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abstract

This paper argues how an increase in minimum wage affects employment, consumption, and social welfare with dynamic general equilibrium model without market frictions. The study demonstrates that a minimum wage hike reduces an actual unemployment rate and has positive effects on an employment rate under the demand-shortage economy whereas they do not under a non-demand shortage economy. The study also shows that optimal minimum wage which maximize social welfare and minimize an actual unemployment rate when the economy faces the demand-shortage initially. These findings imply that the minimum wage can be considered as one of the effective policy for overcoming deflation and stagnation although it increases the natural rate of unemployment.

KEYWORDS: Minimum wage, Unemployment, Natural rate of unemployment, Deflation, Stagnation, Demand shortage, Dynamic general equilibrium model

JEL Classification Codes: E24 E31 J38

1 Introduction

The model of the competitive labor market states that a decline in minimum wage (above competitive wage) increases the employment, firm's profits, and welfare and thus vitalizes the economy. During the depression period, however, this effects do not seem to work well because a decrease in aggregate demand is a significant factor that decline an employment rate as Keynes (1936) explained. What does macro-economy

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respond when the government increases minimum wage in the stagnation? This paper argues that the impact of minimum wage policy on the economy differs in response to the economic situation, and shows that when an economy faces a demand shortage, an increase in minimum wages improves the employment rate, aggregate consumption, and social welfare. On the other hand, in an economy that does not face demand shortage, the increase in wages worsens the employment rate, aggregate consumption, and social welfare. Trade unions often insist on wage hikes during the labor-management negotiations, arguing that wage hikes will stimulate the consumption and aggregate demand. This paper supports these opinions in the context of an sluggish economy but not of an booming economy.

I develop a simple extension of Ono's (2001) dynamic general equilibrium model without market frictions by building in two different types of jobs. In the model economy, single final goods are produced by two labor inputs. Firms pay an efficiency wage and a minimum wage for each job. This assumption gives rise to a positive link between efficiency wage and minimum wages that can be used to analyze the effect of a wage hike (caused by a minimum wage hike) on the economy. This wage setting enables a tractable analysis because the minimum wage hike leaves the relative wage of each job unchanged, which eliminates the effect of substitution of labor demand for each job.[1] Therefore, the model can focus on the other effects of the minimum wage hike such as on inflation and the budget constraint of households that is unnoticed earlier.

Households have utility from consumption and real balances of money. Assumption of insatiable marginal utility of money generates two different equilibria: a demand-shortage and a supply-side (non-demand-shortage) economy as Ono (2001) showed. If the marginal utility of money is insatiable, the households accumulate money more than enough, and hence, the aggregate consumption level falls short of aggregate output level, that is, the demand-shortage equilibrium comes out. If the marginal utility of money is satiable, the supply-side equilibrium shows up. In the analysis, contrasting a demand-shortage and supply-side economy sheds new light on the function of a minimum wage.

In a demand-shortage economy, the minimum wage hike can prominently increase aggregate consumption, decrease an actual unemployment rate, and improve social welfare. The reason for this result is attributable primarily to a firm's labor demand function. When the firm faces the demand-shortage constraint, an equilibrium of underemployment arises in which the marginal product of labor is higher than the wage. Hence, higher aggregate demand induces the firms to increase their labor demand and to decrease the underemployment as Barro and Grossman (1971) and Honkapohja (1980)

[1] Cahuc and Michel (1996) state that the minimum wage hike increases the relative wage of unskilled jobs and induces firms to substitute other jobs including skilled jobs for unskilled jobs.

showed.[2] At the same time, an increase in the minimum wage narrows the disequilibrium gap between demand and supply caused by the stimulation of consumption and then eases deflation. In other words, there is an optimal minimum wage policy that maximizes social welfare and minimizes an actual unemployment rate when the economy faces the demand-shortage initially. The analysis also provides a policy implication for governments concerned about budget deficit–that is, a minimum wage hike can raise the aggregate demand without an increase in government spending.[3]

On the other hand, in a supply-side (non-demand-shortage economy), a minimum wage hike decreases the employment and worsens the social welfare in analogy with the competitive labor market model.

A number of studies are related to this work. As stated above, I use the setting of Ono's (2001) dynamic general equilibrium model with perfect information. Ono shows the reasons for the occurrence of liquidity trap and a stagnation, but he does not discuss the impact of minimum wage.

The minimum wage policy is controversial although its empirical results of employment effects seem elusive (Manning 2016). Lots of empirical studies analyze the minimum wage effects on the employment.[4] Several theoretical studies argue the positive function of minimum wage as opposed to the standard model. The welfare-enhancing minimum wage policy can be obtained by the intensifying capital accumulation. Cahuc and Michel (1996) and Fanti and Gori (2011) consider a growth model in which minimum wage hikes can improve welfare, but for reasons that are different from those we consider here.[5] In their model, a minimum wage hike increases savings and capital accumulation at the cost of increasing unemployment. It then improves economic growth and welfare under generous unemployment benefits and the positive externality of human capital accumulation that stem from the substituting skilled labor for binding minimum-wage low-skilled labor. In a context of search model Flinn (2006) shows that minimum wage

[2]Honkapohja (1980) considers disequilibrium model with endogenous money holdings of household and shows that the steady state effect of an increase in real government expenditure with the case of endogenous money holdings is larger than the fixed price case with exogenous money holdings.

[3]Annual inflation rate in the euro zone and in Japan are 0.2 % and -0.1 % in 2016, respectively. It is also worth considering the minimum wage policy to stimulate the economic activity and to alleviate the diminishing price pressures.

[4]For instance, regarding the minimum wage policy effect on low wage employment, Neumark, Salas and Wascher (2014) conducts a controversy with Allegretto, Dube, and Reich (2011), and Dube, Lester, and Reich (2010). Further Allegretto, Dube, Reich, and Zipperer (2017) offer a counterargument against Neumark et al. (2014).

[5]Some studies focus on the relation economic growth and minimum wage. Irmen and Wigger (2006) consider a two-country overrapping-generations model with capital mobility and endogenous growth, which shows the condition that minimum wage increases the global economic growth. Meckel (2004) also considers an endogenous growth model comprising the sectors of final goods, intermediate goods, and R&D. This model shows that higher minimum wages for unskilled labor leads to increased growth and unskilled unemployment, while possibly reducing unemployment of skilled labor. Tamai (2009) considers a median voter model of heterogeneous households with endogenous growth that determines the minimum wage by voting. He finds that high inequality has a positive effect on the minimum wage but generates a non-monotonic relation between inequality and economic growth.

improves unemployment inefficiency and may increase employment considering the size of the searching participants in response to the minimum wage.[6] Furthermore, in the monopsony model as is well known, a minimum wage hike can increase the employment and improve the welfare.[7] Moreover, Lee and Saez (2012) show the minimum wage hike becomes the social optimal under competitive labor markets with labor market heterogeneity when social welfare function is assumed to value redistribution from high wage to low wage workers.[8] However, none of these studies focus on the optimal minimum wage policy implemented under demand shortage.

The rest of the paper is organized as follows. Section 2 explains the basic setting of the model. The effects of a minimum wage are analyzed in a supply-side economy in section 3 and in a demand-shortage economy in section 4. Section 5 discuss the optimal minimum wage policy. Section 6 concludes the paper.

2 The model

2.1 Firms

Consider an economy without market frictions in which a representative firm produce final goods. Two labor are used in production. One is "high-wage job" which is characterized that workers' effort increase output, and hence the firm pays the efficiency wage for this labor. The other is "low-wage job" characterized that workers' effort is not response to the output, and hence the firm pays a minimum wage for this labor. I assume that the minimum wage is regulated and its level is greater than the competitive wage.[9]

The concave production function is given by

$$y = (en_1)^a \, n_2^b, \quad 0 < a, b < 1, \tag{1}$$

where y denotes the amounts of output produced and n_1 and n_2 stand for the number of employees of high-wage and low-wage job, respectively. e indicates productivity affected

[6] Acemoglu (2001) constructs a search model in which high and low-wage jobs coexist in response to capital intensity of each industry and demonstrates that introducing a minimum wage shifts the composition of employment toward high-wage jobs, increases average labor productivity, and may improve welfare.

[7] Manning (2003) discusses the monopsony in greater detail. Bhaskar and To (1999) construct the monopsonistic competition, where a large number of employers compete for workers, and are able to freely enter or exit. A rise in minimum wage raises employment per firm but causes firm's exit due to the decline of their profit. If the labor market is sufficiently distorted, the rise in minimum wage raises aggregate employment and welfare.

[8] With labor market heterogeneity, Revitzer and Taylor (1995) build on the sharking model of Shapiro and Stiglitz (1984) and show that the increase in minimum wage may raise the employment rate.

