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ABSTRACT

This paper extends the traditional analysis of the output effect under monopoly (third-degree) price discrimination to a multimarket Cournot oligopoly. Under symmetric Cournot oligopoly (all firms selling in all markets) similar results to those under monopoly are obtained: in order for price discrimination to increase total output the demand and inverse demand of the strong market (the high price market) should be, as conjectured by Robinson (1933), more concave than the demand and inverse demand of the weak market (the low price one). When competitive pressure (measured by the number of firms) varies across markets the effect of price discrimination on total output crucially depends on what market, the strong or the weak, is more competitive.

Keywords: Third-Degree Price Discrimination, Output, Oligopoly, Welfare.

JEL Classification: D42, L12, L13.

1   Introduction

With respect to uniform pricing, third-degree price discrimination generates two effects: first, price discrimination causes a misallocation of goods from high to low value users and, second, price discrimination affects total output.[1] Therefore, a necessary condition for third-degree price discrimination to increase social welfare is an increase in total output.[2] As a result, a focal point has been the analysis of the effects of price discrimination on output.[3] Since Robinson (1933) much research has addressed this issue.[4] It is known from Pigou (1920) that under linear demands price discrimination does not change output. In the general non-linear case, however, the effect of price discrimination on output may be either positive or negative. It is also well known (see, for example, Robinson, 1933, Silberberg, 1970, or Schmalensee, 1981) that when all the strong markets (markets where the optimal discriminatory price exceeds the optimal single price) have concave demands and the weak markets (where the optimal discriminatory prices are lower than the single price) have convex demands (with at least one market with strict concavity or convexity), then third-degree price discrimination increases output. When strong markets have convex demands and weak markets concave demands price discrimination reduces output. In the case in which all the demand curves have similar curvature the answer is more complicated. Shih, Mai and Liu (1988) and Cheung and Wang (1994) obtain more general results and Aguirre (2009), Aguirre, Cowan and Vickers (2010) and Cowan (2016) show that the effect of third-degree price discrimination on total output is intrinsically related to both the shape of demands and inverse demands in strong markets compared to the shape of direct and inverse demands in weak markets.

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[1] See, for example, Ippolito (1980), Schmalensee (1981), Aguirre (2012) or Aguirre, Cowan and Vickers (2010) for explicit decompositions of the change in social welfare into these two effects: the misallocation effect and the output effect.

[2] See Schmalensee (1981), Varian (1985) and Schwartz (1990). Ippolito (1980) and Aguirre (2012) provide an alternative proof.

[3] It is assumed throughout the paper that all markets are served under both pricing regimes, uniform pricing and price discrimination. The possibility that price discrimination opens up new markets (and that may even yield Pareto improvements). See, for example, Hausman and Mackie-Mason (1988).

[4] The literature includes Schmalensee (1981), Shih, Mai and Liu (1988), Cheung and Wang (1994), Aguirre (2009), Aguirre, Cowan and Vickers (2010) and Cowan (2016) among others.

Over the last few decades much research has analyzed price discrimination in oligopolistic markets both under price competition and quantity competition.[5] Here we focus on price discrimination under quantity competition on which a quote from Stole (2007) results relevant: "Perhaps the simplest model of imperfect competition and price discrimination is the immediate extension of Cournot's quantity-setting, homogeneous-good game to firms competing in distinct market segments."[6] The Cournot model has been widely used to analyze price discrimination in many different contexts.[7] In this paper, we extend the traditional analysis of the output effect under monopoly third-degree price discrimination to a multimarket Cournot oligopoly.[8] We show that under symmetric Cournot oligopoly (all firms selling in all markets) similar results to those under monopoly are obtained: in order for total output to increase with price discrimination the demand of the strong market (the high price market) should be, as conjectured by Robinson (1933), more concave than the demand of the weak market (the low price one). When competitive pressure (measured by the number of firms) varies across markets the effect of price discrimination on total output crucially depends on what market, the strong or the weak, is more competitive. Importantly, some new unexpected results are obtained, even with linear demand. First, we show that price discrimination in favor of the more competitive market is quite generally output

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[5] Many papers have analyzed oligopolistic price discrimination under price competition. See Holmes (1989), Corts (1998), Dastidar (2006) and Adachi and Matsushima (2014).

