-natural to risk-neutral, where B_t and ν are as defined before. Treating $A_t + nK_t$ as a whole, the dynamic budget constraints (9) and (10) can be manipulated into the following equivalent static budget constraint:

$$\mathbb{E}^Q \left[e^{-rT} \left(A_T + nK_T \right) \right] \leq A_0 + nK_0 \tag{18}$$

where Q denotes the expectation under the risk-neutral measure, with the state-price density defined as $\pi_t \equiv e^{-rt}\xi_t = e^{-(r+\frac{\nu'\nu}{2})t - \nu' B_t}$. Then, (8) can be transformed into the following problem:

$$\sup_{A_T + nK_T} \mathbb{E}^Q \left[\frac{\left(A_T + nK_T \right)^{1-\gamma}}{1-\gamma} - \kappa \pi_T \left(A_T + nK_T \right) \right] \tag{19}$$

where $\kappa \equiv \left(\frac{1}{A_0 + nK_0} \mathbb{E}^Q \left[\pi_T^{\frac{\gamma-1}{\gamma}} \right] \right)^{\gamma}$. Again, treating $A_T + nK_T$ as a whole, the FOC is

$$\left(A_T + nK_T \right)^{-\gamma} = \kappa \pi_T \to A_T + nK_T = \left(\kappa \pi_T \right)^{-\frac{1}{\gamma}} \tag{20}$$

Given the form of π_t, we have

$$\begin{aligned}
\mathbb{E}^Q \left[\pi_T^{\frac{\gamma-1}{\gamma}} \right] &= e^{-\frac{\gamma-1}{\gamma}(r + \frac{1}{2\gamma}\nu'\nu)T} \\
\kappa &= \left(\frac{1}{A_0 + nK_0} e^{-\frac{\gamma-1}{\gamma}(r + \frac{1}{2\gamma}\nu'\nu)T} \right)^{\gamma} \\
A_T + nK_T &= \left(A_0 + nK_0 \right) e^{rT + \frac{1}{\gamma}\nu'\nu T - \frac{1}{2\gamma^2}\nu'\nu T + \frac{1}{\gamma}\nu' B_T}
\end{aligned}$$

Finally, equate the deflated wealths derived in two ways, where the "hats" correspond to the expectations under the risk-neutral measure Q:

$$\begin{aligned}\hat{W}_t &= A_0 + nK_0 + \int_0^t \theta^o_{uA} d\hat{S}^M_u + n\theta^o_{uK} d\hat{S}_u \\ \hat{W}_t &= A_0 + nK_0 + \int_0^t \frac{1}{\gamma}\hat{W}_u d\hat{B}_u\end{aligned}$$

and solve the equation, we have the optimal portfolio weights for the manager:

$$\boldsymbol{\vartheta} \equiv \begin{bmatrix} \vartheta^o_{tA} \\ \vartheta^o_{tK} \end{bmatrix} = \frac{1}{\gamma}(\boldsymbol{\sigma}\boldsymbol{\sigma}')^{-1}\boldsymbol{\lambda} = \frac{1}{\gamma}\begin{bmatrix} \frac{(\sigma^2+\sigma_1^2)\lambda_A - \sigma_M\sigma\lambda_K}{\sigma_M^2\sigma_1^2} \\ \frac{-\sigma_M\sigma\lambda_A + \sigma_M^2\lambda_K}{\sigma_M^2\sigma_1^2} \end{bmatrix}.$$

□

A2. Solution of Problem (15)

Proof. The FOC's of (15) with respect to ϑ^c_{tA} and ϑ^c_{tK} are:

$$[\vartheta^c_{tA}] : J_A A_t \lambda_A + J_{AA} A_t^2 \vartheta^c_{tA} \sigma_M^2 + J_{AK} A_t K_t \vartheta^c_{tK} \sigma_M \sigma = 0 \tag{21}$$

$$[\vartheta^c_{tK}] : J_K K_t \lambda_K + J_{KK} K_t^2 \vartheta^c_{tK} \left(\sigma^2 + \sigma_1^2\right) + J_{AK} A_t K_t \vartheta^c_{tA} \sigma_M \sigma = 0 \tag{22}$$

