Size effect in transitional dynamics of the banking network

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We consider a developed economy banking system, that, when surpass certain size, may destabilize and even enter in chaos. Taking Deposits (D_t), Reserves (R_t), Loans (L_t), the ratio of (R_t) to (D_t) and a parameter γ that weights endogenously the system memory, we analyse stability and the possibility of chaos. Using data for the U.S. between 1960 and 2012 we found that a maximum instability state is verified in 2008 when the crisis hits the U.S. banking system core carrying to a public bailout. A larger system does not necessarily lead to robustness but can expand to greater fragility. A proposed banking system stability indicator is also analysed.

Key Words: Banking System, Financial Crises, Instability, Systemic Risk, Chaos

JEL: G01 - Financial Crises

"Credit driven boom bust cycles are temporally asymmetrical.

The build up is slow and long, the collapse quick and sudden.

In Hemingway's The Sun Also Rises, one of the protagonists

Asks his friend: "How did you go bankrupt?" "Two ways", went

the answer, "gradually, then suddenly." (Leijonhufvud, 2013)

1. Introduction

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Systemic banking crises are rare events that strike in the middle of credit booms and are followed by long and deep recessions (Boissay, Collard and Smets 2013). Bank crises produce, among other consequences, payment chain disruptions and uncertainty on asset values and on debt re-payment capabilities.

Part of the economic literature associates financial (and aggregate economy) instability to fractional reserves banking systems (Cochrane 2012). By contrast, those advocating the benefits of bank credit point to the emergence of credit bubbles that are not sustainable over time as a source of instability (Schularick Moritz, Taylor 2009).

Traditional financial literatures see the origin of systemic banking crises as a classic situation of depositors panic or a mismatch in the interbanking flow of funds. Others (Diamond - Rajan 2005) pointed out to a rapid transmission of liquidity shortages as a cause of contagion that eventually turns into widespread insolvency.

Great efforts have been done in studying the sources of instability and fragility of the economic structure. An aspect of fragility we consider here is the growth of income inequality as a source of insolvency. Authors like Michael Kumhof and Romain Rancière (2010) employed a theoretical framework to explore the link between revenue enhancements enjoyed by high-income families; poor and middle classes increased indebtedness and financial system's vulnerability.

The bank system grows by accumulation of deposits of top income households and loans to maintain lifestyles of middle and low income sectors. The flip of a complex dynamical system from a state of steady growth to a highly unstable one, "*may be initiated by some obvious external event, such as a war, but is more usually triggered by a seemingly minor happenstance or even an unsubstantial rumour*" (May, Levin and Sugihara 2008:893). A minor increase in the bad debt ratio in some banks portfolios in a highly interconnected system with fast feedback mechanisms within it and slight changes in their loan policy, may become suddenly an explosive situation and afterwards *"will exhibit some form of hysteresis, such that recovery is much slower than the collapse. In extreme cases, the changes may be irreversible"* (May, Levin and Sugihara 2008:893). The impact of an unexpected increase in mortgage defaults, next to a drop in property values, may be taken as a minor change in the system's trend but may trigger a chaotic situation.

One way of representing systems is with a network. In this paper we adopt this view of the banking structure as a system. This implies that it possess at least nodes (banks, not necessarily all of equal size) and interconnections of different kinds. The topology of the banking network has been studied in detail in reference to the number of connections, the degree of interconnectedness, and of assortativity (Loepfe, et all and its cites, 2013). In his work the authors affirm that relevant properties of real world banking systems includes high clustering; size heterogeneity and sensibility to contagion even with low density connections. The authors also point out that "*the transition from safe to risky regimes can be very sharp*" and that the range for this "*is situated at low link densities and high values of modularity and size heterogeneity. Crucially all real world examples found in the literature show precisely these characteristics*".

We will adopt those results and analyze the stability of this kind of networks. Of the many topological characteristics of the banking network we select to investigate the relationship between an increasing size of the network and its effect on stability. In this work we take size as the quantity of deposits that the banking system operate. This definition allows comparability and consistency in different moments of time and place[2].

A larger banking system has been seen, before the 2007 crisis in the U.S, as a beneficial element, even by international financial institutions as the IMF. Bank concentration, was a sign of strength (Beck, Demirgüç-Levine 2006). In marked contrast, May, Levin, & Sugihara (2008) presented an argument that the combination of large size and high inter-connectivity can be, as in ecological systems, a source of instability.

As the UK Financial Services Authority point out in the Turner Review (Turner 2009) "*the shift to an increasingly securitised form of credit intermediation and the increased complexity of securitised credit relied upon market practices which, while rational from the point of view of individual participants, increased the extent to which procyclicality was hard-wired into the system*". The systemic risk associated to an increase in the size of the system by keeping unchanged the "hard-wired" interconnection is the subject of our work.

