Is the US stock market getting riskier?

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Working Paper

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In this paper, we compared the distribution of the S&P 500 Index monthly returns of the period 1957-1986 against the period 1987-2016 to evaluate the presence of extreme events. The last 30 years have recorded a higher (lower) probability to exceed a given negative (positive) monthly return compare with the probability of exceedance of the three previous decades.

July 2017

I. S&P 500 Index Distribution

We estimated monthly returns of S&P 500 Index from 1957 to 2016, downloaded from Yahoo Finance, for a total 720 observations.

Figure I depict S&P 500 Index monthly returns, measured as the log return of the first day of the month to the first day of the next month.

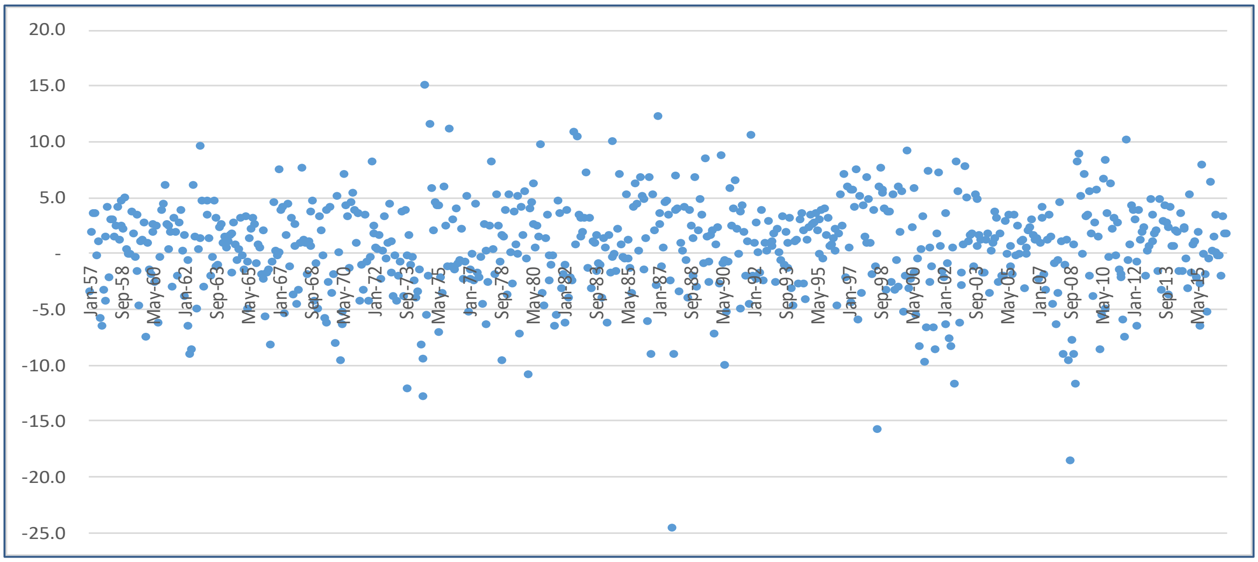


Fig. I. S&P 500 Index monthly returns 1957-2016

We split, ad-hoc, the sample in two periods, 1957-1986 and 1987-2016.

Next table summarized main statistical data:

Period	Obs.	Mean	Std. Dev.
1987-2016	360	0.59%	0.0435
1957-1986	360	0.50%	0.0408

II. Extreme Events

Extreme events occur when a risk takes values from the tail of its distribution (McNeil, 1999).

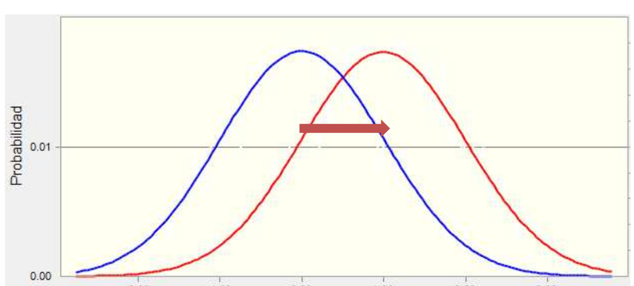
For a deeper mathematical treatment of the extreme value theory, Coles (2001) is recommended.