Conjecture:

$$J(A_t, K_t, t) = g(t) \frac{\left(A_t + we^{-r(T-t)} + nK_t\right)^{1-\gamma}}{1-\gamma}$$

Then, (21) and (22) can be simplified into the following linear equations:

$$\begin{bmatrix} \gamma A_t \sigma_M^2 \sigma & \gamma n K_t \left(\sigma^2 + \sigma_1^2\right) \\ \gamma A_t \sigma_1^2 \sigma & \gamma n K_t \sigma_M \sigma^2 \end{bmatrix} \begin{bmatrix} \vartheta^c_{tA} \\ \vartheta^c_{tK} \end{bmatrix} = \begin{bmatrix} \left(A_t + we^{-r(T-t)} + nK_t\right)\lambda_A \sigma \\ \left(A_t + we^{-r(T-t)} + nK_t\right)\lambda_K \sigma_M \end{bmatrix}$$

which leads to the following optimal portfolio weights:

$$\begin{bmatrix} \vartheta^c_{tA} \\ \vartheta^c_{tK} \end{bmatrix} = \begin{bmatrix} \frac{A_t + we^{-r(T-t)} + nK_t}{\gamma A_t}\left(\frac{\lambda_A\left(\sigma^2+\sigma_1^2\right)}{\sigma_1^2\sigma_M^2} - \frac{\lambda_K}{\sigma_1^2}\frac{\sigma}{\sigma_M}\right) \\ \frac{A_t + we^{-r(T-t)} + nK_t}{\gamma n K_t}\left(\frac{\lambda_K}{\sigma_1^2} - \frac{\lambda_A}{\sigma_1^2}\frac{\sigma}{\sigma_M}\right) \end{bmatrix}.$$

Next, I complete the solution by obtaining the value function. Imposing the Bellman equation (15) gives:

$$\frac{g^{'}(t)}{1-\gamma}+g(t)\left[r+\frac{1}{2}\lambda_K\frac{\lambda_K\sigma_M-\lambda_A\sigma}{\gamma\sigma\sigma_1^2}+\frac{1}{2}\lambda_A\frac{1}{\gamma\sigma_M^2}\left(\lambda_A\frac{\sigma^2+\sigma_1^2}{\sigma_1^2}-\lambda_K\frac{\sigma_M\sigma}{\sigma_1^2}\right)\right]=0\rightarrow g^{'}(t)=-qg(t)$$

where

$$q\equiv(1-\gamma)\left[r+\frac{1}{2}\lambda_K\frac{\lambda_K\sigma_M-\lambda_A\sigma}{\gamma\sigma\sigma_1^2}+\frac{1}{2}\lambda_A\frac{1}{\gamma\sigma_M^2}\left(\lambda_A\frac{\sigma^2+\sigma_1^2}{\sigma_1^2}-\lambda_K\frac{\sigma_M\sigma}{\sigma_1^2}\right)\right].$$

Then, solving for $g(t)$ with the boundary condition that $g(T)=1$, we have:

$$g(t)=e^{q(T-t)}$$

which yields the value function:

$$J(A_t,K_t,t)=e^{q(T-t)}\frac{\left(A_t+we^{-r(T-t)}+nK_t\right)^{1-\gamma}}{1-\gamma}. \tag{23}$$

