Endogenous timing in private and mixed duopolies with emission taxes

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Abstract This paper examines an endogenous timing game in product differentiated duopolies under price competition when emission tax is imposed on environmental externality. We show that a simultaneous-move (sequential-move) outcome can be an equilibrium outcome in a private duopoly under significant (insignificant) environmental externality, but this result can be reversed in a mixed duopoly. We also show that when environmental externalities are significant, public leadership yields greater welfare than private leadership, and that public leadership is more robust than private leadership as an equilibrium outcome. Finally, we find that privatization can result in a public leader becoming a private leader, but this worsens welfare.

Keywords: Emission tax; Endogenous timing; Mixed duopoly; Private duopoly
JEL Classification: L5; D6; Q2

1 Introduction

The earlier literature on duopolistic competition analyzes the endogenous market structure based on whether firms endogenously decide on prices or quantities and whether such decisions are made sequentially or simultaneously. Hamilton and Slutsky (1990) formulated an observable delay game and showed that in a private duopoly with symmetric payoffs, firms decide simultaneously when competing in quantities and sequentially when competing in prices. However, in the literature on mixed duopolies with asymmetric payoffs, where a profit-maximizing private firm competes with a welfare-maximizing public firm, Pal (1998) and Bárcena-Ruiz (2007) showed that the results are surprisingly reversed: firms decide sequentially when competing in quantities and simultaneously when competing in prices.[1]

Besides understanding these conflicting results, recent concerns over environmental quality

[1] Lu (2006), Lu and Poddar (2009) and Heywood and Ye (2009) extended the analysis into a mixed market where a public firm competes with domestic and foreign private firms and obtained similar results. For more extensive analysis, see Bárcena-Ruiz and Garzon (2010), Tomaru and Kiyono (2010), Balogh and Tasnadi (2012), Amir and Feo (2014), Matsumura and Ogawa (2014), Naya (2015) and Din and Sun (2016) among others.

suggest the need for further examination of what allows for environmental externalities and for the possibility of considering public policies such as emission tax or privatization.[2] In the presence of environmental externalities, the analysis of mixed oligopolies has been prominent and thus the possible benefits of public ownership have also motivated recent analyses on mixed markets.[3] For example, Pal and Saha (2014, 2015) and Xu et al. (2016) have recently explored the interaction between privatization and emission tax in order to explain how privatization policies and emission tax affect environmental damage and social welfare. However, previous studies on environmental issues consider an exogenously fixed timing game and hence have very restrictive implications.

This paper is the first to investigate an endogenous timing game in private and mixed duopolies with environmental externalities and emission taxes. Specifically, we examine an observable delay game formulated by Hamilton and Slutsky (1990) in product-differentiated duopoly markets under price competition when an emission tax is imposed on environmental externality. We find that most results in both private and mixed duopolies in the literature without externalities still hold only when environmental externalities are insignificant. For instance, we show that the equilibrium under price competition with an emission tax is a sequential-move outcome in a private duopoly, which is consistent with Hamilton and Slutsky (1990), but a simultaneous-move outcome in a mixed duopoly, which is consistent with Bárcena-Ruiz (2007).

However, when environmental externalities are significant, the results are surprisingly reversed in both private and mixed duopolies. We show that the equilibrium under price competition with an emission tax is a simultaneous-move outcome in a private duopoly but a sequential-move outcome in a mixed duopoly. This is in sharp contrast to the results in the previous literature under price competition without externalities. Therefore, policies concerning environmental quality have a significant effect on the endogenous timing that firms choose for production. This implies that in a mixed duopoly under price competition, the analysis of a

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2 In most countries, mixed markets exist in a broad range of industries such as oil, gas, automobile, steel, chemical, telecommunication, electricity, power plant, and hospital industries, which emit pollutants in the production process. In particular, many state-owned industries in transition economies have relied on highly polluting technologies. Furthermore, EU countries lead the development of environmental policies for the sustainability in a warming planet and have a non-negligible presence of public enterprises in energy-consuming industries such as transportation and automobile industries. More related descriptions can be found in Wang and Wang (2009), Pal and Saha (2014, 2015) and Xu, et al. (2016).

3 Several researchers have recently analyzed the environmental concerns of a mixed market. Beladi and Chao (2006), Bárcena-Ruiz and Garzon (2006), Ohori (2006), Cato (2008), Wang and Wang (2009), and Xu and Lee (2015) provide various discussions on mixed markets. Recently, Clo, et al (2016) supports the positive effect of public ownership on environmental performance in European electricity industry during the two decades since the market-based instrument is introduced in 1980s.

simultaneous-move game can be problematic when environmental externality is significant.

When environmental externalities are significant, we also find that public leadership yields greater welfare than private leadership; moreover, public leadership is more robust than private leadership as an equilibrium outcome. These results are in sharp contrast to those in mixed duopolies without environmental externalities. Pal (1998), Matsumura (2003), and Matsumura and Ogawa (2010) showed that private leadership is more robust and more efficient. However, significant externalities can change the equilibrium outcome between private and public leaderships.

Finally, we investigate an endogenous choice on privatization in order to examine the welfare effect of privatization. We find that privatization can result in a public leader becoming a private leader, but this worsens welfare.

The remainder of this paper is organized as follows. In section 2, we formulate a product-differentiated duopoly model in price competition with environmental externalities. Sections 3 and 4 analyze an endogenous timing game in private and mixed duopolies, respectively. Section 5 examines an endogenous choice on privatization. Finally, Section 6 concludes the paper.

2 The model

We consider a standard differentiated duopoly with linear demand in Sing and Vives (1984), where a representative consumer's utility function is given by

$$U(q_0, q_1) = A(q_0 + q_1) - \frac{1}{2}({q_0}^2 + 2bq_0q_1 + {q_1}^2), \tag{1}$$

where q_i is the output of each firm and $b \in (0, 1)$ measures the degree of product differentiation. A higher value of b represents a lower degree of product differentiation, or higher substitutability.

The inverse demand function of each firm is $p_i = A - q_i - bq_j$, $i = j = 0,1$, $i \neq j$, where p_i is the market price of product *i*. Then, consumer surplus is represented by $\text{CS} = \frac{1}{2}({q_0}^2 + 2bq_0q_1 + {q_1}^2)$. Note that higher substitutability reduces a consumer's willingness to pay for each product but increases consumer surplus. The direct demand function of each firm is expressed as

$$q_0 = \frac{A-Ab-p_0+bp_1}{1-b^2},\ q_1 = \frac{A-Ab+bp_0-p_1}{1-b^2}. \tag{2}$$

We assume that both firms have identical technologies and that the production cost function takes the quadratic form $C(q_i) = F + \frac{q_i^2}{2}$, where $F = 0$ without loss of generality.

In both firms, production leads to pollution e_i, but each firm can reduce pollution by undertaking abatement activities. Suppose that firm i chooses pollution abatement level a_i; then, the emission level of each firm is reduced to $e_i = q_i - a_i$ by investing an amount of $\frac{a_i^2}{2}$ in abatement activities.[4] The extent of environmental damage due to industrial pollution may be given by $ED = d\sum_i e_i$. The government imposes an environmental tax on the emission level, for which the tax rate is t. The resulting total tax revenue is $T = t\sum_i e_i$.

The profit of firm i is given by

$$\pi_i = p_i q_i - \frac{q_i^2}{2} - te_i - \frac{a_i^2}{2}, \ \ i = 0, \ 1. \tag{3}$$

Social welfare is the sum of consumer surplus CS, the profit of both firms $\pi_o + \pi_1$, and tax revenue T, minus environmental damage ED:

$$W = CS + \pi_0 + \pi_1 + T - ED. \tag{4}$$

The game formulated by Hamilton and Slutsky (1990) proceeds as follows. In the first stage, each firm simultaneously chooses whether to move early or late. The basic game played is simultaneous if both firms choose the same period, and sequential otherwise. In the following, we examine respectively a private duopoly where both private firms compete in price and a mixed duopoly where one private firm and one public firm compete, and we compare the results.

3 Private Duopoly

In this section, we first consider a fixed-timing game in private duopolies with two firms competing in prices in a simultaneous-move game and in a sequential-move game, respectively. We then examine the first stage in an endogenous-timing game.

3.1 Simultaneous-move game

In this game, each firm chooses its price and abatement level simultaneously and independently. Assuming interior solutions and simultaneously solving the first-order conditions for

[4] For simplicity of tractability, in line with the literature (Wang and Wang, 2009; Pal and Saha, 2015; Xu et al., 2016), we focus on end-of-pipe abatement, which is additively separable. Implicitly, we assume that both products emit the same type of pollutants.

maximizing the profits of both firms in (3), we obtain the following equilibrium prices and abatement levels:

$$p_0 = p_1 = \frac{2A - Ab^2 + t + bt}{3 + b - b^2},\ a_0 = a_1 = t.$$

The social welfare in equilibrium is

$$W = \frac{A^2(4+b-2b^2) - 2A(3+b-b^2)d + 24d - 2(1-b)(A + Ab - 7bd - b^2d + b^3d)t - 2(11+7b-5b^2-2b^3+b^4)t^2}{(3+b-b^2)^2}. \quad (5)$$

Differentiating social welfare with respect to *t* yields the following optimal emission tax in a simultaneous-move Bertrand game in Private duopolies (BP):[5]

$$t^{BP} = \begin{cases} \frac{12d - (1-b)(A(1+b) - bd(7+b-b^2))}{11+7b-5b^2-2b^3+b^4} & if\ d > d_1 \\ 0 & if\ d \le d_1 \end{cases}, \quad (6)$$

where d_1 is as presented in Appendix A.[6] Note that when $d > d_1$, the optimal emission tax is increasing in both the degree of production differentiation, $\partial t^{BP}/\partial b > 0$, and marginal environmental damage, $\partial t^{BP}/\partial d > 0$. However, it is lower than the marginal environmental damage, $0 < t^{BP} < d$ for $b \in (0,1)$.