[9] I assume that the low-wage job worker does not shirk, because the firm utilizes monitoring technology perfectly in contrast with the high wage job.

by the worker's effort, and its functional form is

$$e = \left(\frac{w_1 - x}{x}\right)^{\theta}, \quad 0 < \theta < 1, \tag{2}$$

where w_1 is a real wage and x is reference point that equals a reservation wage (See below in detail). Equation (2) shows that an increase in wage margin from the reservation wage raises productivity. The firm chooses the level of real wage that minimize cost per unit of the effective labor input, w_1/e, which is modeled by Summers (1988). The optimal real wage and effort level are

$$\bar{w}_1 = \frac{x}{1-\theta}, \tag{3}$$

$$\bar{e} = \left(\frac{\theta}{1-\theta}\right)^{\theta}. \tag{4}$$

The firm is a price taker and sells the final goods at a price P competitively. When they can sell all the output under the exisiting levels of wages and price, employment are determined so that marginal products of each labor equal to their costs. The optimal conditions are

$$\bar{e}a\left(\bar{e}n_1\right)^{a-1} n_2^b = w_1, \tag{5}$$

$$b\left(\bar{e}n_1\right)^{a} n_2^{b-1} = w_2, \tag{6}$$

where w_2 is a real wage of low-wage workers. I refer to this economy as supply-side (non-demand-shortage) regime because the output and employment level are determined by not demand-side factors but supply-side factors.

On the other hand, when the firm is not be able to sell its notional output under the exisiting levels of wages and price, the firm chooses optimal employment subject to aggregate demand constraint as follows.

$$\max_{n_1, n_2} \quad y - w_1 n_1 - w_2 n_2$$
$$s.t. \quad y = \left(\bar{e}n_1\right)^a n_2^b, \qquad y = y^d,$$

where y^d is aggregate demand. The optimal conditions are[10]

$$(\bar{e}n_1)^a n_2^b = y^d, \tag{7}$$
$$\frac{an_2}{bn_1} = \frac{w_1}{w_2}. \tag{8}$$

In (7) and (8), the marginal products of each labor is higher than its cost because firm faces the limiting aggregate demand. This setting is formulated by Barro and Grossman (1971). I refer to this economy as demand-shortage regime. The employment level detemined by the firm under demand shortage regime is smaller than that under supply-side regime.

2.2 Wage determination and employment

The firm's setting of efficiency wage depends on the reservation wage x as in (3), which equals to the worker's expectation wage when he/she loses the present job.[11]

$$x = n_1 w_1 + n_2 w_2. \tag{9}$$

Substituting (9) into (3) yields

$$w_1 = \frac{n_2 w_2}{1 - \theta - n_1}. \tag{10}$$

I assume $\theta < 1 - n_1$ to assure the existence of a solution. Equation (10) predicts that the circumstances of the labor market affect the wage of high-wage job workers. The increase in w_2, n_1, and n_2 raise the reservation wage and the wage of the high-wage job workers.

Using (10), (5) and (6), the equilibrium employment for each type of labor in the supply-side regime economy become

$$n_1 = \frac{a(1-\theta)}{a+b} \equiv n_1^{s*}, \tag{11}$$

[10] As is shown by Ono (2001), the demand shortage arises not due to price rigidity but due to households' preference. The firm knows that the demand shortage cannot be eliminated in spite of price adjustments. Thereby the firm maxmizes their profits given the constraint of demand shortage. The optimal condition of the problem can be written by the Lagrangian multiplier method as follows:

$$\max_{n_1, n_2, \kappa} \quad L = (\bar{e}n_1)^a n_2^b - w_1 n_1 - w_2 n_2 + \kappa((\bar{e}n_1)^a n_2^b - y^d)$$

The optimal conditions are

$$(1+\kappa)\bar{e}a(\bar{e}n_1)^{a-1} n_2^b = w_1,$$
$$(1+\kappa)b(\bar{e}n_1)^a n_2^{b-1} = w_2,$$
$$(\bar{e}n_1)^a n_2^b - y^d = 0.$$

[11] Falk, Kehr, and Zehnder (2006) show minimum wages have significant effects on reservation wages with a laboratory experiment.

$$n_2 = \left(\frac{b}{w_2}\right)^{\frac{1}{1-b}} \left(\frac{\bar{e}a(1-\theta)}{a+b}\right)^{\frac{a}{1-b}} \equiv n_2^{s*}(w_2), \quad \frac{dn_2^{s*}}{dw_2} = n_2^{s*\prime}(w_2) < 0. \tag{12}$$

The increase in minimum wage reduces the employment n_2^{s*} but does not affect the employment n_1^{s*}.[12] This implies that the employment is determined by only the labor market variable. Substituting equilibrium employment level (11) and (12) into production function (1) gives aggregate output level:

$$y = \left(\bar{e}n_1^{s*}\right)^a \left(n_2^{s*}(w_2)\right)^b \equiv y^{s*}(w_2), \quad \frac{dy^{s*}}{dw_2} < 0. \tag{13}$$

On the contrary, aggregate demand affects the employment Under demand shortage. This relation is derived from (10), (7) and (8) as follows.

$$n_1 = \frac{a(1-\theta)}{a+b}, \tag{14}$$

$$n_2 = \left(y^d\right)^{\frac{1}{b}} \left(\frac{\bar{e}a(1-\theta)}{a+b}\right)^{-\frac{a}{b}}. \tag{15}$$

2.3 Households

Infinitely lived households have a utility function of the form

$$U = \int_0^\infty e^{-\rho t} \left[u(c) + v(m)\right] dt, \tag{16}$$

where ρ is a constant rate of time preference, and $u(c)$ and $v(m)$ are a continuous concave instantaneous utilities of real consumption c and real money balances m, respectively. I abbreviate the time notation of each variable to simplify exposition. The households provide one unit of labor inelastically. Population size in a economy is equal to 1. The households are ex ante identical, and the allocation of their labor to high-wage or low-wage jobs is done through a lottery. The households are then divided into two types by their employment status, with each type having different budget constraints. One engages in the high-wage job that receives the efficiency wage w_1, and the other engages in the low-wage job that receives the minimum wage w_2.

Each household chooses the optimal consumption and the real money balances to maximize U, subject to the following flow budget constraint:

$$\dot{m}_1 = w_1 + \frac{q}{n_1} - z_1 - c_1 - \pi m_1, \quad (i = 1), \tag{17}$$

[12] This is because the model assumes the Cobb-Dougulas production function. $\frac{\partial n_1^{s*}}{\partial w_2} > 0$ when $\frac{f_{22}n_2}{f_2} - \frac{f_{12}n_2}{f_1} + 1 > 0$ with concave non-homothetic production function $f(n_1, n_2(w_2))$ or elasticity of substitution is more than 1 with CES production function, where $f_j \equiv \frac{\partial f(n_1,n_2)}{\partial n_j}, \quad (j = 1, 2)$ and $f_{jk} = \frac{\partial f_j(n_1,n_2)}{\partial n_k}, \quad (k = 1, 2)$. I do not consider this case because this paper focuses on the other effects of minimum wage.

$$\dot{m}_2 = w_2 - z_2 - c_2 - \pi m_2, \quad (i = 2). \tag{18}$$

Variables such as c, m, n, w, and , z are denoted by suffix $i = 1$ if he/she has the high-wage job, and $i = 2$ for the low-wage job.[13] $z_i(\geq 0)$ is a lump-sum tax. I assume that a firm's real profit, $q \equiv (\bar{e}n_1)^a n_2^b - w_1 n_1 - w_2 n_2$, is equally distributed to the households of high-wage job.[14] π is a inflation rate. Then, the first-order conditions of each household are

$$\eta_{c_i} \frac{\dot{c}_i}{c_i} = \frac{v'(m_i)}{u'(c_i)} - \rho - \pi, \quad (i = 1, 2), \tag{19}$$

where $\eta_{c_i} \equiv \frac{-u''(c_i)c_i}{u'(c_i)} > 0$. The transversality conditions are

$$\lim_{t \to \infty} \lambda_i(t) m_i(t) e^{-\rho t} = 0, \quad (i = 1, 2), \tag{20}$$

where $\lambda_i(t)$ is a costate variable of m_i.

At any point in time, the money market is in equilibrium.

$$m^s = n_1 m_1 + n_2 m_2, \tag{21}$$

where m^s indicates the real money stock. The percentage change in the money stock depends on the government's money expansion rate, $\mu \equiv \frac{\dot{M}^s}{M^s}$, and the inflation rate, π.

$$\frac{\dot{m}^s}{m^s} = \mu - \pi. \tag{22}$$

Gorvernment spending g is financed by monetary expansion and households' taxation. Therefore, the government's budget constraint is

$$g = m^s \mu + n_1 z_1 + n_2 z_2, \tag{23}$$

where $m^s \mu = \frac{M^s}{P} \frac{\dot{M}^s}{M^s}$.