II.1. Extreme Events over Time

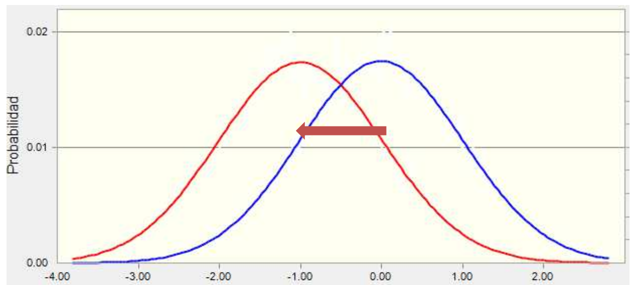
Change in the mean and the variance of the distribution over time will generate change in extreme events occurrence.

Next graphs show the change in negatives and positive value of returns using a comparison against the normalized distribution ($\mu = 0.0$ and $\sigma = 1.0$) as base case:

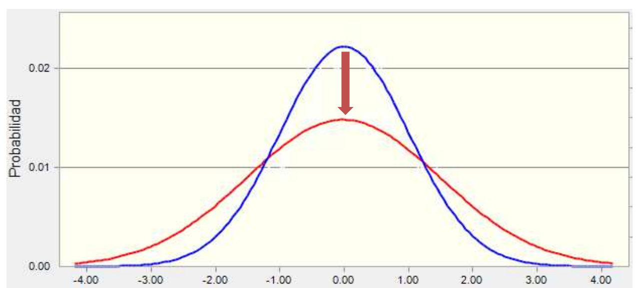
a) An increase in mean and no change in variance ($\mu = 1.0$ and $\sigma = 1.0$), produce less negative returns, and more positive returns along with more record positive returns.



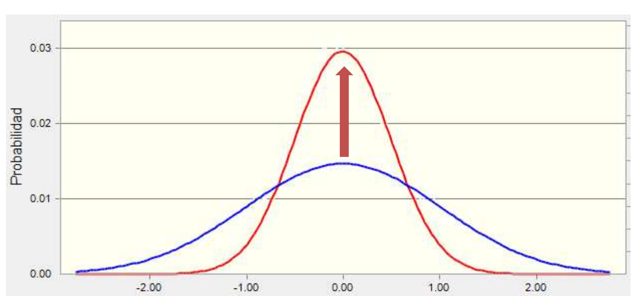
b) A decrease in mean and no change in variance (μ = -1.0 and σ = 1.0), generate more negative returns besides more record negative returns, and less positive returns.



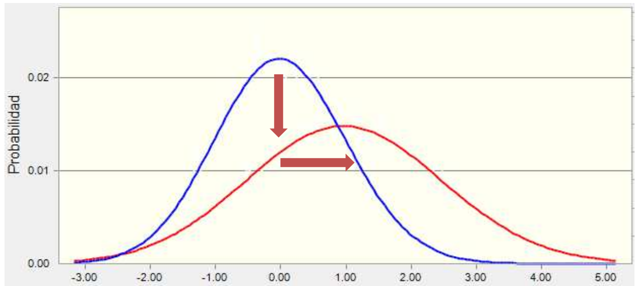
c) Increase in variance and no change in mean (μ = 0.0 and σ = 1.5) make more negative returns and more record negative returns, in addition to, more positive returns and more record positive returns.



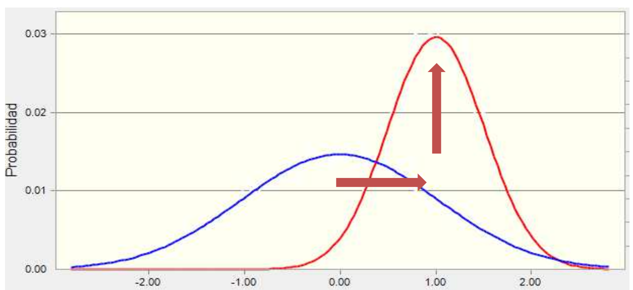
d) Decrease in variance and no change in mean (μ = 0.0 and σ = 0.5) cause less negative returns and less record negative returns, as well as, less positive returns and less record positive returns.



