Strategic Trade and Privatization Policies in Bilateral Mixed Markets

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We consider strategic trade and privatization policies in international bilateral mixed markets where a domestic state-owned enterprise competes with both domestic and foreign private enterprises in each country. We examine the strategic interaction of two countries’ optimal choices of privatization and trade policies with different combinations of production subsidy and import tariff, and find some interesting policy implications. First, a higher social welfare can be achieved with the appropriate degree of privatization when both governments adopt a production subsidy only. Second, FTA can work as a coordination device to solve the prisoner’s dilemma problem. Third, the maximum-revenue privatization, combined with zero subsidy and higher tariff, is higher than optimum-welfare privatization. Finally, the international bilateral equilibrium needs less degree of privatization and lower subsidy rate, even though it is jointly suboptimal from the viewpoint of global welfare.

**Keywords:** strategic privatization; international bilateral mixed market; industrial policy; optimal tariff;

**JEL classifications:** L32; D43; F12

1. Introduction

Since the 1980s, many developed and developing countries have continued to privatize their state-owned enterprises (SOEs) under the global trends of trade liberalization. Yet, SOEs are strongly concentrated in a few strategic sectors and thus, they still control large portions of the world’s resources. Over half (in values terms) of all SOEs in OECD countries are significant players in sectors such as transportation, telecommunications, power generation, electricity, finance, manufacturing, and other energy industries. Along with the open economy and trade liberalization, such as negotiations for joining the WTO or establishing free trade areas, FTA has also inspired foreign firms’ entry into those industries, even with the existence of SOEs.
Although existing literature suggests that there are some gains in the efficiency of a privatized firm, researchers in the fields of industrial organization, international trade and development economics, and especially those who are interested in the privatization of the SOE, want to further explore how the foreign competition affects the desirability of privatization in mixed markets where the SOE competes with domestic and foreign private firms. In particular, economic studies on how to substantially reset production subsidy and import tariff are still increasingly important.

The economic modelling of a mixed oligopoly with domestic and foreign competitors begins with Fjell and Pal (1996), who investigated the effect of introducing foreign private firms on the equilibrium price and allocation of production. White (1996) introduced the production subsidy into the mixed market and found that welfare is unchanged by privatization if subsidies are used before and after privatization. This privatization neutrality theorem was supported by Tomaru (2006) and Kato and Tomaru (2007), who showed that the optimal subsidy, all firms’ output, profits and social welfare are identical regardless of the share in a SOE and the objectives of the firms. However, Matsumura and Tomaru (2012) showed that privatization matters on the welfare even under the optimal tax-subsidy policy if there are foreign competitors.

Other theoretical literature has analyzed import tariff in an international mixed market. Chang (2005) examined a mixed duopoly model with a more efficient foreign firm under Cournot and Stackelberg competition, and showed that the optimal level of privatization depends crucially upon the strategic substitutability-complementarity assumption. Chao and Yu (2006) found that foreign competition lowers the optimal tariff rate but partial privatization raises it. Wang et al. (2012) examined the effect of privatization on the priority of the maximum revenue tariff and the optimum-welfare tariff under Cournot and Stackelberg competitions, and showed that the optimum-welfare tariff will be lower than the maximum-revenue tariff regardless of the order of firms’ move when the asymmetric marginal cost of the privatized firm is higher than a critical value.

Some studies simultaneously consider the relations between privatization policy and dual trade instruments, production subsidy or/and import tariff, in a mixed market. Pal and White (1998) examined the interaction between privatization and strategic trade policies, and found that the welfare is always increased with privatization if production subsidy is used only. However, privatization increases welfare over much of the parameter space if import tariff is used only. Pal and White (2003) also showed that the existence of SOE lowers optimal tariffs and subsidies, but also lowers the total volume of trade between the two countries. The lower volume of trade, however, does not translate into lower levels of welfare for the trading countries. Chang (2007), Yu and Lee (2011), and Han (2012) examined the optimal privatization and trade policies in an international mixed market and showed that full nationalization is the best choice under Cournot competition, but the privatization strategy is affected strongly by trade instruments and cost difference between firms. Wang et al. (2014) examined privatization policy and entry regulation in a mixed oligopoly market with foreign competitors and free entry. It demonstrated that as long as the entry cost is relatively lower, domestic entry is socially excessive whether it is free trade or the domestic government imposes the tariff policy. Wang and Chiu (2015) showed that the welfare effect of privatization will be affected by the trade liberalization policy, and the optimum-welfare tariff and privatization should be higher in the presence of subsidy policy of foreign country than those in the absence of subsidy policy.

FTAs are in the process of being enacted. The well-known inter-regional economic cooperation agreements on FTAs include the European Economic Community (EEC), North American Free Trade Agreement (NAFTA), Southern Common Market (MERCOSUR), and ASEAN Free Trade Area.
All those previous studies have still explored the relationship between privatization and trade policy in a unilateral mixed market framework, where the domestic SOE competes with domestic and foreign firms in the home country. However, as FTA has recently inspired foreign competition into domestic market, the strategic interaction between two governments becomes increasing and more important. The emergence of FTA requires the study on the further analysis of strategic trade policies in the context of international bilateral trade model.

In the strategic trade literature, Brander and Spencer (1984, 1985) firstly showed that government could improve its terms of trade through tariff or subsidy to take a leader position transferring a foreign firm’s revenue to a domestic firm. Eaton and Grossman (1986) and Collie (1993) also analyzed the welfare effects of trade and industrial policies for a range of specifications of an oligopolistic industry and cost asymmetry. Van Long and Stähler (2009) examined that the home government can simultaneously subsidize domestic firms and impose tariffs. It is well-known proposition of trade theory that in the absence of directly trade-related distortions or policy goals, subsidies are superior to tariffs for achieving any economic objective in the pure oligopolistic market.

On the other hand, Barcena-Ruiz and Garzon (2005) firstly considered an international integrated mixed market, comprising of two countries. Assuming that SOEs are less efficient than private firms, they obtained that when the marginal cost of the SOE takes an intermediate value, each government wants the government of the other country to privatize its SOE. In this case, only one government privatizes and that government obtains lower social welfare. Dadpay and Heywood (2006) showed that two competing (domestic and foreign) SOEs play the role of trade barriers and the strategic interaction of the two governments usually serves to reduce welfare. Han and Ogawa (2008), Lee et al. (2013) and Xu and Lee (2015) incorporated import tariff and examined the interaction of two countries regarding strategic choices of privatization policy and import tariff. They demonstrated that the equilibrium degree of privatization depends not only on the relative efficiency of the SOE, but also on choice of trade policy.