[2] Alternatively; "size of the banking system" as "quantity of bank entities" have some drawbacks. It may be misleading in the case when the quantity of bank entities shrinks and at the same time deposits grows or with different definitions of banks across countries. Even the definition of bank itself may change, moving the threshold conditions to consider an entity as bank.

An unexplored approach, intended herein, is to analyze the effects of the banking network size change [3] and explore some critical events (for example, the amount of US banking credit trend break in 2008 (Figure 2); where the system suddenly change from a long stable period of growth to a widespread recession).

Very simple nonlinear systems can have very complex dynamics (Aulbach & Kieninger 2001). Small changes in critical variables produce unexpected and unpredictable results modifying abruptly the development of the system (Fichter 1998). Apparently predictable, the system starts to behave in an unexpected way in a short period of time and stabilizes afterwards, but in a different trajectory.

Vitali, Glattfelder and Battiston (2011) shown that models with systemic-risk propensity are characterized by high connection density (interconnectivity). Instead of strength, concentration may increases fragility.

2. Stylized Dynamics

Over a long period of time (50 years) the US banking sector increased deposits and loans successfully with a declining reserve ratio ρ (see Figure 1). This growth was not seen as a problem neither by the authorities nor by the markets, until, unexpectedly, severe complications emerged.

[3] Stability defined as absence of excessive fluctuations of the variables and convergence when small shocks occurs (see for instance Heijdra, B. J., 2002; Romer, 2006; Shone ,2002).

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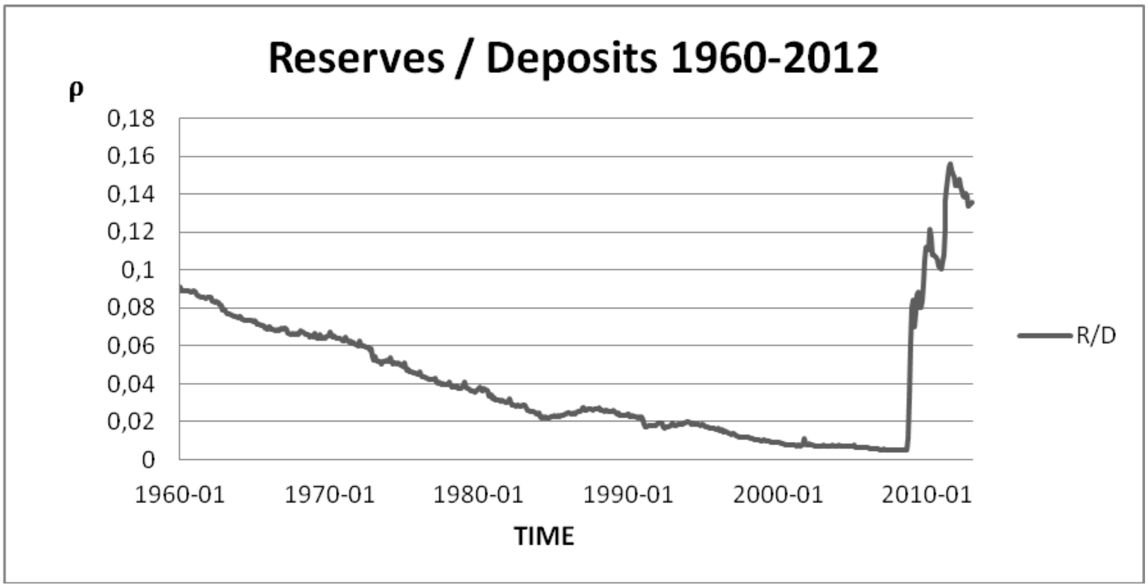
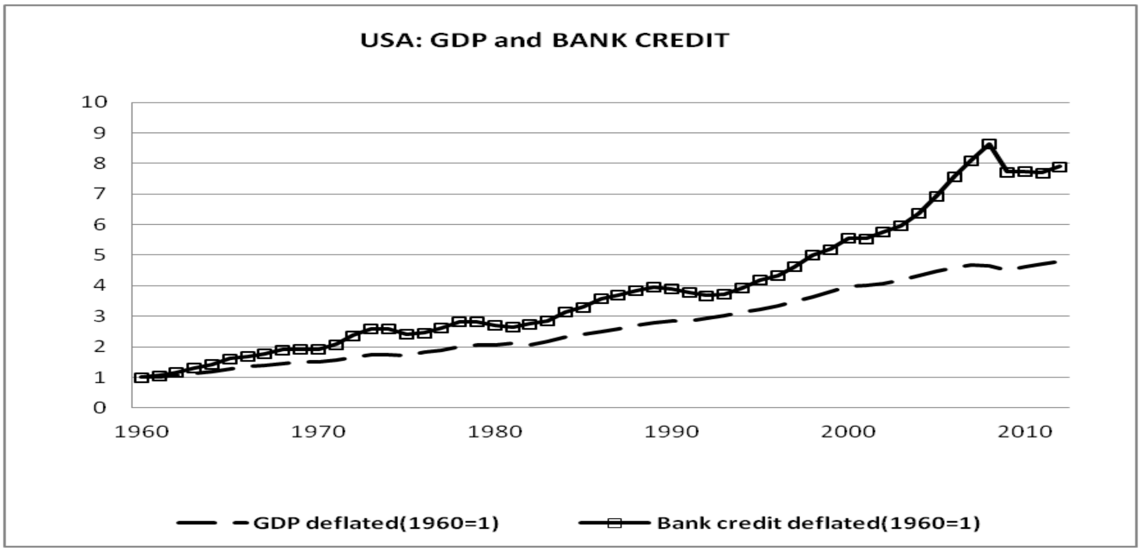
Figure 1