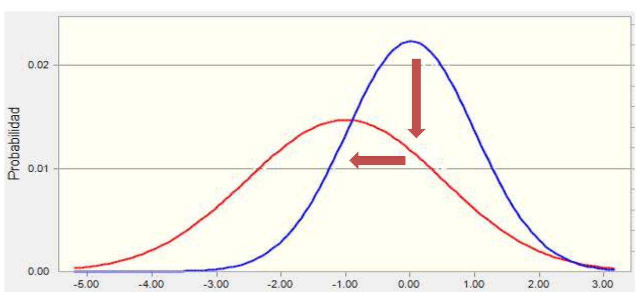
e) An increase in mean and an increase in variance ($\mu = 1.0$ and $\sigma = 1.5$), produce more positive returns and more record positive returns.



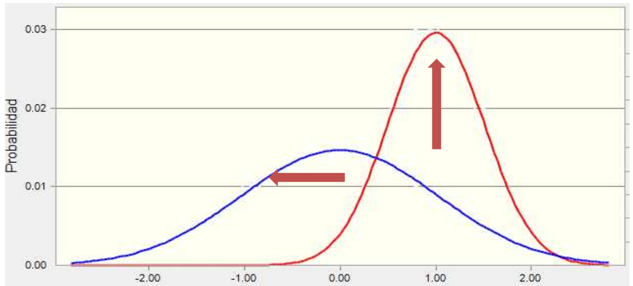
f) An increase in mean and a decrease in variance ($\mu = 1.0$ and $\sigma = 0.5$), generate less negative returns and less record negative returns.



g) A decrease in mean and an increase in variance ($\mu = -1.0$ and $\sigma = 1.5$), cause more negative returns and more record negative returns.



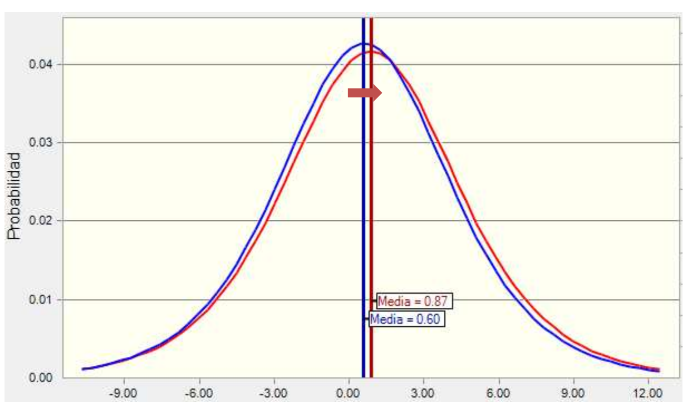
h) A decrease in mean and a decrease in variance (μ = -1.0 and σ = 0.5), create less negative returns and less record negative returns.



Using Oracle Crystal Ball software, we executed the best fit option to determinate the distribution function that apply to S&P 500 Index monthly returns, obtaining a logistic distribution, per the Anderson-Darling test valuation.

Next table summarized parameters (*μ* = mean/location parameter; *s* = scale parameter) computed:

Period	μ	s	σ
1987-2016	0.87	2.31	4.19
1957-1986	0.59	2.26	4.10
σ=s*(π/√3)			



III.3. Return Level

The return level R_k^n is the level expected, on average, to be exceeded in one out of *k* periods of length *n*.

The return period is the amount of time expected to wait for particular return level to be exceed; return period is the inverse of the probability of an event, a called "100 years event" has a 1% (1/100) probability of exceed the record level in any one year.

For a Generalized Pareto Distribution, the *k* year return level is defined:

$$R_k \approx \quad \mu + \frac{\tilde{\sigma}}{\tilde{\xi}}([k*n_y*\Pr(X>\mu)]^{\xi} - 1) \text{ for } \tilde{\xi} \neq 0$$

where μ is the defined threshold, $\tilde{\sigma}$, and $\tilde{\xi}$ are the parameters of the Generalized Pareto Distribution, n_y is the number of observations per year, and Pr (X>μ) is equal to number of exceedances of threshold (*Nu*) divided by total number of observations (*N*).