In this paper, we consider an international bilateral mixed market where a domestic state-owned enterprise competes with both domestic and foreign private enterprises in the context of intra-industry trade. We examine the strategic interaction of two countries’ optimal choices on trade instruments and privatization policy. Specifically, we investigate two different options of production subsidy and import tariff, coupled with partial privatization, and demonstrate the following main results.

First, under Cournot competition, a higher social welfare can be achieved when both governments adopt a production subsidy instrument with partial privatization in international bilateral mixed markets. This is contrasted to the previous results under a unilateral mixed market, such as Yu and Lee (2011) and Han (2012), in which the dual trade instruments of subsidy and tariff with full nationalization is the best choice. Second, when the SOE take a leader position under Stackelberg competition, except for the optimal degrees of privatization, the optimal levels of subsidy, tariff and social welfare are the same with those under Cournot competition. This is interesting in that as far as the optimal degree of privatization is well chosen, the social welfare is independent of the leadership power of the SOE in each country. Third, irrespective of whether symmetric or asymmetric choices of two countries on trade instruments between subsidy and tariff, we show that FTA can work as a coordination device to solve the prisoner’s dilemma, where both countries could achieve higher social welfare if they cooperate and adopt a subsidy instrument only under FTA. Thus, it supports the result in Xu et al. (2016), who showed that privatization policy can play the role of commitment device to encourage parties to agree to an FTA and thus, it can improve both domestic and global welfare. Fourth, we examine and compare the maximum-revenue equilibrium with the optimum-welfare
equilibrium and show that the maximum-revenue privatization, combined with zero subsidy and higher tariff, is higher than optimum-welfare privatization in international bilateral mixed markets. Finally, the international bilateral equilibrium involves less degree of privatization level and lower subsidy rate, even though it is jointly suboptimal from the viewpoint of global welfare. This result is consistent with the result in Lee et al. (2013), who neither consider the subsidy nor FTA agreements.

The remainder of this paper is organized as follows. Section 2 introduces the basic model. In section 3, we investigate four different regime choices of production subsidy and import tariff with partial privatization under Cournot competition. In section 4, we examine the asymmetric choices of two countries on trade instruments between subsidy and tariff, compare the optimal equilibria with four scenarios in a unilateral mixed market, compare the results under Stackelberg competition, and investigate local optimum for maximum-revenue and global optimum for maximum-welfare. Section 5 concludes this paper.

2. The Model

Suppose that there are two countries: one is the home country (country 1) and the other is the foreign country (country 2). The home country and foreign country both have symmetric duopoly situations: each country has a state-owned enterprise (SOE) and a private enterprise (PE), which producing homogeneous products. We assume that the domestic SOE produce output for its domestic market only while the domestic PE can supply not only for domestic but also foreign markets. That is, we consider PEs can export the same products to the other country and two countries are engaged in intra-industry trade.4

Both governments adopt a complete set of trade policy instruments, including a production subsidy \( s_i (\geq 0) \) per unit of output provided to the domestic firms and an import tariff \( t_i (\geq 0) \) per unit of output imposed on the foreign firms. Let us denote the SOE’s outputs and the PE’s outputs in country \( i \) as \( q_{si} \) and \( q_{hi} \), while PE’s export outputs as \( q_{ei} \). The inverse demand functions of both markets are the same and given by \( P_i = a_i - b_i Q_i \), where the price of market \( i \) is denoted by \( P_i \) and the output of market \( i \) is \( Q_i = q_{si} + q_{hi} + q_{ei} \) where \( i \neq j = 1, 2 \).

We assume that the cost functions of SOE and PE are quadratic5 and given as \( C(q_{si}) = \frac{1}{2} q_{si}^2 \) and \( C(q_{hi} + q_{ei}) = \frac{1}{2} (q_{hi} + q_{ei})^2 \). Then, the profits of the SOE and PE in country \( i \) are

\[
\pi_i = (P_i + s_i)q_{si} - \frac{1}{2} q_{si}^2, \tag{1}
\]

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4 In reality, the domestic SOEs have significant market shares in sectors such as transportation, telecommunications, power generation, electricity, finance, manufacturing, and other energy industries. Thus, for some governmental purposes such as to stabilize domestic market prices, SOEs seldom participate in export. Existing literature also shows that international trade will induce only the more productive private firms to enter the export market, while some less productive firms will continue to produce only for the domestic market when export market entry costs exist (Melitz, 2003; Helpman et al. 2004) or cost inefficiency of the SOE (Lee et al. 2013). In the Appendix, even though we allow for SOE to export, we can show that the export outputs of the SOE are zero unless full privatization is achieved at equilibrium.

5 In the mixed market literature, asymmetric costs between SOE and PE proposed the desirability of privatization. See, for example, Lee and Hwang (2003), Chang (2005), Lee (2006), and Wang et al. (2009). However, early studies of mixed oligopoly, including De Fraja and Delbono (1989), Matsumura (1998), and Pal and White (1998), assumed the same increasing marginal cost and showed that partial privatization in mixed oligopoly will improve the welfare.
The consumer surplus is denoted as $CS_i = \frac{1}{2}(Q_i)^2$. And the social welfare is defined as the sum of consumer surplus, domestic industry profits, import tariff revenues, $S_i = s_i(q_{ii} + q_{hi} + q_{wi})$:

$$W_i = CS_i + \pi_{ii} + \pi_{pi} + T_i - S_i.$$  

3. The Policy Analysis

We investigate and compare four regime choices of production subsidy and import tariff, coupled with partial privatization under Cournot competition: no trade instrument, production subsidy, import tariff and dual trade instruments.