Source: Board of Governors of the Federal Reserve System, "Bank credit, all commercial banks".

Banks monitor the ratios between deposits, loans and non-performing loans daily. As far as the net present value of their portfolios remained within the expected level of profitability; in an environment of declining interest rates and low inflation, the expansion can last a long period of time (Figure 2). Loans supply feeds positively the economic growth while the financial system grows in its size.

Figure 2



Source: GDP Bureau of Economic Analysis. Inflation: Federal Reserve of Saint Louis

But future expected values may or may not be achieved. If present value of the portfolio falls due to an increase in non-performing debt, one or more banks may try to reduce loan portfolios exposure and increase reserves. If the system is large and highly interconnected, such individual behavior can spread quickly.

A sustained upward trend of deposits D_t over time can be postulated (necessary condition to increase the size of the system) even with constant money circulation velocity, no money quantity increase and the absence of endogenous money creation. The source of this growth may come, instead, from the accumulation of financial surpluses by economic agents (for simplicity we call them rentiers) that, having a steady flow of income that they cannot consume; turn this resources to various financial instruments. Such agents are, for example, Sovereign Wealth Funds and Pension Funds (Garcia & Nicolini 2012 and they references).

Rentiers decide whether fully or partially placed such surplus in the banking system, changing the volume of deposits in the system. The variation of deposits and therefore the trajectory of D_t is defined by the sum of the individual decisions of each rentier. These surpluses received by banks are in turn placed as loans L_t for those agents who spend above their income, that is to the net debtors of the system (an effect of this asymmetry can be found in the existence of a rate return on financial income which is secularly above the growth rate of the economy (Piketty 2014, Garcia & Nicolini 2012 and they references)).

The behavior of rentiers in deciding whether or not to keep their surpluses in the banking system can be considered as a random walk because of the autonomy of decision of each rentier. However it is possible to think that in certain situations groups may form that act coordinately, increasing deposits or making withdrawals. This approach postulate that they may form coalitions (clusters); each replicating certain strategy, supporting long periods when influx of deposits dominated and other critical periods where the decision to withdraw dominates. Such cases of “behavior copy” (herd behavior) have been studied in models of financial markets (Eguiluz, V. M., & Zimmermann, M. G. 2000). They can be used to analyse situations when portfolio decisions are concentrated in a reduced group of investment fund administrators and banks strategy leads to create products to recover liquidity like securitization schemes of less liquid assets such as costumer loans or mortgages.

A tendency to the growth of the banking system may be seen as a more than proportional increases in deposits relative to GDP. Within this kind of scenarios, the connection of imitating behavior between rentiers can be postulated as probabilistic. This type of models are proposed by Cont, and Bouchaud (2000) and Eguiluz and Zimmermann, M. G. (2000), for an asset market. Decisions to buy, sell or stay inactive are associated with coalitions of actors (clusters) that follow the same strategies. In Cont, and Bouchaud (2000) the size distribution of these coalitions are made dependent on a parameter that arises from the probability that two agents are linked together. A higher probability of connection increases the size of the clusters and reduce the number of clusters in proportion to the size of the system.

With this background in view, we may assume that, in a long period of expansion, a larger number of rentiers surpluses is deposited in banks accounts, forming clusters of agents with the same opinion over business conditions. A concrete dynamic develop as follows. The growth of deposits lead to an increase of long term credits like mortgages. The increase in demand lead to an increase in the value of assets, such as real estates; what in turn elevate the amount of the new credits and exposes the system to a greater risk of bad loans**.** Once big clusters have been formed a critical situation could lead to massive withholdings of funds and then a sharp decrease of deposits take place. The fundamental stylized fact for the model to work is that of deposits growing over time.

In due course, deposits become loans, and all the bank system remains in a steady growth path. Our work analyses what happens when the growth of deposits leads to a growth of loans that cannot be sustained indefinitely; eventually stops abruptly and reversed it trend going (possibly and temporarily) into chaos.

3. The model

Let’s consider the aggregate banking sector of a developed economy and the following variables and parameters.