Using the in2extRemes Toolkit developed by Eric Gilleland and Richard Katz, within statistical software R, we conducted the estimation of the Generalized Pareto Distributions.

III.3.1. Left Tail

According to Figure II and Figure III threshold of 3% monthly lose, is a “good” selection. Indeed, the mean residual life plot is “nearly linear” around that value; and the reparametrized scale and shape estimates seem to be constant on a range around that value.

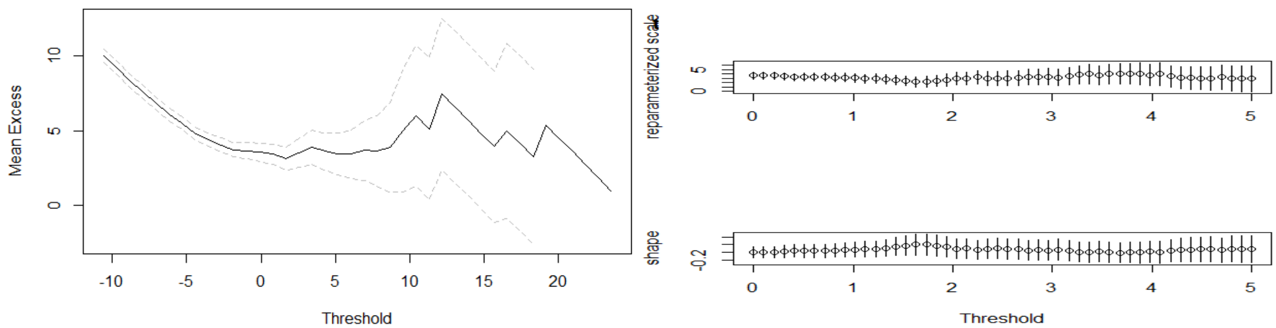


Fig. II. Mean Residual Life and Shape estimates plots, S&P 500 Index monthly returns, 1987-2016.

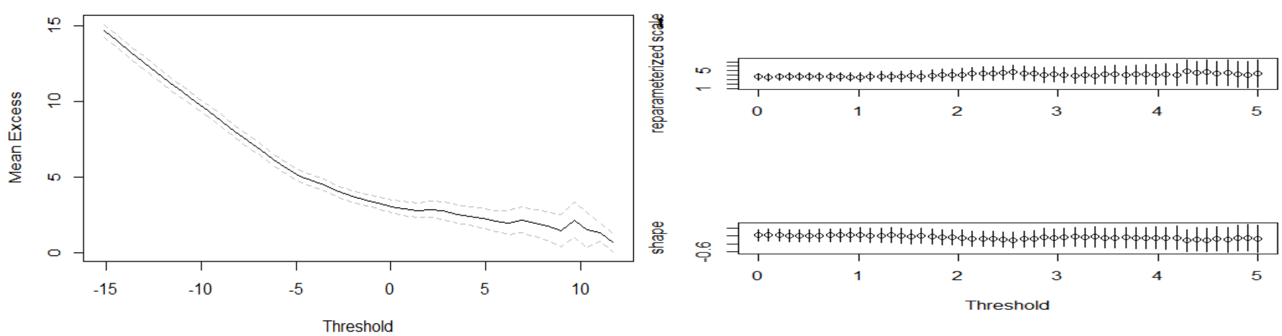


Fig. III. Mean Residual Life and Shape estimates plots, S&P 500 Index monthly returns, 1957-1986.

The selection of 3% negative return as threshold, generated 56 exceedances of threshold for the period 1987-2016 and 64 exceedances of threshold for the period 1957-1986.

For the period 1987 – 2016, the SP 500 Index recorded a total of 134 cases of negatives monthly returns, and a maximum negative monthly return of 24.5%.