3.1. Dual trade instruments

We consider the general case that both governments adopt dual trade instruments of production subsidy and import tariff with the privatization policy. In the second stage, the SOEs maximize their objective functions, $O_i$, and the PEs maximize their own profits, $\pi_{pi}$, after observing the levels of privatization. From the first-order conditions, we have the following equilibrium outputs of SOE and PE:

$$q_{ii} = \frac{1}{\Delta_c} \left[ \frac{3(20 - 7s_i + 2s_j - 5t_i - 5t_j) - (18 - 83s_i + 20s_j - 37t_i - 11t_j)\theta_i}{(27 - 8s_j - s_j - 7t_i - 8t_j)\theta_j + 3(2 + 12s_j - 4s_j + 5t_i + t_j)\theta_j} \right],$$

$$q_{hi} = \frac{1}{\Delta_c} \left[ \frac{3(5 + 12s_i - 4s_j + 10t_i + 10t_j) + (18 - 7s_i + s_j + t_i + 8t_j)\theta_i + (3 + 16s_j + 2s_j + 14t_i + 16t_j)\theta_j + (6 - 4s_i + 4s_j + t_i + 5t_j)\theta_i\theta_j}{(3 + 16s_j + 2s_j + 14t_i + 16t_j)\theta_i + (6 - 4s_i + 4s_j + t_i + 5t_j)\theta_i\theta_j} \right],$$

$$q_{wi} = \frac{1}{\Delta_c} \left[ \frac{3(-5 - 11s_i + s_j + 5t_i + 20t_j) + (-3 - 21s_i + 3s_j + 3t_i + 24t_j)\theta_i + (3(-2 - 4s_i + 4s_j + t_i + 5t_j)\theta_i\theta_j}{(-18 - 20s_i + 26s_j + 11t_i + 37t_j)\theta_i + (3(-2 - 4s_i + 4s_j + t_i + 5t_j)\theta_i\theta_j} \right].$$

Partial public ownership was introduced by Bos (1991) and George and La Manna (1996). Matsumura (1998) formulated theoretical analysis on the mixed market model and investigated the optimal degree of partial privatization. Lee and Hwang (2003) incorporated the agency problem into his model and showed that partial privatization is generally optimal both in public monopoly and in mixed market under moderate conditions.
where \( i = 1,2 \) and \( \Delta_c = 3(45 + 19\theta_i + 19\theta_j + 8\theta_i\theta_j) \).

The market output and price of country \( i \) are

\[
Q_i = \frac{1}{\Delta_c} \left( 9(10 + 2s_i + 3s_j - 5t_i) + (18 + 50s_i + 3s_j + 8t_i)\theta_i + (33 + 5s_i + 22s_j - 17t_i + 5t_j)\theta_j + (6 + 20s_i + 4s_j + t_i + 5t_j)\theta_i\theta_j \right),
\]

\[
P_i = \frac{1}{\Delta_c} \left( 9(5 - 2s_i - 3s_j + 5t_i) + (39 - 50s_i - s_j - t_i - 8t_j)\theta_i + (24 - 5s_i - 22s_j + 17t_i - 5t_j)\theta_j + (18 - 20s_i - 4s_j - t_i - 5t_j)\theta_i\theta_j \right).
\]

The social welfare of country \( i \) is

\[
W_i = \frac{1}{2\Delta_c} \left( 3(15A_i + 6A_i\theta_i + A_i\theta_i^2) + 6(B_i + B_j\theta_j + B_j\theta_j^2)\theta_i + (C_i + C_i\theta_i + 2C_i\theta_i^2)\theta_i^2 \right).
\]

Let superscript “D*” denote the equilibrium outcome in dual trade instruments case. Then the differentiation of \( W_i \) with respect to \( \theta_i \), \( s_i \) and \( t_i \) yields the following optimal degree of privatization, subsidy and tariff rates:

\[
\theta_{i*}^p = \sqrt{2} - 1, \quad s_{i*}^p = \frac{17 - 8\sqrt{2}}{46}, \quad t_{i*}^p = \frac{11 + 7\sqrt{2}}{138}.
\]

Substituting the optimal degree of privatization, subsidy, and tariff rates into the above equations, we obtain the outputs of SOEs and PEs, \( q_{ii*}^p = 0.33 \), \( q_{ij*}^p = 0.22 \) and \( q_{ii*}^p = 0.07 \). Note that \( q_{ii*}^p > q_{iij*}^p + q_{iij*}^p \) which implies that the government will strategically use the SOE to act as trade barriers and promote the domestic market competition for reaching a higher domestic social welfare. It is also noteworthy that the marginal production cost of SOE is higher than that of the PE, which is conferring cost disadvantages of export to the SOE at equilibrium. Then the market output and price are \( Q_i^p = 0.62 \) and \( P_i^p = 0.38 \). Finally, the optimal social welfare is \( W_i^p = 0.331 \).

### 3.2. No trade instrument

We consider the case where both governments do not use any trade instruments but only adopt the privatization policy to maximize their domestic social welfares. Let superscript “N*” denote the equilibrium outcome under no trade instrument regime. Setting \( t_i = t_j = s_i = s_j = 0 \) into the equilibrium and welfare in the previous analysis, we can have the following optimal degree of privatization:

\[
\theta_{i*}^{Np} = \frac{\sqrt{13849 - 99}}{92}.
\]

Substituting the optimal degree of privatization into the above equations, we obtain the outputs of SOEs and PEs, \( q_{ii}^{Np} = 0.39 \) and \( q_{ij}^{Np} = q_{ij}^{N*} = 0.12 \). Then the market output and price are \( Q_i^{Np} = 0.63 \) and \( P_i^{Np} = 0.37 \). Finally, the optimal social welfare is \( W_i^{Np} = 0.327 \).

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7 See Appendix II for the complete expression of the social welfare.
3.3. Production subsidy

We consider the case that both governments adopt a production subsidy instrument with the privatization policy. Let superscript “S*” denote the equilibrium outcome under production subsidy regime. Setting $t_i = t_j = 0$ into the equilibrium and welfare in the previous analysis, we can have the following optimal degree of privatization and subsidy rate:

$$
\theta^*_S = \frac{\sqrt{2929} - 43}{40}, \quad s^*_S = \frac{8459 - 93\sqrt{2929}}{21680}.
$$

Substituting the optimal degree of privatization and subsidy rate into the above equations, we obtain the outputs of SOEs and PEs, $q^*_i = 0.37$ and $q^*_i = 0.16$. Then the market output and price, $Q^*_i = 0.69$ and $P^*_i = 0.31$. Finally, the optimal social welfare is $W^*_i = 0.332$.

3.4. Import tariff

We consider the case that both governments adopt an import tariff instrument with the privatization policy. Let superscript “T*” denote the equilibrium outcome under import tariff regime. Setting $s_i = s_j = 0$ into the equilibrium and welfare in the previous analysis, we can have the following optimal degree of privatization and tariff rates:

$$
\theta^*_T = \frac{\sqrt{2513} - 41}{32}, \quad t^*_T = \frac{3516 - 47\sqrt{2513}}{8553}.
$$

Substituting the optimal degree of privatization and tariff rate into the above equations, we obtain the outputs of SOEs and PEs, $q^*_i = 0.35$, $q^*_i = 0.18$ and $q^*_i = 0.05$. Then the market output and price, $Q^*_i = 0.58$ and $P^*_i = 0.42$. Finally, the optimal social welfare is $W^*_i = 0.324$.

