Patterns of Rebellion: A Model with Three Challengers

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Abstract

This study proposes a dynamic model of rebellion, where three players individually decide to challenge their common adversary. It is formally demonstrated that the pattern of rebellion is determined endogenously, depending on the challengers' resolve and strength. In other words, a stronger challenger with more resolve tends to fight earlier than others do. (JEL: D74; F51)
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1 Introduction

While some scholars regard rebellion as a coordination game, where rebels strive to synchronize their challenges (Granovetter 1978; Weingast 1995, 1997, 2005; Angeletos, Hellwig, and Pavan 2007; Fearon 2011), others presume a rebellion occurs in an uncoordinated manner, where a leader ("vanguard") tries to mobilize opportunistic followers (Roemer 1985; Ginkel and Smith 1999; Bueno de Mesquita 2010). These two distinct approaches have contrasting implications—according to the former, a rebellion breaks out with simultaneous challenges, while the latter suggests a rebellion escalates gradually with sequential challenges.

This theoretical discrepancy is reconciled by Nakao (2015), who endogenized the sequence and timing of challenges in order to demonstrate that a rebellion can occur in either a coordinated or an uncoordinated manner in light of two challengers, depending on their resolve and strength. However, his analysis with more than two challengers remains informal. In extending his model by incorporating three challengers, this study formally analyzes four patterns of rebellion: (i) *snowballing rebellion*, which escalates gradually as more challengers are drawn in (e.g., Napoleonic Wars); (ii) *catalytic rebellion*, in which an instigator provokes a galvanizing event to inspire all others' simultaneous challenges (Boshin War); (iii) *partially coordinated rebellion,* which is initiated by a few of the *ex post* challengers (American Civil War); (iv) *fully coordinated rebellion,* in which all the challengers fight in unison (American Revolution). By delivering the condition for each of the patterns, we show that among three challengers, a stronger challenger with more resolve tends to fight earlier than others do. Even with more than three challengers, a rebellion can still be categorized into one of these four patterns.

The rest of the paper proceeds as follows. Section 2 presents the model, and its assumptions are shown in Section 3. Section 4 portrays four patterns of rebellion. Sections 5 and 6 analyze the players' behavior and solve the model. Section 7 offers numerical example. Section 8 concludes.

2 A Model with Three Challengers

In the model, there are three players ($i, j \in \{\alpha, \beta, \gamma\}$ with $i \neq j$), who are discontented with their government. In the first period $t = 1$, each player decides to "fight" or "acquiesce" to the government. In addition, in subsequent periods $t = 2, 3, \dots,$ any player who has not yet fought the government decides whether or not to fight. If player i chooses to fight, inflicting a lump-sum cost $c_i > 0$, the battle results in one of i's "win," "holdout," and "loss" in that period, depending on the government's (later specified) military strength G relative to the rebels'. If i "wins" ("loses"), the war ends with i's one-off gain $w_i > 0$ (loss $l_i > 0$). If i "holds out" in a battle, the war continues to the next period. The payoff from "acquiesce" is normalized to zero.

Once a player starts fighting, he cannot withdraw its army, and the battle evolves until the player "wins" or "loses" the war. In other words, a player's decision is when to fight or permanently acquiesce. On the other hand, the government is assumed not to make any strategic decision. The game continues until every player's battle ends decisively with "win" or "loss."

The model captures the uncertainty of the government's strength. The players do not know the government's true strength g, which is binary: weak or strong ($g \in \{g^W, g^S\}$ with $0 < g^W < g^S$), but they know the prior probability distribution of the government's strength $\Pr(g)$. Each player i is given a parameter $r_i > 0$, which denotes i's strength. A battle outcome depends on the government's relative military strength:

$$G \equiv g - \sum_{i \in \{\alpha,\beta,\gamma\}} I_i r_i \text{ for } g \in \{g^W, g^S\},$$

where I_i is an indicator which takes the value zero if player i acquiesces or has fought but lost the war (i.e., $I_i = 0$ for i's "acquiesce" or "loss"), while it takes the value one if i is fighting the government or has won the war ($I_i = 1$ for i's "win" or "holdout"). This means that as more players challenge the government, the balance of power shifts away from the government, but it shifts back if a player is defeated. The power balances perfectly when $G = 0$.