For the period 1957 – 1986, the S&P 500 Index recorded a total of 157 cases of negatives monthly returns, and a maximum negative monthly return of 12.7%.

S&P 500 Index monthly returns for that period 1987 – 2016 give a Generalized Pareto Distribution with parameters (μ): 3.0, (σ): 3.40486 and (ξ): 0.06728.

S&P 500 Index monthly returns for that period 1956 – 1986 give a Generalized Pareto Distribution with parameters (μ): 3.0, (σ): 3.33733 and (ξ): -0.23768

Defining an extreme event as a 10% probability of exceed the record level in any one year, such return level R_{10} is 14.01% with 95% confidence interval of (10.417%, 17.611%) for the period 1987-2016, and 10.26% with 95% confidence interval of (8.222%, 12.291%) for the period 1957-1986.

Next table summarized main S&P negative monthly returns events by period:

Period	Max. Loss	# Loss>-3%	# Loss>-10%	# Loss>-14%
1987-2016	-24.54%	56	5.0	3.0
1957-1986	-12.71%	64	3.0	0.0

Figure IV illustrates S&P 500 Index negative monthly report for the two evaluation periods. Given a negative monthly return value, the probability of exceed that negative return value in any one year, defined for a particular return level, was higher in the period 1987-2016 that in the period 1957-2015.

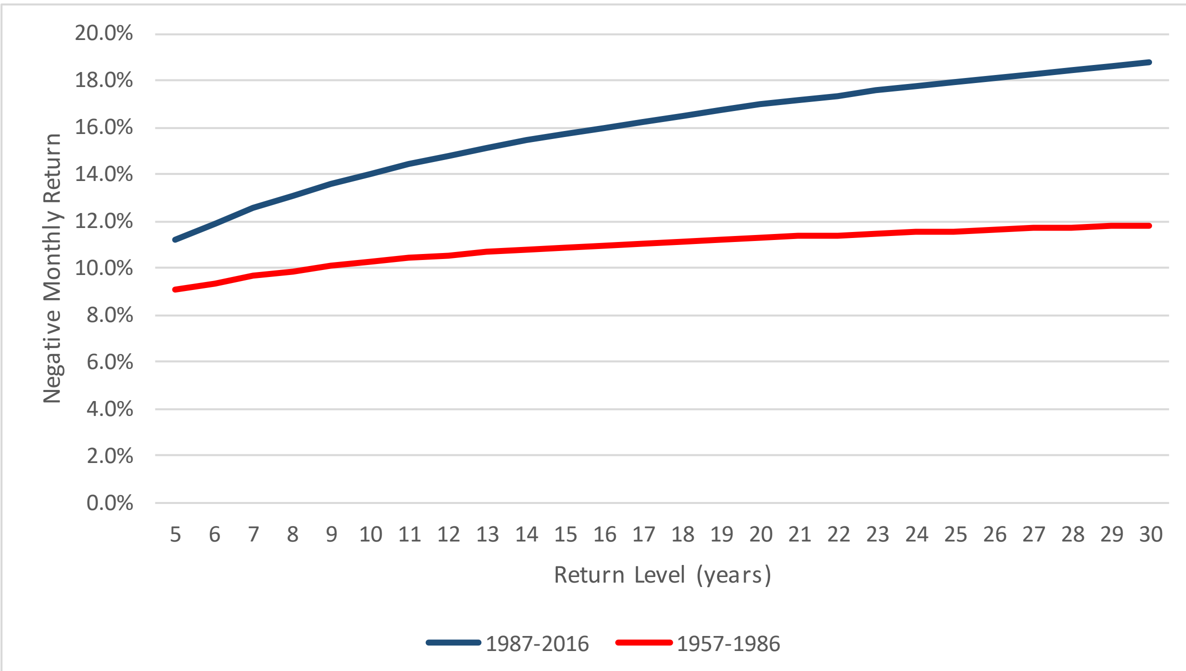


Fig. IV. S&P 500 Index Return Levels Left Tail for the evaluation periods.