In the first case, when $d > d_1$, we obtain the equilibrium prices, abatement levels, and quantities of the two firms, as presented in Appendix B.[7] The equilibrium profit of the private firm, environmental damage, and welfare are, respectively,

$$\pi_0^{BP} = \pi_1^{BP} = \frac{m_1}{2(11+7b-5b^2-2b^3+b^4)^2},\ ED^{BP} = \frac{2d(A(5+b-2b^2) - (4+b-b^2)^2 d)}{11+7b-5b^2-2b^3+b^4} d,$$

$$W^{BP} = \frac{A^2(5+b-2b^2) - 2Ad(5+b-2b^2) + (4+b-b^2)^2 d^2}{11+7b-5b^2-2b^3+b^4}. \quad (7)$$

In the second case, when $d \le d_1$, the optimal emission tax is zero. The equilibrium profit of the private firm, environmental damage, and welfare are, respectively,

$$\pi_0^{BP} = \pi_1^{BP} = \frac{A^2(3-2b^2)}{2(3+b-b^2)^2},\ ED^{BP} = \frac{2Ad}{3+b-b^2},$$

$$W^{BP} = \frac{A(A(4+b-2b^2) - 2(3+b-b^2)d)}{(3+b-b^2)^2} \quad (8)$$

[5] Note that the optimal emission tax can be negative when environmental externality is insignificant under duopolistic competition. Note also that the equilibrium abatement level becomes zero when a non-positive emission tax is imposed. In order to eliminate this trivial and unrealistic situation, we focus on non-negative emission taxes in the remaining analysis.

[6] For the sake of expositional convenience, we provide d_i, m_i, and n_i in Appendix A.

[7] For the sake of expositional convenience, we provide p_i, a_i, and q_i in Appendix B.

3.2 Sequential-move game

In this game, first firm 0 and then firm 1 choose their price and abatement levels sequentially. Then, assuming interior solutions, the first-order conditions of firm 1 to maximize its profits in (3) provide the following reaction function:

$$P_1 = \frac{(1-b)\left(A\left(b^2-2\right)-(1+b)t\right)-b(2-b^2)p_0}{-3+2b^2},\ a_1 = t. \tag{9}$$

Now, the first-order conditions of firm 0 to maximize its profits in (3) with the reaction function of firm 1 in (9) provide the following equilibrium prices and abatement levels:

$$P_0 = \frac{3A(2-b^2)(3-b-b^2)+(3+2b)(3-3b^2+b^3)t}{27-24b^2+5b^4},\ a_0 = t.$$

$$p_1 = \frac{A(2-b^2)(9-3b-4b^2+b^3)+(9+6b-7b^2-5b^3+b^4+b^5)t}{27-24b^2+5b^4}\ ,\ a_1 = t.$$

The social welfare in equilibrium is

$$W = \frac{m_2}{(27-24b^2+5b^4)^2}. \tag{10}$$

Differentiating social welfare with respect to t yields the following optimal emission tax in a sequential-move Leadership game in Private duopolies (LP):

$$t^{LP} = \begin{cases} \frac{m_3}{2(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8)} & if\ d > d_2 \\ 0 & if\ d \le d_2 \end{cases} \tag{11}$$

Note that when $d > d_2$, the optimal emission tax also increases in both the degree of production differentiation, $\partial t^{LP}/\partial b > 0$, and marginal environmental damage, $\partial t^{LP}/\partial d > 0$. Note also that it is lower than the marginal environmental damage, $0 < t^{LP} < d$, for $b \in (0,1)$.

In the first case, when $d > d_2$, the equilibrium profit of the private firm, environmental damage, and welfare are, respectively,

$$\pi_0^{LP} = \frac{m_6}{8(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8))^2},$$

$$\pi_1^{LP} = \frac{m_7}{8(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8))^2},$$

$$ED^{LP} = \frac{A(1620-756b-2268b^2+924b^3+1116b^4-368b^5-220b^6+48b^7+13b^8)-(72-6b-58b^2+2b^3+11b^4)^2d}{2(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8)}d,$$

$$W^{LP} = \frac{m_8}{4(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8)}. \tag{12}$$

In the second case, when $d \le d_2$, the optimal emission tax is zero. The equilibrium profit of the private firm, environmental damage, and welfare are, respectively,

$$\pi_0^{LP} = \frac{A^2(3-b-b^2)^2}{2(27-24b^2+5b^4)},\ \pi_1^{LP} = \frac{A^2(3-2b^2)(9-3b-4b^2+b^3)^2}{2(27-24b^2+5b^4)^2},\ ED^{LP} = \frac{A(18-6b-10b^2+2b^3+b^4)d}{27-24b^2+5b^4},$$

$$W^{LP} = \frac{A^2(324-135b-486b^2+183b^3+251b^4-79b^5-51b^6+11b^7+3b^8)-(3-b^2)(9-5b^2)(18-6b-10b^2+2b^3+b^4)\mathrm{Ad}}{(27-24b^2+5b^4)^2}. \quad (13)$$

3.3 Comparison

Proposition 1 *In private duopolies, the optimal emission tax is lower than marginal environmental damage, and its level in sequential-move games is lower than that in simultaneous-move games.*

Proof: Comparing the values, we have $d_1 < d_2$. Thus, (i) when $0 \leq d \leq d_1$, $t^{LP} = t^{BP} = 0$; (ii) when $d_1 < d \leq d_2$, $t^{LP} = 0 < t^{BP}$; and (iii) when $d > d_2$, $0 < t^{LP} < t^{BP}$. Q.E.D.

This implies that a simultaneous-move game produces more output and thus more emission and higher welfare in price competition. Thus, we have the following proposition.

Proposition 2 *In private duopolies, environmental damage and social welfare are lower in a sequential-move game.*

Proof: We can easily show that $ED^{LP} < ED^{BP}$ and $W^{LP} < W^{BP}$. Q.E.D.

3.4 Endogenous timing game

We now discuss the first-stage choice in an endogenous timing game under price competition in private duopolies. Each firm i (i = 0,1) simultaneously chooses whether to move early (T_i = 1) or late (T_i = 2). If both firms choose the same period, the equilibrium is a simultaneous-move game. Otherwise, the equilibrium is a sequential-move game. Table 1 provides the payoff matrix of the observable delay game in private duopolies.

Table 1: Payoff matrix in private duopolies

Firm 0 /1	T_1 = *1*	T_1 = 2
T_0 = 1	(π_0^{BP}, π_1^{BP})	(π_0^{LP}, π_1^{LP})
T_0 = 2	(π_1^{LP}, π_0^{LP})	(π_0^{BP}, π_1^{BP})

Proposition 3 *In private duopolies,*

(i) *when* $d \in [0, d_3)$, *two sequential-move outcomes,* $(T_0, T_1) = (1,2)$ *and* $(T_0, T_1) = (2,1)$, *are the unique equilibrium outcomes.*

(ii) *when* $d = d_3$, *two sequential-move outcomes,* $(T_0, T_1) = (1,2)$, $(T_0, T_1) = (2,1)$, *and one simultaneous-move outcome,* $(T_0, T_1) = (2,2)$, *are the equilibrium outcomes;*

(iii) *when* $d \in (d_3, d_4)$, *one simultaneous-move outcome,* $(T_0, T_1) = (2,2)$, *is the equilibrium outcome;*

(iv) *otherwise, two simultaneous-move outcomes,* $(T_0, T_1) = (1,1)$ *and* $(T_0, T_1) = (2,2)$, *are the equilibrium outcomes.*

Proof: Comparing the values, we have $d_2 < d_3 < d_4$. Then, the profit ranks are as follows: (i) $\pi_0^{BP} = \pi_1^{BP} \lesseqgtr \pi_0^{LP}$ if $d \lesseqgtr d_3$; and (ii) $\pi_0^{BP} = \pi_1^{BP} \lesseqgtr \pi_1^{LP}$ if $d \lesseqgtr d_4$. Q.E.D.

The proposition represents that private duopolies in price competition with optimal emission tax yield a sequential-move outcome in equilibrium when the environmental externality is insignificant. This result is consistent with the observable delay game without environmental externality, as formulated by Hamilton and Slutsky (1990). On the other hand, under price competition with significant environmental externality, a simultaneous-move outcome appears in equilibrium, which is sharply different from the previous results when environmental externality is not considered.

4 Mixed duopoly

In this section, we examine a mixed duopoly in which firm 0 is a welfare-maximizing public firm and firm 1 is a profit-maximizing private firm. Similarly, we first consider fixed timing games in mixed duopolies where both public and private firms compete in prices in a simultaneous-move game and in two different sequential-move games, public leadership and private leadership. We then examine the endogenous timing game.

4.1 Simultaneous-move game

In this game, both firms choose their prices and abatement levels simultaneously and independently. Assuming interior solutions, we simultaneously solve the first-order conditions of firm 0 to maximize welfare in (4) and those of firm 1 to maximize its profits in (3), to obtain the following equilibrium prices and abatement levels:

$$p_0 = \frac{A(3-2b^2-b^3+b^4)+(1-b)(3-2b^2)d+b(3-b^2)t}{6-4b^2+b^4},$$

$$p_1 = \frac{(2-b^2)(A(2-b)+(1-b)bd)+2t}{6-4b^2+b^4},$$

$$a_0 = d,\ a_1 = t.$$

The social welfare in equilibrium is

$$W = \frac{n_1}{2(6-4b^2+b^4)^2}. \tag{14}$$

Then, differentiating social welfare with respect to t yields the following optimal emission tax in a simultaneous-move Bertrand game in Mixed duopolies (BM):

$$t^{BM} = \begin{cases} \frac{2A(2-b)(b-1)(1+b)+(48-2b-54b^2+2b^3+28b^4-8b^6+b^8)d}{44-50b^2+28b^4-8b^6+b^8} & if\ d > d_5 \\ 0 & if\ \ d \leq d_5 \end{cases}. \tag{15}$$

Note that when $d > d_5$, the optimal emission tax is increasing in both the degree of production differentiation, $\partial t^{BM}/\partial b > 0$, and marginal environmental damage, $\partial t^{BM}/\partial d > 0$. However, it is lower than the marginal environmental damage, $0 < t^{BM} < d$ for $b \in (0,1)$.