Given the relative strength G, "nature" determines player i's battle outcomes in period t ($h_{i|t} \in \{win,\ hold,\ loss\}$). When two or more players are simultaneously fighting the government in a period, they are assumed to operate the same campaign; that is, their battle outcomes are identical probabilistic events (e.g., $h_{\alpha|t} = h_{\beta|t}$).

Once a player initiates a war, other players infer the government's true strength g from a series of reports from battlefields. By Bayes' rule, β and γ's belief based on

α's battles until period T can be shown as:

$$\Pr\left(g^L|H_{\alpha|T}\right) = \frac{1}{1+\frac{\Pr(g^H)}{\Pr(g^L)}\left(\frac{\Pr(hold|G_\alpha^S)}{\Pr(hold|G_\alpha^W)}\right)^{T-1}\frac{\Pr(h_{\alpha|T}|G_\alpha^S)}{\Pr(h_{\alpha|T}|G_\alpha^W)}},$$

where G_i^W (G_i^S) is the weak (strong) government's strength relative to i, $\Pr(h_{i|t}|G)$ is the per-period probability of $h_{i|t}$ conditional on G, and $H_{i|T}$ is i's history ($hold_{i|1}$, $hold_{i|2}$, $\cdots, h_{i|T}$) with $h_{i|T} \in \{win, hold, loss\}$.

Given G, i's per-period payoff from fighting can be denoted as:

$$\pi_i(G) \equiv \Pr(win|G)\, w_i - \Pr(loss|G)\, l_i,$$

while i's continuation payoff as:

$$\Pi_i(G) \equiv \sum_{t=1}^{\infty} \Pr(hold|G)^{t-1}\, \pi_i(G).$$

3 Assumptions

Assumptions are introduced as follows.

Assumption 1 *For $i, j \in \{\alpha, \beta, \gamma\}$, $r_i + r_j < g^W$.*

By Assumption 1, any pairwise coalition is weaker than the government for sure.

Assumption 2 *(i) For $i, j \in \{\alpha, \beta, \gamma\}$, $\Pr\left(win|G_{i,j}^W\right) > 0$ and $\Pr\left(win|G_{i,j}^S\right) = 0$. (ii) Both $\Pr(win|G)$ and $\frac{\Pr(win|G)}{\Pr(win\cup loss|G)}$ decrease with G until they reach zero. (iii) $\Pr(hold|G)$ decreases with $|G|$. (iv) For $i, j \in \{\alpha, \beta, \gamma\}$, $\frac{\Pr\left(hold|G_i^S\right)}{\Pr\left(hold|G_i^W\right)} = \frac{\Pr\left(hold|G_{i,j}^S\right)}{\Pr\left(hold|G_{i,j}^W\right)}$.*

By Assumption 2, (i) no pairwise coalition can defeat a strong government; (ii) stronger players are more likely to beat the government; (iii) a battle tends to remain indecisive if the players are as strong as the government; (iv) two players' joint fighting is as informative as a single player's fighting.[1] By Assumptions 1 and 2-(iii), any two players' "holdout" suggests the government to be weak and thus motivates the other player to join the rebellion.

[1] For instance, Assumption 2-(iv) holds for $\Pr(hold|G) = e^{-a|G|-b}$ with $a, b > 0$.

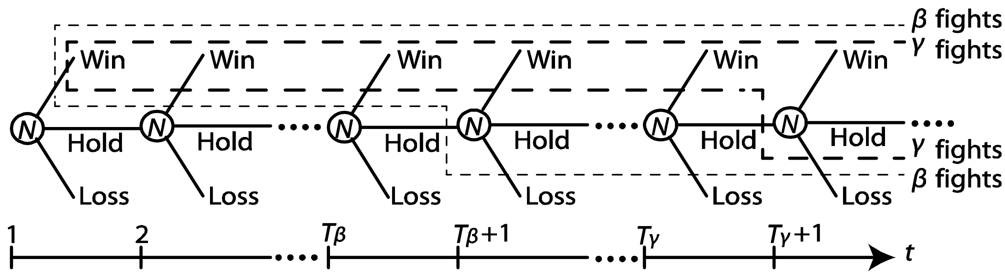

Figure 1: A snowballing rebellion provoked by group α.