The aggregate demand consists of the consumption of households and the gorvernment spending.

$$y^d = c_1 n_1 + c_2 n_2 + g. \tag{24}$$

In the same manner as Ono(2001), the rate of change of prices depends on gap between

[13]The behavior of unemployed people is not considered in the model. That is, the model assumes implicitly that unemployed people are parasites on their friends or relations who have a job.

[14]This assumption is for the simplicity. Meanwhile, it is possible to build in the stock market, in which case the firm's profits are wholly distributed as dividends. This setting makes the model tangled because it needs to endogenize a rate of profit return or stock price.

aggregate supply and demand.

$$\pi = \alpha \left[\frac{y^d}{y^{s*}(w_2)} - 1 \right], \quad \alpha > 0, \tag{25}$$

where π denotes the inflation rate and α stands for the adjustment speed of the price. Excess demand (supply) pushes up (down) the inflation rate.

2.4 Equilibria

The system of consolidated equations is shown as follows.

$$\dot{c}_1 = \frac{c_1}{\eta_{c_1}} \left[\frac{v'(m_1)}{u'(c_1)} - \rho - \alpha \left(\frac{y^d}{y^{s*}(w_2)} - 1 \right) \right], \tag{26}$$

$$\dot{c}_2 = \frac{c_2}{\eta_{c_2}} \left[\frac{v'(m_2)}{u'(c_2)} - \rho - \alpha \left(\frac{y^d}{y^{s*}(w_2)} - 1 \right) \right], \tag{27}$$

$$\dot{m}_1 = \frac{(\bar{e}n_1)^a (n_2)^b - w_2 n_2}{n_1} - z_1 - c_1 - \alpha \left(\frac{y^d}{y^{s*}(w_2)} - 1 \right) m_1, \tag{28}$$

$$\dot{m}_2 = w_2 - z_2 - c_2 - \alpha \left(\frac{y^d}{y^{s*}(w_2)} - 1 \right) m_2, \tag{29}$$

$$y^d = c_1 n_1 + c_2 n_2 + g, \tag{30}$$

where (30) is derived from equation (23) in which I assume $\mu = 0$. Equations (26)-(29) form an autonomous dynamic system with respect to c_1, c_2, m_1 and m_2 under the equation (30), exogeneous minimum wage w_2, the output level $y^{s*}(w_2)$ in (13), and the predetermined variables n_1 and n_2 which are determined in (11) and (12) under non-demand shortage meanwhile in (14) and (15) under demand shortage.

To show the effect of minimum wage on economy clearly, I focus two steady state equilibria such that

$$\dot{c}_1 = 0, \quad \dot{c}_2 = 0, \quad \dot{m}_1 = 0, \quad \dot{m}_2 = 0, \quad \pi = 0, \tag{31}$$

which is called supply-side regime equilibrium and,

$$\dot{c}_1 = 0, \quad \dot{c}_2 = 0, \quad \dot{m}_1 = -\frac{m^s \pi}{n_1}, \quad \dot{m}_2 = 0, \quad \pi < 0, \tag{32}$$

which is called demand-shortage regime equilibrium. The steady state in (32) entails persisting deflation $\pi < 0$ by supposing an addition assumption as explained below.

In the next section the supply-side regime equilibrium is analyzed, and after that, the demand-shortage regime equilibrium is analyzed.

3 Minimum wage effects in supply-side regime equilibrium

The system (26)-(30) reaches to a steady state equilibrium as represented in (31) when the stability conditions (Assumption 1 in Appendix) are satisfied. This is shown in Appendix. In this steady state, consumption and real money balances of each household become constant, the gap between aggregate supply and demand is plugged, that is, $\pi = 0$, and the employment rate of each job is determined by (11) and (12). The steady state equilibrium values are obtained (see Appendix A.2) as follows:

$$c_i^{s*} = c_i^s(w_2), \quad \frac{dc_1^{s*}}{dw_2} < 0, \quad \frac{dc_2^{s*}}{dw_2} = sign(1 + m_1^{s*}\pi_{c_1}^s + m_2^{s*}\pi_{c_1}^s \frac{n_2^{s*}}{n_1^{s*}} - m_2^{s*}\pi_{w_2}^s), \tag{33}$$

$$m_i^{s*} = m_i^s(w_2), \quad \frac{dm_1^{s*}}{dw_2} < 0, \quad \frac{dm_2^{s*}}{dw_2} = sign(1 + m_1^{s*}\pi_{c_1}^s + m_2^{s*}\pi_{c_1}^s \frac{n_2^{s*}}{n_1^{s*}} - m_2^{s*}\pi_{w_2}^s), \tag{34}$$

where the superscript ^{s*} indicates the equilibrium value. The increase in the minimum wage entails an overall wage rise, and which affects consumption and money holdings through each household's budget and the inflation rate. In the steady state equilibrium, the increase in the minimum wage reduces the consumption and money balances of high-wage job households c_1^{s*} and m_1^{s*}, because it is mainly affected by the decreases in real income which consists of their wages and distributed income from profit.[15]

But it may raise the c_2^{s*}, m_2^{s*} when the effect of $\pi_{w_2}^s$ in (33) and (34) is small enough because it is affected by the increase in real income which is just caused by the minimum wage hike. On the whole, the minimum wage hike lowers the aggregate consumption level, that is, $\frac{d(c_1^{s*}n_1^{s*} + c_2^{s*}n_2^{s*})}{dw_2} < 0$, which entails a decrease in aggregate output level, because the negative effects of c_1^{s*} and n_2^{s*} dominates the effects of c_2^{s*}. This result is summed up as follows.

Proposition 1 *In supply-side regime equilibrium, a minimum wage hike reduces the total consumption and aggregate output level.*

Further, unemployment rate in the supply-side regime equilibrium can be expressed as $u^{s*} = 1 - n_1^{s*} - n_2^{s*}(w_2)$ where $\frac{dn_2^{s*}(w_2)}{dw_2} < 0$ and n_1^{s*} is determined by deep parameters. Therefore the minimum wage hike leads to the increase in unemployment rate. This is shown as in Proposition 2:[16]

Proposition 2 *In supply side regime equilibrium, a minimum wage hike raises an unemployment rate.*

[15] The real income of household 1 is $w_1 + \frac{q_1}{n_1} = \frac{y^{*s}(w_2^{s*}) - w_2 n_2^{s*}}{n_1^{s*}}$.

[16] $\frac{\partial u^{s*}}{\partial w_2} > 0$ when $\frac{f_{22}n_2}{f_2} - \frac{f_{12}n_2}{f_1} + 1 + \frac{f_{11}n1}{f_1} - \frac{f_{21}n_1}{f_2} - \frac{n_1}{1-\theta-n_1} > 0$ with a concave non-homothetic production function or elasticity of substitution σ with CES production function satisfies $\frac{2}{\sigma} > 1 - \frac{n_1}{1-\theta-n_1}$.

Moreover, social welfare V^s can be expressed as

$$V^s = \int_0^\infty n_1^{s*}\left(u(c_1^{s*}) + v(m_1^{s*})\right)e^{-\rho t}dt + \int_0^\infty n_2^{s*}\left(u(c_2^{s*}) + v(m_2^{s*})\right)e^{-\rho t}dt$$
$$= \frac{1}{\rho}\left[n_1^{s*}\left(u(c_1^{s*}) + v(m_1^{s*})\right) + n_2^{s*}\left(u(c_2^{s*}) + v(m_2^{s*})\right)\right]. \qquad (35)$$

Differentiating (35) with minimum wage w_2 through $c_i^{s*}, m_i^{s*}, (i = 1, 2), n_2^{s*}$, Proposition 3 is obtained (see Appendix).

Proposition 3 *In supply-side regime equilibrium, a minimum wage hike reduces the social welfare if*

$$\varepsilon > \frac{-1}{\frac{dc_1^{s*}}{dw_2}n_1^{s*}}\left[\left(\frac{dc_1^{s*}}{dw_2}n_1^{s*} + \frac{dc_2^{s*}}{dw_2}n_2^{s*}\right)\left(u'(c_2^{s*}) + \rho\frac{u''(c_2^{s*})}{v''(m_2^{s*})}\right) + \frac{dn_2^{s*}}{dw_2}\left(u(c_2^{s*}) + v(m_2^{s*})\right)\right].$$

Where ε indicates a subtraction $u'(c_2^{s*}) + \rho\frac{u''(c_2^{s*})}{v''(m_2^{s*})}$ from $u'(c_1^{s*}) + \rho\frac{u''(c_1^{s*})}{v''(m_1^{s*})}$. If ε is higher than a certain negative value as in Proposition 3, the ambiguous effects of minimum wage hike on consumption c_2^{s*} in (33) and money balances m_2^{s*} in (34) become lower than the total decreasing effects on c_1^{s*} and m_1^{s*}, and then the minimum wage hike reduces the social welfare. For example when $\varepsilon = 0$, the social welfare deteriorates by the minimum wage hike.