In the first case, when $d > d_5$, we obtain the equilibrium prices, abatement levels, and quantities of the two firms. Note that the price of the public firm is lower than that of the private firm, whereas the output of the public firm is larger; that is, $p_0^{BM} < p_1^{BM}$ and $q_0^{BM} > q_1^{BM}$. This shows that the public firm sets a lower price than the private firm, which does not consider consumer surplus. Furthermore, the abatement of the public firm is larger than that of the private firm, which does not consider environmental damage, $a_0^{BM} > a_1^{BM}$.

The equilibrium profit of the private firm, environmental damage, and welfare are, respectively, as follows:

$$\pi_1^{BM} = \frac{n_2}{2(44-50b^2+28b^4-8b^6+b^8)^2},$$

$$ED^{BM} = \frac{A(42-24b-30b^2+20b^3+9b^4-7b^5-b^6+b^7)-d(130-24b-130b^2+20b^3+65b^4-7b^5-17b^6+b^7+2b^8)}{44-50b^2+28b^4-8b^6+b^8}d,$$

$$W^{BM} = \frac{n_3}{44-50b^2+28b^4-8b^6+b^8}. \tag{16}$$

In the second case, when $d \leq d_5$, the optimal emission tax is zero. This yields the following results in equilibrium:

$$\pi_1^{BM} = \frac{(3-2b^2)(A(2-b)+(1-b)bd)^2}{2(6-4b^2+b^4)^2},$$

$$ED^{BM} = \frac{A(5-3b-b^2+b^3)-(9-4b-4b^2+b^3+b^4)d}{6-4b^2+b^4}d,$$

$$W^{BM} = \frac{n_4}{2(6-4b^2+b^4)^2}. \tag{17}$$

4.2 Sequential-move game with public leadership

In this game, first the public firm and then the private firm choose their price and abatement levels sequentially. Assuming interior solutions, the first-order conditions of firm 1 to maximize its profits in (3) provide the reaction function in (9). Then, the welfare-maximizing prices and pollution abatement levels of the public firm in the second stage yield the following:

$$p_0 = \frac{A(9-2b-7b^2+2b^4)+(3-b-b^2)(3-2b^2)d+b(5-2b^2)t}{2(9-8b^2+2b^4)}, \ a_0 = d.$$

$$p_1 = \frac{(2-b^2)(A(2-b)(3-b^2)+b(3b-b^2)d)+(6-4b^2+b^4)t}{2(9-8b^2+2b^4)}, \ a_1 = t.$$

The social welfare in equilibrium is

$$W = \frac{n_5}{4(9-8b^2+2b^4)}. \tag{18}$$

Now, differentiating social welfare with respect to t yields the following optimal emission tax in a sequential-move public Leadership in Mixed duopolies (LM):

$$t^{LM} = \begin{cases} \frac{A(2-b)(b-1)(1+b)+(24-b-19b^2+b^3+4b^4)d}{22-17b^2+4b^4} & if\ d > d_6 \\ 0 & if\ d \leq d_6 \end{cases}. \tag{19}$$

Note that when $d > d_6$, the optimal emission tax is increasing in both the degree of production differentiation, $\partial t^{LM}/\partial b > 0$, and degree of marginal environmental damage, $\partial t^{LM}/\partial d > 0$. Note also that it is lower than the marginal environmental damage, $0 < t^{LM} < d$, for $b \in (0,1)$.

In the first case, when $d > d_6$, the price of the public firm is lower than that of the private firm, whereas the output of the public firm is larger; that is, $p_0^{LM} < p_1^{LM}$ and $q_0^{LM} > q_1^{LM}$. Furthermore, the abatement of the public firm is larger than that of the private firm, $a_0^{LM} > a_1^{LM}$.

The equilibrium profit of the private firm, environmental damage, and welfare are, respectively,

$$\pi_1^{LM} = \frac{n_6}{2(22-17b^2+4b^4)^2},$$

$$ED^{LM} = \frac{A(21-10b-10b^2+4b^3+b^4)-d(65-10b-44b^2+4b^3+9b^4)}{22-17b^2+4b^4}d,$$

$$W^{LM} = \frac{A^2(21-10b-10b^2+4b^3+b^4)-2Ad(21-10b-10b^2+4b^3+b^4)+(65-10b-44b^2+4b^3+9b^4)d^2}{2(22-17b^2+4b^4)} \quad (20)$$

In the second case, when $d \le d_6$, the optimal emission tax is zero. This yields the following results in equilibrium:

$$\pi_1^{LM} = \frac{(3-2b^2)(A(2-b)(3-b^2)+b(3-b-b^2)d)^2}{8(9-8b^2+2b^4)^2},$$

$$ED^{LM} = \frac{A(15-7b-8b^2+3b^3+b^4)-(27-6b-21b^2+2b^3+5b^4)d}{2(9-8b^2+2b^4)}d,$$

$$W^{LM} = \frac{A^2(17-8b-10b^2+4b^3+b^4)-2A(15-7b-8b^2+3b^3+b^4)d+(27-6b-21b^2+2b^3+5b^4)d^2}{4(9-8b^2+2b^4)}. \quad (21)$$

4.3 Sequential-move game with private leadership

In this game, first the private firm and then the public firm choose their price and abatement levels sequentially. Assuming interior solutions, the first-order conditions of firm 0 to maximize the welfare in (4) provide the following reaction function:

$$P_0 = \frac{(1-b)^2(A+d+bd)+b(3-b^2)p_1}{2}, \; a_0 = d. \quad (22)$$

Then, the profit-maximizing price and pollution abatement level of the private firm in the second stage yield the following results:

$$P_1 = \frac{(4-b^2)(A(2-b)+(1-b)bd)-2(2-b^2)t}{12-8b^2+b^4}, \; a_1 = t,$$

$$P_0 = \frac{A(6-4b^2+b^3)+(1-b)(6-4b^2+b^4)d+b(6-5b^2+b^4)t}{12-8b^2+b^4}, \; a_0 = d.$$

The social welfare in equilibrium is

$$W = \frac{n_7}{2(12-8b^2+b^4)^2}. \quad (23)$$

Now, differentiating social welfare with respect to t yields the following optimal emission tax in a sequential-move private leadership (or public Followership) Mixed duopolies (FM):

$$t^{FM} = \begin{cases} \frac{4A(b-2)+(96-4b-72b^2+16b^4-b^6)d}{(2-b^2)(44-14b^2+b^4)} & if \; d > d_7 \\ 0 & if \; d \le d_7 \end{cases}. \quad (24)$$

In the first case when $d > d_7$, the optimal emission tax first increases and then decreases as the degree of production differentiation increases; that is, $\partial t^{FM}/\partial b \ge 0$ if $0 < b \le 0.35$ and

$\partial t^{FM}/\partial b \leq 0$ if $0.35 \leq b < 1$. However, it is lower than the marginal environmental damage and increases as the marginal environmental damage increases; that is, $0 < t^{FM} < d$ and $\partial t^{FM}/\partial d > 0$ for $b \in (0,1)$.

When we substitute t^{FM} into p_i, a_i, and q_i, the price of the public firm is lower than that of the private firm, but the output of the public firm is larger than that of the private firm; that is, $P_0^{FM} < P_1^{FM}$ and $q_0^{FM} > q_1^{FM}$. Furthermore, the abatement of the public firm is larger than that of the private firm, $a_0^{FM} > a_1^{FM}$.

The equilibrium profit of the private firm, environmental damage, and welfare are, respectively,

$$\pi_1^{FM} = \frac{n_8}{2(2-b^2)^2(44-14b^2+b^4)^2},$$

$$ED^{FM} = \frac{2A(42-24b-21b^2+11b^3+2b^4-b^5)-2d(130-24b-93b^2+11b^3+18b^4-b^5-b^6)}{(2-b^2)(44-14b^2+b^4)}d,$$

$$W^{FM} = \frac{n_9}{(2-b^2)^2(44-14b^2+b^4)} \tag{25}$$

In the second case, when $d \leq d_7$, the optimal emission tax is zero. This yields the following results in equilibrium:

$$\pi_1^{FM} = \frac{(A(2-b)+(1-b)bd)^2}{2(12-8b^2+b^4)},$$

$$ED^{FM} = \frac{2A(5-3b-2b^2+b^3)-(18-8b-8b^2+3b^3)d}{12-8b^2+b^4}d,$$

$$W^{FM} = \frac{n_{10}}{2(12-8b^2+b^4)^2}. \tag{26}$$

4.4 Comparison

Proposition 4 *In mixed duopolies, the optimal emission tax is lower than marginal environmental damage, but its level in the public (private) leadership game is the highest (lowest).*

Proof: Comparing the values, we have $d_6 < d_5 < d_7$. Thus, (i) when $0 \leq d \leq d_6$, $t^{FM} = t^{BM} = t^{LM} = 0$; (ii) when $d_6 < d < d_5$, $t^{FM} = t^{BM} = 0 < t^{LM}$; (iii) when $d_5 \leq d < d_7$, $t^{FM} = 0 < t^{BM} < t^{LM}$; and (iv) when $d \geq d_7$, $0 < t^{FM} < t^{BM} < t^{LM}$. Q.E.D.