3.5. Comparisons

We compare the results of four different regime choices of production subsidy and import tariff, coupled with partial privatization in the international bilateral mixed market.

**Proposition 1:** Under Cournot competition, the highest social welfare can be achieved when both governments adopt a production subsidy policy only with partial privatization in the international bilateral mixed market.

<table>
<thead>
<tr>
<th>TABLE I: Comparisons under Cournot competition in the bilateral market</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_i$</td>
</tr>
<tr>
<td>No instrument</td>
</tr>
<tr>
<td>Subsidy</td>
</tr>
<tr>
<td>Tariff</td>
</tr>
<tr>
<td>Dual instruments</td>
</tr>
</tbody>
</table>

TABLE I shows the comparisons of equilibrium results under the international bilateral Cournot
competition. Three important remarks are noteworthy. First, both governments cannot achieve a higher social welfare when they adopt the dual trade instruments simultaneously, i.e., $W^D_i = 0.331 < W^S_i = 0.332$. This is because the output substitution effect between the SOE and PE is weakened while the welfare-reducing effect from the tariff is strengthened under the dual policy regime. Thus, the equilibrium results of subsidy regime cannot be sustained when both governments have an option to adopt the dual trade instruments when two countries can strategically change the rates of subsidy and tariff together under Cournot competition. Accordingly, the government chooses lower degree of privatization and higher subsidy rates to protect domestic welfare. This is the sharp difference between the competitive equilibrium in the bilateral mixed market and the optimal equilibrium in the unilateral mixed market such as Han (2012), which will be re-examined in the following section. (See Proposition 2)

Second, this competitive equilibrium provides the prisoner’s dilemma situation and thus, the FTA can work for solving this problem. That is, if two countries cooperate and adopt a subsidy instrument only under free trade agreement, then both of them could achieve higher social welfare levels. However, if two countries adopt the dual trade instruments of subsidy and tariff, then the social welfare is lower than those under a single trade instrument with subsidy. Even without subsidy policy under the framework of WTO, compared to the result in the import tariff only regime, the social welfare is higher when both governments choose no trade instrument regime, $W^{NT}_i = 0.327 > W^{T^*}_i = 0.324$. That is, both governments can achieve higher welfare levels when they signed FTA which implies that FTA can work as a coordination device to solve the prisoner’s dilemma in the bilateral mixed market. This result can also be applied to the asymmetric trade instruments case where domestic and foreign governments adopt different trade instruments between subsidy and tariff, which will be discussed in the next section. (See section 4.1)

Third, both governments cannot achieve the maximum welfare in the first-best allocation even under the subsidy regime. When the government decides the direct allocation which maximizes its domestic welfare, where the market price is equal to marginal production cost, both governments can get the maximum welfare level of $W^*_i = 0.333$, which is larger than $W^S_i = 0.332$. It confirms that privatization neutrality theorem does not hold when foreign competitor is included in the international bilateral trade model under the different optimal subsidy/tariff regimes. That is, the effect of privatization on welfare is affected by the response of the foreign country’s policy in bilateral trade. Thus, strategic bilateral trade leads to a significantly different welfare comparison before and after the imposition of production subsidy. In particular, the degree of privatization under subsidy regime should be lower than that under tariff regime, which is also lower than that under dual trade instruments.

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8 It is easy to show that the equilibrium in subsidy regime is not an equilibrium in the dual-instrument regime. For this, suppose that one country $i$ sets the optimal subsidy, tariff, and privatization at the optimal levels in subsidy regime, i.e., $\theta_i^{S^*} = 0.28, s_i^{S^*} = 0.16$ and $t_i^{S^*} = 0$. Then we can find the optimal responses of the other country $j$ in dual-instrument regime are $\theta_j^{T^*} = 0.39, s_j^{T^*} = 0$ and $t_j^{T^*} = 0$. Thus, the resulting social welfare of each country is $W^*_i = 0.325$ and $W^*_j = 0.336$, which induces country $j$ to deviate from the results in subsidy regime.

9 Privatization neutrality theorem states that privatization does not affect welfare regardless of time structure, competition mode, the number of firms, product differentiation, and the degree of privatization under the optimal tax-subsidy policy. This well-known theorem has been discussed in White (1996), Poyago-Theotoky (2001), Tomaru (2006), Hashimzade et al. (2007) and Matsumura and Okumura (2013). However, if there are foreign competitors, privatization matters on the welfare even under the optimal tax-subsidy policy. See, for example, Matsumura and Tomaru (2012).
4. Discussions

In this section, we first analyze the equilibrium with asymmetric trade instruments where
domestic and foreign governments adopt different trade instruments. We next compare the
equilibrium outcomes in the bilateral mixed market with the previous results under a unilateral mixed
market. We then investigate the results under Stackelberg leadership of SOE and compare them with
those under Cournot competition. Finally, we investigate the local optimum for maximum-revenue
and global optimum for maximum-welfare under Cournot competition.

4.1. Asymmetric trade instruments between subsidy and tariff

We have considered the symmetric trade instruments case where domestic and foreign
governments adopt same trade instruments between subsidy and tariff in section 3. Here we analyze
the asymmetric situation where two governments adopt different trade instruments in the international
bilateral mixed market, and provide the similar situation of prisoner’s dilemma game.

Suppose that domestic government adopts a positive production subsidy to its domestic SOE and
PE, \( s_i > 0 \), and foreign government adopts a positive import tariff, \( t_j > 0 \). Then, the profit
functions of SOE and PE, and the social welfare of domestic and foreign countries are as follows:

\[
\pi_i = (P_i + s_i)q_i - \frac{1}{2}q_i^2, \quad \pi_j = P_jq_j - \frac{1}{2}q_j^2,
\]

\[
\pi_{pi} = (P_i + s_i)(q_{hi} + (P_j + s_j - t_j)q_{ei}) - \frac{1}{2}(q_{hi} + q_{ei})^2, \quad \pi_{pj} = P_jq_{ij} + P_jq_{ji} - \frac{1}{2}(q_{ij} + q_{ji})^2,
\]

\[
W_i = CS_i + \pi_i + \pi_{pi} - s_i(q_{si} + q_{hi} + q_{ei}), \quad W_j = CS_j + \pi_j + \pi_{pj} + t_jq_{ji}.
\]

Let superscript “A*” denote the equilibrium outcome in this asymmetric trade instruments case.
The first-order conditions provide the following optimal degree of privatization, subsidy and tariff:
\( \theta_i^{*A} = 0.262, \theta_j^{*A} = 0.314, s_i^{*A} = 0.148 \) and \( t_j^{*A} = 0.171 \). The optimal privatization level of country \( i \)
which adopts product subsidy is lower than that of country \( j \) which adopts import tariff, \( \theta_i^{*A} < \theta_j^{*A} \).
Also, optimal subsidy and tariff levels of both countries are higher than those under dual symmetric
policy instruments, while optimal privatization is lower than that under symmetric dual policy
instruments, \( s_i^{*A} > s_i^{*D}, t_j^{*A} > t_j^{*D} \) and \( \theta_i^{*A} < \theta_i^{*D} \).