□

Appendix B. Figures

Figure 1: Fraction of Firm Capital in Linear Risky Technology-Benchmark

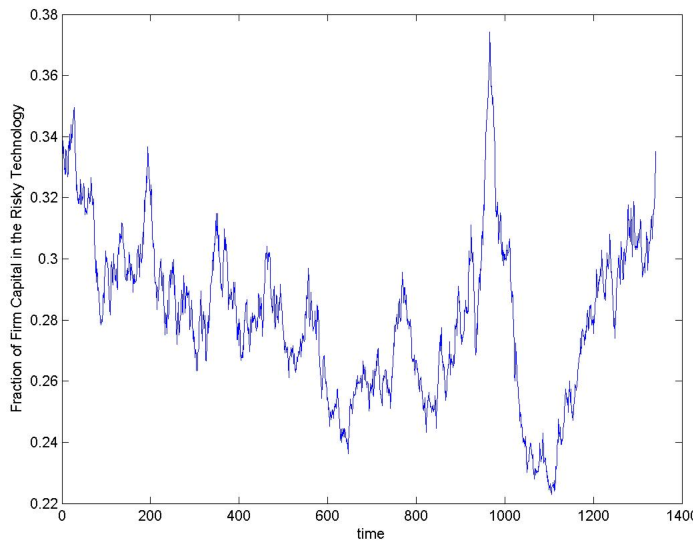

Figure2: Fraction of Personal Wealth in Aggregate Stock Market-Benchmark

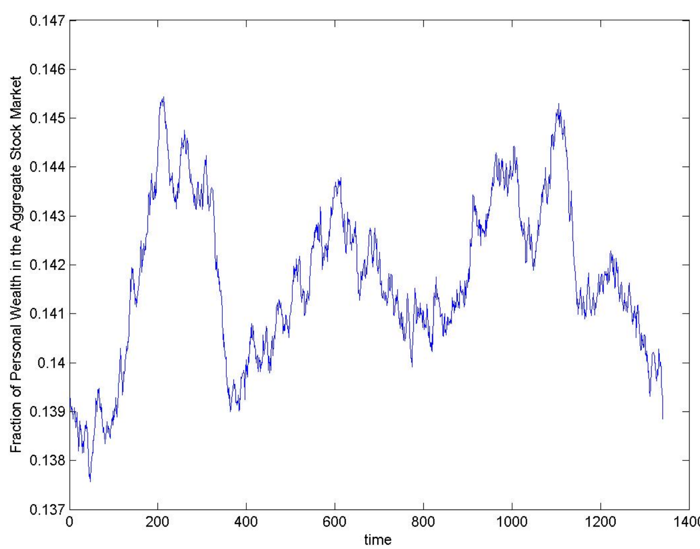

Figure 3: Fraction of Firm Capital in Linear Risky Technology-Comparative Statics with n