Assumption 3 *For $i \in \{\beta, \gamma\}$, (i) $\Pi_i \left(G^S_{\alpha,\beta,\gamma}\right) < c_i < \Pi_i \left(G^W_{\alpha,\beta,\gamma}\right)$ and (ii) $\Pi_i \left(G^W_{\beta,\gamma}\right) < c_i$.*

By Assumption 3, (i) β and γ of the full coalition are willing (unwilling) to fight the weak (strong) government; (ii) β and γ are unwilling to fight once α is defeated.

Assumption 4 *(i) For $i \in \{\beta, \gamma\}$, $\sum_{g \in \{g^W, g^S\}} \Pr(g) \Pr(loss|G_\alpha) (\Pi_i (G_{\alpha,\beta,\gamma}) - c_i) < 0$. (ii) $\sum_{g \in \{g^W, g^S\}} \Pr(g) \Pr(loss|G_{\alpha,\beta}) (\Pi_\gamma (G_{\alpha,\beta,\gamma}) - c_\gamma) < 0$.*

By Assumption 4, (i) both β and γ would like to acquiesce at least once if α alone provokes a (snowballing or catalytic) rebellion; (ii) γ would like to acquiesce at least once if α and β jointly provoke a (partially coalitional) rebellion.[2]

4 Four Patterns of Rebellion

We consider the following four patterns of rebellion. The first pattern depicts the escalation of rebellion in sequence (Figure 1).

Definition 1 *A rebellion is* snowballing *if it is initiated solely by α, followed by β and γ. If α "wins," β and γ simultaneously fight. If α "holds out" for T_β periods, β fights. After β's participation, if α and β "win" or "hold out" until period T_γ ($T_\beta < T_\gamma$), γ fights. Otherwise, β and γ acquiesce.*

[2] To interpret Assumption 4-(i), if α fights alone in the first period, it will be immediately defeated by the government with probability $\sum_{g \in \{g^W, g^S\}} \Pr(g) \Pr(loss|G_\alpha)$; then, $i \in \{\beta, \gamma\}$ will lose the opportunity of having a payoff $\Pi_i (G_{\alpha,\beta,\gamma}) - c_i$. The assumption determines this expected payoff to be negative, implying that by acquiescing once, β and γ could avoid this negative payoff (resulting from fighting a strong government).

The second pattern proceeds as one player fights alone, and the two others then act in unison.

Definition 2 *A rebellion is* catalytic *if α initiates the fighting, and then β and γ simultaneously participate, conditional on α's "win" or $T_{\beta,\gamma}$ rounds of "holdout." Otherwise, β and γ acquiesce.*

The third pattern describes a rebellion that is provoked by a two-player coalition, after which the third player may fight.

Definition 3 *A rebellion is* partially coalitional *if it is initiated jointly by two players α and β. In this rebellion, the third player γ fights if coalition α-β "wins" or if it "holds out" until period T_γ. Otherwise, γ acquiesces.*

The last pattern takes the simplest form.

Definition 4 *A rebellion is* fully coalitional *if all three players fight simultaneously.*

5 Lagged Challenges

We solve the game by backward induction. The following lemma shows the last challenger γ's rational decision.

Lemma 1 *Suppose that α initiates a rebellion and β joins in after α's T_β rounds of "holdout." (i) If T_β is so large that*

$$\sum_{g\in\{g^W,g^S\}} \Pr\left(g|hold_{\alpha|T_\beta}\right)\Pr\left(loss|G_{\alpha,\beta}\right)\left(\Pi_\gamma\left(G_{\alpha,\beta,\gamma}\right)-c_\gamma\right)\geq 0, \tag{1}$$

γ is willing to fight simultaneously with β or even before β. Otherwise, γ continues to acquiesce at least until $T_\beta+1$. Then, (ii) after α and β "win," γ fights immediately, (iii) after α and β "lose," γ acquiesces forever, and (iv) as long as α and β "hold out," γ acquiesces until period T_γ ($T_\beta<T_\gamma$) and then fights in period $T_\gamma+1$, where T_γ equals the smallest T such that Inequality (1) holds with $\Pr\left(g|hold_{\alpha,\beta|T}\right)$ instead of $\Pr\left(g|hold_{\alpha|T_\beta}\right)$. (v) T_γ is independent of T_β.