Substituting them into the equilibrium outputs yields the optimal market output and price in the
two countries, \( Q_i^{*A} = 0.656, Q_j^{*A} = 0.596, P_i^{*A} = 0.344 \) and \( P_j^{*A} = 0.404 \). Then, the resulting social
welfare are \( W_i^{*A} = 0.322 \) and \( W_j^{*A} = 0.337 \). Note that under asymmetric trade instruments, foreign
government which adopts an import tariff with privatization policy can achieve a higher social welfare
than that under symmetric subsidy in TABLE I, i.e., \( W_i^{*A} (\theta_i > 0, \theta_j > 0) = 0.337 > W_j^{*A} = 0.322 \). Thus,
the government which adopts an import tariff policy under asymmetric trade instruments will be better
off than that under symmetry trade instrument. However, this asymmetric trade instrument will harm
the other country which adopts a production subsidy only and thus, will make the other country to
choose tariff, i.e., \( W_i^{*A} (\theta_i > 0, \theta_j > 0) = 0.322 < W_j^{*A} = 0.324 \). It also yields the prisoner’s dilemma
situation: if both countries cooperate to choose the subsidy-only policy with partial privatization, then
both can get higher social welfares. However, the government under asymmetric trade instruments situation will rush to adopt the import tariff in order to obtain a higher social welfare which leads the equilibrium of the symmetric import tariff only with partial privatization. (See footnote 7.)

4.2. Comparison with unilateral mixed market

We examine the equilibria under production subsidy, import tariff, and dual trade instruments choices in the international unilateral mixed market, where there exists one home country in which the SOE compete with the domestic PE and one foreign PE. In particular, from the previous models, if we set \( q_{ij} = 0, q_{ij} = 0, q_{ei} = 0, \theta_j = 0, s_j = 0 \) and \( t_j = \infty \), then we can have the same results with those in Han (2012), which are shown in TABLE II.

<p>| Table II: Comparisons under Cournot competition in the unilateral market |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|</p>
<table>
<thead>
<tr>
<th></th>
<th>( \theta_j )</th>
<th>( t_j )</th>
<th>( s_j )</th>
<th>( W_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsidy</td>
<td>1</td>
<td>0</td>
<td>0.40</td>
<td>0.340</td>
</tr>
<tr>
<td>Tariff</td>
<td>0.36</td>
<td>0.23</td>
<td>0</td>
<td>0.336</td>
</tr>
<tr>
<td>Dual instruments</td>
<td>1</td>
<td>0.15</td>
<td>0.31</td>
<td>0.346</td>
</tr>
</tbody>
</table>

Han (2012) showed that the government prefers the product subsidy to the import tariff (p.589, Proposition 2), and the social welfare is higher when it simultaneously adopts dual trade instruments of subsidy and tariff than that when it only adopts a single-trade instrument whether it is subsidy or tariff (p.591, Proposition 4). Furthermore, it is shown that the optimal regime choice is that the SOE is privatized completely and the dual trade instrument of subsidy and tariff is used jointly in a unilateral mixed market. Therefore, it also confirms that privatization neutrality theorem does not hold when foreign competition is included under the different optimal subsidy/tariff regimes.

The reasoning for the above results is as follows: the role of a subsidy is expanding the total industry output while the role of a tariff is reducing the output of foreign firms. Thus, a subsidy improves the social welfare through increasing consumer surplus while a tariff improves the social welfare through gaining tariff revenue. However, unlike subsidy, the tariff lowers the total industry output and thus lowers consumer surplus. It implies that welfare-maximizing government prefers the product subsidy to the import tariff.\(^{10}\) Comparing the effects of subsidy and tariff, we can find that subsidy can shift production from the high-cost SOE to the low-cost PE, and thus subsidy can induce welfare-improving output substitution effect between the SOE and domestic PE. Accordingly, the government chooses a higher subsidy and a lower tariff, coupled with complete privatization.

However, in the bilateral mixed market where the PEs can export their products to the opposite country, the reduced output of the SOE through privatization will be substituted by the foreign PE and the increased exporting output of domestic PE will increase the cost of domestic PE. Thus, privatization will reduce the welfare-improving output substitution effect. It implies that both governments will choose the partial privatization in the bilateral mixed market and the effectiveness of free trade policy is significant, which is sharply contrasted to the previous results of Han (2012) in a unilateral mixed market. In particular, the strategic interaction under competitive equilibrium in the

---

\(^{10}\) This result parallels the analysis of Pal and White (1998), who found that the subsidy is a better choice for the government when the cost parameter is not so large.
bilateral mixed market requires a less subsidy and no tariff with partial privatization.

**Proposition 2:** Under Cournot competition, a lower production subsidy with partial privatization is the best choice in the international bilateral mixed market while dual trade instruments of higher subsidy and higher tariff with full privatization is the best choice in a unilateral mixed market.

### 4.3. Stackelberg competition

Then, we investigate four different regime choices of production subsidy and import tariff under Stackelberg competition in the international bilateral mixed market where the semi-public firm acts as a leader. Let superscript “SD*”, “SN*”, “SS*” and “ST*” denote the equilibrium outcomes under Stackelberg competition, respectively.

We first consider the case in which both governments adopt dual trade instments with the privatization simultaneously. Under Stackelberg competition with a dominant semi-public firm, after given the announced levels of $\theta_i$, the SOE maximize $O_i$ and then the PE maximize $\pi_i$ sequentially, observing the levels of privatization and the output levels of the SOE.