4 Minimum wage effects in demand-shortage regime equilibrium

Demand shortage regime equilibrium with deflation represented in (32) can be generated by supposing an additional assumption of money utility. In the supply-side regime economy, I assume implicitly that the marginal utility of money converges to zero as the households increase their real money balances, that is, $\lim_{m_i\to\infty} v'(m_i) = 0, \quad (i = 1, 2)$. Here, the marginal utility of money of high-wage households is assumed to be insatiable which is assumed by Ono (2001). This assumption emphasize a role of money holding which generate not only transaction motive but also social power, status and prestages.[17]

Assumption of marginal utility of money

$$\lim_{m_1\to\infty} v'(m_1) = \beta > 0, \qquad (36)$$

[17]Ono, Ogawa and Yoshida (2004) show empirically that the marginal rate of substitution of consumption for money has a strictly positive lower bound. This result implies that the marginal utility of money will will be insatiable.

The lower bound β of marginal utility is coming up as the high-wage job households increase money holdings. This assumption transforms the Euler equation (26) into

$$\eta_{c_1}\frac{\dot{c_1}}{c_1} = \begin{cases} \frac{v'(m_1)}{u'(c_1)} - \rho - \pi, & m_1 \text{ is not big enough,} \\ \frac{\beta}{u'(c_1)} - \rho - \pi, & m_1 \text{ is big enough.} \end{cases} \tag{37}$$

When money holding increases marginally, the present consumption increases in response to the decreasing marginal utility of money in the steady state equilibrium of the first equation of (37), but the present consumption does not respond in the second equation of (37). This invokes a demand shortage and deflation, even if the price adjusts a disequilibrium of demand and supply in the goods market as in (25).

From (15), (27)-(30), and (37), the steady state equilibrium in (32) can be expressed as consolidated two equations.

$$y^d = c_1(y^d, w_2)n_1 + c_2(y^d, w_2)n_2 + g, \tag{38}$$

$$n_2 = \left(y^d\right)^{\frac{1}{b}} \left(\frac{\bar{e}a(1-\theta)}{a+b}\right)^{-\frac{a}{b}}, \tag{39}$$

where Appendix shows the saddle stability conditions of this equilibrium and explains the derivation of (38). (38) indicates the aggregate demand in response to employment n_2 which is depicted as a upward-sloping line AD in Fig.1. (39) represents the relationship between aggregate output level and employment n_2, i.e., the employment determination equation through the production function when firms face the demand shortage. This diagram is similar to the so-called Keynesian cross implying aggregate demand determines aggregate output. E_1 is demand shortage regime equilibrium given a constant minimum wage w_2.

As minimum wage increases, the AD line moves to upper direction and the steady state equilibrium shifts from E_1 to E_1'. This increases the equilibrium aggregate demand y^d and employment n_2 and thus the unemployment rate falls down. At this moment the deflation becomes milder by reaction of the shrinking gap between aggregate demand y^d and potential aggregate output $y^{*s}(w_2)$ which is caused by the increase in y^d and the decrease in $y^{*s}(w_2)$. In this process, the rising inflation rates raises the consumption c_1 in (37). At the same time, the minimum wage hike also raises a real income of household 2 and their consumption. In respose to the increase in the total consumption and aggregate demand, the firm increases the employment.[18] These results are summed

[18]In the initial steady state before the government raises the minimum wage. The government set the nominal minimum wage in proportion to the inflation rate as same as a indexation wage. To implement a minimum wage hike policy, the government raises the nominal minimum wage level transiently. At this moment, the economy goes to the new steady state equilibrium as the government sets the minimum

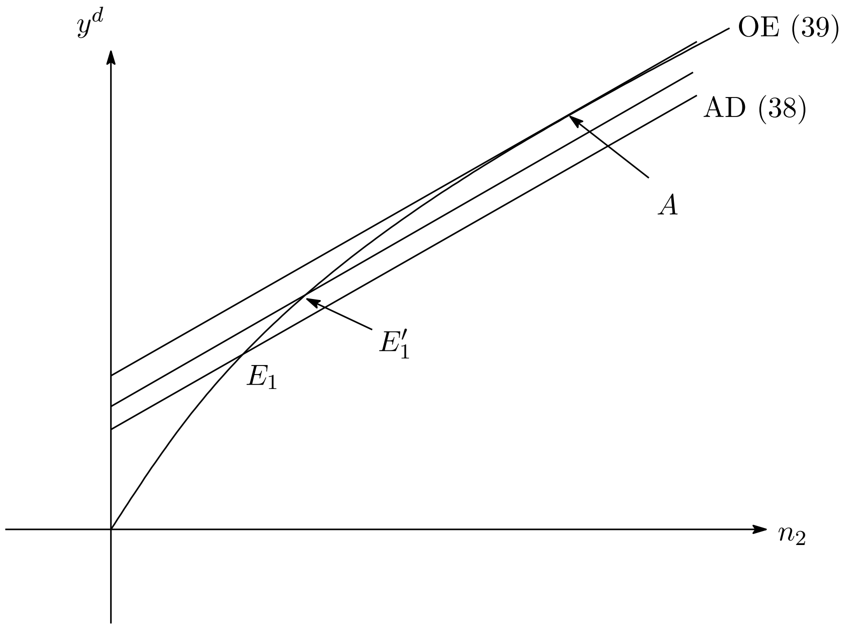

Fig 1: Determination of employment and aggregate demand

up in Proposition 4 and 5.

Proposition 4 *In demand-shortage regime equilibrium, a minimum wage hike raises aggregate demand, total consumption and employment, and also alleviates deflation.*

Proposition 5 *In demand-shortage regime equilibrium, a minimum wage hike decreases unemployment rates.*

Incidentally in point A, the aggregate demand AD becomes tangential to the production function OE in which the point that the aggregate demand y^d equals aggregate output $y^s(w_2)$. To prove this fact, in equilibrium A, it require to show that the slope of aggregate demand AD is just w_2^A refered to the minimum wage in equilibrium A. The budget constraint in (29) becomes $c_2 = w_2^A - z_2$ under inflation rate $\pi = 0$ and then, the AD equation (38) is $y^d = (c_1 + z_1)n_1 + w_2^A n_2$. The slope of this line becomes just w_2. $\frac{dy^d}{dn_2} = w_2^A$.[19]

On the other hand, in the steady state equilibrium of supply side regime, the employment rate $n_2^{s*}(w_2^A)$ is determined by (12) and the marginal productivity of n_2 equals to w_2^A. This implies that in point A the production function has the tangent line that slopes equals to w_2^A. Thus the employment level n_2 under demand shortage in A is just

wage in proportion to the inflation rate that is higher than initial steady state. Note that in the new steady state equilibrium, the inflation rate becomes higher.

[19]Substituting $c_2 = w_2 - z_2$ into aggregate demand yields $y^d = (c_1 + z_1)n_1 + w_2 n_2$, where c_1 in point A does not depend on inflation rate $pi = 0$.

equals to $n_2^{s*}(w_2^A)$, and then the aggregate demand y^d equals to the aggregate output $y^{*s}(w_2^A)$.

Proposition 6 *When minimum wage have increased and the equilibrium has reached to point A, the supply and demand gap has disappeared and then inflation rate π has become zero. In this point the demand shortage regimes alters to supply side regime.*

In analogy with (35), social welfare in the demand-shortage regime equilibrium V^d can be expressed as

$$V^d = \frac{1}{\rho}\left[n_1^{d*}u(c_1^{d*}) + n_2^{d*}u(c_2^{d*}) + n_2^{d*}v(m_2^{d*})\right] + n_1^{d*}\left[\frac{v(m_1(0))}{\rho} - \frac{\beta(m_1(0)n_1^{d*} + m_2^d n_2^{d*})\pi}{\rho(\rho+\pi)n_1^{d*}}\right], \quad (40)$$

where the superscript $d*$ means the equilibrium value in equilibrium (32), $m_1(0)$ indicates the initial ($t = 0$) money holding of high-wage job households and its amount m_1 is increasing in this equilibrium as follows

$$\dot{m}_1 = -\frac{m^s \pi^{d*}}{n_1^{d*}} > 0. \quad (41)$$

The transverserity condition is satisfied even under (41) as shown in Appendix. The last term of (40) is derived from $\int_0^\infty n_1 v(m_1)e^{-\rho t}dt$, which is also presented in Appendix.