This implies that public leadership in a sequential-move game produces more output and thus more emission and higher welfare in price competition. Thus, we have the following proposition.

Proposition 5 *In mixed duopolies, environmental damage and social welfare are the highest (lowest) in a public (private) leadership game.*

Proof: Comparing the results, we can show that $ED^{FM} < ED^{BM} < ED^{LM}$ and $W^{FM} < W^{BM} < W^{LM}$. Q.E.D.

4.5 Endogenous timing game

We now discuss the first-stage choice in an endogenous timing game under price competition in mixed duopolies. Table 2 provides the payoff matrix of the observable delay game in mixed duopolies.

Table 2: Payoff matrix in mixed duopolies

Firm 0 /1	$T_1 = 1$	$T_1 = 2$
$T_0 = 1$	(W^{BP}, π_1^{BP})	(W^{LP}, π_1^{LP})
$T_0 = 2$	(W^{FP}, π_1^{FP})	(W^{BP}, π_1^{BP})

Proposition 6 *In mixed duopolies,*

(i) when $d \in [0, d_8)$, one simultaneous-move outcome, $(T_0, T_1) = (1,1)$, is the unique equilibrium outcome;

(ii) when $d = d_8$, one simultaneous-move outcome, $(T_0, T_1) = (1,1)$, and one sequential-move outcome in which the public is the leader, $(T_0, T_1) = (1,2)$, are the equilibrium outcomes;

(iii) otherwise, one sequential-move outcome in which the public is the leader, $(T_0, T_1) = (1,2)$, is the equilibrium outcome.

Proof: Comparing the values, we have $d_7 < d_9 < d_8$. From Proposition 5, we also have $W^{FM} < W^{BM} < W^{LM}$. Finally, the profit ranks of the private firm are as follows: (i) $\pi_1^{BM} \gtreqless \pi_1^{LM}$ if $d \lesseqgtr d_8$; and (ii) $\pi_1^{FM} \gtreqless \pi_1^{BM}$ if $d \lesseqgtr d_9$. Q.E.D.

The proposition represents that mixed duopolies in price competition with optimal emission tax yield a sequential-move outcome in equilibrium in an endogenous timing game when environmental externality is significant. This result sharply contrasts the previous literature in mixed duopolies without environmental externality. For example, Pal (1998) showed that firms in mixed duopolies decide simultaneously when competing in prices. However, price competition with environmental externality changes the competition structure in mixed duopolies. Thus, the assumption of a simultaneous-move game under significant environmental externality may be problematic because a simultaneous-move outcome does not appear in equilibrium.

Furthermore, we find that if environmental externality is insignificant, public leadership with optimal emission tax will be an equilibrium outcome in an endogenous timing game, yielding higher welfare than private leadership. Thus, welfare-improving public leadership is more robust than private leadership as an equilibrium outcome. Once again, these results sharply contrast those in mixed duopolies without environmental externality whereby private leadership is more robust and more efficient (see, e.g., Pal, 1998; Matsumura and Ogawa, 2010; Capuano and De Feo, 2010).

Remark In Appendix C and D, we provide a numerical example to confirm our main results examine other scenarios.[8] In Appendix C, we compare the equilibrium outcomes between price and quantity competition, and show that most of the results under price competition can be reversed under quantity competition. In Appendix D, we consider the equilibrium outcomes in mixed duopolies under price competition by allowing an agency problem of managers in the public firm and show that some results can be affected by managers' awareness on environmental concern.

5 Endogenous choice on privatization policy

We next examine the endogenous choice on privatization to discuss the welfare effect of privatization policy under price competition.

Proposition 7 *In the region of* $min\{d_3,\ d_8\} < d < max\{d_3,\ d_8\}$, *a privatization policy does not change the equilibrium of an endogenous timing game, and a simultaneous-move outcome in price competition is robust unless two products are highly substitutable.*

Proof: Comparing the equilibrium in the endogenous timing game in private and mixed duopolies, we obtain the following results: (i) When $b \in (0, 0.986)$, we have $0 < d_3 < d_8$. Thus, a simultaneous-move outcome is still an equilibrium under privatization when $d_3 < d < d_8$. (ii) When $b \in (0.986,1)$, we have $0 < d_8 < d_3$. Thus, a sequential-move outcome is still an equilibrium under privatization when $d_8 < d < d_3$. Q.E.D.

[8] Recent research on the endogenous choice between price and quantity contract, see Balogh and Tasnadi (2012), Matsumura and Ogawa (2014), Naya (2015) and Din and Sun (2016). Regarding agency problem of the public firm, see Bárcena-Ruiz and Garzon (2006), Pal and Saha (2015), and Xu, et al. (2016).

This proposition indicates that privatization policy can change the equilibrium of an endogenous timing game when environmental damage is sufficiently small or large. In particular, when environmental damage is sufficiently small (large), a simultaneous-move (sequential-move) outcome will become a sequential-move (simultaneous-move) outcome under privatization. This supports the findings of the previous literature on price competition without environmental externalities that private duopoly firms decide sequentially (see Hamilton and Slutsky, 1990) whereas mixed duopoly firms decide simultaneously (see Bárcena-Ruiz, 2007). However, when environmental externalities are very significant, the results are surprisingly reversed. That is, mixed duopolies decide sequentially but private duopolies decide simultaneously after privatization.

This proposition also indicates that the equilibrium of an endogenous timing game depends on the degree of product differentiation. If $min\{d_3, d_{10}\} < d < max\{d_3, d_{10}\}$, the equilibrium will be a simultaneous-move (sequential-move) outcome when two products are less (more) substitutable. For example, if the two products are almost homogeneous goods, privatization would result in a public leader becoming a private leader.

Proposition 8 *Privatization policy in an endogenous timing game lowers social welfare.*

Proof: Comparing the welfare in the endogenous timing game in private and mixed duopolies, we obtain the following results: (i) When $b \in (0, 0.986)$, we have $0 < d_3 < d_8$. Thus, $W^{LP} < W^{BM}$ when $0 < d < d_7$; $W^{BP} < W^{BM}$when $d_3 < d < d_8$; and $W^{BP} < W^{LM}$ when $d > d_8$. (ii) When $b = 0.986$, we have $d_3 = d_8$. Thus, $W^{LP} < W^{BM}$ when $0 < d < d_3 = d_8$ and $W^{BP} < W^{LM}$ when $d > d_3 = d_8$. (iii) When $b \in (0.986,1)$, we have $0 < d_8 < d_3$. Thus, $W^{LP} < W^{BM}$ when $0 < d < d_8$; $W^{LP} < W^{LM}$ when $d_8 < d < d_3$; and $W^{BP} < W^{LM}$ when $d > d_3$. Q.E.D.

This proposition resembles the results of Fjell and Heywood (2004), who examined a mixed oligopoly with homogenous outputs, to find that without environmental externalities, privatization results in a public leader becoming a private leader and reduces both output and welfare. Furthermore, Heywood and Ye (2009) incorporated endogenous timing into a quantity setting game and demonstrated that privatization will always lower social welfare. We also confirmed that privatization will always lower social welfare in an endogenous timing game with environmental externalities and emission taxes.

6 Concluding remarks

This paper examines an endogenous timing game in private and mixed duopolies with price competition when emission tax is imposed on environmental externality. We show that public concerns on environmental quality affect the equilibrium of an endogenous timing game, and that this depends on the degree of environmental externalities. In particular, we show that in private duopolies, a simultaneous-move (sequential-move) outcome can be an equilibrium under significant (insignificant) environmental externality; however, these results are reversed in mixed duopolies. As expected, the results under insignificant environmental externality are consistent with the results in the previous literature. However, under significant environmental externality, the results sharply contrast those in the previous literature. In fact, public concerns on environmental quality can reverse the equilibrium of an endogenous timing game. We also show that public leadership yields greater welfare than private leadership, and public leadership is more robust than private leadership as an equilibrium. Finally, we show that privatization can result in a public leader becoming a private leader, with welfare-worsening result.

However, a need arises to examine the robustness of the results when there are multiple domestic or foreign private firms under the general functional forms. Subsidy policies on output and/or abatement activities are also important to evaluate the impact of emission tax and other environmental policies, such as trading emission permits and emission standards. The recent research interest in the endogenous choice between price and quantity contract is also a promising topic for future research.