Proof. (i) With Inequality (1), γ fights immediately, given $hold_{\alpha|T_\beta}$. Otherwise, γ acquiesces once, given $hold_{\alpha|T_\beta}$. (ii, iii) By Assumption 2-(i), α and β's "win" guarantees that the government is weak. Then, γ's sequential rationality after α and β's "win" ("loss") is given by Assumption 3-(i) (3-(ii)). (iv) As α and β hold out longer, $\Pr\left(g^W|hold_{\alpha,\beta|T}\right)$ converges to one by Assumptions 1 and 2-(iii), and γ will fight in some period $T+1$ by Assumption 3-(i). (v) T_γ is independent of T_β because γ's payoff is independent of T_β by Assumption 2-(iv). ■

If Inequality (1) does not hold, γ would fight after β, escalating the rebellion as if it snowballs (Definition 1). With Inequality (1), two possibilities emerge: if γ is willing to fight simultaneously with β, the rebellion spreads catalytically (Definition 2); if γ is willing to fight earlier than β, we can replace γ with β, so that without loss of generality, we presume that $T_\beta \leq T_\gamma$. (By Assumption 4-(i), it never happens that both β and γ are willing to fight earlier than the other.)

Given α's history $h_{\alpha|T_\beta}$ and γ's rational strategy, β chooses the best time to fight.

Lemma 2 *Suppose that α initiates a rebellion and γ adopts the strategy of Lemma 1. (i) After α's "win," β fights simultaneously with γ. (ii) After α's "loss," β acquiesces forever. (iii) During α's "holdout," the timing of β's fighting $T_\beta+1$ is determined as the smallest $T+1$ such that $V_{\beta|T+1}\left(hold_{\alpha|T}|T_\gamma\right) \geq V_{\beta|T+\tau}\left(hold_{\alpha|T}|T_\gamma\right)$ for any $\tau \geq 2$, where $V_{\beta|T+\tau}\left(hold_{\alpha|T}|T_\gamma\right)$ is β's expected payoff from fighting in $T+\tau$ with γ's participation from T_γ based on $hold_{\alpha|T}$:*

$$V_{\beta|T+\tau}\left(hold_{\alpha|T}|T_\gamma\right) \equiv \sum_{g\in\{g^W,g^S\}} \Pr\left(g|hold_{\alpha|T}\right)\Pr\left(win \cup hold|G_\alpha\right)^{\tau-1} \left(\begin{array}{l} \sum_{t=T+\tau}^{T_\gamma} \Pr\left(hold|G_{\alpha,\beta}\right)^{t-T-\tau}\pi_\beta\left(G_{\alpha,\beta}\right) \\ +\Pr\left(hold|G_{\alpha,\beta}\right)^{T_\gamma-T-\tau+1}\Pi_\beta\left(G_{\alpha,\beta,\gamma}\right) \end{array}\right) - c_\beta.$$

(iv) During α's "holdout," β fights before γ (i.e., $T_\beta < T_\gamma$) if T_γ satisfies $V_{\beta|T_\gamma}\left(hold_{\alpha|T_\gamma-1}|T_\gamma\right) \geq V_{\beta|T_\gamma+1}\left(hold_{\alpha|T_\gamma-1}|T_\gamma\right)$, or if

$$\begin{aligned} &\sum_{g\in\{g^W,g^S\}} \Pr\left(g|hold_{\alpha|T_\gamma-1}\right)\Pr\left(loss|G_\alpha\right)\left(\Pi_\beta\left(G_{\alpha,\beta,\gamma}\right)-c_\beta\right) \\ \geq &\sum_{g\in\{g^W,g^S\}} \Pr\left(g|hold_{\alpha|T_\gamma-1}\right)\left(\Pi_\beta\left(G_{\alpha,\beta,\gamma}\right)-\pi_\beta\left(G_{\alpha,\beta}\right)-\Pr\left(hold|G_{\alpha,\beta}\right)\Pi_\beta\left(G_{\alpha,\beta,\gamma}\right)\right) \end{aligned} \quad (2)$$

Proof. (i, ii) Group β's decision after α's "win" ("loss") is given by Assumption 3-(i) (3-(ii)). (iii) To determine T_β, β compares its expected payoffs from fighting in all future periods ($T+\tau$ for any $\tau \geq 2$). (Unlike γ, β's incentive to fight does not monotonically increase owing to γ's lagged fighting at $T_\gamma + 1$.) (iv) Inequality (2) checks β's incentive at T_γ. If it holds, β fights at T_γ instead of at $T_\gamma + 1$, or $T_\beta < T_\gamma$.