In the third stage, maximizing $\pi_i$ for a given $\theta_i$ and $q_{id}$, we can obtain the output of each PE

$$q_{id} = \frac{3(1 + t_i + t_f) - 4q_{id} + 4s_i - s_f}{15}, \quad q_{ci} = \frac{3 - 4q_{ci} + 4s_i - s_f - 2t_i - 7t_f}{15}.$$

In the second stage, each SOE sets its output for given $\theta_i$, anticipating the reaction of the domestic and foreign followers as given. We get the solution

$$q_{i} = \Delta_s^{-1} \left( \frac{3008 - 1297t_i - 687t_f + 7(-29 + 391t_i + 66t_f)\theta_i + 23(38 - 17t_i - 17t_f)\theta_f + (-34 + 721t_i)}{+61t_i\theta_i - s_f(233 + 391\theta_i + \theta_f(742 + 299\theta_f)) + s_i(-599 - 115\theta_f + \theta_i(4739 + 1285\theta_i))} \right),$$

where $\Delta_s = 7808 + 2093\theta_i + 2093\theta_f + 561\theta_i\theta_f$.

In the first stage, the differentiation of $W$ with respect to $\theta_i$, $s_i$ and $t_i$ yields the following optimal degree of privatization, subsidy and tariff rates: $\theta^{SD*}_i = 0$, $s^{SD*}_i = 0.12$ and $t^{SD*}_i = 0.15$. Substituting these optimal results, we can obtain the market output and price, $Q_i^{SD*} = 0.62$ and $P_i^{SD*} = 0.38$. The optimal social welfare under Stackelberg competition is $W_i^{SD*} = 0.331$.

When both governments adopt no trade instrument with the privatization policy, using the similar procedures, we can obtain the optimal degree of privatization, $\theta^{SN*}_i = 0$. Substituting the optimal degree of privatization, we can obtain the market output, market price and social welfare under Stackelberg competition, $Q_i^{SN*} = 0.63$, $P_i^{SN*} = 0.37$ and $W_i^{SN*} = 0.327$.

When both governments adopt a production subsidy with the privatization policy, we obtain the optimal degree of privatization and subsidy, $\theta^{SS*}_i = 0$ and $s^{SS*}_i = 0.16$. Substituting the optimal degree of privatization and subsidy, we can obtain the market output, market price and social welfare

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11 Many quantity setting studies are concerned with the case where firms move simultaneously. Nevertheless, much debate among the extensive literature on international trade and industrial organization on the relative merits of the order of moves has been witnessed. See, for example, Pal (1998), Fjell and Heywood (2002), Matsumura (2003), Lu (2006, 2007), Matsumura and Ogawa (2010), Yu and Lee (2011), and Wang et al. (2012) in mixed oligopoly markets.
under Stackelberg competition, $Q_{i}^{SS*} = 0.69$, $P_{i}^{SS*} = 0.31$ and $W_{i}^{SS*} = 0.332$.

When both governments adopt an import tariff coupled with the privatization policy, we obtain the optimal degree of privatization and tariff, $\theta_{i}^{ST*} = 0$ and $t_{i}^{ST*} = 0.14$. Substituting the optimal degree of privatization and tariff, we can obtain the market output, market price and social welfare under Stackelberg competition, $Q_{i}^{ST*} = 0.58$, $P_{i}^{ST*} = 0.42$ and $W_{i}^{ST*} = 0.325$.

TABLE III provides the results under Stackelberg competition in the bilateral market. Comparing the equilibria under Cournot and Stackelberg competition in TABLE I and TABLE III, we find that the optimal degree of tariff, subsidy and social welfare under Stackelberg competition are the same with those under Cournot competition. The only difference is the optimal degree of privatization. The reasoning is as follows: the objective function of SOE is the same with that of the government when both governments retain all the ownership of SOE. Then, because the SOE can set its output in advance of the PE as a Stackelberg leader, the government can enjoy a first-mover-advantage in order to achieve the same level of social welfare with that under Cournot competition. Therefore, as long as the optimal degree of privatization is well chosen by the government, the social welfare in the international bilateral mixed market is independent of the pattern of competition. It is noteworthy that this finding is consistent with the unilateral case in Wang et al. (2009), and Yu and Lee (2011).

**TABLE III: Comparisons under Stackelberg competition in the bilateral market**

<table>
<thead>
<tr>
<th></th>
<th>$\theta_{i}$</th>
<th>$t_{i}$</th>
<th>$s_{i}$</th>
<th>$W_{i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No instrument</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.327</td>
</tr>
<tr>
<td>Subsidy</td>
<td>0</td>
<td>0</td>
<td>0.16</td>
<td>0.332</td>
</tr>
<tr>
<td>Tariff</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0.324</td>
</tr>
<tr>
<td>Dual instruments</td>
<td>0</td>
<td>0.15</td>
<td>0.12</td>
<td>0.331</td>
</tr>
</tbody>
</table>

**Proposition 3:** When the public firm has a leadership position under Stackelberg competition, except for the optimal degree of privatization, the optimal levels of subsidy, tariff and social welfare are all the same with the results under Cournot competition in the international bilateral mixed market.

### 4.4. Maximum-revenue and global optimum

Finally, we investigate the local optimum for maximum-revenue and global optimum for maximum-welfare under Cournot competition.\(^{12}\)

We first consider the local optimum in which both governments maximize the government revenue, $GR_{i} = T_{i} - S_{i}$, instead of the social welfare of each country. Let superscript “GR*” denote the equilibrium outcome for maximum-revenue. The differentiation of the government revenue, $GR_{i}$, with respect to $\theta_{i}$, $s_{i}$ and $t_{i}$ yields the following optimal degree of privatization, subsidy and tariff.

\(^{12}\) The issue of maximum-revenue tariffs versus optimum-welfare tariffs is interesting because tariff revenue is an important income source of the government before building up an efficient tax system. The issue was raised in the literature of strategic tariff analysis under oligopoly, see, for example, Collie (1991), Larue and Gervais (2002), Clarke and Collie (2006). Also, Wang et al. (2010) and Wang et al. (2012) examined the issue of maximum-revenue tariffs versus optimum-welfare tariffs in a different market structure of mixed oligopoly.
rates:

\[ \theta^{GW^*} = 0.29, \quad t_i^{GW^*} = 0.14, \quad s_i^{GW^*} = 0. \]

We next consider the global optimum in which both governments maximize the global welfare, which is defined as the sum of the social welfare of each country\(^{13}\), \(\bar{W} = W_i + W_j\). Let superscript “GW**” denote the equilibrium outcome of global optimum. The differentiation of \(\bar{W}\) with respect to \(\theta_i, s_i\), and \(t_i\) yields the following optimal degree of privatization, subsidy and tariff rates:

\[ \theta_i^{GW^*} = \frac{1}{2}, \quad s_i^{GW^*} = \frac{1}{6}, \quad t_i^{GW^*} = 0. \]

Then, we compare the maximum-revenue equilibrium with the maximum-welfare equilibrium in the following TABLE IV.