[6] Various empirical studies provide support for the assumption that Cournot competition prevails in some markets as, for instance, in the airline market, a market where price discrimination is quite common (see, for example, Brander and Zhang, 1990, and Oum, Zhang and Zhang, 1993).

[7] Neven and Phlips (1985), Howell (1991), Cheung and Wang (1997), Aguirre (2000), Galera and Zaratiegui (2006), Hazledine (2006), Stole (2007), Hazledine (2010), Kutlu (2012), Bakó and Kálecz-Simon (2012), Czerny and Zhang (2014), Kumar and Kutlu (2015) and Aguirre (2016) consider price discrimination under quantity competition in the final good market. In a context of intertemporal price discrimination Dana and Williams (2016) show how inventory controls can facilitate oligopoly price discrimination. There is also an important literature that analyzes quantity competition in contexts of spatial price discrimination: see, for instance, Hamilton, Thisse and Weskamp (1986). It is also common in contexts of international trade to study price discrimination under Cournot competition; see, for instance, the Brander and Krugman's (1983) model of reciprocal dumping, a case of price discrimination, or Krugman (1989) for a general view.

[8] Our model of multimarket Cournot oligopoly can be seen as a particular case of multiproduct Cournot oligopoly (see, for instancem Johnson and Myatt,2006, and Armstrong and Vickers, 2016).

reducing, therefore leading to a welfare deterioration. This result maintains unambiguosly under linear demand or when the strong market exhibits convex demand and the weak market concave demand. Our results are in line with those of Holmes (1989) and Weyl and Fabinger (2013) who suggest that price discrimination against the more competitive markets (measured by the number of firms) might reduce social welfare through decreasing total output. Second, when the competitive pressure is higher in the strong market we obtain the important result that independently of the shape of demands and inverse demands price discrimination can increase total output. In particular we show that even with linear demand price discrimination increases total output.

The paper is organized as follows. Section 2 analyzes the output effect of price discrimination for a Cournot oligopoly showing that the results crucially depend on whether competitive pressure varies across markets and on which market, the strong of the weak, is more competitive. Section 3 presents some concluding remarks.

2   Analysis

Consider a Cournot oligopoly selling a homogeneous product in two perfectly separated markets. Market 1 is served by n_1 firms and market 2 by n_2. The inverse demand function in market i is given by $p_i(q_i)$, where q_i is the quantity sold. Unit cost, c, is assumed constant. The profit function of firm j in market $i = 1, 2$ is given by: $\pi_{ji}(q_{ji}, q_{-ji}) = [\, p_i(q_i) - c]q_{ji}$, where q_{ji} is the quantity sold by firm j in market i and $q_{-ji} = q_i - q_{ji}$, which is assumed to be strictly concave. We shall obtain the change in total output due to a move from third-degree price discrimination to uniform pricing.

Under price discrimination firms present in both markets choose their production in each market independently. By adding first order conditions (assuming an interior equilibrium with second order conditions satisfied) we obtain that the equilbrium total output in market i satisfies:

$$n_i[p_i(q_i^d) - c] + q_i^d p_i^{'}(q_i^d) = 0 \qquad i = 1, 2, \qquad (1)$$

where q_i^d denotes the Cournot total output in market i.[9] From condition (1)