■

When determining the time to fight, β confronts a dilemma—as β delays fighting, α becomes more likely to be defeated (shown in the LHS of Inequality (2)), but γ's support will be introduced sooner (in the RHS). When the former incentive outweighs the latter, β joins α's rebellion.

6 Initial Challenge and Coalition

Taking into account β and γ's strategies in Lemmas 1 and 2, α decides whether to provoke a rebellion.

Proposition 1 *A snowballing rebellion can break out if all players' strategies in Definition 1 are incentive compatible in the sense that for T_γ of Lemma 1 and T_β of Lemma 2, (i) Inequality (1) does not hold, (ii) Inequality (2) holds, and (iii) $V^{Sn}_{\alpha|1}(T_\beta, T_\gamma) \geq 0$, where $V^{Sn}_{\alpha|1}(T_\beta, T_\gamma)$ is α's expected payoff from a snowballing rebellion in Definition 1:*

$$V^{Sn}_{\alpha|1}(T_\beta, T_\gamma) \equiv \sum_{g\in\{g^W, g^S\}} \Pr(g) \left(\begin{array}{c} \sum_{t=1}^{T_\beta} \Pr(hold|G_\alpha)^{t-1} \pi_\alpha(G_\alpha) + \Pr(hold|G_\alpha)^{T_\beta} \\ \left(\begin{array}{l} \sum_{t=T_\beta+1}^{T_\gamma} \Pr(hold|G_{\alpha,\beta})^{t-T_\beta-1} \pi_\alpha(G_{\alpha,\beta}) \\ + \Pr(hold|G_{\alpha,\beta})^{T_\gamma - T_\beta} \Pi_\alpha(G_{\alpha,\beta,\gamma}) \end{array} \right) \end{array} \right) - c_\alpha.$$

Proof. We check the incentive compatibility for each player. (i) For γ, given T_β, γ chooses T_γ such that $T_\beta < T_\gamma$ if Inequality (1) does not hold. (ii) For β, given T_γ, β chooses T_β such that $T_\beta < T_\gamma$ if Inequality (2) holds. In addition, β acquiesces at least once (Assumption 4-(i)). (iii) The rebellion assumes a snowballing pattern if it is initiated by α, or if $V^{Sn}_{\alpha|1}(T_\beta, T_\gamma) \geq 0$. If α "wins," β and γ immediately fight (Assumptions 2-(i) and 3-(i)). If α "loses," they never fight (Assumption 3-(ii)). ■

Proposition 1 suggests that sequential spreading of rebel movements is most likely when the three players are heterogeneous in terms of strength and resolve: the first player leads the rebellion; then, the second overpowers the third. For instance, in period one, while α is willing to fight, the others are not (Assumption 4-(i)); then, in period $T_\beta + 1$, while β is willing to fight, γ is not (Inequality (2) holds, but not (1)). These time lags in fighting stem from the differences in parameters r_i, w_i, l_i, and c_i.

A catalytic rebellion may take place when one player is significantly stronger, while the other two have similar propensities for fighting.

Proposition 2 *A catalytic rebellion can break out if (i) T_γ of Lemma 1 equals T_β of Lemma 2 (so that Inequality (1) holds, while Inequality (2) does not), and (ii) $V^{Ca}_{\alpha|1}(T_{\beta,\gamma}) \geq 0$, as defined with $T_{\beta,\gamma} \equiv T_\beta = T_\gamma$, where $V^{Ca}_{\alpha|1}(T_{\beta,\gamma})$ is α's expected payoff from a catalytic rebellion in Definition 2:*

$$V^{Ca}_{\alpha|1}(T_{\beta,\gamma}) \equiv \sum_{g \in \{g^W, g^S\}} \Pr(g) \left(\begin{array}{c} \sum_{t=1}^{T_{\beta,\gamma}} \Pr(hold|G_\alpha)^{t-1} \pi_\alpha(G_\alpha) \\ + \Pr(hold|G_\alpha)^{T_{\beta,\gamma}} \Pi_\alpha(G_{\alpha,\beta,\gamma}) \end{array} \right) - c_\alpha.$$