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Appendix A: the values of $\mathbf{m_i}$, $\mathbf{n_i}$, and $\mathbf{d_i}$

$m_1 = A^2(49 + 24b - 55b^2 - 22b^3 + 18b^4 + 4b^5 - 2b^6) - 2A(4 + b - b^2)(15 + 4b - 15b^2 - 3b^3 + 3b^4)d + (4 + 2b - b^2)(4 + b - b^2)^2(3 - b^2)d^2.$

$m_2 = A^2(324 - 135b - 486b^2 + 183b^3 + 251b^4 - 79b^5 - 51b^6 + 11b^7 + 3b^8) - A(3 - b^2)(9 - 5b^2)(18 - 6b - 10b^2 + 2b^3 + b^4)d - A(162 - 108b - 270b^2 + 168b^3 + 145b^4 - 80b^5 - 28b^6 + 12b^7 + b^8)t +$

$t(3-b^2)(9-5b^2)(72-6b-58b^2+2b^3+11b^4)d-(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8)t^2.$

$m_3=(3-b^2)(9-5b^2)(72-6b-58b^2+2b^3+11b^4)d-A(162-108b-270b^2+168b^3+145b^4-80b^5-28b^6+12b^7+b^8).$

$m_4=A(1134-432b-1818b^2+522b^3+1109b^4-208b^5-305b^6+27b^7+32b^8)+(3+2b)(3-3b^2+b^3)(72-6b-58b^2+2b^3+11b^4)d.$

$m_5=A(1134-432b-1962b^2+678b^3+1285b^4-406b^5-375b^6+109b^7+41b^8-11b^9)+(72-6b-58b^2+2b^3+11b^4)(9+6b-7b^2-5b^3+b^4+b^5)d.$

$m_6=A^2(1285956-1084752b-3776220b^2+3028752b^3+4863888b^4-3596292b^5-3608784b^6+2353080b^7+1697092b^8-915862b^9-520835b^{10}+211914b^{11}+102262b^{12}-26966b^{13}-11761b^{14}+1454b^{15}+606b^{16})-2A(3-b^2)(9-5b^2)(72-6b-58b^2+2b^3+11b^4)(810-594b-1116b^2+708b^3+582b^4-278b^5-137b^6+36b^7+12b^8)d+(3-b^2)(9-5b^2)(36-6b-29b^2+2b^3+6b^4)(72-6b-58b^2+2b^3+11b^4)^2d^2.$

$m_7=A^2(1285956-1084752b-3776220b^2+3215376b^3+4646160b^4-4020084b^5-3082176b^6+2747856b^7+1169620b^8-1108630b^9-240855b^{10}+263854b^{11}+19230b^{12}-34272b^{13}+1279b^{14}+1872b^{15}-241b^{16})-2A(72-6b-58b^2+2b^3+11b^4)(21870-16038b-49572b^2+35964b^3+43740b^4-31500b^5-18603b^6+13488b^7+3670b^8-2826b^9-203b^{10}+232b^{11}-17b^{12})d+(72-6b-58b^2+2b^3+11b^4)^2(972-162b-1647b^2+234b^3+1002b^4-108b^5-257b^6+16b^7+23b^8)d^2.$

$m_8=A(A-2d)(1620-756b-2268b^2+924b^3+1116b^4-368b^5-220b^6+48b^7+13b^8)+(72-6b-58b^2+2b^3+11b^4)^2d^2.$

$n_1=2A^2(17-8b-18b^2+8b^3+9b^4-4b^5-2b^6+b^7)-4A(3-2b-b^2+b^3)(5+b-3b^2-b^3+b^4)d+(54-12b-74b^2+12b^3+46b^4-8b^5-12b^6+2b^7+b^8)d^2-4A(2-b)(1-b)(1+b)t+2(48-2b-54b^2+2b^3+28b^4-8b^6+b^8)dt-(44-50b^2+28b^4-8b^6+b^8)t^2.$

$n_2=A^2(2-b)^2(196-328b^2+228b^4-88b^6+19b^8-2b^{10})+2A(2-b)(480-196b-844b^2+328b^3+612b^4-228b^5-248b^6+88b^7+56b^8-19b^9-6b^{10}+2b^{11})d-(3072-960b-6268b^2+1688b^3+6172b^4-1224b^5-3916b^6+496b^7+1732b^8-112b^9-545b^{10}+12b^{11}+118b^{12}-16b^{14}+b^{16})d^2.$

$n_3=A^2(21-10b-18b^2+8b^3+9b^4-4b^5-2b^6+b^7)-2A(21-10b-18b^2+8b^3+9b^4-4b^5-2b^6+b^7)d+(65-10b-68b^2+8b^3+37b^4-4b^5-10b^6+b^7+b^8)d^2.$

$n_4=2A^2(17-8b-18b^2+8b^3+9b^4-4b^5-2b^6+b^7)-4A(3-2b-b^2+b^3)(5+b-3b^2-b^3+b^4)d+(54-12b-74b^2+12b^3+46b^4-8b^5-12b^6+2b^7+b^8)d^2.$

$n_5=A^2(17-8b-10b^2+4b^3+b^4)-2A(15-7b-8b^2+3b^3+b^4)d+(27-6b-21b^2+2b^3+5b^4)d^2-2A(2-b)(1-b)(1+b)t+2(24-b-19b^2+b^3+4b^4)dt-(22-17b^2+4b^4)t^2.$

$n_6=A^2(2-b)^2(49-58b^2+20b^4-2b^6)+2A(2-b)(120-49b-155b^2+58b^3+61b^4-20b^5-8b^6+2b^7)d-(768-240b-1087b^2+310b^3+571b^4-122b^5-140b^6+16b^7+14b^8)d^2.$

$n_7=2A^2(68-32b-80b^2+40b^3+24b^4-12b^5-2b^6+b^7)-4A(6-4b-2b^2+b^3)(10+2b-8b^2+b^4)d+(216-48b-296b^2+72b^3+120b^4-24b^5-18b^6+2b^7+b^8)d^2-8A(2-b)(2-b^2)t+2(2-b^2)(96-4b-72b^2+16b^4-b^6)dt-(2-b^2)^2(44-14b^2+b^4)t^2.$

$n_8=A^2(2-b)^2(784-704b^2+204b^4-24b^6+b^8)-2A(2-b)(1920-784b-1696b^2+704b^3+472b^4-204b^5-52b^6+24b^7+2b^8-b^9)d+(12288-3840b-15856b^2+3392b^3+8368b^4-944b^5-2388b^6+104b^7+380b^8-4b^9-31b^{10}+b^{12})d^2.$

$n_9=A^2(84-40b-96b^2+48b^3+28b^4-14b^5-2b^6+b^7)-2A(84-40b-96b^2+48b^3+28b^4-14b^5-2b^6+b^7)d+(260-40b-328b^2+48b^3+132b^4-14b^5-20b^6+b^7+b^8)d^2.$

$n_{10}=2A^2(68-32b-80b^2+40b^3+24b^4-12b^5-2b^6+b^7)-4A(6-4b-2b^2+b^3)(10+2b-8b^2+b^4)d+(216-48b-296b^2+72b^3+120b^4-24b^5-18b^6+2b^7+b^8)d^2.$

$d_1 = \frac{A(1-b^2)}{12+7b-6b^2-2b^3+b^4}$.

$d_2 = \frac{A(162-108b-270b^2+168b^3+145b^4-80b^5-28b^6+12b^7+b^8)}{(3-b^2)(9-5b^2)(72-6b-58b^2+2b^3+11b^4)}$.

$d_3 = A(2405700 + 568620b - 4512996b^2 - 1384128b^3 - 765234b^4 + 656550b^5 + 9694881b^6 + 1304366b^7 - 12397773b^8 - 2198008b^9 + 8283974b^{10} + 1540918b^{11} - 3405551b^{12} - 608600b^{13} + 895524b^{14} + 140930b^{15} - 148072b^{16} - 17922b^{17} + 14169b^{18} + 970b^{19} - 606b^{20})/((3 - b^2)(2566080 + 1846800b - 6043248b^2 - 5765256b^3 + 4368258b^4 + 7474866b^5 + 767313b^6 - 5196596b^7 - 3121668b^8 + 2075716b^9 + 2146562b^{10} - 468206b^{11} - 753879b^{12} + 52350b^{13} + 149403b^{14} - 1516b^{15} - 15919b^{16} - 110b^{17} + 714b^{18}))$.

$d_4 = A(2405700 + 11859372b - 3315492b^2 - 43835256b^3 - 4935546b^4 + 70972830b^5 + 15294213b^6 - 66159898b^7 - 15838541b^8 + 39218870b^9 + 8847024b^{10} - 15376910b^{11} - 2886850b^{12} + 4001238b^{13} + 530575b^{14} - 668514b^{15} - 43655b^{16} + 65238b^{17} - 587b^{18} - 2836b^{19} + 241b^{20})/(7698240 + 16831152b - 19498320b^2 - 61593696b^3 + 14977710b^4 + 98506134b^5 + 3533013b^6 - 90528918b^7 - 13573085b^8 + 52840622b^9 + 10124404b^{10} - 20398162b^{11} - 3889498b^{12} + 5235094b^{13} + 837139b^{14} - 866342b^{15} - 92743b^{16} + 84346b^{17} + 3305b^{18} - 3696b^{19} + 133b^{20})$.

$d_5 = \frac{2A(2-b)(1-b)(1+b)}{48-2b-54b^2+2b^3+28b^4-8b^6+b^8}$.

$d_6 = \frac{A(2-b)(1-b)(1+b)}{24-b-19b^2+b^3+4b^4}$.

$d_7 = \frac{4A(2-b)}{96-4b-72b^2+16b^4-b^6}$.

$d_8 = A(11264 - 28264b^2 + 31556b^4 - 20762b^6 + 8819b^8 - 2473b^{10} + 444b^{12} - 46b^{14} + 2b^{16})/(19008 + 3872b - 46920b^2 - 9328b^3 + 52084b^4 + 10264b^5 - 34146b^6 - 6692b^7 + 14387b^8 + 2784b^9 - 3945b^{10} - 736b^{11} + 672b^{12} + 114b^{13} - 62b^{14} - 8b^{15} + 2b^{16})$.

$d_9 = A(2-b)(70400 - 11776b^2 - 172288b^4 + 249152b^6 - 177584b^8 + 78320b^{10} - 22728b^{12} + 4340b^{14} - 520b^{16} + 35b^{18} - b^{20})/(450560 - 70400b - 690944b^2 + 11776b^3 + 302464b^4 + 172288b^5 + 135168b^6 - 249152b^7 - 232256b^8 + 177584b^9 + 134064b^{10} - 78320b^{11} - 44536b^{12} + 22728b^{13} + 9128b^{14} - 4340b^{15} - 1120b^{16} + 520b^{17} + 74b^{18} - 35b^{19} - 2b^{20} + b^{21})$.