<table>
<thead>
<tr>
<th></th>
<th>(\theta_i)</th>
<th>(t_i)</th>
<th>(s_i)</th>
<th>(GR_i)</th>
<th>(W_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local welfare-Max</td>
<td>0.28</td>
<td>0</td>
<td>0.16</td>
<td>-0.11</td>
<td>0.332</td>
</tr>
<tr>
<td>Global welfare-Max</td>
<td>0.5</td>
<td>0</td>
<td>0.17</td>
<td>-0.11</td>
<td>0.333</td>
</tr>
<tr>
<td>Local revenue-Max</td>
<td>0.29</td>
<td>0.14</td>
<td>0</td>
<td>0.066</td>
<td>0.324</td>
</tr>
</tbody>
</table>

**Proposition 4:** Under Cournot competition, the maximum-revenue privatization, combined with zero subsidy and higher tariff, is higher than local maximum-welfare privatization in the international bilateral mixed market.

The economic reasoning is that privatization decreases the output of SOE which also attracts more exports from the foreign country. Thus, the increased exports from the foreign country will directly lead to an increase in the local government revenue. This finding is still consistent with the previous result of Wang et al. (2010, 2012) in a unilatral mixed market.

**Proposition 5:** The international bilateral equilibrium under Cournot competition yields less degree of privatization and lower subsidy rate, even though it is jointly suboptimal from the viewpoint of global welfare.

This proposition indicates that both governments should choose higher privatization levels and subsidy rates when they maximize the global welfare even though the local country’s optimal degree of tariff is the same as the global optimum, which is zero tariff rate. The intuition comes from the strategic interaction between the two independent countries. There is a business stealing effect from the foreign firms, and thus, concerning its own country’s welfare, each government will strategically reduce the degree of privatization to lessen the business-stealing effect. However, from the perspective of global welfare where both governments do not take the business-stealing effect into

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\(^{13}\) The issue of global welfare-maximization is important to build up an efficient international trade system. In mixed oligopoly setting, see, for example, Barcena-Ruiz and Garzon (2005), Dadpay and Heywood (2006), Han and Ogawa (2008), and Lee et al. (2013).
consideration at all, a higher degree of privatization will increase the cost-saving effect from the PEs, and thus increase both the home country’s welfare and the foreign country’s welfare, i.e., global welfare. This competitive equilibrium can be seen as the prisoner’s dilemma, which was also examined by Han and Ogawa (2008) and Lee et al. (2013).

5. Conclusions

This paper considered an international bilateral mixed market where a domestic state-owned enterprise competes with both domestic and foreign private firms. We examined the strategic interaction of two countries’ optimal choices of trade instruments such as production subsidy and import tariff, coupled with privatization policy.

The main results of our analysis are as follows: First, under Cournot competition, the highest social welfare can be achieved when both governments adopt a production subsidy only with partial privatization in the international bilateral mixed market. This is contrasted to the previous results under the unilateral mixed market, in which dual-trade instruments of subsidy and tariff with full nationalization is the best choice.

Second, we examined the asymmetric choices of two countries on trade instruments between subsidy and tariff, and emphasized the importance of FTA as a coordination device to solve the prisoner’s dilemma at a competitive equilibrium, where both countries could achieve higher social welfare if they cooperate and adopt a subsidy instrument only under FTA.

Third, when the SOE takes a leadership position under Stackelberg competition, except for the optimal degree of privatization, the optimal values of subsidy, tariff and social welfare are the same with the results under Cournot competition. This implies that as far as the optimal degree of privatization is well chosen by the government, the social welfare in the international bilateral mixed market is independent of the pattern of competition. Thus, we showed that the optimal decision on the privatization matters on the welfare even under optimal subsidy/tariff regimes.

Fourth, the maximum-revenue privatization, combined with the zero subsidy and higher tariff, is higher than optimum-welfare privatization in the international bilateral mixed market. However, the international bilateral equilibrium yields less degree of privatization and lower subsidy rate, even though it is jointly suboptimal from the viewpoint of global welfare.

Finally, one might wonder how robust the results are under alternative scenarios for the various modes of competition such as Bertrand competition or/and product differentiation, the number of private firms, and more general specifications of demand and cost functions between the SOEs and the PEs, and so on. We expect that the importance of FTA in the bilateral mixed market is still effective on determining the optimal decisions on the privatization and subsidy rate, even though the competitive effect of other factors will increase the degree of privatization and decrease the level of subsidy. These policy issues will be challenging issue for the future study.

Appendix I

The proof of no-export equilibrium of the SOE

Suppose that SOE can export output to the other country, which is denoted by $q_{oj}^e$. We also assume that the cost function of the SOE is given by $C(q_{h}^S + q_{e}^S) = \frac{1}{2}(q_{h}^S + q_{e}^S)^2$. Then, the inverse
demand function is given by $P_i = 1 - Q_i$, $i = 1, 2$, where $Q_i = q_{hi}^s + q_{ij}^s + q_{hi}^p + q_{ij}^p$, $i \neq j = 1, 2$.

The profit functions of the SOE and the PE, consumer surplus, and welfare of country $i$ are given as, respectively:

$$\pi_{si} = (P_i + s_i)q_{hi}^s + (P_j + s_i - t_j)q_{ci}^s - \frac{1}{2}(q_{hi}^s + q_{ci}^s)^2,$$

$$\pi_{pi} = (P_i + s_i)q_{hi}^p + (P_j + s_i - t_j)q_{ci}^p - \frac{1}{2}(q_{hi}^p + q_{ci}^p)^2,$$

$$CS_i = \frac{1}{2}Q_i^2 = \frac{1}{2}(q_{hi}^s + q_{ij}^p + q_{hi}^p + q_{ij}^p)^2,$$

$$W_i = CS + \pi_{si} + \pi_{pi} + t_i(q_{hi}^s + q_{ij}^p) - s_i(q_{hi}^s + q_{ij}^s + q_{hi}^p + q_{ij}^p).$$

Note that the objective of SOE is given by $O_i = (1 - \theta)W_i + \theta_i\pi_{si}$.

For general solutions, which allow boundary solutions for the SOE’s export output, i.e., zero export output, we should consider Kuhn-Tucker conditions for the maximization problem. However, for the time being, we assume that the optimal output of SOE’s export output is zero and solve the optimality problem of SOE without considering on the export output of SOE. And then we will show that the zero export output of SOE will satisfy the optimal Kuhn-Tucker solutions. Therefore, the SOE would not participate in export at equilibrium.