D_t= Deposits of period t

R_t= Reserves of period t

L_t= Loans of period t

ρ_t= Reserve ratio (reserve holdings of banks to demand deposit liabilities) in period t

γ= "Experience-dependent" parameter that relates Reserve and Loans of period t to ρ_{t+1}

$$R_t = \rho_t D_t \qquad (1)$$

$$L_t = (1 - \rho_t) D_t \qquad (2)$$

Bank decisions on Reserves for each period are related to the Reserves/ Deposits ratio of the previous period and are condensed in equation (3). Usually, the more Loans the larger the Reserves needed (Bagehot, 2004).

$$\rho_{t+1} = f(R_t, L_t) \qquad (3)$$

Therefore,

$$\rho_{t+1} = \gamma R_t L_t \qquad (4)$$

Equation (4) has some properties that a function, that relates the coefficient of reserves to previous reserves and loans, is expected to have. The multiplicative form indicates that both; reserves and loans must not be zero as a necessary condition to the existence of the coefficient. In other words reserves and loans are not separable. The derivative of ρ_{t+1} respect to reserves and loans is positive and related to the other component. For instance, if L_t grows ρ_{t+1} is incremented by the proportion γ of R_t. There is a unit-elasticity of L_t and R_t respect to ρ_{t+1}; witch is one way to review consistency; indicating that when reserves or loans changes, ρ_{t+1} varies proportionally. When D_t varies the function does not define if ρ_{t+1} grows or get smaller, what is consistent with the empirical data of graph 2.

The use of an aggregate and unique γ with an equal decision period for all agents depends on the assumption that the sector has fast and flexible interconnections and efficient transmission mechanisms.

Taking equation (1) one period forward and replacing (4) into it;

$$R_{t+1} = \gamma R_t L_t D_{t+1} \qquad (5)$$

Using equations (2) and (5)

$$R_{t+1} = \gamma R_t(1-\rho_t)D_tD_{t+1} \qquad (6)$$

And by equation (1) together with (6)

$$R_{t+1} = \gamma R_t D_{t+1}(D_t - R_t) \qquad (7)$$

Logistical functions with one lagged period are used in Macroeconomics to account for nonlinearities and at the same time are reasonably simple from the mathematical point of view (Sordi 1996).

Following Shone (2002) we consider the equilibrium for this type of model. For t_0, the solution value is $x(t_0) = x^*$. The fixed point x^* is called “steady state” or “equilibrium point”.

A fixed point is asymptotically stable if any trajectory, started near it, approaches it as $t \to \infty$. We use the concept of equilibrium or fixed point to denote the position where the variables do not change (the first derivative with respect to t is zero) (Shone 2002).

The fixed point in (7) is given by:

$$R^* = \gamma R^* D^*(D^* - R^*) \qquad (8)$$

Where X^* denote the value of the variable X at the fixed point.

$$R^* = \frac{\gamma D^{*2} - 1}{\gamma D^*} \qquad (9)$$

For a positive level of reserves it must be that:

$$\gamma D^{*2} > 1 \qquad (10)$$

From the other side the maximum level of Reserves is given by taking the first derivate of (7) respect to R_t and equating it to Zero.

$$\frac{\partial(R_{t+1})}{\partial R_t} = \gamma D_t D_{t+1} - 2\,\gamma D_{t+1} R_t = 0 \qquad (11)$$

The maximum level in t+1 would be $f\left(\frac{D_t}{2}\right)$. Replacing this result in (7)

$$R_{t+1} = \gamma \frac{D_t}{2} D_{t+1}(D_t - \frac{D_t}{2}) = \frac{\gamma D_t^2 D_{t+1}}{4} \qquad (12)$$

The maximum amount of Reserves could not be greater than the total Deposits;

$$\frac{\gamma D_t^2 D_{t+1}}{4} \leq\ D_{t+1}\ \rightarrow \gamma D_t^2 \leq 4 \qquad (13)$$

This means that

$$1 < \gamma D_t^2 \leq 4 \qquad (14)$$

It is easy to recast equation [4] (7) in the usual form of the logistic equation

$$\rho_{t+1} = \gamma D_t^2(\rho_t - \rho_t^2) \qquad (15)$$

Considering (7) and (14) we find a cycle which includes the possibility of chaos for any value for γD_t^2 above 3.57. Let us call this parameter "stability coefficient", since this number is a parameter of the logistic equation (15) whose characteristics are well known and possess all the information necessary to know the dynamic stability of the relation between deposits, loans and reserves[5]. It is worth to mention that once the stability coefficient is calculated for

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[4] As pointed out by Professor Kawamura, the limit value for chaos (2.57) indicate by Shone (2002) seems to be incorrect. Indeed, equation (7) of our model equation can be written in the general form of the logistic equation that has a well-known limiting value of 3.57 as Shone itself point out writing in general about the logistic equation.