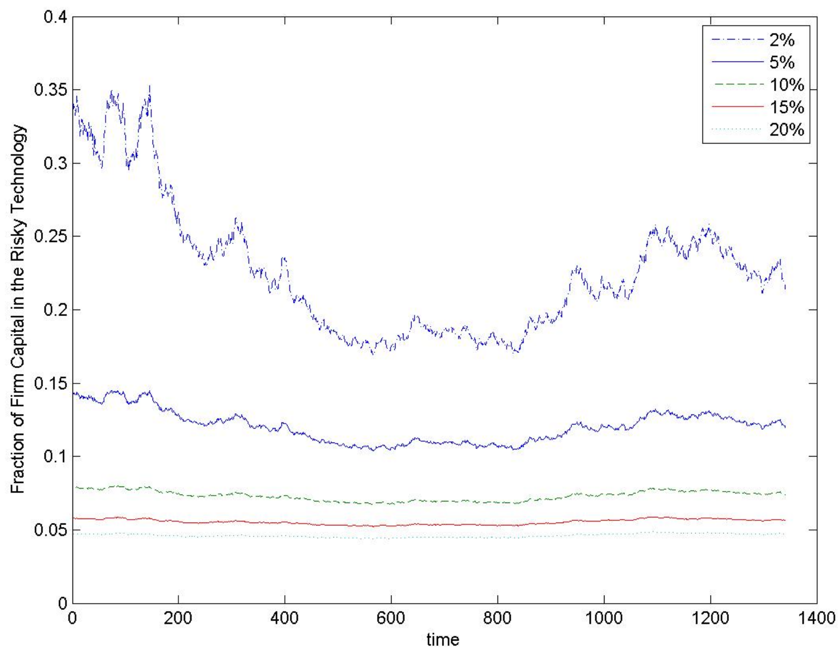

Figure 4: Fraction of Personal Wealth in Stock Market-Comparative Statics with n

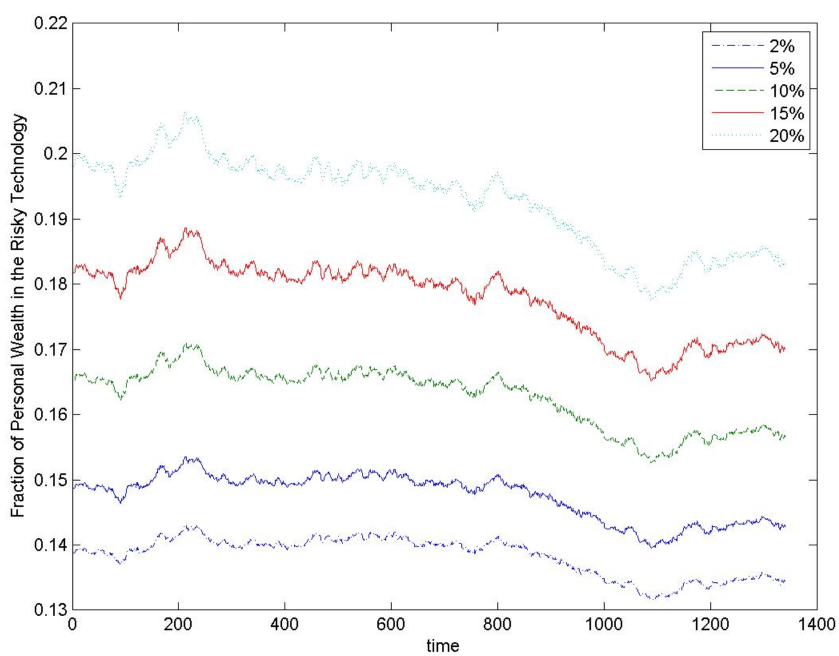

Figure 5: Fraction of Firm Capital in Linear Risky Technology-Comparative Statics with w