Because $c_1^{d*}, c_2^{d*}, n_2^{d*}$, and m_2^{d*} are increasing functions of w_2, a minimum wage hike raise the first parenthesis and the first term of the second parenthesis in (40). However, the effect of the the minimum wage on the last term becomes ambiguous. Differentiating the last term in (40) with w_2 gives

$$\frac{d\left(\frac{-\beta(m_1(0)n_1^{d*}+m_2^{d*}n_2^{d*})\pi^{d*}}{\rho(\rho+\pi^{d*})}\right)}{dw_2} = \frac{d(m_1(0)n_1^{d*} + m_2^{d*}n_2^{d*})}{dw_2}\left(\frac{-\beta\pi^{d*}}{\rho(\rho+\pi^{d*})}\right) + \frac{d\pi^{d*}}{dw_2}\frac{\rho}{(\rho+\pi^{d*})^2}\left[\frac{-\beta(m_1(0)n_1^{d*} + m_2^{d*}n_2^{d*})}{\rho}\right], \quad (42)$$

where $\pi^{d*} < 0$. The first term of (42) becomes positive, However, the second term becomes negative. Assumption 3 (in Appendix) gives rise to the smaller second term effect than the other minimum wage effects. It gives the following Proposition.

Proposition 7 *In the demand-shortage regime economy, a minimum wage hike improves the social welfare.*

5 Discussion

5.1 Optimal minimum wage

What is the optimal minimum wage policy in this context? To answer this question, concepts of natural rate of unemployment rate and actual unemployment rate should be made clear. The natural rate of unemployment has been discussed thoroughly among macroeconomists. Friedman(1968 p8) explains that it "is the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies,$\cdots$". In the supply-side regime equilibrium in (31), the unemployment rate is just determined by the dynamic Walrasian system including market imperfections such as efficiency wage and minimum wage.[20] Thereby the unemployment rate $u^{s*}(= 1 - n_1^{s*} - n_2^{s*})$ can be regarded as the natural unemployment rate.[21] On the other hand, the unemployment rate in the demand-shortage regime equilibrium is not determined by the Walrasian system in the sense that the price adjustment will not be functioned perfectly to plug the demand-supply gap.

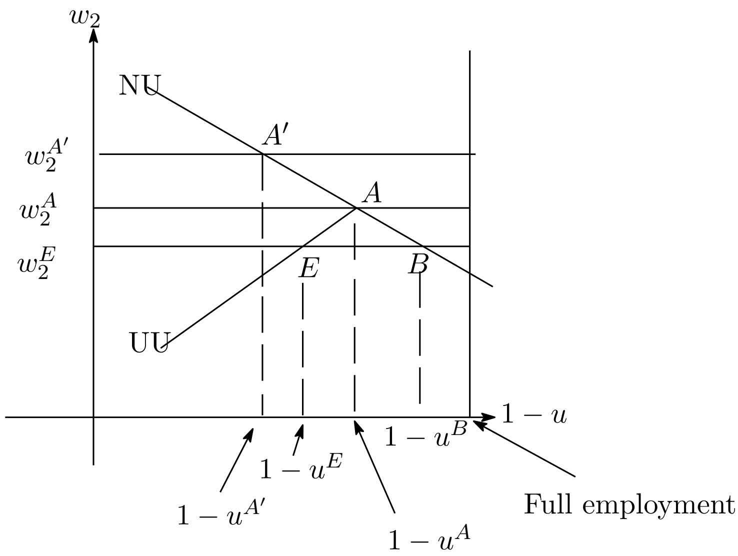

Fig 2: Unemployment rate and minimum wage

As shown above, a minimum wage hike has good or bad effects on unemployment rate and social welfare as shown in Proposition 2, 3, 5 and 7. Fig.2 indicates the relationship

[20]Blanchard and Kats (1997) explain that the natural rate of unemployment is determined by the intersection of demand wage relation (which is assumed to firm's labor demand curve) and supply wage relation (which is assumed to wage setting curve by efficiency wage or bargaining between firm and labor union).

[21]As the supply-side regime equilibrium in (31) stays at zero deflation rate, the unemployment rate of this equilibrium can be regarded as what we call NAIRU (non-accelerating inflation rate of unemployment). Ball and Mankiw (2001) explain this concept as "the unemployment rate at which inflation will be stable, absent the high-frequency shocks $\cdots$".

between minimum wage level and unemployment rate. NU curves represents the natural unemployment rates given by the minimum wage. UU curves represent the unemployment rate u ($= 1 - n_1 - n_2$) determined by the demand-shortage regime equilibrium. When minimum wage level is w_2^E, the general equilibrium of demand shortage regime is at E (which is the same point E in Fig.1) and its unemployment rate becomes u^E. At the same time, the natural unemployment rate is u^B, although u^B is not realized because the economy faces the demand shortage.[22] At this equilibrium E, policy director should increase the minimum wage to w_2^A. It decreases the unemployment rate from u^E to u^A and the economy moves to the equilibrium A (which is the same point A in Fig.1). This policy can also improve social welfare as shown in proposition 7.

The policy director should not increase the minimum wage so high beyond w_2^A. Equilibrium A is not the demand shortage regime any more but the supply side-regime. If the policy director misjudges the economic situation and increases the minimum wage from w_2^A to $w_2^{A'}$, the unemployment rate will go up to $u^{A'}$ with social welfare deteriorating.

What happens when the policy director decreases the minimum wage from w_2^A to w_2^E in the equilibrium A? If no economic shocks happen at this moment, the economy will get trap to demand-shortage regime and the unemployment rate will fall down to u^E rather than to u^B. Therefore the policy director should set the optimal minimum wage so that the demand shortage disappears.

5.2 Deflation and minimum wage

After the global financial crisis in 2008, the monetary authority of various countries has kept interests zero lower bound and many governments have increased the spending. Despite a broad range of measures to overcome the depression, the deflationary concerns does not relieved especially in EU and Japan.

The minimum wage hike is the better policy when the government spending is restricted by its debt limit. The minimum wage hike does not entail any costs, and it is no danger of budget constraint. In our model, as the economy moves from the equilibrium E to A with the increasing minimum wage, the consumption and employment also increases and deflation makes better indeed. That's why a minimum wage hike can be helpful policy in getting rid of deflation and stagnation.

[22] Blanchard and Katz (1997) argue that the increase in the reservation wage shifts up the "supply wage relation" and raises the natural rate of unemployment as same as the result in Proposition 2. Their study also points out that when the increase in the reservation wage is proportion to the productivity growth, the natural rate of unemployment remains constant.

6 Conclusion

This paper analyzes the role of a minimum wage in macroeconomics with dynamic general equilibrium model giving rise to two different equilibria. The study demonstrates that a minimum wage hike has positive effects on an employment rate, aggregate consumption, and social welfare under a demand shortage economy whereas does not under a non-demand shortage economy. In other words there is an optimal minimum wage level that dissolves a demand shortage in macro economy. In this regard, Manning argues that there is some level of the minimum wage at which employment will decline significantly and the literature should re-orient itself towards trying to find that point.

Our theoretical finding implies that the policy director can improve the aggregate economic activity by the increase in the minimum wage without any government spending. However, the increase in the minimum wage may entails the unfavorable side effects that the natural rate of unemployment rate goes up. As for countries that faces the demand shortage or diminishing price pressure, the minimum wage policy can be considered as one of the effective option to stimulate economic activity.

To consider the more realistic minimum wage policy, the extension of incorporating the productivity growth rate may be desirable. Productivity growth affects the natural unemployment rate, the potential output level, the inflation rate and the wage growth rate. Under this situation, the growth rate of minimum wage as a policy tool should be examined.

Acknowledgements

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A Appendix

A.1 Appendix 1 Saddle stability under the supply-side regime in (31)

In order to derive the saddle stability conditions, I first show relations between variables, the inflation rate π, consumption c_i and minimum wage w_2.