[5] This value was originally found by Benhabib and Day (1981) in a consumer choice model that produces chaos when consumer income exceed certain values or due to customs persistence (memory), which could be conceptualized as a chaos problem due to that higher revenue volume implies greater complexity of decisions. The value 3.57 has been found as a general feature of any function that has the form $x_{t+1} = \gamma z x_t(z - x_t)$ where zx_t are generic variables and γ is also a generic parameter.

each period it remain fixed for the projections that can be made using it. Giving the value of the stability coefficient, the reserve percentages may be calculated iteratively for some temporal horizon. Our empirical review for data of commercial banks of the US Federal Reserve Bank system[6] (see below) showed that the nearest point to the limit value of 3.57 was in September 2008 (reaching 2,92), from 2,19 in August going to 1,95 in October of that year.

The relevance of D_t^2 is linked to the importance of the system size (see below). Briefly, γD_t^2 [7] shows how the memory of the system represented by γ reacts to its total size D_t. If the agent's decisions are based on their present common experience (past experience memory), once the system grows, it may enter into instable regimes and even chaos. In practical terms these means that the dynamics of decisions on reserves of the past is no longer a valid guide for the future.

Decisions on bank Reserves level, using the γ parameter, could provide useful information about certain characteristics of the system. If γ is unique and the decision period the same for all agents, the system is highly interconnected and concentrated (at least, at the decision making level). Both characteristics elevate the probability towards a regime that have the potential to be chaotic.

In this way, the γ parameter may not only represents the memory of the system, but also interconnectivity and concentration, characteristics that may be present in the banking system taking in account our discussion in section 2.

4. Empirical Evidence

First, we will evaluate whether the proposed relation in the previous section between R_t and R_{t+1} approximate the empirical data of the Federal Reserve for aggregate data of

[6] By that time several businesses and institutional depositors withdrew money from their accounts in order to drop their balances below the $100,000 insured by the Federal Deposit Insurance Corporation (FDIC) – an event known in banking circles as a "silent run." The fourth bank of the Federal Reserve System, the Wachovia , lost a total of $5 billion in deposits in one day—about one percent of the bank's total deposits- and was pressured by the federal authorities to put itself up for sale over the weekend (Charlotte Observer 2008).

[7] See appendix 1 to see an alternative view of the stability coefficient.

commercial banks of the US. The period t is a month in our calculations for all variables and parameters; we use monthly data at the beginning of it.

Figures 3 and 4

Reserves (in millions of dollars)

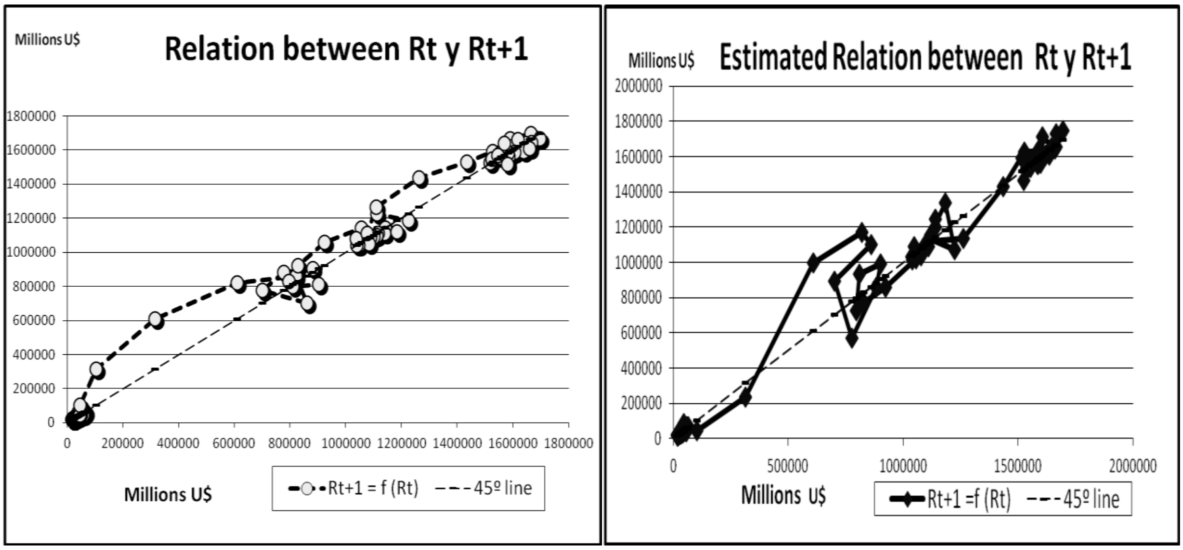

Figure 3, Source: Board of Governors of the Federal Reserve System 1960-2012 adjusted by inflation (Federal Reserve of Saint Louis). Figure 4: based on our model using the relations of γ and R_t, L_t given in equation 4 to calculate R_{t+1} respect to R_t.