Appendix B: the optimal prices, abatements and quantities

Simultaneous-move game in private duopolies

When $d > d_1$, we have:

$$p_0^{BP} = p_1^{BP} = \frac{A(7+2b-5b^2-b^3+b^4)+(4+5b-b^3)d}{11+7b-5b^2-2b^3+b^4},\ a_0^{BP} = a_1^{BP} = \frac{12d-(1-b)(A(1+b)-\mathrm{bd}(7+b-b^2))}{11+7b-5b^2-2b^3+b^4},$$

$$q_0^{BP} = q_1^{BP} = \frac{(4+b-b^2)(A-d)}{11+7b-5b^2-2b^3+b^4}.$$

When $d \le d_1$, $p_0^{BP} = p_1^{BP} = \frac{A(2-b^2)}{3+b-b^2}$, $a_0^{BP} = a_1^{BP} = 0$, $q_0^{BP} = q_1^{BP} = \frac{A}{3+b-b^2}$.

Sequential-move game in private duopolies

When $d > d_2$, we have:

$$p_0^{LP} = \frac{m_4}{2(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8)},$$

$p_1^{LP} = \frac{m_5}{2(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8)}$,

$a_0^{LP} = a_1^{LP} = \frac{m_3}{2(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8)}$,

$q_0^{LP} = \frac{(3-b^2)(3-b-b^2)(72-6b-58b^2+2b^3+11b^4)(A-d)}{2(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8)}$,

$q_1^{LP} = \frac{(9-3b-4b^2+b^3)(72-6b-58b^2+2b^3+11b^4)(A-d)}{2(891-27b-1512b^2+15b^3+952b^4+b^5-263b^6-b^7+27b^8)}$.

When $d \le d_2$, $p_0^{LP} = \frac{3A(2-b^2)(3-b-b^2)}{27-24b^2+5b^4}$, $p_1^{LP} = \frac{A(2-b^2)(9-3b-4b^2+b^3)}{27-24b^2+5b^4}$, $a_0^{LP} = a_1^{LP} = 0$,

$q_0^{LP} = \frac{A(3-b-b^2)}{9-5b^2}$, $q_1^{LP} = \frac{A(9-3b-4b^2+b^3)}{27-24b^2+5b^4}$.

Simultaneous-move game in mixed duopolies

When $d > d_5$, we have:

$p_0^{BM} = \frac{A(22-2b-24b^2-6b^3+17b^4+4b^5-6b^6-b^7+b^8)+22d+bd(2-26b+6b^2+11b^3-4b^4-2b^5+b^6)}{44-50b^2+28b^4-8b^6+b^8}$,

$p_1^{BM} = \frac{A(2-b)(14-14b^2+6b^4-b^6)+(1+b)(16-2b-20b^2+6b^3+10b^4-4b^5-2b^6+b^7)d}{44-50b^2+28b^4-8b^6+b^8}$,

$a_0^{BM} = d$, $a_1^{BM} = \frac{2A(2-b)(b-1)(1+b)+(48-2b-54b^2+2b^3+28b^4-8b^6+b^8)d}{44-50b^2+28b^4-8b^6+b^8}$,

$q_0^{BM} = \frac{(A-d)(22-14b-18b^2+14b^3+7b^4-6b^5-b^6+b^7)}{44-50b^2+28b^4-8b^6+b^8}$, $q_1^{BM} = \frac{(A-d)(2-b)(8-4b^2+b^4)}{44-50b^2+28b^4-8b^6+b^8}$.

When $d \le d_5$, $p_0^{BM} = \frac{A(3-2b^2-b^3+b^4)+(1-b)(3-2b^2)d}{6-4b^2+b^4}$, $p_1^{BM} = \frac{(2-b^2)(A(2-b)+(1-b)bd)}{6-4b^2+b^4}$,

$a_0^{BM} = d$, $a_1^{BM} = 0$, $q_0^{BM} = \frac{A(3-2b-b^2+b^3)-(1-b)(3-b^2)d}{6-4b^2+b^4}$, $q_1^{BM} = \frac{A(2-b)+(1-b)bd}{6-4b^2+b^4}$.

Sequential-move game with public leadership in mixed duopolies

When $d > d_6$, we have:

$p_0^{LM} = \frac{A(11-3b-7b^2+2b^4)+(11+3b-10b^2+2b^4)d}{22-17b^2+4b^4}$,

$p_1^{LM} = \frac{A(2-b)(7-5b^2+b^4)+(1+b)(8-b-6b^2+b^3+b^4)d}{22-17b^2+4b^4}$,

$a_0^{LM} = d$, $a_1^{LM} = \frac{A(2-b)(b-1)(1+b)+(24-b-19b^2+b^3+4b^4)d}{22-17b^2+4b^4}$,

$q_0^{LM} = \frac{(A-d)(11-5b-6b^2+2b^3+b^4)}{22-17b^2+4b^4}$, $q_1^{LM} = \frac{(A-d)(2-b)^2(2+b)}{22-17b^2+4b^4}$.

When $d \le d_6$, we have:

$p_0^{LM} = \frac{A(9-2b-7b^2+2b^4)+(3-b-b^2)(3-2b^2)d}{2(9-8b^2+2b^4)}$, $p_1^{LM} = \frac{(2-b^2)(A(2-b)(3-b^2)+b(3-b-b^2)d)}{2(9-8b^2+2b^4)}$,

$a_0^{LM} = d$, $a_1^{LM} = 0$, $q_0^{LM} = \frac{A(9-4b-6b^2+2b^3+b^4)-(3-b^2)(3-b-b^2)d}{2(9-8b^2+2b^4)}$,

$q_1^{LM} = \frac{A(2-b)(3-b^2)+b(3-b-b^2)d}{2(9-8b^2+2b^4)}$.

Sequential-move game with private leadership in mixed duopolies

When $d > d_7$, we have:

$$P_0^{FM} = \frac{A(44-4b-34b^2+8b^3+4b^4-b^5)+(44+4b-38b^2-8b^3+12b^4+b^5-b^6)d}{(2-b^2)(44-14b^2+b^4)},$$

$$P_1^{FM} = \frac{A(2-b)(28-12b^2+b^4)+(32+28b-48b^2-12b^3+14b^4+b^5-b^6)d}{(2-b^2)(44-14b^2+b^4)},$$

$$a_0^{FM} = d,\ a_1^{FM} = \frac{4A(b-2)+(96-4b-72b^2+16b^4-b^6)d}{(2-b^2)(44-14b^2+b^4)},$$

$$q_0^{FM} = \frac{(A-d)(44-28b-22b^2+12b^3+2b^4-b^5)}{(2-b^2)(44-14b^2+b^4)},\ q_1^{FM} = \frac{(A-d)(2-b)(8-b^2)}{(2-b^2)(44-14b^2+b^4)}.$$

When $d \leq d_7$, $P_0^{FM} = \frac{A(6-4b^2+b^3)+(1-b)(6-4b^2+b^4)d}{12-8b^2+b^4}$, $P_1^{FM} = \frac{(4-b^2)(A(2-b)+(1-b)bd)}{12-8b^2+b^4}$, $a_0^{FM} = d$,

$$a_1^{FM} = 0,\ q_0^{FM} = \frac{A(6-4b-2b^2+b^3)-2(1-b)(3-b^2)d}{12-8b^2+b^4},\ q_1^{FM} = \frac{A(2-b)+(1-b)bd}{6-b^2}.$$

Appendix C

We compare the equilibrium outcomes between price competition and quantity competition by using a numerical example with A=10 and d=1. We then show that most of the results under price competition can be reversed under quantity competition. The followings are the summary of the findings, which are supported by figures.

Proposition C1: *In private duopolies, the optimal emission tax is always lower than the marginal environmental damage. However, the tax level in a sequential-move game under price (quantity) competition is lower (higher) than that in simultaneous-move game.*

Proposition C2: *In private duopolies, environmental damage and social welfare under price (quantity) competition are lower (higher) in a sequential-move game.*

Proposition C3: *In private duopolies, two sequential-move outcomes,* $(T_0, T_1) = (1,2)$ *and* $(T_0, T_1) = (2,1)$*, are the unique equilibrium under price competition while two simultaneous-move outcomes,* $(T_0, T_1) = (1,1)$ *and* $(T_0, T_1) = (2,2)$*, are the unique equilibrium under quantity competition when* $0 < b < 0.897$.