The inflation rate under non-demand shortage becomes

$$\pi^s = \alpha \left[\frac{c_1 n_1^{s*} + c_2 n_2^{s*}(w_2) + g}{y^{s*}(w_2)} - 1 \right], \tag{43}$$

$$\pi_{c_i}^s \equiv \frac{\partial \pi^s}{\partial c_i} = \frac{\alpha n_i^{s*}}{y^{s*}} > 0, \quad \pi_{w_2}^s \equiv \frac{\partial \pi^s}{\partial w_2} = Sign \left[(c_2 - w_2) \frac{n_2^{s*\prime}(w_2) w_2}{y^{s*}} \right] > 0,$$

where I have used (11), (12), (24), and (25). Equation (29) in the steady state gives $c_2 - w_2 < 0$. On the other hand (14), (15), (24), and (25) give the inflation rate under the demand shortage:

$$\pi^d(c_1, c_2, w_2) = \alpha \left[\frac{c_1 n_1^d + c_2 n_2^d(c_1, c_2) + g}{y^s(w_2)} - 1 \right], \quad \pi^d_{c_i} \equiv \frac{\partial \pi^d}{\partial c_i} > 0, \quad \pi^d_{w_2} \equiv \frac{\partial \pi^d}{\partial w_2} > 0. \tag{44}$$

π^d is expressed as a positive function of c_1, c_2, and w_2 because the employment is determined from (14), (15) and (24) as follows.

$$n_1^d = \frac{a(1-\theta)}{a+b} \equiv n_1^{d*}, \tag{45}$$

$$n_2^d = n_2^d(c_1, c_2), \quad n_{2c_1} = \frac{\partial n_2}{\partial c_i} = \frac{c_i n_i}{\frac{\partial((\bar{e}n_1)^a n_2^b)}{\partial n_2} - c_2} > 0, \tag{46}$$

where $\frac{\partial((\bar{e}n_1)^a n_2^b)}{\partial n_2} - c_2 > w_2 - c_2 = z_2 + \dot{m}_2 + \pi m_2 > 0$ because marginal product of n_2 is higher than w_2 because of the demand-shortage. Note that the employment rate of high-wage job n_1^{d*} is constant as is equals n_1^{s*}; whereas the employment rate of low-wage job n_2^d depends not on wage but on consumption of each household, and is significantly less than n_2^s because firms faces the aggregate demand constraint.

(43) and (44) leads to the general form of inflation rate.

$$\pi = \pi(c_1, c_2, w_2), \qquad \pi_{c_i} > 0, \quad \pi_{w_2} > 0. \tag{47}$$

A.1.1 Supply side regime equilibrium in (31) of the stability conditions

Using (47), I next show the local saddle stability conditions of the dynamics described by (26)-(29). Linearizing these equations in the neighborhood of the steady state values $c_i^*, m_i^* \quad (i = 1, 2)$ (which is expressed as superscript ^{s*} in the text) in (31) gives

$$\begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{m}_1 \\ \dot{m}_2 \end{bmatrix} = \begin{bmatrix} \rho + \pi - \frac{c_1^* \pi_{c_1}}{\eta_{c_1}} & -\frac{c_1^* \pi_{c_2}}{\eta_{c_1}} & -\frac{v''(m_1)}{u''(c_1)} & 0 \\ -\frac{c_2^* \pi_{c_1}}{\eta_{c_2}} & \rho + \pi - \frac{c_2^* \pi_{c_2}}{\eta_{c_2}} & 0 & -\frac{v''(m_2)}{u''(c_2)} \\ -m_1^* \pi_{c_1} - 1 & -m_1^* \pi_{c_2} & -\pi & 0 \\ -m_2^* \pi_{c_1} & -m_2^* \pi_{c_2} - 1 & 0 & -\pi \end{bmatrix} \begin{bmatrix} c_1 - c_1^* \\ c_2 - c_2^* \\ m_1 - m_1^* \\ m_2 - m_2^* \end{bmatrix} \tag{48}$$

Noting that, $\frac{\partial\left(\frac{v'(m_1^*)}{u'(c_1^*)\eta_{c_1}}\right)}{\partial c_1^*} = \frac{-u''(c_1^*)v'(m_1^*)}{(u'(c_1^*))^2 \eta_{c_1}} = \frac{v'(m_1^*)}{u'(c_1^*)c_1^*} = \frac{\rho + \pi^*}{c_1^*}$ because I assume $\eta_{c_i} = \frac{-u''(c_i^*)c_i^*}{u'(c_i^*)}$ does not depend on c_i^*. While the real consumption c_1 and c_2 are jumpable variables at any point in time, the real money balances m_1 and m_2 are state valiables. For a stable saddle point in equilibrium, it must have two positive roots (or a pair of complex roots with a positive real part) and two negative real roots (or a pair of complex roots with a negative real part).

Denoting the Jacobian matrix in (53) as A^s, the charactoristic equation can be expressed as

$$\lambda_s^4 - Trace\left(A^s\right)\lambda_s^3 + B^s\lambda_s^2 - C^s\lambda_s + det\left(A^s\right) = 0,$$

where $\lambda_{sk}, \quad (k = 1, 2, 3, 4)$ is the roots of this equation and

$$B^s = \rho^2 - \rho\left(\frac{c_1^*\pi_{c_1}}{\eta_{c_1}} + \frac{c_2^*\pi_{c_2}}{\eta_{c_2}}\right) - \frac{v''(m_1^*)}{u''(c_1^*)}\left(1 + m_1^*\pi_{c_1}\right) - \frac{v''(m_2^*)}{u''(c_2^*)}\left(1 + m_2^*\pi_{c_2}\right),$$

$$det\left(A^s\right) = \left(\frac{v''(m_1^*)}{u''(c_1^*)}\right)^2\left(\frac{v''(m_2^*)}{u''(c_2^*)}\right)^2\left(1 + m_1^*\pi_{c_1} + m_2^*\pi_{c_2}\right) > 0. \tag{49}$$

Note $\pi = 0$ in the supply side regime equilibrium. If the following conditions are satisfied at least, the system has a locally stable saddle point.[23]

$$\lambda_{s1}\lambda_{s2}\lambda_{s3}\lambda_{s4} > 0, \tag{50}$$

$$\lambda_{s1}\lambda_{s2} + \lambda_{s1}\lambda_{s3} + \lambda_{s1}\lambda_{s4} + \lambda_{s2}\lambda_{s3} + \lambda_{s2}\lambda_{s4} + \lambda_{s3}\lambda_{s4} < 0. \tag{51}$$

Considering the relation rule between roots and coefficients of the eigenvalue equation, the LHS of (50) and (51) equal $det\left(A^s\right)$ and B^s, respectively. (50) is satisfied already without additional conditions as in (49). The condition $B^s < 0$ is satisfied under the following assumption.

Assumption 1

$$\rho - \left(\frac{c_1^*\pi_{c_1}}{\eta_{c_1}} + \frac{c_2^*\pi_{c_2}}{\eta_{c_2}}\right) < 0 \tag{52}$$

A.1.2 Demand shortage regime equilibrium in (32) of the stability conditions

The system of demand shortage regime equilibrium is described by (27)-(29) and (37). I show here that this dynamic system has a saddle path to the steady state in (32). Linearizing these equations in the neighborhood of the steady state values $c_i^{d*}, m_i^{d*} \quad (i = 1, 2)$ in (31) give

$$\begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{m}_1 \\ \dot{m}_2 \end{bmatrix} = \begin{bmatrix} \rho + \pi - \frac{c_1^{d*}\pi_{c_1}}{\eta_{c_1}} & -\frac{c_1^{d*}\pi_{c_2}}{\eta_{c_1}} & 0 & 0 \\ -\frac{c_2^{d*}\pi_{c_1}}{\eta_{c_2}} & \rho + \pi - \frac{c_2^{d*}\pi_{c_2}}{\eta_{c_2}} & 0 & -\frac{v''(m_2)}{u''(c_2)} \\ -m_1^{d*}\pi_{c_1} - 1 & -m_1^{d*}\pi_{c_2} & -\pi & 0 \\ -m_2^{d*}\pi_{c_1} & -m_2^{d*}\pi_{c_2} - 1 & 0 & -\pi \end{bmatrix} \begin{bmatrix} c_1 - c_1^{d*} \\ c_2 - c_2^{d*} \\ m_1 - m_1^{d*} \\ m_2 - m_2^{d*} \end{bmatrix} \tag{53}$$

[23] Buiter (1984) discusses saddle-path stability in dynamic general equilibrium model with continuous time model.

Denoting the Jacobian matrix in (53) as A^d, the charactoristic equation can be expressed as

$$\lambda_d^4 - Trace\left(A^d\right)\lambda_d^3 + B^d\lambda_d^2 - C^d\lambda_d + det\left(A^d\right) = 0,$$

where $\lambda_{dk}, \quad (k = 1, 2, 3, 4)$ is the roots of this equation and

$$B^d = (\pi - \rho)\left(\frac{c_1^{d*}\pi_{c_1}}{\eta_{c_1}} + \frac{c_2^{d*}\pi_{c_2}}{\eta_{c_2}} - \rho\right) - \pi(\rho+\pi) - \frac{v''(m_2^{d*})}{u''(c_2^{d*})}\left(1 + m_2^{d*}\pi_{c_2}\right), \quad (54)$$

$$det\left(A^d\right) = -\pi\frac{c_1^{d*}\pi_{c_1}}{\eta_{c_1}}\left(\frac{v''(m_2^{d*})}{u''(c_2^{d*})}\right) + \pi^2(\rho+\pi)\left[\left(\frac{v''(m_2^{d*})}{u''(c_2^{d*})}\right) - (\rho+\pi) + \left(\frac{c_1^{d*}\pi_{c_1}}{\eta_{c_1}} + \frac{c_2^{d*}\pi_{c_2}}{\eta_{c_2}}\right)\right]. \quad (55)$$

The saddle stability conditions require $B^d < 0$ and $\det(A^d) > 0$. To assure this conditions, I assume Assumption 2 in addition to Assumption 1.