Figure 3 shows the relationship between R_t and R_{t+1} observed for the Federal Reserve System for the period between 1960 and 2012. The points above the 45° line indicate that the system accumulates reserves. From 1960 until August 2008, Reserves were kept in the range from 20 to 45 billion dollars and are represented by the small cluster shown on the origin. The first significant point is August 2008 when Reserves jump to almost 103 billion. From then on an unstable loop emerges and remains for one year until Reserves stabilize near the 110 billion dollar mark. The system keeps accumulating Reserves with more fluctuations in the range of 160/170 billion until year 2012.

Figure 4 shows the result of our estimation using γ to calculate R_{t+1} respect to the R_t when instability and subsequent fluctuations emerged in 2008. Since the system calculates an expected reserve ratio based on the levels of reserves and deposits of the previous period, fluctuations are more evident than in historical measurements.

Next, we calculate the stability of the γD_t^2 value. Figure 5 shows the results of our model. It is worth to mention that the period for which the stability coefficient may be useful would be normally very short. The stability coefficient is an initial condition and the logistic equation is very sensible to small changes in the initial conditions[9]. The value for the coefficient obtained may generally be useful in a high frequency environment; that is time slot of minutes and hours. The longer the time slot the less useful the coefficient.

Given that Macroeconomic data does not appear for very short time periods and in the aim of covering the bigger period possible we have taken monthly data to build our empirical review.

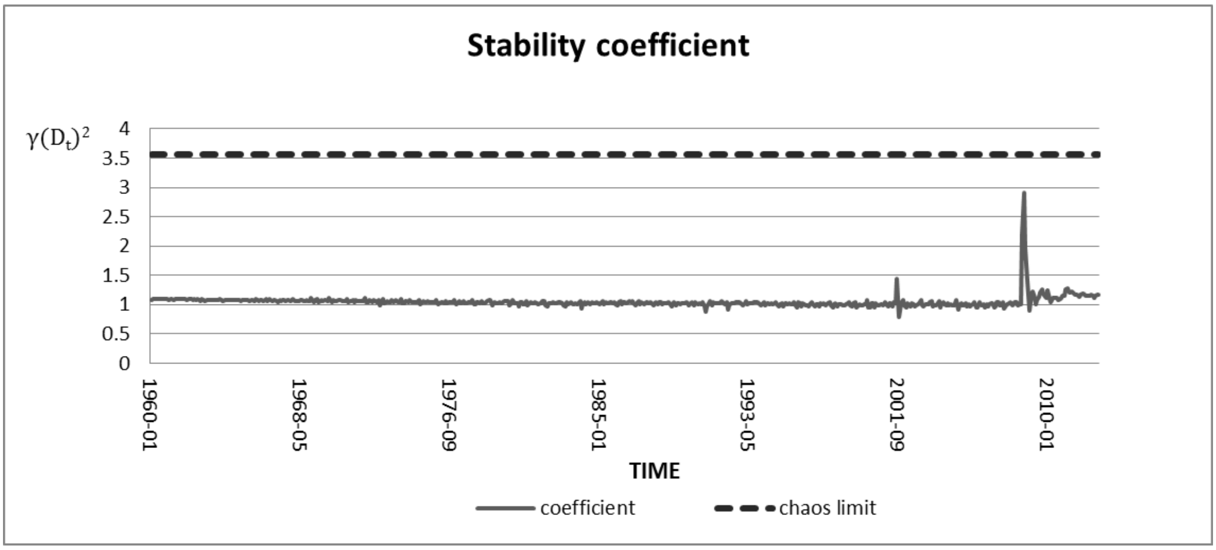

Figure 5

Source: Own calculation, based on statistical data of the Board of Governors of the Federal Reserve System 1960-2012 adjusted by inflation (Federal Reserve of Saint Louis). We estimate γ using the relation of equation 4 taking available data of ρ_{t+1}, R_t and L_t.

During 2008 the system registered the most important instability close to chaos for the considered period. Some instability respect to registered values also emerged in 2001.

[9] As Lorenz point out: *"Chaos: When the present determines the future, but the approximate present does not approximately determine the future"* (Lorenz, 1990).

The system does not go through an adaptive transition phase before the instabilities, which is a hint that a chaotic situation could be considered in the theoretical analysis of the U.S. financial crisis of 2007 – 2008. The agents within the system, probably in grasp of some variables of his environment; may had had difficulties in processing the small changes in information that leads to the crisis.

Another relevant fact is that, despite that the instable situation is quickly surpassed the system appears to behave relatively more unstable and oscillatory than before, as it is shown in figure 5.

Therefore, even with a deterministic model (not stochastic), infinitesimal changes that accumulate (see Figure 1) in a growing system (see Figure 2) may cause a sudden and significant change leading to an instable (near chaotic) situation (see Figure 4 and 5); since it is impossible to know (like in any other chaotic system[10]) with sufficient precision which or how to modify previous decisions practice.