Proof. As in the proof of Proposition 1, we check each players' incentive. (i) During α's "holdout," β and γ fight simultaneously if $T_\beta = T_\gamma$, which requires that Inequality (1) holds, while Inequality (2) does not. In addition, they acquiesce at least once (Assumption 4-(i)). (ii) For α, the outbreak of the catalytic rebellion requires that $V^{Ca}_{\alpha|1}(T_{\beta,\gamma}) \geq 0$. ■

Proposition 2 suggests that when α provokes the rebellion, β and γ are unwilling to take up arms even jointly with others (Assumption 4-(i)). However, as the rebellion evolves, they update their evaluation of the government's strength, so that they convince themselves that the challenge is worthwhile if the other also rebels (Assumption 3-(i)). Then they face the coordination dilemma. After a certain length $T_{\beta,\gamma}$ of α's battles, joint challenges by β and γ in any period $T+1$ can constitute a perfect Bayesian equilibrium as long as $V^{Ca}_{\alpha|1}(T|T \geq T_{\beta,\gamma}) \geq 0$.

A partial coalition is sought by a pair of players who are unwilling to rebel on their own, but who are willing to jointly challenge the government.

Proposition 3 *A partially coalitional rebellion can break out if T_β and T_γ of Lemmas 1 and 2 are such that (i) no player is willing to fight alone ($V^{Sn}_{\alpha|1}(T_\beta, T_\gamma) \ngeq 0$ for $T_\beta < T_\gamma$; $V^{Ca}_{\alpha|1}(T_{\beta,\gamma}) \ngeq 0$ for $T_{\beta,\gamma} \equiv T_\beta = T_\gamma$), but (ii) two players α and β have sufficient resolve that* $\min\left\{V^{Pa}_{\alpha|1}(T_\gamma), V^{Pa}_{\beta|1}(T_\gamma)\right\} \geq 0$*, where $V^{Pa}_{i|1}(T_\gamma)$ for $i \in \{\alpha, \beta\}$*

is i's expected payoff from a partially coalitional rebellion, as given in Definition 3:

$$V_{i|1}^{Pa}(T_\gamma) \equiv \sum_{g\in\{g^W,g^S\}} \Pr(g) \left(\begin{array}{c} \sum_{t=1}^{T_\gamma} \Pr(hold|G_{\alpha,\beta})^{t-1} \pi_i(G_{\alpha,\beta}) \\ +\Pr(hold|G_{\alpha,\beta})^{T_\gamma} \Pi_i(G_{\alpha,\beta,\gamma}) \end{array} \right) - c_i.$$

Proof. Players α and β, but not γ, are willing to jointly fight because (i) a rebellion is impossible without a coalition, and (ii) for α and β, the partially coordinated rebellion is preferred to permanent "acquiesce." On the other hand, γ is willing to acquiesce at least once (Assumption 4-(ii)). If α and β "win," γ fights immediately (Assumptions 2-(i) and 3-(i)). If they "lose," γ never fights (Assumption 3-(ii)). ■

Full coordination is most likely when power is distributed equally across the three players in the sense that no sole player or partial coalition is willing to fight.

Proposition 4 *A fully coalitional rebellion can break out if T_β and T_γ of Lemmas 1 and 2 are such that (i) no player is willing to fight alone ($V_{\alpha|1}^{Sn}(T_\beta, T_\gamma) \ngeq 0$ for $T_\beta < T_\gamma$; $V_{\alpha|1}^{Ca}(T_{\beta,\gamma}) \ngeq 0$ for $T_{\beta,\gamma} \equiv T_\beta = T_\gamma$), (ii) no pair of players are willing to fight ($\min\left\{V_{\alpha|1}^{Pa}(T_\gamma), V_{\beta|1}^{Pa}(T_\gamma)\right\} \ngeq 0$), but (iii) all three players have sufficient resolve to collectively fight the government with unknown strength such that $\min\left\{V_\alpha^{Fu}, V_\beta^{Fu}, V_\gamma^{Fu}\right\} \geq 0$, where V_i^{Fu} for $i \in \{\alpha,\beta,\gamma\}$ is i's expected payoff from a fully coalitional rebellion, as given in Definition 4:*

$$V_i^{Fu} \equiv \sum_{g\in\{g^W,g^S\}} \Pr(g) \Pi_i(G_{\alpha,\beta,\gamma}) - c_i.$$

Proof. The three players are willing to challenge concurrently because (i, ii) no other form of rebellion is incentive compatible, but (iii) they all prefer the fully coalitional rebellion to permanent "acquiesce." ■

If ther are more than three players, even more complicated patterns may emerge, but they still fall into some combination of these four patterns.