Proposition C4: *In mixed duopolies, the optimal emission tax is always lower than the marginal environmental damage. However, the tax level in a public (private) leadership game under price* *competition is the highest (lowest) while that in a sequential-move game under quantity competition is higher than that in a simultaneous-move game.*

Proposition C5: *In mixed duopolies under price competition, environmental damage and social welfare are the highest (lowest) in a public (private) leadership game. However, in mixed duopolies under quantity competition, environmental damage is the highest (lowest) in a private (public) leadership game while social welfare in sequential-move game is higher th an that in a simultaneous-move game.*

Proposition C6: *In mixed duopolies, one simultaneous-move outcome,* $(T_0, T_1) = (1,1)$*, is the unique equilibrium under price competition while one sequential-move outcome,* $(T_0, T_1) = (1,2)$*, is the unique equilibrium under quantity competition.*

<table>
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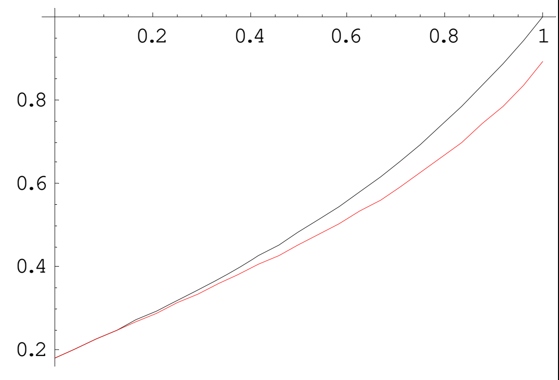
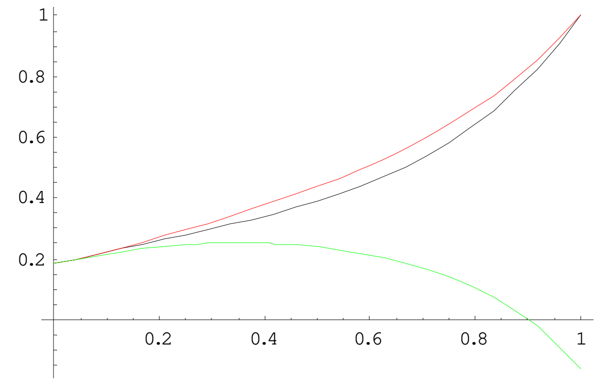
<tr><td rowspan="2">t</td><td>
</td><td>
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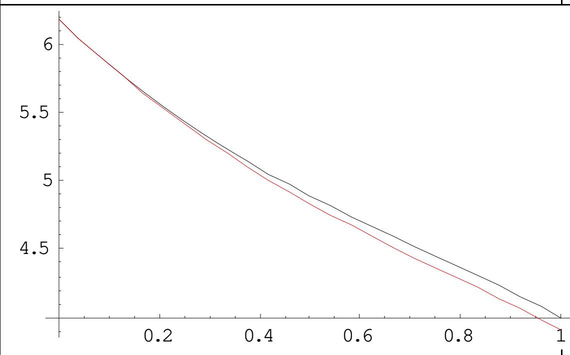
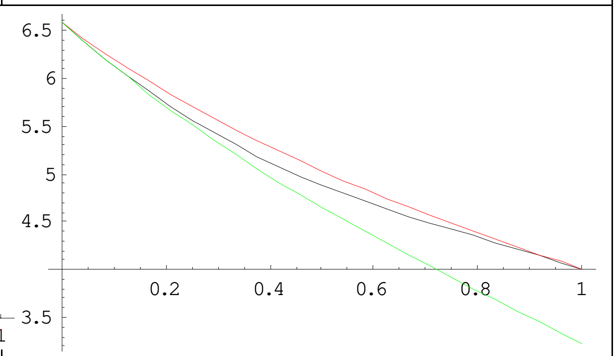
<tr><td rowspan="2">ED</td><td>
</td><td>
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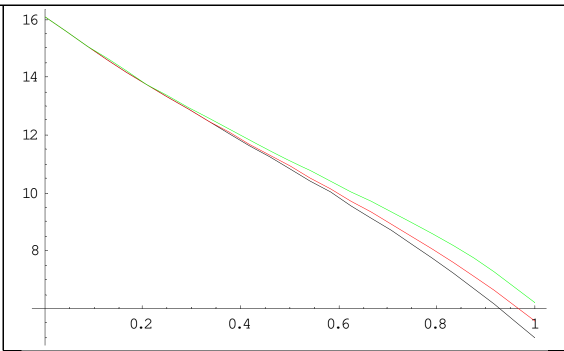
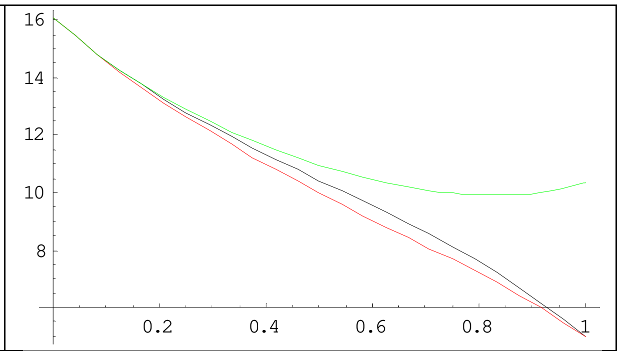
π		
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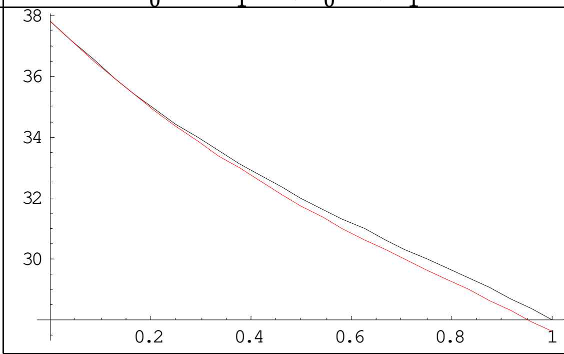
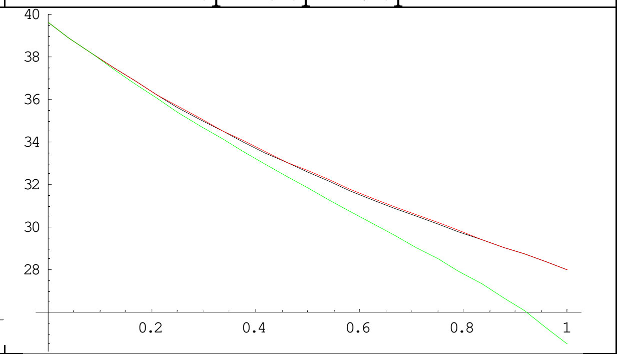
W		
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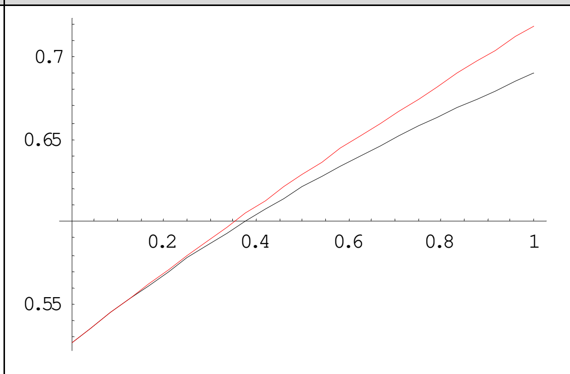
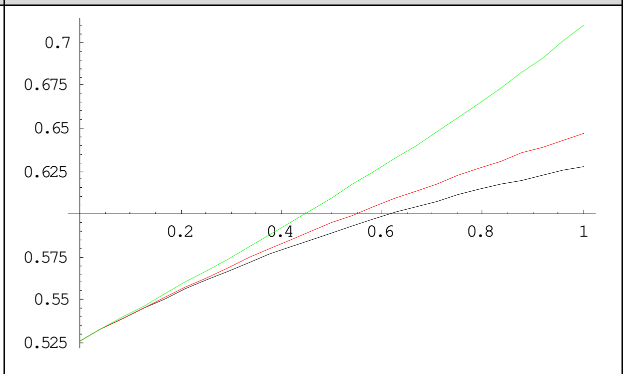
	Quantity Competition	
	Private duopoly	**Mixed duopoly**
t		
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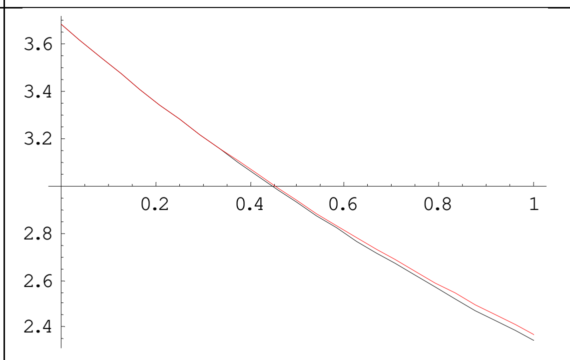
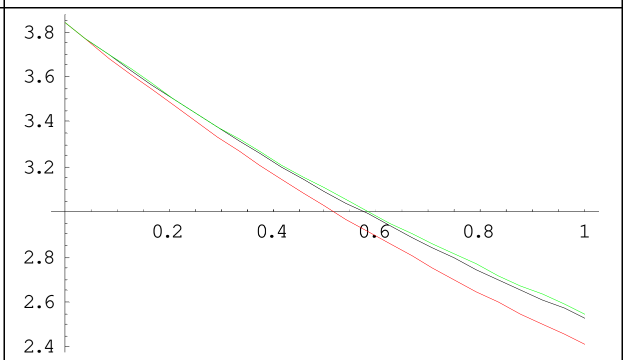
ED		
	$ED^{CP} < ED^{LP}$	$ED^{LM} < ED^{CM} < ED^{FM}$