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[9]Following Kreps and Sheinkman (1983) Cournot competition can be interpreted as the

we obtain that the equilibrium price can be written as:

$$p_i(q_i^d) = \frac{c}{1 - \frac{1}{n_i \varepsilon_i(q_i^d)}} \qquad i = 1, 2, \qquad (2)$$

where $\varepsilon_i(q_i) = -\frac{1}{p_i'(q_i)} \frac{p_i(q_i)}{q_i}$ is the elasticity of demand in market $i = 1, 2$. Therefore, we obtain a generalization of the monopolistic price discrimination rule to a Cournot oligopoly: $p_1(q_1^d) > p_2(q_2^d)$ iff $n_1 \varepsilon_1(q_1^d) < n_2 \varepsilon_2(q_2^d)$. From now on, we assume that market 1 is the strong market, $p_1(q_1^d) > p_2(q_2^d)$. Consequently, if the number of firms does not vary accross markets the Cournot price is higher in the market with the lower elasticity. The total output under price discrimination is $Q^d = \sum_{i=1}^{2} q_i^d$ which, given (1), can be expressed as:

$$Q^d = \sum_{i=1}^{2} q_i^d = -\sum_{i=1}^{2} n_i \frac{[p_i(q_i^d) - c]}{p_i'(q_i^d)}. \qquad (3)$$

In order to solve the problem under uniform pricing, we distinguish between firms that sell in both markets and firms that only sell in one of the two markets. Assume that there are $n_B > 0$ firms selling in both markets, $n_1 - n_B > 0$ firms selling only in market 1 and $n_2 - n_B > 0$ firms selling only in market 2. If we aggregate first order conditions for firms that are only in market i we get:

$$(n_i - n_B)[p_i(q_i^0) - c] + q_{ii}^0 p_i'(q_i^0) = 0 \qquad i = 1, 2, \qquad (4)$$

where q_{ii}^0 is the total output produced by firms only set in market $i = 1, 2$. Under uniform pricing, a firm that sells in both markets has to adjust production in order to maintain the same price in both markets.[10] From the

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reduced form of a two-stage game where firms first choose capacities and then set prices. d'Aspremont, Dos Santos Ferreira and Gérard-Varet (1991), (2007) and d'Aspremont and Dos Santos Ferreira (2009) consider other nice justifications of quantity competition.

[10] Note that price discrimination might be illegal, or impracticable due to regulation or arbitrage and the multimarket firms could be forced to adjust output across markets in order to satisfy price uniformity. Or, equivalently, multimarket firms might sign most-favored-customers (MFC) clauses with their clients committing to price uniformly (see, for instance, Aguirre, 2000). The argument is as follows. Assume that multimarket firm j adopts an MFC policy and that market 1 is strong. If firm j chooses its outputs q_{j1} and q_{j2} so that $p_1(q_{j1} + q_{-j1}) > p_2(q_{j2} + q_{-j2})$ then it must rebate $[p_1(q_{j1} + q_{-j1}) - p_2(q_{j2} + q_{-j2})]q_{j1}$ to market 1 customers. Firm j can increase its profits by choosing an output $q_{j1}' > q_{j1}$, such that given q_{-j1} and q_{-j2}, $p_2(q_{j2} + q_{-j2})$ is the price of both markets obtaining $[p_2(q_{j2} + q_{-j2}) - c](q_{j1}' + q_{j1})$.

first order conditions and by adding over firms set in both markets we get:

$$n_B[p_1(q_1^0)-c]p_2'(q_2^0)+n_B[p_2(q_2^0)-c]p_1'(q_1^0)+q_{B1}^0 p_1'(q_1^0)p_2'(q_2^0)+q_{B2}^0 p_1'(q_1^0)p_2'(q_2^0) = 0,$$
(5)

where q_{Bi}^0 is the total output sold in market $i = 1, 2$ by the firms selling in both markets.[11]

Therefore:

$$q_{B1}^0 p_1'(q_1^0)p_2'(q_2^0)+q_{B2}^0 p_1'(q_1^0)p_2'(q_2^0) = -n_B[p_1(q_1^0)-c]p_2'(q_2^0)-n_B[p_2(q_2^0)-c]p_1'(q_1^0).$$
(6)

$$q_{B1}^0 + q_{B2}^0 = -\frac{n_B[p_1(q_1^0) - c]p_2'(q_2^0) + n_B[p_2(q_2^0) - c]p_1'(q_1^0)}{p_1'(q_1^0)p_2'(q_2^0)}.$$
(7)

$$q_{B1}^0 + q_{B2}^0 = -n_B\frac{[p_1(q_1^0) - c]}{p_1'(q_1^0)} - n_B\frac{[p_2(q_2^0) - c]}{p_2'(q_2^0)}.$$
(8)