5. Is it a real state of chaos?

From the monthly data, although very much unstable than any other period, during the 2000 decade, chaos does not appear. If we take the records from the third of September of 2008 (from the weekly records) instead the of the first of that month (from the monthly data) together with data of the first day of the monthly data of October 2008 the result for the stability coefficient results 3,73. This is due to the well-known characteristic of high sensitivity to initial conditions (Lorenz 1990).

According to Banks, J., Brooks, J., Cairns, G., Davis, G., & Stacey,P (1992) a chaotic system must possess the following conditions ; a) sensitivity to initial conditions, b) possibility for any value of the solution space to be achieved by small variations in parameters and variables c) impossibility of predicting what values will be reached in the future. The three conditions are necessary; sufficiency is achieved if and only if all three are true. The conditions were observed during our empirical verification of a chaotic situation as follow.

[10] See note 9 again

It is possible to observe point (a), in Figure 5, where the set of initial conditions given by the combination of variables and parameters R_t, L_t, ρ_t, and γ is no different than the set for the period 1960 to 2008; therefore the changes in initial conditions that gave rise to the 2008 chaotic situation were very small. As to (b), once chaos is reached, ρ_t increase (see figure 1) is a good indication that the banking system was in a deep crisis, and that the variables involved in the system could have reached any value even from very low levels of Loans, Deposits and Reserves. Finally, in the case of (c) we can consider Figure 1 and 5 together and observe that the increase in Reserves from August 2008 onwards was reached by a dynamic that gave no previous indication of the values that ρ_t would reach later.

The accelerated growth of the system in the five years prior to the crisis is eloquent. The presence of D^2 in equation (14) indicates that any growth increases the possibility of chaos more than proportionately. As shown in Figure 2, the amount of Loans adjusted by inflation increased sevenfold between 1960 and 2012 while GDP did it only five times. The larger and more concentrated the system, the more unstable it becomes, also, the greater the possibility of chaos.

The possibility to represent the system with a model with a single parameter γ could be seen as an indication that the period of decision of all agents in the system is the same and also the information is shared and distributed among the agents so as to produce a single aggregate reaction. Only one-month interval for decisions indicates that there is a very small reaction time for the entire network. A system where decision-making and business strategies are heavily concentrated based either on the volume of loans or on the parameters used for the valuations of portfolios (particularly uncertainty on debtor performance) reduces the diversity of opinions and increases the chances for a rapid spread.

To see what can trigger the possibility of chaos we decompose the stability coefficient and get (see appendix 1, equation (A.4))

$$\gamma D_t^2 = \frac{(1+g_{Rt})D_t}{(1+g_{Dt})L_t} \qquad (16)$$

The higher the ratio $\frac{D_t}{L_t}$ the higher would be γD_t^2. When reserves are very low then the ratio $\frac{D_t}{L_t}$ is closer to one increasing the possibility of chaos, is worth to note that that was precisely the situation near September 2008 (see Figure 1). The other possibility is that the growth rate of reserves between t and $t+1$, g_{Rt}would be much greater than the growth rate of deposits between t and $+1$, g_{Dt}. This was also the situation in September 2008 where from a low value of reserves any addition to those would have a big impact in the growth rate of reserves. Keister, T., & McAndrews, J. (2009) argues that total reserves in the banking system are determined almost entirely by the central bank's actions. These reserves may be created as a byproduct of lending policies designed to mitigate the effects of a disruption in financial markets. Individual banks cannot influence the reserve quantity of the system in a significant way because when a bank interact with other bank one increase his reserves and the deposit of his clients for the same amount that the other decreases his reserves and deposits. In the same way if a client deposit some money on the other side the reserves increases the same amount, being the reserve growth percentage bigger than the growth percentage of the deposits.

If the effect of the amount of currency held by the public and the payment into and out the treasure are small, equation (16) suggest the necessity of a bigger agent as the Central Bank that have the power to increase the amount of reserves very quickly in great amounts in time of crisis. Other significant agents may act and have also significant influence over deposits and reserves, like rentiers (Garcia & Nicolini 2012). This kind of complexities can be worked out within network models that allow different agent size (node) and influences (interconnections).

But what triggered the crisis?. The literature widely linked the behavior of investment banks (shadow banking) and the generation of risky financial products coupled with an extremely permissive attitude of the Federal Reserve (Bernanke 2010). We believe that the size of the commercial bank system may well also be taken into consideration.

A possible explanation of what is going on beyond the equation of the model is that growing Credit / GDP ratio is a strong signal of imbalances (Markeloff, Warner and Wollin, 2012). A stagnant Wages to GDP ratio and rising Debt to GDP ratio can do the rest. The core of the bank network may start a generalized liquidity preference response (growing reserves) to an increase in the level of non-performing loans which will most likely expand throughout the network. From the evidence of section 4, the abrupt change might be adequately illustrated by a chaos dynamic model.