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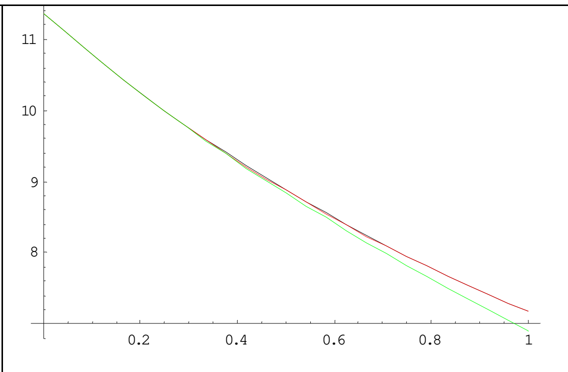
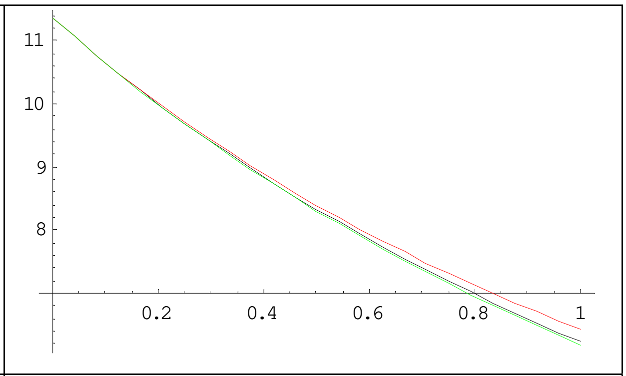
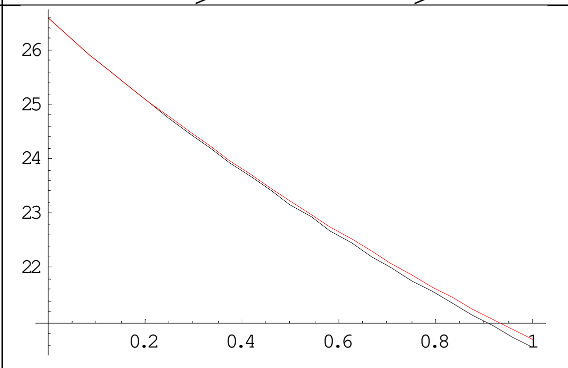
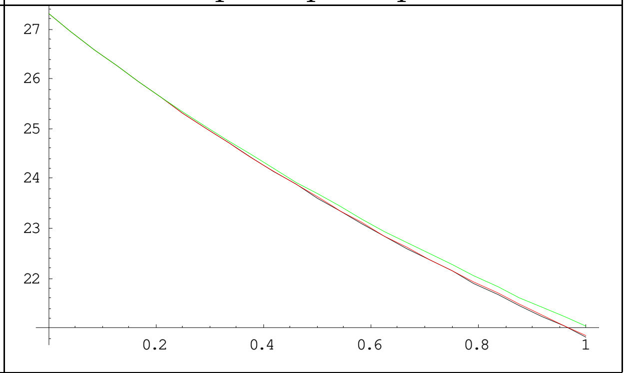
<tr><td>π</td><td></td><td></td></tr>
<tr><td></td><td>$\pi_1^{LP} < \pi_0^{LP} \lesseqgtr \pi_0^{CP} = \pi_1^{CP}$ when $b \lesseqgtr 0.897$</td><td>$\pi_1^{FM} < \pi_1^{CM} < \pi_1^{LM}$</td></tr>
<tr><td>W</td><td></td><td></td></tr>
<tr><td></td><td>$W^{CP} < W^{LP}$</td><td>$W^{CM} < W^{LM} < W^{FM}$</td></tr>
</table>

Appendix D

We consider the equilibrium outcomes in mixed duopolies under price competition by allowing an agency problem of public firm, in which the objective function of the public firm is defined as $G = CS + \pi_0 + \pi_1 + T - \rho ED$ where $\rho(\geq 0)$ represents the political pressure of interest group or managers' awareness on environmental concern in the public firm. That is, $\rho < 1$ implies that managers are more development-oriented while $\rho > 1$ implies that managers are more environment-friendly[9]. Notice that $\rho = 1$ represents a benchmark case without agency problem and thus the public firm maximizes social welfare.

For a comparable analysis, we use the same numerical example in Appendix C where A=10 and d=1. We also describe main outcomes with $\mathrm{b} = \frac{1}{2}$ and provide the figures. Then, in mixed duopolies, the outcomes of three cases are provided as follows:

First, a simultaneous-move game (BM) yields that:

$t^{BM} = \frac{7(177-20\rho)}{2827}$, $\pi_1^{BM} = \frac{159461881+40\rho(178105+2699\rho)}{15983858}$, $ED^{BM} = \frac{17057-3253\rho}{2827}$, and

$W^{BM} = \frac{181723+(5666-3211\rho)\rho}{5654}$.

[9] For positive values in the following analysis, we assume that $0 \leq \rho \leq 2.819$.

Second, a sequential-move game with public leadership (LM) yields that:

$t^{LM} = \frac{7}{16}$, $\pi_1^{LM} = \frac{3488183+720\rho(277+4\rho)}{369664}$, $ED^{LM} = \frac{3883-824\rho}{608}$, and
$W^{LM} = \frac{38867+824(2-\rho)\rho}{1216}$.

Third, a sequential-move game with private leadership or public followership (FM) yields that:

$t^{FM} = \frac{1312-225\rho}{4543}$,$\pi_1^{FM} = \frac{432637200+\rho(19427696+283109\rho)}{41277698}$,$ED^{FM} = \frac{2(13196-2637\rho)}{4543}$, and
$W^{FM} = \frac{1000768+9(3302-2011\rho)\rho}{31801}$.

Fig. D.1 provides the equilibrium results in three cases. Then, we obtain that the rank of optimal emission taxes is not affected by ρ and thus, Proposition 4 still holds.

Proposition D1: *In mixed duopolies, the optimal emission tax is lower than marginal environmental damage, but its level in a public (private) leadership game is the highest (lowest).*

However, the ranks of environmental damage and social welfare depend on the size of ρ, and thus the equilibrium outcomes of endogenous timing game in mixed duopolies are also affected by ρ. In particular, (i) $ED^{FM} < ED^{BM} \leq ED^{LM}$ when $0 \leq \rho \leq 1.725$; $ED^{FM} < ED^{LM} < ED^{BM}$ otherwise; and (ii) $W^{FM} < W^{BM} \leq W^{LM}$ when $0.624 \leq \rho \leq 2.595$; $W^{FM} < W^{LM} < W^{BM}$ otherwise.

Proposition D2: *In mixed duopolies, social welfare in a public leadership game is the highest when* $0.624 \leq \rho \leq 2.595$ *while environmental damage in a public leadership game is the highest when* $0 \leq \rho \leq 1.725$.

It states that (i) when the managers of public firm are much oriented to development, i.e., 0 $\leq \rho \leq 0.624$, a simultaneous-move game yields the highest social welfare but its environmental damage is lower than that in a public leadership game; (ii) when the managers of public firm are much concerned on environments, i.e., 1.725 $\leq \rho \leq 2.595$, a public leadership game yields the highest social welfare but its environmental damage is lower than that in a simultaneous-move game.

Finally, comparing the profit ranks of the private firm, we have: $\pi_1^{FM} > \pi_1^{BM} > \pi_1^{LM}$.

Proposition D3: *In mixed duopolies, one simultaneous-move outcome,* $(T_0, T_1) = (1,1)$, *is the unique equilibrium for any* ρ.

Hence, a simultaneous-move game is the unique equilibrium of an endogenous timing game in mixed duopolies but it is welfare-inferior to a public leadership game when $0.624 \leq \rho \leq 1.725$,

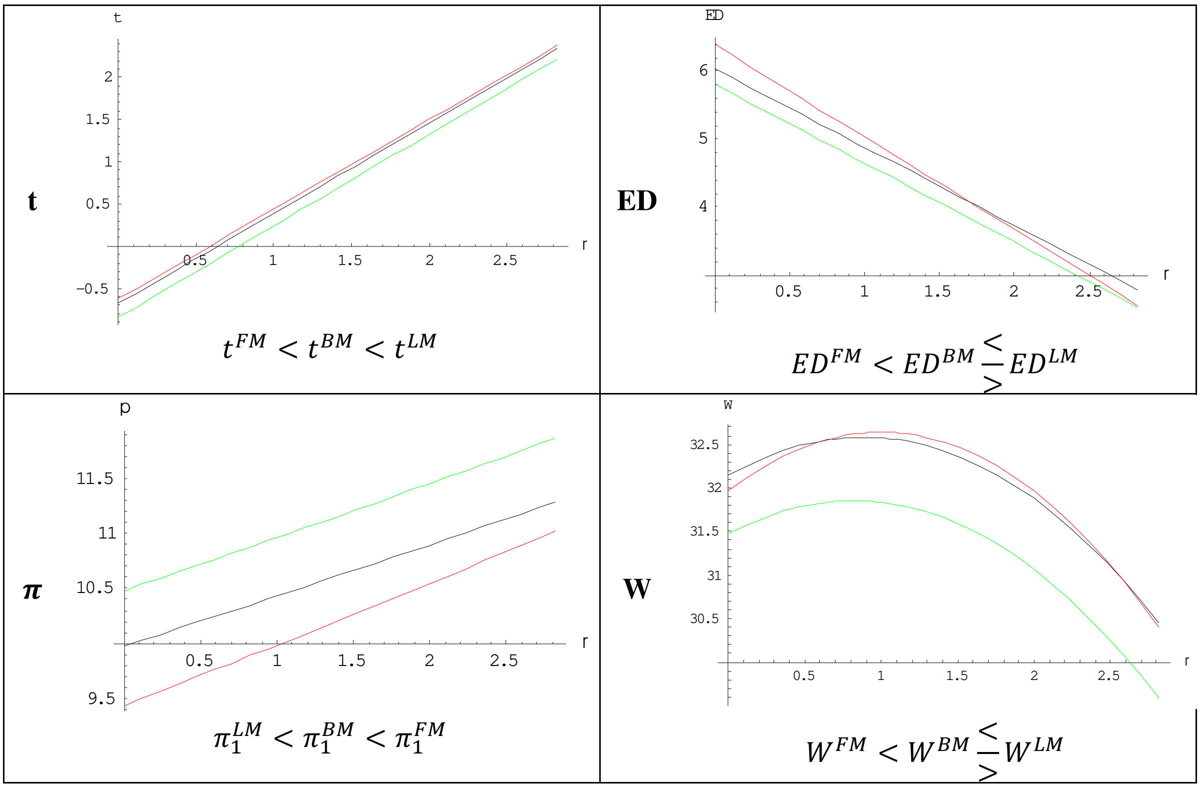


Fig. D1 The equilibrium results in a mixed duopoly