It is satisfied that $p_1(q_1^d) > p_1(q_1^0) = p_2(q_2^0) > p_2(q_2^d)$ and the total output, $Q^0 = \sum_{i=1}^{2} q_i^0$, can be expressed, given (4) and (8), as:

$$Q^0 = \sum_{i=1}^{2} q_i^0 = -\sum_{i=1}^{2} n_i\frac{[p_i(q_i^0) - c]}{p_i'(q_i^0)}.$$
(9)

We follow closely the analysis by Cheung and Wang (1997). Given conditions (3) and (9), the change in total output is given by:

$$\Delta Q = Q^d - Q^0 = -\sum_{i=1}^{2} n_i\frac{[p_i(q_i^d) - c]}{p_i'(q_i^d)} + \sum_{i=1}^{2} n_i\frac{[p_i(q_i^0) - c]}{p_i'(q_i^0)}.$$
(10)

We can rewrite (10) as:

$$\Delta Q = -\sum_{i=1}^{2}\left\{\int_{q_i^0}^{q_i^d} d\left[n_i\frac{[p_i(q_i) - c]}{p_i'(q_i)}\right]\right\}.$$
(11)

Therefore, the change in total output can be expressed as:

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[11]We assume that the bordered Hessian is negative definite.

$$\Delta Q = -\sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i'(q_i)]^2 - [p_i(q_i) - c]p_i''(q_i)}{[p_i'(q_i)]^2} dq_i \right\},$$

$$= -\sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \{1 - \frac{[p_i(q_i) - c]p_i''(q_i)}{[p_i'(q_i)]^2}\} dq_i \right\},$$

$$= \sum_{i=1}^{2} -n_i \Delta q_i + \sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i''(q_i)}{[p_i'(q_i)]^2} dq_i \right\}. \qquad (12)$$

Expression (12) can be written as:

$$\Delta Q = \sum_{i=1}^{2} -n_i \Delta q_i - \sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} L_i(q_i) \varepsilon_i(q_i) C_i^I(q_i) dq_i \right\}, \qquad (13)$$

where $L_i(q_i) = \frac{[p_i(q_i) - c]}{p_i(q_i)}$ is the Lerner index of market i, $\varepsilon_i(q_i) = -\frac{1}{p_i'(q_i)} \frac{p_i(q_i)}{q_i}$ is the elasticity of demand of market i and $C_i^I(q_i) = q_i \frac{p_i''(q_i)}{p_i'(q_i)}$ is the adjusted concavity of the inverse demand in market i (this is analogous to relative risk aversion for a utility function). The adjusted concavity of the direct demand, $C_i^D(q_i) = -\frac{p_i''(q_i)p_i(q_i)}{[p_i'(q_i)]^2}$, is given by $C_i^D(q_i) = \varepsilon_i(q_i)C_i^I(q_i)$. Therefore we can express the change in total output alternatively as:

$$\Delta Q = \sum_{i=1}^{2} -n_i \Delta q_i - \sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} L_i(q_i) C_i^D(q_i) dq_i \right\}. \qquad (14)$$

We next show that the change of total output crucially depends on whether all firms are present in all markets.