6. Conclusions

We use a simple model of a banking system to explore its inter-temporal stability over long periods. We studied the US commercial bank system for the period 1960-2012, which was very stable for 40 years until in 2008 reached a chaotic state. Stability was regained almost instantaneously (given the time scale with which we worked) but with important fluctuations from then on.

The work intends to show the importance of introducing a logistic equation that include the possibility of a chaos considering the size of the system as a relevant variable.

In terms of the model, a single γ means that any alarm signal spreads quickly throughout the system and the average of agents reacts in the same interval of time indicating a highly interconnected and probably concentrated network; additionally, D^2 indicates that the growth in the size of the system increases more than proportionately the possibility of chaos when is characterized by large interconnections and concentration.

The stability of the system could be followed with the simple indicator γD_t^2. It give with a number between 1 and 4 a measure of solidity of the system. The closer this indicator is to one the more stable the system. If it is greater than 3,57 the system reach a chaotic state.

“Deterministic chaos” and “noise-probabilistic exogenous shocks” coexist. To distinguish one from the other is essential to design a consistent banking policy. We made reference to a banking system, where changes in the state of the system could not be anticipated by the agents within it, even if individually, they may control the own variables (D, R, L). In this way it exposes the difficulty and limits of anticipating subsequent development of the banking system.

From the point of view of active policies, considering chaos theory can be seen as a fertile ground where new types of interventions can be explored to stabilize more effectively the system. Following Bullard & Butler (1993), the utilization of this type of nonlinear deterministic models that take into account size, interconnections and complexity could be seen as an interesting starting point for redesigning banking policies. A γ parameter reaching certain high values might be a warning indication that the system should not be regulated only looking at what is happening to each single bank. Larger institutions do not necessarily lead to robustness, but to a greater fragility.

From the model (equation 16) is possible to see that some unstable situation would arise when sudden percentage growth of reserve of the banking are expected to occur while the level of reserve are relatively low. This growth needs some combination of loans payment disruption, deposit retirement and a Central Bank that have the possibility to carry out the reserve expansion.

Other issue to consider is that the origin of the banking system need of additional reserves may very probable be connected to banks liquidity necessities; constrained by not receiving some payment in time (pay chain disruption). Put in the simplest way, the combination of decline in the reserve percentage and growing credits (Figure 1 and 2), in an interconnected system may translate via pay chain disruption very quickly to the connected banks. If investment banks can take credit of traditional bank and eventually the investment bank have some liquidity problem it affect the traditional bank and the banks connected to the latter. A greater interconnected system is more exposed to the failure of one of its component (Size risk) because they are more fragments that can fail and spread his own failure; just the opposite to the known "to big to fail". We may conclude that the ability of a system to grow for a long period of time in a stable manner should not be seen as a sign of robustness.

Due to the financial liberalization of the 1980 and 1990 we live in a world where banks are engaged in virtually every financial market; not just in their home country but around the world. For instance, the Gramm-Leach-Bliley Act of 1999 took down barriers to competition between traditional banks, investment banks, and insurance companies (Wright, & Quadrini, 2009) increasing interconnections of the banking system. Perhaps, as Leijonhufvud (2011) pointed out, we must remember the experience of the Glass-Steagall Act of 1933 that did not allow a highly interconnected banking system and successfully preserved stability.

Appendix 1

Decomposition of the stability coefficient

Another way to gain intuition of the stability coefficient is to take

$$R_{t+1} = \gamma R_t D_{t+1}(D_t - R_t) \qquad (7)$$

and dividing both sides of (7) by $R_t D_{t+1}(D_t - R_t)$ get

$$\gamma = \frac{R_{t+1}}{R_t D_{t+1}(D_t - R_t)} \quad (A.1).$$

Giving that $\frac{R_{t+1}}{R_t} = (1 + g_{Rt})$ where g_{Rt} is the reserves growth between t and t+1;

$$\gamma = \frac{(1 + g_{Rt})}{D_{t+1}(D_t - R_t)} \; (A.2)$$

Multiplying both sides by D_t and giving that $\frac{D_{t+1}}{D_t} = (1 + g_{Dt})$ where g_{Dt} is the deposit growth between t and t+1 and $D_t - R_t = L_t$.

$$\gamma D_t = \frac{(1 + g_{Rt})}{(1 + g_{Dt})L_t} \quad (A.3)$$

Finally,

$$\gamma D_t^2 = \frac{(1 + g_{Rt})D_t}{(1 + g_{Dt})L_t} \quad (A.4)$$

Given some ration between deposits and loans, the more reserves grows between t and t+1 respect to the growth of deposits the higher the coefficient γD_t^2.

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