1. Dual trade instruments ($s_i > 0$ and $t_i > 0$)

In the Table I, we have the welfare-maximizing optimal privatization $\theta_i^{pr} = 0.41$, subsidy $s_i^{pr} = 0.12$ and tariff $t_i^{pr} = 0.15$. Then, we have the following optimal outputs at equilibrium: $q_{hi}^s = 0.33; q_{hi}^p = 0.22; q_{ij}^p = 0.07$. Then, from the Kuhn-Tucker conditions for maximizing the objective of SOE, we have the necessary conditions for having boundary solution for zero the SOE’s export output as follows:

$$\frac{\partial O_i}{\partial q_{ci}^s} = 1 - 2q_{ci}^p - q_{hi}^p - 3q_{ci}^s - q_{hi}^s - q_{ij}^s - t_j + q_{ci}^p\theta_i + s_i\theta_i = -0.09 < 0.$$

2. No trade instrument ($s_i = t_i = 0$)

In the Table I, we have the welfare-maximizing optimal privatization $\theta_i^{pr} = 0.20$ and optimal outputs at equilibrium: $q_{hi}^s = 0.39; q_{hi}^p = 0.12; q_{ij}^p = 0.12$. Then, from the Kuhn-Tucker conditions for maximizing the objective of SOE, i.e., $q_{ij}^s \geq 0$, $\frac{\partial O_i}{\partial q_{ij}^s} \leq 0$ and $q_{ci}^s\frac{\partial O_i}{\partial q_{ci}^s} = 0$, we have the necessary conditions for having boundary solution for zero the SOE’s export output as follows:

$$\frac{\partial O_i}{\partial q_{ci}^s} = 1 - 2q_{ci}^p - q_{hi}^p - 3q_{ci}^s - q_{hi}^s - q_{ij}^s + q_{ci}^p\theta_i = -0.12 < 0.$$
3. Production subsidy \((s_i > t_i = 0)\)

In the Table I, we have the welfare-maximizing optimal privatization \(\theta_i^{s^*} = 0.28\) and subsidy \(s_i^{**} = 0.16\). Then, we have the following optimal outputs at equilibrium: \(q_{hi}^i = 0.37;\) \(q_{gi}^p = 0.16;\) \(q_{ej}^p = 0.16\). Then, from the Kuhn-Tucker conditions for maximizing the objective of SOE, we have the necessary conditions for having boundary solution for zero the SOE’s export output as follows:

\[
\frac{\partial O}{\partial q_{ei}^i} = 1 - 2q_{ei}^p - q_{hi}^p - 3q_{ei}^p - q_{hi}^p + q_{ei}^p\theta_i + s_i\theta_i = -0.13 < 0.
\]

4. Import tariff \((t_i > s_i = 0)\)

In the Table I, we have the welfare-maximizing optimal privatization \(\theta_i^{t^*} = 0.29\) and tariff \(t_i^{*} = 0.14\). Then, we have the following optimal outputs at equilibrium: \(q_{hi}^i = 0.35;\) \(q_{hi}^p = 0.18;\) \(q_{ej}^p = 0.05\). Then, from the Kuhn-Tucker conditions for maximizing the objective of SOE, we have the necessary conditions for having boundary solution for zero the SOE’s export output as follows:

\[
\frac{\partial O}{\partial q_{ei}^i} = 1 - 2q_{ei}^p - q_{hi}^p - 3q_{ei}^p - q_{hi}^p - t_j + q_{ei}^p\theta_i = -0.23 < 0.
\]
Appendix II

The social welfare of country $i$ is

$$W_i = \frac{1}{2\Delta_c} \left( 3(15A_i + 6A_i\theta_j + A_i\theta_j^2) + 6(B_i + B_i\theta_j + B_i\theta_j^2)\theta_j + (C_i + C_i\theta_j + 2C_i\theta_j^2)\theta_j^2 \right),$$

where

$$A_i = -184s_i^2 + 20s_i^2 + 2s_i(29 + 17s_j - 35t_i + 7t_j) + 2s_i(-4 + 70t_i + 13t_j) + 5(52 - 59t_i^2 + 2t_i(8 + t_j))$$
$$+ t_j(-14 + 43t_j),$$

$$A_i = 585 - 400s_i^2 + 107s_i^2 - 85t_i(-1 + 7t_i) + s_i(119 + 103s_j - 175t_i - 58t_j) - 395t_j + 175t_jt_i + 665t_j^2$$
$$+ s_i(-124 + 455t_i + 533t_j),$$

$$A_i = 921 - 485s_i^2 + 821s_i^2 - 106t_i - 697t_i^2 + 2(-541 + 262t_i)t_j + 1244t_j^2 - 2s_i(-143 + 63s_j + 156t_i$$
$$+ 162t_j) + 2s_j(-431 + 503t_i + 925t_j),$$

$$B_i = -706s_i^2 + 235s_i^2 - s_i(238 + 339s_j + 580t_i - 246t_j) + s_j(253 + 765t_i + 64t_j) + 5(366 - 407t_i^2$$
$$+ 55t_i(-2 + 5t_j) + t_j(98 + 6t_j),$$

$$B_i = 1596 - 589s_i^2 + 409s_i^2 + 251t_i - 1662t_i^2 + 5(-205 + 73t_i)t_j + 1675t_j^2 - s_i(149 + 378s_j + 584t_i$$
$$+ 111t_j) + s_j(-1 + 997t_i + 1264t_j),$$

$$B_i = 402 - 104s_i^2 + 392s_i^2 - 23t_i - 33t_i^2 + (-445 + 191t_i)t_j + 518t_j^2 - 4s_i(-11 + 60s_j + 42t_i + 39t_j)$$
$$+ 4s_j(-71 + 94t_i + 191t_j),$$

$$C_i = -13109s_i^2 - 335s_i^2 + 2s_i(5502 + 2573s_j - 6490t_i - 2159t_j) - 3(99 + 482t_i) - 2s_i(1128 - 1888t_i$$
$$- 569t_j) + 2t_i(3147 - 2818t_i + 114t_j),$$

$$C_i = -11136s_i^2 - 6(-29 + 8s_j)(24 + s_j) + (831 + 634s_j)t_j - 775t_i^2 + 36s_i(256 + 128s_j - 307t_i - 109t_j$$
$$+ 6(69 + 422s_j - 25t_i)t_j + 3079t_j^2,$$

$$C_i = 36 - 1168s_i^2 - 176s_i^2 + 468t_i - 475t_i^2 + 8s_i(126 + 52s_j - 149t_i - 61t_j) - 2(54 + 35t_i)t_j + 293t_j^2$$
$$+ 8s_j(-54 + 65t_i + 73t_j).$$
References