III.3.2. Right Tail

Through the mean residual life plot and the reparametrized scale and shape estimates plots analysis, depicted in Figures V and VI, a 3% monthly gain is defined as an acceptable threshold.

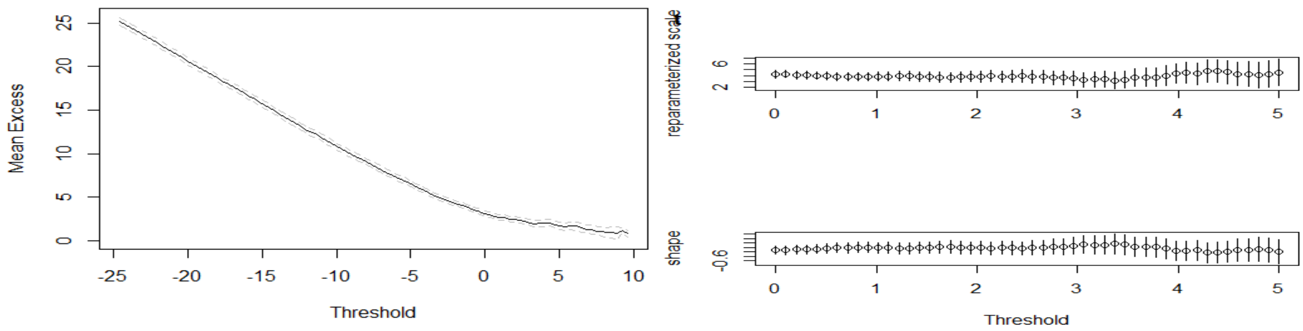


Fig. V. Mean Residual Life and Shape estimates plots, S&P 500 Index monthly returns, 1987-2016.

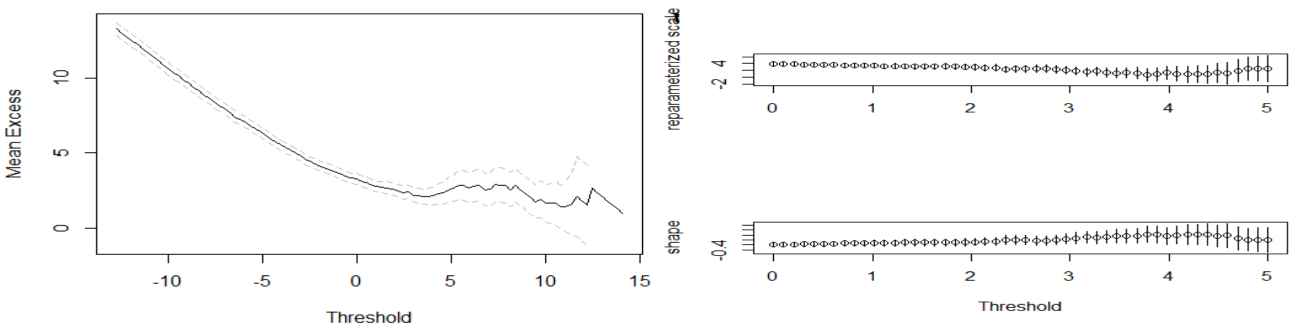


Fig. VI. Mean Residual Life and Shape estimates plots, S&P 500 Index monthly returns, 1957-1986.

The selection of 3% positive return as threshold, generated 102 exceedances of threshold for the period 1987-2016 and 98 exceedances of threshold for the period 1957-1986.

For the period 1987 – 2016, the SP 500 Index recorded a total of 226 cases of positive monthly returns, and a maximum positive monthly return of 10.5%.

For the period 1957 – 1986, the S&P 500 Index recorded a total of 203 cases of positves monthly returns, and a maximum positive monthly return of 15.1%.

S&P 500 Index monthly returns for that period 1987 – 2016 give a Generalized Pareto Distribution with parameters (μ): 3.0, (σ): 2.66987 and (ξ): -0.26119

S&P 500 Index monthly returns for that period 1956 – 1986 give a Generalized Pareto Distribution with parameters (μ): 3.0, (σ): 2.05154 and (ξ): 0.06220

Once again, an extreme event is defined as a 10% probability of exceed the record level in any one year, then R_{10} is 9.16% with 95% confidence interval of (7.732%, 10.593%) for the period 1987-2016, and 11.01% with 95% confidence interval of (8-736%, 13.389%) for the period 1957-1986.