7 Numerical Example

A snowballing rebellion emerges with the following parameter values: $w_\alpha = w_\beta = 12$; $w_\gamma = 8$; $l_\alpha = l_\beta = l_\gamma = 3$; $c_\alpha = c_\beta = c_\gamma = 1$; $r_\alpha = 4$; $r_\beta = r_\gamma = 3$; $g^L = 8$; $g^H = 12$;

$\Pr\left(g^L\right) = \Pr\left(g^H\right) = \frac{1}{2}$; $\Pr\left(hold|G\right) = \frac{99}{100}\exp\left(-\frac{|G|}{100}\right)$; and $\Pr\left(win|G \in [0,5)\right) = \Pr\left(loss|G \in (-5,0)\right) = \frac{1}{200}\exp\left(-\frac{|G|}{2}\right)$. In this equilibrium, α provokes the rebellion, β joins in at $T_\beta = 10$ (with probability 0.509), and γ follows at $T_\gamma = 15$ (with probability 0.428). If β is as bellicose as γ when $w_\beta = 8$ (instead of 12), β and γ fight simultaneously at $T_{\beta,\gamma} \geq 15$, constituting a catalytic rebellion.

8 Conclusion

This study contributes to the theoretical literature on war. Although multilateral war is difficult to theorize (Jackson and Morelli 2011), recent studies have modeled it by simplifying some of its aspects (Krainin and Wiseman 2016). Echoing this theoretical trend, we have formally analyzed a class of war fought between a hegemon and three challengers in order to demonstrate that a rebellion can break out and expand in one of the four patterns. Further theoretical studies of multilateral war are expected.

References

[1] Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan. 2007. "Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks." *Econometrica.* 75(3): 711-756.

[2] Bueno de Mesquita, Ethan. 2010. "Regime Change and Revolutionary Entrepreneurs." *American Political Science Review.* 104(3): 446-466.

[3] Fearon, James D. 2011. "Self-Enforcing Democracy." *Quarterly Journal of Economics.* 126(4): 1661-1708.

[4] Ginkel, John, and Alastair Smith. 1999. " So You Say You Want a Revolution: A Game Theoretic Explanation of Revolution in Representative Regimes." *Journal of Conflict Resolution.* 43(3): 291-316.

[5] Granovetter, Mark. 1978. "Threshold Model of Collective Behavior." *American Journal of Sociology.* 83(6): 1420-1443.

[6] Jackson, Matthew O., and Massimio Morelli. 2011. "The Reasons for Wars - an Updated Survey." In: Christopher J. Coyne & Rachel L. Mathers, eds. *The*

Handbook on the Political Economy of War. Northampton, MA: Elgar Publishing, 34-57.

[7] Krainin, Colin, and Thomas Wiseman. 2016. "War and Stability in Dynamic International Systems." *Journal of Politics.* 78(4): 1139-1152.

[8] Nakao, Keisuke. 2015. "Expansion of Rebellion: From Periphery to Heartland." *Journal of Peace Research.* 52(5): 591-606.

[9] Roemer, John E. 1985. "Rationalizing Revolutionary Ideology." *Econometrica.* 53(1): 85-108.

[10] Weingast, Barry R. 1995. "The Economic Role of Political Institutions: Market Preserving Federalism and Economic Development." *Journal of Law, Economics, & Organization.* 11(1): 1-31.

[11] Weingast, Barry R. 1997. "The Political Foundation of Democracy and the Rule of Law." *American Political Science Review.* 91(2): 245-263.

[12] Weingast, Barry R. 2005. "The Constitutional Dilemma of Economic Liberty." *Journal of Economic Perspectives.* 19(3): 89-108.