(i) *Symmetric Multimarket Cournot Oligopoly.*

First, we consider a symmetric multimarket Cournot oligopoly with all firms selling in all markets and therefore $n_1 = n_2 = n$. The change in total output is (see, Cheung and Wang, 1997):

$$\Delta Q = \frac{n}{1+n} \sum_{i=1}^{2} \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i''(q_i)}{[p_i'(q_i)]^2} dq_i \right\}.$$

$$= -\frac{n}{1+n} \sum_{i=1}^{2} \left\{ \int_{q_i^0}^{q_i^d} L_i(q_i)\varepsilon_i(q_i)C_i^I(q_i)dq_i \right\}. \tag{15}$$

One advantage of Cournot oligopoly is that it converges to the monopoly case when $n = 1$. Under monopoly a move from uniform pricing to third-degree price discrimination leads to (see, Cheung and Wang, 1994, Aguirre, 2009, or Cowan, 2016):

$$\Delta Q = \frac{1}{2} \sum_{i=1}^{2} \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i^{''}(q_i)}{[p_i^{'}(q_i)]^2} dq_i \right\}.$$

$$= -\frac{1}{2} \sum_{i=1}^{2} \left\{ \int_{q_i^0}^{q_i^d} L_i(q_i)\varepsilon_i(q_i)C_i^I(q_i)dq_i \right\}. \tag{16}$$

Therefore, we can immediately extend the results under monopoly obtained by Pigou (1920), Robinson (1933), Schmalensee (1981) and, more recently, by Shih, Mai and Liu (1988), Cheung and Wang, 1994, Aguirre, 2009, Aguirre, Cowan and Vickers, 2010, and Cowan (2016) to a symmetric Cournot oligopoly. For instance, under linear demand total output does not change both for a monopoly and for a symmetric Cournot oligopoly independently of the number of firms (see Neven and Phlips, 1985, and Howell, 1991; Stole, 2007, provides a more elegant proof). The next proposition summarizes the effect of third-degree price discrimination on total output under monopoly and symmetric Cournot oligopoly.[12]

Proposition 1. *Effect of third-degree price discrimination on total output under monopoly and symmetric Cournot oligopoly.*
(i) *If both direct demand curves and inverse demand curves are more concave in strong markets than in weak markets, then third-degree price discrimination increases total output.*
(ii) *If both direct demand curves and inverse demand curves are less (or equally) concave in strong markets than in weak markets, then third-degree price discrimination does not increase total output.*

Proof. With respect to the results under monopoly, see the proof of Theorem 1 in Aguirre (2009) for the n-market case. In Aguirre, Cowan and Vickers

(2010), this result appears as a corollary of their Proposition 4, for the two-market case. Cowan (2016) presents an elegant proof of case (i) for the n-market case.[13] Cheung and Wang (1994) for the case of monopoly and Cheung and Wang (1997) for the case of Cournot oligopoly provide a weaker version of this proposition.∎

(ii) *Asymmetric Multimarket Cournot Oligopoly.* Consider an asymmetric multimarket Cournot oligopoly with $n_1 \neq n_2$. The effect of price discrimination on total output crucially depends on which market, the strong or the weak, exhibits more competitive pressure as measured by the number of firms.

a) $n_2 > n_1$

When the weak market has more firms we can rewrite (12) as

$$
\begin{aligned}
\Delta Q &= \sum_{i=1}^{2} -n_i \Delta q_i + \sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i^{''}(q_i)}{[p_i^{'}(q_i)]^2} dq_i \right\}, \\
&= -(n_2 - n_1)\Delta q_2 - n_1(\Delta q_1 + \Delta q_2) \\
&\quad + \sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i^{''}(q_i)}{[p_i^{'}(q_i)]^2} dq_i \right\}, \\
&= -\frac{(n_2 - n_1)}{1 + n_1}\Delta q_2 + \frac{1}{1 + n_1} \sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i^{''}(q_i)}{[p_i^{'}(q_i)]^2} dq_i \right\} \quad (17)
\end{aligned}
$$

Proposition 2. *Effect of third-degree price discrimination on total output when the weak market is more competitive.*

If the competitive pressure, measured by the number of firms, is higher in the weak market, it is ceteris paribus more probable that price discrimination to reduce total output and therefore social welfare.