Assumption 2

$$\pi(\rho+\pi) + \frac{v''(m_2^{d*})}{u''(c_2^{d*})} > 0. \quad (56)$$

Noting that $\pi < 0$ and $\rho + \pi > 0$ in the neighborhood of the equilibrium, Assumption 1 makes the big bracket in (55) positive and then $det(A) > 0$ is obtained under Assumption 1 substituted c_i^{d*} for c_i^*. Further Assumption 2 makes the second and third term of (54) negative. At the same time Assumption 1 gives the first term of (54) negative. Therefore $B^d < 0$ is obtained under Assumption 1 and 2.

A.2 Appendix 2 Derivations of (33) and (34)

Using the Cramer's rule in the steady state equilibrium, (33) is

$$\frac{dc_1^*}{dw_2} = \frac{1}{det\left(A^s\right)}\frac{v''(m_1^*)}{u''(c_1^*)}\frac{v''(m_2^*)}{u''(c_2^*)}\left[-m_1^*\pi_{c_2} - (1 + m_2^*\pi_{c_2})\frac{n_2^s}{n_1^s} - m_1^*\pi_{w_2}\right] < 0,$$

$$\frac{dc_2^*}{dw_2} = \frac{1}{det\left(A^s\right)}\frac{v''(m_1^*)}{u''(c_1^*)}\frac{v''(m_2^*)}{u''(c_2^*)}\left[1 + m_1^*\pi_{c_1} + m_2^*\pi_{c_1}\frac{n_2^s}{n_1^s} - m_2^*\pi_{w_2}\right],$$

where $det\left(A^s\right) > 0$ obtained by the stability condition. Further, the effect of a minimum wage hike on aggregate consumption is

$$\frac{d(n_1^s c_1^* + n_2^s c_2^*)}{dw_2} = \frac{dc_1^*}{dw_2}n_1^s + \frac{dn_1^s}{dw_2}c_1^* + \frac{dc_2^*}{dw_2}n_2^s + \frac{dn_2^s}{dw_2}c_2^*$$

$$= \frac{1}{det\left(A^s\right)}\frac{v''(m_1^*)}{u''(c_1^*)}\frac{v''(m_2^*)}{u''(c_2^*)}\left[-m_1^* n_1^s\pi_{w_2} - m_2^* n_2^s\pi_{w_2} + \frac{dn_2^s}{dw_2}c_2^*\right] < 0,$$

where $\frac{dn_1^s}{dw_2} = 0$ and $\frac{dn_2^s}{dw_2} < 0$ as shown in (11) and (12). I have used $\pi_{ci} = \frac{-\alpha n_i}{y^s}$.

Differentiating the steady state equilibrium condition $\frac{v'(m_i^*)}{u'(c_i^*)} = \rho$ with w_2 yields $\frac{u''(c_i^*)c_i^*}{u'(c_i^*)}\frac{dc_i^*}{dw_2}\frac{w_2}{c_i^*} = \frac{v''(m_i^*)m_i^*}{v'(m_i^*)}\frac{dm_i^*}{dw_2}\frac{w_2}{m_i^*}$, and then

$$\frac{dm_i^*}{dw_2} = \frac{\rho u''(c_i^*)}{v''(m_i^*)}\frac{dc_i^*}{dw_2}.$$

A.3 Appendix 3 Proof of Proposition 2

In the steady state equilibrium of (31), differentiating (35) with w_2 yields

$$\begin{aligned}\frac{dV^s}{dw_2} &= \frac{1}{\rho}\left[u'(c_1^*)\frac{dc_1^*}{dw_2}n_1^s + v'(m_1^*)\frac{dm_1^*}{dw_2}n_1^s + u'(c_2^s)\frac{dc_2^*}{dw_2}n_2^s + v'(m_2^*)\frac{dm_2^*}{dw_2}n_2^s + \frac{dn_2^s}{dw_2}\left(u(c_2^*)+v(m_2^*)\right)\right] \\ &= \frac{1}{\rho}\left[\frac{dc_1^*}{dw_2}n_1^s\left(u'(c_1^*)+\rho\frac{u''(c_1^*)}{v''(m_1^*)}\right) + \frac{dc_2^*}{dw_2}n_2^s\left(u'(c_2^*)+\rho\frac{u''(c_2^*)}{v''(m_2^*)}\right) + \frac{dn_2^s}{dw_2}\left(u(c_2^*)+v(m_2^*)\right)\right].\end{aligned}$$

Defining $u'(c_1^*) + \rho\frac{u''(c_1^*)}{v''(m_1^*)} = u'(c_2^*) + \rho\frac{u''(c_2^*)}{v''(m_2^*)} + \varepsilon$ yields

$$\frac{dV^s}{dw_2} = \frac{1}{\rho}\left[\left(\frac{dc_1^*}{dw_2}n_1^s + \frac{dc_2^*}{dw_2}n_2^s\right)\left(u'(c_2^*)+\rho\frac{u''(c_2^*)}{v''(m_2^*)}\right) + \varepsilon\frac{dc_1^*}{dw_2}n_1^s + \frac{dn_2^s}{dw_2}\left(u(c_2^*)+v(m_2^*)\right)\right].$$

Noting that $\frac{dc_1^*}{dw_2}n_1^s + \frac{dc_2^*}{dw_2}n_2^s = \frac{1}{det(A^s)}\frac{v''(m_1^*)}{u''(c_1^*)}\frac{v''(m_2^*)}{u''(c_2^*)}\left[-m_1^*n_1^s\pi_{w_2} - m_2^*n_2^s\pi_{w_2}\right] < 0$, the condition of $\frac{dV^s}{dw_2} < 0$ is given by

$$\varepsilon > \frac{-1}{\frac{dc_1^*}{dw_2}n_1^s}\left[\left(\frac{dc_1^*}{dw_2}n_1^s + \frac{dc_2^*}{dw_2}n_2^s\right)\left(u'(c_2^*)+\rho\frac{u''(c_2^*)}{v''(m_2^*)}\right) + \frac{dn_2^s}{dw_2}\left(u(c_2^*)+v(m_2^*)\right)\right].$$

A.4 Appendix 4. Deviations of (38), (39) and equilibrium value in demand-shortage regime

In the steady state equilibrium of (32), total differentials of (27) and (29) give

$$(\rho+\pi)u''(c_2)dc_2 - v''(m_2)dm_2 + u'(c_2)d\pi = 0, \qquad (57)$$

$$dc_2 + \pi dm_2 + m_2 d\pi - dw_2 = 0. \qquad (58)$$

Substituting (58) into (57) yields

$$\left[\pi(\rho+\pi)u''(c_2) + v''(m_2)\right]dc_2 = v''(m_2)dw_2 - \left[m_2 v''(m_2) + u'(c_2)\pi\right]d\pi, \qquad (59)$$

$$\left[\pi(\rho+\pi)u''(c_2) + v''(m_2)\right]dm_2 = (\rho+\pi)u''(c_2)dw_2 + \left[-(\rho+\pi)m_2u''(c_2) + u'(c_2)\right]d\pi. \qquad (60)$$

(59) and (60) represent that consumption c_2 and money holdings m_2 depend on minimum wage w_2 and inflation rate. A minimum wage hike increases income but an increase in inflation rate decreases the value of money holding and the income.