Next table summarized main S&P positive monthly returns events by period:

Period	Max. Gain	# Gain>3%	# Gain>9%	# Gain>11%
1987-2016	10.58%	102	3	0
1957-1986	15.10%	98	8	3

Figure VII shows S&P 500 Index positive monthly results for the two evaluation periods. Given a positive monthly return value, the probability of exceed that positive return value in any one year, defined for a particular return level, was lower in the period 1987-2016 that in the period 1957-2015.

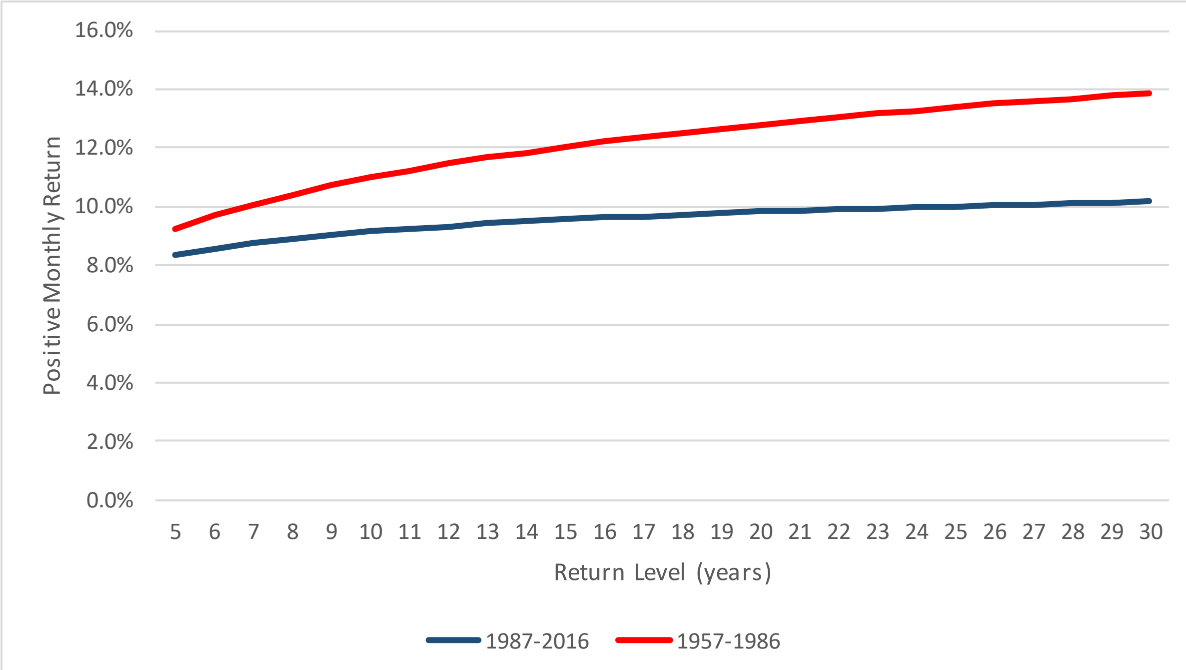


Fig. VII. S&P 500 Index Return Levels Right Tail for the evaluation periods.

IV. CONCLUSION

Through the analysis of the Return Level values, we have concluded that SP&500 index period 1987-2016 are riskier that the period 1957-1986. The last 30 years have recorded more extremes losses and less extremes gains than the three previous decades.

Future lines of research could apply estimation of return level using extreme value theory to others time periods, stocks, portfolio analysis, or markets like commodities and currencies.

REFERENCES

McNeil, A. J. (1999). Extreme Value Theory for Risk Managers. *Internal Modelling and CAD II*. RISK Books, 93-113.

Coles, S. (2001). *An Introduction to Statistical Modeling of Extreme Values*. Springer, London.