Proof. Note that the first term in (17) is negative given that market 2 is the weak market, $\Delta q_2 > 0$, and it exhibits higher competitive pressure, $n_2 > n_1$. Even with inverse and direct demands more concave in the strong market price discrimination might reduce total output.∎

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[13]See Weyl and Fabinger (2013) for a nice interpretation in terms of pass-through.

Given that first term in (17) is negative, we obtain the result that total output (and, therefore, social welfare) might decrease independently of the shape of inverse and direct demands when firms discriminate prices in favor of the market with more competitive pressure. This result is in line with the results of Holmes (1989) and Weyl and Fabinger (2013) who suggest that when discrimination is in favor of individuals for whom competition is more intense, discrimination is more likely to be harmful.

b) $n_1 > n_2$

When the strong market has more firms we can rewrite (12) as

$$
\begin{aligned}
\Delta Q &= \sum_{i=1}^{2} -n_i \Delta q_i + \sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i''(q_i)}{[p_i'(q_i)]^2} dq_i \right\}, \\
&= -(n_1 - n_2)\Delta q_1 - n_2(\Delta q_1 + \Delta q_2) \\
&\quad + \sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i''(q_i)}{[p_i'(q_i)]^2} dq_i \right\}, \\
&= -\frac{(n_1 - n_2)}{1 + n_2}\Delta q_1 + \frac{1}{1 + n_2} \sum_{i=1}^{2} n_i \left\{ \int_{q_i^0}^{q_i^d} \frac{[p_i(q_i) - c]p_i''(q_i)}{[p_i'(q_i)]^2} dq_i \right\} (18)
\end{aligned}
$$

Note that regardless of the shape of the inverse demands there exists a tendency for price discrimination to increase total output given that the first term in (18) is positive because market 1 is the strong market, $\Delta q_1 < 0$, and it exhibits higher competitive pressure, $n_1 > n_2$. The next proposition presents some perhaps unexpected results.

Proposition 3. *Effect of third-degree price discrimination on total output when the strong market is more competitive.*
If competitive pressure measured by the number of firms is higher in the strong market, independently of the shape of direct demands and inverse demands total output can increase with price discrimination..

Proof. Note that the first term in (18) is positive given that market 1 is the strong market, $\Delta q_1 > 0$, and it exhibits higher competitive pressure, $n_1 > n_2$. Even with inverse and direct demands more concave in the weak market price discrimination might increase total output.∎

Note that when the strong market exhibits more competitive pressure, $n_1 > n_2$, general results of part (i) in Proposition 1 maintains: if both direct de-

mand curves and inverse demand curves are more concave in strong markets than in weak markets, then third-degree price discrimination increases total output. However, now it is possible price discrimination leads to an increase in total output when inverse and direct demands are not more concave in the strong markets. In particular, the next corollary states that the most cited result from Pigou (1920) and Robinson (1933) does not maintain yet.

Corollary 1. *Total output increases with price discrimination under linear demand.*

Proof. Note that the second term in (18) is zero under linerar demand. So given that market 1 is the strong market, $\Delta q_1 > 0$, if it exhibits higher competitive pressure, $n_1 > n_2$ then the first term in (18) is strictly positive.∎ These results sharply contrast with the well-known results under monopoly and symmetric Cournot oligopoly that price discrimination maintains total output unchanged with linear demand.

3   Concluding remarks

The analysis of the effects of third-degree price discrimination on total output and, therefore, on social welfare has been the focus of much theoretical research beginning at least from the pioneering work by Robinson (1933). In this paper, we show that the results under monopoly can be directly extended to a Cournot oligopoly with homogeneous product in the case in which all firms are established in all markets. When competitive pressure measured by the number of firms varies across markets we find some perhaps unexpected results. We show that when competitive pressure is higher in the strong market there is a tendency for price discrimination to increase output. On the other hand, price discrimination tends to reduce total output when competitive pressure is higher in the weak market, even with linear demand.

In this paper the market structure is considered as exogenous. Future research may consider extending the analysis to endogenous market structure.

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