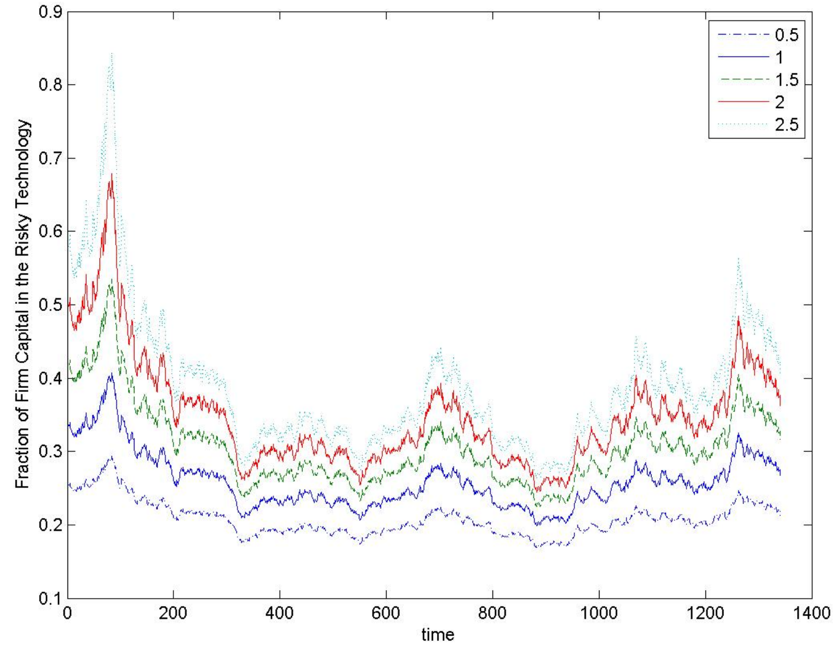

Figure 6: Fraction of Personal Wealth in Stock Market-Comparative Statics with w

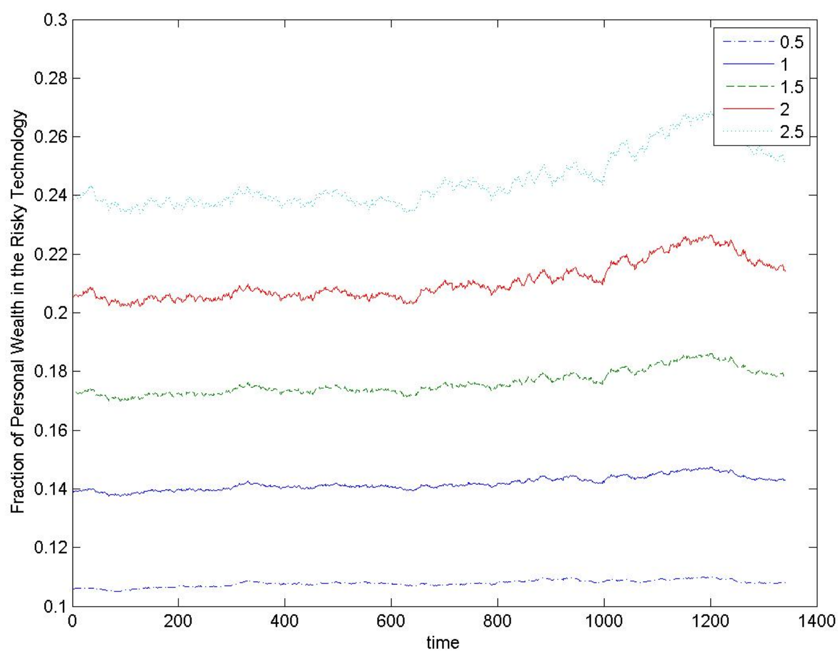

Figure 7: Fraction of Firm Capital in Linear Risky Technology-Comparative Statics with σ_1

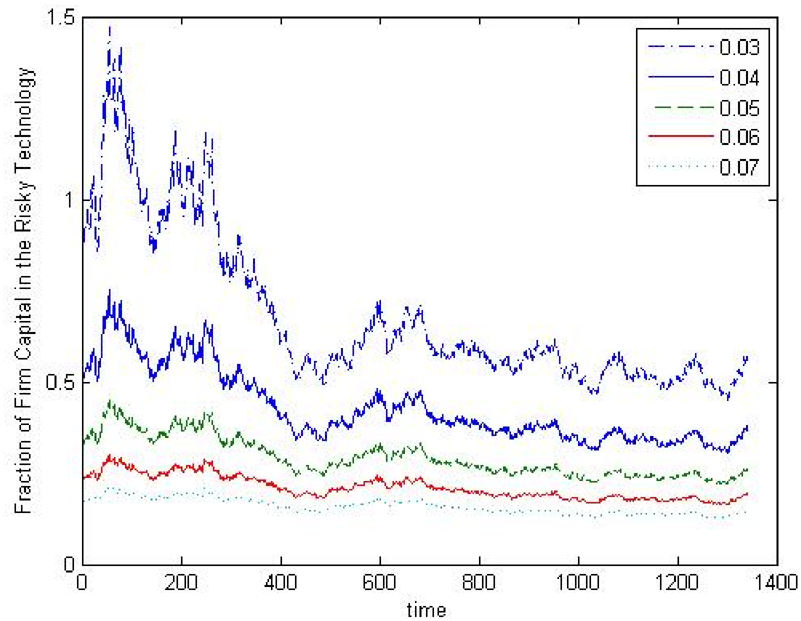

Figure 8: Fraction of Personal Wealth in Stock Market-Comparative Statics with σ_1