In addition, total differential of (37) gives

$$(\rho+\pi)u''(c_1)dc_1 = -u'(c_1)d\pi. \tag{61}$$

Further, inflation rate (25) can be differentiated as follows.

$$d\pi = \frac{\alpha}{y^s(w_2)}dy^d - \frac{\alpha y^d}{(y^s(w_2))^2}\frac{\partial y^s}{\partial w_2}dw_2. \tag{62}$$

Substituting (62) into (59) (60) and (61) yields

$$(\rho+\pi)u''(c_1)dc_1 = u'(c_1)\left[\frac{\alpha y^d}{(y^s(w_2))^2}\frac{\partial y^s}{\partial w_2}dw_2 - \frac{\alpha}{y^s(w_2)}dy^d\right], \tag{63}$$

$$\begin{aligned}[\pi(\rho+\pi)u''(c_2)+v''(m_2)]\,dc_2 = &\left[v''(m_2) + (m_2 v''(m_2) + u'(c_2)\pi)\frac{\alpha y^d}{(y^s(w_2))^2}\frac{\partial y^s}{\partial w_2}\right]dw_2 \\ &- [m_2 v''(m_2) + u'(c_2)\pi]\frac{\alpha}{y^s(w_2)}dy^d,\end{aligned} \tag{64}$$

$$\begin{aligned}[\pi(\rho+\pi)u''(c_2)+v''(m_2)]\,dm_2 = &\left[(\rho+\pi)u''(c_2) - (-(\rho+\pi)m_2 u''(c_2) + u'(c_2))\frac{\alpha y^d}{(y^s(w_2))^2}\frac{\partial y^s}{\partial w_2}\right]dw_2 \\ &+ [-(\rho+\pi)m_2 u''(c_2) + u'(c_2)]\frac{\alpha}{y^s(w_2)}dy^d.\end{aligned} \tag{65}$$

The assumption 2 assures that the square brackets in left-hand side of (64) and (65) become both negative.[24]

In right hand side of both (64) and (65), the cofficient in big bracket of terms dw_2 are influenced by two effects direct minimum wage effect (the first term) and the indirect effect through the inflation (the second term). I assume that the direct effect is larger than the indirect effect (Assumption 3). This assumption implies that wage or income is more important factor than inflation rate when households choose cosumption and money holding.

Assumption 3

$$v''(m_2) + (m_2 v''(m_2) + u'(c_2)\pi)\frac{\alpha y^d}{(y^s(w_2))^2}\frac{\partial y^s}{\partial w_2} < 0, \tag{66}$$

$$(\rho+\pi)u''(c_2) - (-(\rho+\pi)m_2 u''(c_2) + u'(c_2))\frac{\alpha y^d}{(y^s(w_2))^2}\frac{\partial y^s}{\partial w_2} < 0. \tag{67}$$

Equations (63)-(65) and Assumption 3 give the

$$c_1 = c_1(y^d, w_2), \qquad \frac{\partial c_1}{\partial y^d} > 0, \quad \frac{\partial c_1}{\partial w_2} > 0, \tag{68}$$

$$c_2 = c_2(y^d, w_2), \qquad \frac{\partial c_2}{\partial y^d} > 0, \quad \frac{\partial c_2}{\partial w_2} > 0, \tag{69}$$

$$m_2 = m_2(y^d, w_2), \qquad \frac{\partial m_2}{\partial y^d} > 0, \quad \frac{\partial m_2}{\partial w_2} > 0. \tag{70}$$

[24]Multiplying both terms of the assumption 2 by $u''(c_2)$ yields $\pi(\rho+\pi)u''(c_2) + v''(m_2) < 0$. Note $u''(c_2) < 0$.

(68)-(70) generate the AD line in (38) which differential coefficient of n_2, y^d and w_2 is

$$\left[1 - n_1 \frac{\partial c_1}{\partial y^d} - n_2 \frac{\partial c_2}{\partial y^d}\right] dy^d = c_2 dn_2 + \left[n_1 \frac{\partial c_1}{\partial w_2} + n_2 \frac{\partial c_2}{\partial w_2}\right] dw_2. \tag{71}$$

The marginal increment in y^d gives rise to the increase in c_1 and c_2 through the inflation effect in (25). This inflation effects must not be so large in demand shortage equilibrium. If this effects is too large, aggregate demand runs short to satisfy this increasing consumption, this genrates the further inflation. In this process the inflation accerates more and more and steady state equilibrium collapses. To rule out this scenario, it requires following assumption.

Assumption 4

$$1 - n_1 \frac{\partial c_1}{\partial y^d} - n_2 \frac{\partial c_2}{\partial y^d} > 0. \tag{72}$$

Equations (71) and (72) show that the aggregate demand y^d becomes a increasing function of w_2 as also shown in Fig.1. Thereby equations (68)-(70) give conclusions that minimum wage hikes has positive effects on c_1, c_2 and m_2 in the steady state general equilibrium under demand-shortage regime.

A.5 Appendix 5 Derivation of (40)

In the steady state, m_1 increases at defration rate $-\pi^{d*}$ and marginal utility of money becomes constant β. Taylor series expansion for $v(m_1(t))$, evaluated at the steady state equilibrium $m_1^{d*} = m_1(0)$, as follows.

$$\begin{aligned} v(m_1(t)) &= v(m_1(0)) + v'(m_1(0))[m_1(t) - m_1(0)] + v''(m_1(0))[m_1(t) - m_1(0)] \\ &= v(m_1(0)) + \beta[m_1(t) - m_1(0)]. \end{aligned} \tag{73}$$

Using $m_1(t) = -\frac{n_2^{d*}}{n_1^{d*}} m_2^{d*} + \left[m_1(0) + \frac{n_2^{d*}}{n_1^{d*}} m_2^{d*}\right] e^{-\pi^{d*}t}$and (73), the last term in (40) can be derived as follows.

$$\begin{aligned} \int_0^\infty v(m_1(t)) e^{-\rho t} dt &= \int_0^\infty \left[v(m_1(0)) - \frac{n_2^{d*}}{n_1^{d*}} \beta m_2^{d*} + \beta \left(m_1(0) + \frac{n_2^{d*}}{n_1^{d*}} \beta m_2^{d*}\right) e^{-\pi^{d*}t} - \beta m_1(0)\right] e^{-\rho t} dt \\ &= \frac{1}{\rho}\left[v(m_1(0)) - \frac{n_2^{d*}}{n_1^{d*}} \beta m_2^{d*} - \beta m_1(0)\right] + \frac{\beta}{\rho + \pi^{d*}}\left[m_1(0) + \frac{n_2^{d*}}{n_1^{d*}} m_2^{d*}\right] \\ &= \frac{v(m_1(0))}{\rho} - \frac{\beta}{n_1^{d*}\rho}\left[n_1^{d*} m_1(0) + n_2^{d*} m_2^{d*}\right] + \frac{\beta}{n_1^{d*}(\rho + \pi^{d*})}\left[n_1^{d*} m_1(0) + n_2^{d*} m_2^{d*}\right] \\ &= \frac{v(m_1(0))}{\rho} - \frac{\beta(n_1^{d*} m_1(0) + n_2^{d*} m_2^{d*})}{n_1^{d*}(\rho + \pi^{d*})}\left[\frac{\pi^{d*}}{\rho}\right]. \end{aligned}$$

A.6 Appendix 6 Transversality conditon in demand-shortage regime equilibrium

Under (41) in demand-shortage regime equilibrium, the transversality condition will be satisfied as shown below. Considering $\dot{m}_2 = 0$ in the steady state, time differential of the money market equilibrium (21) becomes

$$\dot{m}^s = n_1^{d*}\dot{m}_1. \tag{74}$$

The gorvernment's money expansion rate is assumed to be zero in (22), and thereby money stock increases at the deflation (negative inflation) rate, that is, $\frac{\dot{m}^s}{m^s} = -\pi^{d*}$. Hence, the change in the money holdings of high-wage households (74) can be written as (41), which can be rewritten as:

$$\dot{m}_1(t) = -\pi^{d*}m_1 - \frac{n_2^{d*}}{n_1^{d*}}\pi^{d*}m_2^{d*}, \tag{75}$$

Further, (75) can be solved using the steady state valuables as

$$m_1(t) = -\frac{n_2^{d*}}{n_1^{d*}}m_2^{d*} + \left[m_1(0) + \frac{n_2^{d*}}{n_1^{d*}}m_2^{d*}\right]e^{-\pi^{d*}t}, \tag{76}$$

where $m_1(0)$ indicates the initial ($t = 0$) money holding of high-wage job households.

Using (76), the LHS of the transversality condition (20) is

$$\begin{aligned} &\lim_{t\to\infty} \lambda_1 \left[-\frac{n_2^d}{n_1^d}m_2^{d*} + \left(m_1(0) + \frac{n_2^d}{n_1^d}m_2^{d*}\right)e^{-\pi t}\right]e^{-\rho t} \\ &= \lim_{t\to\infty} u'(c_1^{d*})\left[\frac{n_2^d}{n_1^d}m_2^{d*}e^{-\rho t} + \left(m_1(0) + \frac{n_2^d}{n_1^d}m_2^{d*}\right)e^{-(\rho+\pi)t}\right]. \end{aligned} \tag{77}$$

Since $\frac{n_2^{d*}}{n_1^{d*}}m_2^{d*}e^{-\rho t}$ and $\left(m_1(0) + \frac{n_2^{d*}}{n_1^{d*}}m_2^{d*}\right)e^{-(\rho+\pi^{d*})t}$ are monotonically decreasing as $t \to \infty$, transversality condition (20) converges to zero. Thus, the transversality condition can be satisfied even under (41).

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